Discrete Mathematics
BCSC0010

Module 1

Probability

Introduction

- Probability theory is a mathematical modeling of the phenomenon of chance or randomness.

- **Example**
- If a coin is tossed in a random manner, it can lead head or tail, but we do not know which of these will occur in a single toss.
- Any side of the coin is as likely to occur as the other; hence the chance of getting a head is 1 in 2 which means the probability of getting heads is ½.

Basic Terminologies

- **Experiment:**
- An experiment is any activity from which results are obtained.
- **Random Experiment:**
- In which the outcomes, or results, cannot be predicted with certainty.
- **Examples:**

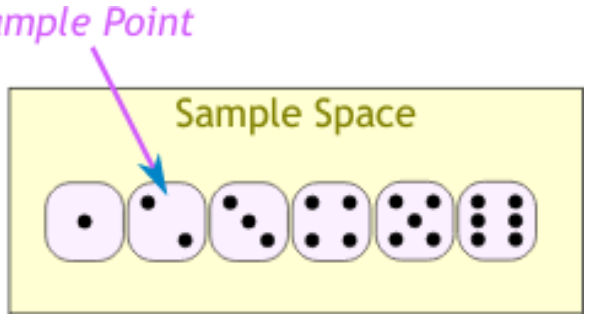
1.Flip a coin

2.Roll a die

Basic Terminologies

- **Sample Space:**
- The set of all possible outcomes of an experiment.

- **Examples:**
- For 52 cards in a deck, the sample space is all 52 cards.



- When you roll 1 die, the sample space is 1, 2, 3, 4, 5, or 6

- Sample space for the **coin** toss experiment is {H,T} .

Basic Terminologies

- **Event:**
- An **event** is a set of outcomes from the sample space or, in other words, a subset of the sample space *S*.
- **Example**
- Getting a Tail when tossing a coin is an event
- Rolling a "5" is an event.
- An event can include several outcomes:
- Choosing a "King" from a deck of cards (any of the 4 Kings) is also an event
- Rolling an "even number" (2, 4 or 6) is an event

Types of Events

- **Independent Events**
- Each event is not affected by any other events.

- **Example:**
- You toss a coin three times and it comes up "Heads" each time ... what is the chance that the next toss will also be a "Head"?
- The chance is simply 1/2, or 50%, just like ANY OTHER toss of the coin.
- What it did in the past will not affect the current toss!
- Next toss of the coin is totally independent of any previous tosses.

Types of Events

- **Dependent Events**
- Event can be affected by previous events.

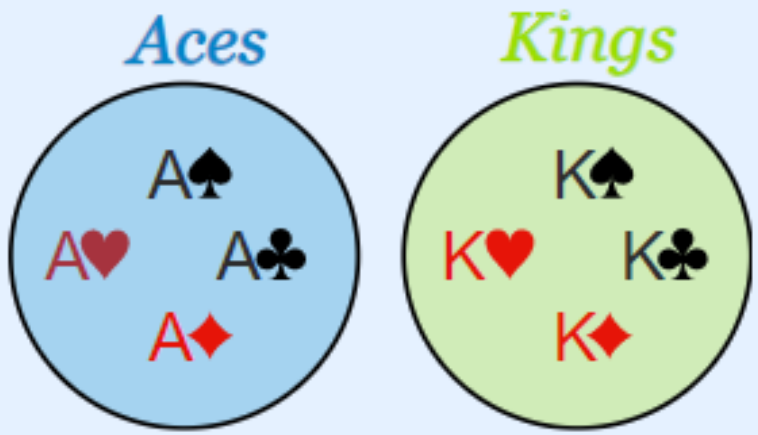
- **Example:**
- Drawing 2 Cards from a Deck
- After taking one card from the deck there are less cards available, so the probabilities change!

- Replacement: When we put each card back after drawing it the chances don't change, and the events are independent.

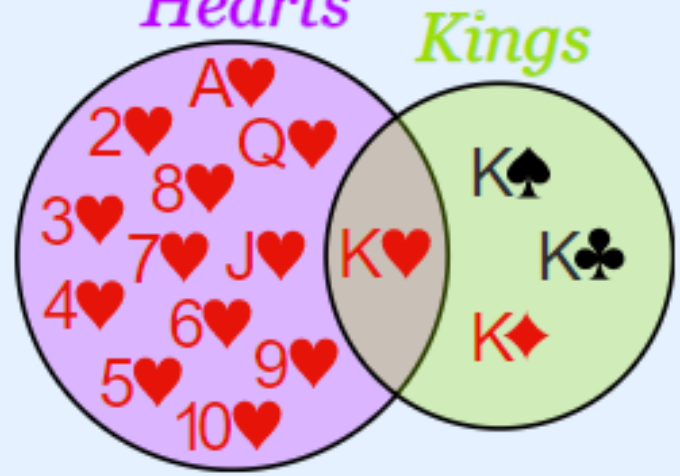
- Without Replacement: The chances will change, and the events are dependent.

Types of Events

- **Mutually Exclusive**
- It means we can't get both events at the same time.
- It is either one or the other, but **not both.**
- **Examples:**
- Turning left or right are Mutually Exclusive
- Heads and Tails are Mutually Exclusive
- Kings and Aces are Mutually Exclusive
- What isn't Mutually Exclusive?
- Kings and Hearts are **not** Mutually Exclusive, because we can have a King of Hearts!



Aces and Kings are
Mutually Exclusive
(can't be both)



Hearts and Kings are
not Mutually Exclusive
(can be both)

Probability

- Chance of happening an event
- If the number of outcomes favorable to event A are denoted by n(A) and total number of outcomes in sample space are denoted by n(S).
- Then probability of an event A, **P(A) = n(A)/n(S)**.

- The probability of an event E can vary between 0 to 1, i.e. $0 \leq P(E) \leq 1$.

- Probability can never be negative.

- Probability of occurrence of an event = 1 – (Probability that it doesn't occur).
- Probability can be expressed as a fraction or %.

- $Probability = \frac{no.of\ favorable\ cases}{total\ cases}$
- $odds\ in\ favor = \frac{favorable\ cases}{unfavorable\ cases}$
- $odds\ against\ favor = \frac{unfavorable\ cases}{favorable\ cases}$
- $favorable\ cases + unfavorable\ cases = Total\ cases$

Probability of word made by APPLE in which both P are together

- $Probability = \frac{no.of\ favorable\ cases}{total\ cases}$

$$= \frac{P(4,4)}{P(5,5)/2!}$$

$$= 2/5$$

3 boys and 3 girls are to arrange in a line. What is the probability of all boys sit together.

- $Probability = \frac{no.of\ favorable\ cases}{total\ cases}$

$$= \frac{P(4,4)xP(3,3)}{P(6,6)}$$

= 1/5

Problems

- Coins
- Dice
- Cards
- Bags and balls

Coins

- Sample Space or Total possibilities= $2^{\text{number of coins}}$
- One coin, Sample space={H,T}
- Two coins, Sample Space={HH,HT,TH,TT}
- Three coins, Total 8 possibilities
- Four coins, Total 16 possibilities

Example (2 coins)

- In a simultaneous toss of 2 coins, find the probability of 2 tails.

- {HH, HT, TH, TT}

- n(S) = 4
- n(E) = 1
- **P(E) = n(E)/n(S)**
- P(E) = 1/4

Example (3 coins)

- Find the probability of all heads
- n(S)=8
- n(A)=1
- **P(A) = n(A)/n(S)**
- P=1/8

- Find the probability of exactly 2 heads
- P=3/8

Example (4 coins)

- Exactly 3 tails
- P=4/16

- At least one tail
- P=15/16

Dice

- Sample Space or Total possibilities= $6^{\text{number of dice}}$
- One die, Total possibilities= $6^1 = 6$
- Two dice, Total possibilities $= 6^2 = 36$
- Thrice dice, Total possibilities $= 6^3 = 216$
- and so on

If one dice is thrown. What is the probability that it shows prime number?

- P =3/6

  =1/2

In a single throw of 2 dice, find probability of getting

- A doublet (both show same number)
- P=6/36

- A total of 11
- P=2/36

- Both face show even number
- P=(3x3)/36
- =1/4

In a single throw of 3 dice, find probability of getting

- Sum of faces is 16
- 6,6,4 => (permutation with repetition) 3 ways
- 6,5,5 => 3 ways
- P=(3+3)/216

- Sum on face is 15
- 3,6,6 => 3 permutations
- 6,5,4 => 6 permutations
- 5,5,5 => 1 permutations
- P=(3+6+1)/216

Cards

• Basic Terminology and information

- Total cards: 52
- Colors: Red (26) and Black (26)
- Shapes: **clubs** (♣), **diamonds** (♦), **hearts** (♥) and **spades** (♠)
- 13 cards of each shape (2 to 10 , Ace, Jack, King, Queen)
- Each number → 4 in total (eg. 4 Ace in 52)
- Face cards : 16

Examples (Cards)

- One card is drawn at random from 52 cards. What is the probability of picking a black card?
- P=26/52=1/2

- Picking an Ace of spades or Jack of Diamonds
- P=(1+1)/52=1/26

Examples (Cards)

- What is the probability of both are red, if :

- 2 cards drawn one by one.
- P=(26/52)x (25/51)
- = 25/102

- If 2 cards drawn at random.
- P=C(26,2)/C(52,2)

1 Bag and 2 color balls

A bag contains 6 red balls and 4 yellow balls

4 balls are picked at random. What is the probability that 3 are red and 1 is yellow OR 2 are red and 2 are yellow?

$$P=\frac{C(6,3)xC(4,1)+C(6,2)xC(4,2)}{C(10,4)}$$

$$=\frac{17}{21}$$

1 Bag and 3 color balls

A bag contains 6 red , 4 yellow and 2 green balls

- 4 balls are picked . Find probability for 2 red,1 yellow and 1 green ball.

$$P=\frac{C(6,2)xC(4,1)xC(2,1)}{C(12,4)}$$

$$=\frac{8}{33}$$