Discrete Mathematics
BCSC0010

Module 1

Combinations

Find the number of combinations of 4 objects, A, B, C, D, taken 3 at a time. Each combination of three objects determines 3! = 6 permutations of the objects as follows:

$ABC:$	$ABC,$	$ACB,$	$BAC,$	$BCA,$	$CAB,$	CBA
$ABD:$	$ABD,$	$ADB,$	$BAD,$	$BDA,$	$DAB,$	DBA
$ACD:$	$ACD,$	$ADC,$	$CAD,$	$CDA,$	$DAC,$	DCA
$BCD:$	$BDC,$	$BDC,$	$CBD,$	$CDB,$	$DBC,$	DCB

Thus the number of combinations multiplied by 3! gives us the number of permutations; that is,

$$C(4,3)\cdot 3! = P(4,3) \quad \text{or} \quad C(4,3) = \frac{P(4,3)}{3!}$$

P (4, 3) = 4 · 3 · 2 = 24 and 3! = 6; hence C(4, 3) = 4

COMBINATIONS

- Each of the different group or selections which can be formed by taking some or all of a number of objects, irrespective of their arrangement, is called **combination.**

Or

- Selection of objects from a group of objects where the order of selection does not matter.

COMBINATIONS

- Let S be a set with n elements.
- A combination of these n elements taken r at a time is any selection of r elements where order does not count. Such a selection is called an r-combination;
- it is simply a subset of S with r elements.
- The number of such combinations will be denoted by C(n, r)
- (other texts may use $_nC_r$, $C_{n,r}$, or C_r^n)

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r!(n-r)!}$$

We shall use $C(n, r)$ and $\binom{n}{r}$ interchangeably.

Example

- A farmer buys 3 cows, 2 goats, and 4 hens from a man who has 6 cows, 5 goats, and 8 hens. Find the number of choices that the farmer has.
- **Solution**
- The farmer can choose the cows in C(6, 3) ways,
- the goats in C(5, 2) ways,
- and the hens in C(8, 4) ways.
- Thus the number m of choices follows:

$$m = \binom{6}{3}\binom{5}{2}\binom{8}{4} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \cdot \frac{5 \cdot 4}{2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 20 \cdot 10 \cdot 70 = 14\,000$$

Example

- Find the number of subsets of the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11} having 4 elements.

We can do this by using the combination formula as:

- $^{11}C_4 = 11!/4!(11-4)!$

  $= 11!/\ 4!7!$

  $= (11.10.9.8)/4.3.2.1$

  $= 330$ ways

Example

A box contains 8 blue socks and 6 red socks. Find the number n of ways two socks can be drawn from the box if:

(a) They can be any color.

(b) They must be the same color.

(a) There are "14 choose 2" ways to select 2 of the 14 socks. Thus:

$$n = C(14, 2) = \binom{14}{2} = \frac{14 \cdot 13}{2 \cdot 1} = 91$$

Example

A box contains 8 blue socks and 6 red socks. Find the number n of ways two socks can be drawn from the box if:

(*a*) They can be any color.

(*b*) They must be the same color.

(*b*) There are $C(8, 2) = 28$ ways to choose 2 of the 8 blue socks, and $C(6, 2) = 15$ ways to choose 2 of the 4 red socks. By the Sum Rule, $n = 28 + 15 = 43$.

Example

- **The Indian Cricket team consists of 16 players. It includes 2 wicketkeepers and 5 bowlers. In how many ways can you select a cricket team of 11 players if you have to select 1 wicketkeeper and at least 4 bowlers?**

- Case1: number of ways of selecting 1 wicket keeper, 4 bowlers and 6 other players $= {}^2C_1 \times {}^5C_4 \times {}^9C_6 = 840$.
- Case2: the number of ways of selecting 1 wicket keeper, 5 bowlers and 5 other players$= {}^2C_1 \times {}^5C_5 \times {}^9C_5 = 252$
- Therefore, the total number of ways of selecting the team
- $= 840 + 252 = 1092$.

Combination with Repetition

- There are C(n + r - 1, r) r-combinations from a set with n elements when repetition of elements is allowed.

- $C(n + r - 1, r) = \dfrac{(n + r - 1)!}{r!\,(n - 1)!}$

Example

- Suppose that a cookie shop has 4 different kinds of cookies. How many different ways can 6 cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters.
- **Solution**
- The number of ways to choose 6 cookies is the number of

  6-combinations of a set with 4 elements.
- This equals C (4 + 6 - 1, 6) = C (9, 6) = 84
- There are 84 different ways to choose the six cookies.

Next Topic

Probability