# COL 351: Analysis and Design of Algorithms

## Lecture 13

# String Matching Problem

**Given:** String S = [$s_1, \ldots, s_n$] and a pattern P = [$p_1, \ldots, p_k$], represented as arrays of size $n$, $k$. (Here $k < n$).

**Find:** Does there exists a sub-string of S that is identical to P.

Example:

S = "cuckoo hashing is efficient"

P = "hash"

Yes

S = "cuckoo hashing is efficient"

P = "hash-table"

No

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*Can you have an O(nk) time algorithm?*

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**For** $i = 1$ to $n$:

Flag = True

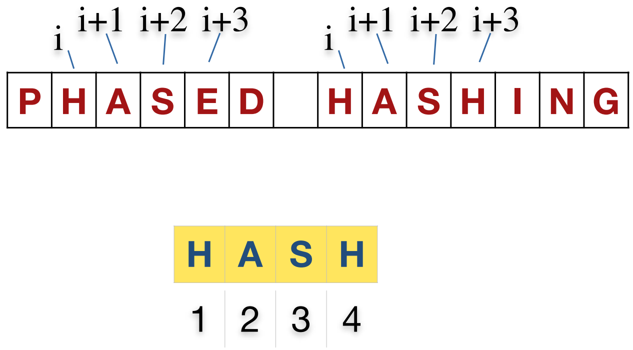
**For** $j = 1$ to $k$:

**If** $S[i + j - 1] \neq P[j]$ **then**
Flag= False

If (Flag) Return True

Return False

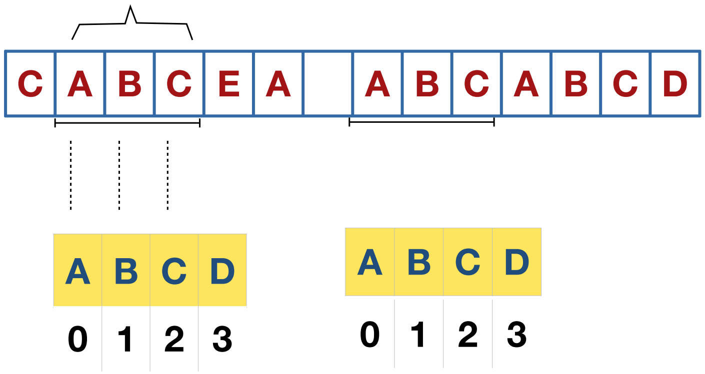
O($nk$) time algorithm



# Special Scenario: All characters in " pattern P " are different!

*Can you have an*
*O(n) time algorithm?*

The substring matched with prefix of P contains only one copy of P[1]=A as no characters are repeated in P



# Special Scenario: All characters in " pattern P " are different!

$i, j \leftarrow 1$;
**While** ($i \leq n$):
  **If** S[$i$] = P[$j$]):
    **If** ($j = k$): **Return** True
    Increment $i$ and $j$ by 1;
  **Else** :
    $j \leftarrow 1$;
    Increment $i$ by 1;
**Return** False;

O($n$) time algorithm
for special scenario

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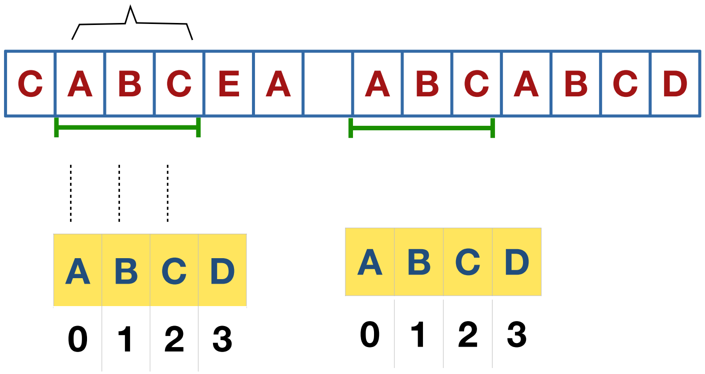
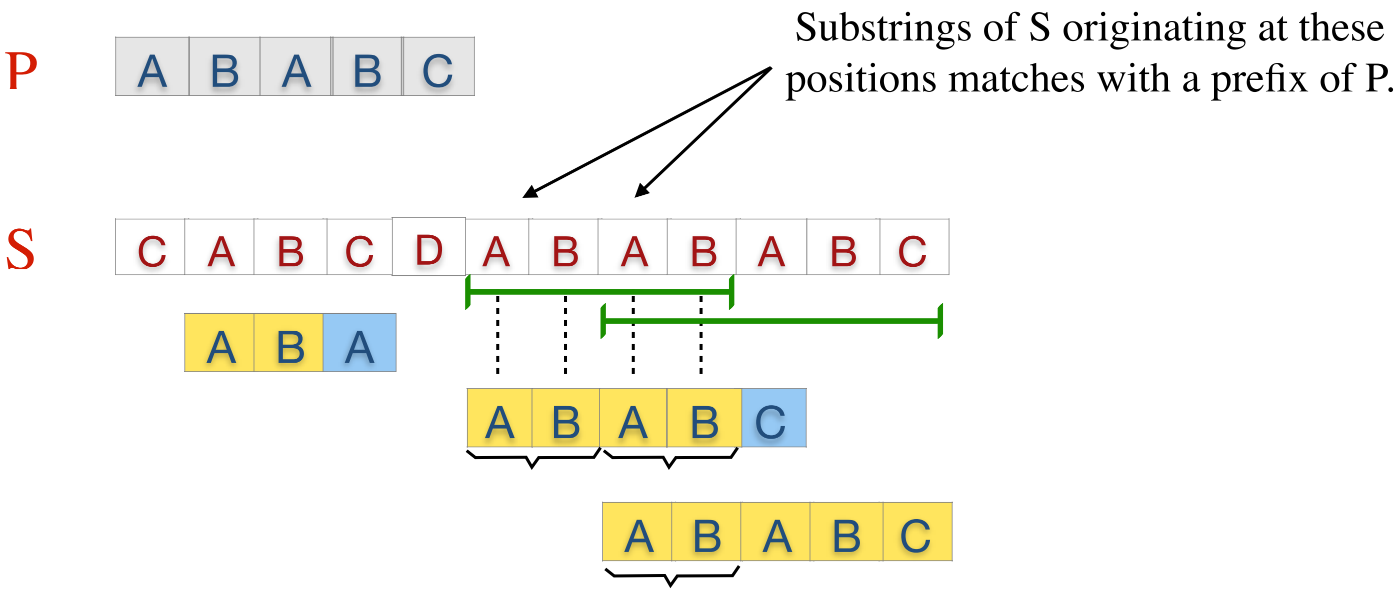


# An Example where P has repeated characters




**Remark:** Key Idea to obtain Linear time algorithm is <u>Pattern preprocessing.</u>

# Sub-Problem

**Given:** Pattern $P = [p_1, \ldots, p_k]$.

**Find:** Table of size $k$ satisfying

Table[$i$] = **Length of longest ~~non-trivial~~ prefix of P[1, i] that is also a suffix of P[1, i]**

PROPER

**Examples:**

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P[$i$] | A | B | C | A | B | B |
| Table[$i$] | 0 | 0 | 0 | 1 | 2 | 0 |

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| P[$i$] | A | A | B | A | A | B | A | A | A |
| Table[$i$] | 0 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 2 |

# Sub-Problem

$\text{Table}[i]$ = Length of longest (non-trivial) common prefix and suffix of $P[1, i]$

**Lemma:** For any $i \geq 1$, we have $\underline{\text{Table}[i+1] \leqslant 1 + \text{Table}[i].}$

**Proof Sketch:**

(H. W.)

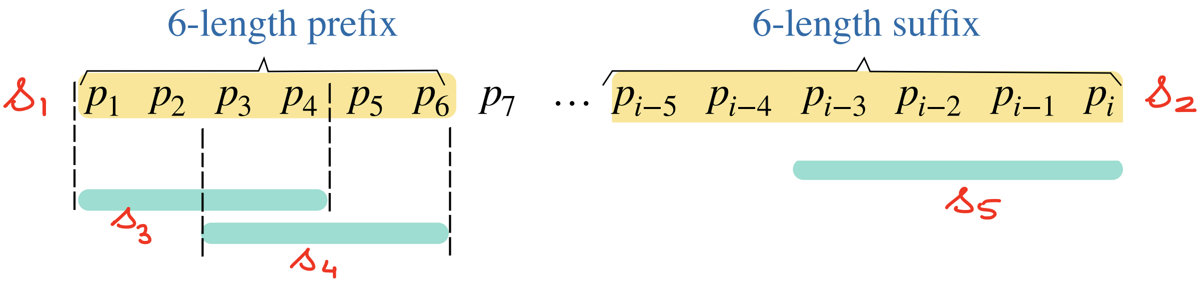
# Prefix Suffix subproblem

Table[$i$] = Length of longest (non-trivial) common prefix and suffix of P[1, $i$]

**Lemma:** Suppose $L \geq 1$ satisfy that $L$-length prefix and suffix of $P[1,\ i]$ are identical.

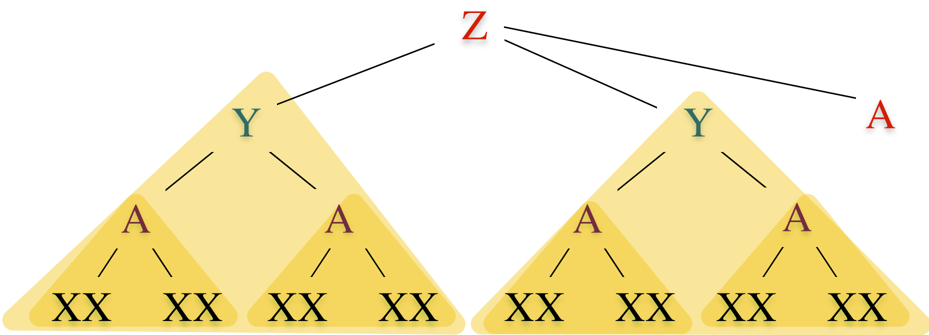
Then, the length of longest common prefix-suffix of $P[1,\ i]$ of size just smaller than $L$ is "Table[$L$]".

**Proof Sketch:**




If $s_1 = s_2$ and $s_3 = s_4$, then $s_3 = s_5$

**We will study this example to understand the intuition for an O($k$) time algorithm to solve our sub-problem.**




| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | | | | | | | | | | | |
| P[$i$] | X | X | A | X | X | Y | X | X | A | X | X | Z | X | X | A | X | X | Y | X | X | A | X | X | A |
| Table[$i$] | 0 | 1 | 0 | 1 | 2 | 0 | 1 | 2 | 3 | 4 | 5 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | ? |

# Example

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | … | | | | | | | | | 22 | $i$ | $i$+1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P[$i$] | X | X | A | X | X | Y | X | X | A | X | X | Z | X | X | A | X | X | Y | X | X | A | X | X | A |
| Table[$i$] | 0 | 1 | 0 | 1 | **2** | 0 | 1 | 2 | 3 | 4 | **5** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |

1) We have computed Table values upto an index $i = 23$.

2) Note Table[23] =11. Thus, 11 is length of longest identical suffix-prefix of P[1, 23].

3) Now, Z = P[11+1] $\neq$ P[23+1] = A, so, Table[23+1] cannot be 12.
   We compute length of longest identical suffix-prefix of P[1, 23] smaller than 11.
   This is just P[11] = 5.

4) Again, Y = P[5+1] $\neq$ P[23+1] = A.
   Therefore, we compute length of longest identical suffix-prefix of P[1, 23] smaller than 5.
   This is just P[5] = 2.

5) Compare P[2+1] and P[23+1]. Both are A. Thus, Table[23+1] is 2+1=3.

# Prefix Suffix subproblem

Table[$i$] := Length of longest (non-trivial) common prefix and suffix of P[1, $i$]

**Lemma:** Suppose Table is computed for indices 1 to $i$.
Let $len$ = Table[$i$] and suppose P[$i$+1] $\neq$ P[$len$+1].

Then, Table[$i$+1] can be computed as follows:

Recursively update $len$=Table[$len$] until either P[$i$+1]=P[$len$+1] or $len = 0$.

—If $len$ becomes 0, then Table[$i$+1] must be 0.

—If $len > 0$, then Table[$i$+1] must be one larger than current value of $len$.