Q1 Solution:

$$H(w) = A(w) + jB(w) \ where, A(w) = \pi\delta(w). \tag{1}$$

Given that the system is causal. Therefore,

$$h(t) = h(t)u(t). \tag{2}$$

Taking Fourier transform on both sides,

$$H(w) = \frac{1}{2\pi}H(w) * \left(\pi\delta(w) + \frac{1}{jw}\right). \qquad \textbf{(1 mark)} \tag{3}$$

$$= \frac{H(w)}{2} + \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{H(\tau)}{w-\tau}d\tau. \qquad \textbf{(1 mark)} \tag{4}$$

$$H^*(w) = \frac{H^*(w)}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{H^*(\tau)}{w-\tau}d\tau. \tag{5}$$

$$H(w) - H^*(w) = \frac{H(w) - H^*(w)}{2} + \frac{1}{\pi j}\int_{-\infty}^{\infty}\frac{H(\tau) + H^*(\tau)}{2(w-\tau)}d\tau. \qquad \textbf{(1 mark)} \tag{6}$$

From eq.(1) we can write the above expression as:

$$2jB(w) = jB(w) + \frac{1}{\pi j}\int_{-\infty}^{\infty}\frac{A(\tau)}{w-\tau}d\tau. \qquad \textbf{(1 mark)} \tag{7}$$

$$B(w) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{A(\tau)}{w-\tau}d\tau. \tag{8}$$

$$= \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\tau)}{w-\tau}d\tau. \tag{9}$$

$$= \frac{-1}{w}. \qquad \textbf{(1 mark)} \tag{10}$$

$$\therefore H(w) = \pi\delta(w) + \frac{1}{jw}. \tag{11}$$

Page No.
Date

Ques 2: 1. $x(t)$ is real and even.

(1 marks)

2. $a_{k+3} = \frac{1}{T}\int x(t)\, e^{-j(k+3)\omega_0 t}\, dt$

$= \frac{1}{T}\int x(t)\, e^{-jk\omega_0 t}\, dt$

$\Rightarrow x(t) e^{-j3\omega_0 t} = x(t)$

$\Rightarrow x(t)$ is build using impulses

$\Rightarrow e^{-j3\omega_0 t} = e^{j2\pi m}$ (or) $x(t)=0$

$\Rightarrow t = \frac{2\pi m}{3\,2\pi} \times T \Rightarrow t = \frac{mT}{3}$

$$\Rightarrow \boxed{x(t) = a_0\,\delta(t) + a_1\,\delta(t-1) + a_2\,\delta(t-2)}$$

(3 marks)

3. $\int_1^4 x(t)\,dt = 2 \Rightarrow \int_0^3 x(t)\,dt = 2$

$\Rightarrow a_0 + a_1 + a_2 = 2$ — (1)

$\int_1^3 x(t)\,dt = 1.5 \Rightarrow a_1 + a_2 = 1.5$ — (2)

From (1) and (2), we have

$\Rightarrow \boxed{a_0 = 0.5}$ & $a_1 = a_2 = 0.75$

(0.5 marks) (0.5 marks)

$x(t) = 0.5\,\delta(t) + 0.75\,\delta(t-1) + 0.75\,\delta(t-2)$

Solⁿ:-3. Given

(a) $y[n] - \frac{1}{2} y[n-1] = x[n] + \frac{1}{2} x[n-1]$

Taking DTFT both side we will get,

$$Y[e^{j\Omega}] - \frac{1}{2} e^{-j\Omega} Y[e^{j\Omega}] = X(e^{j\Omega}) + \frac{1}{2} X[e^{j\Omega}]$$ (1 mark)

$$\left(1 - \frac{1}{2} e^{-j\Omega}\right) Y[e^{j\Omega}] = X(e^{j\Omega}) \left(1 + \frac{1}{2} e^{-j\Omega}\right)$$

We know that

$$H(e^{j\Omega}) = \frac{Y[e^{j\Omega}]}{X[e^{j\Omega}]} = \frac{1 + \frac{1}{2} e^{-j\Omega}}{1 - \frac{1}{2} e^{-j\Omega}}.$$

$$\boxed{H(e^{j\Omega}) = \frac{2 + e^{-j\Omega}}{2 - e^{-j\Omega}}}$$ (1 mark)

(b) Method - I :- $H(e^{j\Omega}) = \frac{2 + e^{-j\Omega}}{2 - e^{-j\Omega}} = \frac{1}{1 - \frac{e^{-j\Omega}}{2}} + \frac{e^{-j\Omega}}{2\left[1 - \frac{e^{-j\Omega}}{2}\right]}$

As we know

if $x[n] = a^n u[n]$

$$\boxed{X(e^{j\Omega}) = \frac{1}{1 - a e^{-j\Omega}}}$$

Similarly Taking inverse D.T.F.T of above eqⁿ we will get,

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$ (By using Time shifting Property)

$$\boxed{h[n] = \left(\frac{1}{2}\right)^n [u[n] + u[n-1]]}$$ (2 marks)

Method - II :- $H(e^{j\Omega}) = -1 + \frac{2}{1 - \frac{1}{2} e^{-j\Omega}}$

$1 \xleftrightarrow{IDTF} \delta[n]$

$\delta[n] \xleftrightarrow{DTFT} 1.$

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Taking inverse DTFT we will get

$$= -\delta[n] + \left(\tfrac{1}{2}\right)^n u[n]$$

$$\boxed{h[n] = \left(\tfrac{1}{2}\right)^n u[n] - \delta[n]}$$

(2 marks)

(c)

$$H(e^{j\Omega}) = \frac{2 + e^{-j\Omega}}{2 - e^{-j\Omega}}$$

Given

$$x[n] = \cos\tfrac{\pi}{2} n = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$$

at $\Omega = \frac{\pi}{2}$.

$$H(e^{j\pi/2}) = \frac{2 + e^{-j\pi/2}}{2 - e^{-j\pi/2}}$$

$$e^{-j\pi/2} = \cos\tfrac{\pi}{2} - j\sin\tfrac{\pi}{2}$$

$$= 0 - j = -j$$

$$\boxed{H(e^{j\pi/2}) = \frac{2 - j}{2 + j}}$$

(2 marks)

Method - I

By using Eign function $y[n]$ can be written as -

$$y[n] = \tfrac{1}{2} e^{j\frac{\pi}{2}n} H(e^{j\pi/2}) + \tfrac{1}{2} e^{-j\frac{\pi}{2}n} H(e^{-j\pi/2})$$

$$\boxed{y[n] = \tfrac{1}{2} e^{-j\frac{\pi}{2}n}\left[\frac{2+j}{2-j}\right] + \tfrac{1}{2} e^{j\frac{\pi}{2}n}\left[\frac{2-j}{2+j}\right]}.$$

(4 marks)

Method - II.

$$x[n] = \cos\tfrac{\pi}{2}n = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$$

$$X(e^{j\Omega}) = \frac{1}{2}\left[2\pi\left[\delta(\omega - \tfrac{\pi}{2}) + \delta(\omega + \tfrac{\pi}{2})\right]\right]$$

$$X(e^{j\Omega}) = \pi\left[\delta(\omega - \tfrac{\pi}{2}) + \delta(\omega + \tfrac{\pi}{2})\right]$$

Periodic in $[-\pi, \pi]$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$Y(e^{j\Omega}) = H(e^{j\Omega}) \cdot X(e^{j\Omega})$$

$$= \left(\frac{2+e^{-j\Omega}}{2-e^{-j\Omega}}\right) \pi \left[\delta(\omega - \pi/2) + \delta(\omega + \pi/2)\right] \text{ in } [-\pi, \pi].$$

$$Y(e^{j\Omega}) = \left(\frac{2+e^{-j\Omega}}{2-e^{-j\Omega}}\right) \pi \delta(\omega - \pi/2) + \pi \left(\frac{2+e^{-j\Omega}}{2-e^{-j\Omega}}\right) \delta(\omega + \pi/2) \quad \text{in } [-\pi, \pi]$$

$$Y(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} \left[\frac{1}{2}\left(\frac{2-j}{2+j}\right) \delta(\omega - \pi/2 - 2\pi k) + \frac{1}{2}\left(\frac{2+j}{2-j}\right) \delta(\omega + \pi/2 - 2\pi k)\right]$$

Taking inverse DTFT. we will get.

$$\boxed{y[n] = \frac{1}{2}\left(\frac{2-j}{2+j}\right) e^{j\frac{\pi}{2} n} + \frac{1}{2}\left(\frac{2+j}{2-j}\right) e^{-j\frac{\pi}{2} n}}$$

(4 marks)

4.) DTFS:

a) $$y_4[n] = \begin{cases} x[n/2] & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

$$b_k = \frac{1}{2N} \sum_{n=0}^{2N-1} y_4[n] e^{-jk\frac{2\pi}{2N}n}$$ (i)

$$b_k = \frac{1}{2N} \sum_{m=0}^{N-1} x[m] e^{-jk\frac{\pi}{N}2m}$$ 1

$$\boxed{b_k = \frac{a_k}{2}}$$ 1

b) $y_4[n] + y_4[n-1]$

$$\frac{a_K}{2} + \frac{a_K}{2} e^{-jK\frac{\pi}{N}}$$ 3

c) $y[n] = x[2n]$

First calculate coefficients of $w[n]$

$$= \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Let coefficients of $w[n]$ be c_K, then

$$c_K = \begin{cases} \frac{1}{2}\left(a_K + a_{K-\frac{N}{2}}\right) & N \text{ even} \quad 1 \\ \begin{cases} \frac{a_{K-N}}{2} & K \text{ odd} \\ a_{K/2} & K \text{ even} \end{cases} & N \text{ odd} \quad 1 \end{cases}$$

$$b_k = 2c_k$$

1

d) $(-1)^n x[2n]$

3

let $(-1)^n y[n]$

Coefficients of $y[n]$: $b_k =$

$$\begin{cases} a_k + a_{k-\frac{N}{2}} & N \text{ even} \\ \left.\begin{matrix} 2a_{k-\frac{N}{2}} & k \text{ odd} \\ 2a_{k/2} & k \text{ even} \end{matrix}\right\} & N \text{ odd} \end{cases}$$

Then coefficients of $(-1)^n y[n]$:

$$\begin{cases} b_{k-\frac{\tilde{N}}{2}} & \text{if } \tilde{N} \text{ is even} \\ \begin{cases} b_{k-\frac{\tilde{N}}{2}} & k \text{ odd} \\ 0 & k \text{ even} \end{cases} & \text{if } \tilde{N} \text{ is odd} \end{cases}$$

DTFT

Marks

a). $X(e^{j2\Omega})$ 2

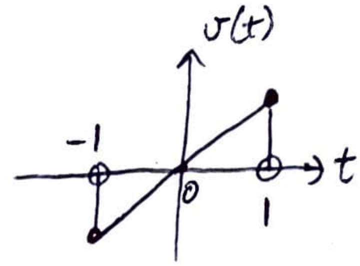
b) $Xe^{(j2\Omega)}(1+e^{-j\Omega})$ 2

c) $\frac{1}{2}\sum_{k=-\infty}^{\infty} X\left(e^{j\frac{(\Omega-k2\pi)}{2}}\right)$ 2

d) $\frac{1}{2}\sum_{k=-\infty}^{\infty} X\left(e^{j\frac{(\Omega-k2\pi-\pi)}{2}}\right)$ 2

Question -5

a) $v(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$



$$= \begin{cases} t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt = \int_{-1}^{1} t e^{-j\omega t} dt$$

$$= \frac{t e^{-j\omega t}}{-j\omega} \Bigg|_{-1}^{1} - \int_{-1}^{1} \frac{e^{-j\omega t}}{-j\omega} dt$$

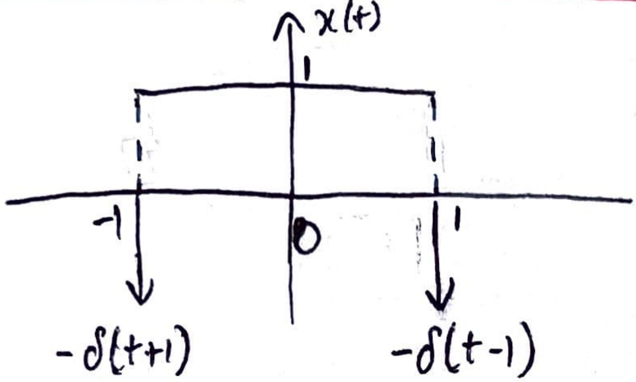
$$= -\frac{1}{j\omega} \left[e^{-j\omega} - (-e^{j\omega}) \right] + \frac{1}{j\omega} \frac{\left[e^{-j\omega} - e^{j\omega} \right]}{-j\omega} \quad \text{—— } 2$$

$$x(t) = \frac{d}{dt} v(t) \longleftrightarrow X(\omega) = j\omega V(\omega)$$

$$X(\omega) = -\left[e^{j\omega} + e^{-j\omega} \right] + \frac{\left[e^{j\omega} - e^{-j\omega} \right]}{j\omega}$$

$$\boxed{\begin{aligned} X(\omega) &= -2\cos\omega + \frac{2\sin\omega}{\omega} \\ &= -2\cos\omega + 2\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right) \end{aligned}} \quad \text{—— } 3$$

b) $\boxed{x(t) = \text{rect}\left(\frac{t}{2}\right) - \delta(t+1) - \delta(t-1)}$ ——— 2



——— 3

c) i) $\int_{-\infty}^{\infty} x(t)\cos\left(\frac{\pi t}{6}\right)dt = \int_{-\infty}^{\infty} x(t)\left[\frac{e^{j\pi t/6} + e^{-j\pi t/6}}{2}\right]dt$

$= \frac{1}{2}\left[X\left(\frac{-\pi}{6}\right) + X\left(\frac{\pi}{6}\right)\right]$ ——— 1

$= \frac{1}{2}\left[-2\cos\left(\frac{-\pi}{6}\right) + \frac{2\sin(-\pi/6)}{-\pi/6} - 2\cos\left(\frac{\pi}{6}\right) + \frac{2\sin(\pi/6)}{\pi/6}\right]$

$= \boxed{-\sqrt{3} + \frac{6}{\pi}}$ ——— 2

ii) $\int_{-\infty}^{\infty} X(\omega)e^{j\omega/2}d\omega = 2\pi\, x(1/2) = 2\pi \times 1 = \boxed{2\pi}$ ——— 2

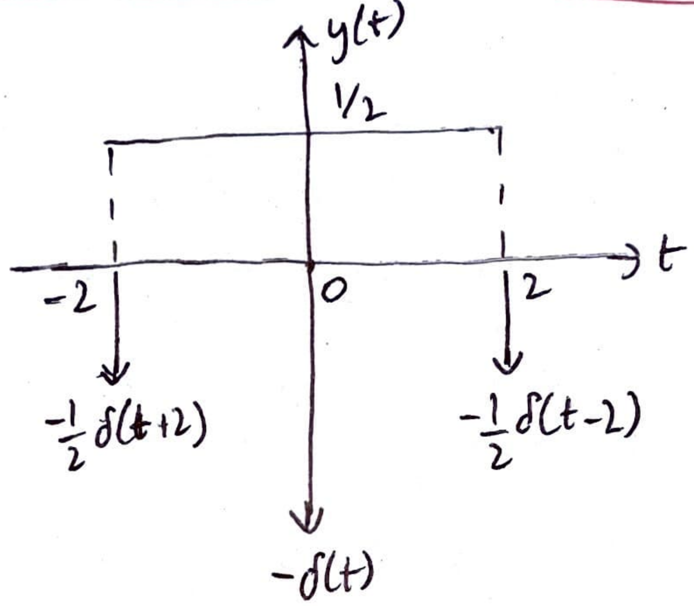
d) $Y(\omega) = H(\omega)\,X(\omega) = \cos\omega\left[-2\cos\omega + \frac{2\sin\omega}{\omega}\right]$

$$= -2\cos^2\omega + \frac{2\sin\omega\cos\omega}{\omega}$$

$$= \frac{\sin 2\omega}{\omega} - 1 - \cos 2\omega = \frac{2\sin 2\omega}{2\omega} - 1 - \cos 2\omega$$

$$Y(\omega) = 2\,\mathrm{sinc}\left(\frac{2\omega}{\pi}\right) - 1 - \left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right) \text{ ——— } 2$$

$$\boxed{y(t) = \frac{1}{2}\,\mathrm{rect}\left(\frac{t}{4}\right) - \delta(t) - \frac{\delta(t+2)}{2} - \frac{\delta(t-2)}{2}} \text{ ——— } 1$$



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