COL 351 Quiz 1B

PrathamAgrawal

TOTAL POINTS

9 / 10

QUESTION 1

1 Question 1 **9 / 10**

✓ **+ 3 pts** *Correct Greedy Algorithm Idea*

✓ **+ 1 pts** *Proper Algorithm Description*

+ 4 pts Proof Mostly correct, but some Mistake/Informalness in Proof

✓ **+ 5 pts** *Proof Mostly correct, Slight Mistake/Informalness in Proof*

+ 6 pts Correct Proof

- 1 pts Only proved for first element. Did not extend it using induction

+ 0 pts Incorrect / Not Attempted

1 First need to show that there exists an optimal solution in which s_1 is picked

Quiz I (COL 351)

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Give precise arguments. Needlessly long explanations will not fetch any marks.

You have n friends, call them $1, \ldots, n$. Person i has *unfriendliness* value s_i. You are given a target T. You need to divide these n friends into teams of two each (assume n is even, and so there will be $n/2$ teams). A team consisting of i and j is said to be good if $s_i + s_j \leq T$. Give a greedy algorithm to divide the n friends into teams of two each such that the number of good teams is maximized.

Greedy Algorithm:

sort in increasing order of unfriendliness value's s_i.
it becomes $s_1, s_2, \ldots s_n$ in sorted order.
starting from s_1, we find the friend with maximum s_j s.t $s_1 + s_j \leq T$. Then, we continue from the left. number of such pairs found will be the maximum answer possible.

Proof of Correctness:

Example $\longrightarrow$ 1 3 4 2 8 7 6 5, $T = 9$
sort $\longrightarrow$ 1 2 3 4 5 6 7 8 $T = 9$
$\rightarrow$ 4 such possible pairs
$\therefore$ answer $= 4$.

1. ~~for~~ order doesn't matter as we are just chosing 2 friends.
$\therefore$ sorting doesn't affect the problem.

$\longrightarrow$ 2. let $s_1, s_2, \ldots s_n$ be the sorted order.
if s_1 is matched with s_j (according to my greedy alg$\hat{}$ G)
$s_1 + s_j \leq T$ and s_j is the max such value.
i.e, $s_1 + s_{j+1} > T$, $s_1 + s_{j+2} > T$, ..., $s_1 + s_n > T$.

$\longrightarrow$ • in the optimal sol^n O, say s_1 is matched to s_k* where $k \neq j$.
~~if k~~ clearly $k < j$. as if $k > j \longrightarrow s_1 + s_k > T$.
• if $k < j \longrightarrow s_1 + s_k \leq T$. Now we reduce our sample space (S_1) to $s_2 \ldots, s_{k-1}$. However if we chose s_j instead of s_k, our sample space (S_2) would have been $s_2 \ldots, s_{j-1}$.
$\therefore$ $\underline{S_1 \subseteq S_2}$ (subset notation). To prove, our greedy, G is as much optimal as O, we need to show, number of pairs in $S_2 \geq$ number of pairs in S_1.
since $S_1 \subseteq S_2 \longrightarrow$ all possible pairs in S_1 are also possible in S_2, because S_2 contains all values of S_1. $\therefore$ no. of pairs in $S_2 \geq$ number of pairs in S_1. $\therefore$ G is as optimal as O. $\therefore$ our greedy algo. G works.

* if $k = j$, we can use induction for new sample space $\{s_2 \ldots, s_{j-1}\}$

more examples:

$n = 8 \longrightarrow$ 1 3 3 4 4 6 7 8 , $T = 7$

answer = 3

$n = 4 \longrightarrow$ 1 2 4 8 , $T = 6$

answer = 1

1 3 4 5 8 2

1 2 3 4 5 8 $T = 7$