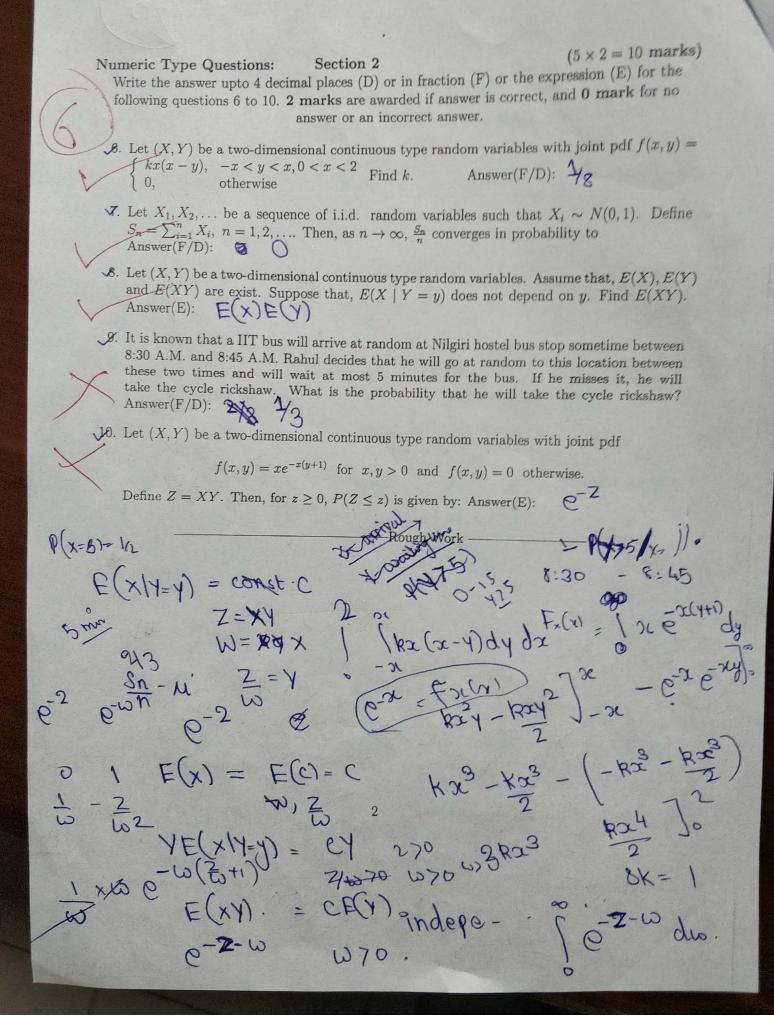
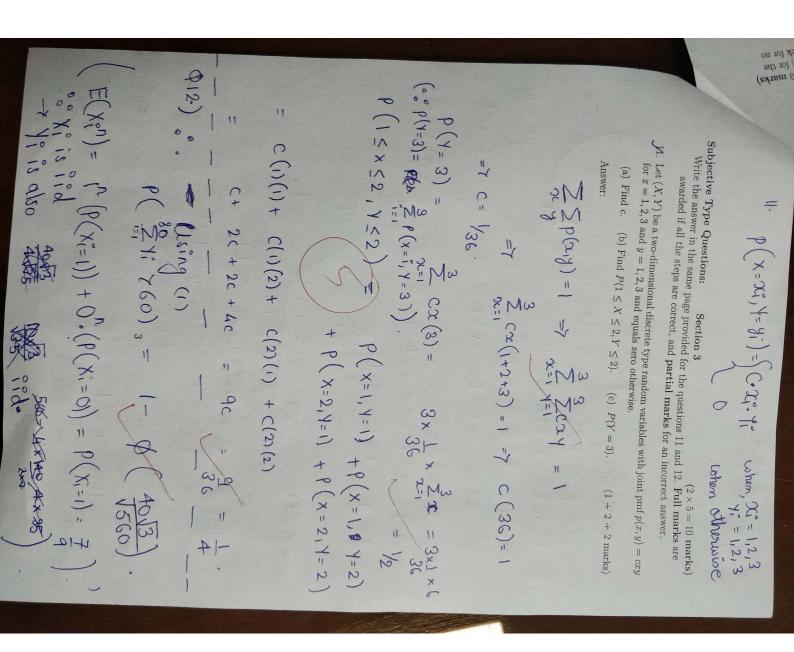
MTL 106 (Introduction to Probability Theory and Stochastic Processes) Minor 2 Examination Time allowed: 1 hour Name: Shivam Jadhav Entry Number: 2017 CS Signature: $(1 \times 5 = 5 \text{ marks})$ Multiple Selection Questions: Section 1 Each of the following questions 1 to 5 has four options out of which one or more options can be correct. Write A, B, C or D which corresponds to the correct option for the first correct answer followed by space and the next correct answer and so on. 1 mark is awarded if all correct answers are written, 0 mark for no answer or partial correct answers or any incorrect answer. \mathcal{X} . Let $X_1 \sim Exp(3)$ and $X_2 \sim Exp(5)$ be two independent random variables. The distribution of $X_{(1)} = \min\{X_1, X_2\}$ is (A) Exp (8) (B) Gamma (8,3/5) (C) Erlang (8, 3/5) (D) Exp (3/5)Answer: A \mathcal{Z} . Let X and Y be continuous type random variables with joint pdf given by $f(x,y) = \frac{1}{2\pi}e^{-\frac{(x^2+y^2)}{2}}, -\infty < x < \infty, -\infty < y < \infty.$ Then, the correlation coefficient between X and Y is (A) 0 (B) 1 (C) $0_{\bullet}5$ (D) -1 Answer: \mathcal{A} . Let (X,Y) be a two-dimensional random variables with joint pdf f(x,y). Then, the pdf of U = X + Y is (A) $g(u) = \int_{-\infty}^{\infty} f(u, u - v) dv$ (B) $g(u) = \int_{-\infty}^{\infty} f(u - v, v) dv$ (C) $g(u) = \int_{-\infty}^{\infty} f(u-v, u-v) dv$ (D) $g(u) = \int_{-\infty}^{\infty} f(v, u-v) dv$ Answer: Let N be a positive integer random variable and X_1, X_2, \ldots be a sequence of iid random variables. N is independent of X_i 's. Let $S_N = \sum_{i=1}^N X_i$. Then, $Var(S_N)$ is (A) $E(N)Var(X) + Var(N)[E(X)]^2$ (B) $E(N^2)Var(X) + Var(N)E(X^2)$ (C) E(N)Var(X) + Var(N)E(X) (D) $[E(N)]^2Var(X) + Var(N)[E(X)]^2$ Answer: **5.** Let (X_1, X_2, \ldots, X_n) be a *n*-dimensional random variables. Let \sum be a $n \times n$ matrix where the entry in row i, column j, \sum_{ij} is given by $Cov(X_i, X_j)$. If y is any $1 \times n$ vector, then (A) $y \sum y^T < 0$ (B) $y \sum y^T > 0$ (C) $y \sum y^T \le 0$ (D) $y \sum y^T \ge 0$ Answer: -Rough Work E(Sw)





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= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
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= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P((\frac{x}{2}, \frac{x}{2}) + x \sqrt{Var(\frac{x}{2}, \frac{x}{2})} + E(\frac{x}{2}, \frac{x}{2}) - Q)
= \frac{1 - \phi(x)}{3} = P(\frac{x}{2}, \frac{x}{2}) + \frac{1 - \phi(x)}{3} = \frac{x}{3} + \frac{1 - \phi(x)}{3} = \frac{x}{3}
= \frac{1 - \phi(x)}{3} = \frac{1 + \phi(x)}{3} + \frac{1 - \phi(x)}{3} = \frac{x}{3} + \frac{1 - \phi(x)}{3} 
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Var(Y_{1}^{2}) = E(Y_{1}^{2}) - E(Y_{1}^{2})^{2} = 4 \times \frac{1}{2} = \frac{28}{3}
Var(Y_{1}^{2}) = E(X_{1}^{2}) - E(Y_{1}^{2})^{2} = 4 \times \frac{1}{2} = \frac{28}{3}
Var(Y_{1}^{2}) = E(X_{1}^{2}) - E(Y_{1}^{2})^{2} = \frac{252 - 196}{81} = \frac{56}{81}
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= 7 \qquad 1 - P\left(\left(\frac{2}{2}Y_{1}\right) > \infty \sqrt{Var(\frac{2}{2}Y_{1})} + E\left(\frac{2}{2}Y_{1}\right)\right) = \phi(\infty)
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Let X_1, X_2, \ldots be iid random variables, each having pmf P(X_i = 1) = \frac{7}{9} = 1 - P(X_i = 0). Let Y_i = X_i + X_i^2, i = 1, 2, \ldots Use central limit theorem to evaluate P\left(\sum_{i=1}^{30} Y_i > 60\right) approximately. Final answer can be in terms of \Phi(z) where \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt.
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