Group 1.

(1)

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Multiple Selection Questions: Section 1 $(1 \times 5 = 5 \text{ marks})$

Each of the following questions 1 and 2 has four options out of which one or more options can be correct. Write A, B, C or D which corresponds to the correct option for the first correct answer followed by space and the next correct answer and so on. **1 mark** is awarded if all correct answers are written, **0 mark** for no answer or partial correct answers or any incorrect answer.

1. Let $\Omega = \{a, b, c, d\}$. Which of the following sets are NOT σ-fields on Ω.
   (A) $\{\emptyset, \{a\}, \{b\}, \{c, d\}, \Omega\}$ (B) $\{\emptyset, \{a\}, \{b, c, d\}, \Omega\}$
   (C) $\{\emptyset, \{a, b, d\}, \{c, d\}, \Omega\}$ (D) $\{\emptyset, \{b\}, \{a, c, d\}, \Omega\}$ Answer: AC

2. Let X be uniform distributed random variable on the interval $(0, 1)$. Suppose $Y = g(X)$ is uniform distributed random variable on the interval (a, b) with $-\infty < a < b < \infty$. Then, $g(X)$ is (A) $aX + (b - a)$ (B) $a + (b - a)X$ (C) $bX + (a - b)$ (D) $b + (a - b)X$
   Answer: B

3. The probability generating function for $B(n, p)$ is
   (A) $(qt+p)^n$ (B) $(pt+1-p)^n$ (C) $(pt+p)^n$ (D) $(qt+1-p)^n$ Answer: B

4. Which of the following distributions NOT satisfy the memoryless property?
   (A) Poisson   (B) Bernoulli   (C) Geometric   (D) Exponential   Answer: ABC

5. If X is a random variable on a measurable space $(\Omega, \mathcal{F})$ where $\mathcal{F}$ is the largest σ-field on Ω, then which of the following real-valued functions will be random variables on this measurable space $(\Omega, \mathcal{F})$. (A) X^- (B) $|X|$ (C) X^2 (D) X^+ Answer: ABCD

─────────────────── Rough Work ───────────────────

$X \leq \frac{y-b}{a-b}$

19

$P(x > S+t \,/\, x > S)$

13

16

$g(x)$ in creasing $a < y < b$

$0 < x < 1$

$\frac{1-F_X(S+t)}{1-F_X(S)}$

$= \qquad X \geq 1$

15

$\frac{15\pm}{30}$

$y \leq y$

$P(g(x) \leq y)$

11.11 → 4
1.112
1.121
12 w   → 5
4.111   1

1.122
1.212
2.112   6
2.211
2.121
2.1221

$t^k \times p^k$
$p^t \times 1-p$

$\frac{L!}{16}t$

$b + (a-b)x \leq y$

$w \neq ?^w$

2.221
2.212   7
2.122
1.222
2.222 → 8

$1 + 4 + 6 + 4$

4, 5.

0

$4t2$

space $(\Omega, \mathcal{F})$.

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(5 × 2 = 10 marks)

Numeric Type Questions: Section 2

Write the answer upto 4 decimal places (D) or in fraction (F) or the expression (E) for the following questions 6 to 10. **2 marks** are awarded if answer is correct, and **0 mark** for no answer or an incorrect answer.

6. Let $\Omega = \{s_1, s_2, s_3, s_4\}$ and $P\{s_1\} = \frac{1}{5}$, $P\{s_2\} = \frac{1}{10}$, $P\{s_3\} = \frac{2}{5}$, $P\{s_4\} = \frac{3}{10}$. Define,

$$A_n = \begin{cases} \{s_1, s_3\} & \text{if } n \text{ is odd} \\ \\ \{s_2, s_4\} & \text{if } n \text{ is even} \end{cases}$$

. Find $P(\limsup A_n)$. Answer(F/D): **1**

7. Suppose that the number of passengers for a limousine pickup is thought to be either 1, 2, 3, or 4, each with equal probability, and the number of pieces of luggage of each passenger is thought to be 1 or 2, with equal probability, independently for different passengers. What is the probability that there will be five or more pieces of luggage? Answer(F/D): **19/30**

8. Suppose X has a geometric distribution with parameter p. Then, the moment generating function for X is given by: Answer(E): $\dfrac{1-p}{1-e^t p}$

9. Let $X \sim P(\lambda)$ such that $P(X = 0) = e^{-3}$. Find $Var(X)$. Answer(E): $\dfrac{1-p}{1-e^t p}$ → **3**

10. Let X be a normal distributed random variable with mean 0 and variance 9. Then, the pdf of $\ln(X)$ is given by:

Answer(E): $\left[\dfrac{1}{y} \times \dfrac{1}{\sqrt{2\pi}\,6} e^{-1/2\left(\frac{\ln y}{6}\right)^2}, 0 < y < \infty \quad \text{where} \quad 6 = 3. \right]$

Rough Work

$$1 \quad 2 \quad 3 \quad 4 \longrightarrow$$
$$1,2 \quad 2,3,4 \quad 3,4,4,4,5,5,5,6 \quad \mu = 0 \quad 6^2 = 9 \quad 6 = 3$$

$y = \ln(x)^3$

$e^y = x \cdot$

$E(x) = \lambda$
$E(x^2) = \lambda + \lambda^2$

$i \geq 0$

$\int 2x e^{-\lambda x}\left(\dfrac{e^{-\lambda}\lambda^k}{k!}\right)$

$\dfrac{1-p}{1-e^t p}$

$p^i(1-p)$

$\lambda = E(\lambda)$

$(e^t p)^i \lambda \sum_{k=0}^{\infty} e^{-\lambda} \dfrac{\lambda^k}{k!}$

$\lambda = 3$

$x = 3$

$E(x^2) = \dfrac{e^{-\lambda}}{k-1+1}$

$ey \dfrac{1}{\sqrt{2\pi}6} e^{-1/2\left(\frac{e^t 0}{6}\right)^2}$

$0 < y < \infty$

$1 - e^{-\lambda x}$

$0 < y < \infty$

Subjective Type Questions: **Section 3**   (2 × 5 = 10 **marks**)

Write the answer in the same page provided for the questions 11 and 12. **Full marks** are awarded if all the steps are correct, and **partial marks** for an incorrect answer with wrong steps.

1. (a) Write the Kolmogorov axiomatic definition of probability.   (2 marks)

    (b) Let $\Omega = \Re$ and $\mathcal{F}$ be the Borel σ-field on $\Re$. For each interval $I \subseteq \Re$ with end points c and d $(c \leq d)$, let

$$P(I) = \int_c^d \frac{1}{\pi} \frac{1}{1+x^2}\, dx \ .$$

Does P define a probability on the measurable space $(\Omega, \mathcal{F})$? Justify your answer.

    (3 marks)

Answer:

(a)   if S is the sample space & E_i are the events

① $0 \leq P(E_i) \leq 1$ ② $\sum_{i=1}^{n} P(E_i) = 1$ where n is total no. of events in sample space

③   if E_i are mutually exclusive events then $\quad 1 \leq i \leq m$

$$P\left(\bigcup_{i=1}^{m} E_i\right) = \sum_{i=1}^{m} P(E_i) \quad \text{(1)}$$

true for Countable collection

(b) $P(I) = \int_c^d \frac{1}{\pi} \frac{1}{1+x^2} dx$

$\int_{-\infty}^{\infty} \frac{1}{\pi} \times \frac{1}{1+x^2} dx = \frac{2}{\pi} \int_0^{\infty} \frac{1}{1+x^2} dx$ ②

$= 1$

$P(I)_{max} = 1$ & $P(I)_{min} = 0$ at $c = d$.

$\Rightarrow \quad 0 \leq P(I) \leq 1$

$\therefore \ P(\Omega) = 1$ (for $(c,d) = (-\infty, \infty)$ for $\Omega = \Re$).

$\frac{1}{\pi}\left(\frac{1}{1+x^2}\right) = f(x)$

$f(x) \geq 0$

$\Rightarrow \int_c^d f(x)$ is max for $c = -\infty$, $d = \infty$.

$(\because P(I) > 0 \ \because f(x) > 0)$

$I_i = (c,d) \quad I_j = (a,b)$

$I_i \cap I_j = \phi \Rightarrow I_i \text{ & } I_j$ are mutually exclusive.

show for Countable union

$\Rightarrow \quad P(I_1 \cup I_2 \cup I_3 \ldots \cup I_m) = \sum_{i=1}^{m} P(I_i) \quad (\because \ I_i \cap I_j = \phi)$

$\therefore P$ satisfies Kolmogorov axiomatic def^n of probability

Hence P defines a probability on the measurable space $(\Omega, \mathcal{F})$.

12. (b) $Var(X) = 0 \rightleftharpoons E(x) = k$ k is a const. $k \in R$.

$\Rightarrow P(x=k) = 1.$

12. (a) Let X be a continuous type random variable with pdf

$$f(x) = \begin{cases} \beta + \alpha x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

If $E(X) = 3/5$, find the value of α and β.   (2 marks)

(b) Prove that, for any random variable X, $E(X^2) \ge [E(X)]^2$. When does one have equality?   (2 + 1 marks)

Answer: (a)

$\because f(x)$ is pdf

$\int_{-\infty}^{\infty} f(x) dx = 1.$ $\Rightarrow \int_0^1 f(x) dx = 1$ $\Rightarrow \int_0^1 (\beta + \alpha x^2) dx = 1$

$\beta + \dfrac{\alpha}{3} = 1 \quad - (i)$

$E(x) = \int_0^1 x(\beta + \alpha x^2) dx + \int_{-\infty}^0 x \times 0 \, dx \quad \int x \cdot (0) dx$

$= \int_0^1 \beta x + \alpha x^3 \, dx = \dfrac{\beta}{2} + \dfrac{\alpha}{4}$

$\Rightarrow \quad \dfrac{\beta}{2} + \dfrac{\alpha}{4} = \dfrac{3}{5} \quad \Rightarrow \quad 2\beta + \alpha = \dfrac{12}{5} \quad - (ii)$

$3\beta + \alpha = 3$

$2\beta + \alpha = 12/5 \qquad \Rightarrow \quad \beta = 3/5, \quad \alpha = 6/5.$ ③

(b) $E(x^2) - E(x)^2 = Var(x) = E(|x-\mu|^2)$

$Var(x) = \sum_{k=0}^{n} |x_k - \mu|^2 P(x=x_k)$ if x is discrete type

$\qquad P(x=x_k) \ge 0.$

$\qquad \& \quad |x_k - \mu|^2 \ge 0$

$\Rightarrow \quad Var(x) \ge 0$

if x is cont. type

$Var(x) = \int_{-\infty}^{\infty} f(x) |x_k - \mu|^2 dx \qquad \qquad \therefore f(x) = F f_x(x) \ge 0$

$\qquad \qquad \qquad \Rightarrow Var(x) \ge 0 \qquad \& \quad |x_k - \mu|^2 \ge 0$

$\Rightarrow \quad E(x^2) - E(x)^2 \ge 0 \qquad \Rightarrow \quad E(x^2) \ge E(x)^2.$

Equality $\Rightarrow Var(x) = 0 \Rightarrow$ mass distributed