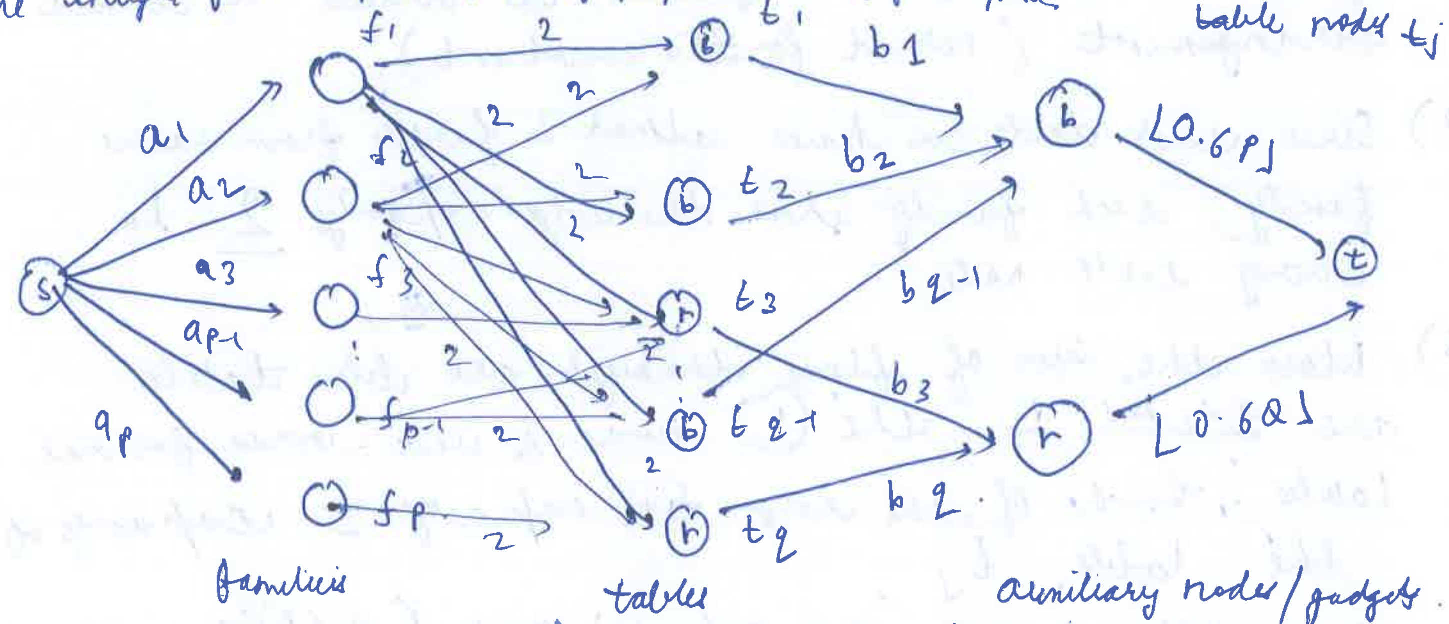
Quiz5A (COL 351)

Name KUSHAGRA GUPTA Entry No. 2021CS50592

Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that at most **two** members of the same family are at the same table. Further, each table has a capacity of how many people can sit there and is coloured either red or blue. We would like to ensure that at most 60 percent of the people are seated at a blue table, and similarly, at most 60 percent of the people are seated at a red table. Show how to formulate a seating arrangement that meets these objectives as a maximum flow problem. Assume that the dinner contingent has p families and the i^{th} family has a_i members. Also assume that there are q tables available and the j^{th} table has a seating capacity of b_j and is coloured either red or blue. You should clearly state the set of vertices and edges, the edge capacities and briefly explain why the maximum flow formulation solves the desired problem.

We model our ~~solution~~ arrangement as a solution to the integral max flow problem (with all edge capacities as integers).
Consider the following flow network, with source & dest node s & t & family nodes f_i, table nodes t_j.
The integer flow represents no. of people with 1 flow = 1 person.



Connect all families to the source node(s) and assign the family size a_i to the capacity of outedge from s.

Add edges from f_i ($\forall i \in [p]$) to t_j ($\forall j \in [q]$) with capacity 2.

Now add 2 nodes b & r and connect all blue tables to the b node and all red tables to the r node.

Add the edges with capacity $b_1, b_2 \ldots$ to their respective node (b or r).

Now consider the sum of capacities of tables which are blue $= \sum_{j \in Blue} b_j = P$. & sum of capacities of red table $= Q$.

Add edges from b & r to t with capacity $\lfloor 0.6P \rfloor$ & $\lfloor 0.6 Q \rfloor$ ($\lfloor x \rfloor$ denotes greatest int $\leq x$).

If the max flow to such a flow network has a flow $= \sum_{i \in \{1..p\}} a_i = a_1 + a_2 .. a_p$ we obtain the arrangement by taking the flows on edges from f_i to t_j

We will prove now why all constraints are satisfied iff such a flow exists.

(1) The flow through every family node f_i is atmost a_i. Also if such a flow has value a_i, all members of the family are assigned a table & hence if it is true for all families we obtain a correct arrangement (correct for this constraint).

(2) Since each table can have atmost 2 people from same family, each family has the edge capacity 2 to every table node.

(3) Now the sum of flows through all blue tables are directed to the (b) node & vice-versa for red tables. Each of the edge has capacity = capacity of the table t_j.

The ~~Each~~ edge ~~b-t~~ from b node to t has capacity $\lfloor 0.6P \rfloor$ & vice-versa from r node to t. This ensures ~~mean~~ flow through r & b cannot exceed the ~~no~~ $0.6 \times$ capacity of red / blue tables. Hence, our flow will satisfy all constraints (all red tables $\leq 60\%$ & all blue tables $\leq 60\%$ full)

(Note: we give b_j so that one ~~red~~ table may be full but not the total blue/red sum as 0.8)

The Integral flow with integer values represent the people & only 1 person contributes to 1 unit of flow.