

Started on Wednesday, 16 March 2022, 7:00 PM

State Finished

Completed on Wednesday, 16 March 2022, 7:45 PM

Time taken 45 mins

Marks 19.67/30.00

Grade 2.62 out of 4.00 (66%)

Question 1

Correct

Mark 1.00 out of 1.00

The Fourier Transform of $x(t) = t^3 e^{-at} u(t)$, $\text{Re}\{a\} > 0$ is given by

Select one:

☒ a. $\frac{6}{(a+j\omega)^4}$

☐ b. $\frac{1}{(a+j\omega)^4}$

☐ c. $\frac{3}{(a+j\omega)^4}$

☐ d. $\frac{2}{(a+j\omega)^4}$

Your answer is correct.

The correct answer is: $\frac{6}{(a+j\omega)^4}$

Question 2

Correct

Mark 1.00 out of 1.00

The CTFT of $x(t) = \frac{2a}{a^2+t^2}$ is

$a > 0$

Select one:

☐ a. $\pi e^{-a|\omega|}$

☐ b. $4\pi e^{-a|\omega|}$

☒ c. $2\pi e^{-a|\omega|}$

☐ d. $e^{-a|\omega|}$

Your answer is correct.

The correct answer is: $2\pi e^{-a|\omega|}$

Question 3

Correct

Mark 2.00 out of 2.00

Let $x[n] = (\frac{1}{2})^n u[n]$

$y[n] = x^2[n]$ and $Y(e^{j\Omega})$ be the Fourier Transform of $y[n]$

Value of $Y(e^{j0})$ is

Select one:

☒ a. $\frac{4}{3}$



☐ b. $\frac{1}{4}$

☐ c. 4

☐ d. 2

Your answer is correct.

The correct answer is: $\frac{4}{3}$

Question 4

Correct

Mark 1.00 out of 1.00

Let $x[n] = \{1, 1, 0, 5 (n = 0), 1\}$ which has the discrete-time Fourier Transform given by $X(e^{j\Omega})$.

The value of $\int_{-\pi}^{\pi} X(e^{j\Omega}) d\Omega$ is

Select one:

☐ a. 5

☒ b. 10π



☐ c. 16π

☐ d. 4

Your answer is correct.

The correct answer is: 10π

Question 5

Correct


Mark 1.00 out of 1.00

A discrete-time signal $x[n]$ has DTFT given by $X(e^{j\Omega}) = 2 - 2e^{-j2\Omega}$.

Given that $y[n] = n \times x[n - 2]$ and its DTFT is denoted by $Y(e^{j\Omega})$

The value of $Y(e^{j\Omega})$ will be

Select one:

- ☐ a. $e^{-j2\Omega} - 8e^{-j4\Omega}$
- ☒ b. $4e^{-j2\Omega} - 8e^{-j4\Omega}$
-  c. $4e^{-j2\Omega} - 4e^{-j4\Omega}$
- ☐ d. $2e^{-j2\Omega} - 4e^{-j4\Omega}$

Your answer is correct.

The correct answer is: $4e^{-j2\Omega} - 8e^{-j4\Omega}$

Question 6

Correct


Mark 1.00 out of 1.00

Consider a discrete-time signal $x[n]$ given by

$$x[n] = \{1, 0, -1, 3, -2 \text{ (} n = 0), 1, -1, -2, 3, 2, 1\}$$

Value of $\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega$ will be

Select one:

- ☐ a. 120π
- ☐ b. 100π
- ☒ c. 70π
-  d. 20π

Your answer is correct.

The correct answer is: 70π

Question 7

Correct

Mark 1.00 out of 1.00

A signal is represented as: $x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$. The Fourier transform of the convolved signal

$y(t) = x(2t) * x(\frac{t}{2})$ will be:

Select one:

- ☐ a. $\frac{4}{\omega^2} \sin(\omega)$
- ☐ b. $\frac{4}{\omega^2} \sin(\frac{\omega}{2})$
- ☒ c. $\frac{4}{\omega^2} \sin(\frac{\omega}{2}) \sin(2\omega)$
- ☐ d. $\frac{4}{\omega^2} \sin(2\omega) \sin(\omega)$

Your answer is correct.

$$x(t) = \text{rect}(\frac{t}{2})$$

$$\begin{aligned} Y(\omega) &= \mathcal{F}\{x(2t)\} \mathcal{F}\{x(t/2)\} \\ &= \frac{\sin(\omega/2)}{\omega/2} \frac{2\sin(2\omega)}{\omega} \\ &= \frac{4}{\omega^2} \sin(\omega/2) \sin(2\omega) \end{aligned}$$

The correct answer is: $\frac{4}{\omega^2} \sin(\frac{\omega}{2}) \sin(2\omega)$

Question 8

Correct

Mark 1.00 out of 1.00

Let $x(t) = \text{rect}(t - \frac{1}{2})$ where $\text{rect}(y) = \begin{cases} 1 & -\frac{1}{2} \leq y \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$.

Let $\text{sinc}(y)$ be defined as $\text{sinc}(y) = \frac{\sin(\pi y)}{\pi y}$, then the Fourier transform of $x(t) + x(-t)$ will be:

Select one:

☒ a. $2\text{sinc}(\frac{\omega}{2\pi}) \cos(\frac{\omega}{2})$



☐ b. $2\text{sinc}(\frac{\omega}{2\pi})$

☐ c. $\text{sinc}(\frac{\omega}{2\pi})$

☐ d. $\text{sinc}(\frac{\omega}{2\pi}) \sin(\frac{\omega}{2})$

Your answer is correct.

$$x(t) = \text{rect}(t - \frac{1}{2}) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \mathcal{F}\{x(t)\} &= \int_0^1 e^{-j\omega t} dt \\ &= -\frac{1}{j\omega} [e^{-j\omega} - 1] \\ &= \frac{1}{j\omega} \left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{e^{j\omega/2}} \right] \\ &= \frac{\sin(\omega/2)}{\omega/2} e^{-j\omega/2} \end{aligned}$$

$$\mathcal{F}\{x(-t)\} = X(-\omega) = \frac{\sin(\omega/2)}{\omega/2} e^{j\omega/2}$$

$$\begin{aligned} \mathcal{F}\{x(t) + x(-t)\} &= \frac{\sin(\omega/2)}{\omega/2} (e^{j\omega/2} + e^{-j\omega/2}) \\ &= 2\text{sinc}(\omega/2\pi) \cos(\omega/2) \end{aligned}$$

The correct answer is: $2\text{sinc}(\frac{\omega}{2\pi}) \cos(\frac{\omega}{2})$

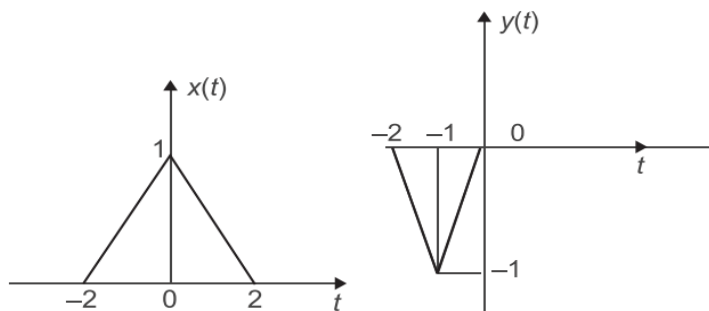
Question 9

Incorrect

Mark 0.00 out of
1.00

Two signals $x(t)$ and $y(t)$ are related as shown in the figure having Fourier transform as $X(\omega)$ and $Y(\omega)$ respectively.

Then $Y(\omega)$ in terms of $X(\omega)$ is:



Select one:

- ☐ a. $-\frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j\omega}$
- ☒ b. $-X\left(\frac{\omega}{2}\right)e^{j\frac{\omega}{2}}$
- ☐ c. $-X\left(\frac{\omega}{2}\right)e^{-j\frac{\omega}{2}}$
- ☐ d. $-\frac{1}{2}X\left(\frac{\omega}{2}\right)e^{j\omega}$

Your answer is incorrect.

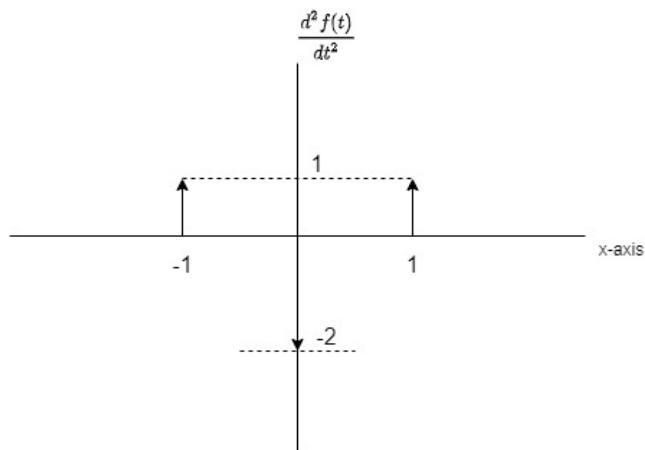
The correct answer is: $-\frac{1}{2}X\left(\frac{\omega}{2}\right)e^{j\omega}$

Question 10

Correct

Mark 1.00 out of 1.00

Shown in the figure is the second derivative of a function $f(t)$. The Fourier transform of $f(t)$ is:



Select one:

- ☐ a. $\frac{2(1+\cos(\omega))}{\omega^2}$
- ☐ b. $1 + \cos(2\omega)$
- ☐ c. $1 + \sin(\omega)$
- ☒ d. $\frac{2(1-\cos(\omega))}{\omega^2}$



Your answer is correct.

$$\frac{d^2 f(t)}{dt^2} = \delta(t+1) - 2\delta(t) + \delta(t-1)$$

$$\mathcal{F}\left(\frac{d^2 f(t)}{dt^2}\right) = e^{-j\omega} - 2 + e^{j\omega}$$

$$(j\omega^2)\mathcal{F}(f(t)) = 2\cos(\omega) - 2$$

$$\mathcal{F}(f(t)) = \frac{2(1 - \cos\omega)}{\omega^2}$$

The correct answer is: $\frac{2(1-\cos(\omega))}{\omega^2}$

Question 11

Correct

Mark 1.00 out of 1.00

The DTFT of a signal $g[n] = \{p(n=0), q, r, s\}$ is $G(\Omega)$. The inverse DTFT of $G(\Omega - \pi)$ is:

Select one:

- ☐ a. $\{p(n=0), q, r, s\}$
- ☐ b. $\{-p(n=0), q, -r, s\}$
- ☐ c. $\{-p(n=0), -q, -r, -s\}$
- ☒ d. $\{p(n=0), -q, r, -s\}$



Your answer is correct.

$$\mathcal{F}\{G(\Omega - \pi)\} = e^{j\pi n} x(n)$$

The correct answer is: $\{p(n=0), -q, r, -s\}$

Question 12

Correct

Mark 1.00 out of 1.00

For a signal $x(t)$ the Fourier transform is $X(f)$. Then the inverse Fourier transform of $X(3f + 2)$ is given by

Select one:

- ☐ a. $\frac{1}{2}x(\frac{t}{2})\exp(j3\pi t)$
- ☒ b. $\frac{1}{3}x(\frac{t}{3})\exp(-j4\pi t/3)$
- ☐ c. $x(3t + 2)$
- ☐ d. $3x(3t)\exp(-j4\pi t)$



Your answer is correct.

$$X[3(t + \frac{2}{3})] = \frac{1}{3}x(\frac{t}{3})\exp(-j4\pi t/3).$$

Applying scaling and shifting property.

The correct answer is: $\frac{1}{3}x(\frac{t}{3})\exp(-j4\pi t/3)$

Question 13

Incorrect

Mark 0.00 out of 1.00

The Fourier transform of a signal $h(t)$ is $H(\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega}$. The values of $h(0)$ is

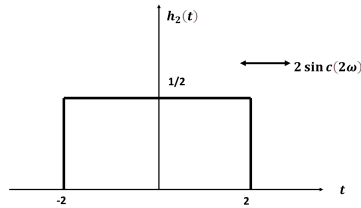
Select one:

- ☐ a. 1/4
- ☒ b. 1/2 ✗
- ☐ c. 1
- ☐ d. 2

Your answer is incorrect.

$$H(\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega} \dots(1)$$

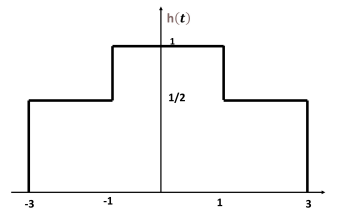
$$H(\omega) = 2\cos\omega \left(\frac{2\sin 2\omega}{2\omega} \right) = H_1(\omega)H_2(\omega) \dots(2)$$



where, $H_1(\omega) = 2\cos\omega = \exp(j\omega) + \exp(-j\omega)$

$$H_2(\omega) = \frac{2\sin 2\omega}{2\omega} = 2\text{sinc}(2\omega)$$

So, $H(\omega) = \exp(j\omega)H_2(\omega) + \exp(-j\omega)H_2(\omega)$



Therefore, $h(t) = h_2(t+1) + h_2(t-1)$.

The correct answer is: 1

Question 14

Correct

Mark 1.00 out of 1.00

Let $g(t) = \exp(-\pi t^2)$, and $h(t)$ is a filter matched to $g(t)$, i.e. $h(t) = g(-t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is

Select one:

- ☐ a. $\exp(-\pi|f|)$
- ☐ b. $\exp(-\pi f^2/2)$
- ☒ c. $\exp(-2\pi f^2)$ ✓
- ☐ d. $\exp(-\pi f^2)$

Your answer is correct.

$$\text{Fourier transform of the output} = G(f)H(f) = G(f)G(-f) = \exp(-2\pi f^2)$$

The correct answer is: $\exp(-2\pi f^2)$

Question 15

Incorrect

Mark 0.00 out of 2.00

If a signal $x[n] = \{2 \ (n = 0), 1, 0, -1, 3\}$ is applied at input to a system whose impulse response is $h[n]$, where $h[n] = \{1 \ (n = 0), -3, 2\}$. The output of the system is denoted by $y[n]$ with DTFT $Y(e^{j\Omega})$. The value of $Y(e^{j0})$ is

Answer: ❌

The correct answer is: 0

Question 16

Incorrect

Mark 0.00 out of 1.00

Consider an LTI system with magnitude response as:

$$H(f) = \begin{cases} 1 - \frac{|f|}{10} & |f| \leq 10 \\ 0 & |f| > 10 \end{cases}$$

and phase response: $\arg(H(f)) = -3f$

If the input of the system is:

$$x(t) = 8\cos(5\pi t + \pi/4) + 10\sin(20\pi t + \pi/8) + 14\cos(40\pi t + \pi/16).$$

Then the average power of the output signal y(t) is:

Answer: ❌

For 2.5Hz frequency: $|H(f)| = \frac{3}{4}$

For 10Hz frequency: $|H(f)| = 0$

For 20Hz frequency: $|H(f)| = 0$

Therefore output amplitude $= 8|H(f)|_{f=2.5Hz} = 6$

Average power $= \frac{6^2}{2} = 18$

The correct answer is: 18

Question 17

Correct

Mark 1.00 out of 1.00

The Fourier transform of a discrete time signal $x[n]$ is given as $X(e^{j\omega}) = \cos^2(\omega) + \sin^2(3\omega)$. The value of $4x[0] + 8x[2]$ is

Answer: ✔️

$$\begin{aligned} X(e^{j\omega}) &= \cos^2(\omega) + \sin^2(3\omega) \\ &= 1 + \sin(4\omega)\sin(2\omega) \\ &= 1 + \left(\frac{e^{j4\omega} - e^{-j4\omega}}{2j} \right) \left(\frac{e^{j2\omega} - e^{-j2\omega}}{2j} \right) \\ &= -\frac{1}{4}e^{j6\omega} + \frac{1}{4}e^{j2\omega} + 1 + \frac{1}{4}e^{-j2\omega} - \frac{1}{4}e^{-j6\omega} \end{aligned}$$

Comparing with standard Fourier Transform equation: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

$$x[0] = 1, \quad x[2] = \frac{1}{4}$$

$$4x[0] + 8x[2] = 6$$

The correct answer is: 6

Question 18

Correct

Mark 1.00 out of 1.00

Consider the signal $x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2]$. If $X(e^{j\omega})$ is the discrete-time Fourier transform of $x[n]$, then $\frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \sin^2(2\omega) d\omega$ is equal to

Answer: 8



$$\begin{aligned} & \frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \frac{1 - \cos 4\omega}{2} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cos 4\omega d\omega \end{aligned}$$

First term $= x[0] = 8$

$$\text{Second term} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \frac{e^{j4\omega} + e^{-j4\omega}}{2} d\omega$$

$$= \frac{1}{2} (x[4] + x[-4]) = \frac{1}{2} (0 + 0) = 0$$

$$\frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \sin^2(2\omega) d\omega = 8$$

The correct answer is: 8

Question 19

Incorrect

Mark 0.00 out of 1.00

A Fourier transform pair is given by

$$\left(\frac{2}{3}\right)^n u[n+3] \rightarrow \frac{A \exp(j6\pi f)}{1 - \frac{2}{3} \exp(-j2\pi f)}$$

where, $u[n]$ denotes the unit step sequence, the value of A is

Answer: 2



The correct answer is: 3.375

Question 20

Incorrect

Mark 0.00 out of 1.00

Consider the function $g(t) = e^{-t} \sin(2\pi t) u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is

Answer: 2



The correct answer is: 0.155

Question 21

Incorrect

Mark 0.00 out of 1.00

A continuous time signal $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$, where t is in seconds, is the input to a linear time invariant filter with the impulse response

$$h(t) = \begin{cases} \frac{2\sin(300\pi t)}{\pi t}, & t \neq 0 \\ 600, & t = 0 \end{cases}$$

Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is

Answer: 2



The correct answer is: 8

Question 22

Incorrect

Mark 0.00 out of
1.00

The value of the integral $\frac{1}{2\pi} \int_{-\infty}^{\infty} \text{sinc}^2(5t) dt$ is ❌ where, $\text{sinc}(t)$ denotes the sinc function

defined as

$$\text{sinc}(t) = \frac{\sin(t)}{t}$$

Let $x(t) = \text{sinc}(5t)$

Using Parseval

$$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$\text{Therefore, } X(\omega) = \frac{2\pi}{10} \text{rect}\left(\frac{\omega}{10}\right)$$

where, $\text{rect}\left(\frac{\omega}{10}\right) = 1$ for $-5 \leq \omega \leq 5$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} x^2(t) dt = \left(\frac{1}{2\pi}\right)^2 \frac{4\pi^2}{100} \int_{-5}^5 1 d\omega = 0.1$$

The correct answer is: 0.1

Question 23

Correct

Mark 2.00 out of
2.00

Consider a continuous-time real signal $x(t)$ with Fourier Transform $X(\omega)$

Which of the following is/ are true

Select one or more:

- ☐ a. $X(\omega) = -X^*(-\omega)$
- ☒ b. $X(\omega) = X^*(-\omega)$
- ☒ c. $\text{Re}\{X(\omega)\} = \text{Re}\{X(-\omega)\}$
- ☒ d. $\text{Im}\{X(\omega)\} = \text{Im}\{X(-\omega)\}$

Your answer is correct.

The correct answers are: $X(\omega) = X^*(-\omega)$

, $\text{Re}\{X(\omega)\} = \text{Re}\{X(-\omega)\}$

Question 24

Correct

Mark 2.00 out of 2.00

Let $x(t) = \text{sgn}(t)$, the signum function. Which of the following is/ are true regarding its Fourier Transform $X(\omega)$

Select one or more:

☐ a. $X(\omega)$ is even

☐ b. $X(\omega) = \frac{-1}{\pi\omega}$

☒ c. $X(\omega) = \frac{-2}{\omega}j$



☐ d. $\text{Im}\{X(\omega)\} = 0$

☒ e. $\text{Re}\{X(\omega)\} = 0$



Your answer is correct.

The correct answers are: $X(\omega) = \frac{-2}{\omega}j$

, $\text{Re}\{X(\omega)\} = 0$

Question 25

Partially correct

Mark 0.67 out of 2.00

Consider an imaginary signal $x(t)$ such that $x(t) = -x(-t)$. Out of the following options, which are true regarding its Fourier Transform $X(\omega)$

Select one or more:

☒ a. $X(\omega) = -X(-\omega)$



☐ b. $X(\omega) = X^*(\omega)$

☐ c. $X(\omega) = -X^*(-\omega)$

☒ d. $X(\omega) = -X^*(\omega)$



☒ e. $X(\omega) = X^*(-\omega)$



☐ f. $X(\omega) = X(-\omega)$

Your answer is partially correct.

You have correctly selected one option.

The correct answers are: $X(\omega) = -X(-\omega)$

, $X(\omega) = X^*(\omega)$

, $X(\omega) = -X^*(-\omega)$