ELL205: Problem Sheet 3

1. Initial conditions being zero

a) Show that if a system is either additive or homogeneous, it has the property that if the input is identically zero, then the output is also identically zero.

b) Determine a system (either in continuous or discrete time) that is neither additive nor homogeneous but which has a zero output if the input is identically zero.

c) From part (a), can you conclude that if the input to a linear system is zero between times $t_1$and t_2 in continuous time or between times $n_1$and n_1 in discrete time, then its output must also be zero between these same times? Explain your answer.

2. Periodicity of a time-invariant system

a) Consider a time-invariant system with input x(t) and output y(t). Show that if x(t) is periodic with period T, then so is y(t). Show that the analogous result also holds in discrete time.

b) Give an example of a time-invariant system and a non-periodic input signal x(t) such that the corresponding output y(t) is periodic.

3. Condition of initial rest

a) Show that causality for a continuous-time linear system is equivalent to the following statement:

   For any time t_0 and any input x(t) such that x(t) = 0 for $t<t_0$, the corresponding output y(t) must also be zero for $t <t_0$. The analogous statement can be made for a discrete-time linear system.

b) Find a nonlinear system that satisfies the foregoing condition but is not causal.

c) Find a nonlinear system that is causal but does not satisfy the condition.

d) Show that invertibility for a discrete-time linear system is equivalent to the following statement:

   The only input that produces y[n] = 0 for all n is x[n] = 0 for all n. The analogous statement is also true for a continuous-time linear system.

e) Find a nonlinear system that satisfies the condition of part (d) but is not invertible.

4. Correlation

Correlation, a function which gives a measure of similarity of two signals when shifted by a certain (t) amount, for two signals, x(t) and y(t) is defined as

$$\varphi_{xh}(t) = \int_{-\infty}^{\infty} x(t+\tau)h(\tau)d\tau$$

It is often important in practice to compute the correlation function $\varphi_{hx}(t)$, where h(t) is a fixed given signal, but where x(t) may be any of a wide variety of signals. In this case, what is done is to design a system S with input x(t) and output φ_{hx} (t).

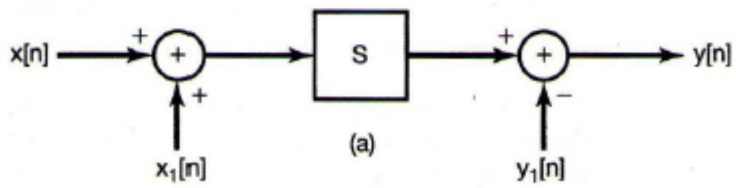
a) Is S linear? Is S time invariant? Is S causal? Explain your answers.

b) Do any of your answers to part (a) change if we take as the output $\varphi_{xh}(t)$ rather than $\varphi_{hx}(t)$?

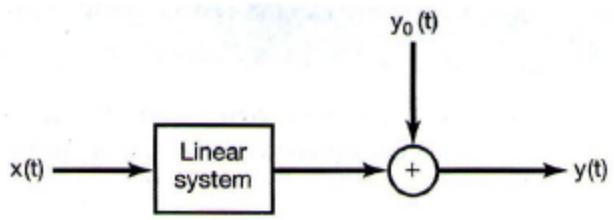
5. Incrementally linear systems

Let S denote an incrementally linear system, and let x_1[n] be an arbitrary input signal to S with corresponding output y_1[n]. Consider the system illustrated in figure below.

a) Show that this system is linear and that, in fact, the overall input-output relationship between x [n] and y[n] does not depend on the particular choice of x_1[n].



b) Use the result of part (a) to show that S can be represented in the form shown in figure below



c) Which of the following systems are incrementally linear? Justify your answers and if a system is incrementally linear, identify the linear system Land the zero-input response y_0[n] or y_0(t) for the representation of the system as shown in the figure above

(i) y[n] = n + x[n] + 2x[n + 4]

(ii) $y[n] = \begin{cases} \frac{n}{2}, & n \text{ even} \\ \frac{n-1}{2} + \sum_{k=-\infty}^{\frac{n-1}{2}} x[k] & , n \text{ odd} \end{cases}$

6. System properties

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless
(2) Time invariant
(3) Linear
(4) Causal
(5) Stable

Determine which of these properties hold and which do not hold. Justify your answers, symbols have the usual meaning.

a) $y(t) = x(t - 2) + x(2 - t)$

b) $y(t) = [\cos(3t)]x(t)$

c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

d) $y[n] = \sum_{k=-\infty}^{3n} x[k]$

e) $y(t)=\begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t > 0 \end{cases}$

f) $y(t)=\begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) > 0 \end{cases}$

g) $y(t) = x(t/3)$

h) $y(t) = \frac{d\,x(t)}{dt}$

i) $y(t) = t^2x(t-1)$

j) $y[n] = x^2[n- 2]$

k) $y[n] = x[n + 1] - x[n- 1]$

l) $y[n] = \text{Odd}\{x(t)\}$

m) $y(t) = x(\sin(t))$

n) $y[n] = \sum_{k=n-n_o}^{n+n_o} x[k]$ where n_o is a finite positive integer

7. Convolution of continuous signals

Determine the convolution of the following two signals.

a) u(t) and u(t)

b) x(t) and $\delta(t - \tau)$

c) $e^{\alpha t}u(t)$ and u(t)