

# Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

## Chapter 4: Substitution and Income Effects

# Outline

- Income Changes
- Price Changes
- Income and Substitution Effects
- Putting Income and Substitution Effects Together
- Appendix A. Not All Goods Can Be Inferior
- Appendix B. Alternative Representation of Income and Substitution Effects

# Income Changes

# Income Changes

- We analyze how the demand for a good (optimal consumption bundle) changes as the consumer's income increases.
- *Four ways* to measure a **change in demand**:
  1. Using the Derivative of Demand.
  2. Using Income Elasticity.
  3. Using the Income-Consumption Curve.
  4. Using the Engel Curve.

# Income Changes

## 1. Using the Derivative of Demand.

- Formally,  $x(p_x, p_y, I)$  represents consumer demand for good  $x$ .
- Normal goods.** A consumer's demand for good  $x$  is normal if

$$\frac{\partial x(p_x, p_y, I)}{\partial I} > 0.$$

- She demands more units of good  $x$  as her income increases.  
*Example:* holiday packages.

- Inferior goods.** A consumer's demand for good  $x$  is inferior if

$$\frac{\partial x(p_x, p_y, I)}{\partial I} < 0.$$

- She cut her consumption as soon as she can afford to do so.  
*Example:* food staples.

# Income Changes

- *Example 4.1: Increasing income in a Cobb-Douglas utility function.*

- Consider an individual with  $u(x, y) = xy$ , who faces prices  $p_x, p_y$ , and income  $I$ .
- Her optimal consumption for good  $x$  (i.e., her demand) is

$$x = \frac{I}{2p_x}.$$

- This demand is increasing in income because  $\frac{\partial x}{\partial I} = \frac{1}{2p_x} > 0$ .
- Hence good  $x$  is normal in consumption.
- Similarly, the demand of good  $y$ ,  $y = \frac{I}{2p_y}$ , is also increasing in income.

# Income Changes

## 2. Using Income Elasticity.

- We can represent the relationship between income and demand by using the formula of income elasticity,

$$\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \underbrace{\frac{I}{x(p_x, p_y, I)}}_{> 0},$$

which measures the % change in quantity demanded per 1% change in income.

- $\varepsilon_{x,I} > 0$  when the good is *normal*,  $\frac{\partial x(p_x, p_y, I)}{\partial I} > 0$ .
- $\varepsilon_{x,I} < 0$  when the good is *inferior*,  $\frac{\partial x(p_x, p_y, I)}{\partial I} < 0$ .

# Income Changes

## 2. Using Income Elasticity (cont.).

- A good with  $\varepsilon_{x,I} > 1$ , is regarded as *luxury*.
  - An 1% increase in income produces a more-than-proportional increase in the quantity demanded of the good.
  - *Example*: electronic gadgets, yachts.
- A good with  $0 < \varepsilon_{x,I} < 1$ , is regarded as *necessity*.
  - A 1% increase in income yields a less-than-proportional increase in demand.
  - *Example*: water, electricity.
- When  $\varepsilon_{x,I} = 0$ , the consumer purchases the same amount of the good regardless of her income.



# Income Changes

## 2. Using Income Elasticity (cont.).

- *Summary.* Types of goods according to their income elasticity.

Income Elasticity, $\varepsilon_{x,I}$	Type of Good	Example
$\varepsilon_{x,I} < 0$	Inferior	Canned food
$0 < \varepsilon_{x,I} < 1$	Necessity	Water
$\varepsilon_{x,I} > 1$	Luxury	Yachts

Table 4.1

# Income Changes

- *Example 4.2: Finding income elasticity in the Cobb-Douglas scenario.*

- From example 4.1, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ , and  $\frac{\partial x}{\partial I} = \frac{1}{2p_x}$ .
- We can evaluate the income elasticity of good  $x$  as

$$\begin{aligned}\varepsilon_{x,I} &= \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)} \\ &= \frac{1}{2p_x} \frac{I}{\frac{I}{2p_x}} = \frac{1}{2p_x} 2p_x = 1.\end{aligned}$$

- The good is normal ( $\varepsilon_{x,I} > 0$ ), but it is neither a luxury (which requires  $\varepsilon_{x,I} > 1$ ) nor a necessity (which needs  $\varepsilon_{x,I} < 1$ ).

# Income Changes

## 3. Using the Income-Consumption Curve.

- Depict the optimal consumption bundle at initial income  $I_1$ , Bundle  $A$  (where  $IC_1$  is tangent to  $BL_1$ )
- When income increases, budget line shifts to  $BL_2$ , Bundle  $B$  is optimal.
- Income increases again producing  $BL_3$ , Bundle  $C$  is optimal.
- The “income-consumption curve” yields after connecting optimal consumption bundles.

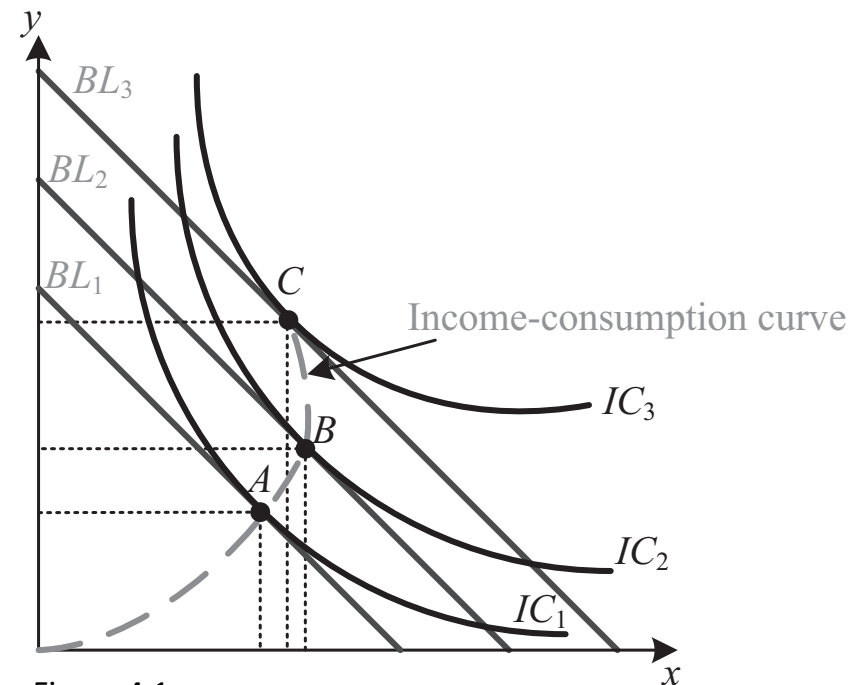


Figure 4.1

# Income Changes

## 3. Using the Income-Consumption Curve (cont.).

- When the slope of the **income-consumption curve** is:
  - Positive* (segment  $A - B$ ), the consumer increases her purchases of both  $x$  and  $y$  → normal goods.
  - Negative* (segment  $B - C$ ), she decreases her purchases of  $x$  but increases purchases of  $y$  → one of the goods must be inferior.

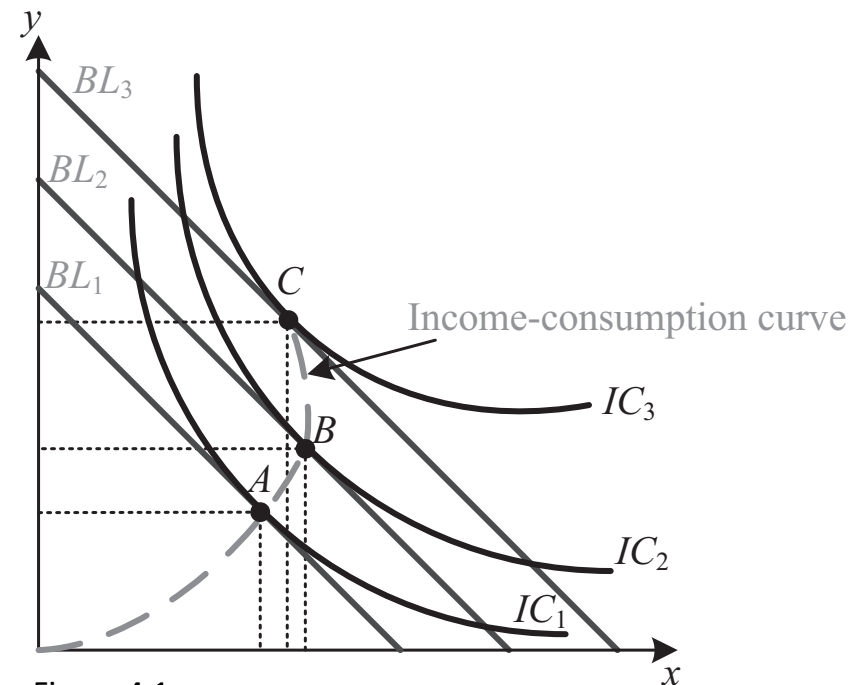


Figure 4.1

# Income Changes

- *Example 4.3: Finding income-consumption curves.*

- From example 4.1, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ , and the demand of good  $y$  is  $y = \frac{I}{2p_y}$ .
- The ratio of these demands is

$$\frac{y}{x} = \frac{\frac{1}{2p_y}}{\frac{1}{2p_x}} = \frac{p_x}{p_y}.$$

which is the slope of the ray connecting the origin  $(0,0)$  with any optimal consumption bundle.

- *Example:* When  $p_x = \$4$  and  $p_y = \$2$ , this ratio is  $\frac{y}{x} = \frac{4}{2} = 2$ .
  - The optimal consumption of goods  $y$  and  $x$  maintain a two-to-one relationship. Graphically, the income-consumption curve is a straight line.

# Income Changes

## 4. Using the Engel Curve.

- The **Engel curve** represents how income affects the demand of a good by plotting the demand for the good on the vertical axis, and the income on the horizontal axis.

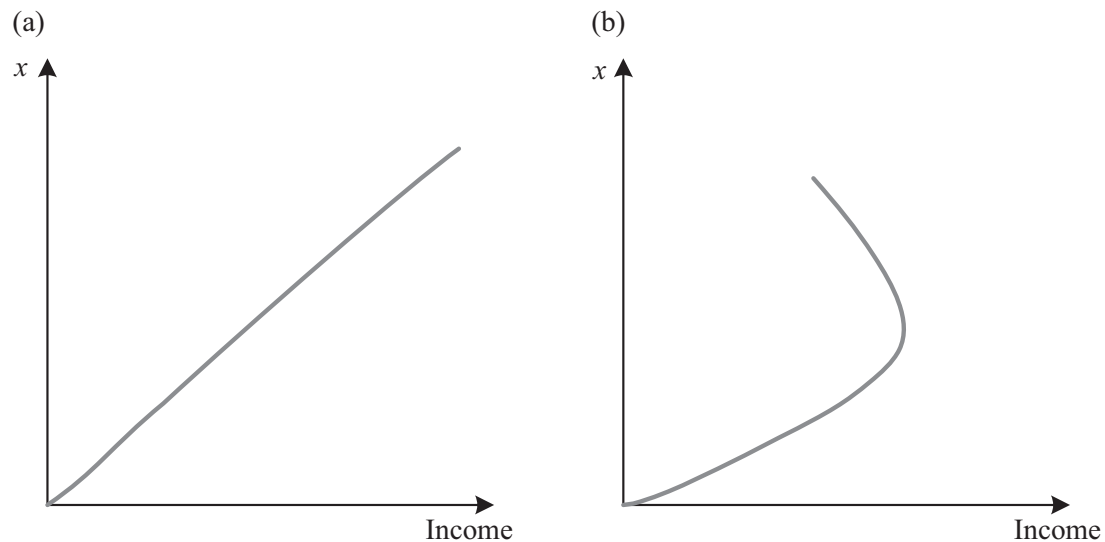


Figure 4.2

# Income Changes

## 4. Using the Engel Curve (cont.).

- Figure 4.2a depicts a positively sloped Engel curve, which implies the good is normal.
  - The number of units purchased increases with income.
  - *Example:* Products such as real state.

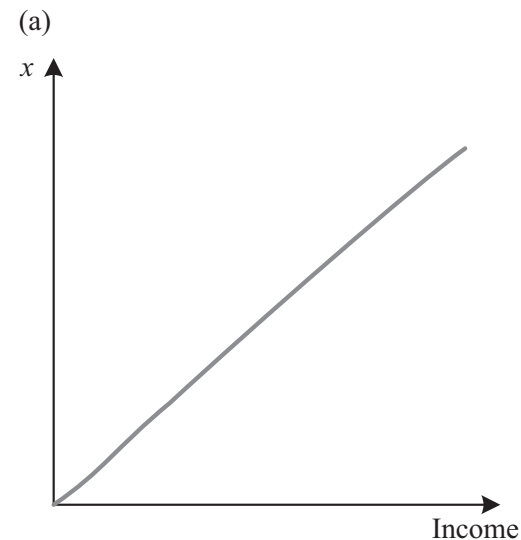


Figure 4.2a

# Income Changes

## 4. Using the Engel Curve (cont.).

- Figure 4.2b depicts an Engel curve that has a positive slope for low-income levels, but eventually becomes negatively sloped.
  - The good is normal when the individual is not very rich.
  - She starts regarding the good as inferior once she is sufficiently rich.
  - Example:* Canned food or public transportation.

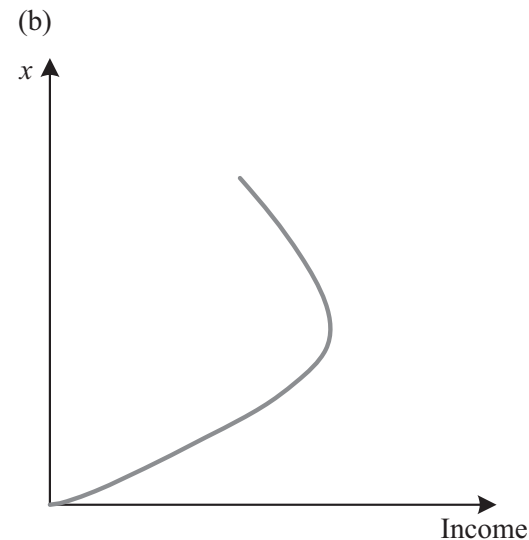


Figure 4.2b



# Income Changes

- *Example 4.3: Finding Engel curves.*

- From example 4.1, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ .
- Solving for  $I$ , we obtain an Engel curve of

$$I = (2p_x)x.$$

- This Engel curve originates at zero, and has a slope of  $2p_x$  (e.g., a slope of 6 if  $p_x = \$3$ ).
- This slope is positive and constant in  $x$ , indicating the consumer regards good  $x$  as normal (demand increases in income) for all income levels.

# Income Changes

- Remark – Not All Goods Can be Inferior.

- Figure 4.3 depicts an individual facing income  $I_1$  at budget line  $BL_1$ .
- When her income increases to  $I_2$ , budget line shifts to  $BL_2$ .

*Which bundle  $B$  does the consumer select?*

- If  $B$  lies in region  $a$ , she increases consumption of  $x$  and  $y$ .
- If  $B$  lies in region  $b$ , she purchase more of  $y$  (normal) but few of  $x$  (inferior).
- If  $B$  lies in region  $c$ , she buys few of  $y$  (inferior) but more of  $x$  (normal).

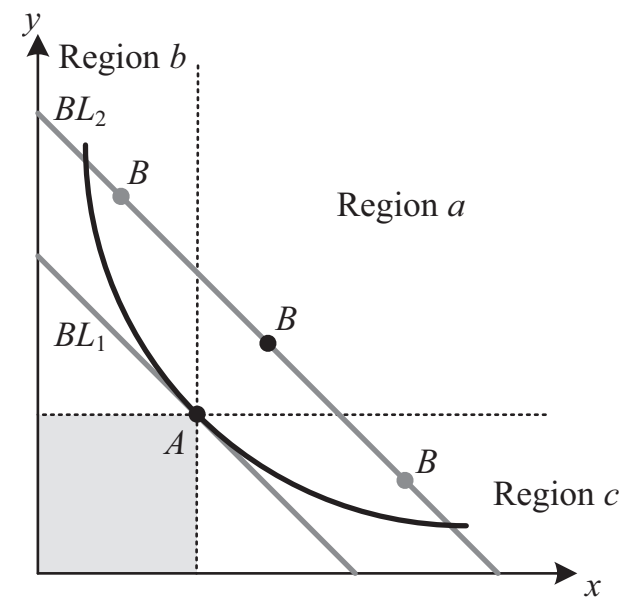


Figure 4.3

- Hence, either both goods are normal, or only one of them is inferior.

# Prices Changes

# Price Changes

- We analyze how demand changes as the price of one good increases.
- *Three ways* to measure a **change in price**:
  1. Using the Derivative of Demand.
  2. Using Income Elasticity.
  3. Using the Income-Consumption Curve.

# Price Changes

## 1. Using the Derivative of Demand.

- Formally,  $x(p_x, p_y, I)$  represent a consumer demand for good  $x$ .
- Her demand curve for the good is negatively sloped if

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} < 0.$$

- She purchase fewer units as the good becomes more expensive, keeping her income and price of all other goods constant.
- Her demand curve is positively sloped if

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} > 0.$$

- Quantity demanded and price go in the same direction. This types of goods are referred as “Giffen goods.”

# Price Changes

- *Example 4.5: Demand and price changes.*

- From example 4.1, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ .
- If the price of good  $x$  increases by a small amount, the consumer's purchases respond as follows:

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} = -\frac{I}{2p_x^2} < 0,$$

given that  $p_x, I > 0$ .

- The demand function  $x = \frac{I}{2p_x}$  decreases in price .
- Graphically, this demand function has a negative slope.

# Price Changes

## 2. Using the Price-Elasticity of Demand.

- We can represent the relationship between price of good  $x$  and its demand by using the formula of price-elasticity,

$$\varepsilon_{x,p_x} = \frac{\partial x(p_x, p_y, I)}{\partial p_x} \underbrace{\frac{p_x}{x(p_x, p_y, I)}}_{> 0}.$$

- If we increase price  $p_x$  by 1%, quantity demanded changes by  $\varepsilon_{x,p_x}$  %.
- For most goods, the demand function has a negative slope, entailing that the price-elasticity must be also negative.

# Price Changes

- *Example 4.6: Price elasticity and demand.*

- From example 4.1, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ .
- Using the formula of price elasticity,

$$\begin{aligned}\varepsilon_{x,p_x} &= \frac{\partial x(p_x, p_y, I)}{\partial p_x} \frac{p_x}{x(p_x, p_y, I)} \\ &= -\frac{I}{2p_x^2} \frac{p_x}{\frac{I}{2p_x}} = -1.\end{aligned}$$

- A 1% increase in price  $p_x$  produces a proportional reduction in its own demand (i.e., purchases of good  $x$  decrease by exactly 1%).



# Price Changes

## 2. Using the Price-Elasticity of Demand (cont).

- The “cross-price elasticity” of demand for good  $y$  to  $p_x$  is,

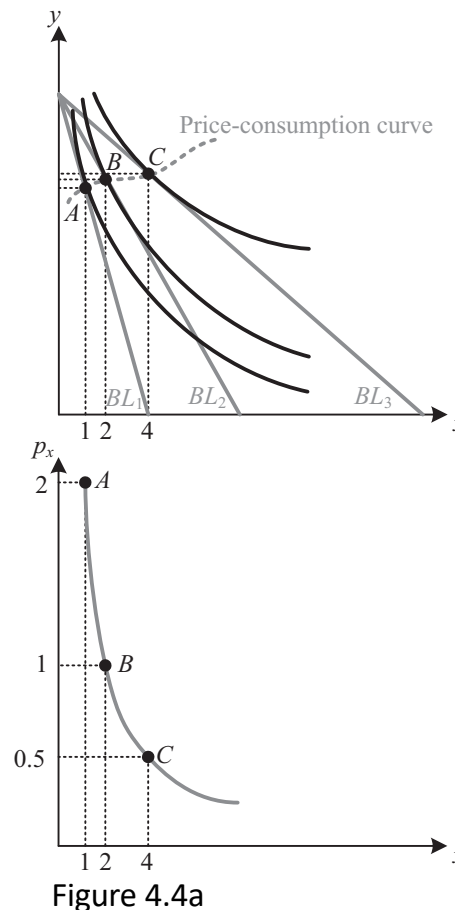
$$\varepsilon_{y,p_x} = \frac{\partial y(p_x, p_y, I)}{\partial p_x} \frac{p_x}{y(p_x, p_y, I)}.$$

- If we increase the price of good  $x$  by 1%, the quantity demanded of good  $y$  changes by  $\varepsilon_{y,p_x}$  %.
- In example 4.1, the demand for good  $y$  is  $y = \frac{I}{2p_y}$ , implying the demand of good  $y$  is independent of  $p_x$ .
- Therefore,  $\frac{\partial y(p_x, p_y, I)}{\partial p_x} = 0$ , entailing  $\varepsilon_{y,p_x} = 0$ . A 1% increase in the price of good  $x$  does not affect the demand of good  $y$ .

# Price Changes

## 3. Using the Price-Consumption Curve.

- Figure 4.4a illustrates a decrease in the price of good  $x$ , from  $p_x = \$2$  in  $BL_1$  to  $p_x = \$1$  in  $BL_2$ , and to  $p_x = \$0.5$  in  $BL_3$ .
- This figure also depicts the optimal consumption bundles the consumer selects at each price.
- The graph connects optimal bundles  $A - C$  with a curve, which is referred to as “**price-consumption curve**,” which has a positive slope at all levels of price  $p_x$ .
  - *Example:* Good  $x$  is housing.



# Price Changes

## 3. Using the Price-Consumption Curve (cont.).

- Figure 4.4b illustrates a situation in which the individual also increases her consumption of  $x$ , the good that becomes cheaper.
- For good  $y$ ,
  - she decreases her purchases when moving from bundle  $A$  to  $B$  (when  $p_x$  decreases from \$2 to \$1);
  - she increases her purchases afterwards (when  $p_x$  further decrease to \$0.5).

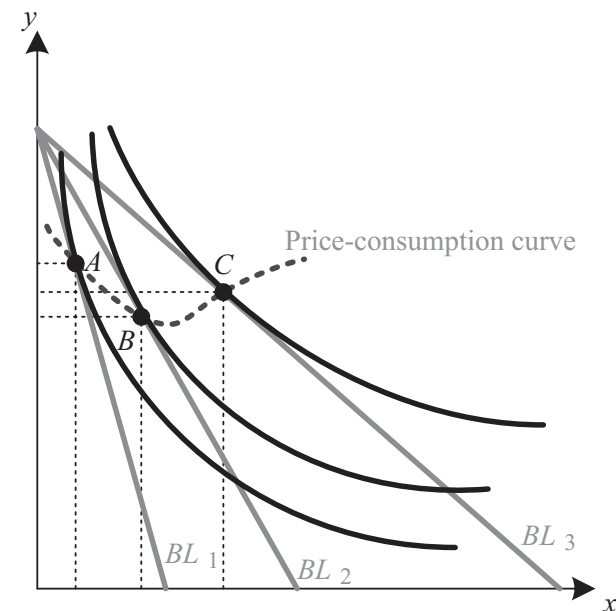


Figure 4.4b

# Price Changes

- *Example 4.7: Finding price-consumption curves.*

- From example 4.3, the demand for good  $x$  is  $x = \frac{I}{2p_x}$ , and for good  $y$  is  $y = \frac{I}{2p_y}$ .
- The ratio of these demands is

$$\frac{y}{x} = \frac{\frac{I}{2p_y}}{\frac{I}{2p_x}} = \frac{p_x}{p_y},$$

which gives the slope of the ray connecting the origin  $(0,0)$  with any optimal consumption bundle.

- An increase in  $p_x$ , increases the value of the ratio. The consumer moves to optimal bundles with more  $y$  and less  $x$ .
- An increase in  $p_y$ , decreases the value of the ratio. The consumer purchases more of  $x$  but less of  $y$ .

# Income and Substitution Effects

# Income and Substitution Effects

- An increase in the quantity demanded after a price decrease produces 2 simultaneous effects:
  - Income effect.
  - Substitution effect.

# Income Effect

- **Income effect (IE).** The change in the quantity demanded due to an *increase in purchasing power*, with the price of the item held constant.
  - A cheaper good  $x$  allows the individual to afford more units of all goods (i.e., larger purchasing power).
- An increase in income can induce the consumer to increase (decrease) her purchases of a good if she regards it as normal (inferior), allowing for positive (negative) income effects.

# Substitution Effect

- **Substitution effect (SE).** The change in the quantity demanded due to a change in its price, holding the utility level constant.
  - The quantity change is due to a change in the relative price for the two goods, and not due an increase in the consumer's purchasing power.
- After a price decrease (increase), the substitution effect always lead to an increase (decrease) in the quantity demanded of the good that became relatively less (more) expensive.



# Putting Income and Substitution Effects Together

# Putting IE and SE Together

- IE and SE of a price decrease for normal goods.

- When facing  $BL_1$ , the consumer selects  $A$ , where she reaches  $IC_1$ , purchasing  $x_A$ .
- When the price of  $x \downarrow$ , the budget line pivots upward to  $BL_2$ , and she chooses bundle  $C$  with  $x_C$ .
- The difference  $x_C - x_A$ , measures the total effect (TE).

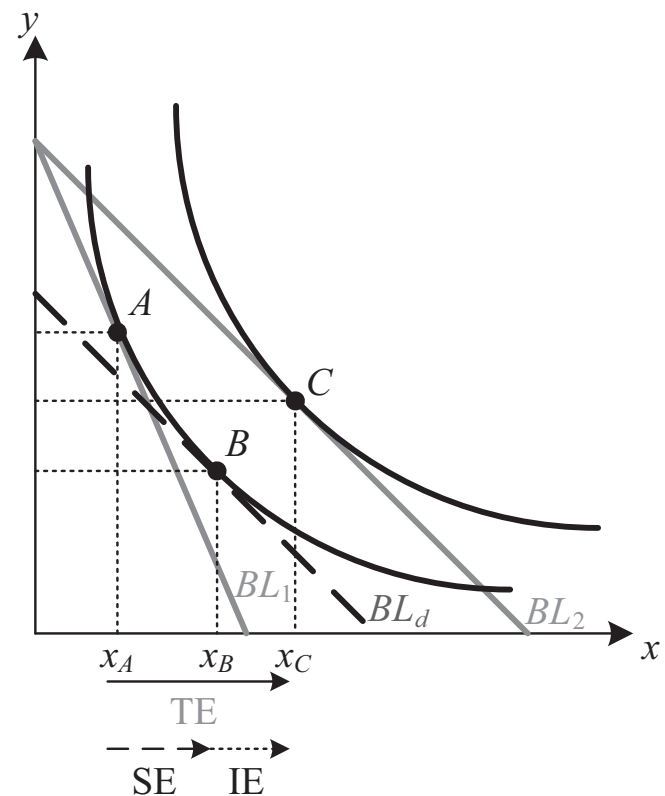


Figure 4.5

# Putting IE and SE Together

- IE and SE of a price decrease for normal goods (cont.).
  - To separate TE into substitution effects (SE) and income effects (IE), we need to shift  $BL_2$  downward (reducing her income) to make her reach the same utility level as before the price change.
  - The resulting  $BL_d$  is parallel to  $BL_2$ , having the final price ratio. And it is tangent to  $IC_1$  at bundle B, where she purchases  $x_B$ .

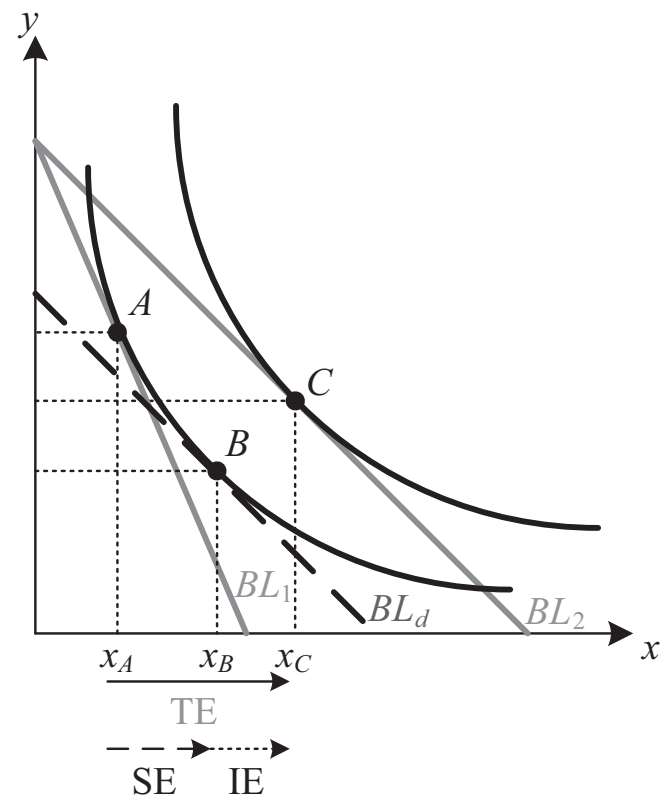


Figure 4.5

# Putting IE and SE Together

- IE and SE of a price decrease for normal goods (cont.).

1. The increase in consumption from  $x_A$  to  $x_B$  reflects the *substitution effect (SE)*.
2. The increase in consumption from  $x_B$  to  $x_C$  reflects the *income effect (IE)*.
3. The sum of SE and IE is the *total effect (TE)*.

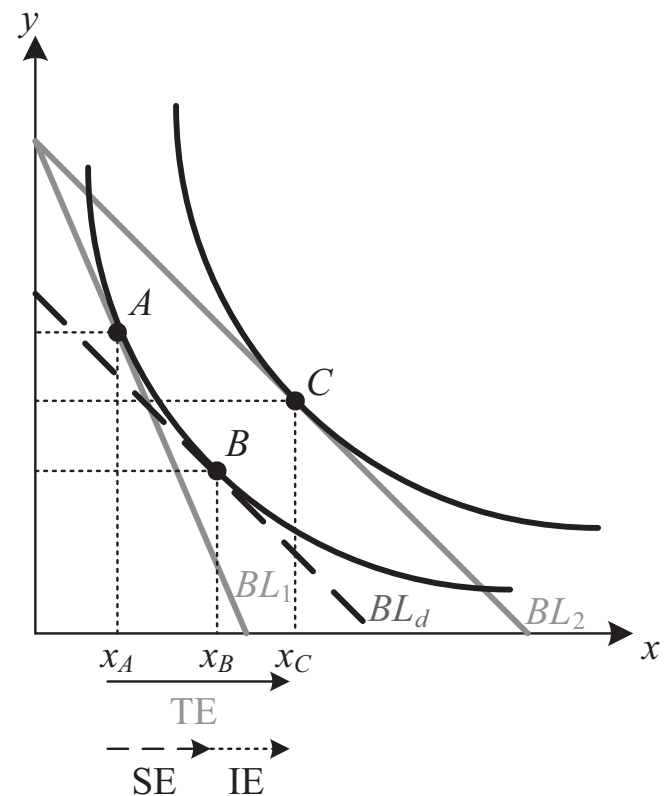


Figure 4.5

# Putting IE and SE Together

- IE and SE when good  $x$  is regarded as inferior.
- The income effect (IE) is negative, but it only partially offsets the substitution effect (SE), producing a positive total effect (TE).

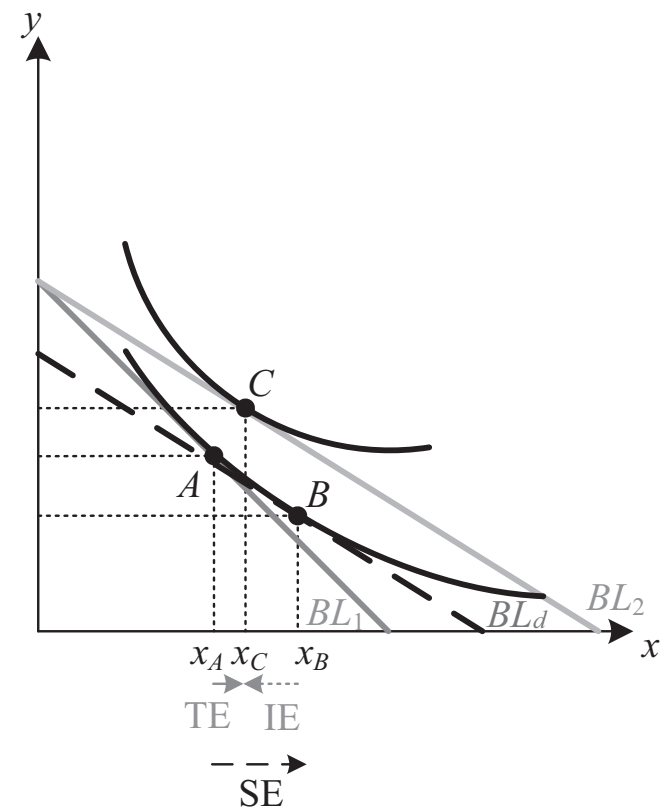


Figure 4.6a

# Putting IE and SE Together

- IE and SE with a Giffen good.
  - The income effect (IE) is negative, but large enough to offset the substitution effect (SE) and produce a negative total effect (TE).

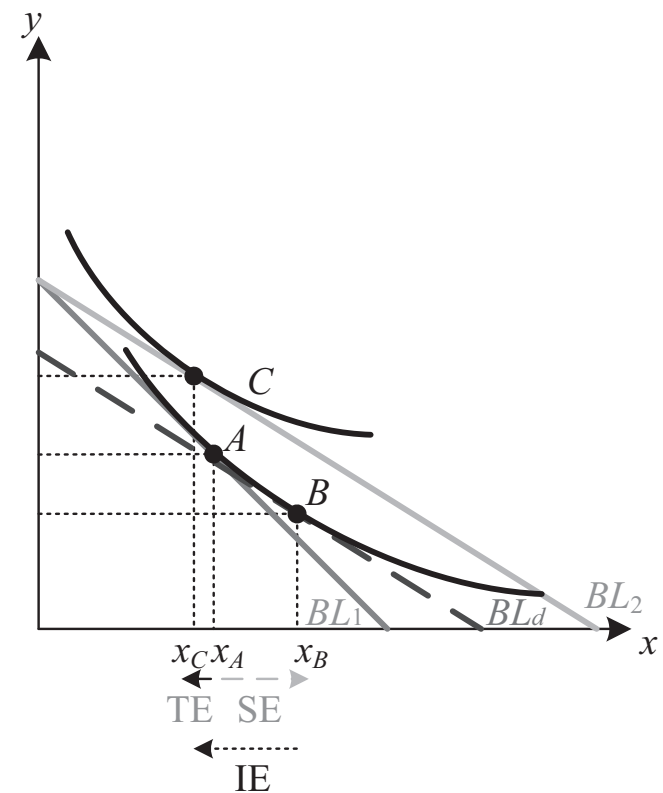


Figure 4.6b

# Putting IE and SE Together

- Summary:

Table 4.2

Price Decrease				Price Increase			
Type of good	<i>SE</i>	<i>IE</i>	<i>TE</i>	Type of Good	<i>SE</i>	<i>IE</i>	<i>TE</i>
Normal	+	+	+	Normal	−	−	−
Inferior	+	−	−	Inferior	−	+	+
Giffen	+	−	−	Giffen	−	+	+

# Income and Substitution Effects

- *Example 4.8: Finding IE and SE with a Cobb-Douglas utility function.*
  - Consider  $u(x, y) = xy$ ,  $I = \$100$ , and  $p_y = \$1$ . And assume  $p_x = \$3$  decreases to  $p'_x = \$2$ .
  - *Finding initial bundle A (at  $p_x = \$3$ ):*
    - The tangency condition is
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{3}{1} \Rightarrow y = 3x.$$
    - Inserting this result into the budget line,  $3x + y = 100$ ,
$$3x + 3x = 100 \Rightarrow x = \frac{100}{6} \text{ units,}$$
$$y = 3 \frac{100}{6} = 50 \text{ units.}$$
  - At  $p_x = \$3$ , optimal bundle is  $A = \left(\frac{100}{6}, 50\right)$ .



# Income and Substitution Effects

- *Example 4.8* (continued):

- *Finding final bundle C* (at  $p'_x = \$2$ ):

- The tangency condition is

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{2}{1} \Rightarrow y = 2x.$$

- Inserting this result into the new budget line,  $2x + y = 100$ ,

$$2x + 2x = 100 \Rightarrow x = \frac{100}{4} = 25 \text{ units,}$$

$$y = 2 \times 25 = 50 \text{ units.}$$

- At  $p'_x = \$2$ , optimal bundle is  $C = (25, 50)$ .

- The **total effect (TE)** of the decrease in  $p_x$  is an increase of

$$TE = x_C - x_A = 25 - \frac{100}{6} \cong 8.3 \text{ units.}$$

# Income and Substitution Effects

- *Example 4.8* (continued):

- *Finding decomposition bundle B:*

- Bundle  $B$  satisfies 2 conditions:

1. At  $B$  the consumer reaches the same utility level as at  $A$

$$u\left(\frac{100}{6}, 50\right) = xy = \frac{100}{6} \times 50 \cong 833.3.$$

2. The decomposition budget line  $BL_d$ , has the same slope as  $BL_2$ , and is tangent to the indifference curve. That is,

$$\frac{MU_x}{MU_y} = \frac{p'_x}{p_y},$$
$$\frac{y}{x} = \frac{2}{1} \Rightarrow y = 2x.$$

# Income and Substitution Effects

- *Example 4.8* (continued):
  - *Finding decomposition bundle B* (cont.):
    - In summary, previous conditions state that,

$$xy = 833.3 \text{ and } y = 2x.$$

- Inserting one equation into the other,

$$x(2x) = 833.3,$$

$$x^2 = 416.6,$$

$$\sqrt{x^2} = \sqrt{416.6},$$

$$y = 2 \times 20.4 = 40.8 \text{ units.}$$

- Then, bundle  $B$  is  $B = (20.4, 40.8)$ .

# Income and Substitution Effects

- *Example 4.8* (continued):

- The **substitution effect** of the decrease in  $p_x$  is

$$SE = x_A - x_B = 20.4 - \frac{100}{6} \cong 3.74 \text{ units.}$$

The consumer increases purchases of good  $x$  by 3.74 units only due to the lower price of this good, but still reaches the same utility level as before the price change.

- The **income effect** of this price decrease is

$$IE = x_C - x_B = 25 - 20.4 = 4.6 \text{ units}$$

For a given price ratio, the consumer increases her consumption of good  $x$  by 4.6 units because a cheaper good  $x$  increases her purchasing power.

# Income and Substitution Effects

- *Example 4.9: Finding IE and SE with quasilinear utility.*
  - Consider  $u(x, y) = 2x^{1/2} + y$ ,  $I = \$100$ , and  $p_y = \$1$ . And assume  $p_x = \$3$  decreases to  $p'_x = \$2$  as in example 4.8.
  - *Finding initial bundle A (at  $p_x = \$3$ ):*

- The tangency condition  $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$  becomes

$$\frac{\frac{1}{x^{1/2}}}{1} = \frac{3}{1} \Rightarrow \frac{1}{x^{1/2}} = \frac{3}{1},$$

$$\frac{1}{3} = x^{1/2} \Rightarrow \left(\frac{1}{3}\right)^2 = (x^{1/2})^2,$$

$$x = \frac{1}{9} \cong 0.11 \text{ units.}$$

# Income and Substitution Effects

- *Example 4.9* (continued):

- *Finding initial bundle A* (at  $p_x = \$3$ ) (cont.):

- Inserting this result into the budget line,

$$3x + y = 100,$$

$$(3 \times 0.11) + y = 100,$$

$$y = 100 - 0.33 \cong 99.67 \text{ units.}$$

- At  $p_x = \$3$ , optimal bundle is  $A = (0.11, 99.67)$ .

# Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding final bundle C* (at  $p'_x = \$2$ ):
    - The tangency  $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$  condition yields

$$\frac{\frac{1}{x^{\frac{1}{2}}}}{1} = \frac{2}{1} \Rightarrow \frac{1}{x^{\frac{1}{2}}} = 2$$

$$\frac{1}{2} = x^{1/2} \Rightarrow \left(\frac{1}{2}\right)^2 = (x^{1/2})^2,$$

$$x = \frac{1}{4} \cong 0.25 \text{ units.}$$

# Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding final bundle C* (at  $p'_x = \$2$ ) (cont.):
    - Inserting this result into the new budget line,
$$2x + y = 100,$$
$$(2 \times 0.25) + y = 100$$
$$y = 100 - 0.5 = 95.5 \text{ units.}$$
    - At  $p'_x = \$2$ , optimal bundle is  $C = (0.25, 95.5)$ .
  - The **total effect** of the decrease in  $p_x$  is an increase of
$$TE = x_C - x_A = 0.25 - 0.11 = 0.14 \text{ units.}$$



# Income and Substitution Effects

- *Example 4.9* (continued):

- *Finding decomposition bundle B:*

1. At  $B$  the consumer reaches the same utility level as at  $A$   
 $u(0.11, 99.67) = (2 \times 0.11^{1/2}) + 99.67 \cong 100.33$ .
2. The decomposition budget line  $BL_d$ , has the same slope as  $BL_2$ , and is tangent to the indifference curve. That is,

$$\frac{MU_x}{MU_y} = \frac{p'_x}{p_y},$$
$$\frac{\frac{1}{x^{1/2}}}{1} = \frac{2}{1} \Rightarrow \frac{1}{\frac{1}{x^2}} = 2 \Rightarrow \frac{1}{2} = x^{1/2},$$
$$\left(\frac{1}{2}\right)^2 = (x^{1/2})^2 \Rightarrow x = \frac{1}{4} \cong 0.25 \text{ units.}$$

# Income and Substitution Effects

- *Example 4.9* (continued):

- *Finding decomposition bundle B* (cont.):

- In summary, previous conditions state that,

$$2x^{1/2} + y = 100.33 \text{ and } x = 0.25.$$

- Inserting one equation into the other,

$$(2 \times 0.25^{1/2}) + y = 100.33,$$

$$(2 \times 0.5) + y = 100.33,$$

$$y = 100.33 - 1 = 99.33 \text{ units.}$$

- Then, bundle  $B$  is  $B = (0.25, 99.33)$ .

# Income and Substitution Effects

- *Example 4.9* (continued):

- The **substitution effect** of the decrease in  $p_x$  is

$$SE = x_A - x_B = 0.22 - 0.11 = 0.14 \text{ units.}$$

- The **income effect** of this price decrease is

$$IE = x_C - x_B = 0.25 - 0.25 = 0 \text{ units.}$$

- $SE = TE$ :

- The consumer, after experiencing a cheaper good  $x$ , uses her increased purchasing power to buy more units of good  $y$  alone, rather than increasing purchases of good  $x$ .

# IE and SE on the Labor Market

- We apply the analysis of IE and SE to the case of hours of leisure an individual enjoys,  $L$ .
- Because the day has only 24, the analysis of leisure choices allows us to examine its counterpart, working hours,  $H$ .

$$L + H = 24,$$

$$H = 24 - L.$$

# IE and SE on the Labor Market

- Figure 4.7a represents an individual facing a salary of  $w$  per working hour.
  - $Bl_1$  originates at the horizontal intercept at  $L = 4\text{h/day}$  ( $H = 0$ ).
  - At this point,  $I = 0$ . She cannot buy any unit of the composite good  $y$ , in the vertical axis.
  - If  $H = 1$ ,  $I$  increases to  $w$ . If we works all day, her income is  $24w$ . She does not enjoy leisure but can purchase the largest amount of  $y$ .

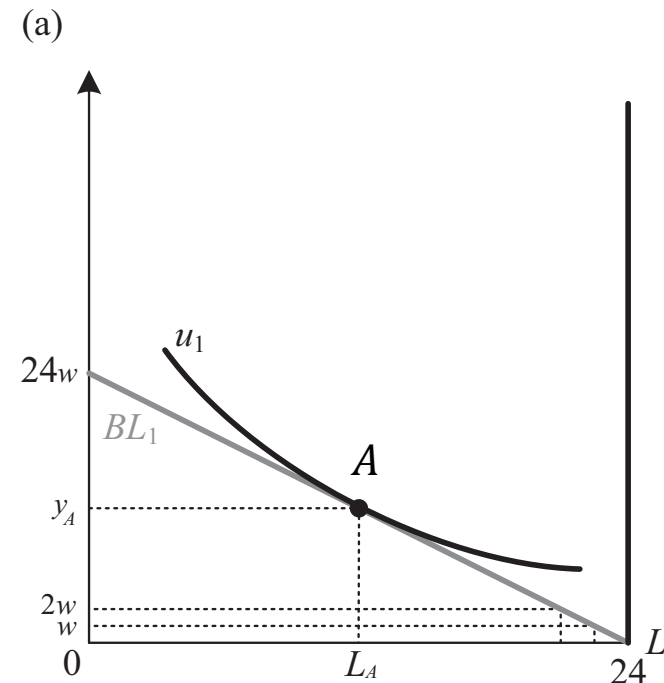


Figure 4.7a

# IE and SE on the Labor Market

- Figure 4.7a represents an individual facing a salary of  $w$  per working hour.
  - The indifference curve moves northeast. Her utility increases as she enjoys more  $L$  and  $y$ .
  - At hourly wage of  $w$ , she chooses the optimal bundle  $A$ , in which  $BL_1$  is tangent to her indifference curve  $u_1$ . She enjoys  $L_A$  hours of leisure and  $y_A$  goods.

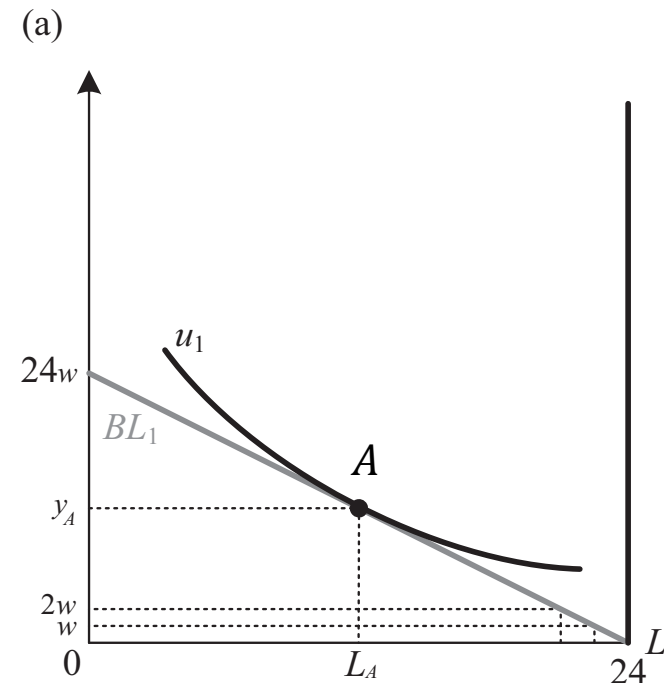


Figure 4.7a

# IE and SE on the Labor Market

- Figure 4.7b depicts an increase in the worker's hourly salary from  $w$  to  $w'$ .
  - The new budget line  $BL_2$  becomes steeper than  $BL_1$ .
  - Working 24h her income becomes  $24w'$ , which lies above  $BL_1$  because  $24w' > w$ .
  - With this salary, she chooses a new optimal bundle  $C$ , where she enjoys  $L_C$ .
  - The total effect from the salary increase is  $TE = L_C - L_A$ .

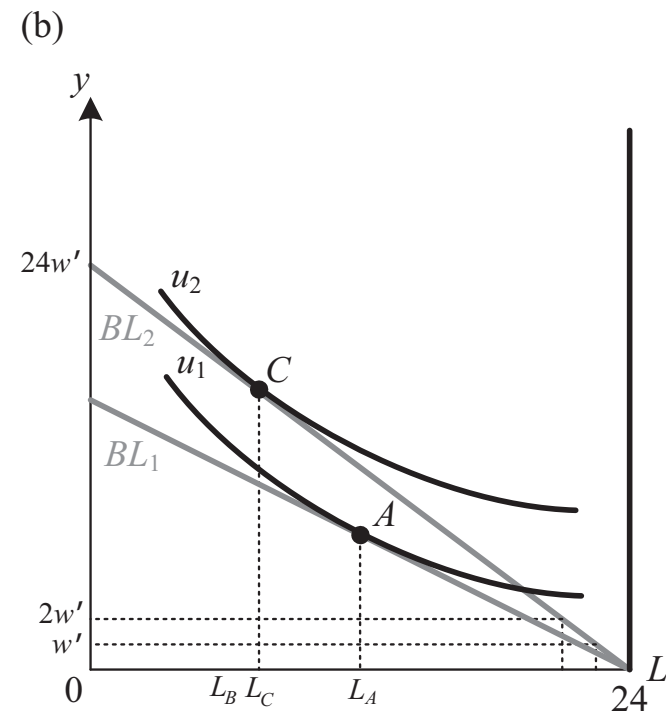


Figure 4.7b

# IE and SE on the Labor Market

- Figure 4.8 decomposes total effect into substitution and income effects.

- To examine SE and IE we find the decomposition budget line  $BL_D$ , which is tangent to initial indifference curve,  $u_1$ , at bundle  $B$ , where she enjoys  $L_B$  hours of leisure.

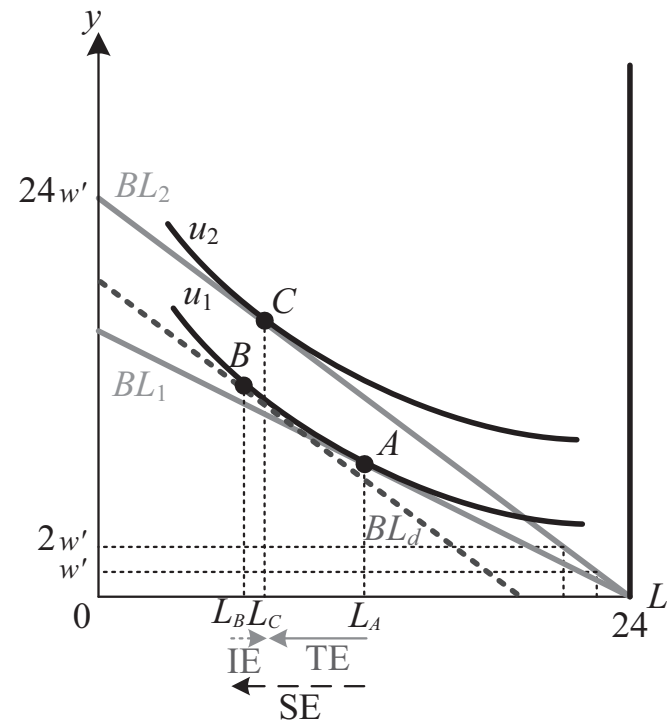


Figure 4.8



# IE and SE on the Labor Market

- Figure 4.8 decomposes total effect into substitution and income effects.

- The substitution effect is

$$SE = L_A - L_B.$$

A higher salary/h induces her to work more hours.

- The income effect is

$$IE = L_C - L_B.$$

As she becomes richer, she can afford to work less and enjoy more leisure.

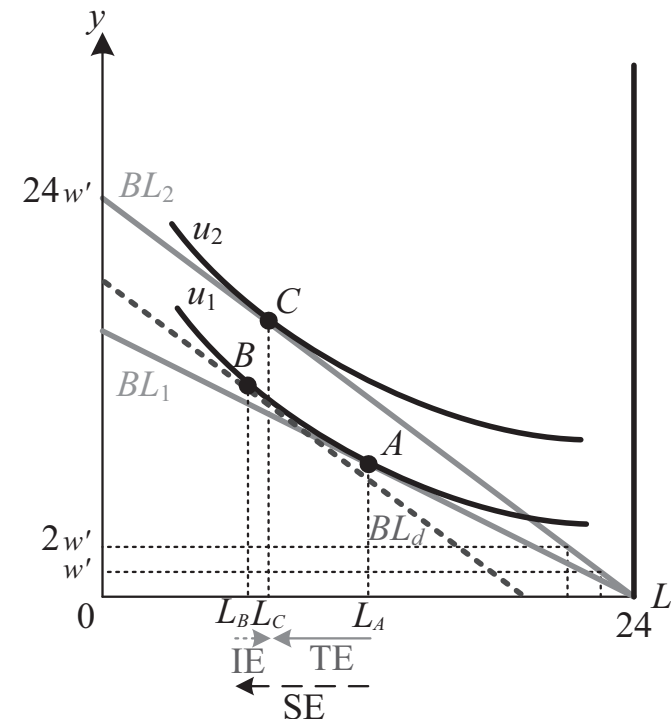


Figure 4.8

# Appendix A.

## Not All Goods Can be Inferior

# Not All Goods Can Be Inferior

- We can use income elasticities to prove that NOT all goods can be inferior.
- When the consumer chooses her optimal bundle, this bundle must lie on the budget line.
- Formally,

$$p_x x(p_x, p_y, I) + p_y y(p_x, p_y, I) = I.$$

- We want to analyze the effect of an income change. We differentiate the budget line with respect to  $I$ :

$$p_x \frac{\partial x(p_x, p_y, I)}{\partial I} + p_y \frac{\partial y(p_x, p_y, I)}{\partial I} = 1.$$

# Not All Goods Can Be Inferior

- We obtain the expression of income elasticity,

- We multiply the first term by  $\frac{x(p_x, p_y, I) \times I}{x(p_x, p_y, I) \times I}$ ;
- We multiply the second term by  $\frac{y(p_x, p_y, I) \times I}{y(p_x, p_y, I) \times I}$ ,

$$p_x \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{x(p_x, p_y, I) I}{x(p_x, p_y, I) I} + p_y \frac{\partial y(p_x, p_y, I)}{\partial I} \frac{y(p_x, p_y, I) I}{y(p_x, p_y, I) I} = 1.$$

- Rearranging,

$$\underbrace{\frac{p_x x(p_x, p_y, I)}{I}}_{\theta_x} \underbrace{\frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)}}_{\varepsilon_{x, I}} + \underbrace{\frac{p_y y(p_x, p_y, I)}{I}}_{\theta_y} \underbrace{\frac{\partial y(p_x, p_y, I)}{\partial I} \frac{I}{y(p_x, p_y, I)}}_{\varepsilon_{y, I}} = 1.$$

# Not All Goods Can Be Inferior

- More compactly,

$$\theta_x \varepsilon_{x,I} + \theta_y \varepsilon_{y,I} = 1.$$

where  $\theta_x = \frac{p_x x(p_x, p_y, I)}{I}$  is the budget share spent on good  $x$  ( $\theta_x$  is a percentage,  $0 \leq \theta_x \leq 1$ )

$\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)}$  is the definition of income elasticity.

- Can *all* goods be inferior? If it would be the case,  $\varepsilon_{x,I} < 0$  and  $\varepsilon_{y,I} < 0$ ,

$$\underbrace{\overbrace{\theta_x}^{+} \underbrace{\varepsilon_{x,I}}^{-}}_{-} + \underbrace{\overbrace{\theta_y}^{+} \underbrace{\varepsilon_{y,I}}^{-}}_{-} \neq 1.$$

**NO!** One of the goods must be normal.

# Appendix B.

## Alternative Representation of IE and SE

# Alternative IE and SE

- From the discussion of the utility maximization problem (UMP), and the expenditure minimization problem (EMP),
  - the demand function from the UMP is  $x^U(p_x, p_Y, I)$ ,
  - which is evaluated at income level  $I = e(p_x, p_y, \bar{u})$ . This is the necessary income to purchase optimal bundle solving EMP.

# Alternative IE and SE

- Therefore,

$$x^U(p_x, p_y, e(p_x, p_y, \bar{u})) = x^E(p_x, p_y, \bar{u}), \quad (4.1)$$

where  $e(p_x, p_y, \bar{u}) = p_x x^E(p_x, p_y, \bar{u}) + p_y y^E(p_x, p_y, \bar{u})$ .

The optimal bundle solving the UMP (left side) coincides with the bundling solving the EMP (right side).



# Alternative IE and SE

- Because the IE and SE measure how purchases of good  $x$  are affected by a change in its price,  $p_x$ , we differentiate equation (4.1) with respect to  $p_x$ ,

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial e} \frac{\partial e}{\partial p_x} = \frac{\partial x^E}{\partial p_x}. \quad (4.2)$$

# Alternative IE and SE

- To understand the left side of equation (4.2), recall that price  $p_x$  shows up:
  - in the first and second argument of  $x^U(p_x, p_y, e(p_x, p_y, \bar{u}))$ , meaning we need to differentiate separately each of them.
  - in  $e(p_x, p_y, \bar{u}) = p_x x^E(p_x, p_y, \bar{u}) + p_y y^E(p_x, p_y, \bar{u})$ , meaning we need to apply the chain rule.

# Alternative IE and SE

- Differentiating  $e(p_x, p_y, \bar{u}) = p_x x^E(p_x, p_y, \bar{u}) + p_y y^E(p_x, p_y, \bar{u})$ ,  
with respect to with respect to  $p_x$

$$\frac{\partial e}{\partial p_x} = x^E(p_x, p_y, \bar{u})$$

$$\text{and } x^U(p_x, p_y, e(p_x, p_y, \bar{u})) = x^E(p_x, p_y, \bar{u}).$$

# Alternative IE and SE

- We can insert this result at the end of the left side of equation (4.2),

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial e} x^U = \frac{\partial x^E}{\partial p_x}. \quad (4.3)$$

- Finally, because  $e(p_x, p_y, \bar{u}) = I$ , we express equation (4.3) as

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial I} x^U = \frac{\partial x^E}{\partial p_x}.$$

# Alternative IE and SE

- Rearranging yields the so-called **Slutsky equation**:

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE},$$

The total effect of a decrease in  $p_x$  (measured by the effect on the demand function from solving UMP) is given by:

- the substitution effect (as captured by the change in demand found solving the EMP)
- and the income effect.

# Alternative IE and SE

- The substitution effect is measured by  $\frac{\partial x^E}{\partial p_x}$ .
- A decrease in price  $p_x$  implies  $\frac{\partial x^E}{\partial p_x} < 0$ .
- This result does not rely on goods being normal or inferior.

# Alternative IE and SE

- For the income effect, however, the sign depends on the goods being normal or inferior.

- When goods are normal ( $\frac{\partial x^U}{\partial I} > 0$ ), then

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE \text{ is } -} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE \text{ is } -} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE \text{ is } -}.$$

- When goods are inferior ( $\frac{\partial x^U}{\partial I} < 0$ ), then

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE \text{ is } ?} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE \text{ is } -} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE \text{ is } +}.$$

# Income and Substitution Effects

- *Example 4.10: Applying the Slutsky equation to the Cobb-Douglas case.*

- Consider the Cobb-Douglas utility function from example 4.1.
- After solving the UMP, we found  $x^U(p_x, p_y, I) = \frac{I}{2p_x}$ .
- In that situation,

$$\frac{\partial x^U}{\partial p_x} = -\frac{I}{2(p_x)^2} \quad \text{and} \quad \frac{\partial x^U}{\partial I} = -\frac{1}{2p_x}$$

- Applying the Slutsky equation,

$$\underbrace{-\frac{I}{2(p_x)^2}}_{TE} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE} - \underbrace{\frac{1}{2p_x} \frac{I}{2p_x}}_{IE},$$

implying that SE is  $\frac{\partial x^E}{\partial p_x} = -\frac{I}{4(p_x)^2}$ .



# Income and Substitution Effects

- *Example 4.10* (continued):

- For instance, if  $p_x = \$3$  and  $I = \$100$ :

- SE is  $-\frac{I}{4(p_x)^2} = -\frac{100}{4(3)^2} = -\frac{25}{9}$ .

- IE is  $-\frac{1}{2p_x} \frac{I}{2p_x} = -\frac{1}{2 \times 3} \frac{100}{2 \times 3} = -\frac{25}{9}$ .

- The two effects reinforce each other, producing a TE of

$$-\frac{I}{2(p_x)^2} = -\frac{100}{2(3)^2} = -\frac{50}{9} \cong 5.55 \text{ units.}$$

# Income and Substitution Effects

- *Example 4.10* (continued):
  - This results implies that:
    - A marginal increase of the price of good  $x$  decreases the quantity demanded by 5.55 units,
    - where half of this decrease can be attributed to SE alone (change in price ratio).
    - The remaining half is explained by the IE (smaller purchasing power).

# Using Elasticities to Represent the Slutsky Equation

- First, multiply the left and right sides by  $\frac{p_x}{x^U}$ ,

$$\frac{\partial x^U}{\partial p_x} \frac{p_x}{x^U} = \frac{\partial x^E}{\partial p_x} \frac{p_x}{x^U} - \frac{\partial x^U}{\partial I} x^U \frac{p_x}{x^U}. \quad (4.4)$$

- Next, multiply the second term in right side of equation (4.4) by  $\frac{I}{I} = 1$ .
- And note that  $x^U(p_x, p_y, e(p_x, p_y, \bar{u})) = x^E(p_x, p_y, \bar{u})$  in the first term of the right side.

# Using Elasticities to Represent the Slutsky Equation

- Equation (4.4) becomes

$$\frac{\partial x^U}{\partial p_x} \frac{p_x}{x^U} = \frac{\partial x^E}{\partial p_x} \frac{p_x}{x^E} - \frac{\partial x^U}{\partial I} x^U \frac{p_x}{x^U} \frac{I}{I}, \quad (4.5)$$

where the left side corresponds to the definition of price elasticity,  $\varepsilon_{x,p_x} = \frac{\partial x^U}{\partial p_x} \frac{p_x}{x^U}$ ; and the first term in the right side is

$$\varepsilon_{x,p_x}^E = \frac{\partial x^E}{\partial p_x} \frac{p_x}{x^E}.$$

# Using Elasticities to Represent the Slutsky Equation

- Furthermore, the second term on the right side can be arranged as:

$$\frac{\partial x^U}{\partial I} x^U \frac{p_x I}{x^U I} = \underbrace{\frac{\partial x^U}{\partial I} \frac{I}{x^U}}_{\varepsilon_{x,I}} \underbrace{\frac{p_x x^U}{I}}_{\theta_x},$$

where  $\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)}$  represents the income-elasticity of demand, and  $\theta_x = \frac{p_x x^U}{I}$  denotes the budget share that the individual spends on good  $x$ .

# Using Elasticities to Represent the Slutsky Equation

- As a consequence, equation (4.5)

$$\underbrace{\frac{\partial x^U}{\partial p_x} \frac{p_x}{x^U}}_{\varepsilon_{x,p_x}} = \underbrace{\frac{\partial x^E}{\partial p_x} \frac{p_x}{x^E}}_{\varepsilon_{x,p_x}^E} - \underbrace{\frac{\partial x^U}{\partial I} \frac{I}{x^U}}_{\varepsilon_{x,I}} \underbrace{\frac{p_x x^U}{I}}_{\theta_x}, \quad (4.6)$$

can be rewritten as

$$\varepsilon_{x,p_x} = \varepsilon_{x,p_x}^E - \theta_x \varepsilon_{x,I}.$$

# Using Elasticities to Represent the Slutsky Equation

- Consider two extreme examples:

1. *Demand for garlic.*

The budget share of most consumers is negligible (i.e.,  $\theta_x \cong 0$ ), implying that equation (4.6) reduces to

$$\varepsilon_{x,p_x} \cong \varepsilon_{x,p_x}^E.$$

- The IE is close to zero, and hence, the SE coincides with TE.

# Using Elasticities to Represent the Slutsky Equation

- Consider two extreme examples:

## 2. *Demand for housing.*

The budget share is much larger (i.e.,  $\theta_x = 0.3$ ). If we have estimates of  $\varepsilon_{x,p_x} = -0.6$ , and  $\varepsilon_{I,p_x} = 1.3$ , then we can find  $\varepsilon_{x,p_x}^E$  by using equation (4.6):

$$\begin{aligned} -0.6 &= \varepsilon_{x,p_x}^E - (0.3 \times 1.3), \\ \varepsilon_{x,p_x}^E &= -0.21. \end{aligned}$$

- A 1% increase in the price reduces demand by 0.6% if wealth is left unaffected.
- However, if the consumer receives additional wealth (to maintain utility before the price change), demand reduces to 0.21%.