A

Department of Mathematics

MTL 106 (Probability and Stochastic Processes)

Quiz 1

Time: 20 minutes

Max. Marks: 10

Date: 11/09/21

**Note: The exam is closed-book, and all the questions are compulsory.**

1. Let the PMF of a random variable $X$ is given by

$$P\{X = k\} = e^{-20}\frac{20^k}{k!}, \quad k = 0, 1, 2, \cdots, \infty.$$

Show that (a) $P\{X \leq 10\} \leq \frac{1}{5}$; (b) $P\{X \geq 40\} \leq \frac{1}{20}$.

(3 + 3 marks)

2. Consider a probability space $(\Omega, \mathcal{F}, P)$, where $\Omega = \{(u, v) \in \mathbb{R}^2 | 0 \leq u \leq 1, 0 \leq v \leq 1\}$, and $\mathcal{F}$ is a Borel $\sigma$-field on $\Omega$, and $P(A) = \frac{\text{area of } A}{\text{area of } \Omega}$ for every $A \in \mathcal{F}$. Define a random variable $X : \Omega \to \mathbb{R}$ such that $X(u, v) = \frac{u+v}{4}$ for all $(u, v) \in \Omega$. Find the probability density function of $X$.

(4 marks)

(1)

(a) $E(X) = 20$ ; $Var(X) = 20$

$P\{|X - 20| \geq 20\} \leq \frac{1}{5}$

$P\{X \leq 10\} \leq \frac{1}{5}$

(b) $P\{|X - 20| \geq 20\} \leq \frac{1}{20}$

$P\{X \geq 40\} \leq \frac{1}{20}$

(2)

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ 8x^2, & 0 \leq x < \frac{1}{4} \\ 1 - \frac{1}{2}(2-4x)^2, & \frac{1}{4} \leq x < \frac{1}{2} \\ 1, & x \geq \frac{1}{2} \end{cases}$$

$$f(x) = \begin{cases} 16x, & 0 < x < \frac{1}{4} \\ 4(2-4x), & \frac{1}{4} \leq x < \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$$

B

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1. Let the PMF of a random variable $X$ is given by

$$P\{X = k\} = e^{-16}\frac{16^k}{k!}, \quad k = 0, 1, 2, \cdots, \infty.$$

Show that (a) $P\{X \leq 8\} \leq \frac{1}{4}$; (b) $P\{X \geq 32\} \leq \frac{1}{16}$.

(3 + 3 marks)

2. Consider a probability space $(\Omega, \mathcal{F}, P)$, where $\Omega = \{(u, v) \in \mathbb{R}^2 | 0 \leq u \leq 1, 0 \leq v \leq 1\}$, and $\mathcal{F}$ is a Borel $\sigma$-field on $\Omega$, and $P(A) = \frac{\text{area of } A}{\text{area of } \Omega}$ for every $A \in \mathcal{F}$. Define a random variable $X : \Omega \to \mathbb{R}$ such that $X(u, v) = \frac{u+v}{3}$ for all $(u, v) \in \Omega$. Find the probability density function of $X$.

(4 marks)

(1) a) $E(X) = 16$; $Var(X) = 16$

$$P\{|X - 16| \geq 8\} \leq \frac{1}{4}$$

$$P\{X \leq 8\} \leq \frac{1}{4}$$

(b)

$$P\{|X - 16| \geq 16\} \leq \frac{1}{16}$$

$$P\{X \geq 32\} \leq \frac{1}{16}.$$

(2)

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{9}{2}x^2 & , 0 \leq x < \frac{1}{3} \\ 1 - \frac{1}{2}(2-3x)^2 & , \frac{1}{3} \leq x < \frac{2}{3} \\ 1 & , x \geq 2/3 \end{cases}$$

$$f(x) = \begin{cases} 9x & , 0 < x < \frac{1}{3} \\ 3(2-3x) & , \frac{1}{3} < x < \frac{2}{3} \\ 0 & - \text{otherwise} \end{cases}$$

C

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1. Let the PMF of a random variable $X$ is given by

$$P\{X = k\} = e^{-12}\frac{12^k}{k!}, \quad k = 0, 1, 2, \cdots, \infty.$$

Show that (a) $P\{X \le 6\} \le \frac{1}{3}$; (b) $P\{X \ge 24\} \le \frac{1}{12}$.

(3 + 3 marks)

2. Consider a probability space $(\Omega, \mathcal{F}, P)$, where $\Omega = \{(u, v) \in \mathbb{R}^2 | 0 \le u \le 1, 0 \le v \le 1\}$, and $\mathcal{F}$ is a Borel $\sigma$-field on $\Omega$, and $P(A) = \frac{\text{area of } A}{\text{area of } \Omega}$ for every $A \in \mathcal{F}$. Define a random variable $X : \Omega \to \mathbb{R}$ such that $X(u, v) = \frac{u+v}{2}$ for all $(u, v) \in \Omega$. Find the probability density function of $X$.

(4 marks)

(1) (a) $E(x) = 12$ ; $Var(x) = 12$

$$P\{|x - 12| \geq 6\} \leq \frac{1}{3}$$

$$P\{x \leq 6\} \leq \frac{1}{3}$$

(b)

$$P\{|x - 12| \geq 12\} \leq \frac{1}{12}$$

$$P\{x \geq 24\} \leq \frac{1}{12}$$

(2)
$$F_X(x) = \begin{cases} 0 & , x < 0 \\ 2x^2 & , 0 \le x < \frac{1}{2} \\ 4x - 2x^2 - 1 & , \frac{1}{2} \le x < 1 \\ 1 & , x \ge 1 \end{cases}$$

$$f(x) = \begin{cases} 4x & , 0 < x < \frac{1}{2} \\ 4 - 4x & , \frac{1}{2} < x < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

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1. Let the PMF of a random variable $X$ is given by

$$P\{X = k\} = e^{-8}\frac{8^k}{k!}, \quad k = 0, 1, 2, \cdots, \infty.$$

Show that (a) $P\{X \leq 4\} \leq \frac{1}{2}$; (b) $P\{X \geq 16\} \leq \frac{1}{8}$.

(3 + 3 marks)

2. Consider a probability space $(\Omega, \mathcal{F}, P)$, where $\Omega = \{(u, v) \in \mathbb{R}^2 | 0 \leq u \leq 1, 0 \leq v \leq 1\}$, and $\mathcal{F}$ is a Borel $\sigma$-field on $\Omega$, and $P(A) = \frac{\text{area of } A}{\text{area of } \Omega}$ for every $A \in \mathcal{F}$. Define a random variable $X : \Omega \to \mathbb{R}$ such that $X(u, v) = \frac{u+v}{5}$ for all $(u, v) \in \Omega$. Find the probability density function of $X$.

(4 marks)

(1) (a) $E(X) = 8$, $Var(X) = 8$

$P\{|X - 8| \geq 4\} \leq \frac{1}{2}$

$P\{X \leq 4\} \leq \frac{1}{2}$

(b) $P\{|X - 8| \geq 8\} \leq \frac{1}{8}$

$P\{X \geq 16\} \leq \frac{1}{8}$

(2)

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{25}{2}x^2 & , 0 \leq x < \frac{1}{5} \\ 1 - \frac{1}{2}(2-5x)^2 & , \frac{1}{5} \leq x < \frac{2}{5} \\ 1 & , x \geq 2/5 \end{cases}$$

$$f(x) = \begin{cases} 25x & , 0 < x < \frac{1}{5} \\ 5(2-5x) & , \frac{1}{5} \leq x < \frac{2}{5} \\ 0 & , \text{otherwise} \end{cases}$$