COL202 Minor exam

Aaveg Jain

TOTAL POINTS

20 / 27

QUESTION 1

1 Problem 1 3 / 3

✓ **+ 3 pts** *Correct*

- 0.5 pts Each Minor mistake/ Undefined variable used

+ 0 pts Incorrect/Not attempted

mention that you are doing for minimum k

QUESTION 2

2 Problem 2 2 / 2

✓ **+ 0.5 pts** *Mentioned proof method, and concluded the proof*

✓ **+ 1.5 pts** *Considered all cases of A and shown there is a y*

+ 0 pts Incorrect/Not attempted

QUESTION 3

3 Problem 3 6 / 6

✓ **- 0 pts** *Correct answer for both statements*

- 3 pts Wrong truth table for statement 1

- 3 pts Wrong conclusion for statement 1

- 1 pts Not written concluding statement for 1

- 3 pts Wrong truth table for statement 2

- 3 pts Wrong conclusion for statement 2

- 1 pts Not written concluding statement for 2

QUESTION 4

4 Problem 4 7 / 7

✓ **+ 7 pts** *Correct*

+ 1 pts Using the proof by contradiction.

+ 1 pts Assuming S to be non-empty.

+ 1 pts There exist some $$n_0$$ (smallest element in the S)

+ 3 pts Correct by cases and using the contradiction of the minimality of S.

+ 1 pts Concludes S is empty and proved.

+ 0 pts Unattempted/Completely wrong.

QUESTION 5

5 Problem 5 2 / 9

Proof that G' is connected

+ 1.5 pts Partially correct

+ 3 pts Correct

Proof that G’ is acyclic

+ 1 pts Without using maximally acyclic concept

✓ **+ 2 pts** *Using maximally acyclic concept - considered edges of G*

+ 5 pts Using maximally acyclic concept - considered both edges and non-edges of G

+ 1 pts G’ is connected and acyclic => spanning tree

+ 0 pts Incorrect/Not Attempted

COL202: Discrete Mathematical Structures. I semester, 2022-23.
Minor exam.
28 September 2022, Maximum Marks: 29.

Name (In CAPITAL letters as on Gradescope)	I	**Ent. No.**
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Important: Please write within the box given for your answer. Answers written elsewhere on the paper will not be graded.

Problem 1 (5 marks)

We say that graph $G = (V, E)$ is *k-edge colourable* for integer $k > 0$ if there is a function $f : E \to \{1, \ldots, k\}$ such that no two edges incident on a vertex have the same "colour." i.e., the same value of $f(\cdot)$. The *edge colouring number* of G, χ_G, is the maximum k for which G is k-edge colourable. For $k > 0$, let us denote by $\mathcal{F}_k$ the set of all functions from E to $\{1, \ldots, k\}$. Use this notation to write the following statement as a predicate: $\chi_G = 10$. Note that your predicate must take only $G = (V, E)$ as an argument. Use only logic notation. You may use set inclusion, e.g. $x \in A$, if required.

let predicate be P

$P(G(V,E)) : \left(\exists f \in \mathcal{F}_{10} : \forall e_1, e_2 \in E : e_1 \cap e_2 \neq \phi \Rightarrow f(e_1) \neq f(e_2)\right)$

$\wedge \left(\forall k \in \mathbb{N} \setminus \{0,1,2, \ldots 10\} : \forall f \in \mathcal{F}_k : \exists e_1, e_2 \in E :\right.$
$\left. e_1 \cap e_2 \neq \phi \Rightarrow f(e_1) = f(e_2)\right)$

Problem 2 (2 marks)

We are given sets $A = \{a, b, c, d\}$ and $B = \{e, f, g, h\}$ and we are given predicates $p : A \times B \to \{T, F\}, r : A \to \{T, F\}, q : B \to \{T, F\}$ for which *only* the following are True: $p(a, e), p(a. f), (\forall y : p(b, y)), p(c, g), r(b), r(d), q(e), q(g), q(h)$.

Prove or disprove

$$\forall x : \exists y : r(x) \Rightarrow (p(x, y) \Rightarrow q(y)).$$

claim - $\forall x : \exists y : r(x) \Rightarrow (p(x,y) \Rightarrow q(y))$. to prove this,
for each x, we will find one such y s.t
$\big(r(x) \Rightarrow (p(x,y) \Rightarrow q(y))\big) = z$ is T

for $x = a$, take $y = e$ $\quad r(a)$ is F ~~,~~
$p(a,e)$ is T, $q(e)$ is T $\Rightarrow$ z is T (from TT of $\Rightarrow$)

for $x = b$, take $y = e$ $\quad r(b)$ is T, $p(b,e)$ is T and $q(e)$ is T $\Rightarrow$ z is T (from TT of $\Rightarrow$)

for $x = c$, take $y = e$; $r(c)$ is F, $q(e)$ is T, $p(c,e)$ is F $\Rightarrow$ z is T (from TT of $\Rightarrow$)

for $x = d$, take $y = e$ $\quad r(d)$ is T, $q(e)$ is T, $p(d,e)$ is F $\Rightarrow$ z is T (from TT of $\Rightarrow$)

since $\forall x \in A$, $\exists y$ s.t z is T, we conclude our claim.

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Problem 3 (6 marks)
Prove or disprove the following logical statements using the truth table method.

1. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$.

2. $((P \wedge Q) \Rightarrow R) \Leftrightarrow (P \Rightarrow (Q \Rightarrow R))$

1.

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

since TT of $\neg(P \vee Q)$ and $\neg P \wedge \neg Q$ is same, thus
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

2.

P	Q	R	$P \wedge Q$	$Q \Rightarrow R$	$(P \wedge Q) \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

since TT of $(P \wedge Q \Rightarrow R)$ and $(P \Rightarrow (Q \Rightarrow R))$ is same,
thus $((P \wedge Q) \Rightarrow R) \Leftrightarrow (P \Rightarrow (Q \Rightarrow R))$

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Problem 4 (7 marks)
Use the Well Ordering Principle to show that $17^n > 0$ for every $n \in \mathbb{N} \cup \{0\}$.

We use Well Ordering Lemma (WOP) and contradiction to show the claim.
Let $C = \{ n \in \mathbb{N} \cup \{0\} ; \ 17^n \leq 0 \}$ be the set of countery to the claim.
Assume C is non-empty. We now show by contradiction that ~~Cis~~ our assumption is false.
Since C ~~is~~ is non-empty and has non negative el. (defn of C) by WOP there exists $n_0 \in \mathbb{N} \cup \{0\}$ such that n_0 is minimum el. of C. ~~a~~ observe ~~ex~~ $17^0 = 1 > 0$ thus ~~is~~ $n_0 > 0$ ~~ie~~ $n_0 - 1 \in \mathbb{N} \cup \{0\}$

~~no~~ now, $17^{n_0 - 1} = \frac{17^{n_0}}{17^1}$ (law of exp.). since $17^{n_0} \leq 0$ ($n_0 \in C$) and $17 > 0$, we have $17^{n_0-1} \leq 0$.
thus $n_0 - 1 \in C$. ~~but n_0 is the least el. of~~ this contradicts the fact that n_0 is least el. of C as $n_0 - 1 < n_0$. Thus, by assuming C is non empty we arrive at a contradiction. Thus our assumption is F. i.e C is empty. Hence we conclude our claim [$17^n > 0 \ \forall n \in \mathbb{N} \cup \{0\}$ as C is empty]

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Problem 5 (9 marks)

Suppose that $G = (V, E)$ is a connected graph. Here is an algorithm we run on G:

> Create a new graph $G' = (V, E' = \emptyset)$. Now go through the edges from E in some order and try to add them into the edgeset E' one at a time. If the addition of an edge creates a cycle in G', discard the edge and move on to the next edge of E. The algorithm ends when we have either added or discarded every edge of E. Return G'.

Using the fact that every maximally acyclic graph is a tree, prove that the above algorithm returns a spanning tree of G. *If you don't use this fact you will get a 0 even if your proof is correct.*

Let the order in which the edge are considered be $E_1, E_2, \ldots E_n$
n = no. of edges = $|E|$.
To prove that G' is a spanning tree, we first prove G' is a maximally acyclic graph on V.
Claim - G' is maximally acyclic.
Pf - By strong inductn. Let G_i be graph obtained after considering edges $E_1, \ldots E_i$. S_i be set of discarded edges upto E_i. If E_j is a discarded edge, let C_j be cycle formed by addn of E_j.
Indn predicate $P(i)$ - after considering E_i, G_i is acyclic and addn of any edge from S_i forms a cycle.
Indn hypothesis - $P(1) \wedge P(2) \ldots \wedge P(i)$. to prove $P(i+1)$.
from $P(i)$, G_i is acyclic. After considering E_{i+1}, no cycle is formed. thus G_{i+1} is also acyclic - (1)
Let E_j be any edge in S_{i+1} ($j \leq i+1$) since G_{i+1} is acyclic, no more than 1 path can exist b/w any 2 vertices of C_j. (otherwise a cycle is formed from $P(j)$) This 1 path is the path in C_j which already exists. thus, let $E_j = (x, y)$. addn of E_j forms 2 paths b/w x and y and thus a cycle is formed. ie thus addn of any edge from S_{i+1} causes a cycle to get formed. - (2)
from (1) and (2), $P(i+1)$ is T. Thus
Base case - G_1 is a single edge graph or no-edge graph. in either case $P(1)$ is T. Thus from indn $P(i)$ is T for $1 \leq i \leq n$. from $P(n)$ $G_n = G'$ is acyclic and addn of any edge from S_n = set of discarded edges, forms a cycle. Thus G' is maximally acyclic. Also G' spans G ($V(G') = V(G)$). Thus G' is a spanning tree of G.