# PYL 102

Thursday, Oct. 17, 2024

Dielectric Constant: Frequency dependence

# Total Polarization

In the presence of electronic, ionic, and dipolar polarization mechanisms, the average induced dipole moment per molecule will be the sum of all the contributions in terms of the local field. Each effect adds linearly to the net dipole moment per molecule,

$$P_{av} = \alpha_e \bar{E}_{local} + \alpha_i \bar{E}_{local} + \alpha_d \bar{E}_{local}$$

It is possible for one or more of these contributions to the total polarization to be either absent or negligible in magnitude relative to the others. For example, ionic polarization will not exist in covalently bonded materials in which no ions are present.

# Frequency dependence of Dielectric constant

We have studied the static dielectric constant that is an effect of polarization under dc conditions. When the applied field is a sinusoidal signal, then the polarization of the medium under these ac conditions leads to an ac dielectric constant.

Consider a dielectric material that is subject to polarization by an ac electric field. Let us consider example of orientational polarization involving dipolar molecules. With each direction reversal, the dipoles attempt to reorient with the field in a process requiring some finite time. For each polarization type, some minimum reorientation time exists, which depends on the ease with which the particular dipoles are capable of realignment. **A relaxation frequency** is taken as the reciprocal of this minimum reorientation time.

Why is there reorientation time? There are two factors opposing the immediate alignment of the dipoles with the field. (1)Thermal agitation tries to randomize the dipole orientations. (2) The molecules rotate in a viscous medium by virtue of their interactions with neighbors, which is particularly strong in the liquid and solid states and means that the dipoles cannot respond instantaneously to the changes in the applied field.

If the instantaneous induced dipole moment p per molecule can instantaneously follow the field variations, then at any instant $p = \alpha_d E$

and the polarizability $\alpha_d$ has its expected maximum value from dc conditions, that is $\alpha_d = \frac{p_0^2}{3kT}$

If the field changes rapidly, then the dipoles cannot follow the field and as a consequence, remain randomly oriented. At high frequencies, therefore $\alpha_d$ will be zero as the field cannot induce a dipole moment. At low frequencies the dipoles can respond rapidly to follow the field and $\alpha_d$ has its maximum value.
Let us determine the behavior of $\alpha_d$ as a function of frequency so that dielectric constant can be determined by the Clausius-Mossotti equation.

The applied dc field is suddenly changed from $E_0$ to E at time t = 0. The induced dipole moment p has to decrease from $\alpha_d(0)$ $E_0$ to a final value of $\alpha_d(0)$ E. The decrease is achieved by random collisions of molecules in the gas.

The decrease, or the relaxation process in the induced dipole moment is achieved by random collisions. Assuming that $\tau$ is the average time, called the relaxation time, between molecular collisions, then this is the mean time it takes per molecule to randomize the induced dipole moment. [p - $\alpha_d(0)$ E] is the excess dipole moment, which must eventually disappear to zero through random collisions as t→∞. It would take an average $\tau$ seconds to eliminate the excess dipole moment. The rate at which the induced dipole moment is changing is

$$\frac{dP}{dt} = -\frac{p - \alpha_d(0)\,E}{\tau}$$
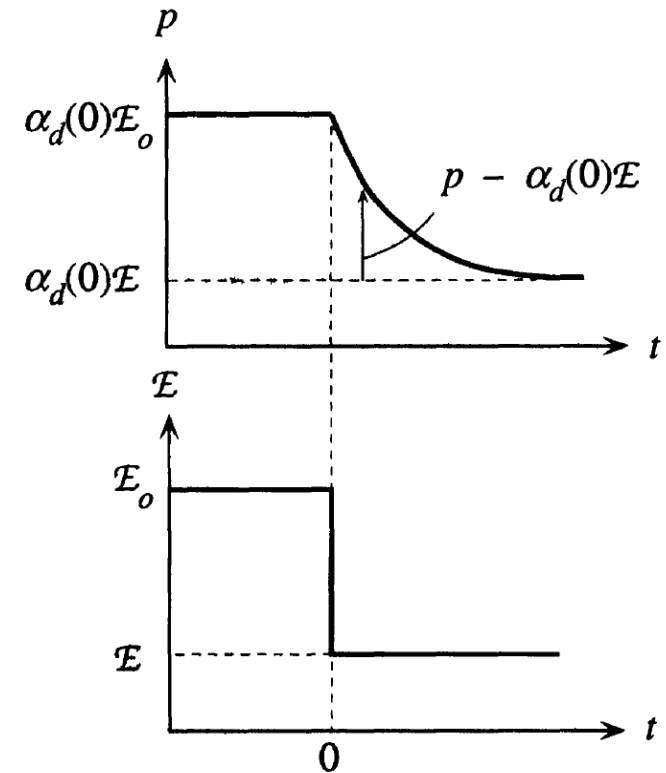
$$E = E_s\, e^{-i\omega t} \quad (\text{applied ac field})$$

Induced dipole moment $\quad p = \alpha_d(\omega) E \, e^{-i(\omega t - \phi)}$

where orientational polarizability under ac field $\alpha_d(\omega) = \boxed{\dfrac{\alpha_d(0)}{1 + i\omega\tau}}$

Polarization P lags behind E by some angle $\phi$.

At low frequencies, $\tau\omega<<1$, p is in phase with E. The rate of relaxation $1/\tau$ is much faster than the frequency of the field or the rate at which the polarization is being changed.
At very high frequencies, $\tau\omega >>1$, the rate of relaxation $1/\tau$ is much slower than the frequency of the field and p can no longer follow the variations in the field.
**(This gives rise to energy absorption)**

$P = P_o \sin(\omega t - \phi)$

$E = E_o \sin \omega t$



$v = V_o \sin \omega t$
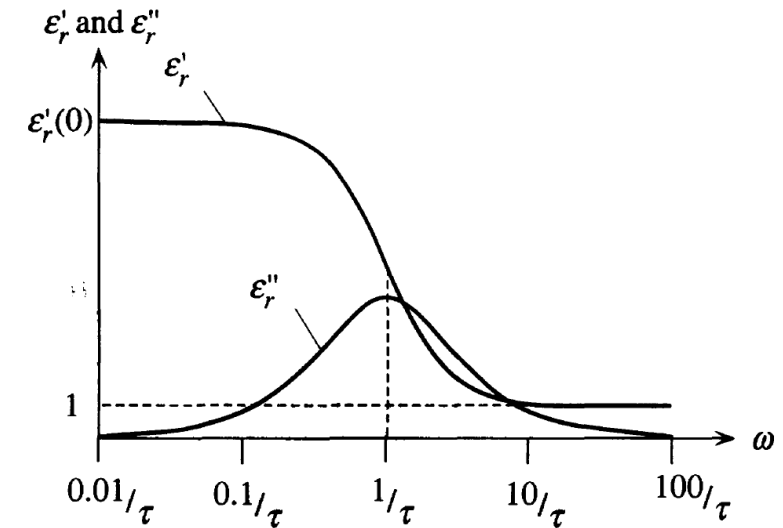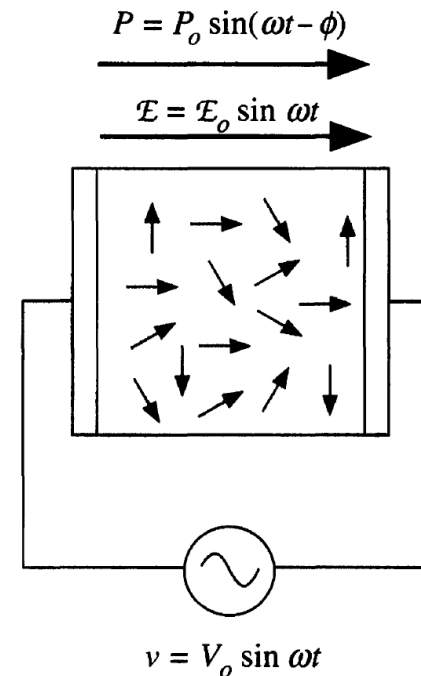
$\boxed{\varepsilon_r = 1 + \dfrac{N\alpha_d}{\varepsilon_o}}$ Since $\alpha_d$ is a complex number, so dielectric constant $\varepsilon_r$ is also a complex number

complex dielectric constant $\boxed{\varepsilon_r = \varepsilon_r' - i\varepsilon_r''}$

where $\varepsilon_r'$ is the real part and $\varepsilon_r''$ is the imaginary part, both being frequency dependent.

$\varepsilon_r'$ decreases from its maximum value $\varepsilon_r'(0)$ corresponding to $\alpha_d(0)$ to 1 at high frequencies when $\alpha_d = 0$ as $\omega \to \infty$.

The imaginary part $\varepsilon_r''$ is zero at low and high frequencies but peaks when $\tau\omega = 1$.

The real part $\varepsilon_r'$ represents the relative permittivity, for example in capacitance calculation.
The imaginary part $\varepsilon_r''$ represents the energy lost in the dielectric medium as the dipoles are oriented against random collisions by the field.
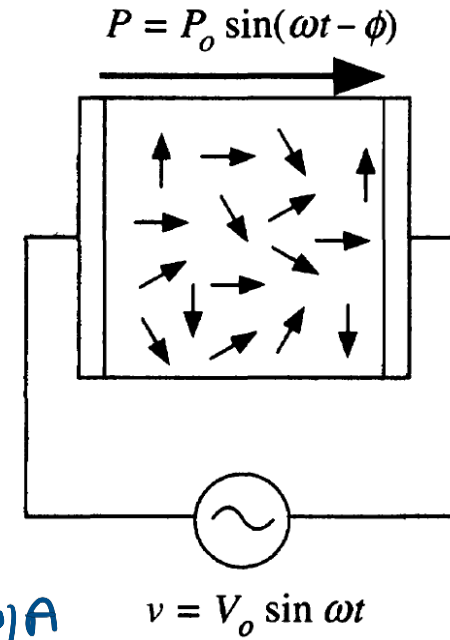
Let us consider a capacitor which has this dielectric medium between the plates. Then the admittance Y, i.e., the reciprocal of impedance of this capacitor
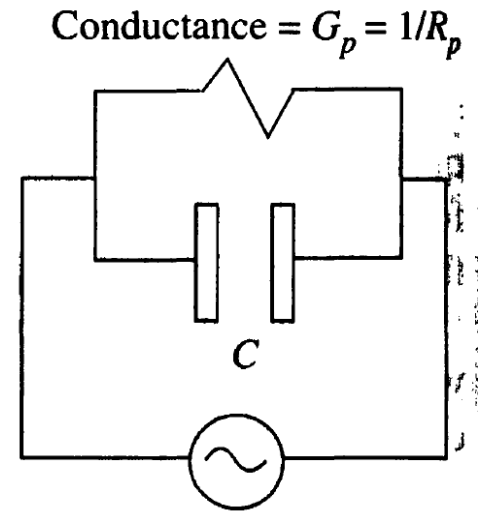
$$Y = jWC = jW \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

$$Y = \frac{jw\varepsilon_0 \varepsilon_r'(w) A}{d} + \frac{w\varepsilon_0\varepsilon_r''(w)A}{\lambda}$$

$$\boxed{Y = jwC + G_p}$$

$$P = P_o \sin(\omega t - \phi)$$

$$v = V_o \sin \omega t$$

Conductance $= G_p = 1/R_p$

The dielectric medium behaves like an ideal (lossless) capacitor of capacitance C which is in parallel with a conductance $G_p$.

There is no real electric power dissipated in C, but there is indeed real power dissipated in $R_p = 1/ G_p$

Input power = IV = Y V² = j$\omega$CV² + V²/R$_P$   second term is real

The power dissipated in the dielectric medium is related to $\omega$ and peaks when $\omega = 1/\tau$. The rate of energy storage by the field is determined by $\omega$ whereas the rate of energy transfer to molecular collisions is determined by $1/\tau$. When $\omega = 1/\tau$, the two processes, energy storage by the field and energy transfer to random collisions, are then occurring at the same rate, and hence energy is being transferred to heat most efficiently. The peak in $\varepsilon_r''$ versus $\omega$ is called a relaxation peak, which is at a frequency when the dipole relaxations are at the right rate for maximum power dissipation. This process is known as dielectric resonance.

We define the relative magnitude of $\varepsilon_r'$ with respect $\varepsilon_r''$ through a quantity, $\tan\delta$, called the loss tangent (or loss factor).

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

The actual value of $1/\tau$ depends on the material, but typically for liquid and solid media it is in the gigahertz range (microwave frequencies). The power-dissipated per unit volume as dielectric loss in the medium.

$$W_{vol} = \frac{Power\ loss}{Volume} = \frac{V^2}{R_p} \times \frac{1}{dA} = \omega E^2 \varepsilon_o \varepsilon_r' \tan\delta \qquad (E = V/d)$$
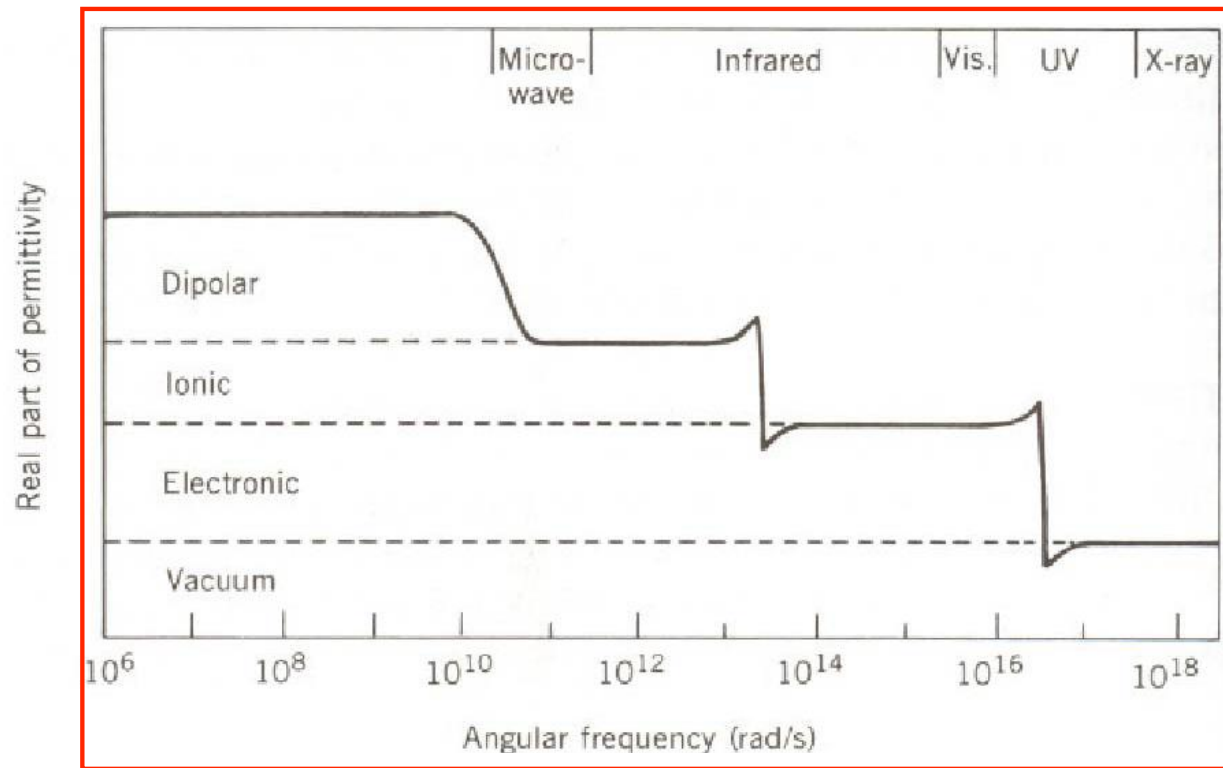
We considered only orientational polarization, in general a dielectric medium will also exhibit other polarization mechanisms. If we were to consider the ionic polarizability in ionic solids, we would also find $\alpha_i$ to be frequency dependent and a complex number. In this case, lattice vibrations in the crystal, typically at frequencies in the infrared region of the electromagnetic spectrum, will dissipate the energy stored in the induced dipole moments. Thus, the energy loss will be greatest when the frequency of the polarizing field is the same as the lattice vibration frequency $\omega = \omega_i$ which tries to randomize the polarization.