

Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Quiz 2 Examination

Time: 20 min
Max. Marks: 10

Date: 05/10/2021

Note: The exam is closed-book, and all the questions are compulsory and of descriptive type. The standard normal distribution table is appended with the question paper.

1. The grading scheme of a Mathematics professor is as follows:

- A: Top 12% of scores

$$(A \geq 81.558)$$

- B: Scores below the top 12% and above the bottom 60%

$$(74.106 < B < 81.558)$$

- C: Scores below the top 40% and above the bottom 25%

$$(66.573 < C < 74.106)$$

- D: Scores below the top 75% and above the bottom 8%

$$(60.66 < D < 66.573)$$

- F: Bottom 8% of scores.

$$(F \leq 60.66)$$

Scores on the test follow normal distribution with a mean of 72 and a standard deviation of 8.1. Find the numerical limits for all the grades. (5 marks)

2. Let X_1, X_2, \dots be iid random variables, each having exponential distribution with parameter $\frac{1}{4}$. Define $Y_i = X_i - X_i^2, i = 1, 2, \dots$. Use central limit theorem to evaluate $P\left(\sum_{i=1}^{40} Y_i \geq 40\right)$ approximately. (5 marks)

$$E(Y_i) = -28$$

$$\text{Var}(Y_i) = 4624$$

$$\approx 0.0036$$

Other Papers

$$\begin{aligned} A &\geq 80.4287 \\ 77.508 &< B < 80.4 \\ 63.657 &< C < 77.5 \\ 58.716 &< D < 63.6 \\ F &\leq 58.716 \\ E(Y_i) &= -45 \\ \text{Var}(Y_i) &= 11,525 \\ P(\dots) &\approx 0.0012 \end{aligned}$$

$$\begin{aligned} A &\geq 82.449 \\ 75.159 &< B < 82.4 \\ 65.196 &< C < 75.1 \\ 61.713 &< D < 65.1 \\ F &\geq 61.713 \\ E(Y_i) &= -6 \\ \text{Var}(Y_i) &= 260 \\ P(\dots) &\approx 0.0262 \end{aligned}$$

$$\begin{aligned} A &\geq 83.421 \\ 76.293 &< B < 83.4 \\ 67.788 &< C < 76.293 \\ 62.523 &< D < 67.7 \\ F &\geq 62.5 \\ E(Y_i) &= -15 \\ \text{Var}(Y_i) &= 1413 \\ P(\dots) &\approx 0.0099 \end{aligned}$$