Problem Sheet 0

1. If $z = 2 + 3j$. Find solutions of $z^{\frac{1}{4}}$
2. If $z1 = 2 + 3j$ and $z2 = 4 + 3j$. Find $z1/z2$
3. Let z be an arbitrary complex number.

   a) Show that $\mathrm{Re}\{z\} = \dfrac{z + z^*}{2}$

   b) Show that $j\,\mathrm{Im}\{z\} = \dfrac{z - z^*}{2}$

   where Re{z} is real part of z, Im{z} is imaginary part of z, and z^* is complex conjugate of z.

4. Using Euler's relation $e^{j\theta} = \cos\theta + j\sin\theta$, show that

   a) $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

   b) $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

   c) $2\cos A \sin B = \sin(A + B) - \sin(A - B)$

5. Let $z = re^{j\theta}$. Express in polar form the following functions of z

   a) z^2

   b) z^{-1}

   c) z^*

   d) jz

   e) zz^*

   f) z / z^*

6. Evaluate the following integrals

a) $\int_0^a e^{-5t} dt$

b) $\int_a^\infty e^{-2t} dt$

7. A relation from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

   a) The relation *f* is defined by: $f(x) = \begin{cases} x^2, 0 \le x \le 1 \\ 3x, 1 \le x \le 5 \end{cases}$

   Show that *f* is not a function.

   b) Find the domain and the range of the real function *f* defined by $f(x) = \sqrt{x-1}$

8. Solve the differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = 1 \; t \ge 0$$

9. Solve the difference equation:

$$y[n] - 2y[n-1] = 1 \; n \ge 0$$

10. Explain the concept of equivalence classes
11. Expand the following functions and indicate the values of *a* for which the function converges

   a) $\dfrac{1}{1-a}$

   b) $\dfrac{1}{(1-a)^2}$