Department of Mathematics
MTL 106 (Introduction to Probability and Stochastic Processes)
Tutorial Sheet No. 3
Answer for Selected Problems

1. $F(y) = \begin{cases} 0, & -\infty < y < 2 \\ \frac{y}{10}, & 2 \leq y < 4 \\ 1, & 4 \leq y < \infty \end{cases}$

2. (i)$f_Y(y) = \frac{1}{|b|\sqrt{2\pi}\sigma} e^{\frac{-1}{2}\left(\frac{y-(a+\mu b)}{b\sigma}\right)^2}, \quad -\infty < y < \infty$

   (ii)$f_Z(z) = \begin{cases} \frac{1}{\sqrt{2\pi z}} e^{\frac{-z}{2}}, & z > 0 \\ 0, & \text{otherwise} \end{cases}$

3. Y is uniformly distributed random variable on the interval (a, b)

4. $f_Y(y) = \frac{\alpha e^y}{(e^y+1)^{\alpha+1}}, \quad -\infty < y < \infty$

5. (a) Y is a continuous type random variable. (b) $f_Y(y) = \begin{cases} e^{-y} + \frac{1}{y^2} e^{-1/y}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$.

6. $\alpha = e^{-\lambda}$

7. (a) 91.6% (b)56.8%

8. $f_Y(y) = \sqrt{\frac{2}{\pi}} \mid y \mid e^{-\frac{1}{2}y^4}, \quad -\infty < y < \infty$

9. $f_Y(y) = \begin{cases} \frac{1}{\pi\sqrt{1-y^2}}, & -1 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

10. Z has mixed type distribution where pmf is given by

$$P[Z = z] = \begin{cases} \frac{1}{4}, & z = -1, 1 \\ 0, & \text{otherwise} \end{cases}$$

and density function given by

$$f_Z(z) = \begin{cases} \frac{1}{\pi(1+z^2)}, & -1 < z < 1 \\ 0, & \text{otherwise} \end{cases}$$

11. $P[X = x] = \begin{cases} \dfrac{\binom{n}{x} p^x q^{n-x}}{\sum_{i=0}^{r-1} \binom{n}{i} p^i q^{n-i}}, & x = 0, 1, \ldots, r-1 \\ 0, & \text{otherwise} \end{cases}$

12. $f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma(\phi(\beta)-\phi(\alpha))} exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2), & \alpha < x < \beta \\ 0, & \text{otherwise} \end{cases}$

13. (a)False (b)True (c)False

14. If $E(X^2) = (E(X))^2$, then X is a degenerate random variable taking a fixed value with probability 1.

15. $X =$ No. of games played, $P(X = k) = p_k(> 0), k = 4, 5, 6, 7$ $E(X) = \frac{93}{16}$

17. a) $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} < y < \sqrt{3} \\ 0, & \text{otherwise} \end{cases}$; $P(-2 < Y < 2) = 1$.

    b) 50

19. (a)$Y \sim U(0, 1)$ (b)$\frac{1}{12}$

20. (a) $P(X = 1) = 1$; $E[(X - E(X))^4] = 0$ (b) $P(-1/2 < X \leq 3) = 1$ and $P(X = 0) = 0$

21. (a) $X \sim P(\mu)$ (b) $\sum_{k=1}^{7} \frac{e^{-\mu}\mu^k}{k!}$; $\mu = 4$

22. (a) $e^{-\lambda}$

23. ?

24. $e^{\sigma^2 t^2/2}$

    $E(X^n) = \begin{cases} 0 & n - \text{odd} \\ \frac{n!}{(n/2)!2^{n/2}}\sigma^n & n - \text{even} \end{cases}$

25. $P(-1.062 < X < 0.73) = \frac{2}{3}$