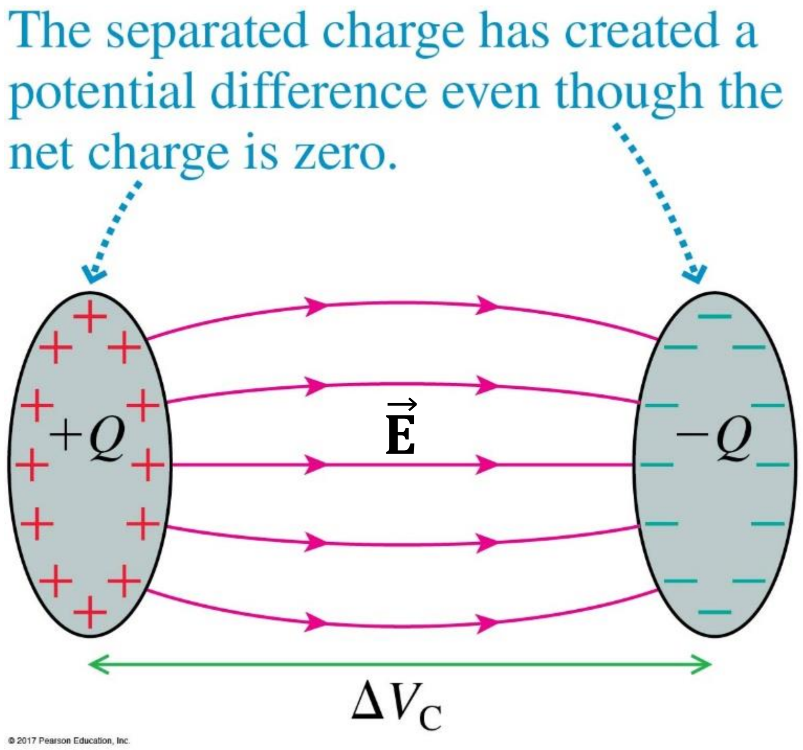
PYL 102

Thursday, Oct. 03, 2024

Dielectric properties of solids

Capacitors and Capacitance

- A capacitor consists of two conductors separated by empty space or an insulator.
- Charges are transferred, giving the conductors equal but opposite charges.
- Capacitance measures the magnitude of the charge for a given potential difference.
- The separation of charge establishes an electric field, which stores energy.



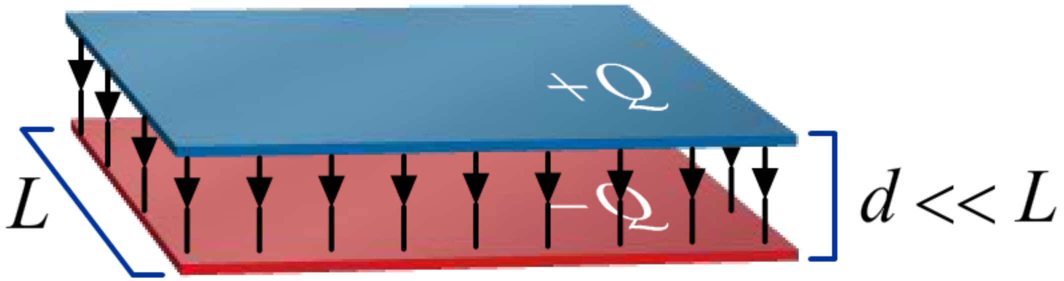
$$\boxed{C := \frac{Q}{\Delta V}}$$

SI unit: $1\ \text{farad} = 1\ \text{F} = 1\ \text{C/V}$

Parallel-plate Capacitors

A common, simple type of capacitor is the parallel-plate capacitor. It consists of two plates of surface area A, separated by a distance d.

We generally make the assumption that the separation d is very small compared to the (perpendicular) dimensions of the plates.



$$E = \frac{Q}{\varepsilon_0 A}$$

$$C = \frac{\varepsilon_0 A}{d}$$ (Parallel-plate)

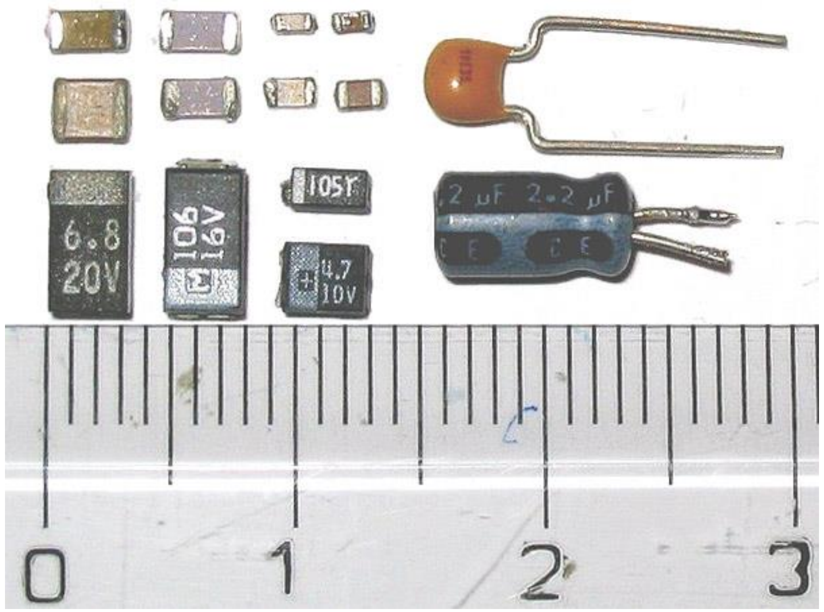
Capacitance depends only on the geometry of the conductors:

- Surface area of the conductors and the distance they are separated.
- Other shapes of capacitors have different formulas, but the qualitative behavior is the same.

Capacitor Examples

There are many varieties of capacitors:

https://commons.wikimedia.org/wiki/File:Capacitors_(7189597135).jpg



http://commons.wikimedia.org/wiki/File:Photo-SMDcapacitors.jpg

The general properties of a parallel-plate capacitor – that the capacitance increases as the plates become larger and decreases as the separation increases – are common to all capacitors.

Capacitance

How can we increase the capacitance?

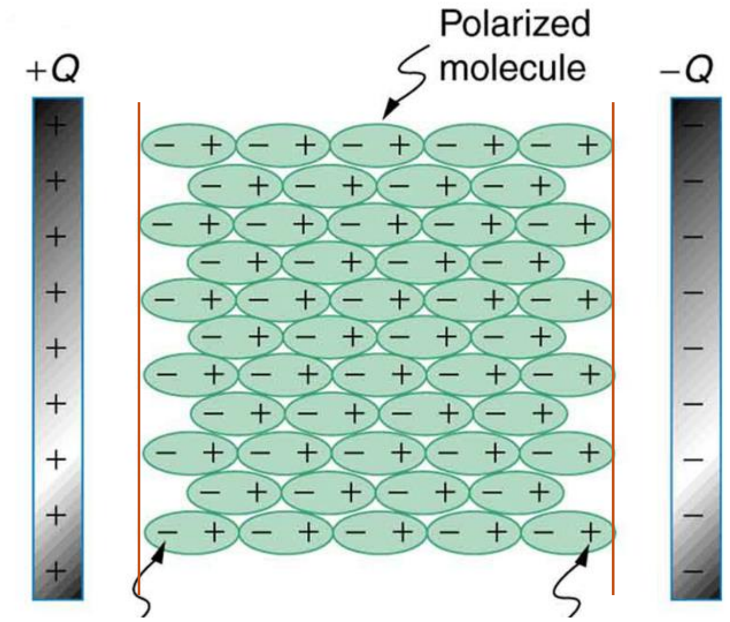
$$C := \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Increase the area of the plates so that they can store more charge.
- Another way would be to decrease the separation of the plates.
- Physically, is there some difficulty in doing so?

Assume a 1.0 F parallel plates capacitor having plates 1.0 mm apart. Find the area of the plates.

Capacitors and Dielectrics

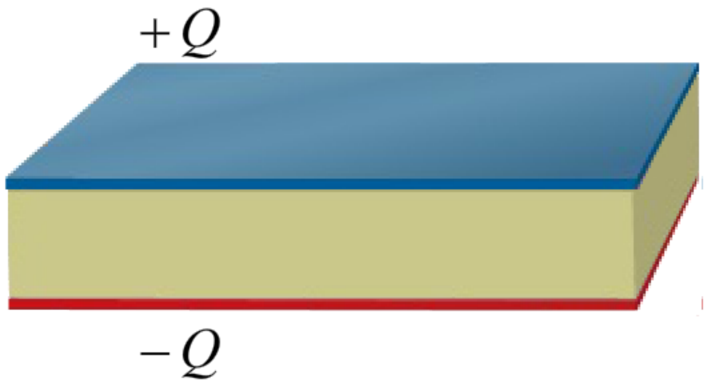
- Instead of just empty space, it can be beneficial to add an insulating material called a *dielectric* between the plates
- In general, electrical insulators in an electric field will be polarized:
  - The dipoles align with the E-field from the capacitor plates, polarizing the material.
  - This reaction induces a secondary E-field due to the polarization
  - What can we say about the total electric field between the capacitor plates now? Did it increase or decrease?



Capacitors and Dielectrics

- Consider an isolated capacitor (i.e. Q is constant).
- Since, $C := \frac{Q}{\Delta V}$ and, capacitance will increase.
- Dielectrics will increase capacitance!

Adding a dielectric to an isolated capacitor:



E decreases: $E = E_0/\kappa$

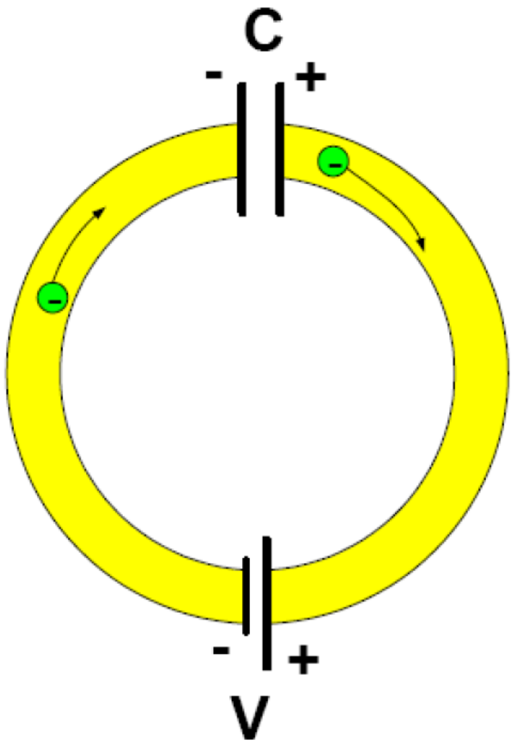
V decreases: $V = V_0/\kappa$ $\Delta V = -E\Delta s$

C increases: $\boxed{C = \kappa C_0}$

Dielectric constant of material: κ

Energy Stored in a Capacitor

External work needs to be done in order to charge a capacitor. Assume at one moment during the charging of capacitor, the charge is q and the potential difference is ΔV, if amount of dq charge is moving from one plate to another, the work done by external force (or battery) is



$$dW = \Delta V dq$$

The total work done (equal to the total potential energy gained) by the external force is

$$W = \int_0^Q \Delta V dq$$

Energy Stored in a Capacitor

Since $\Delta V = \dfrac{q}{C}$

Then $$W = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

So, a capacitor separated by electric potential *V* has a potential energy

$$U = \frac{1}{2}\frac{Q^2}{C}$$

Since $Q = CV$, so $$U = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Properties of Dielectrics

Redistribution of charge – called polarization

$$K = \frac{C}{C_0}$$ dielectric constant of a material

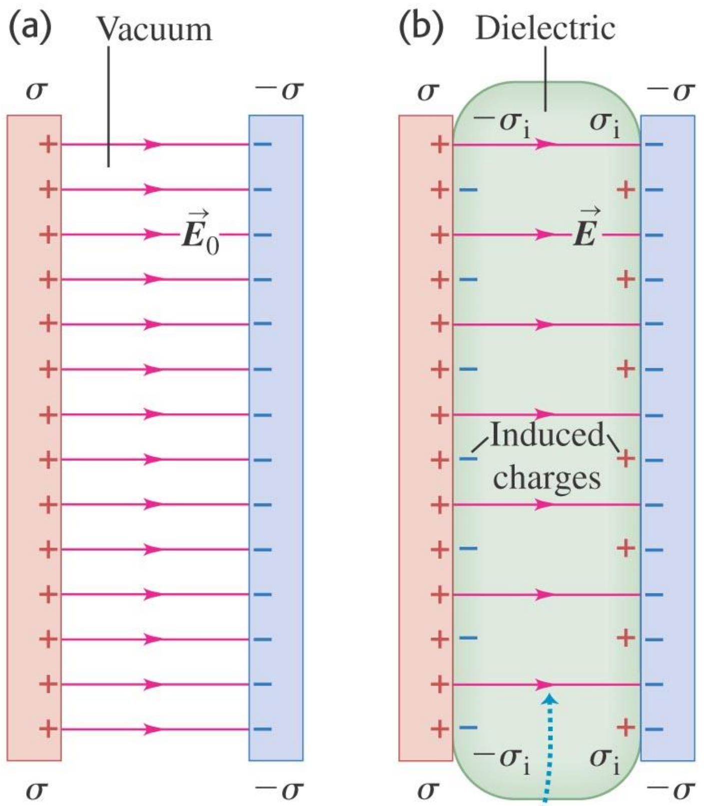
We assume that the induced charge is directly proportional to the E-field in the material

$$E = \frac{E_0}{K}$$

when Q is kept constant

$$V = \frac{V_0}{K}$$

In dielectrics, induced charges do not exactly compensate charges on the capacitance plates



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$$E_0 = \frac{\sigma}{\varepsilon_0}; \qquad E = \frac{\sigma - \sigma_i}{\varepsilon_0} \qquad \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$ Induced charge density

$$\left\{ \begin{array}{l} \varepsilon = K\varepsilon_0 \\ \\ E = \dfrac{\sigma}{\varepsilon} \end{array} \right.$$

$\varepsilon = K\varepsilon_0$ Permittivity of the dielectric material

$E = \frac{\sigma}{\varepsilon}$ E-field, expressed through charge density σ on the conductor plates (not the density of induced charges) and permittivity of the dielectric ε (effect of induced charges is included here)

$$u = \frac{1}{2}\varepsilon E^2$$ Electric field density in the dielectric

The relative permittivity (or the dielectric constant) ε_r is defined to reflect this increase in the capacitance or the charge storage ability by virtue of having a dielectric medium.

It is important to remember that when the dielectric medium is inserted, the electric field remains unchanged, provided that the insulator fills the whole space between the plates. The voltage V remains the same and therefore so does the gradient V/d, which means that E_o remains constant.

Dielectric Breakdown

If the electric field in a dielectric becomes too large, it can rip electrons from atoms, enabling the now-ionized material to conduct.

This is called *dielectric breakdown*; the field at which this happens is called the *dielectric strength*.

TABLE 20-2 Dielectric Strengths

Substance	Dielectric Strength (V / m)
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Pyrex glass	14×10^6
Neoprene rubber	12×10^6
Air	3.0×10^6

Capacitor Energy and Separation

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. Suppose the plates are now pulled apart until they are separated by a distance $D > d$. The electric potential energy stored in the capacitor

1) Decreases
2) Is unchanged
3) Increases

Capacitor Energy and Separation

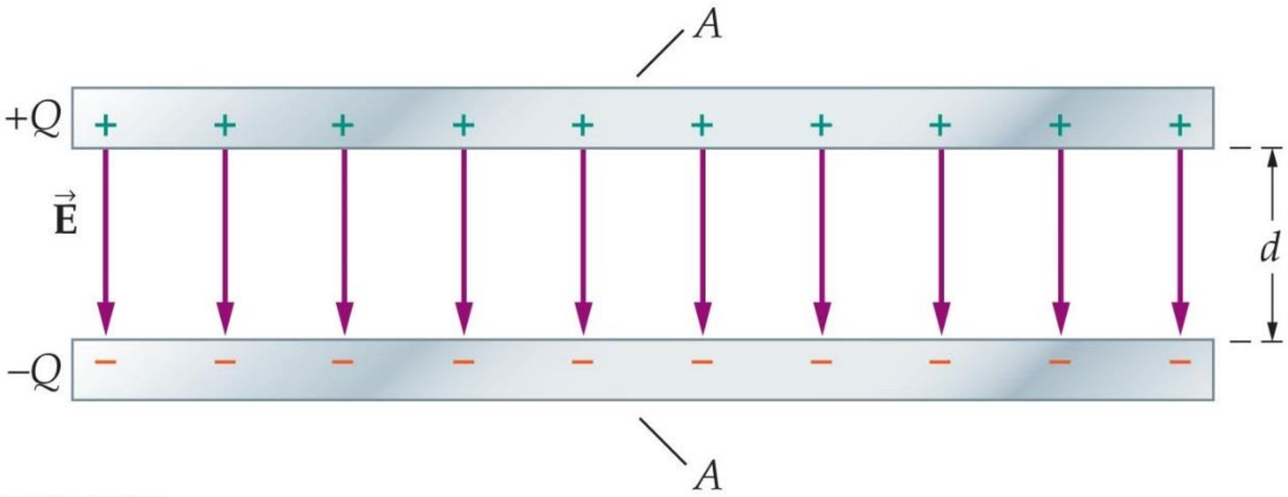
Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. Suppose the plates are now pulled apart until they are separated by a distance $D > d$. The electric potential energy stored in the capacitor

1) Decreases
2) Is unchanged
3) Increases

Example: Capacitor

A parallel-plate capacitor is constructed with plates of area $0.0360\ \mathrm{m}^2$.

a) If a $10.0\ \mathrm{nC}$ charge is to result in a potential difference of $25.0\ \mathrm{V}$, what separation between the plates is needed?
b) How many electrons does this charge correspond to?
c) What is the electric field between the plates?
d) What if a sheet of mica ($\kappa = 5.4$) is inserted into the capacitor, while maintaining the same potential difference?



Example: Capacitor

Known quantities:

$$A = 0.0360\ \mathrm{m}^2, \qquad Q = 10.0\ \mathrm{nC}, \qquad V = 25.0\ \mathrm{V}$$

a)

$$C = \frac{\varepsilon_0 A}{d}, \qquad C = \frac{Q}{V} \quad \Rightarrow \quad \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

$$d = \frac{\varepsilon_0 AV}{Q} = \frac{\left(8.85 \times 10^{-12}\ \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2\right)\left(0.0360\ \mathrm{m}^2\right)(25.0\ \mathrm{V})}{1.00 \times 10^{-8}\ \mathrm{C}}$$

$$= 7.965 \times 10^{-4}\ \mathrm{m} \approx 0.797\ \mathrm{mm}$$

b)

$$\mathrm{N}umber\ of\ e^- = \frac{Q}{e} = \frac{1.00 \times 10^{-8}\ C}{1.60 \times 10^{-19}\ C} = 6.24 \times 10^{10}$$

Example: Capacitor

c)

$$E = \frac{Q}{\varepsilon_0 A} = \frac{1.00 \times 10^{-8}\ \mathrm{C}}{(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(0.0360\ \mathrm{m^2})}$$

$$= 31387\ \mathrm{N/C} \approx 3.14 \times 10^{4}\ \mathrm{V/m}$$

d) $Q = \kappa Q_0 = (5.4)(10.0\ \mathrm{nC}) = 54.0\ \mathrm{nC}$

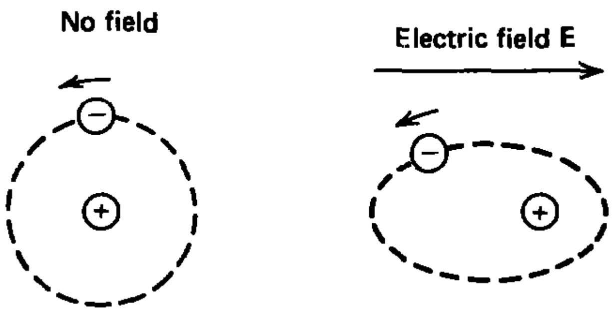
$$C = \kappa C_0$$

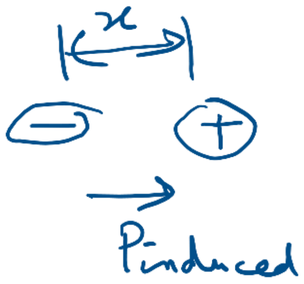
$$C_0 = \frac{Q_0}{V_0} = \frac{1.00 \times 10^{-8}\ \mathrm{C}}{25.0\ \mathrm{V}} = 4.00 \times 10^{-10}\ \mathrm{F} = 400\ \mathrm{pF}$$

$$C = (5.4)(400\ \mathrm{pF}) = 2.16\ \mathrm{nF}$$

Furthermore, on average, the center of negative charge of the electrons coincides with the positive nuclear charge, which means that the atom has no net dipole moment.

When this atom is placed in an external electric field, it will develop an induced dipole moment. The electrons, being much lighter than the positive nucleus, become easily displaced by the field, which results in the separation of the negative charge center from the positive charge center. This separation of negative and positive charges and the resulting induced dipole moment are termed polaization. An atom is said to be polaized if it possesses an effective dipole moment, that is, if there is a separation between the centers of negative and positive charge distributions.





Induced Dipole moment

Polaizability relates the induced dipole moment to the E field causing it as $\boxed{P_{induced} = \alpha E}$
where α is a coefficient called the polarizability of the atom. Since the polarization of a neutral atom involves the displacement of electrons, α_e is called electronic polarization.

When an electric field E is applied, the light electrons become displaced in the opposite direction to E, so their center of mass is shifted by some distance *x* with respect to the nucleus. As the electrons are "pushed " away by the applied ield, the Coulombic attraction between the electrons and nuclear charge " pulls in " the electrons.

$$\boxed{\text{Restoring force } F_r = -\beta x = ZeE \qquad \textit{at equilibrium}}$$

Therefore the magnitude of the induced electronic dipole moment $P_e = Ze(x)$ is given by

$$\boxed{P_{induced} = P_e = \frac{Z^2e^2E}{\beta}}$$

pe is proportional to the applied field.

$$\boxed{\alpha_e = \frac{Ze^2}{m_e\omega_o^2}}$$

Suppose the applied electric field polarizing the atom is removed.

The equation of motion of the negative charge center is then $-\beta x = Zm_e\dfrac{d^2x}{dt^2}$

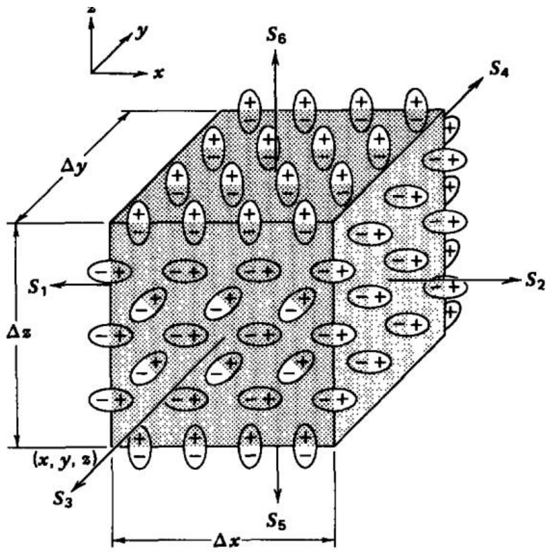
The displacement at any time is $x = x_o\cos(\omega_o t)$ SHM

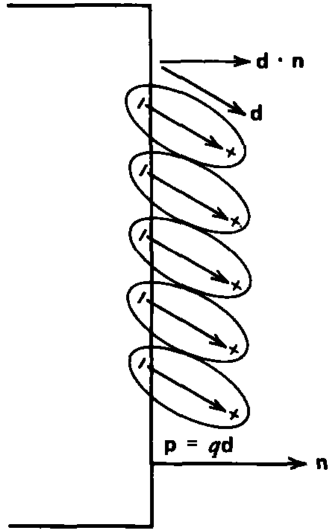
$$\boxed{\omega_o = \sqrt{\frac{\beta}{Zm_e}}}$$

ω_o is called the electronic polarization resonance frequency.

Polarization P

- When a material is placed in an electric field, the atoms and the molecules of the material become polarized, so we have a distribution of dipole moments in the material.
- Charges +Qp and -Qp appear on the opposite surfaces of a material when it becomes polarized in an electric field. These charges are bound and are a direct result of the polarization of the molecules. They are termed surface polarization charges.





All totally enclosed dipoles contribute no net charge.

Polarization P is defined as the total dipole moment per unit volume

$$\boxed{P = \frac{1}{volume}[p_1 + p_2 + p_3 + \cdots \ldots + p_N}$$ ☆

where p_1, p_2,….,p_N are the dipole moments induced at N molecules in the volume.

We can view this arrangement as one big dipole moment ptotai from -Qp to +Qp. Thus $\boxed{P_{total} = Q_p d}$

The magnitude of $\boxed{P = Q_p d/Ad = Q_p/A}$

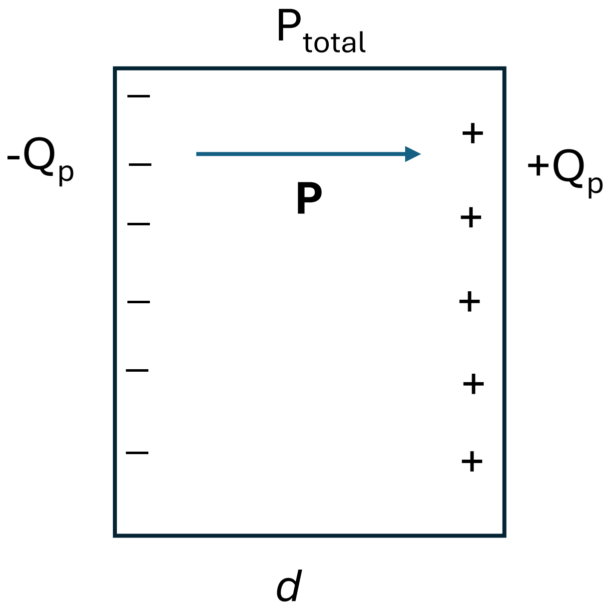
$\boxed{\sigma_p = Q_p/A}$ is the surface polarization charge density

$\boxed{P = \chi \varepsilon_o E}$ where χ is called the electric susceptibility

N -> number of dipoles per unit volume

$$P = \chi \varepsilon_o E = N P_{induced} = N \alpha_e E$$

$$\boxed{\chi = \frac{N\alpha_e}{\varepsilon_o}}$$ approx



The field E before the dielectric was inserted is given by $E = \frac{V}{d} = \frac{Q_o}{\varepsilon_o A} = \frac{\sigma_o}{\varepsilon_o}$

where σ_o is the free surface charge density without any dielectric medium between the plates

After the insertion of the dielectric, this field remains the same V/d, but the free charges on the plates are different. Additional free charges Q - Qo needed on the plates to neutralize the opposite polarity polarization charges Qp appearing on the dielectric surfaces.

$$Q = Qo + Qp$$

Q = Qo + Qp

Dividing by A, defining = Q/A as the free surface charge density on the plates with the dielectric inserted

$$\sigma = \varepsilon_o E + \sigma_p$$

since σ_p = P and $P = \chi \varepsilon_o E$

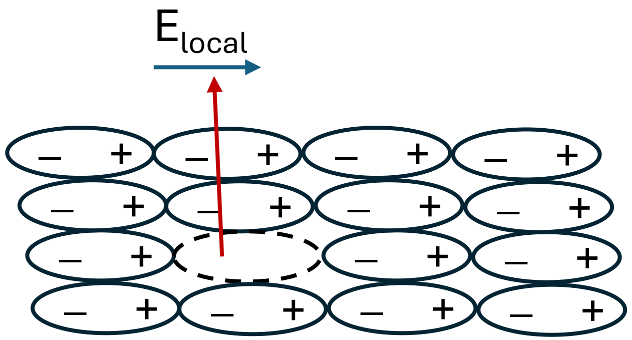
$$\sigma = \varepsilon_o (1 + \chi) E$$

The relative permittivity is defined as $\boxed{\varepsilon_r = \frac{Q}{Q_o} = \frac{\sigma}{\sigma_o}}$

$$\boxed{\varepsilon_r = 1 + \chi}$$

$$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_o}$$

microscopic polarization α_e determines the macroscopic property ε_r



The electric field inside a polarized dielectric at the atomic scale is not uniform. The local field is the actual field that acts on a molecule. It can be calculated by removing that molecule and evaluating the field at that point from the charges on the plates and the dipoles surrounding the point.