2202 COL 352 Quiz1

Viraj Agashe

TOTAL POINTS

10 / 10

QUESTION 1

True or False 6 pts

1.1 Containment Regular language 3 / 3

+ 0 pts Incorrect/No Explanation

+ 1 pts Partially correct with some right ideas

✓ **+ 3 pts** *Correct*

1.2 3 / 3

✓ **- 0 pts** *Correct*

- 3 pts Incorrect

QUESTION 2

2 Divisibility by 4 4 / 4

✓ **+ 4 pts** *Completely Correct*

+ 0 pts Completely Incorrect/Unattempted

+ 3 pts Correct construction, but mistakes in proof of correctness

+ 2 pts Correct construction, but no proof of correctness

+ 1 pts There is an attempt, but construction is wrong

Name: VIRAJ AGASHE Roll No: 2020CS10567

(COL 352) Introduction to Automata and Theory of Computation Jan 25, 2023

Quiz 1

Duration: 40 minutes **(10 points)**

Beware: Be clear in your writing. If you use a statement proved in class or in the problem set, then write down the entire statement before using it. You will not get a new sheet, so make sure you are certain when you write something (maybe use a dark pencil). Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space. If you cheat, you will surely get an F in this course.

---

1. **(2 × 3 = 6 points)** For the questions that follow answer whether they are True/False with a brief justification. Each question carries 4 points. Simply writing True or False will not get you any points.

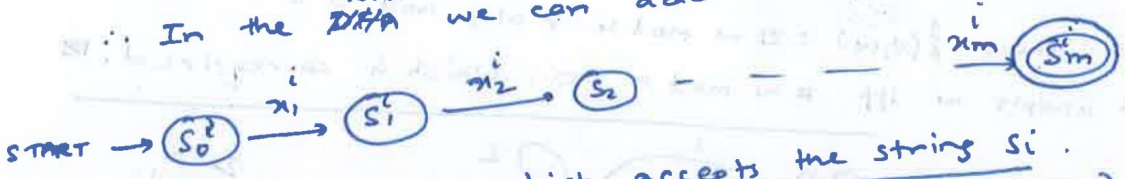
   (a) If L is a regular language and $L' \subseteq L$, then L' is also a regular language.

FALSE.

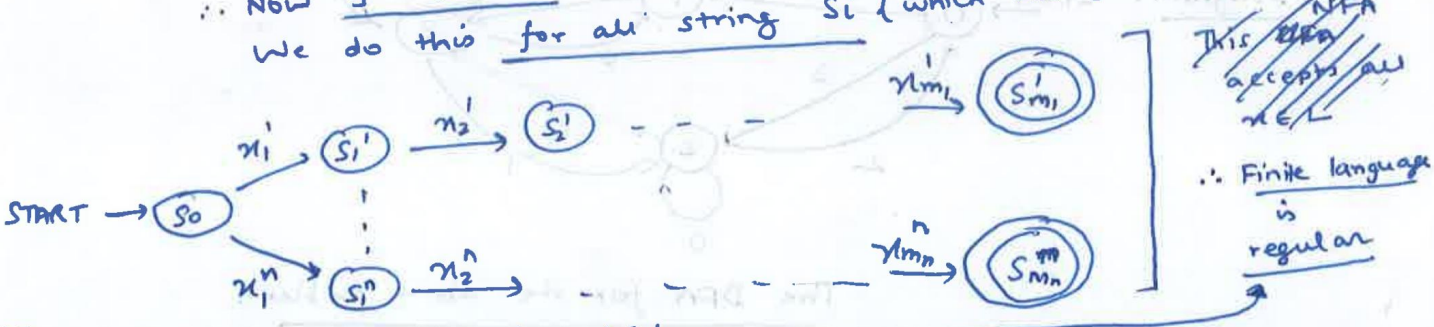
Consider the language Σ^* (set of all strings over the characters Σ)
By closure properties we know that Σ^* is regular since Σ is regular (finite characters). For simplicity take $\Sigma = \{0,1\}$. Now, note that any language over $\Sigma \subseteq \Sigma^*$.
However, we have already proved an impossibility result in class that $\exists$ languages over Σ which are not regular. Take any such language L'. Now $L' \subseteq \Sigma^*$ but L is NOT regular - Hence disproved.
(For example: $L = 0^*1^*$, $L' = \{0^n 1^n \mid n \geq 0\}$, L' is not regular but L is, and $L' \subseteq L$)

   (b) Any finite language is a regular language.

Sol. TRUE. Any finite language is a regular language.
We can enumerate all strings of the language as
$s_1, s_2, \ldots s_n$ (say $|L| = n$). For each $s_i \in L$,
say $s_i = x_1^i x_2^i \ldots x_m^i$ (say)
∴ In the NFA we can add states as:



∴ Now ∃ a run which accepts the string s_i.
We do this for all string s_i (which are finite).



2. (4 **points**) Construct a DFA that works over the alphabet $\{0, 1, 2\}$ which recognizes base-3 representations of numbers divisible by 4. Give a brief proof of correctness of your construction.

Let #w represent base-3 value of string.

Sol. Let the DFA $\equiv (Q, \Sigma, \delta, q_0, F)$, where:

$Q = \{0, 1, 2, 3\}$

$\Sigma = \{0, 1, 2\}$

$\delta(x, a) = (3\#x + \#a) \bmod 4$

$q_0 = 0$

$F = \{0\}$

Proof of correctness: To prove the correctness, we claim that $\hat{\delta}(0, w) = \#w \bmod 4$

Proof. By induction on $|w|$.

Base case $|w| = 0 \Rightarrow w = \epsilon$

$\therefore \hat{\delta}(0, \epsilon) = \delta(0, \epsilon) = 0 = \#0 \bmod 4$

I.H Assume true for all $|w| < k$

I.S Consider any $|\bar{w}| = k$ string $\bar{w}$. Then, let $\bar{w} = w \cdot a$

$\hat{\delta}(0, wa) \equiv \delta(\hat{\delta}(0, w), a)$ (by defn)

$\equiv \delta(\#w \bmod 4, a)$ (by I.H)

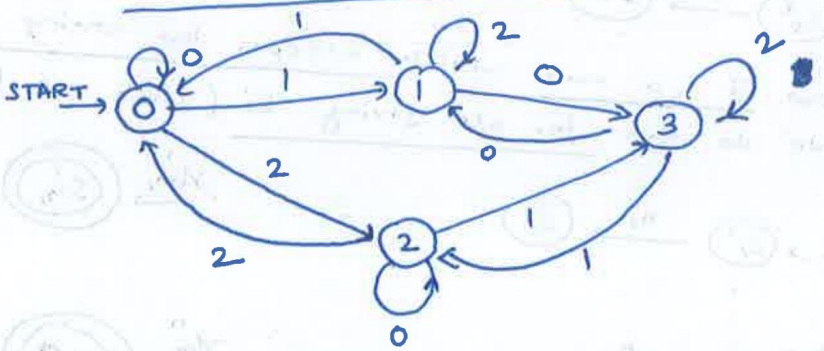
$\equiv (3\#w \bmod 4 + \#a) \bmod 4$ (by defn of δ)

$\equiv ((3\#w + \#a) \bmod 4) \bmod 4$ (by modular arithmetic)

$\equiv \#\bar{w} \bmod 4$ (since $\#\bar{w} = 3\#w + \#a$ (base 3 notation))

Hence, since $\hat{\delta}(0, w) = \#w \bmod 4 \ \forall w$, and 0 is the only accepting state, DFA accepts w iff $\#w \bmod 4 = 0$, which is as required. ∎

Diagram:



The DFA for the above problem.