COL 351 : Analysis and Design of Algorithms
Semester I, 2022-23, CSE, IIT Delhi

Assignment - 2 (due on 22nd September, 11:00 PM)

Important Guidelines:

- Each assignment must be done in a group of size at most two.
- Handwritten submissions will not be accepted. Solutions must be typed-up (in Latex, Microsoft Word, etc.), and submitted in pdf format. Each solution must start on a new page.
- **Your answer to each question must be formal and have a proper correctness proof.** No marks will be granted for vague answers with intuition or for algorithms without proof. You must be very rigorous in providing mathematical detail in support of your arguments.
- Cheating of any form will lead to strict penalty.

1 Maximum sum

Let $D = (d_1, \ldots, d_n)$ be a disc with n sectors such that sector d_i in disc stores an integer p_i, for $i \in [1, n]$. Design an $O(n)$ time algorithm to compute a contiguous collections of sectors in D such that the sum of integers associated with the chosen sectors is maximum. [18 marks]

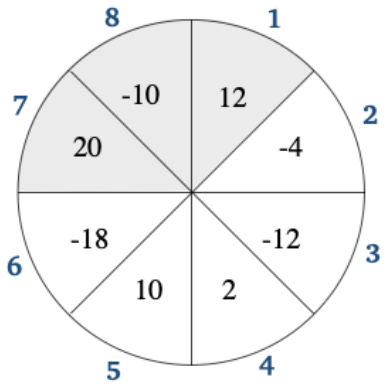

Figure 1: Depiction of a disc with $n = 8$ sectors. The contiguous collection of sectors that has maximum associated sum is $\{1, 7, 8\}$.

2 Forex Trading

Suppose you are a trader aiming to make money by taking advantage of price differences between different currencies. You model the currency exchange rates as a weighted graph G, wherein, the vertices correspond to n currencies - $c_1, \ldots, c_n$, and the edge weights correspond to exchange rates between these currencies. In particular, for a pair (i, j), the weight of edge (i, j), say $R(i, j)$, corresponds to total units of currency c_j received on selling 1 unit of currency c_i.

1. Design an algorithm to verify whether or not there exists a cycle $(c_{i_1}, \ldots, c_{i_k}, c_{i_{k+1}} = c_{i_1})$ such that exchanging money over this cycle results in positive gain, or equivalently, the product $R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1]$ is larger than 1. [10 marks]

   (Hint: Use the fact that a number x is strictly larger than 1 if and only if $\log(1/x) < 0$).

2. Present a cubic time algorithm to print out such a cyclic sequence if it exists. [7 marks]

Remark: For simplicity you can assume that the graph G is strongly-connected.

3 Disjoint collection of Paths

Let $T = (V, E)$ be a rooted binary tree on n vertices, and S be a set of paths in the tree T. Two paths $P, Q \in S$ are said to be disjoint if they do not have a common edge.

Design a polynomial in n time algorithm which finds a subset X of S of maximum size such that each pair of paths in X is disjoint. [25 marks]

Remark: The size of set S is at most ${}^nC_2$ since T is acyclic. Note that each path in S can be identified by just its endpoints.