ELL205 Signals and Systems — 40 Marks

Major Examination — April 2022

Useful Formulas:

1.

DTFS

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\Omega_o n}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\Omega_o n}$$

DTFT

$$x[n] = \frac{1}{2\pi}\int_0^{2\pi} H(e^{j\Omega}) e^{jn\Omega} d\Omega$$

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

CTFS

$$a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_o t} dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

CTFT

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

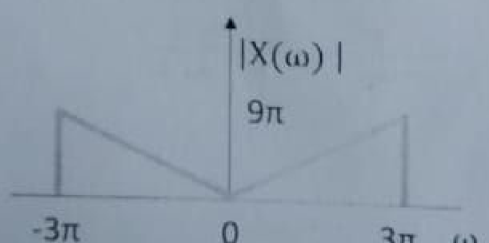
Laplace: $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

Z Transform: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

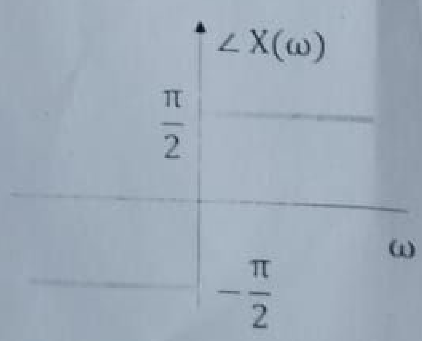
Important Instructions:

Each question carries 8 marks.

1. Find $x(t)$ whose Fourier transform $X(\omega)$ has the following magnitude and angle



a. Magnitude response



b. Phase response

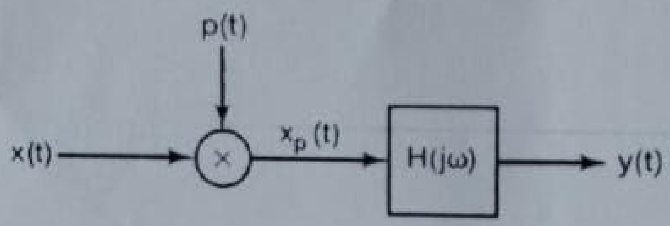
2. Shown in the figure below is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

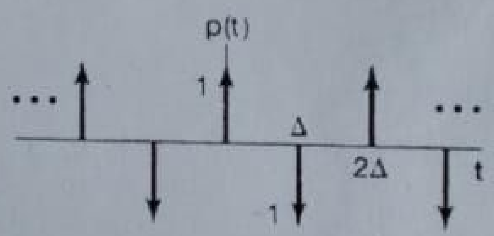
(a) For $\Delta < \pi/(2w_M)$, sketch the Fourier transform of x_p(t) and y(t).

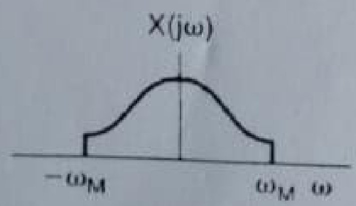
(b) For $\Delta < \pi/(2w_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.

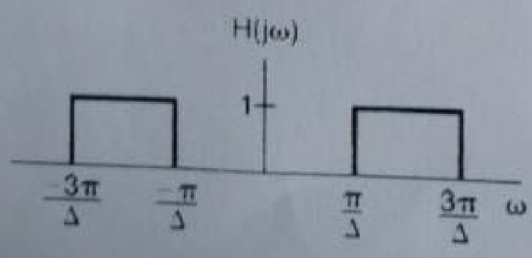
(c) For $\Delta < \pi/(2w_M)$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the maximum value of Δ in relation to w_m for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$?









3. Given system function $H(s) = \frac{1}{(s+0.6421)}$, and the input $x(t) = \alpha\delta(t) + \cos(t)\,u(t)$, determine constant α such that system satisfies condition of initial rest and for $t \geq 0$ the system's response contains the sinusoidal steady-state only.

4. $x(t)$ has a discrete representation $x[n]$ if $x[n] = x(nT)$ and similarly $y(t)$ has a discrete representation $y[n]$ if $y[n] = y(nT)$. Assuming that Nyquist sampling theorem is satisfied in the previous equations with equality, find the discrete representation of $(x * y)(t)$. (Answer should involve $x[n]$ and $y[n]$)

5. A $Nth-$order Butterworth filter has a frequency response the square of whose magnitude is given by

$$|B(j\omega)|^2 = \frac{1}{1 + \left(\frac{j\omega}{j\omega_c}\right)^{2N}}$$

where N is the order of the filter.

a) If the impulse response of the Butterworth filter is real, causal and stable, find the poles of the filter for $N = 2$.

b) Draw the system details of this filter.

c) The discrete-time filter is obtained by the bilinear transformation (where $s = \frac{1-z^{-1}}{1+z^{-1}}$) of the continuous-time filter. Find the poles in the z-plane.

d) Is the resultant discrete-time filter causal and stable? Justify.