2202 COL 352 Quiz 3

CHINMAY MITTAL

TOTAL POINTS

10 / 10

QUESTION 1

1 Unary undecidable 4 / 4

- **+ 0 pts** Incorrect/Not Attempted
- ✓ **+ 1 pts** *Statement is True*
- ✓ **+ 3 pts** *Correct Proof*
- **+ 2 pts** Partially correct proof

QUESTION 2

2 Unary PCP 6 / 6

- ✓ **+ 6 pts** *Correct*
- **+ 0 pts** Incorrect
- **+ 3 pts** Partially Correct
- **+ 1 pts** Decidable

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(COL 352) Introduction to Automata and Theory of Computation April 19, 2023

Quiz 3

Duration: 40 minutes **(10 points)**

Beware: Be clear in your writing. If you use a statement proved in class or in the problem set, then write down the entire statement before using it. You will not get a new sheet, so make sure you are certain when you write something (maybe use a dark pencil). Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space. If you cheat, you will surely get an F in this course.

1. **(4 points)** Prove or disprove: There exists a unary language (i.e., $L \subseteq \{1\}^*$) which is undecidable.

Consider the set of all unary string $= \{1\}^*$. clearly, This set is countable infinite. We can make an surjection from $\mathbb{N} \cup \{0\}$ to $\{1\}^*$ (map n to string with n 1's)

The set of all unary Languages $A = \{ L \mid L \subseteq \{1\}^* \}$ is the power set of $\{1\}^*$ and as shown in class - it has the same cardinality as the set of Real Numbers $\mathbb{R}$.

Let $M = \{ \langle m_i \rangle \mid m$ is a Turing machine decider for a unary languages over $\{1\} \}$

clearly $|M| \leq |\mathbb{N}|$ since M is a subset of $\{0,1\}^*$ and hence is countable. ($\{0,1\}^*$ is countable)

Clearly $|M| \leq |\mathbb{N}| < |\mathbb{R}| = |A|$

Hence no surjective mapping exists M to A

$\Rightarrow$ there exists a unary language for which there is no turing machine decider.

and Hence there exists a unary language which is undecidable.

☆ M $\longrightarrow$ compute $a_i = |x_i| - |y_i| \ \forall i$

if all $a_i > 0$ or all $a_i < 0$ $\longrightarrow$ Halt and reject

otherwise Halt and accept.

2. **(6 points)** Recall the Post Correspondence problem (PCP): You are given as input, a set of pairs of strings $P = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ over the alphabet $\Sigma = \{0, 1\}$. You are required to decide if there exists a finite sequence $\langle i_1, i_2, \ldots, i_m \rangle \in \{1, 2, \ldots, n\}^m$ such that $x_{i_1} \ldots x_{i_m} = y_{i_1} \ldots y_{i_m}$ (we stated the problem a little differently in class, but these two formulations are equivalent). Prove or disprove: Unary PCP (i.e., PCP where $\Sigma = \{1\}$) is undecidable.

This problem can be mapped to a similar problem (the key idea being that since there is only one letter in the alphabet the ordering of dominoes in the match doesn't matter)

for each i, we compute $|x_i| - |y_i| = a_i$ (extra 1's that the numerator string has compared to the denominator string)

The problem is equivalent to finding $\alpha_i \in \mathbb{N}$ such that

$$\sum \alpha_i a_i = 0 \quad \text{— (1)}$$

since order of dominoes does not matter we always keep all the dominoes of type (x_1, y_1) first then all of (x_2, y_2) and so on....

A match corresponds to an equal number of one's in the numerator and denominator which is equivalent to equation (1).

if any $a_i = 0$ we can keep $\alpha_i = 1$ and $\alpha_j = 0$ for $j \neq i$ $\rightarrow$ YES

if all $a_i > 0$ Then no match exists $\rightarrow$ No (all α_i need to be 0 which is not possible and otherwise eq^n is >0)

if all $a_i < 0$ then no match exists $\rightarrow$ No (all α_i need to be 0 which is not possible and otherwise eq^n is <0)

remaining case is when one $a_i < 0$ and one $a_j > 0$ exists (others can be anything)

in this case a match always exists

put $\alpha_i = |a_i|$ and $\alpha_j = |a_j|$, others are zero

$$\sum \alpha_i a_i = -a_i a_j - a_i a_j = 0$$

Hence the unary PCP decidable by the following M which implements the above algorithm

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