Hints to problem sheet 12

SECTION: A = all questions are property based.

SECTION: B

Ques 1.

From Clue 1, we know that $x[n]$ is real. Therefore, the poles and zeros of $X(z)$ have to occur in conjugate pairs. Since Clue 4 tells us that $X(z)$ has a pole at $z = (1/2)e^{j\pi/3}$, we can conclude that $X(z)$ must have another pole at $z = (1/2)e^{-j\pi/3}$. Now, since $X(z)$ has no more poles, we have to assume that $X(z)$ has 2 or less zeros. If $X(z)$ has more than 2 zeros then $X(z)$ have poles at infinity. Since Clue 3 tell us that $X(z)$ has 2 zeros at the origin, we know that $X(z)$ must be of the form

$$X(z) = \frac{Az^2}{\left(z-\frac{1}{2}e^{\frac{i\pi}{3}}\right)\left(z-\frac{1}{2}e^{-\frac{i\pi}{3}}\right)}$$

Since Clue 5 tell us that $X(1) = 8/3$, We may conclude that A=2. Therefore,

$$X(z) = \frac{2z^2}{\left(z-\frac{1}{2}e^{\frac{i\pi}{3}}\right)\left(z-\frac{1}{2}e^{-\frac{i\pi}{3}}\right)}$$

Since $x[n]$ is right sided, the ROC must be $|z| > 1/3$.

Ques 2.

(a) Using the shift property, we get

$$Z\{\Delta x[n]\} = X(z) - z^{-1} X(z) = (1 - z^{-1}) X(z)$$

(b) The z-transform $X_1(z)$ is given by

$$X_1(z) = \sum_{-\infty}^{\infty} x_1(n)\, z^{-n}$$
$$= \sum_{-\infty}^{\infty} x(n)\, z^{-2n}$$
$$=X(2z).$$

(a) Let us define a signal $g(n) = \{x(n) + (-1)^n x(n)\}/2$. Note that $g[2n] = x[2n] = 0$ For n odd. Also, using Table 10.1, we get

$$G(z) = \frac{1}{2}X(z) + \frac{1}{2}X(-z).$$

The z-transform $X_1(z)$ is given by

$$X_1(z) = \sum_{-\infty}^{\infty} x_1(n)\, z^{-n}$$
$$= \sum_{-\infty}^{\infty} g(2n)\, z^{-n}$$
$$= \sum_{-\infty}^{\infty} g(n)\, z^{-n/2}$$
$$= G(z^{1/2}).$$

$$= \frac{1}{2}X(z^{1/2}) + \frac{1}{2}X(-z^{1/2}).$$

Ques 3.

In each part of this problem, we assume that the signal obtained by taking the inverse z-transform is called $x[n]$.

(a) Yes. The order of the numerator is equal to the order of the denominator in the given z-transform. Therefore, we can perform the long division to expand the z-transform such that the highest power of z in the expansion is 0. This would make $x[n] = 0$ for $n < 0$.

(b) No. This z-transform can be obtained by multiplying the z-transform of the previous part by z. Hence, its inverse is the inverse of the previous part shifted by 1 to the left. This implies that the resultant signal is not zero at $n = -1$.

(c) Yes. We can perform the long division to expand the z-transform such that the highest power of z in the expansion is -1.This would make $x[n] = 0$ for $n \leq 0$.

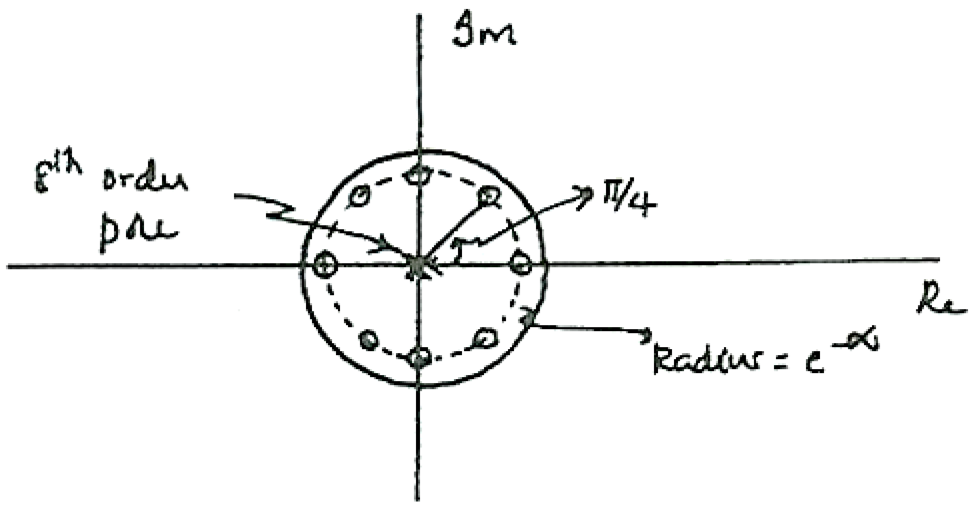
(d) No. When long division is used to expand the z-transform, the highest power of z in expansion is 1.This would make $x[-1] \neq 0$.

Ques 4.

(a) Taking the z-transform of both the sides of the difference equation relating $x[n]$and $s[n]$ and simplifying, we get

$$H_1(z) = \frac{X(z)}{S(z)} = 1 - z^{-8}e^{-8\alpha} = (z^8 - e^{-8\alpha})/z^8.$$

The system has an 8^{th} power pole at $z = 0$ and 8 zeros distributed around a circle of radius $e^{-\alpha}$. This is shown in figure below. The ROC is everywhere on the z-plane except at $z = 0$.



(b) We have

$$H_2(z) = \frac{Y(z)}{X(z)} = \frac{S(z)}{X(z)} = \frac{1}{H_1(z)}$$

Therefore,

$$H_2(z) = \frac{1}{(1-z^{-8}e^{-8\alpha})} = \frac{z^8}{(z^8-e^{-8\alpha})}$$

There are two possible ROCs for $H_2(z)$: $|z| < e^{-\alpha}$ or $|z| > e^{-\alpha}$. If the ROC is $|z| < e^{-\alpha,}$ then the ROC does not include the unit circle. This in turn implies that the system would be unstable and anti-causal. If the ROC is $|z| > e^{-\alpha}$, then the ROC includes the unit circle .This in turn implies that the system would be stable and causal.

(c) We have

$$H_2(z)= \frac{1}{(1-z^{-8}e^{-8\alpha})}$$

We need to choose the ROC to be $|z| > e^{-\alpha}$ in order to get a stable system. Now consider

$$P(z)= \frac{1}{(1-z^{-1}e^{-8\alpha})}$$

with ROC $|z| > e^{-\alpha}$. Taking the inverse z-transform , we get

$$p[n] = e^{-8\alpha n}u[n].$$

Now, note that $H_2(z) = P(z^8)$.

From Table 10.1 we know that

$$h_2[n] = p[n/8] = e^{-\alpha n}, \qquad n = 0, \pm 8, \pm 16, \ldots..$$

$$= 0, \qquad \text{otherwise}$$