Department of Mathematics
MTL 106/MAL 250 (Introduction to Probability Theory and Stochastic Processes)
Minor 1 Test (II Semester 2014 - 2015)

Time allowed: 1 hour **Max. Marks: 25**

1. (a) Write the axiomatic definition of probability. (2 marks)

   (b) Consider $\Omega = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Let $\mathcal{F}$ be the largest σ-field over Ω. Define
$$P(R) = \text{area of } R = (b-a)(d-c)$$
where R is the rectangular region that is a subset of Ω of the form $R = \{(u, v) : a \leq u < b, c \leq v < d\}$. Let T be the triangular region $T = \{(x, y) : x \geq 0, y \geq 0, x + y < 1\}$. Show that T is an event, and find $P(T)$, using the axioms. (1+2 marks)

2. A random walker starts at 0 on the x-axis and at each time unit moves 1 step to the right or 1 step to the left with probability 0.5. Find the probability that, after 4 moves, the walker is more than 2 steps from the starting position. (3 marks)

3. A student arrives to the bus stop at 6:00 AM sharp, knowing that the bus will arrive in any moment, uniformly distributed between 6:00 AM and 6:20 AM.

   (a) What is the probability that the student must wait more than five minutes? (2 marks)

   (b) If at 6:10 AM the bus has not arrived yet, what is the probability that the student has to wait at least five more minutes? (2 marks)

4. State **True** or **False** with valid reasons for the following statements. Without **valid reasons**, marks will NOT be given.

   (a) A box contains a double-headed coin, a double-tailed coin and an unbiased coin. A coin is picked at random and flipped. It shows a head. The conditional probability that it is the double-headed coin is 0.5 .

   (b) Define the $(100p)$th percentile of a random variable X is the smallest value of x such that $P(X \leq x) \geq p$. Then, 50th percentile is called the *median* of X.

   (c) Consider the following game: you flip an unbiased coin, until the first head appears. If the head appears on the nth flip of the coin, you will receive 2^n rupees. The expected gain for playing the game is 0.5.

   (d) The characteristic function $\phi_X(t)$ of a random variable X satisfies the property $\phi_{-X}(t) = \overline{\phi_X(-t)}$ where bar denotes complex conjugation.

   (1 + 1 + 1 + 1 marks)

5. Let X be a random variable with $N(0, \sigma^2)$. Find the moment generating function for the random variable X. Deduce the moments of order n about zero for the random variable X from the above result.

   (2 + 2 marks)

6. (a) Let X be a uniformly distributed random variable on the interval $[a, b]$ where $-\infty < a < b < \infty$. Find the distribution of the random variable $Y = \frac{X-\mu}{\sigma}$ where $\mu = E(X)$ and $\sigma^2 = Var(X)$. Also, find $P(-2 < Y < 2)$. (3 + 1 marks)

   (b) Suppose Shimla's temperature is modeled as a random variable which follows normal distribution with mean 10 Celcius degrees and standard deviation 3 Celcius degrees. Find the mean if the temperature of Shimla were expressed in Fahreneit degrees. (1 mark)