# COL202 Quiz 2

Aaveg Jain

TOTAL POINTS

## 2.5 / 5

QUESTION 1

## Bandwidth 5 pts

### *1.1* Definition predicate **0 / 2**

**- 0 pts** Correct

**- 0.5 pts** Did not mention $$\exists f \in F$$ for which the predicate is true.

**- 0.5 pts** Did not mention $$\forall (u, v) \in E$$

✓ **- 2 pts** *Incorrect Predicate, one correct predicate is:*

*$$ \exists f \in \mathcal{F}: \forall (u,v) \in E: |f(u) - f(v)| \leq k$$*

**- 2 pts** Did not attempt

### *1.2* The bandwidth of a cycle **2.5 / 3**

**- 0 pts** Correct

✓ **- 0.5 pts** *Incorrect/No argument that bandwidth cannot be 1*

**- 0.5 pts** Did not follow proof guidlines

**- 2 pts** Did not show construction/Incorrect Construction for bandwidth = 2

**- 1 pts** DId not show proof of construction/Incorrect proof of construction

**- 3 pts** Did not attempt

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Quiz 2, 12 September 2022, Maximum Marks: 5

| Name AAVEG JAIN | Ent. No. 2021CS10073 |
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**Important:** Answer within the boxes. Anything written outside the box will be treated as rough work.

**Problem 1.1 (2 marks)**
The *bandwidth* of a graph is defined as follows: Find a numbering of the vertices of a graph such that the maximum difference between the numbers assigned to two vertices connected by an edge in the graph is minimized. This minimum value is called the bandwidth of the graph. Write the following as a predicate: The bandwidth of $G = (V, E)$ is at most $k$. You *must* use the following notation: $\mathcal{F}$ is the set of functions from $V$ to $\{1, \ldots, |V|\}$; bandwidth$(G, k)$ is the name of the predicate you define.

bandwidth (G, k): consider any 2 adj vertices $V_1, V_2$ of G - then

$\min_{f \in \mathcal{F}} \max_{V_1, V_2 \in V(G);\, V_1, V_2 \text{ are adj.}} |f(V_1) - f(V_2)| \leq k$

**Problem 1.2 (3 marks)**
Prove that a cycle on $n$ vertices has bandwidth 2.

Claim - there exists a numbering of vertices of G s.t max. diff. b/w any 2 vertices is 2

Pf. by inductⁿ.

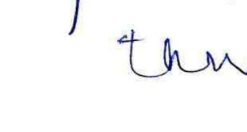
P(n): ∃ a num. of vert. of G, s.t max diff. b/w any 2 verti is 2 (Indⁿ pred.)

Base case : n = 3

a in any no. of vert. of G, max diff. is 2 ~~to~~ hence proved trivially.

Inductⁿ step - Indⁿ hypothesis - P(n) is true.

Consider a cycle of n, Consider the vertice v, ~~whose num~~ to which n is assigned. its neighbours ~~can~~ be have to be atleast n-2, n-1. insert the vertex (n+1) at ~~n~~ the max. of the neighbours of n as shown.




n-2 . The max diff. is 2. Hence P(n+1) is T

Thus by indⁿ ∃ a numbering of n+1