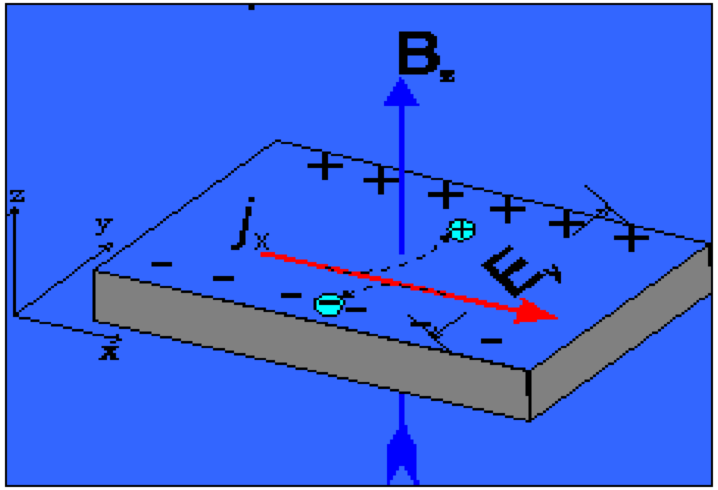
PYL 102

Monday, Sept. 30, 2024

Quantum Hall effect

The qunatum Hall effect

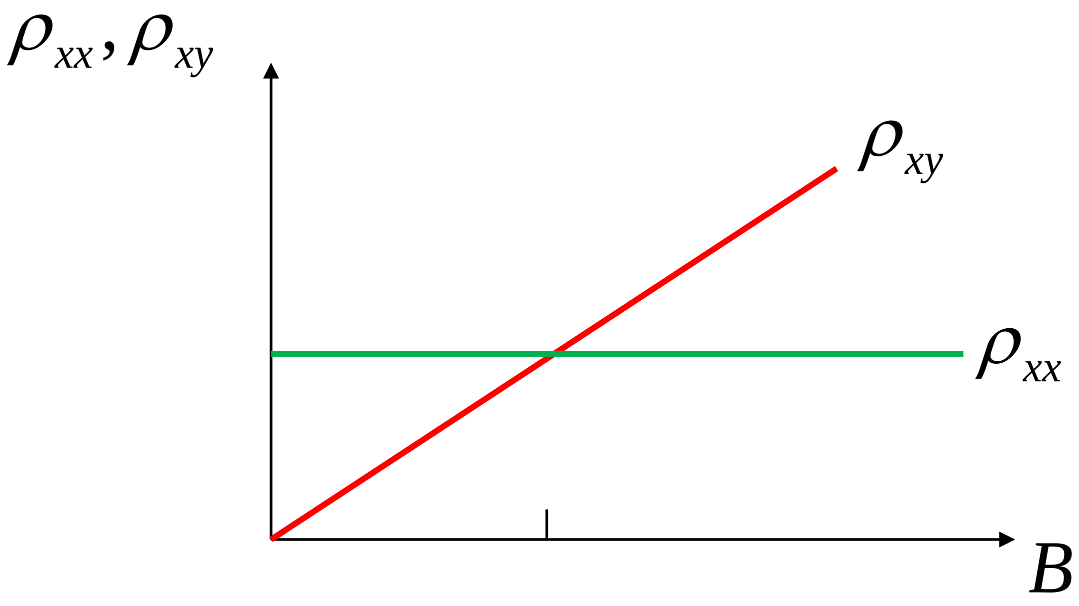
- When an electric current passes through a metal strip with a perpendicular magnetic field, the electrons are deflected towards one edge and a potential difference is created across the strip. This phenomenon is termed the ***Hall Effect***.



Hall coefficient $R_H = E_y / B_z J_x = 1/ne$. The magnitude and sign of R_H allow determination of the free carrier density and type of the majority carrier.

Experimentally the electric field along the sample is determined by measuring V_x. This allows two resistivities to be defined $\rho_{xx} = E_x / J_x$ and $\rho_{xy} = E_y / J_x$. ρ_{xy} increases linearly with increasing magnetic field while ρ_{xx} remains constant.

However, for 2D system, ρ_{xy} increases with magnetic field in a step-like manner, in addition ρ_{xx} oscillates between 0 and non-zero values with 0's fields here ρ_{xy} forms plateau. This surprising behavior is known as quantum Hall effect.



$$\rho_{xy} = \frac{B}{ne}$$

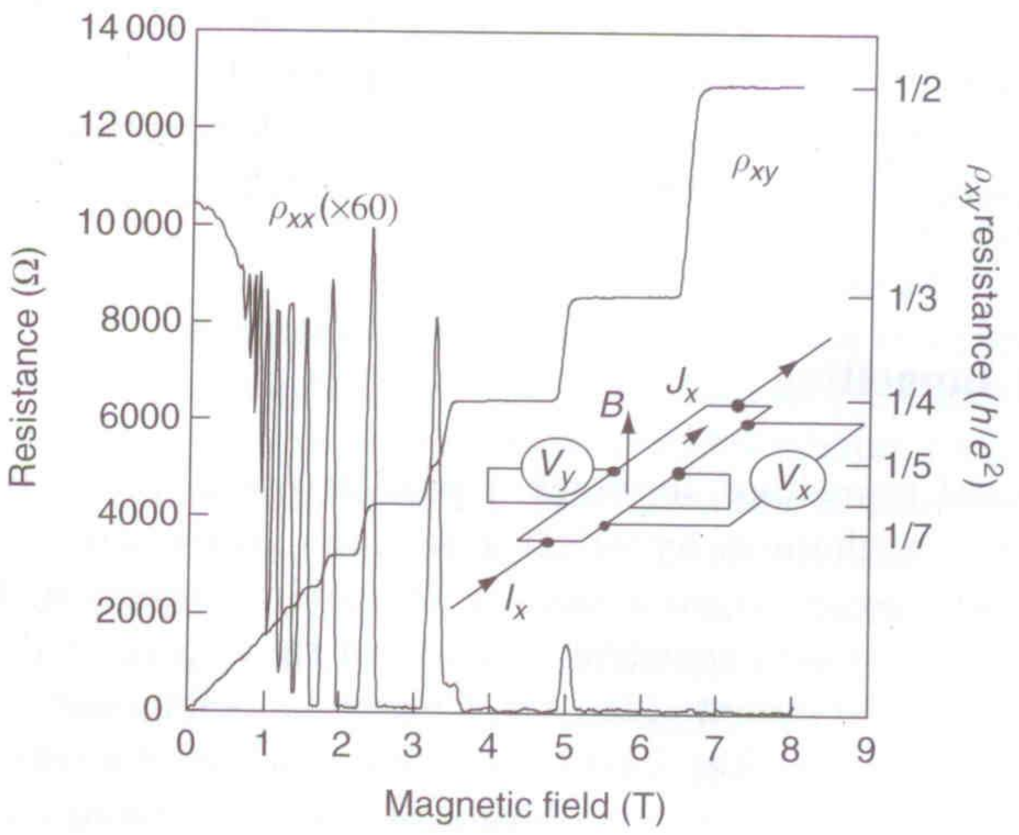
"Hall effect" – depends only on *B* and *n*, can be used to measure n

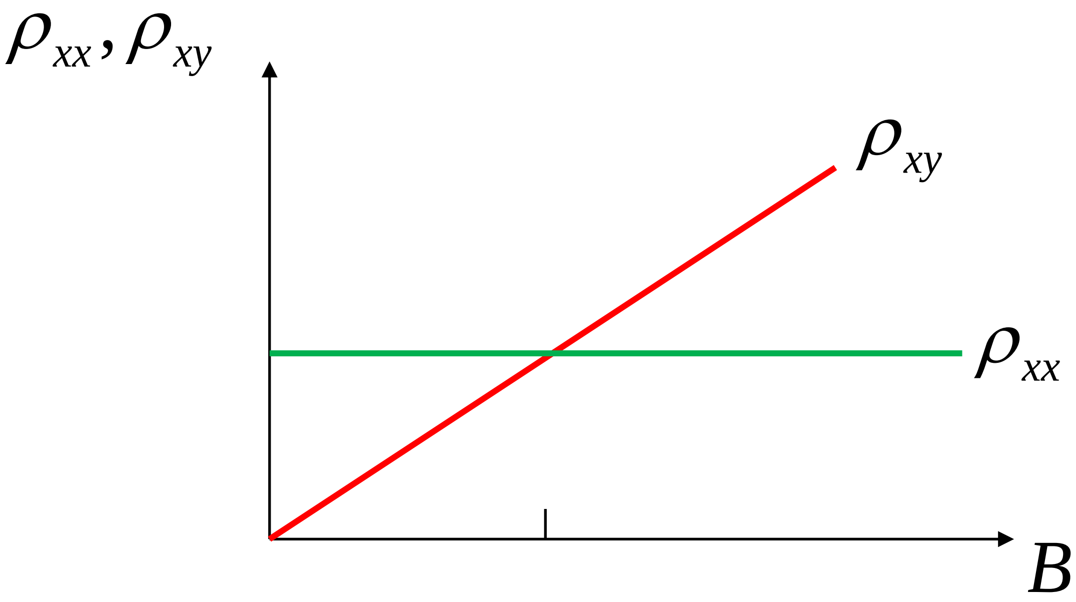
$$\rho_{xx} = \frac{m}{ne^2\tau}$$

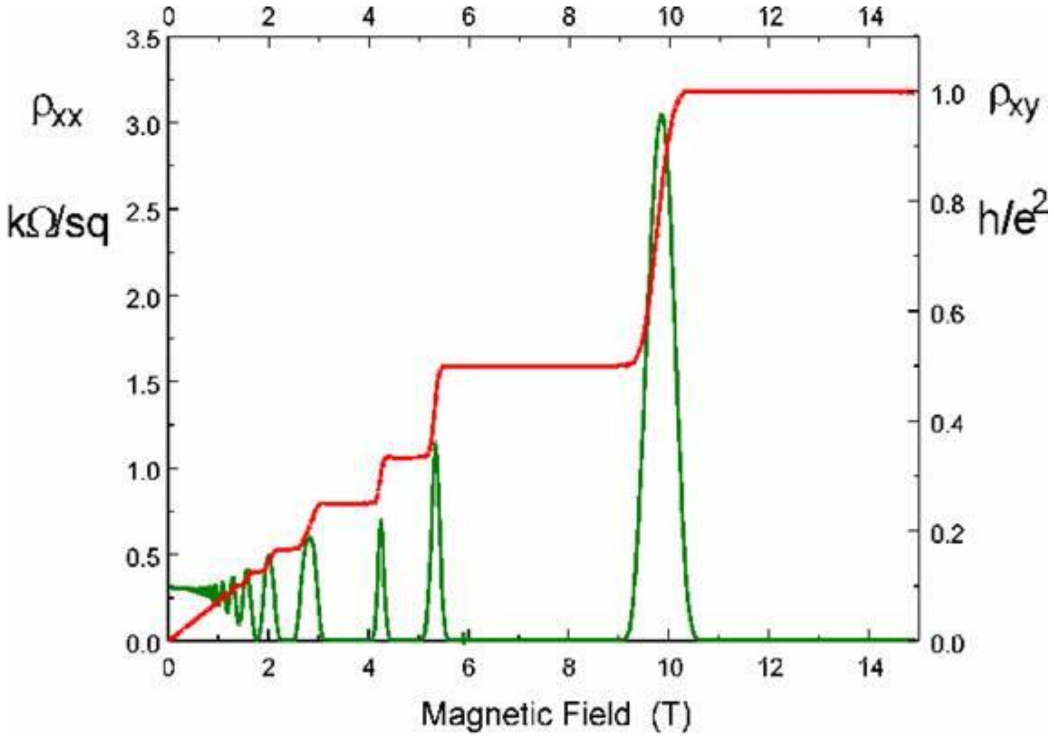
Resistivity in 2D does not depend on *B*

Observed QHE in 2DEG GaAs-AlGaAs heterojunction at temperature of 60mK

(Ref: Plaanen et al. Phys. Rev. B 25, 5566 (1982)).

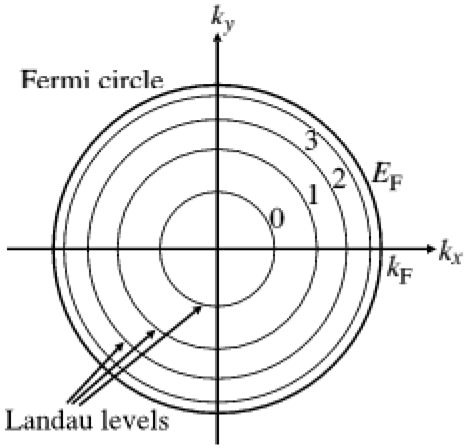






At higher magnetic fields, strong deviations from semi-classical expectation

At high magnetic field B, the electronic density of states becomes a set of discrete Landau levels due to the confinement produced by the field. The following diagram shows the Fermi circle in two dimensional k-space, with a series of Landau levels inside it.



If a magnetic field is applied perpendicular to the plane of 2DEG the e trajectories will be a set of circles around the lines of field.

- In a strong magnetic field the energy of the electron is quantized (Landau levels).

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

- These discrete energy values are given by $n = 0,1,2,\ldots$

  Where $\omega_c = \dfrac{eB}{m}$ is the Cyclotron frequency

Each level is highly degenerate due to independence of energy on k_x

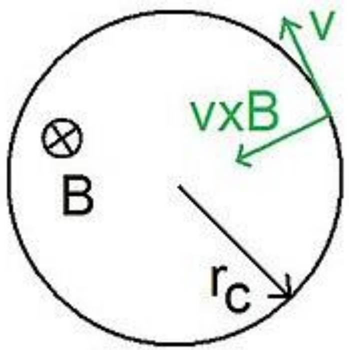


Diagram of a cyclotron orbit of a particle with speed **v**, which is the classical trajectory of a charged particle (here positive charge) under a uniform magnetic field **B**.

$R_H = E_y / B_z J_x = 1/ne$

$$v_x = \frac{E_y}{B_z}$$

How much is the degeneracy of Landau levels?

$n h = m v r_c$

$$\omega_c = \frac{eB}{m} = v/r_c$$

$$j_x = -nev_x \qquad n = \frac{eB}{h}$$

degeneracy of Landau level

Let $\Phi = BA$ is the magnetic flux through the system Flux will be quantized, $\Phi = n\Phi_0$ where $\Phi_0 = h/e$ is the fundamental magnetic flux quantum

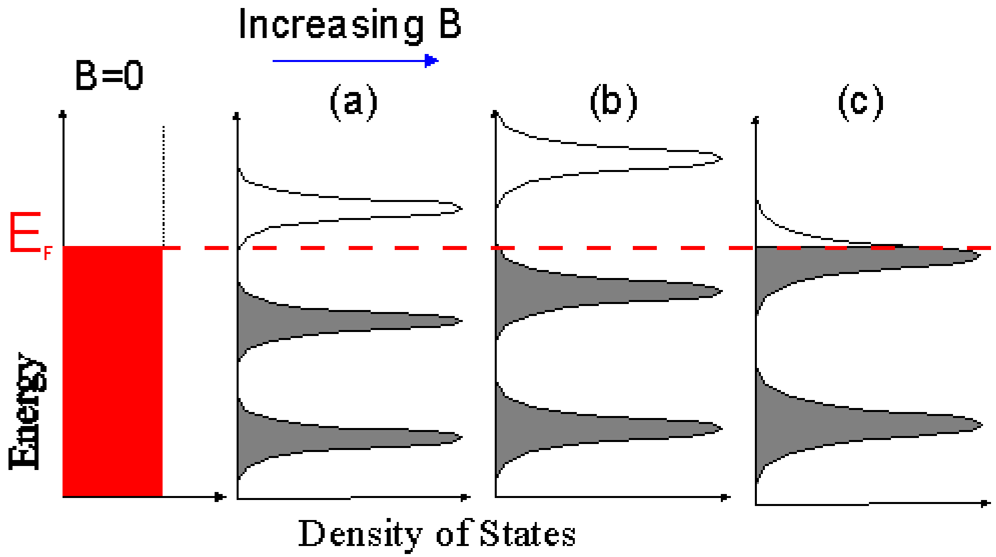
$$j_x = \frac{e^2}{h} E_y$$

Quantum Hall effect

$\rho_{xy} = E_y / J_x$

$$\rho_{xy} = \frac{h}{e^2}$$

In the absence of magnetic field the density of states in 2D is constant as a function of energy, but in field the available states clump into Landau levels separated by the cyclotron energy, with regions of energy between the LLs where there are no allowed states.



As the magnetic field is swept the LLs move relative to Fermi energy.

When the Fermi energy lies in a gap between LLs electrons can not move to new states and so there is no scattering.

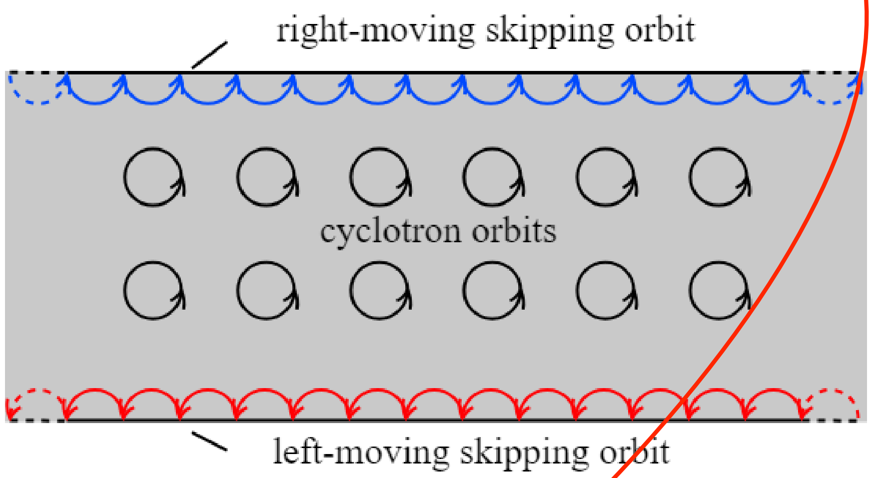
Thus the transport is dissipation less and the resistance falls to zero.

- When the Fermi energy is in a gap, i.e. between the fields (a) and (b) in the diagram, Hall resistance cannot change from the quantized value for the whole time, and so a plateau results.

If the Fermi energy in the Landau level, i.e. the field (c) is reached in the diagram, it is possible to change the voltage and a finite value of resistance will be appeared. In this situation the step like behavior of the Hall conductivity is observed.

The effects of Landau levels may only be observed when the mean thermal energy kT is smaller than the energy level separation, $kT \ll \hbar\omega_c$, meaning low temperatures and strong magnetic fields.

- Confining potential pushes up Landau levels at edges of sample
- Edges are metallic – charge can move in/out
- 1D edge states carry current in **E** x **B** direction



The number of electrons at one edge can increase, because electrons are being depleted from the other edge. In quantum hall effect the current transport is through edge states.

Same as classical result for electron in crossed E, B fields: