2202-COL226 Minor 2

Utkarsh Singh

TOTAL POINTS

14 / 40

QUESTION 1

1 Q1 **0 / 8**

+ **+ 1 pts** mentioning type of len with justification
+ **+ 1 pts** mentioning type of val with justification
+ **+ 1 pts** correct rules for production $$S\longrightarrow L$$
+ **+ 1.5 pts** correct rules for production $$S\longrightarrow L.L$$
+ **+ 1 pts** correct rules for production $$L\longrightarrow B$$
+ **+ 1.5 pts** correct rules for production $$L\longrightarrow LB$$
+ **+ 0.5 pts** correct rules for production $$B\longrightarrow 0$$
+ **+ 0.5 pts** correct rules for production $$B\longrightarrow 1$$

✓ **+ 0 pts** *Incorrect/Unattempted*

QUESTION 2

2 Q2 **0 / 12**

+ **+ 12 pts** All Correct
+ **+ 6 pts** Converted the expressions into prefix notation instead of postfix
+ **+ 6 pts** Correct translation rules for productions with operators, brackets
+ **+ 6 pts** Correct translation rules for other productions
+ **+ 0 pts** Translation rules correspond to expression evaluation

✓ **+ 0 pts** *Incorrect / Not Attempted*

QUESTION 3

3 Q3 **7 / 8**

+ **+ 0 pts** Incorrect / Not Attempted

✓ **+ 4 pts** *Correct application of initial beta reductions with correct notations.*

+ **+ 3 pts** Correct application of initial beta reductions with minor errors in notations.
+ **+ 4 pts** Correct proof for $$((\lambda x [(z^n x)])^m x)$$ beta reduces to $$\lambda x[(z^{mn} x)]$$ (used induction).

✓ **+ 3 pts** *Partially correct proof for $$((\lambda x [(z^n x)])^m x)$$ beta reduces to $$\lambda x[(z^{mn} x)]$$ (used vague arguments like '...').*

QUESTION 4

4 Q4 **7 / 12**

✓ **+ 3.5 pts** *Proof-by-cases for basic $$\eta$$-reduction: If $$L \rightarrow_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.*

Case 1: $$L \equiv \lambda y \;[(\; M \; y \;)]$$ where $$y \not\equiv x$$.

✓ **+ 3.5 pts** *Proof-by-cases for basic $$\eta$$-reduction: If $$L \rightarrow_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.*

Case 2: $$L \equiv \lambda x \;[(\; M \; x \;)]$$.

+ 1 pts Proof for $$1$$-step $$\eta$$-reduction by induction on the derivation of $$L \rightarrow^1_{\eta} M$$: If $$L \rightarrow^1_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.

Case 1: $$L \rightarrow_{\eta} M$$.

+ 1 pts Proof for $$1$$-step $$\eta$$-reduction by induction on the derivation of $$L \rightarrow^1_{\eta} M$$: If $$L \rightarrow^1_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.

Case 2: $$L \equiv \lambda y \;[\; L' \;]$$, $$y \not\equiv x$$, $$L' \rightarrow^1_{\eta} M'$$ and $$M \equiv \lambda y \;[\; M' \;]$$.

+ 1 pts Proof for $$1$$-step $$\eta$$-reduction by induction on the derivation of $$L \rightarrow^1_{\eta} M$$: If $$L \rightarrow^1_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.

Case 3: $$L \equiv \lambda x \;[\; L' \;]$$, $$L' \rightarrow^1_{\eta} M'$$ and $$M \equiv \lambda x \;[\; M' \;]$$.

+ 1 pts Proof for $$1$$-step $$\eta$$-reduction by induction on the derivation of $$L \rightarrow^1_{\eta} M$$: If $$L \rightarrow^1_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.

Case 4: $$L \equiv (\; L_1 \; L_2 \;)$$, $$L_1 \rightarrow^1_{\eta} M_1$$ and $$M \equiv (\; M_1 \; L_2 \;)$$.

+ 1 pts Proof for $$1$$-step $$\eta$$-reduction by induction on the derivation of $$L \rightarrow^1_{\eta} M$$: If $$L \rightarrow^1_{\eta} M$$ and $$x \not\in FV(L)$$ then $$x \not\in FV(M)$$.

Case 5: $$L \equiv (\; L_1 \; L_2 \;)$$, $$L_2 \rightarrow^1_{\eta} M_2$$ and $$M \equiv (\; L_1 \; M_2 \;)$$.

+ 0 pts Incorrect.

COL226: Programming Languages

II semester 2022-23

Sun 26 Mar 2023 Minor 2 LH325 60 minutes Max Marks 40

Instructions.
1. Answer only in the space provided for each question in the question paper itself.
2. Write your name and your **IITD Loginid** on the top line of every page
3. No extra sheets will be provided. You may do your rough work in the separately provided sheets. 4. Answers will be judged for correctness, efficiency and elegance.
5. If there are minor mistakes in the question, correct them explicitly and answer the question accordingly.
5. If the question is totally wrong, give adequate reasons why it is wrong with detailed counter-examples, if necessary.

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1. **[8 marks]** Consider the following CFG G which generates bit-sequences optionally separated by a single "binary point" (denoted by ".") which are to be interpreted as unsigned integers (if there is no binary point) or as unsigned rational numbers (if there is a binary point). $G = \langle N, T, P, S \rangle$ where $T = \{0, 1, .\}$, $N = \{S, L, B\}$ and P the set of productions is given by

$$\begin{array}{lcl} S & \rightarrow & L \mid L.L \\ L & \rightarrow & B \mid LB \\ B & \rightarrow & 0 \mid 1 \end{array}$$

Assume *val* (for the value denoted by a bit string) and *len* (for the length of a bit sequence) are attributes.

(a) Which of the attributes is synthesised and which is inherited. Justify your answer.

(b) write L-attributed definitions to compute the value of each string generated by this grammar.

2. **[12 marks]** It is well known that given the arity of each operator in a programming language, an expression with infix, prefix, postfix and/or mixfix operators may be transformed into a semantically equivalent bracket-free expression in which all the operators are used in postfix form. That is,

- bracketing symbols are not required,
- associativity and precedence rules are not required to capture the order of operations,

Many stack architectures actually used postfix evaluation as the preferred means of evaluating expressions. Assume the usual rules of associativity and precedence for integer operators (in a language like SML). Consider the following augmented grammar

$$G = \langle \{S, E, T, F, U, I\}, \{\$, -, /, \sim, (,), \mathbf{y}, \mathbf{z}\}, P, S \rangle$$

whose set P of productions is given below.

$$\begin{array}{lcl} S & \rightarrow & E\$ \\ E & \rightarrow & E - T \mid T \\ T & \rightarrow & T/F \mid F \\ F & \rightarrow & U \mid \sim F \\ U & \rightarrow & I \mid (E) \\ I & \rightarrow & \mathbf{y} \mid \mathbf{z} \end{array}$$

Define *syntax-directed translation rules* for transforming expressions into semantically equivalent bracket-free postfix notation.

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3. **[8 marks]** Recall that for any non-negative integer n its Church numeral is defined as the function $\underline{\mathsf{n}} \stackrel{df}{=} \lambda\ f\ x[(f^n x)]$. For all natural numbers m and n express the β-normal form of the expression $(\lambda\ x\ y\ z[(x\ (y\ z))]\ \underline{\mathsf{m}}\ \underline{\mathsf{n}})$ as a Church numeral.

Sol^n) $\underline{m} = \lambda f x [(f^m x)]$

$(\lambda x\ y\ z[(x\ (y\ z))]\ \underline{m}\ \underline{n})$

$\xrightarrow[\beta]{} (\lambda y\ z[(\underline{m}\ (y\ z))]\ \underline{n}) \xrightarrow[\beta]{} \lambda z[(\underline{m}\ (\underline{n}\ z))]$

$\downarrow \beta$

$\lambda z[(\underline{m}\ \lambda x[(z^n x)])]$

$\downarrow n$

$\lambda z[(\underline{m}\ z^n)]$

$|||_\alpha$

$\lambda z[\lambda x[(z^{mn}\ x)]] \xleftarrow[\beta]{} \lambda z[(\lambda f x[(f^m x)]\ z^n)]$

$|||_\alpha$

$\lambda z\ x[(z^{mn}\ x)] \equiv_\alpha \lambda f\ x[(f^{mn}\ x)] \equiv_\alpha \underline{mn}$

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4. **[12 marks]** Prove that if $L \rightarrow^1_\eta M$ and $x \notin FV(L)$ then $x \notin FV(M)$.

Sol^n) $L \rightarrow^1_\eta N \Rightarrow (L\ a) \rightarrow_\beta (M\ a)$

$\therefore$ L can be of the form $\lambda x[M\ x]$ or $\lambda t[M\ t]$ where t is any arbitary variable other than x.

We can say that our choice of L is exhaustive.

Case 1) $L = \lambda x[M\ x]$

$\because (L\ a) \rightarrow_\beta (M\ a)$ where a can be any variable, then say $a = x$

$\therefore (L\ x) \rightarrow_\beta (M\ x)$

Assume that M has independent variable x in it.

$\because$ L cannot have independent variable and all the occurance of x inside L would represent the bound variable x then it is not possible for M to have x as free variable.

this creates contradiction, hence M cannot have x as FV.

Case 2) $L = \cancel{\lambda y[m}\ \lambda t[M\ t]$

Assume that in this condition $x \in FV(M)$

This implies that ~~either there~~ there would be occurance of x in M.

Since L has just a single arguement t which is definately not equal to x, therefore I can say that x is a free variable, this creates a contradiction

So our assumption is wrong.

$\therefore x \notin FV(M)$

Since both cases were exhaustive, therefore I have proved the above statement.

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