COL 202 Discrete Mathematics **Diwali 2023**

Tutorial 10

1. [Submission Problems for Group 1] Problem 19.34 in LLM Book
2. [Submission Problems for Group 2] Problem 19.26 in LLM Book
3. [Submission Problems for Group 3] Problem 19.18 in LLM Book
4. [Submission Problems for Group 4] Problem 19.30 in LLM Book
5. [Bonus] Problem 19.8, 19.15, 19.19, 19.21, 19.24, 19.35, 19.37, in LLM Book
6. [Bonus] Let n be a random integer, chosen uniformly between 1 and N. What is the expected number of distinct prime divisors of n? Show that the result is asymptotically equal to $\ln \ln N$ (as $N \to \infty$).
7. [Bonus] For a permutation π of the set $[n]$, let $c_k(\pi)$ denote the number of k-cycles[1] in the cycle decomposition of π. (For instance, if $n = 7$ and $\pi = (18)(256)(3)(47)(9)$ then $c_1(\pi) = 2, c_2(\pi) = 2, c_3(\pi) = 1$, and $c_k(\pi) = 0$ for all $k \neq 1, 2, 3$.) Pick π at random from all permutations of $[n]$.
   (a) Calculate $E[c_k(\pi)]$. Your answer should be a very simple expression (no factorials, no binomial coefficients, no summation).
   (b) Calculate the expected number of cycles (including cycles of length 1) in the cycle decomposition of a random permutation (This will be a simple sum, not a closed-from expression). Prove that this number is $\sim \ln n$.

[1] You can read more about the cycle notation for a permutation here