Department of Mathematics
Minor I Examination
MTL 106: Probability and Stochastic Processes

Venue: LH 121 **Date: 29-08-2017** **Time 2:30 – 3:30 PM** **Full Marks 20**

Q1. (i) What are the three axioms for defining probability of an event E?
(Note: These are called **Kolmogorov's Axioms of Probability**)

(ii) Prove or Disprove: Conditional probability of Event A given that Event B has occurred, i.e. P (A | B) satisfies the above axioms.

(iii) Suppose X and Y are independent and identically distributed (***iid***) random variables both following **Bin(2, 0.4).** Draw the graph of the Cumulative Distribution Function (***cdf***) of Z = X + Y.

[3 + 3 + 2 = 8]

Q2. Suppose X is a random variable with pdf $k.e^{\frac{-(x-2)^2}{2}}$ $\forall\, x \in (2,\infty)$

(i) Find the value of *k.*

(ii) Calculate the Expected value of X.

(iii) Obtain the MGF of X

[1.5 + 1.5 + 3 = 6]

Q3. (i) Let X be a random variable following **Beta$_1$ (2, 3)** distribution. Note that the pdf of a random variable **X ~ Beta$_1$ (m, n)** is:

$$f(x) = \frac{1}{\beta(m,n)} x^{m-1}(1-x)^{n-1}, \qquad for\ 0 < x < 1$$

Note that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Find Mean and Variance of X.

(ii) Suppose X is a Binomial (10, p) random variable, where p is determined by the trial of a variable P ~ ***BETA$_1$*** **(2, 3).**

What is the Expected value of X?

[2.5 + 3.5 = 6]