Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 4
Answer for selected Problems

1. (a) $p_x(1) = 0.2,\ \ p_x(3) = 0.5\ \ p_x(4) = 0.3\ \ p_y(1) = 0.4,\ \ p_y(2) = 0.6$ (b) 0.5
2. (a) $\frac{1}{36}$ (b) $\frac{9}{36}$ (c) $\frac{1}{2}$
4. $k = \frac{1}{8},\quad \frac{5}{81},\ \ 1$
5. (a) $k = 4$ (b) $f_{Y_1,Y_2}(y_1, y_2) = \frac{2y_2}{y_1},\ \ 0 < y_1 < 1, 0 < y_2 < \sqrt{y_1} < 1$
6. Let Y: r.v. denoting no. of 1's transmitted. $P(Y = n) = \frac{e^{-\lambda(1-p)}(\lambda(1-p))^n}{n!},\ n = 0, 1, \ldots$
7. (a) $\frac{3^5-1^5}{4^5}$ (b) $\frac{1}{20}$
8. Yes
9. $\frac{\lambda\mu}{(\mu+\nu)(\lambda+\mu+\nu)}$
10. 0.3214
11. 0.27
12. (a) $P_X(j) = qp^{j-1},\ j = 1, 2, \ldots\ \ P_Y(k) = q^k\left(\frac{\frac{p}{q}-\left(\frac{p}{q}\right)^k}{1-\frac{p}{q}}\right),\ \ k = 2, 3, \ldots$

    $P_{X/Y}(j/k) = \frac{(p/q)^j(1-(p/q))}{(p/q)-(p/q)^k},\ \ j = 1, 2, \ldots, k-1\ P_{Y/X}(k/j) = q^{k-j-1}p,\ \ k = j+1, j+2, \ldots$

    (b) $X \sim B(1/2, 15),\ \ Y \sim B(1/3, 15)$, $Y/(X = j) \sim B(2/3, 15-j),\ \ X/(Y = k) \sim B(3/4, 15-k)$
13. $k = 12,\quad (1 - e^{-8})(1 - e^{-3})$
14. $Z = X + Y,\ \ f_Z(z) = \begin{cases} z\ \ , & 0 < z < 1 \\ 2 - z, & 1 \le z \le 2 \\ 0, & \text{otherwise} \end{cases}$
15. $f_z(z) = \begin{cases} \frac{\lambda}{2}e^{\lambda z}, & z < 0 \\ \frac{\lambda}{2}e^{-\lambda z}, & z \ge 0. \end{cases}$
16. $f_V(v) = -\ln(v),\ 0 < v < 1; f_W(w) = \frac{1}{2\sqrt{w}}, 0 < w < 1; f_{V,W}(v, w) = f_V(v)f_W(w)$
17. $f_{R,\theta}(r, \theta) = \begin{cases} r, & 0 < \theta < \pi/4, 0 < r < \sec\theta \text{ or } \pi/4 < \theta < \pi/2, 0 < r < cosec\theta \\ 0, & \text{otherwise} \end{cases}$

    $f_\theta(\theta) = \begin{cases} \frac{1}{2}\sec^2\theta, & 0 < \theta < \pi/4 \\ \\ \frac{1}{2}cosec^2\theta, & \pi/4 < \theta < \pi/2 \end{cases}$

    $f_R(r) = \begin{cases} \frac{\pi}{2}r, & 0 < r < 1 \\ \\ r(cosec^{-1}(r) - \sec^{-1}(r)), & 1 < r < \sqrt{2} \end{cases}$
18. $\frac{1}{8}$
19. $f_{X,Y}(x, y) = 1/2,\ 1 \le x \le 1,\ -1 \le y \le 1.$
20. $P(Z \le z) = \begin{cases} z(1 - e^{-z}) & 0 \le z < 1 \\ (1 - e^{-1})(e^{-1} - e^{-z}) & 1 \le z < \infty \end{cases}$.

21. $\frac{5}{9}$

22. $f_X(x) = exp{-x},\ x > 0.$

23. $\frac{5}{36} + \frac{1}{12}\ln 4$

24. (a) $\frac{3}{8}$ (b) $\frac{3}{8}e^{-\frac{1}{3}}$ (c) $\frac{3}{8}e^{-\frac{1}{3}} + \frac{5}{8}e^{-\frac{1}{5}}$

25. $\frac{5}{9}$

27. $P(X = x) = {}^{x+n-1}C_x \frac{1}{2^{x+n}},\quad x = 0, 1, 2, 3, \ldots$

28. Geometric$\left(\frac{1}{4\times 10^5}\right)$

29. 0.7222

30. $P(Z \le z) = (1 - e^{-z}),\quad 0 \le z < \infty$

31. $N\left(\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1),\ \sigma_2^2(1 - \rho^2)\right)$

32. (a) $2 \times \left(\frac{3}{5}\right)^3$
(b) $\frac{36}{125}$