Hints to problem sheet 2

Ques. 1:

a) $\delta(2t)$ has half area as $\delta(t)$

b) Discussed in class

c) The answer depends upon how we define $u_\Delta(t)$. $u_\Delta(t)$ in limiting form is known as a generalized function (as per the function is not specific). We can prove the given equality if we assume $u_\Delta(t)$ as t/Δ for $0 \le t \le \Delta$.

$$\lim_{\Delta\to 0}[\mathrm{u}_\Delta(t)\delta(\mathrm{t})] = \lim_{\Delta\to 0}[\mathrm{u}_\Delta(0)\delta(\mathrm{t})] = \lim_{\Delta\to 0}[0] = 0$$

d) $\boldsymbol{\lim_{\Delta\to 0}}[\boldsymbol{u_\Delta}(t)\delta_{\boldsymbol{\Delta}}(\boldsymbol{t})] = \boldsymbol{\lim_{\Delta\to 0}}\left[\frac{t}{\Delta}\times\frac{1}{\Delta}\right] = \boldsymbol{\lim_{\Delta\to 0}}\left[\frac{t}{\Delta^2}\right] = \frac{1}{2}\delta(\boldsymbol{t})$

Note that as t is between 0 and Δ and as Δ is very small $t \approx \Delta$, and thus $\frac{\partial t}{\partial \Delta} \approx 1$.

We can also prove the above equation by taking integration. By taking area on of the $\boldsymbol{u_\Delta}(t)\delta_{\boldsymbol{\Delta}}(\boldsymbol{t})$ where $\boldsymbol{\Delta} \to 0$**,** we can observe that the area remains constant i.e., ½ and the function will transform into impulse function at t = 0.

Ques. 2: $E_\infty = \int_{-\infty}^{\infty}|y(t)|^2 dt = \int_{-2}^{2} dt = 4$

Ques.3:

a) S1: $y_1(\mathrm{t}) = 2x_1(2\mathrm{t}+2)$

This is invertible function, so we can find inverse

$$x_1(t) = 2y_1(2t+2)$$

Let $2\mathrm{t}+2 = \mathrm{z}$ then

$$x_1((z-2)/2) = 2y_1(z)$$

Now change the variable

$$y_1(t) = \frac{1}{2}x_1((t-2)/2) : S2$$

b) The system is not invertible as time scaling with factor more than 1 leads to decimation of samples.

c) The inverse system is non-causal so this system is realizable only for offline processing.

Ques.4

a) Memory, Time variant, Linear, Non causal, Stable

b) Memory less, Time variant, Linear, Causal, Stable

c) Memory, Time variant, Linear, Non causal, Unstable
d) Memory, Time variant, Linear, Non causal, Unstable
e) Memory, Time variant, Linear, Causal, Stable
f) Memory, Time Invariant, Non-Linear, Causal, Stable
g) Memory, Time variant, Linear, Non causal, Stable
h) Memory less[1], Time Invariant, Linear, Non causal, Unstable
i) Memory, Time variant, Linear, Causal, Unstable
j) Memory, Time Invariant, Nonlinear, Causal, Stable
k) Memory, Time Invariant, Linear, Non causal, Stable
l) Memory, Time Variant, Linear, Non causal, Stable
m) Memory, Time variant, Linear, Non causal, Stable
n) Memory, Time Invariant, Linear, Non causal, Stable