Department of Mathematics, IIT Delhi

2201-MTL106: Minor Exam.

Time: 1 hour **Date:** 27-09-2022 **Total Marks:** 25

Q.1) Let X and Y be **i.i.d** random variables with common probability density function $f(x) = 2e^{-2x}1_{\{x\geq 0\}}$. Find the probability density function of $W := \min\{X, Y^2\}$. 6 marks

Q.2) Suppose that the travel time by Auto from Hauz Khas metro station to IIT Delhi Math office is normally distributed with mean 13.5 minutes and standard deviation 10 minutes^2. If you want to be 95% certain that you will not be late for an appointment at 10.30 am, what is the latest time that you should leave the metro station? You may use the following: for any $Z \sim \mathcal{N}(0,1)$, $\mathbb{P}(Z > 1.65) = 0.05$. 5 marks

Q.3) Let X and Y be continuous random variables with joint probability density function

$$f_{(X,Y)} = \begin{cases} \frac{x}{5} + cy, & 0 < x < 1,\ 1 < y < 5, \\ 0, & \text{otherwise.} \end{cases}$$

i) What is the value of c ?

ii) Are X and Y independent? Justify your answer.

iii) Find $\mathbb{P}(X + Y > 3)$.

1+3+3 marks

Q.4) Let X and Y be random variables with mean 0, variance 1 and correlation coefficient ρ.

a) Show that the random variables $W := X - \rho Y$ and Y are not correlated.

b) Show that $\mathbb{E}[\mathrm{Var}(Y|X)] \leq (1 - \rho^2)$.

3+4 marks

Best of Luck!!!