Problem sheet–10

1. Frequency Domain Analysis of functions of a signal

Let x(t) be a signal with Nyquist rate w_o . Determine the Nyquist rate for each of the following signals:

a) x(t) + x(t - 1)

b) $\frac{dx(t)}{dt}$

c) $x^2(t)$

d) x(t)

2. Frequency Convolution with Ideal Sampling Function

Let x(t) be a signal with Nyquist rate w_o. Also, let

$$y(t) = x(t)p(t - 1)$$

where,

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), and\ T < \frac{2\pi}{w_o}$$

Specify the constraints on the magnitude and phase of the frequency response of a filter that gives x(t) as its output when y(t) is the input.

3. Properties of Sampled signal

Let. $x_c(t)$ be a continuous-time signal whose Fourier transform has the property that

$$x_c(jw) = 0\ for\ |w| \geq 2000\pi$$

A discrete-time signal is obtained. For each of the following constraints on the Fourier transform of $x_d[n]$, determine the corresponding constraint on $X_c(jw)$:

(a) $X_d(e^{jw})$ is real.

(b) The maximum value of $X_d(e^{jw})$ over all w is 1.

(c) $X_d(e^{jw})$ = 0 for $\frac{3\pi}{4} \leq |w| \leq \pi$

(d) $X_d(e^{jw}) = X_d(e^{j(w-\pi)})$).

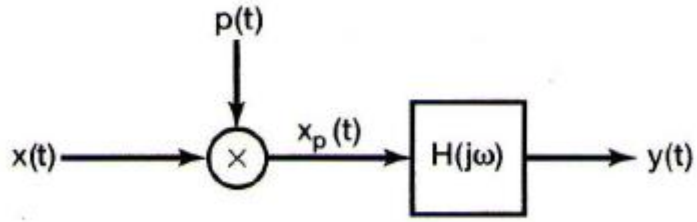
4. Variation in Sampling Function

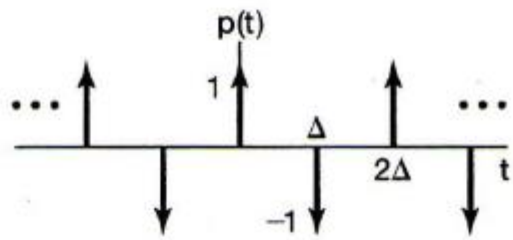
Shown in the figure below is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

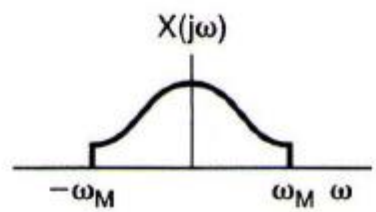
(a) For $\Delta < \pi/(2w_M)$, sketch the Fourier transform of x_p(t) and y(t).

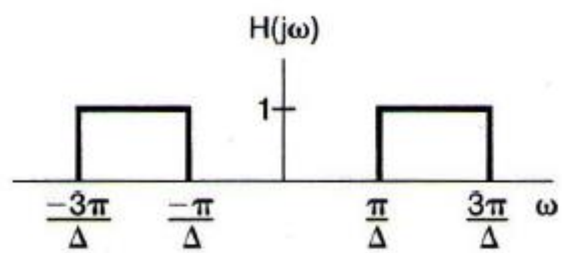
(b) For $\Delta < \pi/(2w_M)$, determine a system that will recover x(t) from x_p(t).

(c) For $\Delta < \pi/(2w_M)$, determine a system that will recover x(t) from y(t).
(d) What is the maximum value of Δ in relation to w_m for which x(t) can be recovered from either x_p (t) or y(t)?









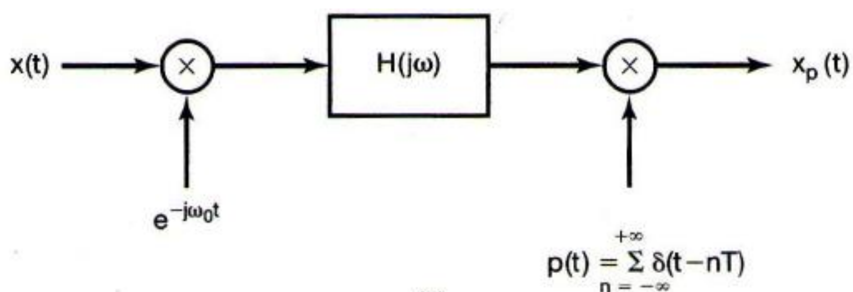
5. **Band Pass Sampling**

A procedure for band pass sampling and reconstruction, used when x(t) is real, consists of multiplying x(t) by a complex-exponential and then sampling the product. The sampling system is shown in figure(a) below. With x(t) real and with $X(jw)$ nonzero only for $w_1 < |w| < w_2$, the frequency is chosen to be w_0 = (1/2)($w_1 + w_2$),and the lowpass filter $H_1(jw)$ has cutoff frequency
$(1/2)(w_2 - w_1)$.

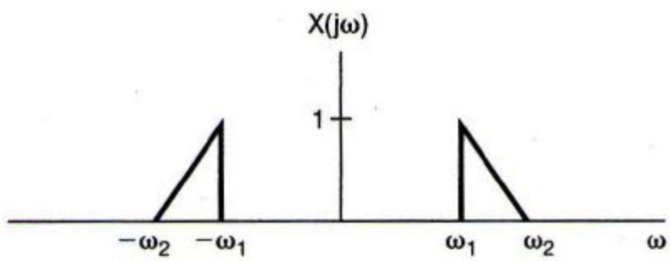
(a) For $X(jw)$ as shown in figure (b), sketch $X_p(jw\ )$.

(b) Determine the maximum sampling period T such that x(t) is recoverable from $X_p(t)$.

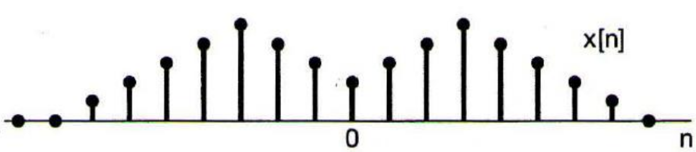
(c) Determine a system to recover x(t) from $x_p(t)$.



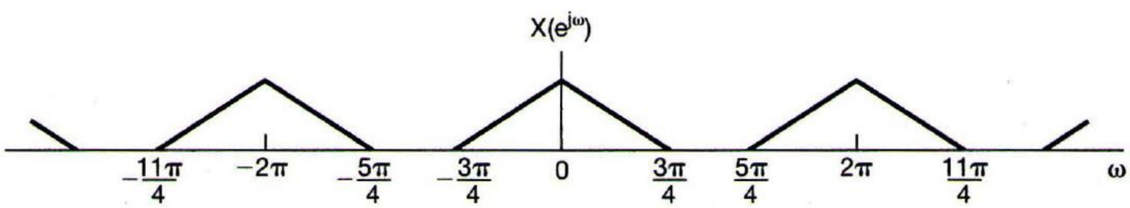
(a)



(b)



(a)



(b)