

COL351 Holi2023: Tutorial Problem Set 10

1. Recall the **CircuitSAT** problem discussed in class. The input to the problem is a boolean circuit over variables x_1, \dots, x_n , where each input of each gate is a variable, a constant (**true** or **false**), or the output of an “earlier” gate. One gate is designated as the output gate, and we are required to determine whether there exists a boolean assignment to the variables that makes the output of the circuit **true** (a.k.a. satisfying assignment). The **CircuitSATSearch** problem has the same input as **CircuitSAT**, but we are required to output a satisfying assignment if such an assignment exists, and otherwise output “NO”. Design a Cook-reduction from **CircuitSATSearch** to **CircuitSAT**.
2. A *dominating set* of a graph is a subset D of vertices such that every vertex that is not in D is adjacent to some vertex in D . The input to the **DominatingSet** problem is a graph G and an integer k , and the output is whether G has a dominating set of size at most k . Prove that **DominatingSet** is in NP, and that **VertexCover** is Karp-reducible to **DominatingSet**.
3. The input to the **DominatingSetSearch** problem is a graph G and an integer k , and the output is a dominating set of G having at most k vertices, if such a set exists, and “NO” otherwise. Design a Cook-reduction from **DominatingSetSearch** to **DominatingSet**.
4. Let U be a finite set and, let $\mathcal{S} \subseteq 2^U$ be a collection of subsets of U . We say that a set $H \subseteq U$ is a *hitting set* of \mathcal{S} if for all $A \in \mathcal{S}$, we have $H \cap A \neq \emptyset$, or in other words, H intersects every set in \mathcal{S} . In the **HittingSet** problem, we are given U , $\mathcal{S} \subseteq 2^U$, and an integer k as input. We are required to determine whether there is a hitting set of \mathcal{S} having size at most k . Prove that **HittingSet** is in NP, and that **VertexCover** is Karp-reducible to **HittingSet**.
5. In the **HittingSetOpt** problem, we are given a finite set U and a collection $\mathcal{S} \subseteq 2^U$ of subsets of U , and we are required to output a minimum cardinality hitting set of \mathcal{S} . Design a Cook-reduction from **HittingSetOpt** to **HittingSet**.
6. Design a Karp-reduction from **HittingSet** to **CircuitSAT**.
7. Recall that in the **GraphIso** problem, we are given two undirected graphs and we wish to determine whether they are isomorphic. Design a Karp-reduction from **GraphIso** to **CircuitSAT**.