COL202 Quiz 4

Aaveg Jain

TOTAL POINTS

5 / 5

QUESTION 1

1 Problem 1 **5 / 5**

✓ **+ 2 pts** *Correct approach and reasoning for LHS*

+ 1 pts Correct approach but reasoning not specified clearly for LHS

✓ **+ 3 pts** *Correct approach and reasoning for RHS*

+ 2 pts Correct approach but reasoning not specified clearly for RHS

+ 0 pts Not using combinatorial argument/Incorrect

COL202: Discrete Mathematical Structures. I semester, 2022-23.
Quiz 3, 31 October 2022, Maximum Marks: 5

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Important: Answer within the box. Anything written outside the box will be treated as rough work.

Problem 1

Prove using *only* a combinatorial argument that, for all $n \geq 1$,

$$n2^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}.$$

Hint: Consider all sequences of length n that contain letters a and b and just one c.

~~this will prove this~~

Consider set S = set of all seq. of length n having only one c and remaining a, b.

(We will prove this by counting S in 2 diff. ways.)

① no. of ways to choose position of c
= no. of one-el. subsets of $\{1, 2, \ldots n\} = \binom{n}{1} = n$

set of all possible ways to assign remaining $n-1$ positions is in biject^n with $\{a,b\}^{n-1}$

$\Rightarrow$ no. of ways to assign a, b in remaining positions $= |\{a,b\}|^{n-1} = 2^{n-1}$ (product rule)

$\Rightarrow$ no. of seq. $= |S| =$ no. of ways to assign c $\times$ no. of ways to assign a, b
$= n \cdot 2^{n-1}$ (product rule)

② Let k be no. of positions occ. by c and b together.
note $n \geq k \geq 1$. Thus a occupies all the remaining $(n-k)$ pos^n.
no. of ways of selecting k pos^n $= \binom{n}{k}$ (subset rule).
out of these k positions, c has to occur in exactly one position, while b occupies all remaining $k-1$ pos^n.
$\Rightarrow$ no. of ways of assigning b, c to the k selected pos^n
= no. of ways of choosing pos^n of c $= \binom{k}{1} = k$

$\Rightarrow$ total no. of sequences with b, c occupying k pos^n
= no. of ways of choosing k pos^n $\times$ no. of ways of ass. b, c $\times$ no. of ways of assigning a $= \binom{n}{k} \times k \times 1$

Since $n \geq k \geq 1$ (c has to occur atleast once), total no. of seq. with one occ. of c $= \sum_{k=1}^{n} \binom{n}{k} k$; hence $|S| = \sum_{k=1}^{n} k\binom{n}{k}$

since ①, ② count the same set, we have

$$n \cdot 2^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}.$$