COL 202 Discrete Mathematics **Diwali 2023**

Tutorial 10

1. [Submission Problems for Group 1] Problem 18.2 in LLM Book
2. [Submission Problems for Group 2] Problem 18.11 in LLM Book
3. [Submission Problems for Group 3] Problem 17.7 in LLM Book
4. [Submission Problems for Group 4] Problem 17.7 in LLM Book
5. [Bonus] Problem 17.5, 17.8, 18.18, 18.21, 18.25, 18.35 in LLM Book
6. [Bonus] Let Γ_n denote the set of graphs with vertex set $V = [n]$. In the next sequence of problems we consider the uniform probability space over the sample space Γ_n. This is called the uniform Erdös-Rényi model of "random graphs". Let $A(i, j)$ denote the event that vertices i and j are adjacent ($1 \leq i, j \leq n, i \neq j$). Note that $A(i, j) = A(j, i)$ so we are talking about $\binom{n}{2}$ events.

   (a) Determine $Pr(A(i, j))$.

   (b) Prove that these $\binom{n}{2}$ events are independent.

   (c) What is the probability that the degrees of vertex 1 and vertex 2 are equal? Give a simple closed-form expression.

   (d) If p_n denotes the probability calculated in item (c), prove that $p_n\sqrt{n}$ tends to a finite positive limit and determine its value.

   (e) How are the following two events correlated: A_n = "vertex 1 has degree 3; B_n = "vertex 2 has degree 3"? Find the limit of the ratio $Pr(A_n \mid B_n)/Pr(A_n)$ as $n \to \infty$.

   Recall that the diameter of a graph is the maximum distance between all pairs of vertices. So if a graph has diameter d then $(\forall x, y \in V)(dist(x, y) \leq d)$ and $(\exists x, y \in V)(dist(x, y) = d)$. If G is disconnected, we say that we say that $diam(G) = \infty$. Let p_n denote the probability that a random graph on n vertices has a certain property. We say that almost all graphs have the property if $lim_{n\to\infty} p_n = 1$.

   (f) Prove: almost all graphs have diameter 2.