Department of Mathematics
MTL 106 (Introduction to Probability and Stochastic Processes)
Tutorial Sheet No. 7
Answer for Selected Problems

Note: $\rho = \frac{\lambda}{\mu}$ everywhere

1. $P'(t) = P(t)Q$ *K.F.E* with $P(t) = (P_0(t), P_1(t), P_2(t))$ and
   $P'(t) = QP(t)$ *K.B.E* with $P(t) = (P_0(t), P_1(t), P_2(t))^T$
   where $P_i(t) = P[X(t) = i]$.

2. (a) $\pi_0 = \frac{1}{1+\rho+\frac{\rho^2}{2}}, \pi_1 = \rho\pi_0, \pi_2 = \frac{\rho^2}{2}\pi_0$ (b) π_0.

3. (a) $\frac{\sum_{n=0}^{M} n(\lambda/\mu)^n M!/(M-n)!}{1+\sum_{n=1}^{M}(\lambda/\mu)^n M!/(M-n)!}$ (b) $1 - \sum_{n=0}^{M} \frac{nP_n}{M}$ where P_n is the probability that n machines will not be in use.

4. $S = \{0, 1, 2\}$ $P'(t) = P(t)Q$ *K.F.E* where $P(t) = (P_0(t), P_1(t), P_2(t))$
   $Q = \begin{pmatrix} -2 & 2 & 0 \\ 1 & -3 & 2 \\ 0 & 2 & -2 \end{pmatrix}$.
   Then, $\pi_0 = \frac{1}{5}$, $\pi_1 = \frac{2}{5}$, $\pi_2 = \frac{2}{5}$.

5. (a) $S = \{0, 1, 2\}$, $Q = \begin{pmatrix} -3 & 3 & 0 \\ 2 & -4.5 & 2.5 \\ 0 & 1.5 & -1.5 \end{pmatrix}$. Then, $\pi = (\frac{1}{5}, \frac{3}{10}, \frac{1}{2})$
   (c) $1 - \pi_0 = \pi_1 + \pi_2 = \frac{4}{5}$.

6. $S = \{0, \pm 1, \pm 2, \ldots\}$, $P'(t) = P(t)Q$
   K.F.E where $P(t) = (P_N(t), \ldots, P_{-2}(t), P_{-1}(t), P_0(t), P_1(t), P_2(t), \ldots, P_N(t))$
   (c) $\rho = 2 > 1$, therefore equilibrium probability distribution does not exists.

8. (a) $P(N(t+s) - N(t) = k \mid N(u); 0 \le u \le t) = \frac{e^{-\lambda s}(\lambda s)^k}{k!}$, $k = 0, 1, \ldots$
   (b) $P(N(2.5) = 15, N(3.7) = 21, N(4.3) = 21) = \frac{e^{-5\times 2.5}(5\times 2.5)^{15}}{15!} \cdot \frac{e^{-5\times 1.2}(5\times 1.2)^6}{6!} \cdot \frac{e^{-5\times 0.6}(5\times 0.6)^0}{0!}$

9. ??

10. ?

11. ?

12. (a) $gamma(8, \lambda)$ (b) $\frac{8}{\lambda}$.

13. ?

14. ?