COL202: Discrete Mathematical Structures **Spring 2023**

Tutorial Sheet 3

Announced on: Jan 19 (Thurs)

1. Show that ordinary induction implies well ordering principle.

2. **[Submission Problem for Group 1]** Problem 2.5 in [LLM17].

   Use the Well Ordering Principle to prove that there is no solution over the positive integers for the following equation:
   $$4a^3 + 2b^3 = c^3.$$

3. **[Submission Problem for Group 2]** Problem 2.7 in [LLM17].

   Use the Well Ordering Principle to prove that any integer greater than or equal to 8 can be represented as the sum of nonnegative integer multiples of 3 and 5.

4. **[Submission Problem for Group 3]** Consider the *selection sort* algorithm (you may have seen this in COL106). The input to this algorithm is an array of n integers. The desired output is a sorted list of these integers arranged in ascending order.

   The algorithm runs for n rounds. In the i^{th} round (where $i \in \{1, 2, \ldots, n\}$), the algorithm finds the smallest element between (and including) the positions i and n in the array, and swaps it with the element at position i.

   a) What property is satisfied by the array maintained by selection sort at the end of the i^{th} round? (The property may depend on i.)

   b) Use the property from part (a) to prove that selection sort returns a sorted array after n rounds.

5. **[Submission Problem for Group 4]** Consider the *insertion sort* algorithm (you may have seen this in COL106). The input to this algorithm is an array of n integers. The desired output is a sorted list of these integers arranged in ascending order.

   The algorithm runs for n rounds. In the i^{th} round (where $i \in \{1, 2, \ldots, n\}$), the algorithm inserts the element at position i into the subarray between (and including) the positions 1 and $i - 1$ at the correct location, say position j, and shifts all elements between (and including) the positions j and $i - 1$ by one position each to their right side.

   a) What property is satisfied by the array maintained by insertion sort at the end of the i^{th} round? (The property may depend on i.)

   b) Use the property from part (a) to prove that insertion sort returns a sorted array after n rounds.

References

[LLM17] Eric Lehman, Tom Leighton, and Albert R Meyer. *Mathematics for Computer Science*. 2017. URL: `https://courses.csail.mit.edu/6.042/spring18/mcs.pdf`.