Name:

COL202: Quiz-5

Maximum marks: 40

Kerberos id:

Instructions.

1. For each problem, you will receive 10% marks for leaving it blank. However, there will be penalty for submitting bogus proofs.
2. Please write your proofs clearly (marks will be deducted for skipping steps).
3. Clearly mark whether you have attempted the problem or not. In case there are multiple versions in a particular problem, clearly specify which version you have attempted.

   If nothing is marked, we will assume that you have not attempted the problem.

Question 1 (10 marks). Consider the following statement : *if every vertex in a graph has positive degree, then the graph is connected.* The following is a flawed proof using (regular) induction.

$P(n)$: for every undirected graph G on n vertices, if every vertex has positive degree, then G is a connected graph.

Base case: $n = 2$ is true.
Induction step: $P(n) \implies P(n+1)$. Consider any n-vertex graph $G = (V, E)$ where every vertex has positive degree. Now, consider an $(n + 1)$ vertex graph G' that is obtained from G by taking a new vertex x, and adding edges from x to a subset $S \subseteq V$ (since x must have positive degree, S is a non-empty set). Now, we will prove that G' is a connected graph. From our induction hypothesis, for all $a, b \in V$, there exists a path from a to b. The new vertex x is connected to at least one vertex in V. As a result, there exists a path from x to every $v \in V$.

Where is the flaw in this proof? Identify a counter-example to show that $P(n)$ can be false for some $n > 2$.

☑ **ATTEMPTED**

☐ **NOT ATTEMPTED**

Question 2: Eulerian Tours and Closed Eulerian Tours (15 marks) Let $G = (V, E)$ be an undirected connected graph with $n = |V|$, $m = |E|$. An Eulerian tour in G is a walk W, starting from some vertex u and ending at some vertex v (may or may not be same as u) such that W visits each edge in the graph exactly once. A closed Eulerian tour is one that starts and ends at the same vertex.

In class, we showed that if every vertex in G has even degree, then G has a closed Eulerian tour. Prove that if $n - 2$ vertices have even degree and two vertices have odd degree, then the graph has an Eulerian tour.

Question 3: (15 marks) Consider a 2-edge-connected undirected graph $G = (V, E)$ (recall, a 2-edge-connected graph is one where for every edge $e \in E$, the graph remains connected even after the removal of e). Consider any two vertices u, v such that u and v have a common neighbor w. Prove that there exist two edge-disjoint paths from u to v. (Recall, two paths P and Q from u to v are said to be edge-disjoint if they share no common edges).