Ques 1: (a)

$$y[n] - \alpha y[n-1] = x[n]$$

$$\implies Y - \alpha RY = X$$

$$\implies Y(1-\alpha R) = X$$

$$\implies \frac{Y}{X} = \frac{1}{1-\alpha R}$$

$$\implies h[n] = \alpha^n u[n]$$

(4 marks)

Stability :$|\alpha| < 1$ (2 marks)

(b) Method 1:

$$h_1[n] = \alpha^{n-1}u[n-1] - \alpha^{2+n-3}u[n-3]$$

$$h_1[n] = \alpha^{n-1}u[n-1] - \alpha^{n-1}u[n-3]$$

$$h_1[n] = \alpha^{n-1}[u[n-1] - u[n-3]]$$

$$h_1[n] = \alpha^{n-1}[\delta(n-1) + \delta(n-2)]$$

(2 marks)

$$h_1[n] = (1+R)R\alpha^{n-1}$$

and,

$$h_2[n] = \alpha^n u[n] - \alpha^3\alpha^{n-3}u[n-3]$$

$$h_2[n] = \alpha^n[u[n] - u[n-3]]$$

$$h_2[n] = \alpha^n[\delta(n) + \delta(n-1) + \delta(n-2)]$$

(2 marks)

$$h_2[n] = (1+R+R^2)\alpha^n$$

$$h_1[n] * h_2[n] = \alpha^{n-1}(R+R^2)\alpha^n(1+R+R^2)$$

$$h_1[n] * h_2[n] = \alpha^{2n-1}(R+R^2+R^3+R^2+R^3+R^4)$$

$$h_1[n] * h_2[n] = \alpha^{2n-1}(\delta(n-1) + 2\delta(n-2) + 2\delta(n-3) + \delta(n-4))$$

(2 marks)

Method 2:

$$g[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[n-k]h_2[k]$$

$$g(n) = \begin{cases} 0, & n \leq 0, n \geq 5 \\ 1, & n = 1 \\ 2\alpha, & n = 2 \\ 2\alpha^2, & n = 3 \\ \alpha^3, & n = 4 \end{cases}$$

Q2 Solution:

$$x[n] = \sum_{k=-\infty}^{+\infty} a_k \ \phi[n-k]. \tag{1}$$

$$\phi[n] = \left(\frac{1}{2}\right)^n u[n]. \tag{2}$$

$$\phi[n-1] = \left(\frac{1}{2}\right)^{n-1} u[n-1]. \tag{3}$$

$$\phi[n] - \frac{1}{2}\phi[n-1] = \left(\frac{1}{2}\right)^n \Big[u[n] - u[n-1]\Big] = \delta[n]. \qquad \textbf{(2 marks)} \tag{4}$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]. \qquad \textbf{(1 mark)} \tag{5}$$

$$= \sum_{k=-\infty}^{+\infty} x[k]\Big[\phi[n-k] - \phi[n-k-1]\Big]. \tag{6}$$

$$= \sum_{k=-\infty}^{+\infty} x[k]\left(1 - \frac{R}{2}\right)\phi[n-k]. \qquad \textbf{(2 marks)} \tag{7}$$

$$= \sum_{k=-\infty}^{+\infty} \left(x[k] - \frac{x[k-1]}{2}\right)\phi[n-k]. \tag{8}$$

By comparing (1) and (8) we get

$$a_k = x[k] - \frac{x[k-1]}{2}. \qquad \textbf{(1 mark)} \tag{9}$$

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Qus 3> $y(t) + 2\int y(t) = x(t)$

I/P/O/P X,

Method - 1

$$Y + 2AY = X$$

$$\Rightarrow \frac{Y}{X} = \frac{1}{1+2A} \longrightarrow \text{② Marks.}$$

$$= \frac{1}{A}\left(\frac{A}{1+2A}\right)$$

$$= \frac{d}{dt}\left(e^{-2t}u(t)\right) \longrightarrow \text{② Marks}$$

$$= -2 \times e^{-2t} u(t) + \delta(t)$$

$$= \delta(t) - 2e^{-2t}u(t) \longrightarrow \text{② Marks}$$

Method - 2

$$\Rightarrow \frac{dy}{dt} + 2y(t) = \frac{dx(t)}{dt} \longrightarrow \text{① Mark}$$

Put $y(t) = A e^{st} u(t) \longrightarrow$ ① Mark

$$\Rightarrow A s e^{st} u(t) + \cancel{t}\,\delta(t) + 2Ae^{st}u(t) = \delta(t)$$

By ~~of~~ the following equation, we get

$A = 1$

and $s = -2$ $\longrightarrow$ ② Marks

$$\Rightarrow y(t) = e^{-2t}u(t)$$

The final answer is $\frac{d}{dt}\left(e^{-2t}u(t)\right)$

$\longrightarrow$ ② Marks

$$= \delta(t) - 2e^{-2t}u(t)$$

Q.4

Ans. Given

$$x[n] = \alpha^n u[n] \qquad for\ 0 < \alpha < 1$$
$$h[n] = \beta^n u[n] \qquad for\ 0 < \beta < 1$$

here $x[n]$ and $h[n]$ are casual. Therefore,

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k]$$

$$= \sum_{k=0}^{n} \alpha^k \beta^{n-k} \qquad for\ n \geq 0 \qquad\qquad (2\ marks)$$

$$= \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$$

$$= \beta^n \left[\frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\frac{\alpha}{\beta}}\right] = \left(\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\right)$$

Hence,

$$y[n] = \begin{cases} \left(\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\right), & for\ n \geq 0 \\ 0, & for\ n < 0 \end{cases} \qquad\qquad (2\ marks)$$

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Ques 5> $$x(t) = e^{j\pi N\frac{t}{T}} \frac{\sin\left(\frac{\pi t}{T}(N+1)\right)}{\sin\left(\frac{\pi t}{T}\right)}$$

To simplify this expression,

$$= e^{j\pi N\frac{t}{T}} \frac{\left(e^{j\frac{\pi t}{T}(N+1)} - e^{-j\frac{\pi t}{T}(N+1)}\right)}{e^{j\pi t/T} - e^{-j\pi t/T}}$$

$$= \frac{e^{j\pi\frac{Nt}{T}} \times e^{j\frac{\pi t}{T}(N+1)}}{e^{j\pi t/T}} \underbrace{\left[\frac{1 - e^{-2j\frac{\pi t}{T}(N+1)}}{1 - e^{-2j\pi t/T}}\right]}_{\text{Sum of G.P.}}$$

$$= e^{2j\frac{\pi t}{T}\times N}\left[1 + e^{-2j\frac{\pi t}{T}} + e^{-4j\frac{\pi t}{T}} + \cdots + e^{-2Nj\frac{\pi t}{T}}\right]$$

$$= 1 + e^{2j\frac{\pi t}{T}} + \cdots + e^{2Nj\frac{\pi t}{T}}$$

$$= \sum_{K=0}^{N} e^{2j\frac{\pi t}{T}\times K}$$

a> Since, the signal is the summation of exponentials, the fundamental time period of the signal

$$= LCM\left(T, T/2, \cdots, T/N\right)$$
$$= T$$

Marking scheme →

→ If only, it is shown that $x(t+T) = x(t)$ and no question about the fundamentality of T is raised then 1 mark is awarded.

→ If three time periods $2T$, $\frac{2T}{N+1}$ and $\frac{2T}{N}$ is written and final answer is written to be $2T$, then 2 marks are awarded.

→ For any other correct approach, full marks 3 are awarded. Even 1 mark is awarded for decent effort.

b) The signal is 0 when $\frac{\pi t}{T}(N+1) = m\pi$ — (1)

except for $m=0$ because it is $\frac{0}{0}$ format of the signal. So total points $= N+1-1 = N$

Marking Scheme.

→ For writing eqn (1), 1 marks has been given.

→ For writing N+1 points or not showing the 0/0 format (1) mark has been deducted otherwise full marks are given.

c) $$a_k = \begin{cases} 1 & \forall\ k = 0, 1, \ldots, N \\ 0 & \text{otherwise} \end{cases}$$

Marking Scheme.

→ For only writing $a_k = 1$ or not writing correct range for k only (2) marks are given.

→ For writing $a_k = 1$ for $k = 0, 1, \ldots N$ and not specifying $a_k = 0$, (1) mark is deducted.

→ No marks for writing Fourier series equation.

→ Full marks for correct values of a_k for given range of k.

d) $$\frac{1}{T}\int_0^T x(t)\,dt = a_0 \Rightarrow \int_0^T x(t)\,dt = T \times a_0 = T \times 1 = T$$

Marking Scheme

→ (1) Mark for writing a_0 or 1 as final answer

→ Zero marks for any efforts or writing 0 as ans.

→ (2) Marks for writing T as final answer.