COL202: Discrete Mathematical Structures **Spring 2023**

Tutorial Sheet 2

Announced on: Jan 12 (Thurs)

1. For what values of $n \in \mathbb{N} \cup \{0\}$ is $2^n > n^2$? Use induction to show that the inequality holds for the reported values of n. Repeat the exercise for $2^n \leqslant n!$.

2. **[Submission Problem for Group 1]** A *graph* (or a network) is a structure consisting of a set of objects (also known as *vertices* or *nodes*) some pairs of which are connected via *edges*. Assume that there are no self-edges (or loops). See Figure 1 for an example.

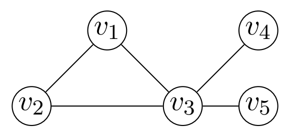

Figure 1: A graph with five vertices v_1, v_2, v_3, v_4, v_5, five edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\{v_1, v_3\}$, $\{v_3, v_4\}$, and $\{v_3, v_5\}$, and maximum degree four.

   The *degree* of a vertex is the number of other vertices that share an edge with it.

   Say we are given a set of k colors. A graph is said to be k*-colorable* if each vertex can be assigned one of the k colors in a way that all neighboring vertices (i.e., any pair of vertices that share an edge) have different colors. (Some of the k colors may be left unused.)

   Prove via induction that any graph with maximum degree d is $(d+1)$-colorable.

3. **[Submission Problem for Group 2]** Prove the following statements using induction:

   a) The number of subsets of an n-element set is 2^n.

   b) The number of ways of ranking n different objects is $n!$.

4. **[Submission Problem for Group 3]** The sequence of Fibonacci numbers $\{F_n\}_{n \in \mathbb{N} \cup \{0\}}$ is defined as follows:

$$F_0 = 0$$
$$F_1 = 1$$
$$\forall n \geqslant 2, F_n = F_{n-1} + F_{n-2}$$

   Let r be a positive real number satisfying $r^2 = r + 1$. Using induction, show that for all $n \in \mathbb{N}$, $F_n \geqslant r^{n-2}$.

5. **[Submission Problem for Group 4]** Prove the following using induction:

   a) For all $n \in \mathbb{N}$ such that $n \geqslant 3$, the sum of internal angles of a regular n-gon is $180(n-2)$.

   b) For all $n \in \mathbb{N}$, the last digit in the decimal expansion of 6^n is 6.