2202 COL 352 Minor1

CHINMAY MITTAL

TOTAL POINTS

30 / 35

QUESTION 1

True or False 15 pts

1.1 SqRootL 0 / 5

✓ **+ 0 pts** *Wrong, ans is $$False$$ **or** Not-attempted*

+ 0 pts Incorrect justification **or** No Justification at all

+ 2.5 pts Slightly correct justification

+ 5 pts Correct Justification

1.2 Fibonacci 5 / 5

✓ **+ 5 pts** *fully correct with proper proof*

+ 0 pts not attempted/completely incorrect answer

+ 3 pts correct answer but incomplete proof

+ 1 pts correct answer but no proof

1.3 Evenregular 5 / 5

✓ **+ 5 pts** *Correct*

+ 2.5 pts Partially correct

+ 0 pts Incorrect

+ 4.5 pts Minor mistake

+ 1 pts Correct True/False

QUESTION 2

2 CountingABBA 10 / 10

✓ **+ 4 pts** *Correct regular expression*

✓ **+ 6 pts** *Correct proof/DFA for regular language*

+ 4 pts Partially Correct Proof/DFA

+ 0 pts Wrong DFA/Proof

+ 0 pts Wrong Regular Expression

QUESTION 3

3 RegularExpressionRegular 10 / 10

+ 0 pts Incorrect(proved that the statement is true)/No attempt

✓ **+ 2 pts** *The Statement is true or false.*

✓ **+ 8 pts** *Correct Proof*

+ 4 pts Partially correct proof

+ 2 pts The proof is not correct but some proof ideas are correct.

+ 0 pts Incorrect Proof/Proof not present

Name: CHINMAY MITTAL

Roll No: 2020CS10336

(COL 352) Introduction to Automata and Theory of Computation

Feb 7, 2023

Minor 1

Duration: 60 minutes **(35 points)**

Beware: Be clear in your writing. If you use a statement proved in class or in the problem set, then write down the entire statement before using it. You will not get a new sheet, so make sure you are certain when you write something (maybe use a dark pencil). Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space. If you cheat, you will surely get an F in this course.

Notation: By $\#_x(w)$ we denote the number of occurrences of string x in w.

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1. ($3 \times 5 = 15$ **points**) For the questions that follow answer whether they are True/False with a brief justification. Each question carries 4 points. Simply writing True or False will not get you any points.

   (a) $\sqrt{L} = \{w \in \Sigma^* \mid ww \in L\}$ is not a regular language. L is regular.

(b) The n-th Fibonacci number is defined as $F_1 = 1, F_2 = 1$, and for all $n \geq 3, F_n = F_{n-1} + F_{n-2}$. Let $\Sigma = \{a\}$. Then $L_2 = \{a^m \mid m = F_n\}$ is regular.

FALSE. for unary languages to be regular they have to be ultimately periodic. and the set of fibbonacci numbers is not ultimatley periodic. Let's say the set of the fibonacci numbers is ultimately periodic with period p. Since the difference b/w successive fibonacci numbers is increasing. We can always find a large n such that $F_{m+1} - F_m > p \quad \forall\, m \geq n$

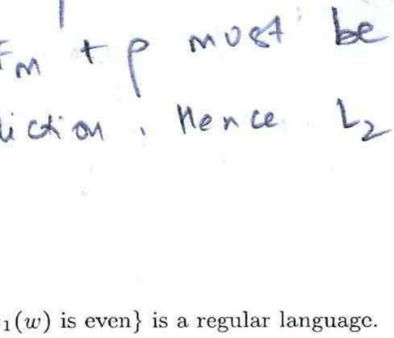
but since our assumption is that fibonacci numbers are ultimately periodic if $F_m + p$ must be a fibonacci number but this is a contradiction, Hence L_2 is not regular.

(c) $L_2 = \{w \in \{0,1\}^* \mid \#_0(w) \cdot \#_1(w) \text{ is even}\}$ is a regular language.

TRUE. L_2 can be seen as the union of L_0 and L_1 where L_0 is the languages with even number of 0's and L_1 is a language with even number of ones. Since any language with either even number of 0s or even number of 1s will be in L_2. Both L_0 and L_1 are regular and hence $L_2 = L_0 \cup L_1$ is regular

DFA for L_0

DFA for L_1



represent seeing odd number of zeros

represents seeing even number of zeros.

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2. (10 **points**) Show that $L = \{w \in \{a, b\}^* \mid \#_{ab}(w) = \#_{ba}(w) + 1\}$ is regular. Give a regular expression for L.

Consider any string in $\{a, b\}^*$

it will be of one of the following forms → ϵ, $(a^+b^+)^+$, $(a^+b^+)^+a^+$, $(b^+a^+)^+$, $(b^+a^+)^+b^+$, a^+, b^+

for ϵ $\quad \#_{ab}(\epsilon) = \#_{ba}(\epsilon) = 0$

for $(a^+b^+)^+$ $\quad \#_{ab}((a^+b^+)^+) = \#_{ba}((a^+b^+)^+) + 1$

for $(b^+a^+)^+$ $\quad \#_{ab}((b^+a^+)^+) = \#_{ba}((b^+a^+)^+) - 1$

for $(a^+b^+)^+a^+$ $\quad \#_{ab}((a^+b^+)^+a^+) = \#_{ba}((a^+b^+)^+a^+)$

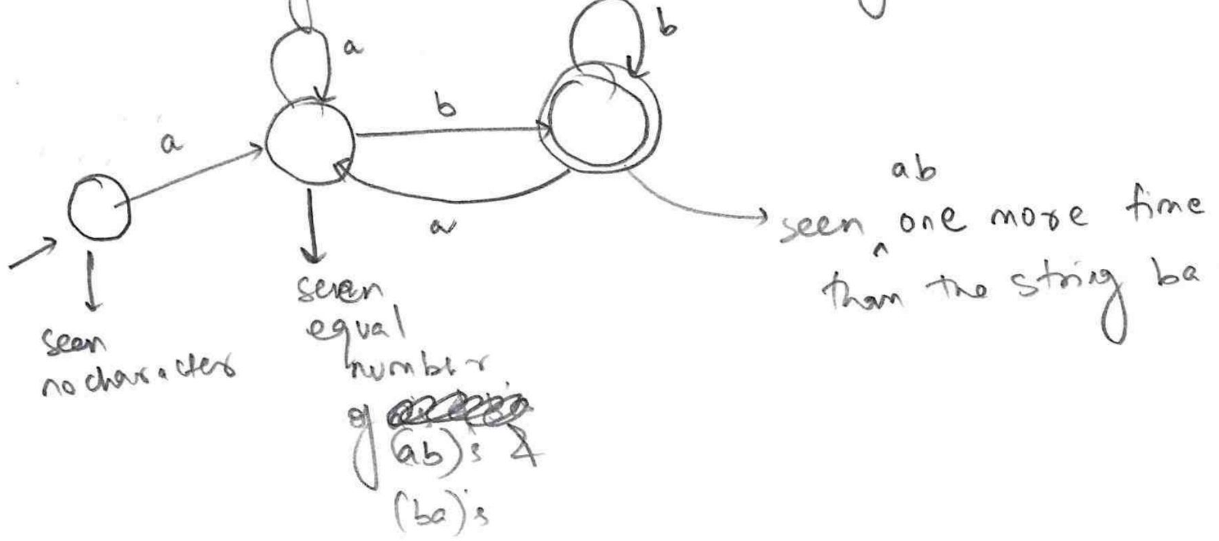
for $(b^+a^+)^+b^+$ $\quad \#_{ab}((b^+a^+)^+b^+) = \#_{ab}((b^+a^+)^+b^+)$

for a^+ $\longrightarrow \#_{ab}(a^+) = \#_{ab}(a^+) = 0$

for b^+ $\longrightarrow \#_{ab}(b^+) = \#_{ab}(b^+) = 0$

The only strings in $\{a, b\}^*$ that satisfy $\#_{ab}(w) = \#_{ba}(w) + 1$ are of the form $(a^+b^+)^+$ → regex. and hence this language is regular

DFA



set of regular expressions over an alphabet is not a regular language

3. **(10 points)** A regular expression over an alphabet Σ can be seen as a string over the alphabet $\Sigma \cup \{\varnothing, \varepsilon, +, *, (,)\}$. Prove or disprove: the set of regular expressions over an alphabet Σ is a regular language.

FALSE note that $(^n a)^n$ is a valid set of regular expressions but is not a regular language

let $a \in \Sigma$

<u>Formal proof</u> by pumping Lemma.

choose any $n \geq 1$

consider the string $w = (^n a)^n$

$|(^n a (^n| = 2n+1 > n$

consider any breakup of w into $w = xyz$ such that $y \neq \epsilon$ and $|xy| \leq n$

the only possible breakups of this form are of the form $x = (^i$

$y = (^j$ $\quad i+j \leq n$

$j \neq 0$

and $z = (^{n-i-j} a)^n$

We need to show a $k \geq 0$ exists such that $xy^k z \notin L$

for $k = 2$ $\quad xy^2z = (^i (^{2j} (^{n-i-j} a)^n$

$= (^{n+j} a)^n \notin L$ $\longrightarrow$ mismatched brackets

hence L is not regular