PYL 102

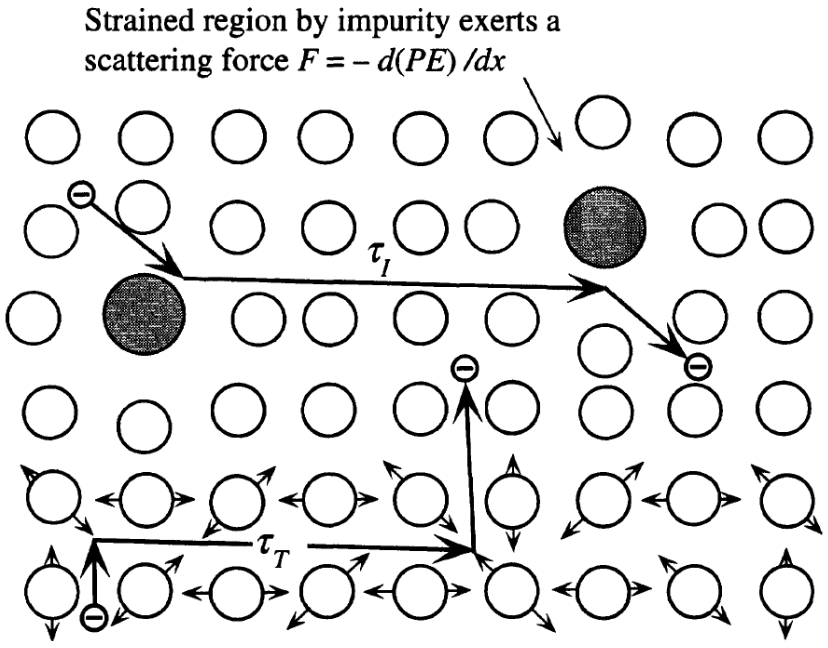
Monday, Nov. 11, 2024

Matthiessen and Nordheem rules for resistivity

Matthiessen's Rule

- Consider a metal alloy that has randomly distributed impurity atoms. An electron can now be scattered by the impurity atoms because they are not identical to the host atoms.
- As long as the impurity atom results in a local distortion of the crystal lattice, it will be effective in scattering. We now effectively have two types of mean free times between collisions: one for scattering from thermal vibrations only, and the other for scattering from impurities only. Here, the effective mean scattering time r is clearly smaller than both mean free times
- In unit time, the overall number of collisions ($1/\tau$) is the sum of the number of collisions with thermal vibrations alone ($l/\ \tau_T$) and the number of collisions with impurities alone ($1/\ \tau_I$). The drift mobility μ_d depends on the effective scattering time τ via $\mu_d = e\ \tau\ /m_e$

The first term is temperature dependent but the second term is not.



$$\frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I}$$

$$\rho = \rho_T + \rho_I$$

$$\rho = \frac{1}{en\mu_d} = \frac{1}{en\mu_L} + \frac{1}{en\mu_I}$$

There may also be electrons scattering from dislocations and other crystal defects, as well as from grain boundaries. All of these scattering processes add to the resistivity of a metal, just as the scattering process from impurities. We can therefore write the effective resistivity of a metal as

$$\boxed{\rho = \rho_T + \rho_R}$$

where ρ_R is called the residual resistivity and is due to the scattering of electrons by impurities, dislocations, interstitial atoms, vacancies, grain boundaries, etc. The residual resistivity shows very little temperature dependence, whereas $\rho_{\cancel{R}T} = AT$, so the effective resistivity ρ is given by

$$\boxed{\rho \approx AT + B}$$

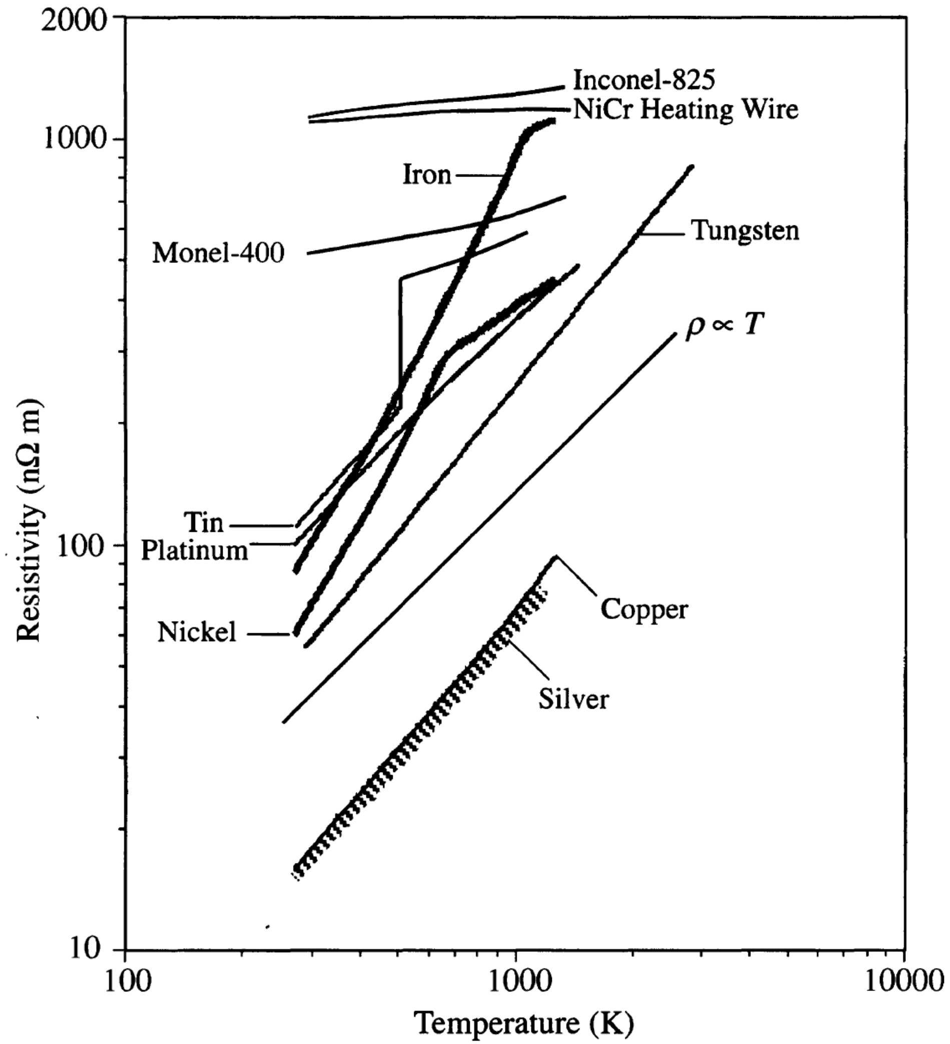
where A and B are temperature-independent constants.

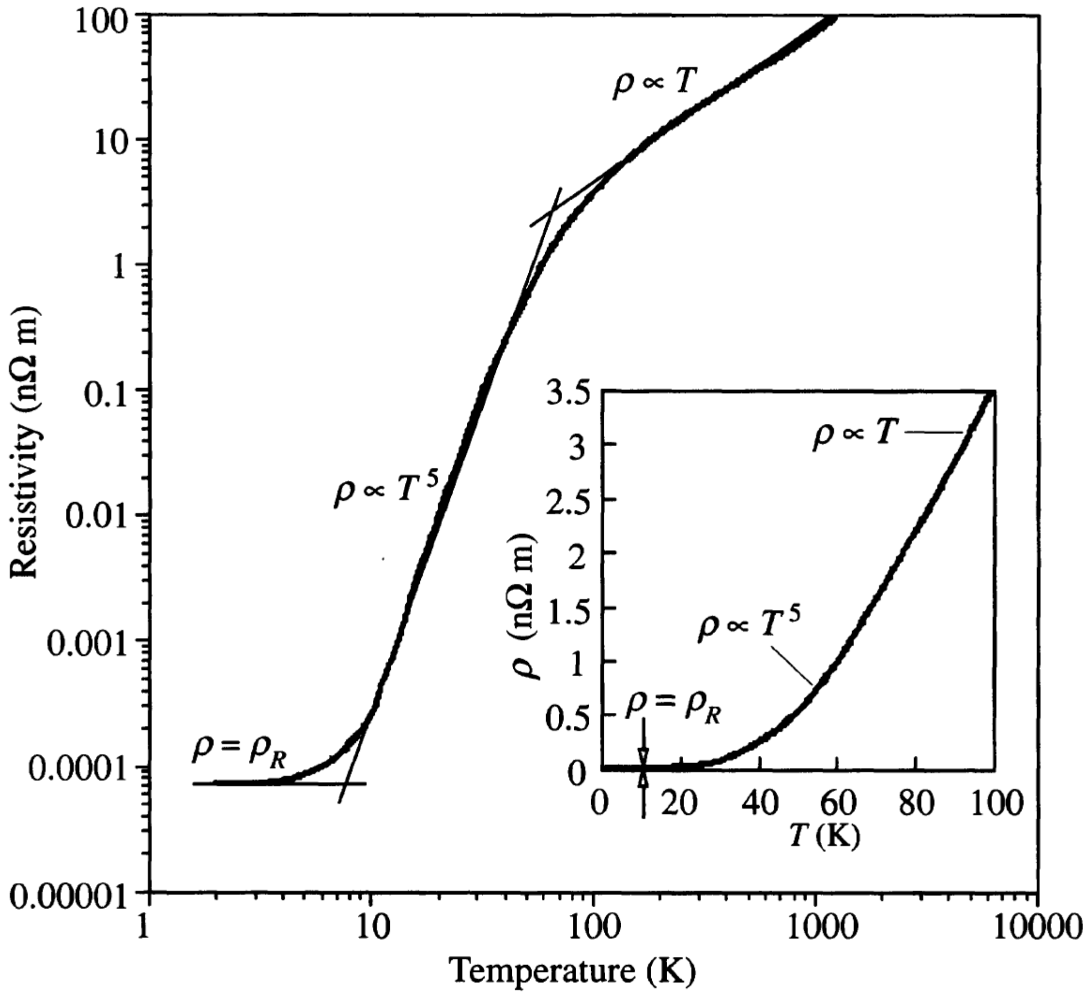
The temperature coefficient of resistivity (TCR) α_o is defined as the fractional change in the resistivity per unit temperature increase at the reference temperature T_o, that is,

$$\boxed{\alpha_0 = \frac{1}{\rho_0}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}}$$

If is α_o constant over a temperature range T_o to T then

$$\boxed{\rho = \rho_0[1 + \alpha_0(T - T_0)]}$$

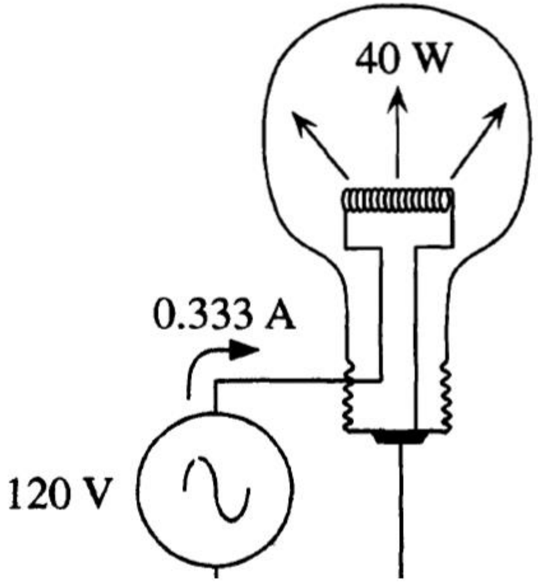




For small temperature (typically below 100 K for many metals) all the atoms are not vibrating with a constant frequency. Indeed, the number of atoms that are vibrating with sufficient energy to scatter the conduction electrons starts to decrease rapidly with decreasing temperature. So, the resistivity becomes strongly temperature dependent. The mean free time becomes longer leading to a smaller resistivity. At low temperature, from Matthiessen's rule, the resistivity becomes $\boxed{\rho = DT^5 + \rho_R}$ where D is a constant.

For most of the metals the resistivity obeys the equation: $\rho = \rho_0 \left[\frac{T}{T_0} \right]^n$

Example: Consider a 40 W, 120 V incandescent light bulb. The tungsten filament is 0.381 m long and has a diameter of 33 μm. Its resistivity at room temperature is 5.51 x 10^{-8} Ω m. Given that the resistivity of the tungsten filament varies at $T^{1.2}$, estimate the temperature of the bulb when it is operated at the rated voltage, that is, when it is lit directly from a power outlet, as shown schematically in Figure. Note that the bulb dissipates 40 W at 120 V.



The current is $I = \frac{P}{V} = \frac{40\ \text{W}}{120\ \text{V}} = 0.333\ \text{A}$

The resistance of the filament at the operating temperature T must be

$$R = \frac{V}{I} = \frac{120}{0.333} = 360\ \Omega$$

Since R = ρL/A, the resistivity of tungsten at temperature T must be

$$\rho(T) = \frac{R(\pi D^2/4)}{L} = \frac{360\ \Omega\ \pi(33 \times 10^{-6}\ \text{m})^2}{4(0.381\ \text{m})} = 8.08 \times 10^{-7}\ \Omega\ \text{m}$$

But, $\rho(T) = \rho_0(T/T_0)^{1.2}$, so that

$$T = T_0\left(\frac{80.8 \times 10^{-8}}{5.51 \times 10^{-8}}\right)^{1/1.2}$$

$$= 2746\ \text{K} \qquad \text{or} \qquad 2473\ °\text{C}$$

Resistivity for Solid Solutions: Nordheem's Rule

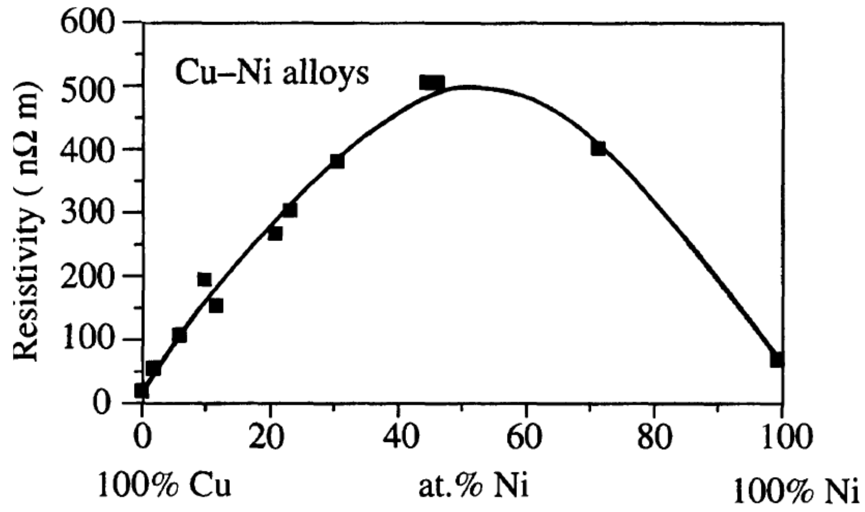
$$\rho = \rho_T + \rho_I$$

A binary alloy that forms a solid solution have the temperature-independent impurity contribution increasing with the concentration of solute atoms. This means that as the alloy concentration increases, the resistivity ρ_I increases and becomes less temperature dependent as ρ overwhelms p_I

The semiempirical equation that can be used to predict the resistivity of an alloy is Nordheim's rule which relates the impurity resistivity to the atomic fraction X of solute atoms in a solid solution, as follows:

$$\rho_I = CX(1 - X)$$

where C is the constant termed the Nordheim coefficient



The Nordheim's rule assumes that the alloying does not significantly vary the number of conduction electrons per atom in the alloy. This is true for alloys with the same valency, that is, from the same column in the Periodic Table (Cu-Au, Ag-Au), it will not be true for alloys of different valency, such as Cu and Zn. Therefore, the resistivity predicted by Nordheem rule will be greater than the actual value.

The resistivity of an alloy of composition X is $\rho = \rho_{\text{matrix}} + CX(1 - X)$ where $\rho_{\text{matrix}} = \rho_T + \rho_R$

Nordheim rule only applies to solid solutions that are single-phase solids. It is valid for homogeneous mixtures in which the atoms are mixed at the atomic level.

Example: The alloy 90 wt.% Au-10 wt.% Cu is sometimes used in low-voltage dc electrical contacts, because pure gold is mechanically soft and the addition of copper increases the hardness of the metal without sacrificing the corrosion resistance. Predict the resistivity of the alloy.

$$\rho_{Au} = 22.8 \text{ n}\Omega \text{ m and } C = 450 \text{ n}\Omega \text{ m},$$

With 10 wt.% Cu converted to the atomic fraction for X. If w is the weight fraction of Cu, w = 0.1, and if M_{Au} and M_{cu} are the atomic masses ofAu and Cu, then the atomic fraction X of Cu is given by

$$X = \frac{w/M_{Cu}}{w/M_{Cu} + (1-w)/M_{Au}} = \frac{0.1/63.55}{(0.1/63.55) + (0.90/197)} = 0.256$$

$$\rho = \rho_{Au} + CX(1-X) = (22.8 \text{ n}\Omega \text{ m}) + (450 \text{ n}\Omega \text{ m})(0.256)(1-0.256)$$

$$= 108.5 \text{ n}\Omega \text{ m}$$