COL751: Quiz-1 Name:

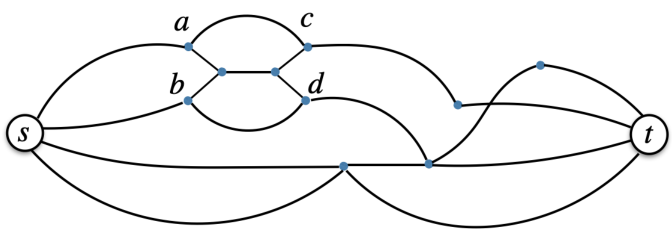
Maximum marks: 10 Entry number: | 2 | 0 | | | | | | | | |

Question 1 Let $G = (V, E)$ be a graph on n vertices, and $(x_1, y_1), \ldots, (x_n, y_n)$ be n vertex pairs. Present an algorithm to compute a subgraph $H = (V, E_H \subseteq E)$ with $O(n^{1.5} \log n)$ edges such that $distance(x_i, y_i, G) = distance(x_i, y_i, H)$, for $i \in [1, n]$. [3 marks]

Question 2 Let G be an undirected unit-capacited graph on n vertices and G_f be residual graph with respect to some (s, t)-max-flow.

A. Prove that the number of directed edges entering SCC of s is same as (s, t)-max-flow value. [2 marks]

B. Explain what are (i) SCCs of G_f, (ii) intra-cluster edges, (iii) the DAG G_f^{scc}, if G is as shown below. [1 marks]



Question 3 Let $G = (V, E)$ be an n vertex weighted undirected graph and $Z \subseteq V$ be a set of size k.

A. Explain how can you compute a $Z \times Z$ distance oracle with stretch 5, $O\big((nk)^{2/3} \log n\big)$ space, and $O(1)$ query time. [3 marks]

B. Let $H_Z = (Z, E_Z)$ be a complete graph such that $wt(x, y, H_Z) = distance(x, y, G)$, for each $x, y \in Z$. Explain how can you use this new graph along with result of (i) as black-box to improve space of $Z \times Z$ oracle to $O(n)$ when $k = n^{2/3}$. [1 marks]