PROBLEM SHEET# 8

Ques.1 Periodic a_k with period $N = 10$

$a_{11} = a_1 = a_{-1} = 5$ (Since real and even)

$\sum_{k=-1}^{8} |a_k|^2 = 50$

$$a_1{}^2 + a_{-1}{}^2 + a_0{}^2 + \sum_{k=2}^{8} |a_k|^2 = 50$$

$$a_0{}^2 + \sum_{k=2}^{8} |a_k|^2 = 0$$

$$a_0, a_2, a_3 \ldots \ldots .. a_8 = 0$$

$$x(n) = 5e^{\frac{j2\pi n}{10}} + 5e^{-\frac{j2\pi n}{10}}$$

$$x(n) = 10\cos\left(\frac{\pi n}{10}\right)$$

$$A = 10, B = \pi/5$$

Ques.2 $x(n)$ is periodic with period $N = 4$

$$a_k = \frac{1}{4}\sum_{n=0}^{3} x(n)e^{-j2\pi nk/4}$$

$$x(n) = \delta(n)\ periodic\ \ 0\ to\ 3$$

$$y(n) = \sum_{k=0}^{3} a_k\, H\left(e^{\frac{j2\pi k}{4}}\right)\, e^{\frac{j2\pi kn}{4}}$$

$$y(n) = \frac{1}{4}\, H\left(e^{0j}\right)\, e^{0j} + \frac{1}{4}\, H\left(e^{j\pi/2}\right)\, e^{\frac{j\pi n}{2}} + \frac{1}{4}\, H\left(e^{j3\pi/2}\right)\, e^{\frac{j3\pi n}{2}} + \frac{1}{4}\, H\left(e^{j\pi}\right)\, e^{j\pi}$$

$$y(n) = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$$

$$y(n) = \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$$

$$y(n) = \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})}$$

$$y(n) = \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{j(\frac{3\pi}{2}n-\frac{\pi}{4})}$$

By comparing

$$H\left(e^{0j}\right) = H\left(e^{j\pi}\right) = 0$$

$$H\left(e^{j\pi/2}\right) = 2\, e^{j\pi/4}\ \ H\left(e^{j3\pi/2}\right) = 2\, e^{-j\pi/4}$$

Ques.3

a) $x_1(n) = (-1)^n$ period $N = 2$

$$a_k = \frac{1}{2}(1 - e^{-j\pi k})$$

$$a_0 = 0 \; a_1 = 1$$

$$y_1(n) = \sum_{k=0}^{1} a_k \, H\left(e^{\frac{j2\pi k}{2}}\right) \, e^{\frac{j2\pi k}{2}}$$

$$y_1(n) = 0 + a_1 H\left(e^{j\pi}\right) e^{j\pi}$$

$$y_1(n) = 0$$

b)

$$x_2(n) = \sin\left(\frac{3\pi}{8} n + \frac{\pi}{4}\right)$$
$$N = 16$$
$$w_0 = \frac{2\pi}{N} = \frac{2\pi}{16} = \frac{\pi}{8}$$

$$x_2(n) = \frac{1}{2j}\left[e^{j\left(\frac{3\pi}{8}n + \frac{\pi}{4}\right)} - e^{-j\left(\frac{3\pi}{8}n + \frac{\pi}{4}\right)}\right]$$

$$a_3 = \frac{1}{2j} e^{j\pi/4} \quad a_{-3} = \frac{-1}{2j} e^{-j\pi/4}$$

$$y_2(n) = \sum_{k=0}^{15} a_k \, H\left(e^{\frac{j2\pi k}{N}}\right) \, e^{\frac{j2\pi kn}{N}}$$

$$H\left(e^{\frac{j\pi k}{8}}\right) \begin{cases} \neq 0 & for \; a_3 \, , a_{13} \\ = 0 & k = 0{,}1 \ldots .14 \end{cases}$$

$$y_2(n) = a_3 e^{\frac{j3\pi n}{8}} + a_{13} e^{\frac{-j3\pi n}{8}}$$

$$a_{13} = a_{-3}$$

$$y_2(n) = \frac{1}{2j} e^{j\pi/4} \; e^{\frac{j3\pi n}{8}} + \left(\frac{-1}{2j}\right) e^{-j\pi/4} e^{\frac{-j3\pi n}{8}}$$

$$y_2(n) = \sin\left(\frac{3\pi}{8} n + \frac{\pi}{4}\right)$$

c)

$x_3(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-4k} u(n - 4k)$ can be written as

$$x_3(n) = \left[\left(\frac{1}{2}\right)^n u(n)\right] * \sum_{k=-\infty}^{\infty} \delta(n-4k) = g(n) * r(n)$$

$$y_3(n) = g(n) * r(n) * h(n)$$

$$q(n) = r(n) * h(n)$$

We pass $r(n)$ through the filter and convolve result with F.S. coeff. Of $r(n)$ are $a_k = \frac{1}{4}$ for all value of k

$$q(n) = \sum_{k=0}^{3} a_k\, H\left(e^{\frac{j2\pi k}{4}}\right) e^{\frac{j2\pi kn}{4}} = 0$$

$$y_3(n) = g(n) * 0 = 0$$

Ques.4

a) $x(n-n_0)$

$$a_k = \frac{1}{N} \sum_{n=<N>} x(n) e^{-\frac{j2\pi nk}{N}}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n-n_0) e^{-\frac{j2\pi nk}{N}}$$

$$n - n_0 = l$$

$$a_k = \frac{1}{N} \sum_{l=-n_0}^{N-1-n_0} x(l) e^{-jk(\frac{2\pi}{N})(l+n_0)}$$

$$= e^{-jk(\frac{2\pi}{N})n_0} \frac{1}{N} \sum_{l=<N>} x(l) e^{-jk(\frac{2\pi}{N})l}$$

$$b_k = e^{-jk(\frac{2\pi}{N})n_0} a_k$$

b) $x(n) - x(n-1)$

$$x(n-1) \rightarrow e^{-jk(\frac{2\pi}{N})} a_k$$

$$x(n) \rightarrow a_k$$

$$b_k = a_k[1 - e^{-jk\left(\frac{2\pi}{N}\right)}]$$

c) $x(n) - x(n - \frac{N}{2})$

$$b_k = a_k[1 - e^{-j\pi k}]$$

$$b_k = \begin{cases} 2a_k \ odd\ k \\ 0 \ \ even\ k \end{cases}$$

d) $x(n) + x(n + \frac{N}{2})$

Hint: $b_k = \frac{2}{N} \sum_{n=0}^{\frac{N}{2}-1} \{x(n) + x(n + \frac{N}{2})\} e^{-jk(\frac{4\pi}{N})n}$

Taking only $x(n+\frac{N}{2})$ part,

$$=\frac{2}{N}\sum_{n=0}^{\frac{N}{2}-1} x(n+\frac{N}{2})e^{-jk(\frac{4\pi}{N})n}$$

Let's take m= n+N/2

$$=\frac{2}{N}\sum_{m=N/2}^{N-1} x(m)e^{-jk(\frac{4\pi}{N})m}$$

So from above hint you can get an answer

$$b_k = 2a_{2k}$$

e) $x^*(-n)$

$$a_k = a_k^*$$

f) $(-1)^n\, x(n)$

$$b_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j(\frac{2\pi n}{N})(k-\frac{N}{2})}$$

$$b_k = a_{k-\frac{N}{2}}$$

g) $(-1)^n\, x(n)$

$$b_k = \frac{1}{2N}\sum_{n=0}^{2N-1} x(n)e^{-j(\frac{2\pi n}{N})(\frac{k-N}{2})}$$

$$b_k = \begin{cases} a_{\frac{k-N}{2}} & k\ \ odd \\ 0 & k\ even \end{cases}$$

h) $y(n) = \begin{cases} x(n) & n\ even \\ 0 & n\ odd \end{cases}$

$$y(n) = \frac{1}{2}[x(n) + (-1)^n\, x(n)]$$

For N even

$$b_k = [a_k + a_{\frac{k-N}{2}}]$$

For N odd

$$b_k = \begin{cases} \frac{1}{2}\left[a_k + a_{\frac{k-N}{2}}\right] & k\ odd \\ \frac{1}{2}a_k & k\ even \end{cases}$$

Ques.5

a) N even $x(n) = -x(n+\frac{N}{2})$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{\frac{N}{2}-1} x(n)e^{-\frac{j2\pi nk}{N}} + \frac{1}{N}\sum_{n=N/2}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[1 - e^{-j\pi k}\right] x(n)e^{-\frac{j4\pi nk}{N}} = 0$$

b) $x(n) = -x(n+\frac{N}{4})$

$$a_k = \frac{1}{N}\sum_{n=0}^{\frac{N}{4}-1} x(n)e^{-\frac{j2\pi nk}{N}} + \frac{1}{N}\sum_{n=\frac{N}{4}}^{\frac{N}{2}-1} x(n)e^{-\frac{j2\pi nk}{N}} + \frac{1}{N}\sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} x(n)e^{-\frac{j2\pi nk}{N}}$$
$$+ \frac{1}{N}\sum_{n=\frac{3N}{4}}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}}$$

$$a_k = \frac{1}{N}\ [\sum_{n=0}^{\frac{N}{4}-1} x(n) + x\left(n+\frac{N}{4}\right)e^{-\frac{j\pi k}{2}} + x\left(n+\frac{2N}{4}\right)e^{-\frac{j2\pi k}{2}}$$
$$+ x\left(n+\frac{3N}{4}\right)e^{-\frac{j3\pi k}{2}}\ ]e^{-\frac{j2\pi nk}{N}}$$

$a_k = 0$ For k multiple of 4.

c)

$$a_k = \frac{1}{N}\ [\sum_{n=0}^{\frac{N}{M}-1}(1 - e^{-j2\pi r} + e^{-j4\pi r} \dots\dots\dots\dots + e^{-j(m-1)\pi r}]\ e^{-\frac{j2\pi nk}{N}}$$

$$r = k/m$$

$a_k = 0$ For k multiple of m.

Ques.6

a) $x_m(n+Nm) = x_m(n)$

$$x_m(n+Nm) = x(\frac{n+mN}{m})$$
$$= x(\frac{n}{m}+N)$$
$$= x\left(\frac{n}{m}\right) = x_m(n)$$
$$n = 0, \pm m, \pm 2m$$

$x_m(n)$ is periodic with period mN.

b) $x(n) = v(n) + w(n)$

$$x\left(\frac{n}{m}\right) = v\left(\frac{n}{m}\right) + w\left(\frac{n}{m}\right)$$
$$x_m(n) = v_m(n) + w_m(n)$$

c) $x(n) = e^{-\frac{j2\pi n k_0}{N}}$

$$x_m(n) = e^{-\frac{j2\pi n k_0}{mN}}$$
$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi n k_0}{N}}$$
$$y(n) = \frac{1}{m}\sum_{l=0}^{m-1} e^{\frac{j2\pi((k_0+lN)}{mN}n}$$
$$y(n) = \frac{1}{m} e^{\frac{j2\pi n k_0}{mN}} \sum_{l=0}^{m-1} e^{\frac{j2\pi n l}{m}}$$
$$y(n) = \begin{cases} e^{\frac{j2\pi n k_0}{mN}} & for\ n = 0, \pm m, \pm 2m \\ 0 & else \end{cases}$$

d) $x_m(n) \rightarrow b_k$

$$b_k = \frac{1}{mN}\sum_{n=0}^{mN-1} x_m(n)\, e^{-\frac{j2\pi n k}{mN}}$$

Only mth value in above is non zero.

$$b_k = \frac{1}{mN}\sum_{n=0}^{N-1} x_m(mn)\, e^{-\frac{j2\pi m n k}{mN}}$$
$$b_k = \frac{1}{mN}\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi n k}{N}}$$
$$b_k = \frac{1}{m} a_k$$

Ques.7

a) $x(n) \rightarrow a_k$

$$x^*(-n) \rightarrow a^*_{-k}$$
$$x(n) \times x^*(-n) = |x(n)|^2$$
$$b_k = \sum_{l=<N>} a_l\, a^*_{l+k}$$

b) Yes

Ques.8

a) Not LTI
b) LTI Unique

$$H(e^{jw}) = \frac{1 - \left(\frac{1}{2}\right) e^{-jw}}{1 - \left(\frac{1}{4}\right) e^{-jw}}$$

c) LTI Unique

$$H\left(e^{jw}\right) = \frac{1 - \left(\frac{1}{2}\right) e^{-jw}}{1 - (\frac{1}{4}) e^{-jw}}$$

d) LTI Not Unique

$$H\left(e^{\frac{j}{8}}\right) = 2$$

e) LTI Unique

$$H\left(e^{jw}\right) = 2$$

f) LTI Not Unique

$$H\left(e^{j\pi/2}\right) = 2(1 - e^{\frac{j\pi}{2}})$$

g) LTI Not Unique

$$H\left(e^{j\pi/3}\right) = 1 - j\sqrt{3}$$

h) LTI Not Unique

i) Not LTI