COL 202 Discrete Mathematics **Diwali 2023**

Tutorial 9

1. [Submission Problems for Group 1] Problem 10.48 in LLM Book
2. [Submission Problems for Group 2] Problem 10.24 in LLM Book
3. [Submission Problems for Group 3] Problem 10.7 in LLM Book
4. [Submission Problems for Group 4] Problem 10.6 in LLM Book
5. [Bonus] Some of the problems could be hard and maynot fit into the tutorial slot - use piazza for discussing those.

   (a) A digraph G is strongly connected if there is a (directed) path between each pair of vertices, i. e., all vertices belong to the same strong component. (There is just one strong component.) An undirected walk (path, cycle, etc.) in a digraph is a walk (path, cycle, etc.) in the undirected graph obtained by ignoring orientation. A digraph is weakly connected if there is an undirected path between each pair of vertices.

   Prove: if $(\forall v \in V)(outdegree(v) = indegree(v))$ and G is weakly connected then G is strongly connected.

   (b) Problem 10.13 in LLM Book

   (c) Tournaments are orientations of complete graphs. So in a tournament $G = (V, E)$, for every pair of vertices $u, v \in V$, exactly one of the following holds: (a) $u = v$; (b) $u \to v$; (c) $v \to u$. We can think of the vertices of a tournament as players in a round-robin tournament without ties or rematches. Each player plays against every other player exactly once; $u \to v$ indicates that player u beat player v.

   A **Hamilton cycle** in a digraph is a (directed) cycle of length n, i. e., a cycle that passes through all vertices. G is **Hamiltonian** if it has a Hamilton cycle. A **Hamilton path** in a digraph is a (directed) path of length $n - 1$, i. e., a path that passes through all vertices.

   i. Count the tournaments on a given set of n vertices.
   ii. Let T be a tournament with n vertices. Prove: if all vertices have the same out-degree then n is odd.
   iii. Prove that every tournament has a Hamilton path.
   iv. Prove that every strongly connected tournament is Hamiltonian.