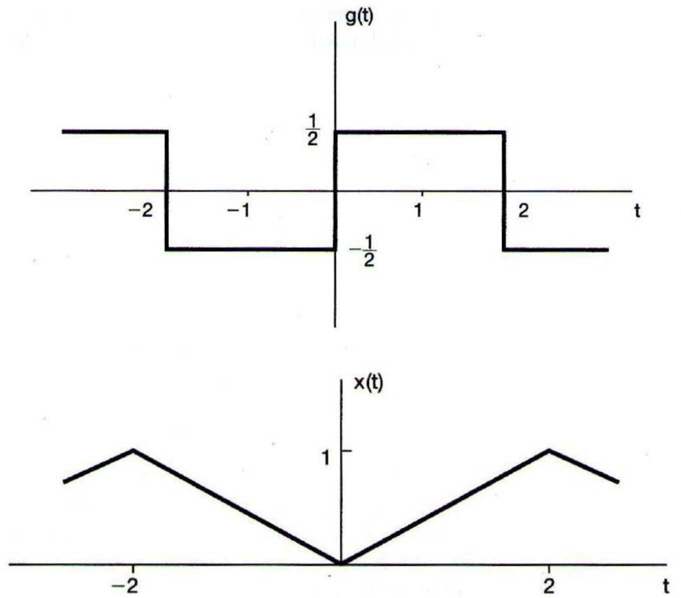
Problem sheet – 6

1. Using properties of Fourier Series

Consider the signals *g(t)* and *x(t)* given below of fundamental period $T = 4$. Note that the derivative of signal *x(t)* is the same as signal *g(t)*. Find the Fourier coefficients of signal $g(t)$. Using the derivative property of Fourier Series coefficients, find the same for signal *x(t).*



2. Fourier representation of an Impulse Train

a) Find the Fourier series representation of the impulse train given by

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

b) Represent the derivative of *g(t)* in the question above in terms of a shifted impulse train and hence find the Fourier coefficients of *g(t)* in terms of Fourier Coefficients of *x(t).* Verify it with your answer for question 4.

3. Fourier series representation of a chess board

A chess board image has to be represented using a Fourier series. Think about a possible representation.

(Hint: image is a two dimensional signal)

4. Signal Determination from Fourier coefficient properties

Suppose we are given the following information about a signal x(t):

a. x(t) is real and odd.

b. x(t) is periodic with period T = 2 and has Fourier coefficients a_k.
c. a_k= 0 for | k | > 1.
d. $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

Specify 2 different signals with the above properties.

5. Orthogonal representation of signals

Two functions u(t) and v(t) are said to be orthogonal over the interval (a,b)

$$\int_a^b u(t)v^*(t)dt = 0$$

If, in addition,

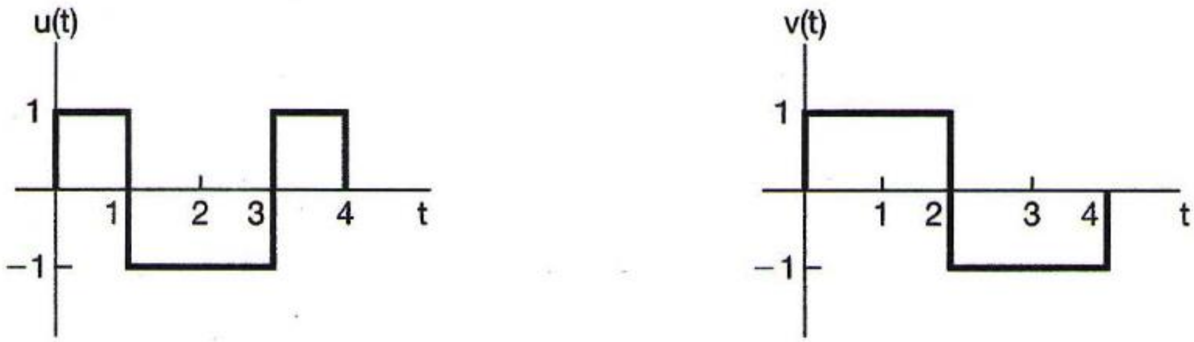
$\int_a^b |u(t)|^2 dt = 1, \quad \int_a^b |v(t)|^2 dt = 1$

the functions are said to be normalized and hence are called orthonormal. A set of functions $\{\phi_k(t)\}$ is called an orthogonal (orthonormal) set if each pair of functions in the set is orthogonal (orthonormal).

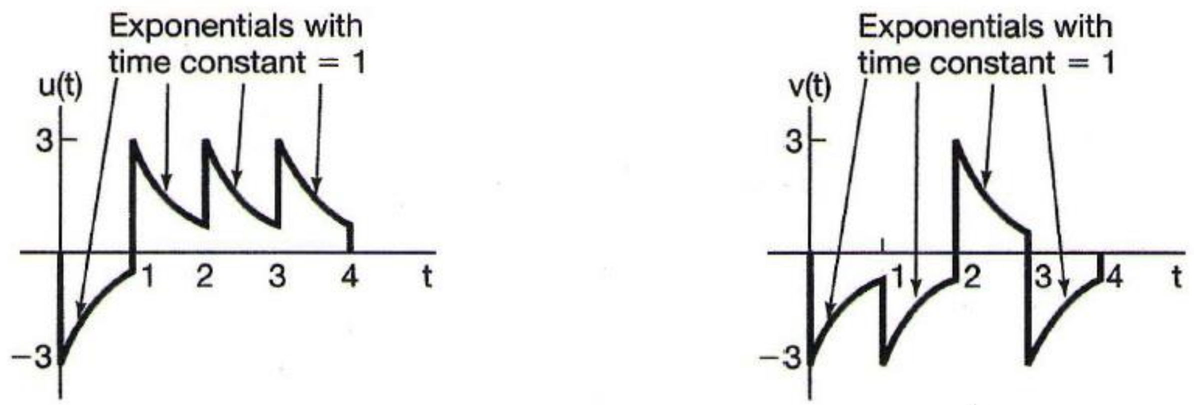
a) Consider the pairs of signals u(t) and v(t) depicted in Figure below. Determine whether each pair is orthogonal over the interval (0, 4).
b) Are the functions $\sin(mw_0t)$ and $\sin(nw_0t)$ orthogonal over the interval (0, T), where T $= {}^{2\pi}/_{w_0}$? Are they also orthonormal?
c) Repeat part (b) for the functions $\phi_m(t)$ and $\phi_n(t)$, where $\varphi_k(t) = \frac{1}{\sqrt{T}}[\cos(kw_0t) + \sin(kw_0t)]$.
d) Show that the functions $\phi_k(t) = e^{jkw_0t}$ are orthogonal over any interval of length $T = 2\pi/w_0$. Are they orthonormal?
e) Let x(t) be an arbitrary signal, and let $x_o(t)$ and $x_e(t)$ be, respectively, the odd and even parts of x(t). Show that $x_o(t)$ and $x_e(t)$ are orthogonal over the interval (-T, T) for any T.
f) Show that if $\{\phi_k(t)\}$ is a set of orthogonal signals over the interval (a, b), then the set $\{\frac{1}{\sqrt{A_k}}\phi_k(t)\}$, where

$$A_k = \int_a^b |\varphi_k(t)|^2 dt$$

is orthonormal.


u(t)
1
−1
1
2
3
4
t
v(t)
1
−1
1
2
3
4
t

(a)


Exponentials with
time constant = 1
u(t)
3
−3
1
2
3
4
t
Exponentials with
time constant = 1
v(t)
3
−3
1
2
3
4
t

(b)

(c)

(d)

g) Let $\{\phi_i(t)\}$ be a set of orthonormal signals on the interval (a, b), and consider a signal of the form

$$x(t) = \sum_i a_i \varphi_i(t)$$

where the a_i are complex constants. Show that

$$\int_a^b |u(t)|^2 dt = \sum_i a_i^2$$

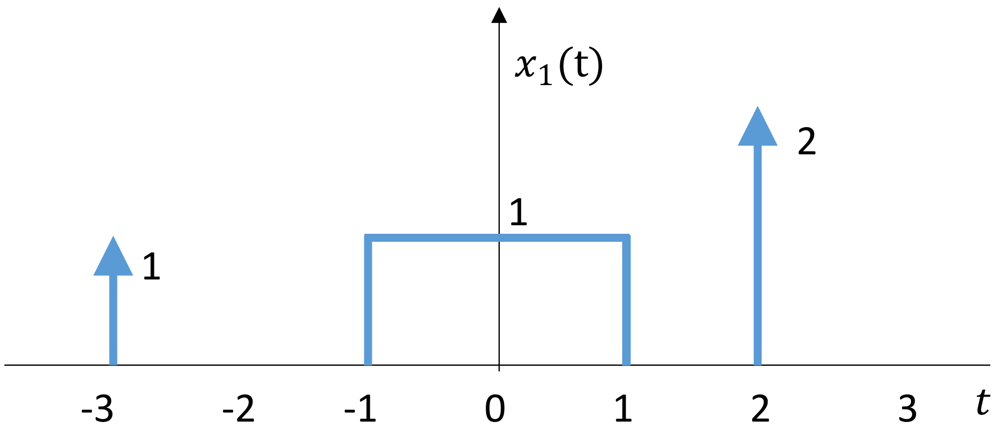
h) Suppose that $\phi_1(t), \dots \phi_n(t)$ are nonzero only in the time interval $0 < t < T$ and that they are orthonormal over this time interval. Let L_i denote the LTI system with impulse response

$$h_i(t) = \varphi_i(T - t)$$

Show that if $\phi_j(t)$ is applied to this system, then the output at time T is 1 if i = j and 0 if i≠j. The system with impulse response given by eq. above was referred to in Problems 2.66 and 2.67(Oppenhiem) as the matched filter for the signal $\phi_i(t)$.

6. Convolution of two signals

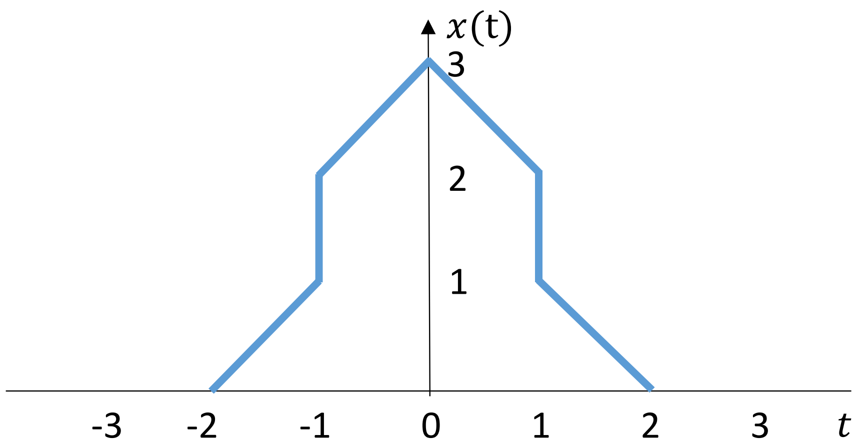
Let $x_1(t)$ and $x_2(t)$ be two continuous-time signals as given below. Find and plot $y(t) = x_1(t) * x_2(t)$.



$$x_2(t) = u(t + 1.5) - u(t - 1.5)$$

7. Output of a periodic signal

Consider the periodic signal, $x(t)$, given in the figure below

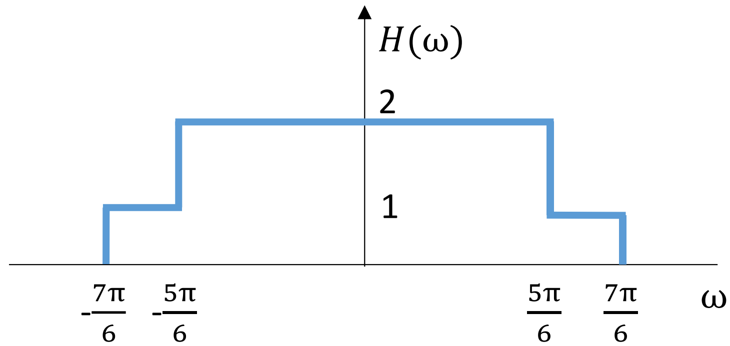


One period of $x(t)$ is :

$$x(t) = \begin{cases} 0 & -3 \le t \le -2 \\ as\ given\ in \text{ the fig.} & -2 \le t \le 2 \\ 0 & 2 \le t \le 3 \end{cases}$$

(a) Find Fourier series representation for $x(t)$.

(b) Find $y(t) = x(t) * h(t)$, given that $h(t)$ has a Fourier transform $H(\omega)$, where $H(\omega)$ is as given in the figure below



H(ω)
2
1
-7π/6
-5π/6
5π/6
7π/6
ω