# Department of Mathematics
# MTL 106 (Introduction to Probability and Stochastic Processes)
# Tutorial Sheet No. 4
### Answer for Selected Problems

1. $(a)$ $p_x(1) = 0.2$, $p_x(3) = 0.5$ $p_x(4) = 0.3$ $p_y(1) = 0.4$, $p_y(2) = 0.6$ $(b)$ $0.5$

2. $(a)$ $P_X(j) = qp^{j-1},\ j = 1, 2, \ldots$ $P_Y(k) = q^k \left( \frac{\frac{p}{q} - \left(\frac{p}{q}\right)^k}{1 - \frac{p}{q}} \right),\ k = 2, 3, \ldots$

   $P_{X/Y}(j/k) = \frac{(p/q)^j(1-(p/q))}{(p/q)-(p/q)^k},\ j = 1, 2, \ldots, k-1$ $P_{Y/X}(k/j) = q^{k-j-1}p,\ k = j+1, j+2, \ldots$

   $(b)$ $X \sim B(1/2, 15)$, $Y \sim B(1/3, 15)$, $Y/(X = j) \sim B(2/3, 15 - j)$, $X/(Y = k) \sim B(3/4, 15 - k)$

4. $k = 12$, $(1 - e^{-8})(1 - e^{-3})$

5. Let $Y$: r.v. denoting no. of 1's transmitted. $P(Y = n) = \frac{e^{-\lambda(1-p)}(\lambda(1-p))^n}{n!},\ n = 0, 1, \ldots$

6. $Z = X + Y$, $f_Z(z) = \begin{cases} z \quad , & 0 < z < 1 \\ 2 - z, & 1 \le z \le 2 \\ 0, & \text{otherwise} \end{cases}$

7. $f_z(z) = \begin{cases} \frac{\lambda}{2} e^{\lambda z}, & z < 0 \\ \frac{\lambda}{2} e^{-\lambda z}, & z \ge 0. \end{cases}$

8. $f_V(v) = -\ln(v),\ 0 < v < 1; f_W(w) = \frac{1}{2\sqrt{w}}, 0 < w < 1; f_{V,W}(v, w) = f_V(v) f_W(w)$

9. $f_{R,\theta}(r, \theta) = \begin{cases} r, & 0 < \theta < \pi/4, 0 < r < \sec\theta \text{ or } \pi/4 < \theta < \pi/2, 0 < r < cosec\theta \\ 0, & \text{otherwise} \end{cases}$

   $f_\theta(\theta) = \begin{cases} \frac{1}{2}\sec^2\theta, & 0 < \theta < \pi/4 \\ \\ \frac{1}{2} cosec^2\theta, & \pi/4 < \theta < \pi/2 \end{cases}$

   $f_R(r) = \begin{cases} \frac{\pi}{2} r, & 0 < r < 1 \\ \\ r(cosec^{-1}(r) - \sec^{-1}(r)), & 1 < r < \sqrt{2} \end{cases}$

10. $\frac{1}{8}$

11. (a) Let $Y$: r.v. denoting the waiting time of passenger. $f_Y(y) = \begin{cases} \frac{1}{10}, & 0 < y < 5, \\ \frac{1}{20}, & 5 < y < 15, \\ 0, & \text{otherwise.} \end{cases}$ (b) $\frac{25}{4}$ min.

12. $k = \frac{1}{8}$, $\frac{5}{81}$, $1$

13. $\frac{5}{16} + \frac{1}{12}\ln 4$

14. (a) $\frac{3}{8}$ (b) $\frac{3}{8}e^{-\frac{1}{3}}$ (c) $\frac{3}{8}e^{-\frac{1}{3}} + \frac{5}{8}e^{-\frac{1}{5}}$

15. (a) Yes (b) $1 + \frac{t^2}{2}$ (c) 12

17. 31

18. a) $\frac{3^5 - 1^5}{4^5}$ b) $\frac{1}{20}$

19. (a) $\frac{\sigma^2}{n}$ (b) $\sigma^2$

20. $\frac{X^2}{3}$