MTL103
Tutorial 5 Hints

1. The following payoff matrix corresponds to a modified version of the Prisoner's Dilemma problem called the DA's brother problem. In this problem prisoner 1 is related to the District Attorney. How is this problem different? How many Nash equilibria are there? Does player 2 really have a choice?

1	2	
	NC	C
NC	$0, -2$	$-10, 1$
C	$-1, -10$	$-5, -5$

   Hint: Notice that Player 2 will never play NC no matter what player 1 does. Hence there is only one Nash equilibrium unlike the classical Prisoner's Dilemma case. Even though (NC, NC) has better payoffs for both 1 and 2, it will never be played.

2. Consider any arbitrary two player game of the following type (with a, b, c, d any arbitrary real number):

	A	B
A	a, a	b, c
B	c, b	d, d

   It is known that the game has a strongly dominant strategy equilibrium. Now prove or disprove: The above strongly dominant strategy equilibrium is the only possible mixed strategy equilibrium of the game.

   Hint: In a strongly dominant strategy equilibrium, each player plays his strictly dominant strategy. Notice only (a, a) or (d, d) can be the strongly dominant strategy equilibrium.

   If (d, d) is strongly dominant strategy equilibrium, then $d > b,\ c > a$. If (a, a) is strongly dominant strategy equilibrium, then $d < b,\ c < a$.

   If a player is randomizing between two alternatives then he must be indifferent between them. If player 2 plays A with prob q, then expected payoff of player 1 must be same if he plays pure strategy A or if he plays pure strategy B. That is $aq + b(1-q) = cq + d(1-q)$. But this is not possible if (a, a) or (d, d) is a strictly dominant strategy equilibrium because either $d > b,\ c > a$ or $d < b,\ c < a$.

3. An $m \times m$ matrix is called a latin square if each row and each column is a permutation of $(1, ..., m)$. Compute pure strategy Nash equilibria, if they exist, of a two person game for which a latin square is the payoff matrix.

   Hint: Only (m, m) can be Nash equilibria and all (m, m) are Nash equilibria.

4. Consider the following instance of the prisoners' dilemma problem.
   Find the values of x for which:

   (a) the profile (C,C) is a strongly dominant strategy equilibrium.

1	2	
	NC	C
NC	$-4, -4$	$-2, -x$
C	$-x, -2$	$-x, -x$

(b) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.

(c) the profile (C,C) is a not even a weakly dominant strategy equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

Hint: By definition of strongly/weakly dominant strategy equilibrium

(a) $x > -2$

(b) $x = -2$

(c) $x < -2$

5. Find the pure strategy Nash equilibrium of the following game.

	X	Y	Z
X	$6, 6$	$8, 20$	$0, 8$
Y	$10, 0$	$5, 5$	$2, 8$
Z	$8, 0$	$20, 0$	$4, 4$

6. Find the mixed strategy Nash equilibria for the following games:

   (a) *(Matching Pennies Game)*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

   Hint: Again, if a player is randomizing between two alternatives then he must be indifferent between them. If player 2 plays heads with probability q,
   $q + -1(1-q) = -q + (1-q) \implies q = 1/2$

   (b) *(Rock-Paper-Scissors Game)*

1	3		
	Rock	Paper	Scissors
Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
Scissors	$-1, 1$	$1, -1$	$0, 0$