TUTORIAL- 9

Solution 1:

$x[n] \overset{DTFT}{\longleftrightarrow} X(\Omega)$

$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega$ …………(1)

$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$ …………..(2)

Where $X(\Omega)$ is continuous and periodic (w/period 2π)

$X(\Omega)$ has a Fourier series

$X(\Omega) \leftrightarrow a_k$

$X(\Omega) = \sum_{k=-\infty}^{\infty} a_k\ e^{-jk\Omega}d\Omega$ $(w_0 = 2\pi/2\pi = 1)$ ………..(3)

$a_k = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{-jk\Omega}$ …………….(4)

Where a_k are the Fourier series coefficient of X(Ω)

Comparing equation (1) and (4)

$$x[n] = a_{-n}$$

Or (5)

$$x[-n] = a_n$$

The original discrete signal is the reversed Fourier series of Its DTFT.

Applying Parsevel's Relation on X(Ω) and a_k

$\frac{1}{2\pi}\int_{2\pi} |X(\Omega)|^2\, d\Omega = \sum_{n=-\infty}^{\infty} |a_n|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$ ………………….(using Eq. 5)

Solution 2: Accumulation in discrete-time is like integration in continuous-time. So, there is a DC component in the DTFT of accumulation of $x[n]$.

Taking the difference equation $y[n] - y[n-1]$, this DC component will be lost.

Solution 3:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

(a) Assuming DTFT of x[n] and y[n] exists

$$Y(\Omega)\left[1 - \frac{3}{4}e^{-j\Omega} + \frac{1}{8}e^{-2j\Omega}\right] = 2X(\Omega)$$

$$\frac{Y(\Omega)}{X(\Omega)} = H(\Omega) = \frac{2}{1 - \frac{3}{4}e^{-j\Omega} + \frac{1}{8}e^{-2j\Omega}}$$

$$H(\Omega) = \frac{4}{1 - \frac{e^{-j\Omega}}{2}} - \frac{2}{1 - \frac{e^{-j\Omega}}{4}}$$

$$H[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

(b) No, we didn't do any error similar to part-2. The given system is invertible as $H(\Omega) \neq \infty \; \forall \, \Omega$. No poles.

(c) Using DTFT methods, we Assume on LTI system with zero initial condition $\leftrightarrow$ causal system.

These initial conditions x[n]=0 for all $n < n_0$

y[n]=0

form the auxiliary conditions

(d) Using the delay operator D{x[n]}=x[n-1]

$$\gg \left(1 - \frac{3}{4}D + \frac{1}{8}D^2\right) y[n] = 2x[n]$$

$$\gg Y(\Omega) = \frac{2X(\Omega)}{1 - \frac{3}{4}D + \frac{1}{8}D^2}$$

is the frequency response.

(e) Yes, unstable solution of the difference equation cannot be found using this method (as taking the DTFT assumes stability)

Solution 4:

$x[n] \leftrightarrow a_k$ ………………..(1)

$x[n] = \sum_{k=<N>} a_k \, e^{-jk2\pi \;\; /N}$

$a_k = \frac{1}{N}\sum_{<N>} x[n]e^{jk2\pi n/N}$

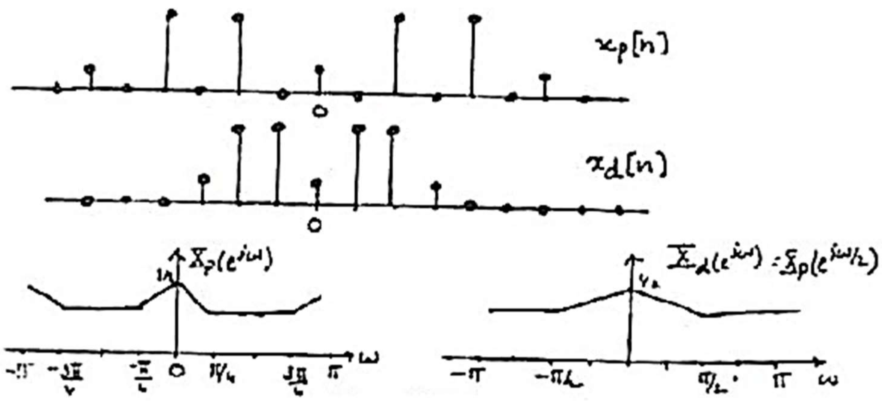
$a_k = \frac{1}{N}\sum_{<N>} \frac{x[-n]}{N} e^{-jk2\pi n/N}$ ………………..(2)

Comparing (2) with (1)

a_k has Fourier series coefficient $\dfrac{x[-n]}{N}$

a_n has Fourier series coefficient $\dfrac{x[-k]}{N}$

Solution 5:



(a) The signals $x_p[n]$ and $x_d[n]$ are sketched in the above figure.
(b) $X_p(e^{jw})$ and $X_d(e^{jw})$are sketched in the above figure.

Solution 6:

The Fourier transform of x[n] is given by

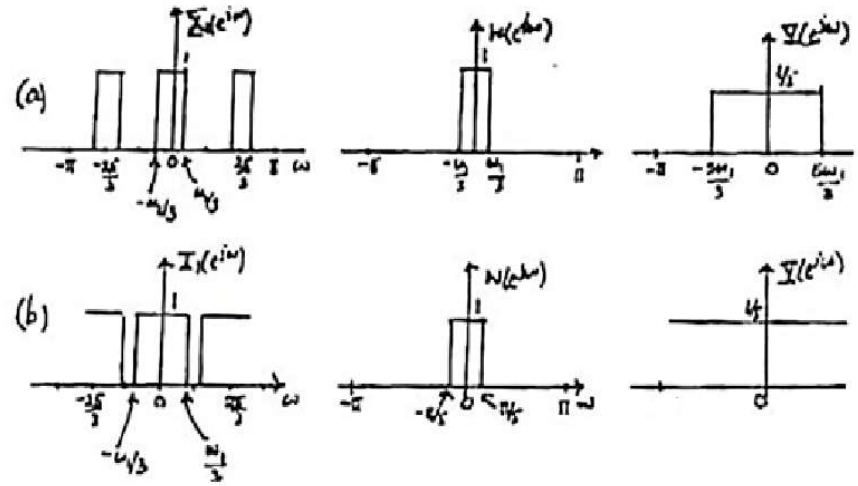
$$X(e^{jw}) = \begin{cases} 1, & |w| \le w_1 \\ 0, & other\ wise \end{cases}$$

This is shown in fig

(a) When $w_1 \le 3\pi/5$, the Fourier transform $X_l(e^{jw})$ of the output of the zero-insertion system is as shown in the fig. The output $W(e^{jw})$ of the lowpass filter is as shown is as shown in fig. The Fourier transform of the output of the decimation system $Y(e^{jw})$. This is as shown fig.
Therefore,

$$Y[n] = \frac{\sin\left(\frac{5w_1 n}{3}\right)}{5\pi n}$$

(b) When $w_1 \ge 3\pi/5$, the Fourier transform $X_1(e^{jw})$ of the output of the zero-insertion system is as shown in Fig. The output $W(e^{jw})$ of the lowpass filter is as shown in Fig

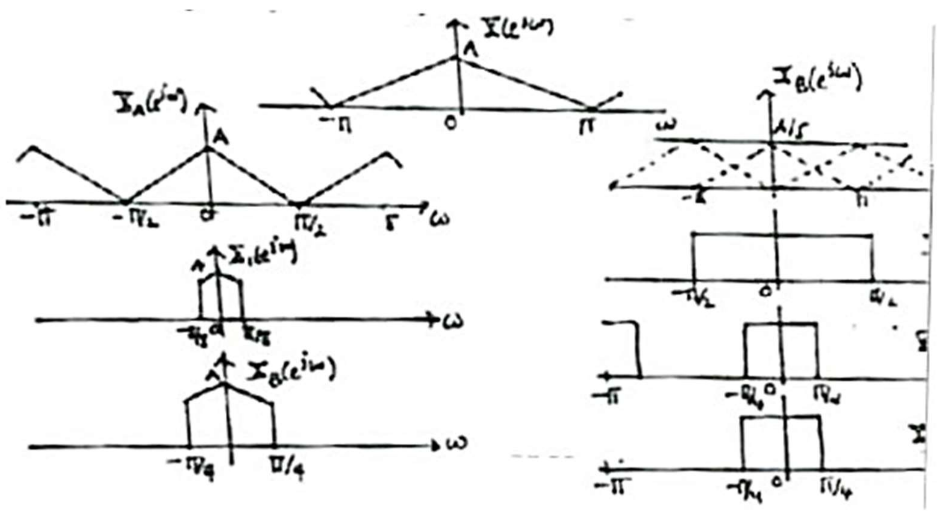


The Fourier transform of the output decimation system $Y(e^{jw})$ is an expanded or stretched out version of $W(e^{jw})$.This is as shown in figure. Therefore,

$$y[n] = \frac{1}{5}\,\delta[n]$$

Solution: 7

(a) Suppose that $X(e^{jw})$ is as shown in the figure, then the Fourier transform X of the output of S_A , the Fourier transform $X_1(e^{jw})$ of the output of the lowpass filter and the Fourier transform $X_B(e^{jw})$ of the output of S_B are all shown in the fig. below. Clearly this system accomplishes the filtering task.



(b) Suppose that $X(e^{jw})$ is as shown in the figure, then the Fourier transform X of the output of S_B, the Fourier transform $X_1(e^{jw})$ of the output of the first lowpass filter and

the Fourier transform $X_A(e^{jw})$ of the output of S_A , the Fourier transform of the output of the first lowpass filter are all shown in the fig below.
Clearly this system doesn't accomplish the filtering task.

Solution 8:

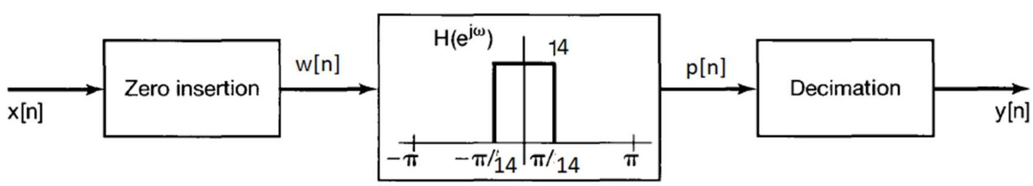
Let $y[n] = x[n]\sum_{k=-\infty}^{\infty} \delta[n-3k]$. Then

$$Y(e^{jw}) = \frac{1}{3}\sum_{k=0}^{3} X(e^{j\left(w-\frac{2\pi k}{3}\right)})$$

Note that $\sin\left(\frac{n\pi}{3}\right)/(n\pi/3)$ is the impulse response of an ideal low pass filter with cut-off frequency π/3 and passband gain of 3. Therefore, we now require that y[n] when passed through this filter should yield x[n]. Therefore, the replicas of $X(e^{jw})$ contained in $Y(e^{jw})$ should not overlap with one another. This is possible only if $X(e^{jw})=0$ for $\pi/3 \le |w| \le \pi$

Solution 9:

In order to make $X(e^{jw})$ occupy the entire region from -π to π, the signal x[n] must be down sampled by a factor of 14/3. Since it is not possible to directly down sample by a non-integer factor, we first up sample the signal by a factor of 3. Therefore, after the up sampling we will need to reduce the sampling rate by14/3 *3 =14. Therefore, the overall system for performing the sampling rate conversion is shown in fig.



$$w[n] = \begin{cases} x[n], n = 0. \pm 3, \pm 6, \dots \\ 0, \quad otherwise \end{cases}$$

y[n]=p[14n]