

# COL 351 Quiz 1B

PrathamAgrawal

TOTAL POINTS

**9 / 10**

QUESTION 1

1 Question 1 9 / 10

✓ + **3 pts** *Correct Greedy Algorithm Idea*

✓ + **1 pts** *Proper Algorithm Description*

+ **4 pts** *Proof Mostly correct, but some Mistake/Informalness in Proof*

✓ + **5 pts** *Proof Mostly correct, Slight Mistake/Informalness in Proof*

+ **6 pts** *Correct Proof*

- **1 pts** *Only proved for first element. Did not extend it using induction*

+ **0 pts** *Incorrect / Not Attempted*

1 First need to show that there exists an optimal solution in which  $s_1$  is picked

## Quiz I (COL 351)

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Give precise arguments. Needlessly long explanations will not fetch any marks.

You have  $n$  friends, call them  $1, \dots, n$ . Person  $i$  has *unfriendliness* value  $s_i$ . You are given a target  $T$ . You need to divide these  $n$  friends into teams of two each (assume  $n$  is even, and so there will be  $n/2$  teams). A team consisting of  $i$  and  $j$  is said to be good if  $s_i + s_j \leq T$ . Give a greedy algorithm to divide the  $n$  friends into teams of two each such that the number of good teams is maximized.

**Greedy Algorithm:**

sort in increasing order of unfriendliness value's  $s_i$ .  
it becomes  $s_1, s_2, \dots, s_n$  in sorted order.

starting from  $s_1$ , we find the friend with maximum  $s_j$  s.t.  
 $s_1 + s_j \leq T$ . Then, we continue from the left. number of  
such pairs found will be the maximum answer possible.

**Proof of Correctness:**

Example  $\rightarrow$  1 2 4 2 8 7 6 5,  $T=9$

sort  $\rightarrow$  1 2 3 4 5 6 7 8,  $T=9$

1. ~~sort~~ order doesn't matter  
as we are just choosing 2 friends.  $\therefore$  answer = 4.  
 $\therefore$  sorting doesn't affect the problem.

$\rightarrow$  2. let  $s_1, s_2, \dots, s_n$  be the sorted order.

if  $s_1$  is matched with  $s_j$  (according to my greedy algo  $G$ )

$s_1 + s_j \leq T$  and  $s_j$  is the max such value.

i.e.  $s_1 + s_{j+1} > T, \dots, s_1 + s_{j+2} > T, \dots, s_1 + s_n > T$

$\rightarrow$  in the optimal sol<sup>n</sup>  $O$ , say  $s_1$  is matched to  $s_k$  where  $k \neq j$ .

if  $k$  clearly  $k < j$ . as if  $k > j \rightarrow s_1 + s_k > T$ .

if  $k < j \rightarrow s_1 + s_k \leq T$ . Now we reduce our sample space<sup>n</sup>  
to  $s_2, \dots, s_{k-1}$ . However if we chose  $s_j$  instead of  
 $s_k$ , our sample space<sup>n</sup> would have been  $s_2, \dots, s_{j-1}$ .

$\therefore s_1 \subseteq s_2$  (subset notation). To prove, our greedy  $G$

is as much optimal as  $O$ , we need to show, number of  
pairs in  $s_2 \geq$  number of pairs in  $s_1$ .

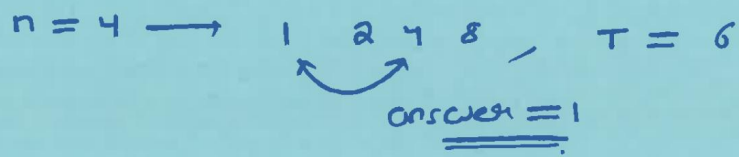
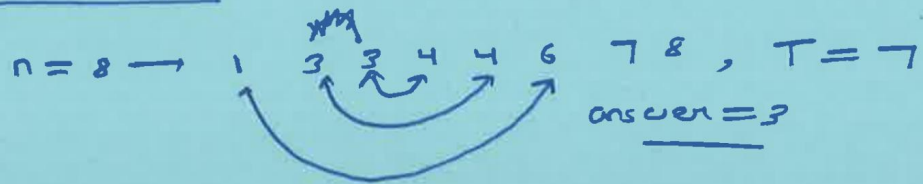
since  $s_1 \subseteq s_2 \rightarrow$  all possible pairs in  $s_1$  are also possible  
in  $s_2$ , because  $s_2$  contains all values of  $s_1$ .  $\therefore$  no. of

pairs in  $s_2 \geq$  number of pairs in  $s_1$ .  $\therefore G$  is as optimal

as  $O$ .  $\therefore$  our greedy algo.  $G$  works.

\* if  $k=j$ ,  
we can use  
induction for  
new sample space  
 $\{s_2, \dots, s_{j-1}\}$

more examples:



1 3 4 5 8 2  
1 2 3 4 5 8       $T = 7$