1 (a). The crypto pledge (1 mark)

*I promise that once I see how simple cryptographic constructions really are, I will not implement them in **production code** even though it will be really fun. This agreement will remain in effect until I learn all about side-channel attacks and countermeasures to the point where I lose all interest in implementing them myself.*[1]

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[1] Taken from the course textbook, originally due to Jeff Moser.

1 (b). True/False (4 marks)

For each of the following questions, indicate whether the statement is true or false, and **provide a one-line justification for your answer**.

1. (1 mark) The signing algorithms of all MAC systems are also secure pseudorandom functions.

2. (1 mark) Let $G : \{0,1\}^n \to \{0,1\}^{2n}$ be a secure pseudorandom generator. Since it is a secure PRG, the statistical distance of the following two distributions is bounded by a negligible function of n:

$$\mathcal{D}_0 := \Big\{\text{Sample uniformly random } s \leftarrow \{0,1\}^n, \text{ output } G(s)\Big\}$$
$$\mathcal{D}_1 := \Big\{\text{Output uniformly random } u \leftarrow \{0,1\}^{2n}\Big\}$$

3. (1 mark) If $(\mathsf{Sign}, \mathsf{Verify})$ is a secure MAC system, then at least some bits of m are guaranteed to be hidden if I give out $\sigma = \mathsf{Sign}(m, k)$.

4. (1 marks) If $G : \{0,1\}^n \to \{0,1\}^{2n}$ is a secure pseudorandom generator, then so is $H : \{0,1\}^n \to \{0,1\}^{2n}$ where $H(s) = G(s) \oplus G(0^n)$ for all $s \in \{0,1\}^n$.

1 (c). Multiple Choice Questions (8 marks)

For each of the following MCQs, select the correct option. **Provide a short justification**.

1. (2 marks) Consider a symmetric key encryption scheme $\mathcal{E}$ that satisfies Semantic Security. Then,

   (A) $\mathcal{E}$ is No-Query-Semantic-Security secure against **ALL (computationally unbounded)** adversaries.

   (B) There exists a **computationally unbounded** adversary that wins the No-Query-Semantic-Security game against scheme $\mathcal{E}$ with probability greater than 1/2.

   (C) Neither of the above. It depends on the scheme $\mathcal{E}$.

Let $F : \mathcal{X} \times \mathcal{K} \rightarrow \mathcal{Y}$ be a secure pseudorandom function with $\mathcal{X} = \mathcal{K} = \mathcal{Y} = \{0,1\}^n$ (here $\mathcal{K}$ is the key space). Consider the following keyed functions. For each of them, indicate whether the keyed function is a provably secure PRF, may be/may not be a secure PRF (depends on some additional properties of F), or is definitely insecure.

2. (2 marks) $F'(x,k) = F(x,k) \mid\mid F(x, F(x,k))$

   (A) F' is provably secure, assuming F is.

   (B) F' may/may not be a secure PRF. It requires some additional properties of F.

   (C) F' is an insecure PRF.

3. (2 marks) $F'(x, k) = F(x, k) \oplus x$

(A) F' is provably secure, assuming F is.

(B) F' may/may not be a secure PRF. It requires some additional properties of F.

(C) F' is an insecure PRF.

4. (2 marks) $F'(x, k) = F(x, k) \mid\mid F(x, x \oplus k)$

(A) F' is provably secure, assuming F is.

(B) F' may/may not be a secure PRF. It requires some additional properties of F.

(C) F' is an insecure PRF.

2. PRGs with large expansion (4 marks)

Let $G_1 : \{0,1\}^n \to \{0,1\}^{2n}$, and $G_2 : \{0,1\}^n \to \{0,1\}^{3n}$ be secure pseudorandom generators. Consider the following function $H : \{0,1\}^n \to \{0,1\}^{6n}$ defined as follows:

$$H(s) = y_1 \mid\mid y_2 \mid\mid \ldots \mid\mid y_5 \mid\mid y_6$$
where
$$G_1(s) = s_0 \mid\mid s_1$$
$$y_1 \mid\mid y_2 \mid\mid y_3 = G_2(s_0)$$
$$y_4 \mid\mid y_5 \mid\mid y_6 = G_2(s_1)$$

Here, each y_i, s_0 and s_1 are n bit strings.

We will show that H is a secure pseudorandom generator, assuming G_1 and G_2 are secure PRGs. The proof will go via a sequence of hybrids. In World-0, the challenger samples a uniformly random $s \leftarrow \{0,1\}^n$ and sends $H(s)$. In World-1, the challenger chooses a $6n$-bit uniformly random string $u \leftarrow \{0,1\}^{6n}$ and sends u.

Define the intermediate hybrids required to prove security of H. No need to provide reductions for showing indistinguishability of hybrids.

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3. A broken encryption scheme (4 marks)

Let $P : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a secure pseudorandom permutation, and let P^{-1} be its inverse (that is, for all x, k, $P^{-1}(P(x,k),k) = x$). Consider the following encryption scheme with message space $\{0,1\}^{\ell \cdot n}$ for some $\ell > 3$:

- KeyGen: choose a random PRP key $k \leftarrow \{0,1\}^n$.
- $\mathsf{Enc}\Big(m = (m_1, \ldots, m_\ell), k\Big)$: choose a uniformly random string $x_0 \leftarrow \{0,1\}^n$ and set $\mathsf{ct}_0 = P(x_0, k)$.

  For $i = 1$ to ℓ, do the following:

  - set $x_i = m_i \oplus x_{i-1}$
  - compute $\mathsf{ct}_i = P(x_i, k)$.

  Output $\mathsf{ct} = (\mathsf{ct}_0, \mathsf{ct}_1, \mathsf{ct}_2, \ldots, \mathsf{ct}_\ell)$ as the final ciphertext.
- $\mathsf{Dec}\Big(\mathsf{ct} = (\mathsf{ct}_0, \mathsf{ct}_1, \ldots, \mathsf{ct}_\ell), k\Big)$: Let $y_0 = P^{-1}(\mathsf{ct}_0, k)$. For each $i = 1$ to ℓ, do the following:

  - compute $y_i = P^{-1}(\mathsf{ct}_i, k)$.
  - set $m_i = y_i \oplus y_{i-1}$

  Output $m = (m_1, m_2, \ldots, m_\ell)$ as the final decryption.

Show that the above scheme does not satisfy $\mathsf{No\text{-}Query\text{-}Semantic\text{-}Security}$**.**

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4. A new MAC scheme (4+1 marks)

Let $\mathsf{MAC} = (\mathsf{Sign}, \mathsf{Verify})$ be a MAC scheme with message space $\mathcal{M}$, key space $\mathcal{K}$ and signature space $\mathcal{T}$, satisfying **weak** unforgeability. Consider the following MAC scheme $\mathsf{MAC}' = (\mathsf{Sign}', \mathsf{Verify}')$:

- $\mathsf{Sign}'(m, k) =$ Compute $\sigma_1 \leftarrow \mathsf{Sign}(m, k)$, $\sigma_2 \leftarrow \mathsf{Sign}(m, k)$, output (σ_1, σ_2).
- $\mathsf{Verify}'(m, (\sigma_1, \sigma_2), k)$: Output 1 if either $\mathsf{Verify}(m, \sigma_1, k) = 1$ or $\mathsf{Verify}(m, \sigma_2, k) = 1$.

1. Is MAC' a **weakly** unforgeable MAC scheme, assuming MAC is?

2. Suppose MAC is a **strongly** unforgeable message auth. code. Can we conclude that MAC' is also **strongly** unforgeable?

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Definitions

Definition 04.01. *A function $\mu : \mathbb{N} \to [0,1]$ is said to be negligible if, for any polynomial $p(\cdot)$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$, $\mu(n) < 1/p(n)$.*

Definition 04.02. *An encryption scheme* $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ *is said to satisfy no-query-semantic-security if, for any probabilistic polynomial time adversary $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ such that for all n,*

$$\Pr\left[\mathcal{A} \text{ wins the } \mathsf{No\text{-}Query\text{-}Semantic\text{-}Security} \text{ game}\right] \leq 1/2 + \mu(n)$$

where the No-Query-Semantic-Security *game is defined in Figure 1.*

No-Query-Semantic-Security
1. Adversary sends two messages m_0, m_1 to the challenger, such that $\lvert m_0\rvert = \lvert m_1\rvert$. 2. The challenger chooses a bit $b \leftarrow \{0,1\}$, key $k \leftarrow \mathcal{K}$ and sends $\mathsf{Enc}(m_b, k)$ to the adversary. 3. The adversary sends its guess b', and wins the security game if $b = b'$.

Figure 1: The No-Query Semantic Security Game

Definition 05.01. *A deterministic polynomial time computable function $G : \{0,1\}^n \to \{0,1\}^\ell$ is a secure pseudorandom generator (PRG) if $\ell > n$, and for any prob. poly. time adversary $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ such that for all n,*

$$\Pr\left[\mathcal{A} \text{ wins the PRG security game against } G\right] \leq 1/2 + \mu(n),$$

where the PRG game is defined in Figure 2.

PRG-Security
1. The challenger chooses a bit $b \leftarrow \{0,1\}$, string $s \leftarrow \{0,1\}^n$, $u_1 \leftarrow \{0,1\}^\ell$. It computes $u_0 = G(s)$, and sends u_b to the adversary. 2. The adversary sends its guess b', and wins the security game if $b = b'$.

Figure 2: The PRG Security Game

Definition 09.01. *A keyed function $F : \mathcal{X} \times \mathcal{K} \to \mathcal{Y}$ is a pseudorandom function (PRF) if, for any p.p.t. adversary $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ s.t. for all n,*

$$\Pr\left[\mathcal{A} \text{ wins the PRF security game}\right] \leq 1/2 + \mu(n),$$

where the PRF security game is defined in Figure 3.

PRF-Game

1. Challenger chooses a bit $b \leftarrow \{0,1\}$. If $b = 0$, challenger chooses a key $k \leftarrow \mathcal{K}$ and sets function $F_0 \equiv F(\cdot, k)$. If $b = 1$, challenger chooses a truly random function F_1 from the set of all functions mapping $\mathcal{X}$ to $\mathcal{Y}$. [a]
2. The adversary sends polynomially many queries. For each query $x \in \mathcal{X}$, the challenger sends $F_b(x)$.
3. Finally, after polynomially many queries, the adversary sends a guess b' and wins if $b' = b$.

[a] Note: there are only $|\mathcal{K}|$ keys, but the number of functions from $\mathcal{X}$ to $\mathcal{Y}$ is much more. In some sense, this is similar to what we saw for PRGs: in one case, the set of possible outputs was equal to the size of input domain, while in the other case, it was equal to size of output domain.

Figure 3: PRF Security Game

Definition 13.01. *Given two distributions $\mathcal{D}_0$ and $\mathcal{D}_1$ over the same sample space Ω, the statistical distance of $\mathcal{D}_0$ and $\mathcal{D}_1$ is defined as*

$$\mathsf{SD}(\mathcal{D}_0, \mathcal{D}_1) = \frac{1}{2} \left(\sum_{i \in \Omega} \left| \Pr_{x \leftarrow \mathcal{D}_0} [x = i] - \Pr_{x \leftarrow \mathcal{D}_1} [x = i] \right| \right)$$

Definition 15.01. *An encryption scheme $\mathcal{E} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ is said to satisfy* Semantic Security *if, for any p.p.t. adversaries $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ such that for all n,*

$$\Pr\left[\mathcal{A} \textit{ wins the } \mathsf{semantic\ security} \textit{ game } \right] \leq 1/2 + \mu(n)$$

where the probability is over the choice of key k, randomness used in Enc*, and the adversary's randomness.*

Semantic Security

1. **Setup:** Challenger chooses an encryption key $k \leftarrow \mathcal{K}$ and a bit $b \leftarrow \{0,1\}$.
2. **Challenge encryption queries:** Adversary sends polynomially many challenge encryption queries (adaptively). The i^{th} challenge pair consists of two messages $m_{i,0}, m_{i,1}$. The challenger sends $\mathsf{ct}_i = \mathsf{Enc}(m_{i,b}, k)$ to the adversary.

   Note that the bit b and key k were chosen during setup, and the same bit and key are used for all queries.
3. **Guess:** The adversary sends its guess b', and wins the security game if $b = b'$.

Figure 4: Semantic Security Game

Definition 18.01. *A MAC scheme $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ is said to satisfy* Strong-UF-CMA *if, for any p.p.t. adversary $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ such that for all n,*

$$\Pr\left[\mathcal{A} \textit{ wins the strong unforgeability game}\right] \leq \mu(n).$$

Strong-UF-CMA

1. **Setup:** Challenger chooses a signing key k.
2. **Signature queries:** Adversary sends polynomially many signing queries (adaptively). The i^{th} query is a message m_i. The challenger sends $\sigma_i = \mathsf{Sign}(m_i, k)$ to the adversary.

   Note that the key k was chosen during setup, and the same key is used for all queries.
3. **Forgery:** The adversary sends a message m^* together with a signature σ^*, and wins if $(m^*, \sigma^*) \neq (m_i, \sigma_i)$ for all i, and $\mathsf{Verify}(m^*, \sigma^*, k) = 1$.

Figure 5: Security Game for MACs: Strong Unforgeability under Chosen Message Attack