Problem sheet – 1
Understanding signals

1. Determine whether the following signals are periodic or not. For the signals which are periodic, find their fundamental period.

a) $x(\mathrm{t}) = 2\, sin\left(\frac{2}{3}\mathrm{t}\right) + 2\, sin\left(\frac{2\,\pi}{3}\mathrm{t}\right)$

b) $\mathrm{x(t)} = \mathrm{E_V\{sin(4\pi t).\,u(t)\}}$

where, Ev {.} denotes the even part of the given signal, and u(t) represents unit-step signal.

c) $\mathrm{x[n]} = 2 \sin\,(0.8\;\pi\;\mathrm{n})$

d) $\mathrm{x[n]} = 2\cos(4\mathrm{n})$

e) $\mathrm{x[n]} = 2\cos\left(\frac{\pi}{4}\mathrm{n}\right) + \sin\left(\frac{\pi}{8}\mathrm{n}\right) - 2\cos\left(\frac{\pi}{2}\mathrm{n} + \frac{\pi}{6}\right)$

f) $\mathrm{x[n]} = \sum_{\mathrm{k}=-\infty}^{\infty}\{\delta[\mathrm{n}-4\mathrm{k}] - \delta[\mathrm{n}-1-4\mathrm{k}]\}$

where, $\delta[n]$ represents unit-impulse signal.

2. Let x[n] be a signal with x[n] = 0 for n < -2 and n > 4. For each signal given below find the values of n for which it is guaranteed to be zero

a) x[n-3]

b) x[n+4]

c) x[-n]

d) x[-n+2]

e) x[-n-2]

3. Find the transformations.

$$\mathrm{x[n]} = \{2, 6, 3, \underset{\substack{\uparrow \\ \mathrm{n}=0}}{2}, 5, 7, 9, 3\}$$

a) $\mathrm{y[n]} = \mathrm{x[2n]}$

b) $\mathrm{y[n]} = \begin{cases} \mathrm{x}\left[\frac{\mathrm{n}}{3}\right], & \mathrm{n\ is\ multiple\ of\ 3} \\ 0, & \mathrm{otherwise} \end{cases}$

4. Let x(t) be a continuous-time signal, and let

$$y_1(\mathrm{t}) = \mathrm{x(2t)} \text{ and } y_2(\mathrm{t}) = \mathrm{x(t/2)}.$$

The signal y_1(t) represents a speeded up version of x(t) in the sense that the duration of the signal is cut in half. Similarly, y_2(t) represents a slowed down version of x(t) in the sense that the duration of the signal is doubled. Consider the following statements:

(1) If x(t) is periodic, then y_1(t) is periodic.
(2) If y_1(t) is periodic, then x(t) is periodic.
(3) If x(t) is periodic, then y_2(t) is periodic.
(4) If y_2(t) is periodic, then x(t) is periodic.

For each of these statements, determine whether it is true, and if, so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counter-example to it.

5. Let x[n] be a discrete-time signal, and let

$$y_1[n] = x[2n] \text{ and } y_2[n] = \begin{cases} x\left[\frac{n}{2}\right] & , n \text{ even} \\ 0, n \text{ odd} \end{cases}$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in some sense the speeded up and slowed down versions of x[n]. However, it should be noted that the discrete-time notions of speeded up and slowed down have subtle differences with respect to their continuous-time counterparts. Consider the following statements:

(1) If x[n] is periodic, then $y_1[n]$ is periodic.
(2) If $y_1[n]$ is periodic, then x[n] is periodic.
(3) If x [n] is periodic, then $y_2[n]$ is periodic.
(4) If $y_2[n]$ is periodic, then x [n] is periodic.

For each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

6. In this problem, we explore several of the properties of even and odd signals.

(a) Show that if x[n] is an odd signal, then

$$\sum_{n=-\infty}^{\infty} x[n] = 0.$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]\, x_2[n]$ is an odd signal.

(c) Let x[n] be an arbitrary signal with even and odd parts denoted by

$$x_e[n] = \text{Even}\{x[n]\}$$

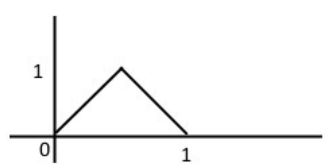
and

$$x_o[n] = \text{Odd}\{x[n]\}.$$

Show that

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

7. Find the even and odd part of the following signal



8. In this problem, we examine a few of the properties of the unit-step and unit-impulse function. Show that

a) $\delta(2t) = \frac{1}{2}\delta(t)$
b) $u(t) = \int_0^{\infty} \delta(t-\sigma)d\sigma$
c) $\lim_{\Delta\to 0}[u_\Delta(t)\, \delta(t)] = 0$
d) $\lim_{\Delta\to 0}[u_\Delta(t)\, \delta_\Delta(t)] = \frac{1}{2}\delta(t)$

9. Consider the continuous time signal

$$x(t) = \delta(t+2) - \delta(t-2)$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$