COL202: Discrete Mathematical Structures **Spring 2023**

Tutorial Sheet 4

Announced on: Jan 26 (Thurs)

1. **[Submission Problem for Group 1]** Based on Problem 9.10 in [LLM17].

   Prove or disprove the following:

   a) For any triple of integers a, b, c, if $\gcd(a, b) \neq 1$ and $\gcd(b, c) \neq 1$, then $\gcd(a, c) \neq 1$.

   b) For any triple of integers a, b, c, $\gcd(ab, ac) = a \cdot \gcd(b, c)$.

   c) For any pair of integers a and b and any natural number n, $\gcd(a^n, b^n) = \gcd(a, b)^n$.

2. **[Submission Problem for Group 2]** Recall the water filling puzzle discussed in Lecture 10. Imagine that now you are given *three* jugs of capacities a, b, and c litres, where $a, b, c \in \mathbb{N}$. Further, as with the two-jugs puzzle discussed in class, there is a faucet with an unlimited supply of water and a drain with an unlimited capacity.

   What are all possible water levels that you can create in a jug using a sequence of standard moves and why? (As with the two-jugs puzzle, a standard move comprises of drawing water from the faucet, discarding water into the drain, and pouring water from one jug into the other.)

3. **[Submission Problem for Group 3]** Based on Problems 9.18, 9.21, and 9.26 in [LLM17].

   Prove or disprove the following:

   a) For any triple of integers a,b, and c, $\gcd(a, \gcd(b, c)) = \gcd(\gcd(a, b), c)$.

   b) Let

   $$a = 2^9 \cdot 5^{24} \cdot 7^4 \cdot 11^7,$$
   $$b = 2^3 \cdot 7^{22} \cdot 11^{211} \cdot 19^7, \text{ and}$$
   $$c = 2^5 \cdot 3^4 \cdot 7^{6042} \cdot 19^{30}.$$

   Then, $\gcd(a, b, c) = 2 \cdot 7 \cdot 19$.

   c) For any integers a and b, if $a \equiv b \pmod 5$ and $a \equiv b \pmod{14}$, then $a \equiv b \pmod{70}$.

4. **[Submission Problem for Group 4]** Based on Problems 9.7 and 9.31 in [LLM17].

   Find:

   a) $\gcd(3^{101}, 21)$

   b) $\gcd(13a + 8b, 5a + 3b)$ - $\gcd(a, b)$ for arbitrary integers a and b

   c) $\gcd(m, n)/\gcd(m/2, n/2)$ for arbitrary even natural numbers m and n.

   In each case, provide the reasoning behind your answer.

References

[LLM17] Eric Lehman, Tom Leighton, and Albert R Meyer. *Mathematics for Computer Science*. 2017. URL: `https://courses.csail.mit.edu/6.042/spring18/mcs.pdf`.