1.1.

Let x be an arbitrary element of S. We need to prove (x,x) \in R_f^+.

Let x_0 = x and x_I = f^i(x) for each I \in N. Since |S|=n, some two of x_0, x_1, …, x_n must be equal. Say x_i = x_j, where i<j. Applying f^{-1} j-i times, we get x = x_0 = x_{j-i} = f^{j-i}(x), where j-I>0.

Thus, (x,x) \in R_f^+.

1.2

Suppose (x,y) \in R_f^+. We need to prove (y,x) \in R_f^+.

From part 1, for every x in S there exists a k in N such that f^k(x) = x. Also, since (x,y) \in R_f^+, there exists some l in N such that y = f^l(x). Let m be a multiple of k greater than l. Then f^{m-l}(y) = f^m(x) = x. Thus, (y,x) \in R_f^+.

2.1.

P^* = Q^n_k intersection {f | f is a bijection from S to S and f(n) = n} is in bijective correspondence with Q_{n-1}^{k-1} as follows. f in P^* is mapped g in Q_{n-1}^{k-1}, where g(x) = f(x) for all x in {1,…,n-1}. R_g^+ has k-1 equivalence classes, namely, the equivalence classes of R_f^+ other than {n} (nothing else is in the equivalence class of n). Note that every g in Q_{n-1}^{k-1} has a unique pre-image f in P^* under this mapping, where f(x) = g(x) for all x in {1,…,n-1}, and f(n) = n.

2.2

For a fixed z in {1,…,n-1} let us count the size of the set P_z = Q^n_k intersection {f | f is a bijection from S to S and f(n) = z}. P_z is in bijective correspondence with Q^{n-1}_k as follows. f in P_z is mapped to g in Q_{n-1}_k, where g(f^{-1}(n)) = z, and for all other x in {1,…,n-1}, g(x) = f(x). Note that the equivalence classes of R_g^+ are the same as those of R_f^+, except that n disappears from its equivalence class, which contains more elements (eg. z). Given a g in Q^{n-1}_k, the unique f in P_z which maps to g is given by f(g^{-1}(z)) = n, f(n) = z, and for all other x in {1,…,n}, f(x) = g(x). Putting together the facts |P_z| = |Q^{n-1}_k|, P_z's are disjoint, and their union is Q^n_k, the claim stands proved.