1. (i) False. Downward sloping due to "More is better" assumption.
e.g. Mon / Strong Mon.

(ii) False. Interior of the budget set costs less.

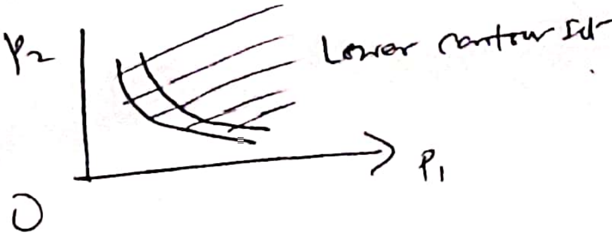
(iii) It is the ratio of market prices: Market exchange rate.
MRS is the psychological exchange rate.

(iv) True. Strong monotonicity: $x \geq y,\ x \neq y \Rightarrow x \succ y$.
Mon: $x \gg y \Rightarrow x \succ y$.

(v) True. MU may vary if a monotonic transformation is taken. But MRS does not.

2. Prove that indirect utility functions are quasi-convex (done in the class).

Next step: Since indirect utility functions are quasi-convex, the lower contour set is convex, i.e. $\{(p_1, p_2) \mid v(p_1, p_2, I) \leq \bar{v}\}$ is convex. Price indifference curve is defined as: $\{(p_1, p_2) \mid v(p_1, p_2) = \bar{v}\}$. Also, note that for $p_1' \geq p_1$, $p_2' \geq p_2$ and $p \neq p'$, $v(p') < v(p)$, where $p = (p_1, p_2)$, $p' = (p_1', p_2')$. Thus price indifference curves are convex to the origin:



S3.

$$MU_1 = x_2$$

$$MU_2 = x_1 + 10$$

$$\frac{P_1}{P_2} = \frac{1}{3}$$

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2} \quad \text{(tangency condition)}$$

or, $$\frac{x_2}{x_1 + 10} = \frac{1}{3}$$

$$3x_2 = x_1 + 10$$

further $10x_1 + 30x_2 = 5$

after you solve, you get

$$x_1 = -95/20$$

Since x_1 cannot be negative, so consumer would like to reduce his / her consumption of x_1 as much as possible i.e.

$$x_1 = 0$$

$$x_2 = \frac{5}{30} = \frac{1}{6}$$

$\boxed{\text{corner solution.}}$

(84.)

If $x_1 \leq 2$ units

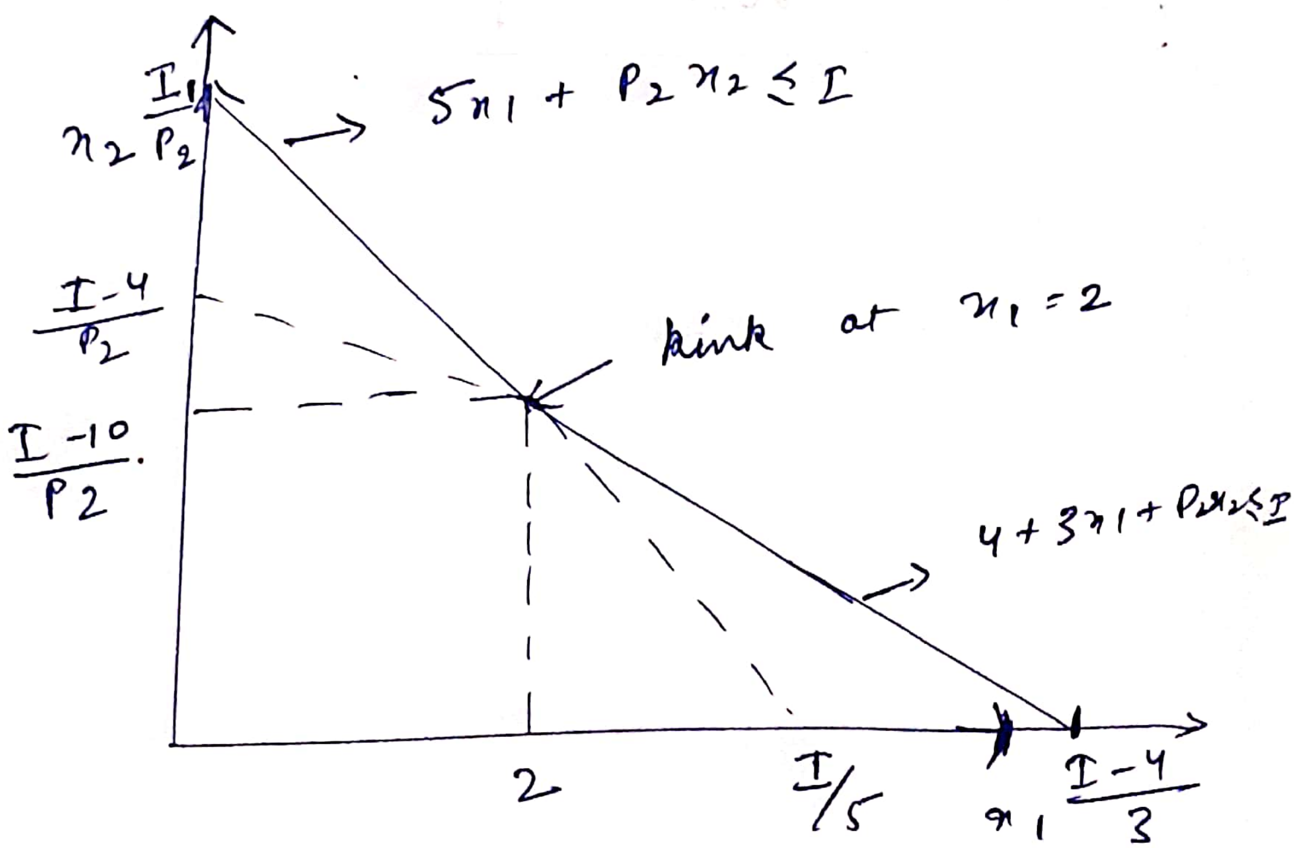
then

$$5x_1 + P_2 x_2 \leq I$$

As $x > 2$

$$10 + 3(x_1 - 2) + P_2 x_2 \leq I$$

$$4 + 3x_1 + P_2 x_2 \leq I$$



Q5)

Constant Elasticity of Substitution [CES] Preferences

$$u(\mathbf{x}) = (x_1^\rho + x_2^\rho + \cdots + x_n^\rho)^{1/\rho}$$

where $-\infty < \rho < 1$ and $\rho \neq 0$

marginal utilities

$$\frac{\partial u}{\partial x_i} = \frac{1}{\rho}(x_1^\rho + x_2^\rho + \cdots + x_n^\rho)^{1/\rho - 1} \rho x_i^{\rho - 1}$$

first–order conditions for utility maximization

$$(x_1^\rho + x_2^\rho + \cdots + x_n^\rho)^{1/\rho - 1} x_i^{\rho - 1} - \lambda p_i = 0 \quad i = 1, 2, \cdots, n$$

along with the budget constraint

$$\sum_j p_j x_j = y$$

manipulating the equations

Take the first–order condition for consumption of commodity i, and divide both sides by the first–order condition for the consumption of commodity 1. What results is

$$(\frac{x_i}{x_1})^{\rho-1} = \frac{p_i}{p_1}$$

or

$$x_i = (\frac{p_i}{p_1})^{1/(\rho-1)} x_1 \tag{1}$$

which implies that

$$\begin{aligned} p_i x_i &= p_i (p_i)^{1/(\rho-1)} p_1^{-1/(\rho-1)} x_1 \\ &= (p_i)^{\rho/(\rho-1)} (p_1)^{-1/(\rho-1)} x_1 \\ &= p_i^r p_1^{1-r} x_1 \end{aligned} \tag{2}$$

Now let

$$r \equiv \frac{\rho}{\rho - 1}$$

Add up equation 2 over all n commodities to get

$$\sum_{j=1}^{n}(p_j x_j) = [\sum_{j=1}^{n} p_j^r](p_1)^{1-r} x_1 \tag{3}$$

The budget constraint says that the left side of equation 3 is y, which means that

$$x_1 = \frac{p_1^{r-1} y}{\sum_{j=1}^{n} p_j^r}$$

which is the Marshallian demand function for commodity number 1. Substituting back into equation (1) shows that, for any commodity i,

$$x_i(\mathbf{p}, y) = \frac{p_i^{r-1} y}{\sum_{j=1}^{n} p_j^r}$$

defining the Marshallian demand functions when preferences are CES.

CES : Expenditure Function and Hicksian Demands

expenditure minimization

minimize $\mathbf{p} \cdot \mathbf{x}$ subject to

$$[\sum_{i=1}^{n} x_i^{\rho}]^{1/\rho} \geq u \qquad (1)$$

so the Lagrangean is

$$\mathbf{p} \cdot \mathbf{x} + \mu[u - [\sum_{i=1}^{n} x_i^{\rho}]^{1/\rho}] \qquad (2)$$

with first–order conditions

$$p_i = \mu[\sum_{k=1}^{n} x_k^{\rho}]^{1/\rho - 1} x_i^{\rho - 1} \quad i = 1, 2, \ldots, n \qquad (3)$$

re–arranging (3),

$$\frac{x_i}{x_j} = [\frac{p_i}{p_j}]^{1/(\rho-1)} \tag{4}$$

for any 2 goods i and j, so that, in particular

$$x_i = [\frac{p_i}{p_1}]^{1/(\rho-1)} x_1 \tag{5}$$

which means that

$$u = x_1 [\sum_{j=1}^{n} (\frac{p_j}{p_1})^{\rho/(\rho-1)}]^{1/\rho} \tag{6}$$

which, in turn, can be re–arranged to

$$x_1 = p_1^{-1/(1-\rho)} [\sum_{j=1}^{n} p_i^{\rho/(\rho-1)}]^{-1/\rho} u \tag{7}$$

which is a Hicksian demand function

since

$$r \equiv \frac{\rho}{\rho - 1}$$

so that

$$\rho = -\frac{r}{1 - r}$$

equation (7) can be written

$$x_1^h(\mathbf{p}, u) = p_1^{r-1}[\sum_{j=1}^{n} p_j^r]^{1/r-1} u \tag{8}$$

and the Hicksian demand function for any other good i is

$$x_i^h(\mathbf{p}, u) = p_i^{r-1}[\sum_{j=1}^{n} p_j^r]^{1/r-1} u \tag{9}$$

CES : Expenditure Function

the expenditure function is the sum of expenditure $p_i x_i^h(\mathbf{p}, u)$ on all the goods ; from equation (9),

$$\mathbf{p} \cdot \mathbf{x}^h(\mathbf{p}, u) = \sum_{i=1}^{n} p_i (p_i^{r-1}) [\sum_{j=1}^{n} p_j^r]^{1/r-1} u \tag{10}$$

or

$$\mathbf{p} \cdot \mathbf{x}^h(\mathbf{p}, u) = [\sum_{i=1}^{n} p_i^r][\sum_{i=1}^{n} p_i^r]^{1/r-1} \tag{11}$$

meaning that the expenditure function for CES preferences is

$$e(\mathbf{p}, u) = [\sum_{i=1}^{n} p_i^r]^{1/r} u \tag{12}$$

8b.

a)

- Weak axiom says if x is chosen over y when y is available, then there can be no budget set containing both alternatives for which y is chosen and x is not. And thus.

$$100 \cdot 120 + 100y \geq 100 \cdot 100 + 100 \cdot 100$$

and

$$100 \cdot 100 + 80 \cdot 100 \geq 100 \cdot 120 + 80y$$

$$y \leq 75$$

& $\quad y \geq 80$

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- We shall prove that if $y < 75$, then good 1 is an inferior good.

So suppose that $y < 75$, then

$$100 \cdot 120 + 100y \leq 100 \cdot 100 + 100 \cdot 100$$

and

$$100 \cdot 100 + 80 \cdot 100 > 100 \cdot 120 + 80y$$

Hence the wealth decreases from year 1 to 2. Also the relative price of good 1 increases. But the demand for good 2, y, decreases because $y < 75 < 100$.

This means that the wealth effect on good 1 must be negative. Hence it is an inferior good.

b) Answer is on lecture 5-6 (slide 18 or slide 59-61)