Problem sheet–9

1. Using duality between Continuous-time Fourier Series and Discrete-time Fourier Transform, prove

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} \left|X(e^{j\Omega})\right|^2 d\Omega$$

2. John evaluated the DTFT of the output of an accumulator, that is, $y[n] = \sum_{m=-\infty}^{n} x[m]$, by thinking about the output as the convolution of the input with the unit-step response as

$$y[n] = \sum_{m=-\infty}^{n} x[m]\, u[n-m] = x[n] * u[n]$$

By substituting the frequency response of the input and the unit-step response, he obtained the following

$$Y\left(e^{j\Omega}\right) = \frac{X\left(e^{j\Omega}\right)}{1-e^{-j\Omega}} + \pi X(e^{j0}) \sum_{p=-\infty}^{\infty} \delta(\Omega - 2\pi p)$$

Amit evaluated the DTFT of the output of an accumulator by writing corresponding difference equation

$$y[n] - y[n-1] = x[n]$$

By substituting frequency responses of each part, he obtained the following result:

$$Y\left(e^{j\Omega}\right) = \frac{X\left(e^{j\Omega}\right)}{1-e^{-j\Omega}}$$

Explain the differences between the two answers.

3. **Difference equations**

Consider an LTI system that is characterized by the difference equation:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

a) Find the impulse response of the above system.
b) Do we do any error similar to what Amit did in solving a difference equation in question 2?
c) Why did we not require any auxiliary conditions for solving the impulse response?
d) Using operators (converting system into polynomials), find the frequency response.
e) Does system stability influence our solutions?

4. Prove if a periodic-time signal $\boldsymbol{x[n]}$ with period *N* has Fourier series coefficients $\boldsymbol{a_k}$, then $\boldsymbol{a_n}$ has Fourier series coefficients $\frac{1}{N}x[-k]$

5. **Comparison of decimation and low frequency sampling**

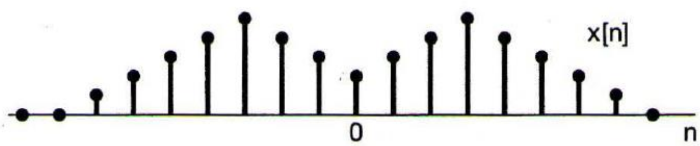
Consider a discrete-time sequence x[n] from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to sampling x[n] with a sampling period of 2 and $x_d[n]$ corresponds to decimating x[n] by a factor of 2, so that

$$x_p[n] = \begin{cases} x[n], & n = 0, \pm 2, \pm 4 \ldots \\ 0, & n = \pm 1, \pm 3 \ldots \end{cases}$$

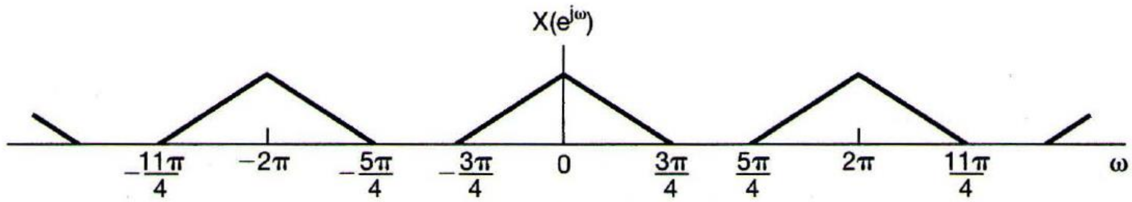
and

$$x_d[n] = x[2n]$$

a. If x[n] is as illustrated in Figure (a), sketch the sequences $x_p[n]$ and $x_d[n]$.
b. If $X(e^{jw})$ is as shown in Figure (b), sketch $X_p(e^{jw})$ and $X_d(e^{jw})$.



(a)



(b)

6. **Fourier transform of decimated signal**

Consider the system shown in the figure below, with input x[n] and the corresponding output y[n]. The zero-insertion system inserts two points with zero amplitude between each of the sequence values in x[n]. The decimation is defined by

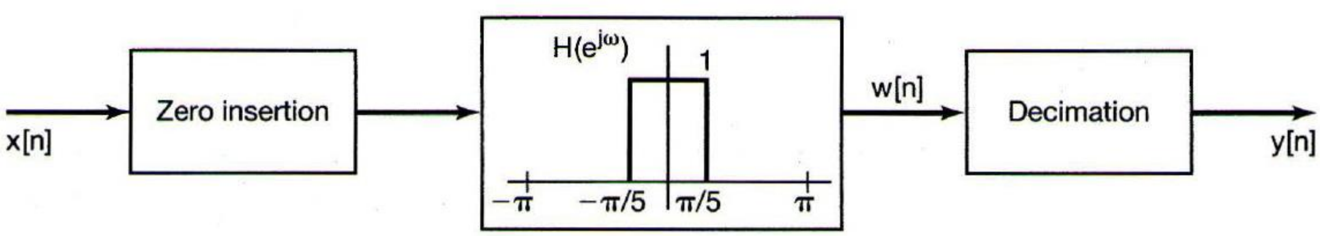
$$y[n] = w[5n]$$

where w[n] is the input sequence for the decimation system. If the input is of the form

$$x[n] = \frac{\sin(w_1 n)}{\pi n}$$

determine the output y[n] for the following values of w_1 :

a) $w_1 \leq \frac{3\pi}{5}$
b) $w_1 > \frac{3\pi}{5}$

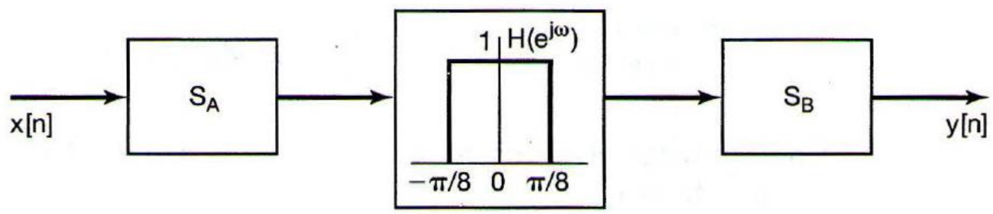


7. **Graphical analysis of Decimation and Interpolation function**

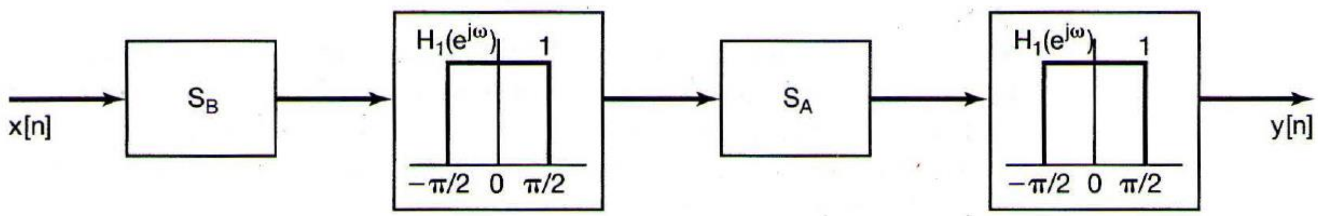
Two discrete-time systems S_1 and S_2 are proposed for implementing an ideal low-pass filter with cutoff frequency $\pi/4$. System S_1 is depicted in figure (a) below. System S_2 is depicted in the figure (b). In these figures, S_A corresponds to a zero insertion system that inserts one zero after every input sample, while S_B corresponds to a decimation system that extracts every second sample of its input.

(a) Does the proposed system S_1 correspond to the desired ideal low-pass filter?

(b) Does the proposed system S_2 correspond to the desired ideal low-pass filter?



(a)



(b)

8. **Graphical Analysis of Low pass Filter**

A signal x[n] with Fourier transform $X(e^{jw})$ has the property that

$$\left(x[n]\sum_{k=-\infty}^{\infty}\delta[n-3k]\right)*\left(\frac{\sin\frac{\pi}{3}n}{\frac{\pi}{3}n}\right)=x[n]$$

For what values of w is it guaranteed that $X(e^{jw})$ = 0?

9. **Graphical Analysis of Interpolation and Decimation**

A real-valued discrete-time signal x[n] has a Fourier transform $X(e^{jw})$ that is zero for $3\pi/14 \le w \le \pi$. The nonzero portion of the Fourier transform of one period of $X(e^{jw})$ can be made to occupy the region $|w| < \pi$ by first performing up-sampling by a factor of L and then performing down-sampling by a factor of M . Specify the values of L and M