Problem Sheet 0
Solutions

Q1. $13^{\frac{1}{8}}e^{j(\frac{\theta}{4}+k\frac{\pi}{2})}$**,** $k = 0,1,2$ and $\theta = \arctan(\frac{3}{3}) \approx 56.3°$

Q2. $\frac{z_1}{z_2} = \frac{2+3j}{4+3j}\frac{4-3j}{4-3j}$
$= \frac{1}{25}(17-6j)$

Q3. Assume $z = x + jy$**,** where $x, y \in \mathbb{R}$, rest is pretty straight forward.

Q4. (a) and (b) can be obtained quite easily by simple substitution.

For (c), notice that,

$\sin(A+B) - \sin(A-B) = \mathfrak{I}(e^{j(A+B)} - e^{j(A-B)})$
$= \mathfrak{I}(e^{jA}(e^{jB} - e^{-jB})) = \mathfrak{I}(e^{jA}2j\sin B) = \mathfrak{R}(2\sin Be^{jA})$
$= 2\cos A \sin B$

Q5. (a) $r^2e^{2j\theta}$ (b) $\frac{1}{r}e^{-j\theta}$ (c) $re^{-j\theta}$ (d) $re^{j(\theta+\frac{\pi}{2})}$ (e) r^2 (f) $e^{2j\theta}$

Q6. (a) $\frac{1-e^{-5a}}{5}$ (b) $\frac{e^{-2a}}{2}$

Q7. (a) Clearly, since f(1) takes two values. (b) Domain is $[1,\infty)$ and Range is $[0,\infty)$

Q8. The solution is composed of two parts, the particular and the homogeneous solution. That is,

$$y(t) = y_p(t) + y_h(t)$$

$y_h(t)$ can be solved for, and is a solution to the homogeneous differential equation,

$$\frac{dy}{dt} + 2y = 0$$

Substituting $y = Ae^{st}$ gives us that $y_h(t) = Ae^{-2t}$, where A is a constant. The particular solution is a solution to the original differential equation. Making the substitution for $y = a$ where a is a constant, and substituting, we obtain that $a = \frac{1}{2}$. Thus,

$$y(t) = \frac{1}{2} + Ae^{-2t}$$

Where the value of A can be calculated if the initial conditions are known.

Q.9 $y[n] - 2y[n-1] = 1$**, for** $n \geq 0$

We need to figure out what $y[n]$ is, in terms of n. Substituting $n = 1$, we get that

$y[1] = 1 + 2y[0]$
$y[2] = 1 + 2y[1] = 1 + 2(1 + 2y[0])$
$= 1 + 2 + 4y[0]$

$y[3] = 1 + 2y[2] = 1 + 2(1 + 2 + 4y[0])$
$y[3] = 1 + 2 + 4 + 8y[0]$

Thus, we see that a pattern emerges, and $y[n]$ can be written as,

$$y[n] = \sum_{k=0}^{n-1} 2^k + 2^n y[0]$$

$$\implies y[n] = 2^n - 1 + 2^n y[0]$$
$$y[n] = 2^n(y[0] + 1) - 1$$

Alternate method:
As in the case of solving the differential equation, here the final signal (sequence) $y[n]$ can be split into two parts: a particular solution and an homogeneous solution. Mathematically,

$$y[n] = y_p[n] + y_h[n]$$

$y_h[n]$ is a solution to the homogeneous difference equation,

$$y[n] - 2y[n-1] = 0$$

Assuming $y[n] = Az^n$, and substituting,
$Az^n - 2Az^{n-1} = 0$
$\implies A = 0$ or $z = 0$ or $z = 2$ $(A \neq 0)$

Thus $y_h[n]$ can be written as $y_h[n] = A2^n$, where A is a constant.

The particular solution is a solution to the original difference equation. Substituting $y[n] = a$ where a is a constant, we get that $a = -1$. Which means that $y_p[n] = -1$. Thus, we get that,

$$y[n] = -1 + A2^n$$

Which is what we had obtained before.

Q10. A relation R defined on a set $\mathcal{S}$ is said to be an equivalence relation, iff it is reflexive, transitive and symmetric. Any equivalence relation partitions the set it is defined upon into various equivalence classes. All elements within a particular equivalence class are equivalent to each element in that class, but is not equivalent to any element from some other equivalence class. Such a partition is disjoint.

Q11. (a) $\dfrac{1}{1-a} = 1 + a + a^2 + \ldots$, this sum converges for $|a| < 1$

(b) $\dfrac{1}{(1-a)^2} = 1 + 2a + 3a^2 + \ldots$, this sum converges for $|a| < 1$

The expressions can be obtained either by performing a Taylor Series expansion about the point $a = 0$. Or, one can use the binomial expansion,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$