

Department of Mathematics  
MTL 106 (Introduction to Probability Theory and Stochastic Processes)  
Minor 1 (I Semester 2015 - 2016)

Time allowed: 1 hour

Max. Marks: 25

1. (a) Write axiomatic definition of probability.  
(b) Show that the conditional probability  $P(A/B)$  satisfies the three axioms of probability.  
(3 + 3 marks)

2. Let  $X$  be a random variable such that  $P(X = 2) = \frac{1}{4}$  and its distribution function is given by

$$F_X(x) = \begin{cases} 0, & x < -3 \\ \alpha(x+3), & -3 \leq x < 2 \\ \frac{3}{4}, & 2 \leq x < 4 \\ \beta x^2, & 4 \leq x < 8/\sqrt{3} \\ 1, & x \geq 8/\sqrt{3} \end{cases}$$

- (a) Find  $\alpha, \beta$  if 2 is the only jump discontinuity of  $F$ .  
(b) Compute  $P(X < 3/X \geq 2)$ .  
(1 + 1 + 2 marks)

3. Suppose the length of a telephone conversation between two persons is a random variable  $X$  with cumulative distribution function

$$P(X \leq t) = \begin{cases} 0, & -\infty < t < 0 \\ 1 - e^{-0.04t}, & 0 \leq t < \infty \end{cases}$$

where the time is measured in minutes.

- (a) Given that the conversation has been going on for 20 minutes, compute the probability that it continues for at least another 10 minutes.

(b) Show that, for any  $t > 0$ ,  $E(X/X > t) = t + 25$ .

(3 + 2 marks)

4. Consider a random variable  $X$  with  $E(X) = 1$  and  $E(X^2) = 1$ .

- (a) Find  $E[(X - E(X))^4]$  if it exists.  
(b) Find  $P(-1/2 < X \leq 3)$  and  $P(X = 0)$ .

(3 + 1 + 1 marks)

5. Suppose that  $X$  is a continuous random variable with pdf  $f_X(x) = e^{-x}$  for  $x > 0$ . Define

$$Y = \begin{cases} X, & X < 1 \\ \frac{1}{X}, & X \geq 1 \end{cases}$$

- (a) Discuss whether the distribution of  $Y$  is discrete or continuous or mixed type.  
(b) Determine the pmf/pdf as applicable to this case.

(1 + 4 marks)

8 + 8 = 16