Hints to problem sheet 1

Ques. 1:

a) Aperiodic
b) Aperiodic
c) Periodic with period N = 5
d) Aperiodic
e) Periodic with period N = 16
f) Periodic with period N = 4

Remark: LCM rule is not always applicable. For example, when functions are co-functions.

Ques. 2:

a) n < 1 & n > 7
b) n < -6 & n > 0
c) n < -4 & n > 2
d) n < -2 & n > 4
e) n < -6 & n > 0

Ques. 3:

f) $y[n] = \{6,2,7,3\}$
g) $y[n] = \{\ldots..6,0,0,3,0,0,2,0,0,5,0,0,7 \ldots..\}$

Ques. 4: All statements are true

h) x(t) is periodic with period T; $y_1(t)$ is periodic with T/2
i) $y_1(t)$ is periodic with period T; x(t) is periodic with 2T
j) x(t) is periodic with period T; $y_2(t)$ is periodic with 2T
k) $y_2(t)$ is periodic with period T; x(t) is periodic with T/2

Ques. 5:

l) True. $x[n] = x[n+N]; y_1[n] = y_1[n+N_0]$. i.e., periodic with $N_0 = N/2$ if N is even, and with period $N_0 = N$ if N is odd.
m) False. $y_1[n]$ Periodic does not imply $x[n]$ is periodic. i.e. let $x[n] = g[n] + h[n]$ where

$$g[n] = \begin{cases} 1, & n\ even \\ 0, & n\ odd \end{cases} \text{ and } h[n] = \begin{cases} 0, & n\ even \\ (1/2)^n, & n\ odd \end{cases}$$

Then $y_1[n] = x[2n]$ is periodic but $x[n]$ is clearly not periodic.
n) True. $x[n+N] = x[n]; y_2[n+N_0] = y_2[n]$ where $N_0 = 2N$
o) True. $y_2[n+N_0] = y_2[n];\ x[n+N_0] = x[n]$ where $N_0 = N/2$

Ques. 6:

a) Consider

$$\sum_{n=-\infty}^{n=\infty} x[n] = x[0] + \sum_{n=1}^{n=\infty} x[n] + x[-n]$$

For odd function, $x[n] + x[-n] = 0$, and $x[0] = 0$

So $\sum_{n=-\infty}^{n=\infty} x[n] = 0$

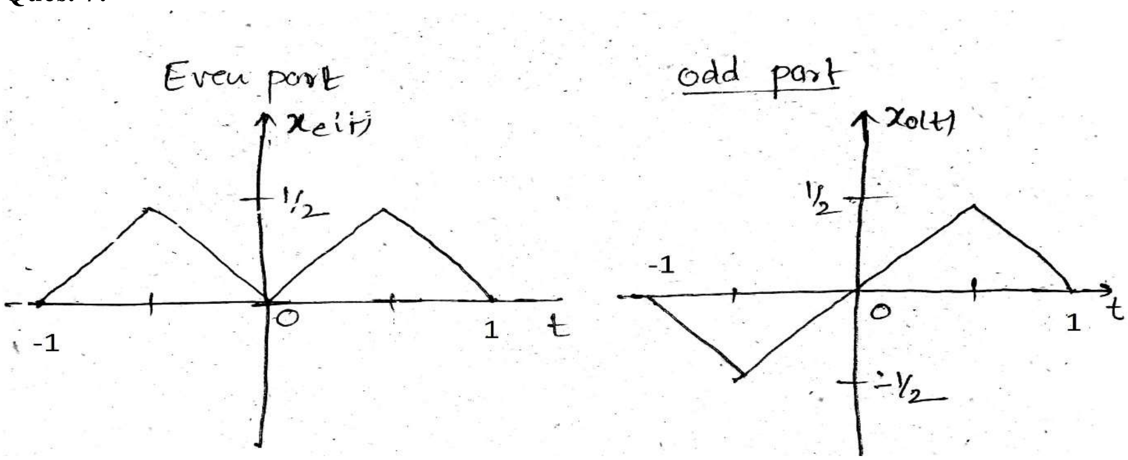
b) Let $y[n] = x_1[n].x_2[n]$

Then $y[-n] = x_1[-n].x_2[-n] = -x_1[n].x_2[n] = -y[n]$ then this is odd signal

c) $\sum_{n=-\infty}^{n=\infty} x^2[n] = \sum_{n=-\infty}^{n=\infty}(x_e[n] + x_o[n])^2$

Use $\sum_{n=-\infty}^{n=\infty}(x_e[n]x_o[n]) = 0$ and obtain the result.

Ques. 7:



Ques. 8:

a) $\delta(2t)$ has half area as $\delta(t)$

b) Discussed in class

c) The answer depends upon how we define $u_\Delta(t)$. $u_\Delta(t)$ in limiting form is known as a generalized function (as per the function is not specific). We can prove the given equality if we assume $u_\Delta(t)$ as t/Δ for $0 \le t \le \Delta$.

$$\lim_{\Delta \to 0}[\mathrm{u}_\Delta(t)\delta(\mathrm{t})] = \lim_{\Delta \to 0}[\mathrm{u}_\Delta(0)\delta(\mathrm{t})] = \lim_{\Delta \to 0}[0] = 0$$

d) $\lim_{\Delta \to 0}[\mathrm{u}_\Delta(t)\delta_\Delta(\mathrm{t})] = \lim_{\Delta \to 0}\left[\frac{t}{\Delta} \times \frac{1}{\Delta}\right] = \lim_{\Delta \to 0}\left[\frac{t}{\Delta^2}\right] = \frac{1}{2}\delta(\mathrm{t})$

Note that as t is between 0 and Δ and as Δ is very small $t \approx \Delta$, and thus $\frac{\partial t}{\partial \Delta} \approx 1$.

We can also prove the above equation by taking integration on both sides.

Ques. 9: $E_\infty = \int_{-\infty}^{\infty} y(t)dt = \int_{-2}^{2} dt = 4$