Tutorial-1 MTL103

1. Prove that the set of all convex combinations of a finite number of L.I. vectors is a convex set.

2. Examine the convexity of the following sets:

   - $\{(x, y, z) : x^2 + y^2 \leq 4, 0 \leq z \leq 3\}$
   - $\{(x, y) : |x| \leq 1, |y| \leq 1\}$
   - $\{(x, y) : y \geq 3 - x^2\}$
   - $\{(x, y) : y \leq e^{-x}\}$

3. Examine the convexity of the following functions:

   - e^{-x}
   - $|x|$
   - $f(x) = -x_1^2 - 4x_2^2 - 9x_3^2 + 2x_1x_2 + 3x_1x_3 + 6x_2x_3$
   -
   $$f(x) = \begin{cases} x^2 & \text{for} \quad -1 \leq x < 1, \\ 2 & \text{for} \quad x = 1. \end{cases}$$

     What if $f(x) = 1/2$ at $x = 1$?

4. If f is convex on R^n, show that $f(Ax + b)$ is also convex on R^n.

5. If f and g are convex functions, show through an example that fg is not necessarily a convex function.

6. Let h be a non-decreasing convex function on R and f be a convex function on $T \subset R^n$. Then prove that the composite function hof is a convex function on T. Hence or otherwise show that $e^{2x_1^2 + x_2^2}$ is a convex function on R^2.

7. If $S = \{(x, y) : -x + y \leq 4, 3x - 2y \leq 9, x, y \geq 0\}$, solve graphically the following problems:

   - max $x - 5y$ over S
   - max $6x + 4y + 18$ over S

8. Solve the following problems graphically:

   - max $2x_1 + x_2$ subject to:
     $0 \leq x_1 \leq 2$
     $x_1 + x_2 \leq 3$
     $x_1 + 2x_2 \leq 5$
     $x_2 \geq 0$
   - min $5x_1 + 2x_2$ subject to:
     $x_1 + 4x_2 \geq 4$
     $5x_1 + 2x_2 \geq 10$
     $x_1, x_2 \geq 0$

9. Write the following minimization problem as Linear Optimization Problem.

min $||Ax - b||_\infty$ subject to:

$c^T x \leq d$

$x \geq 0$

where A $= \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$, b $= \begin{bmatrix} 5 \\ -6 \end{bmatrix}$, c $= \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, d $= 30$ In the LPP, set $x_2 = 0$ and solve the LPP with graphical methods.

10. An investor is considering investing in two securities A and B. The risk and return associated with these securities is different. Security A gives returns of 9% and a risk factor of 6 on scale of 0-10 and security B gives returns of 15% but has risk factor of 8. Mr. X wants to invest a total capital of 50,000\$. He expects a minimum returns of 12% and will trade only if the combined risk factor doesn't exceed 6. Mr. X wants to find the split the capital investing in stocks A and B and maximize the returns. Formulate this as an LPP and solve it graphically.

11. A factory manufactures two products A and B. To manufacture one unit of A, 1.5 machine hours and 2.5 labour hours are required. To manufacture product B, 2.5 machine hours and 1.5 labour hours are required. In a month, 300 machine hours and 240 labour hours are available. Profit per unit for A is Rs. 50 and for B is Rs. 40. The factory owner wants to plan the production of A and B such that he bags maximum profit. Formulate as LPP.