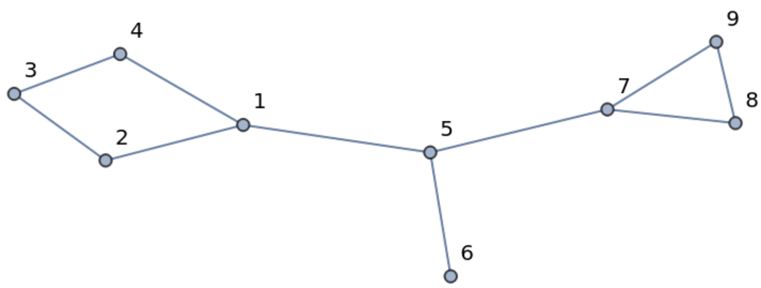
COL 751 : Practice Sheet - 5

1. Matching in Streaming Model
Prove that there exists a streaming algorithm that for any undirected graph G computes a matching of size at least $|M_{opt}|/2$ in $O(|M_{opt}|)$ working space.

2. Approximate Bipartite matching
Argue that for any $\epsilon \in (0,1)$ and any connected bipartite graph $G = (X, Y, E)$ with m edges a matching of size at least $(1-\epsilon) \cdot |M_{opt}|$ in $O(\epsilon^{-1}m)$ time.

3. Augmenting path
Let P be a shortest augmenting path with respect to a matching M and let Q be an augmenting path with respect to $M \oplus P$. Then argue $|Q| \geqslant |P| + 2|P \cap Q|$. Explain how this can be used to obtain alternate proof of Hopcraft Karp algorithm.

4. Counting Maximum matchings
Compute the Gallai–Edmonds decomposition of the following graph, and use it to compute all possible distinct maximum matchings.



5. Gallai–Edmonds decomposition
Let $G = (V, E)$ be an undirected graph and (W, X, Y) be its Gallai–Edmonds decomposition. Prove the following: Every subset $A \subseteq X$ has neighbors in at least $|A| + 1$ components in $G[Y]$.

6. Peterson's theorem
Prove Petersen's theorem: If G is 3-regular (all degrees are exactly 3) and $G - e$ is connected for all edges e, then G has a perfect matching. (Hint: Show that for a subset C of vertices of odd size, then there are at least three edges from C to $V \setminus C$.)