ELL205 Signals and Systems **40 Marks**

Minor Examination **February 2023**

Important Instructions: Do not carry mobile phones with you.

1. A causal LTI system with input $x[n]$ and output $y[n]$ follows the input-output relation

   $y[n] - \alpha y[n-1] = x[n]$, where $\alpha \in \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers.

   a. Find $h[n]$ the impulse response of the system and the range of α for which the system is stable. (6 marks).

   b. Consider a system which is cascade of two systems $h_1[n]$ and $h_2[n]$ given by

   $h_1[n] = h[n-1] - \alpha^2 h[n-3]$ and $h_2[n] = h[n] - \alpha^3 h[n-3]$. Find the impulse response $g[n]$ of this cascaded system. (6 marks).

2. Consider the following set of signals: $\Phi[n] = \left(\frac{1}{2}\right)^n$ u[n]. Show that an arbitrary signal can be represented in the form

$$x[n] = \sum_{k=-\infty}^{+\infty} a_k \, \Phi[n-k]$$

   by determining an explicit expression for the coefficient a_k in terms of the values of the signal $x[n]$. (6 marks).

3. Determine the unit impulse response $h(t)$ of a Causal LTI system specified by the differential equation: (6 marks)

$$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

4. Compute the convolution $y[n] = x[n] * h[n]$ when

   $x[n] = \alpha^n u[n] \quad 0 < \alpha < 1$

   $h[n] = \beta^n u[n] \quad 0 < \beta < 1$

   Assume that α, β are not equal. (4 marks)

5. Let $x(t) = exp(j\pi Nt/T) \frac{\sin(\pi(N+1)t/T)}{\sin(\pi t/T)}$ with N even.

a. What is the time-period of $x(t)$? (3 marks)

b. How many points of time does $x(t)$ take the value 0 in one time period? (3 marks)

c. What are the Fourier series coefficients of $x(t)$? (4 marks)

d. What is $\int_0^T x(t)dt$? (2 marks)