COL202 Quiz 5

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TOTAL POINTS

0 / 5

QUESTION 1

1 Problem 1 **0 / 5**

✓ **+ 0 pts** *Incorrect*

+ 0.5 pts Base case for recurrence

$$f(1, k) = 1$$ if $$k=1, 0$$ otherwise

$$f(n, 0) = 0$$

+ 1.5 pts Correct recurrence

$$f(n, k) = (n-1)*f(n-1, k) + f(n-1, k-1)$$

+ 1 pts Correct argument for recurrence

+ 2 pts Solve to find the generating function

from the recurrence and concluding the proof

+ 0.5 pts Correct Base case for induction on n

+ 0.5 pts Correct Base case for induction on k

+ 2 pts Correct hypothesis for induction on n

+ 2 pts Correct hypothesis for induction on k

1 2 3 4

$T(1) = \cancel{n}_1 (n-1)!$

$n(n+1)$

$-(n+n-1)$

$\frac{(n-1)n}{2}$

$= T(2)$

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Important: Answer within the box. Anything written outside the box will be treated as rough work.

Problem 1

Given a permutation $\pi : [n] \to [n]$, we say that $j \in [n]$ is a *winner* in π if $\pi(j) > \pi(i)$ for all $1 \le i \le j$. We assume that 1 is a winner in every permutation. Prove that for each $k \in [n]$, the number of permutations of $[n]$ with exactly k winners is the coefficient of x^k in $\prod_{i=0}^{n-1}(i + x)$.

let $T(k)$ be no. of ~~per~~ permutat^n with exactly k winners. and ~~if~~ S_k be set of such perm.

$\Rightarrow S_k$ bij V = set of $f : [n] \to [n]$, & f has k local maxima with each maxima greater than prev.

let $A_n(x)$ be ogf of $T(n)$

$$\Rightarrow A_n(x) = A_{n-1}(x)(x+n)$$

$$\Rightarrow A_n(x) = \prod_{i=0}^{n-1}(x+i) \quad \Rightarrow \text{no. of } \pi$$

$= \text{coeff. of } x^k$