PROBLEM SHEET# 5

Ques.1

$$\frac{2\pi}{\frac{\pi}{7}} * k = 250$$

$$k = 17.85$$

$$a_k = 0 \quad for\ |k| < 18$$

Ques.2

a) $y(t) = \sum_{k-\infty}^{\infty} a_k\, e^{jw_0kt} H(w_0k)$

$$a_k = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

$$y(t) = \sum_{k-\infty}^{\infty} a_k = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}\ e^{j\pi kt}$$

b)

$$H(w) = \int_{-\infty}^{\infty} h(\tau)\, e^{-jw\tau} d\tau$$

$$H(-w) = \int_{-\infty}^{\infty} h(\tau)\, e^{jw\tau} d\tau$$

$$= \int_{-\infty}^{\infty} h(-\tau)\, e^{-jw\tau} d\tau$$

If *h(t)* is even then

$$h(\tau) = h(-\tau)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{-jw\tau} d\tau$$

$$= H(w)$$

Hence *H(-w)=H(w)*

c) No, it is not causal. For a causal system $h(t) = 0\ for\ t < 0$
And if *h(t)* is also even, that means, $h(t) = 0$ for all values of *t*, or

$$h(t) = \delta(t)$$

If $h(t) = 0$ for all values of *t* $H(w) = 0\ \forall$ w
If

$$h(t) = \delta(t)$$

H(w) is constant.

Ques.3

$x(t) = x'(t) - 1/2$; Where $x'(t)$ is as given in question 1.

So,

$$b_k = a_k - \frac{1}{2} \ if \ k = 0$$

$$= a_k \ if \ k \neq 0$$

Where

$$a_k = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

$$x(t) \leftrightarrow b_k$$

$$x'(t) \leftrightarrow a_k$$

Ques.4

$$x_2(t) = x_1(1-t) + x_1(t-1)$$

$$x_2(t) = x_1(-t+1) + x_1(t-1)$$

$$x_1(-t+1) \rightarrow a_k e^{+jw_1k}$$

$$a_{-k} e^{-jw_1k}$$

$$x_1(t-1) \rightarrow a_k e^{-jw_1k}$$

$$b_k = e^{-jw_1k}[a_k + a_{-k}]$$

Ques.5

a) $x(t-t_0) + x(t+t_0)$

$$\leftrightarrow a_k e^{-jkw_0t_0} + a_k e^{jkw_0t_0}$$

b) $Ev\{x(t)\} = \frac{[x(t)+x(-t)]}{2}$

$$\leftrightarrow \frac{1}{2}[a_k + a_{-k}]$$

c) $Re\{x(t)\} = \frac{[x(t)+\overline{x(t)}]}{2}$

$$\leftrightarrow \frac{1}{2}[a_k + \overline{a_{-k}}]$$

d) $\frac{d^2x(t)}{dt^2} \leftrightarrow (jkw_0)^2 a_k$

e) $x(3t-1) \leftrightarrow a_k e^{-jkw_0}$

Ques.6

a) $a_k = a_{k+2}$

$$a_k = \frac{1}{T}\int x(t)\, e^{-jw_0kt}\, dt$$

$$a_{k+2} = \frac{1}{T}\int x(t)\, e^{-jw_0kt}e^{-j2w_0t}\, dt$$

$$x(t) = x(t)e^{-j2\left(\frac{2\pi}{3}\right)t}$$

$$x(t) = x(t)e^{-\frac{j4\pi}{3}t}$$

x(t) can be non-zero only when

$$e^{-\frac{j4\pi}{3}t} = 1$$

This implies

$$e^{-\frac{j4\pi}{3}t} = e^{j2\pi m}$$

$$\frac{4\pi}{3}t = 2\pi m$$

$$t = \frac{3}{2}m$$

$$t = 0, \pm 1.5, \pm 3, \ldots \ldots ..$$

$$x(t) = a\delta(t) + b\delta(t-1.5) + c\delta(t+1.5)$$

b)

As $a_k = a_{-k}$, $b = c$

$$x(t) = a\delta(t) + b\delta(t-1.5) + b\delta(t+1.5)$$

c)

$$\int_{-0.5}^{0.5} a\delta(t)dt = 1$$

$$a = 1$$

d)

$$\int_{1}^{2} b\delta(t)dt = 2$$

$$b = 2$$

Ques.7

The unknown FS coefficients are $a_1, a_{-1,} a_2 and\ a_{-2}$

Since *x(t)* is real $a_1 = a_{-1}^*$ and $a_2 = a_{-2}^*$

Since a_1 is real $a_1 = a_{-1}$

$$x(t) = A_1 \cos(w_0 t) + A_2 \cos(2w_0 t + \theta)$$

$$w_0 = 2\pi/6$$

From this we get

$$x(t-3) = A_1 \cos(w_0 t - 3w_0) + A_2 \cos(2w_0 t + \theta - 6w_0)$$

Now, for $x(t) = -x(t-3)$, $3w_0$and $6w_0$ should both be odd multiples of π.

Therefore $a_2 = a_{-2} = 0$ and

$$x(t) = A_1 \cos(w_0 t)$$

Using Parseval's relation

$$\sum_{k=-\infty}^{\infty} |a_k|^2 = |a_1|^2 + |a_{-1}|^2 = 1/2$$

Then $|a_1| = 1/2$, since a_1 is positive

$$a_1 = a_{-1} = 1/2$$

$$x(t) = \cos(\pi t/3)$$

Ques.8

a) [**NOTE: The part (a) in problem sheet represents the question itself**]

b)

$$x(t) = \sum a_k\, e^{\frac{jk2\pi}{T}t}$$

$$x(t + T/2) = \sum a_k\, e^{\frac{jk2\pi}{T}t} e^{j\pi k}$$

$$e^{j\pi k} = -1\ \ if\ k\ is\ odd$$

Thus,

$$x(t + T/2) \leftrightarrow\ - a_k$$
$$x(t + T/2) \leftrightarrow\ - x(t)$$

c)

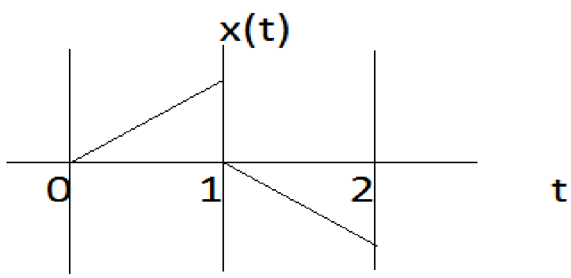
$$a_k = \frac{1}{T}\int x(t)\, e^{-jw_0kt}\, dt$$

Put t = t + T/2 and we know from the above equation that x(t) = -x(t+T/2)
Therefore,

$$a_k = \frac{(-1)^{k+1}}{T}\int x(t)\, e^{-jw_0kt}\, dt$$
$$a_k = (-1)^{k+1}a_k$$

Therefore, for even values of k, $a_k = 0$

d)



e)

$$x(t + T/2) = x(t)$$
$$x(t + T/2) = \sum a_k\, e^{\frac{jk2\pi}{T}t} e^{j\pi k}$$

If k is even, then $e^{j\pi k} = 1$

Hence

$$x(t + T/2) = x(t)$$

Fundamental period is T/2

f)

$$x(t) = a_1 e^{jw_0t} + a_{-1}e^{-jw_0t} + \cdots$$

Then the fundamental period is T. Or

$$x(t) = a_k e^{jkw_0t} + a_l e^{jlw_0t}$$

If both of these conditions are absent then for any two coefficients a_m and a_n such that m = pn then the fundamental time period would be T/n