COL202: Discrete Mathematical Structures **Spring 2023**

Tutorial Sheet 7

Announced on: Feb 23 (Thurs)

1. **[Submission Problem for Group 1]** Based on Problem 6.25 in [LLM17].

   In the stable matching problem, call a person (a man or a woman) *lucky* if they are matched with someone in the top half of their preference list under some stable matching. Prove that there must be at least one lucky person.

2. **[Submission Problem for Group 2]**

   Show that under the men-proposing deferred acceptance algorithm, there is always at least one woman who receives exactly one proposal during the execution of the algorithm.

3. **[Submission Problem for Group 3]**

   Consider any input to the DA algorithm consisting of n men and n women, where n is an arbitrary natural number. As a function of n, what is the maximum number of rounds for which the DA algorithm can run before it terminates?

   Construct an instance of the stable matching problem with n men and n women (again, for a general n) where the DA algorithm runs for the number of rounds specified in your answer above. Explain the *correctness* of your answer—specifically, why does your instance satisfy the stated bound and why is it the optimal bound.

4. Prove that an instance has a unique stable matching if and only if the men-optimal and women-optimal stable matchings are identical. Can you give an algorithm for quickly determining if a given instance has a unique stable matching?

5. **[Submission Problem for Group 4]** Based on Problem 12.47 in [LLM17].

   a) Prove that the average degree of a tree is less than 2.

   b) Suppose every vertex in a graph has degree at least k. Explain why the graph has a path of length at least k. Does such a graph also have a path of length *exactly* k?

References

[LLM17] Eric Lehman, Tom Leighton, and Albert R Meyer. *Mathematics for Computer Science*. 2017. URL: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf.