Problem sheet – 5

1. Filter output evaluation using Fourier Series

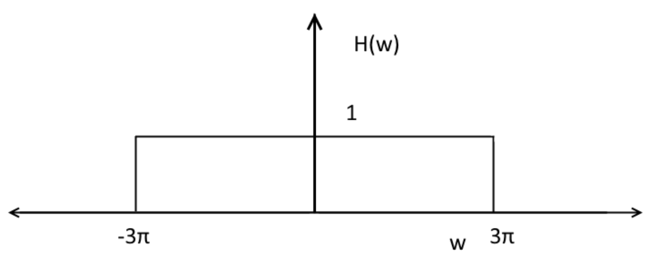
Consider a continuous-time LTI system S whose frequency response is

$$H(w) = \begin{cases} 1, & |w| \geq 250 \\ 0, & \text{otherwise} \end{cases}$$

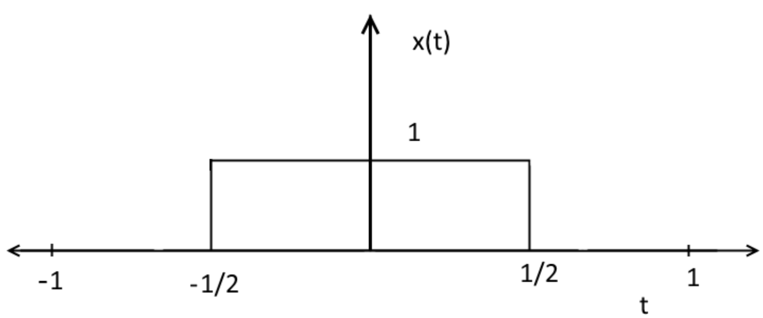
When the input to this system is a signal x(t) with fundamental period T = $\pi/7$ and Fourier series coefficients a_k, it is found that the output y(t) is identical to x(t). For what values of k is it guaranteed that a_k = 0?

2. System response using Fourier Transform

a) Determine the output of LTI system with the following frequency response.



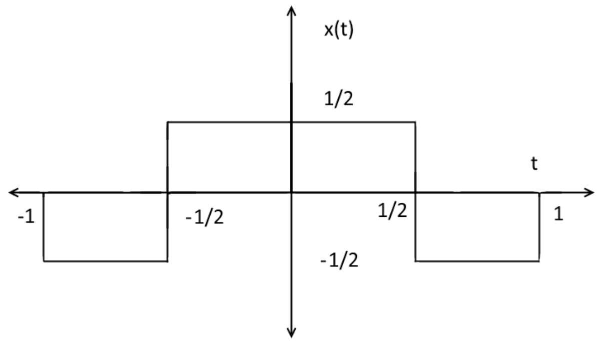
Assume *x(t)* is a periodic signal with period 2.



b) Prove that if *h(t)* is even then *H(ω)* is also even.
c) Is *H(ω)* causal?

3. Fourier series representation of a signal

Determine a_k for the following signal with period 2



4. Relationship between Fourier coefficients

Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that

$$x_2(t) = x_1(1-t) + x_1(t-1)$$

how is the fundamental frequency ω_2 of $x_2(t)$ related to ω_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k. You may use the properties of the Fourier series coefficients.

5. Properties of Fourier series coefficient

Let x(t) be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of each of the following signals in terms of a_k:

(a) $x(t - t_0) + x(t + t_0)$

(b) Ev(x(t))

(c) Re{x(t)}

(d) $\frac{d^2x(t)}{dt^2}$

(e) x(3t - 1) [for this part, first determine the period of x(3t - 1)]

6. Signal Determination from Fourier coefficient properties

Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients a_k:

a) $a_k = a_{k+2}$.
b) $a_k = a_{-k}$.
c) $\int_{-0.5}^{0.5} x(t)dt = 1$
d) $\int_{1}^{2} x(t)dt = 2$

Determine x(t)

7. Signal Determination from Fourier coefficient properties

Suppose we are given the following information about a signal x(t):

a) x(t) is a real signal.
b) x(t) is periodic with period T = 6 and has Fourier coefficients a_k.

c) a_k= 0 for k = 0 and k > 2.
d) x(t) = - x(t - 3).
e) $\frac{1}{6}\int_{-3}^{3} x(t)dt = \frac{1}{2}$
f) a_1 is a positive real number.

Show that x(t) = A cos(Bt + C), and determine the values of the constants A, B, and C.

8. Even and Odd Harmonic Signals

a) A continuous-time periodic signal x(t) with period T is said to be odd harmonic if, in its Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk(2\pi/T)t}$$

a_k = 0 for every even integer k.

b) Show that if x(t) is odd harmonic, then

$$x(t) = -x(t + \frac{T}{2})$$

c) Show that if x(t) satisfies eq. above, then it is odd harmonic.
d) Suppose that x(t) is an odd-harmonic periodic signal with period 2 such that

$$x(t) = t \qquad \text{for } 0 < t < 1.$$

Sketch x(t).

e) Analogously, to an odd-harmonic signal, we could define an even-harmonic signal as a signal for which a_k= 0 for k odd in the Fourier series representation. Could T be the fundamental period for such a signal? Explain your answer.
f) More generally, show that T is the fundamental period of x(t) in eq. in part(a) if one of two things happens:
   i. Either a_1 or a_{-1} is nonzero; or
   ii. There are two integers k and l that have no common factors and are such that both a_k and a_l are nonzero