MTL103: Optimization Methods and Applications Date: February 27, 2023

Tutorial 3

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Question 1. *Consider the problem of minimizing $c^T x$ over a polyhedron P. Prove the following:*

1. *A feasible solution x is optimal if and only if $c^T d \geq 0$ for every feasible direction d at x.*
2. *A feasible solution x is the unique optimal solution if and only if $c^T d > 0$ for every nonzero feasible direction d at x.*

Question 2. *Let $P = \{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 1,\ x \geq 0\}$ and consider the $x = (0, 0, 1)$. Find the set of feasible directions at x.*

Question 3. *Consider the problem*

$$\begin{aligned} \textit{Minimize } & -2x_1 - x_2 \\ \textit{Subject to } & x_1 - x_2 \leq 2 \\ & x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0. \end{aligned}$$

1. *Convert the problem into standard form and construct a basic feasible solution at which $(x_1, x_2) = (0, 0)$.*
2. *Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part (1).*

Question 4. *Let x be a basic feasible solution associated with some basis matrix A_B. Prove the following:*

1. *If the reduced cost of every nonbasic variable is positive, then x is the unique optimal solution.*
2. *If x is the unique optimal solution and is nondegenerate, then the reduced cost of every nonbasic variable is positive*

Question 5. *Use the simplex method to find an improved solution for the linear programming problem represented in the tableau.*

variables	b	x_1	x_2	s_1	s_2	s_3
	0	−4	−6	0	0	0
s_1	11	−1	1	1	0	0
s_2	27	1	1	0	1	0
s_3	90	2	5	0	0	1

The objective function for this problem is $z = 4x_1 + 6x_2$.

Question 6. *Let x be an element of the standard form polyhedron $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$. Prove that a vector $d \in \mathbb{R}^n$ is a feasible direction at x if only if $Ad = 0$ and $d_i \geq 0$ for every i such that $x_i = 0$.*

Question 7. *Raju holds two part-time jobs, Job 1 and Job 2. He never wants to work more than a total of 12 hours a week. He has determined that for every hour he works at Job 1, he needs two hours of preparation time, and for every hour he works at Job 2, he needs one hour of preparation time, and he cannot spend more than 16 hours on preparation. If he makes Rs. 40 an hour at Job 1 and Rs. 30 an hour at Job 2, how many hours should he work per week at each job to maximize his income?*

Question 8. *Find a solution using the Two-Phase method.*

$$\begin{aligned}
&\textit{Minimize } z = x_1 + x_2 \\
&\textit{Subject to } 2x_1 + x_2 \geq 4 \\
&\qquad\qquad\quad x_1 + 7x_2 \geq 7 \\
&\qquad\qquad\quad x_1, x_2 \geq 0.
\end{aligned}$$