Problem Sheet-2

1. In this problem, we examine a few of the properties of the unit-step and unit-impulse function. Show that

a) $\delta(2t) = \frac{1}{2}\delta(t)$

b) $u(t) = \int_0^{\infty} \delta(t-\sigma)d\sigma$

c) $\lim_{\Delta\to 0}[u_\Delta(t)\,\delta(t)] = 0$

d) $\lim_{\Delta\to 0}[u_\Delta(t)\,\delta_\Delta(t)] = \frac{1}{2}\delta(t)$

2. Consider the continuous time signal

$$x(t) = \delta(t+2) - \delta(t-2)$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

3. **Identity system/Invertibility**

Consider a system S with input x(t) and output y(t). This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 is

$$S_1 : \; y_1(t) = 2x_1(2t+2),$$

where $x_1(t)$ denote input signal.

a) Determine S_2 if a cascade of S_1 and S_2 is an identity system.

b) Do the same exercise for its discrete counterpart.

c) Assume now that the system S_1 response is $y_1(t) = 2x_1(2t-2)$, under what conditions this system can be inversed? Does it conflict with the definition that a system is invertible if distinct inputs leads to distinct outputs?

4. **System properties**

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless
(2) Causal
(3) Stable

Determine which of these properties hold and which do not hold. Justify your answers, symbols have the usual meaning.

a) $y(t) = x(t-2) + x(2-t)$

b) $y(t) = [\cos(3t)]x(t)$

c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

d) $y[n] = \sum_{k=-\infty}^{3n} x[k]$

e) $y(t)=\begin{cases} 0, & t<0 \\ x(t)+x(t-2), & t>0 \end{cases}$

f) $y(t)=\begin{cases} 0, & x(t)<0 \\ x(t)+x(t-2), & x(t)>0 \end{cases}$

g) $y(t) = x(t/3)$

h) $y(t) = \frac{d\,x(t)}{dt}$

i) $y(t) = t^2x(t-1)$

j) $y[n] = x^2[n-2]$

k) $y[n] = x[n+1] - x[n-1]$

l) $y[n] = Odd\{x(t)\}$

m) $y(t) = x(\sin(t))$

n) $y[n] = \sum_{k=n-n_o}^{n+n_o} x[k]$ where n_o is a finite positive integer.

MATLAB Exercises

Please download MATLAB either from the MathWorks website, or from the IITD software repo if you are on the IITD network (make sure to download it via LAN to save yourself a trip to the CSC!). You may also choose to use MATLAB Online or download Octave, an open source alternative to MATLAB. To familiarise yourself with the language and the relevant commands, watch the video uploaded in files section in Teams.

a) Open an audio file in MATLAB and plot its waveform.
b) Generate an upsampled version of this audio signal. Listen to the audio signal and observe the effects of upsampling.
c) Now, downsample the upsampled signal. Verify that you have obtained the original signal back.
d) Plot the waveforms for the upsampled/downsampled signals, and compare with the original signal.