**Name**: ____________________________ **Entry number**: ________________

COL351 Quiz 3 Duration: 1 hour

---

**Read the following instructions before you begin writing.**

1. Keep a pen, your identity card, and optionally a water bottle with you. Keep everything else away from you, at the place specified by the invigilators.
2. Do not detach sheets. Write your entry number and name on every page. (We will detach sheets prior to grading.)
3. Answer only in the designated space. Think before you use this space. No additional space will be provided for writing answers. Blank pages will be provided for rough work on demand, but they cannot be used for writing answers.
4. No clarifications will be given during the exams. If something is unclear or ambiguous, make reasonable assumptions and state them clearly. The instructor reserves the right to decide whether your assumptions were indeed reasonable.
5. You may use any result discussed in class or tutorials without proving it. Similarly, you can use any algorithm discussed in class or tutorials as a "black-box" i.e., without reproducing any details of how it works.

---

1. (10 points) You are helping the National Highway Authorities of India in planning the construction of a network consisting of exactly $n-1$ roads to connect $n$ villages, say $1, 2, \ldots, n$, located remotely in the Himalayas. Obviously, being an expert in data structures, you decide to treat villages as vertices, roads as (undirected) edges, and the road network as a tree connecting these vertices. You are aware that roads in this region are prone to blockages due to landslides. Let $p_{i,j} = p_{j,i}$ denote the probability that the road between villages $i$ and $j$ would get blocked on any given day, if it were a part of your network. Assume that blockages happen on all roads independently. A road network is said to have failed on a given day if at least one road in the network is blocked. Design a polynomial-time algorithm which takes the numbers $p_{i,j}$ for all $i \neq j$ as input, and outputs a network with minimum failure probability.

**Name**: ______________________________ **Entry number**: ____________________

COL351 Quiz 3 Duration: 1 hour

2. (10 points) Let $\mathcal{M} = (S, \mathcal{I})$ be a matroid. Recall that a *basis* of $\mathcal{M}$ is a maximal independent set, that is, it is an independent set that does not have an independent strict superset. Let $w : S \longrightarrow \mathbb{R}$ be an assignment of weights to the elements of $S$. The *bottleneck cost* of a subset $A$ of $S$ is the weight of a heaviest element in $A$. Given $w$, we are interested in finding a basis with minimum bottleneck cost. Here is an algorithm.

**Algorithm 1** Kruskal's algorithm

```
Sort the elements of S in non-decreasing order of weight.
I ← ∅.
for each element e of S in the sorted order do
    if I ∪ {e} ∈ 𝓘 then
        I ← I ∪ {e}.
    end if
end for
```

**Rigorously** prove or disprove the following statement: for every $w$, the above algorithm finds a basis with minimum bottleneck cost.