

## Tutorial 5

1. [Submission Problem for Group 1] A collection  $\mathcal{C}$  of sets is called a chain when, given any two sets in  $\mathcal{C}$ , one is a subset of the other. Prove that if  $\mathcal{F}$  is chain of finite sets, then  $\cup \mathcal{F}$  is countable. (Notice that without the chain condition, every set is the union of its finite subsets.) Problem 8.10 in LLM Book)
2. [Submission Problem for Group 2] Let  $\{0, 1\}^\omega$  be the set of infinite binary sequences. Call a sequence in  $\{0, 1\}^\omega$  lonely if it never has two 1s in a row. For example, the repeating sequence  $\{0, 1, 0, 1, 0, 1, 0, 1, 0, \dots\}$  is lonely, but the sequence  $\{0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, \dots\}$  is not lonely because it has two 1s next to each other. Let  $F$  be the set of lonely sequences. Show that  $F$  is uncountable. (Problem 8.17 in LLM Book)
3. [Submission Problem for Group 3] Prove that if  $\{A_0, A_1, \dots, A_n, \dots\}$  is an infinite sequence of countable sets, then so is

$$\bigcup_{n=0}^{\infty} A_n$$

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A complex number  $\alpha$  is called algebraic if there exists a univariate polynomial  $p(x)$  with rational coefficients such that  $p(\alpha) = 0$ . Conclude using the first part of the question that the set of *algebraic numbers* is countable.

4. [Submission Problem for Group 4] Prove that the set of all finite subsets of positive integers is countable. (Problem 8.9 in LLM Book)
5. [Bonus] Problems 8.14 (Schröder-Bernstein Theorem), 8.19, 8.20, 8.22.
6. Watch Veritasium's amazing video about foundations of mathematics.