Hints and Solution: 3

Ques.1

a)

i) If the system is additive, following relation is true.

$$x_1(t) + x_2(t) = y_1(t) + y_2(t)$$

$$0 = 0$$

Now if the system is homogenous, we proceed as follows:

$$ax(t) = ay(t)$$

If $x(t) = 0$ then y(t) = 0

If a = 0, zero-input gives zero-output

b) $y(t) = \sin\{x(t)\}$

c) No. Consider x(t)=$\begin{cases} 0 \;\; for\; t < 0 \\ 1 \;\; otherwise \end{cases}$ and y(t) = x(t+2). Then y(t) is not zero for t < 0

Ques.2

a) Let x(t) gives y(t)

If system is time-invariant then $x(t+T) = y(t+T)$

$x(t) = x(t+T) implies\; y(t+T) = y(t))$

b) Consider the time invariant system y(t) = sin[x(t)]. Let x(t) = t (non-periodic). Then $y(t) = sint$ which is periodic signal.

Ques.3

a) Assumption: If $x(t) = 0$ for $t < t_0$, then $y(t) = 0$ for $t < t_0$.
To prove that: The system is causal.
Let us consider an arbitrary signal $x_1(t)$. Then, let us consider another signal $x_2(t)$ which is the same as $x_1(t)$ for $t < t_0$.
Since the system is linear,

$$x_1(t) \;-\; x_2(t) \;\to\; y_1(t) \;-\; y_2(t)$$

Since $x_1(t) - x_2(t) = 0$ for $t < t_0$, by our assumption $y_1(t) - y_2(t) = 0$ for $t < t_0$. This implies $y_1(t) = y_2(t)$ for $t < t_0$, 2 which is a property of a causal system.

Assumption: The system is causal.
To prove that: If $x(t) = 0$ for $t < t_0$, then $y(t) = 0$ for $t < t_0$.
Let us assume that the signal $x(t) = 0$ for $t < t_0$. Then we may express $x(t)$ as $x(t) = x_1(t) - x_2(t)$, where $x_1(t) = x_2(t)$ for $t < t_0$. Since the system is linear, the output to $x(t)$ will be $y(t) = y_1(t) - y_2(t)$. Now, since the system is causal, $x_1(t) = x_2(t)$ for $t < t_0$ implies that $y_1(t) = y_2(t)$ for $t < t_0$. Therefore, $y(t) = 0$ for $t < t_0$

b) $y(t) = x(t)x(t+5)$
c) $y(t) = x(t) + 10$
d) Let us assume that $x[n] \neq 0$ gives $y[n] = 0$

Then $ax[n] \rightarrow 0$ (linearity) which implies that for distinct values of a, we will get still the same output 0, which contradicts with the definition of invertibility that the distinct inputs should lead to distinct outputs.
e) $y[n] = x^2[n]$

Ques.4

a) Linear, Time variant and Non causal

$$\emptyset_{hx}(t) = \int_{-\infty}^{\infty} h(t+\tau)x(\tau)d\tau$$

Replacing x(t) by cx(t) where c is a real number

$$\widetilde{\emptyset}_{hx}(t) = \int_{-\infty}^{\infty} h(t+\tau)cx(\tau)d\tau = c\emptyset_{hx}(t)$$

Similarly putting $\tilde{x}(t) = x_1(t) + x_2(t)$ we will get $\widetilde{\emptyset}_{hx}(t) = \emptyset_{hx_1}(t) + \emptyset_{hx_2}(t)$

Thus $\emptyset_{hx}(t)$ obeys homogeneity and additivity. That's implies $\emptyset_{hx}(t)$ is linear.

Now substitute $x(t)$ by $x(t\text{-}t_0)$

$$\widetilde{\emptyset}_{hx}(t) = \int_{-\infty}^{\infty} h(t+\tau)x(\tau-\tau_0)d\tau$$

Which can be arranged to

$$\text{LHS} = \int_{-\infty}^{\infty} h(t-t_0+\tau)x(\tau)d\tau = \emptyset_{hx}(t-t_0)$$

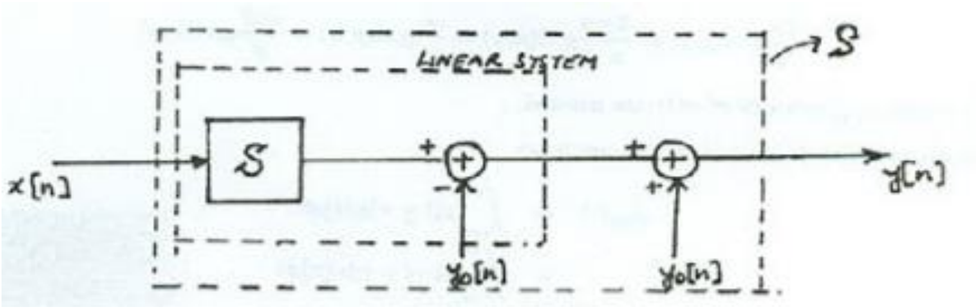
Hence the system is invariant

As the system output depends upon the values of input from [-∞, +∞], the system is casual.

b) Linear, Time-invariant and non-causal.

Ques.5

a) $\boldsymbol{y}[\mathrm{n}] = \mathrm{H}\{\mathrm{x}[\mathrm{n}] + \mathrm{x}_1[\mathrm{n}]\} - y_1[\mathrm{n}] = \mathrm{H}\{\mathrm{x}[\mathrm{n}]\}$
b) If x_1 [n] = 0 for all n, then $y_1[n]$ will be the zero input response y_0[n]. S may then be redrawn as shown in below



i) Incrementally linear

$$x[n] \rightarrow x[n] + 2x[n+4] \;\; and \; y_0[n] \; = n$$

ii) Incrementally linear

$$x[n] \rightarrow \begin{cases} 0\,, \; n \; even \\ \sum_{k=-\infty}^{(n-1)/2} x[k] \; n \; odd \end{cases} \text{ and } \quad y_0[n] = \begin{cases} \frac{n}{2} & n \; even \\ \frac{n-1}{2} & n \; odd \end{cases}$$

Ques.6

a) Memory, Time variant, Linear, Non causal, Stable
b) Memory less, Time variant, Linear, Causal, Stable
c) Memory, Time variant, Linear, Non causal, Unstable
d) Memory, Time variant, Linear, Non causal, Unstable
e) Memory, Time variant, Linear, Causal, Stable
f) Memory, Time Invariant, Non-Linear, Causal, Stable
g) Memory, Time variant, Linear, Non causal, Stable
h) Memory less / Memory, Time Invariant, Linear, Non causal / Causal , Unstable
i) Memory, Time variant, Linear, Causal, Unstable
j) Memory, Time Invariant, Nonlinear, Causal, Stable
k) Memory, Time Invariant, Linear, Non causal, Stable
l) Memory, Time Variant, Linear, Non causal, Stable
m) Memory, Time variant, Linear, Non causal, Stable
n) Memory, Time Invariant, Linear, Non causal, Stable

Ques.7

a) $y(t) = u(t) * u(t)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau$$

$$y(t) = \int_{0}^{\infty} u(t-\tau)d\tau$$

$$y(t) = \int_0^t d\tau$$

$$y(t) = \begin{cases} t \ for \ t > 0 \\ 0 \ for \ t < 0 \end{cases}$$

$$y(t) = tu(t)$$

b) $y(t) = x(t) * \delta(t - \tau)$

$$y(t) = \int_{-\infty}^{\infty} x(m)\delta(t - m - \tau)dm$$

$$y(t) = \int_{-\infty}^{\infty} x(t - \tau)\delta(-m(t - \tau))dm$$

$$y(t) = x(t - \tau)\int_{-\infty}^{\infty} \delta(t - m - \tau)dm$$

$$y(t) = x(t - \tau)$$

c) $y(t) = e^{\alpha}u(t) * u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{\alpha\tau}u(\tau)u(t - \tau)\, d\tau$$

$$y(t) = \int_0^t e^{\alpha\tau}u(\tau)\, d\tau$$

$$y(t) = \frac{(e^{\alpha t} - 1)}{a}u(t)$$