Department of Mathematics
MTL 106 (Introduction to Probability and Stochastic Processes)
Tutorial Sheet No. 5
Answer for Selected Problems

1. $E(Y/x) = 2(1+x), \quad x \geq 0$

2. $E(X/y) = \frac{a+y}{n+a+b}, \quad y = 0, 1, \ldots, n$

3. $P(X = k) = \frac{\Gamma(k+n)}{\Gamma(n)\Gamma(K+1)} \left(\frac{1}{2}\right)^{k+n}, \quad k = 0, 1, \ldots$

4. $np\left(\frac{1-(1-q)^{n-1}}{1-(1-q)^{n}}\right); \quad p = q = 1/3$

6. $E(Y^k/x) = \frac{x^k}{k+1}, \quad E(Y^k) = \frac{1}{(k+1)^2}$

7. $X/y \sim N\left(\frac{y}{1+\sigma^2}, \frac{\sigma^2}{1+\sigma^2}\right), \quad E(X/y) = \frac{y}{1+\sigma^2}$

10. No, use Chebyshev's inequality

11. (a) Yes (b) yes (c) Yes (d) No (e) Yes (f) Yes

11. Geometric(p) where $p = \frac{1}{4 \times 10^5}$

12. $P^{(n)}(t) = P(P(...(P(t))...))$ where $P(t) = \frac{1}{4} + \frac{t}{4} + \frac{t^2}{2}$. $P_{Z_n}(t) = [P^{(n)}(t)]^{Z_n}$. $E(Z_{51}) = 1250$.

15. (a) $M_Y(t) = \frac{pM_X(t)}{1-(1-p)M_X(t)}$ where $M_X(t) = \frac{1}{3}\left(1 + e^t + e^{2t}\right)$ b) $E[Y] = 3$

16. (a) $f(y/x) = \begin{cases} e^{-y+x}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$

    (b)$E(Y/X = x) = (1+x), \quad x > 0$

17. $n \geq 162$

18. $1 - \phi(0.913) = 0.1814$

19. $\mu = 2\alpha \sum_{i=1}^{n} i^2, \qquad \sigma^2 = \alpha^2 \sum_{i=1}^{n} i^2$

    $E(W) = e^{\mu + \frac{1}{2}\sigma^2}$

    $Var(W) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$

    $f(w) = \frac{1}{w\sqrt{2\pi}\sigma} e^{-\frac{(\ln w - \mu)^2}{2\sigma^2}}, \qquad w > 0$

21. (a) $1 - \phi(4.93) \approx 0$ (b) 0.003599

22. $n \geq 50,000$

24. (a) $P(Y \geq 900) \leq \frac{1}{9}$ (b) $P(Z \geq 2) = 1 - \Phi(2) = 0.0228$