# Problem Set

HUL 212

October 19, 2022

Question 1

Exactly 24 hours are left for Lauren's physics major. She has an economics major directly after the physics major and has no time to study in between. Lauren wants to be a physicist, so she places more weight on her physics test score. Her utility function is given by

$$u(p, e) = 0.6ln(p) + 0.4ln(e)$$

where p is the score on the physics major and e is the score on the economics final. Although she cares more about physics, she is better at economics; for each hour spent studying economics she will increase her score by 3 points, but her physics score will only increase by 2 points for every hour spent studying physics. Studying zero hours results in a score of zero on both subjects (although $ln(0)$ is not defined, assume her utility for a score of zero is negative infinity).

(a)What constraints does Lauren face in her test score maximization problem?

(b)How many hours should Lauren optimally spend studying physics? How many hours studying economics? (hours are divisible)

(c)What economics and physics test scores will she achieve (i.e. what are e and p)?

(d)What utility level will she achieve?

(e)Suppose Lauren can get an economics tutor. If she goes to the tutor, she will increase her economics test score by 5 points for every hour spent studying instead of 3 points, but will lose 4 hours of study time by going to the tutor. She cannot study while at the tutor, and going to the tutor does not directly improve her test score. Should Lauren go to the tutor?

Question 2

Confirm that if a consumer's utility function is described by $U = 2X + Z$, and prices are $P_X = 2$ and $P_Z = 1$, there is no unique utility maximizing solution regardless of income level. What does this tell you about $X$ and $Z$ as commodities?

Question 3

Assume that the firm has the production function $y = AL^{\alpha}K^{\beta}F^{\gamma}$. In the short-run, however, the quantity of land farmed is fixed to F, so there effectively are only two factors of production with respect to which the firm maximizes.
1. Write down the cost minimization problem with respect to L and K and the first order conditions with respect to L and K.
2. Solve for $L^*(w, r, s, y, \bar{F})$ and $K^*(w, r, s, y, \bar{F})$.
3. How does $L^*$ vary as $w$ increases? Compute $\partial L^*/\partial w$ using the solution in point 2. Does it make sense? How about if there is technological progress and A increases? What happens to $L^*$?
4. Write down the cost function $c(w, r, s, y, \bar{F})$ and derive the expressions for the marginal cost $c_y'(w, r, s, y, \bar{F})$ and the average cost $c(w, r, s, y, \bar{F})/y$.
5. Assume $\alpha + \beta < 1$ and plot the marginal cost $c_y'(w, r, s, y, \bar{F})$ and the average cost $c(w, r, s, y, \bar{F})/y$ as a function of $y$. Draw the supply curve of the firm, that is, $y = y^*(w, r, s, y, \bar{F}$. Is the supply curve (weakly) increasing in $p$?
6. Assume $\alpha + \beta = 1$. Repeat exercise 5. Why does the firm never want to produce a positive and finite quantity of output despite constant returns to scale in labor and capital?
7. Assume $\alpha + \beta > 1$. Repeat exercise 5.
8. Consider $\alpha + \beta < 1$. Under perfect competition setting (for now just take $p = MC$). Write the equation for firm supply and industry supply function.
9. Now, the consumer side. Assume that consumers have Cobb-Douglas preferences for mangoes (good $y$ with price $p$) and all the other goods (good $m$ with price 1). Their utility function is

$$u(y, m) = y^{\alpha}m^{1-\alpha}$$

. Compute aggregate demand function.

10. Calculate market clearing price (where market supply equals aggregate demand) and then the output.
11. Determine now the sign of the following changes on $p_M$ and $Y$. Provide intuition. Also, make sure that the results square with your graphical intuition about moving supply and demand function.
(a) an increase in total income $\sum_{j=1}^{J} M_j$
(b) an increase in productivity $A$.

Question 4

Consider a quasi-linear utility function $U(x, y) = \sqrt{x} + y$. Show that the good which doesn't enter linearly in the utility function has an Income effect of zero.

Question 5

Show that it is simultaneously not possible for two goods to be gross substitutes and net complements of each other if both goods are normal goods? Check the same for inferior goods.

Question 6

All goods cannot be net complements at the same time. Explain. (Two goods can be net complements in a model with more than two good.)

Question 7

'A giffen good is necessarily an inferior good'. Check the validity of the statement using Slutsky equation.