ELL205 Signals and Systems — **60 Marks**

Minor 2 Examination — **March 2023**

Important Instructions: Do not carry mobile phones with you.

Useful Formulas:

1.	**DTFS** $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\Omega_o n}$ $a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_o n}$ **CTFS** $a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_o t} dt$ $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$	**DTFT** $x[n] = \frac{1}{2\pi} \int_0^{2\pi} H(e^{j\Omega}) e^{jn\Omega} d\Omega$ $H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$ **CTFT** $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

1. Consider a causal continuous-time LTI system with frequency response

   $H(\omega) = A(\omega) + jB(\omega)$, where $j = \sqrt{-1}$. If $A(\omega) = \pi\delta(\omega)$, find $H(\omega)$. (5 marks)

2. Let $x(t)$ be a periodic signal with fundamental period $T = 3$ and Fourier series coefficients a_k. Suppose we are given the following information about Fourier series coefficients a_k and the signal $x(t)$.

   1. a_k are real and even
   2. $a_k = a_{k+3}$
   3. $\int_1^4 x(t)dt = 2$
   4. $\int_1^{3} x(t)dt = 1,5$

   Determine $x(t)$. (5 marks)

3. Consider a discrete-time LTI system described by

$$y[n] - \frac{1}{2}y[n-1] = x[n] + \frac{1}{2}x[n-1]$$

   a) Determine the frequency response $H(e^{j\Omega})$ of the system. (2 marks)

b) Find the impulse response $h[n]$ of the system. (2 marks)

c) Determine its response $y[n]$ to the input, $x[n] = \cos\frac{\pi}{2}n$. (6 marks)

4. A discrete-time signal $x[n]$ is non-zero for only N samples and everywhere else it is zero. If $x[n]$ has a Fourier transform $X(e^{j\Omega})$ and Fourier series coefficients a_k, find the Fourier transforms and Fourier series coefficients of

d) $y_u[n] = \begin{cases} x[n/2] & if\ n\ is\ even \\ 0 & otherwise \end{cases}$ [5 marks]

e) $y_u[n] + y_u[n-1]$ [5 marks]

f) $x[2n]$ [5 marks]

g) $(-1)^n x[2n]$ [5 marks]

5. A signal $x(t)$ satisfies the relation

$$\int_{-\infty}^{t} x(\tau)d\tau = \begin{cases} t, |t| \leq 1 \\ 0, |t| > 1 \end{cases}$$

a) Find $X(\omega)$, the CTFT of $x(t)$. [5]

b) Find $x(t)$ and sketch its plot, labeling the relevant positions. [5]

c) Calculate $\int_{-\infty}^{\infty} x(t) \cos\left(\frac{\pi t}{6}\right) dt$ and $\int_{-\infty}^{\infty} X(\omega) e^{j\frac{\omega}{2}} d\omega$. [5]

d) If $x(t)$ is passed through an LTI system with transfer function $H(\omega) = cos\omega$, then find the output $y(t)$ of this system and sketch its plot, labeling the relevant portions. [5]