Problem sheet–8

1. Closed form of signal from its Fourier Series properties

Suppose we are given the following information about a signal x[n]:

a) x[n] is a real and even signal.
b) x[n] has period N = 10 and Fourier coefficients a_k.
c) $a_{11} = 5$.
d) $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$

Show that $x[n] = A\cos(Bn + C)$, and specify numerical values for the constants A,B, and C.

2. Fourier Series decomposition and determination of output using eigenvalue relation

When the impulse train

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

is the input to a particular LTI system with frequency response $H(e^{jw})$, the output of the system is found to be

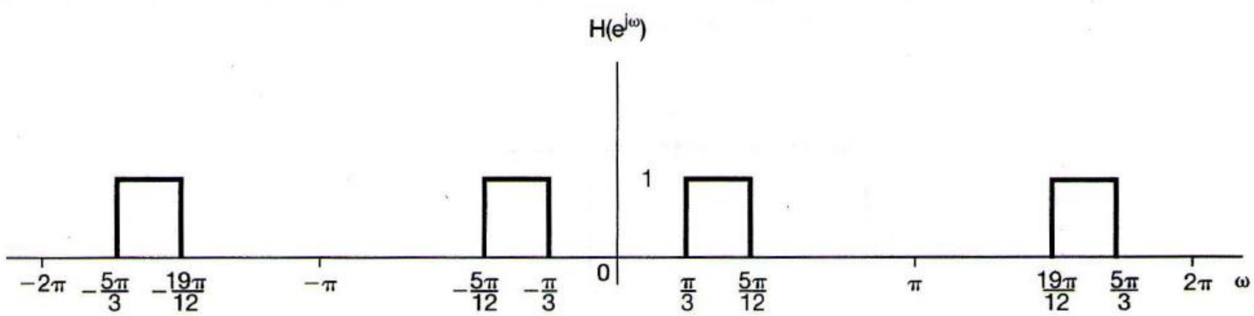
$$y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$$

Determine the values of $H(e^{jk\frac{\pi}{2}})$ for k = 0, 1, 2, and 3.

3. Band pass filter in Discrete signals

Determine the output of the filter shown in figure below for the following periodic inputs:

a) $x_1[n] = (-1)^n$
b) $x_2[n] = \sin\left(\frac{3\pi}{8}n + \frac{\pi}{4}\right)$
c) $x_3[n] = \sum_{k=-\infty}^{\infty}\left(\frac{1}{2}\right)^{n-4k} u[n-4k]$



4. Properties related to Fourier Series

Let x[n] be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

The Fourier series coefficients for each of the following signals can be expressed in terms of a_k in equation above. Derive the expressions for the below signals.

a) $x[n-n_0]$
b) $x[n]-x[n-1]$
c) $x[n]-x\left[n-\frac{N}{2}\right]$ (assume N to be even)
d) $x[n]+x\left[n+\frac{N}{2}\right]$ (assume N to be even, also note that signal is periodic with period N/2)
e) $x^*[-n]$
f) $(-1)^n x[n]$ (assume N is even)
g) $(-1)^n x[n]$ (assume N is odd, also note that signal is periodic with period 2N)
h) $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

5. Fourier properties of Wave symmetric signals

Let x[n] be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

a) Suppose that N is even and that x[n] in eq. above satisfies

$$x[n] = -x\left[n+\frac{N}{2}\right] \text{ for all n,}$$

Show that a_k= 0 for all even integers k.

b) Suppose that N is divisible by 4. Show that if

$$x[n] = -x\left[n+\frac{N}{4}\right] \text{ for all n,}$$

then $a_k = 0$ for every value of k that is a multiple of 4.

c) More generally, suppose that N is divisible by an integer M. Show that if

$$\sum_{r=0}^{\frac{N}{M}-1} x\left[n+r\frac{N}{M}\right] = 0 \text{ for all n,}$$

then $a_k = 0$ for every value of k that is a multiple of M.

6. Time Scaling

Let x[n] be a periodic signal with fundamental period N and Fourier series coefficients a_k. In this problem, we derive the time-scaling property.

$$x_{(m)}[n] = \begin{cases} x\left[\frac{n}{m}\right], & n = 0, \pm m, \pm 2m \\ 0, & \text{elsewhere} \end{cases}$$

a) Show that $x_{(m)}[n]$ has period of mN.

b) Show that if

$$x[n] = v[n] + w[n]$$

then

$$x_{(m)}[n] = v_{(m)}[n] + w_{(m)}[n]$$

c) Assuming that x[n] = $e^{\frac{j2\pi k_0 n}{N}}$ for some integer k_0, verify that

$$x_{(m)}[n] = \frac{1}{m} \sum_{l=0}^{m-1} e^{j2\pi(k_0+lN)n/(mN)}$$

That is, one complex exponential in x[n] becomes a linear combination of m complex exponentials in $x_{(m)}[n]$

d) Using the results of parts (a), (b), and (c), show that if x[n] has the Fourier coefficients a_k, then $x_{(m)}[n]$ must have the Fourier coefficients $\frac{1}{m} a_k$.

7. Conjugate and Multiplication properties in Fourier Coefficients

Let x[n] be a periodic signal with period N and Fourier coefficients a_k

a) Express the Fourier coefficients b_k of $|x[n]|^2$ in terms of a_k

b) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real?

8. Frequency response in Discrete time LTI systems

Consider the following pairs of signals x[n] and y[n]. For each pair, determine whether there is a discrete-time LTI system for which y[n] is the output when the corresponding x[n] is the input. If such a system exists, determine whether the system is unique (i.e., whether there is more than one LTI system with the given input output pair). Also, determine the frequency response of an LTI system with the desired behavior. If no such LTI system exists for a given x[n], y[n] pair, explain why.

a) $x[n] = \left(\frac{1}{2}\right)^n$, $y[n] = \left(\frac{1}{4}\right)^n$

b) $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $y[n] = \left(\frac{1}{4}\right)^n u[n]$

c) $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $y[n] = 4^n u[-n]$

d) $x[n] = e^{jpi/8}$, $y[n] = 2e^{jpi/8}$

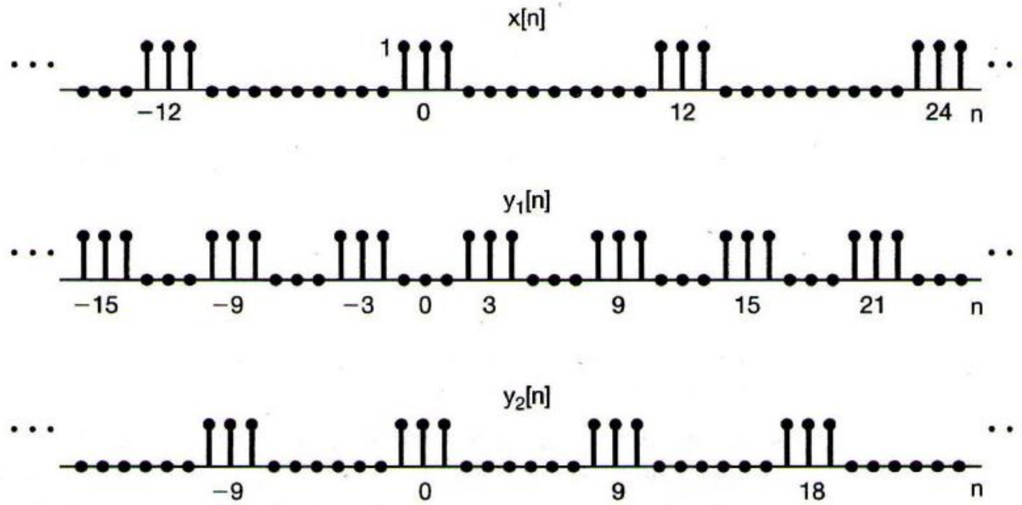
e) $x[n] = e^{jpi/8} u[n]$, $y[n] = 2e^{jpi/8} u[n]$

f) $x[n] = j^n$, $y[n] = 2\, j^n (1 - j)$

g) $x[n] = \cos(pi\, n/3)$, $y[n] = \cos(pi\, n/3) + \sqrt{3} \sin\,(pi\, n/3)$

h) $x[n] = y_1[n]$, as in figure below

i) $x[n] = y_2[n]$, as in figure below


x[n]
1
-12
0
12
24
n
y1[n]
-15
-9
-3
0
3
9
15
21
n
y2[n]
-9
0
9
18
n