MTL103
Tutorial 5

Definitions

- **B strictly/strongly dominates A:** choosing B always gives a better outcome than choosing A, no matter what the other players do.
- **B weakly dominates A:** choosing B always gives at least as good an outcome as choosing A, no matter what the other players do.
- **Strongly dominant strategy equilibrium:** A strategy profile s^D is a strongly dominant strategy equilibrium if s_i^D is a strongly dominant strategy $\forall\, i$. In other words, if every player plays their strictly dominant strategy, the resulting strategy profile is called a strongly dominant strategy equilibrium

1. The following payoff matrix corresponds to a modified version of the Prisoner's Dilemma problem called the DA's brother problem. In this problem prisoner 1 is related to the District Attorney. How is this problem different? How many Nash equilibria are there? Does player 2 really have a choice?

1	2	
	NC	C
NC	$0, -2$	$-10, 1$
C	$-1, -10$	$-5, -5$

2. Consider any arbitrary two player game of the following type (with a, b, c, d any arbitrary real number):

	A	B
A	a, a	b, c
B	c, b	d, d

   It is known that the game has a strongly dominant strategy equilibrium. Now prove or disprove: The above strongly dominant strategy equilibrium is the only possible mixed strategy equilibrium of the game.

3. An $m \times m$ matrix is called a latin square if each row and each column is a permutation of $(1, ..., m)$. Compute pure strategy Nash equilibria, if they exist, of a two person game for which a latin square is the payoff matrix.

4. Consider the following instance of the prisoners' dilemma problem.

1	2	
	NC	C
NC	$-4, -4$	$-2, -x$
C	$-x, -2$	$-x, -x$

   Find the values of x for which:

(a) the profile (C,C) is a strongly dominant strategy equilibrium.

(b) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.

(c) the profile (C,C) is a not even a weakly dominant strategy equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

5. Find the pure strategy Nash equilibrium of the following game.

	X	Y	Z
X	$6, 6$	$8, 20$	$0, 8$
Y	$10, 0$	$5, 5$	$2, 8$
Z	$8, 0$	$20, 0$	$4, 4$

6. Find the mixed strategy Nash equilibria for the following games:

(a) *(Matching Pennies Game)*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

(b) *(Rock-Paper-Scissors Game)*

1	3		
	Rock	Paper	Scissors
Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
Scissors	$-1, 1$	$1, -1$	$0, 0$