Department of Mathematics
MTL 106 (Introduction to Probability and Stochastic Processes)
Tutorial Sheet No. 8

Answer for Selected Problems

1. $\mu = 3.939$

2. $M/M/1$ model with $\lambda = 8\ hour^{-1}, \quad \mu = 10\ hour^{-1}$
(a) $1 - \pi_0$
(b) $\frac{1}{\mu-\lambda}\ hours.$

3. Given $\frac{1}{\mu} = 10$ ms, $\pi_0 + \pi_1 = 0.8$
(a) $M/M/1$ model
(b) $E(W) = \frac{\rho}{\mu-\lambda}$.

4. $p_k = \sum_r \pi_r \pi_{k-r}, k = 0, 1, \ldots$ where $\pi_r = (1-\rho)\rho^r, r = 0, 1, \ldots$

6. $M/M/1$ model with $\lambda = 5\ hour^{-1}, \quad \mu = 6\ hour^{-1}$
(a) $\sum_{n=1}^{\infty} n\pi_n$
(b) $\sum_{n=2}^{\infty}(n-1)\pi_n$
(c) π_0
(d) $1 - (\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5)$.

7. $M/M/3$ model with $\lambda = 6\ hour^{-1}, \quad \mu = 3\ hour^{-1}$.
(a) $L_q = \sum_{n=4}^{\infty}(n-3)\pi_n$
(b) $\frac{L_q}{\lambda} + \frac{1}{\mu}$.

8. $M/M/3$ model with $\lambda = 20\ hour^{-1} \quad \mu = 20\ hour^{-1}$.
(a) Steady state probability that no waiting time to land $= \pi_0 + \pi_1 + \pi_2$
(b) Expected no. of airplanes waiting to land $L_q = \sum_{n=4}^{\infty}(n-3)\pi_n$
(c) $E(W) = \frac{L_q}{\lambda}$.

9. $M/M/n/n$ model
(a) $\pi_n = (\frac{\lambda}{\mu})^n \frac{1}{n!}\pi_0 = \frac{(\frac{\lambda}{\mu})^n \frac{1}{n!}}{1 + \frac{\lambda}{\mu} + (\frac{\lambda}{\mu})^2 \frac{1}{2!} + \ldots + (\frac{\lambda}{\mu})^n \frac{1}{n!}}$
(b) Minimum n such that $\pi_n \leq 0.02$, $n = 26$.

10. $M/M/2/3$ model with $\lambda = 3\ hour^{-1}, \qquad \mu = 2\ hour^{-1}$.
(a)$1 - \pi_3$
(b)$1 - \pi_3$ for the M/M/1/3 model with $\lambda = 3\ hour^{-1}, \qquad \mu = 4\ hour^{-1}$.

11. $M/M/3/7$ model with $\lambda = 1\ min^{-1}, \qquad \mu = \frac{1}{6}\ min^{-1}$.
(a) $L = L_q + \dfrac{\lambda}{\mu}(1 - \pi_7)$
(b) $\frac{L}{\lambda_{eff}} = \frac{L}{\lambda(1-\pi_7)}$
(c) $60\lambda\pi_7$

12. $$\Lambda = \begin{pmatrix} -\lambda & \lambda & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda+\mu) & \lambda & \cdots & 0 & 0 \\ 0 & 2\mu & -(\lambda+2\mu) & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & -(\lambda+(N-1)\mu) & \lambda \\ 0 & 0 & 0 & \cdots & N\mu & -N\mu \end{pmatrix}$$

$$\pi_k = \frac{\rho^k/k!}{1+\displaystyle\sum_{i=1}^{N}\frac{\rho^i}{i!}}, k=0,1,2,\cdots,N \text{ where } \rho=\lambda/\mu.$$

13. X_t= number of vehicles at time t.
$S=\{0,1,2,\ldots\}$
$\lambda \ = \ 8\ min^{-1}, \qquad \mu \ = \ \frac{1}{1}=1\ min^{-1}$.
(a) The state transition diagram is equivalent to following generator matrix.

$$\Lambda = \begin{pmatrix} -\lambda & \lambda & 0 & \cdots & 0 & 0 & 0 & \cdot \\ \mu & -(\lambda+\mu) & \lambda & \cdots & 0 & 0 & 0 & \cdot \\ 0 & 2\mu & -(\lambda+2\mu) & \cdots & 0 & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & -(\lambda+9\mu) & \lambda & 0 & \cdot \\ 0 & 0 & 0 & \cdots & 10\mu & (\lambda-10\mu) & \lambda & \cdot \\ 0 & 0 & 0 & \cdots & 0 & 10\mu & (\lambda-10\mu) & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

(b) $\pi_n \ = \ (\frac{\lambda}{\mu})^n \frac{1}{n!}\pi_0 \qquad \text{if} \ \ n \le 10$
$\pi_n \ = \ (\frac{\lambda}{\mu})^n \frac{1}{n!(10)^{n-10}}\pi_0 \qquad \text{if} \ \ n > 10$
with $\sum_i \pi_i = 1$
(c) $\lambda < 12\mu = 12min^{-1}$

14. X_t= number of busy terminals at time t.
$S=\{0,1,2,3,4\}$
$\lambda \ = \ 25\ hour^{-1}, \qquad \mu \ = \ 24\ hour^{-1}$.
(a) $\pi_n \ = \ (\frac{\lambda}{\mu})^n \frac{1}{n!}\pi_0 \qquad n=1,2,3,4$
with $\sum_i \pi_i = 1$
(b) $1/\mu$
(c) n such that $\pi_n \le .05$