Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 5
Answer for selected Problems

1. (a) 1/81 (b) $exp(9)$ (c) 61/180
2. $E(X) = bE(Y) + a$
3. (a)$f(t) = 2^{-3/2}(1 + t^2/2)^{-3/2}, \quad t \in (-\infty, \infty)$ (b) $E(T) = 0, \quad Var(T) = \infty$
5. 12
6. $c = \sqrt{3/2}, \quad E(T) = 0$
7. (a) Let Y: r.v. denoting the waiting time of passenger. $f_Y(y) = \begin{cases} \frac{1}{10}, & 0 < y < 5, \\ \frac{1}{20}, & 5 < y < 15, \\ 0, & \text{otherwise.} \end{cases}$

   (b) $\frac{25}{4}$ minutes.
8. $\frac{5}{2}\log(3/2)$
9. 31
10. (a) $\frac{\sigma^2}{n}$ (b) σ^2
11. $\frac{X^2}{3}$
12. $E(Y/x) = 2(1 + x), \quad x \geq 0$
13. Regression of X on Y is
    $E(X/y) = \frac{a+y}{n+a+b}, \quad y = 0, 1, \ldots, n.$
    Yes.
14. $E[X/X > y] = \frac{1}{\lambda} + y; \qquad E[X - y/X > y] = \frac{1}{\lambda}$
15. $X_2 \sim Binomial(n, p_2)$,
    $np\left(\frac{1-(1-q)^{n-1}}{1-(1-q)^n}\right); \quad p = q = 1/3$
17. $E(Y^k/x) = \frac{x^k}{k+1}, \quad E(Y^k) = \frac{1}{(k+1)^2}$
18. $X/y \sim N\left(\frac{y}{1+\sigma^2}, \frac{\sigma^2}{1+\sigma^2}\right), \quad E(X/y) = \frac{y}{1+\sigma^2}$
19. 1/2
20. $P^{(n)}(t) = P(P(...(P(t))...))$ where $P(t) = \frac{1}{4} + \frac{t}{4} + \frac{t^2}{2}$. $P_{Z_n}(t) = [P^{(n)}(t)]^{Z_n}$. $E(Z_{51}) = 1250$.
21. $M_{S_N}(t) = M_N(\log(M_X(t)))$
23. (a) $M_Y(t) = \frac{pM_X(t)}{1-(1-p)M_X(t)}$ where $M_X(t) = \frac{1}{3}\left(1 + e^t + e^{2t}\right)$ b) $E[Y] = 3$
24. (a) $f(y/x) = \begin{cases} e^{-y+x}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$

    (b)$E(Y/X = x) = (1 + x), \quad x > 0$

25. $\mu = 2\alpha \sum_{i=1}^{n} i^2, \qquad \sigma^2 = \alpha^2 \sum_{i=1}^{n} i^2$

$E(W) = e^{\mu + \frac{1}{2}\sigma^2}$

$Var(W) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$

$f(w) = \frac{1}{w\sqrt{2\pi}\sigma} e^{-\frac{(\ln w - \mu)^2}{2\sigma^2}}, \qquad w > 0$

26. $E(XY) = E(X|Y)E(Y)$

27. $P(X = x) = \frac{1}{2^{x+1}},\ x = 0, 1, 2, \ldots$

28. Yes, X and Y are independent.

29. (a) $e^{-0.4}$

32. No, use Chebyshev's inequality