PYL 102

Thursday, Sept. 26, 2024

Hall effect

The vector equation of motion for a particle of charge q and mass m in a solid, undergoing an external force $\vec{F}_e$ and a friction $\vec{f} = -k\vec{v}$, is given by:

$$m\frac{d\vec{v}}{dt} = \vec{F}_e - \frac{m}{\tau}\vec{v}$$

The friction force describes the relaxation of the particle due to its interactions (collisions) with the ions of the crystal lattice and with the other charge carriers. Here, we suppose that if the external forces go back to zero, the state returns to its equilibrium position exponentially with a relaxation time. τ

$$\frac{d\vec{v}}{\vec{v}} = -\frac{1}{\tau}dt \Rightarrow \vec{v} = \vec{v}_0 e^{-t/\tau}$$

If the external forces remain constant, the system goes to a stationary state, i.e. dv/dt = 0. Supposing that the external force is due to a homogeneous electric field $\vec{E}$, the new stationary velocity, or drift velocity v_d of the charge carrier becomes:

$$0 = q\vec{E} - \frac{m}{\tau}\vec{v}_d \qquad \vec{v}_d = \frac{q\tau}{m}\vec{E} = \mu\vec{E}$$

The algebraic quantity $\mu = q\tau/m$ represents the speed per unit electric field, and is defined as the mobility of the charge carriers. For a solid containing n charge carriers per unit volume, under the action of an electric field E and a permanent regime, then the charge carriers move with an average drift velocity in the same direction as the electric field. In this case, a constant electric current appears, given by:

$$\therefore \quad \vec{j} = q n \vec{v}_d = \frac{q^2 n \tau}{m} \vec{E} = q n \mu \vec{E} = \sigma \vec{E}$$

σ is the conductivity.

Current density j in the presence of an electromagnetic field Assume a random $\vec{E}$ and $\vec{B} = B_z \hat{z}$ along the z axis. By defining the cyclotron frequency $\omega=(q/m)B_z$ and the electric conductivity defined for B = 0, then it can be shown that in a stationary regime, the average drift velocity of the particle is given by:

$$\vec{v}_d = \frac{q\tau}{m} \vec{E} + \frac{q\tau B_z}{m} (\vec{v}_d \times \hat{z})$$

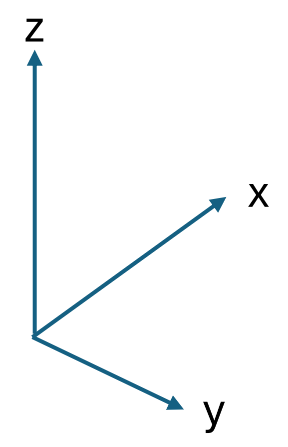
$$v_x = \frac{q\tau}{m} E_x + \frac{q\tau}{m} B_z v_y \qquad v_y = \frac{q\tau}{m} E_y - \frac{q\tau}{m} B_z v_x \qquad v_z = \frac{q\tau}{m} E_z$$

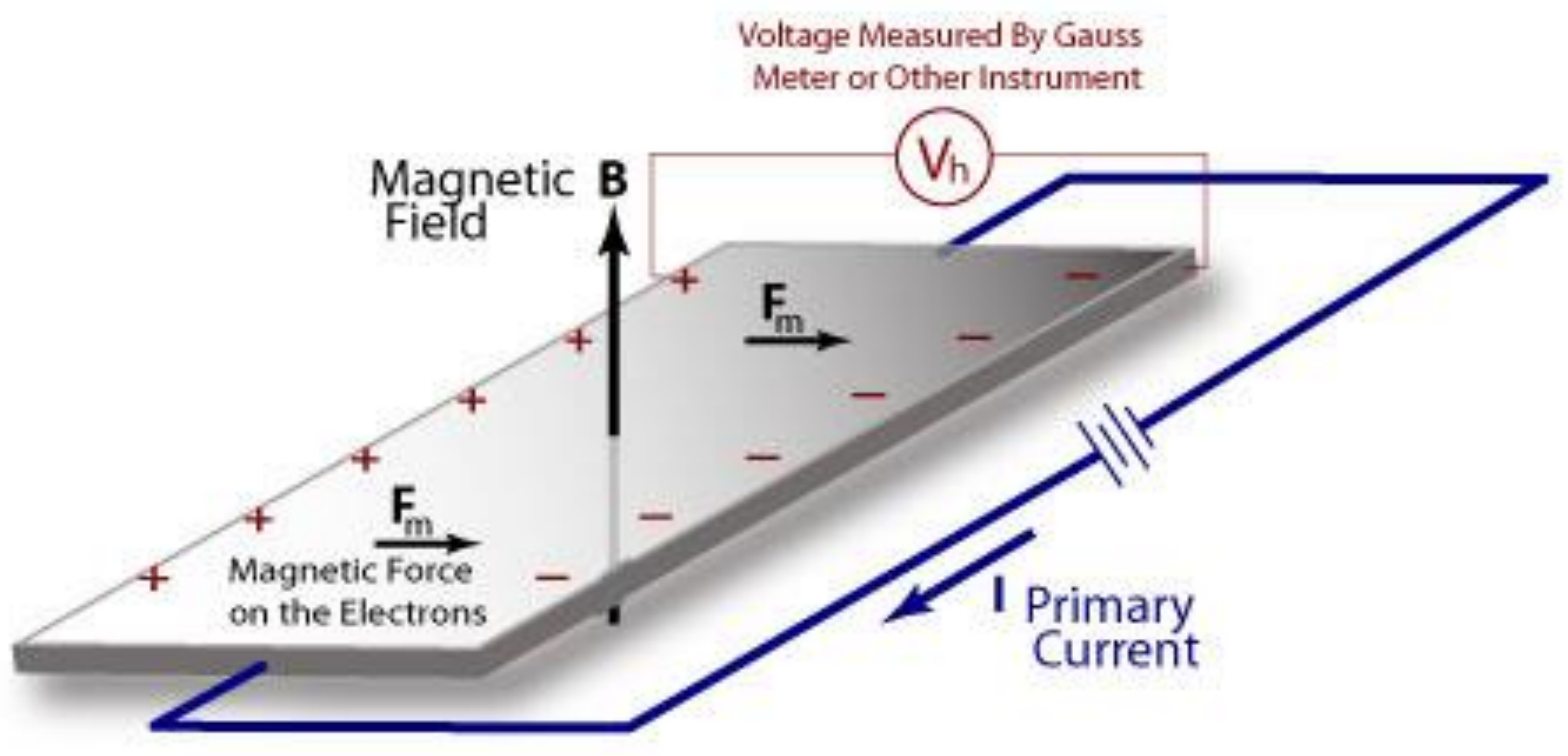
$$j_x = \frac{\sigma\,(E_x + \omega\tau E_y)}{(1+\omega^2\tau^2)} \quad , \quad j_y = \frac{\sigma\,(E_y - \omega\tau E_x)}{(1+\omega^2\tau^2)}$$

$$j_z = \sigma E_z$$

In the presence of a magnetic field, the current density j is generally not parallel to the electric field E . However, for metals, even under a large applied magnetic field B, the corresponding anisotropy is very small, in such a way that $j \sim \sigma E$, i.e. Ohm's law remains valid. The consequences of anisotropy are mostly relevant in semiconductors, and depend on the geometry of the system.

Hall effect





Mobile charges pressed to one side from Lorentz force, immobile charges unaltered

Creates internal electric potential, known as Hall voltage.

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

According to the current density equations (below) under a magnetic field, the charge carriers are deflected towards the sides of the sample. On the other side, a lack of charges carriers creates an effective charge of opposite side. This charge separation continues until the voltage generated this way, called Hall Voltage, counters the magnetic force. At the equilibrium state, there is no longer a drift velocity along the y axis. Therefore, the Hall field E_H is defined by the condition $j_y = 0$

$$j_x = \frac{\sigma (E_x + \omega\tau E_y)}{(1 + \omega^2\tau^2)} \quad , \quad j_y = \frac{\sigma (E_y - \omega\tau E_x)}{(1 + \omega^2\tau^2)} \quad j_z = \sigma E_z$$

$$E_H = E_y = \frac{j_x}{\sigma}\omega\tau = \left(\frac{1}{qn}\right) j_x B_z = R_H j_x B_z$$

$$R_H = \frac{1}{qn}$$

R_H is therefore an experimental measure of the algebraic quantity describing the mobile charge carrier density in a conductor, and the sign of the carrier. Also, if σ is known, then a measure of R_H can be used to determine the mobility $\mu = \sigma R_H$, as long as there is only one type of carrier.

Different types of charge carriers:

In semiconductors, the electric conductivity is often the result of two charge carriers of charges q1 and q2, of density n1 and n2, respectively. The total conductivity and the current can thus be written:

$$\sigma = \sigma_1 + \sigma_2 = q_1 n_1 \mu_1 + q_2 n_2 \mu_2$$

$$j = j_1 + j_2 = q_1 n_1 v_1 + q_2 n_2 v_2$$

At room temperature, the relaxation time τ is of the order 10^{-14}-10^{-15} s, so the term $(\omega\tau)^2$ are negligible.

$$j_x = \sigma E_x + (\sigma_1 \mu_1 + \sigma_2 \mu_2) B_z E_y$$

$$j_y = \sigma E_y - (\sigma_1 \mu_1 + \sigma_2 \mu_2) B_z E_x$$

and that the Hall condition $j_y = 0$ implies

$$E_y = \frac{(\sigma_1 \mu_1 + \sigma_2 \mu_2)}{(\sigma_1 + \sigma_2)} E_x$$

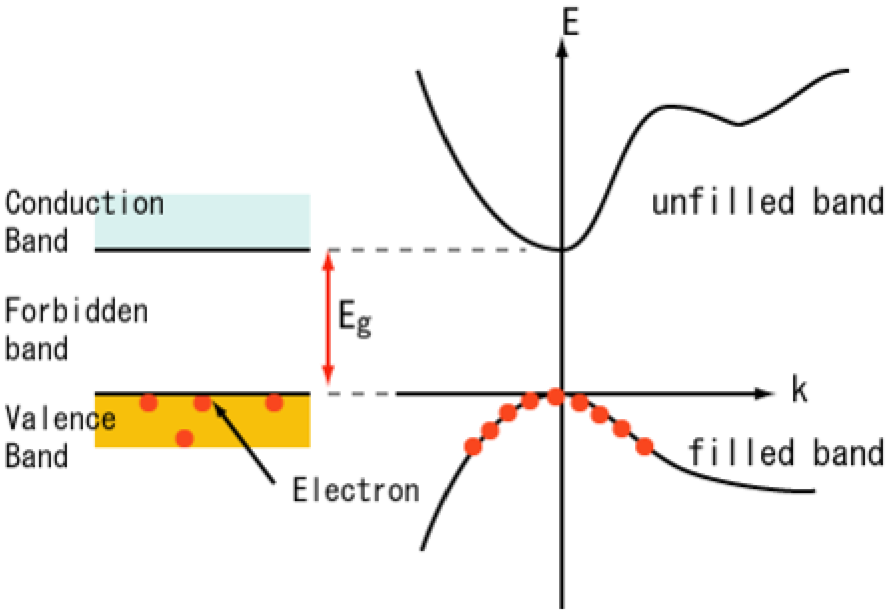
two different charge carriers; electrons and holes with $q1 = -q2$ and n1= n and n2 = p

$$R_H = \frac{p * \mu_p^2 - n * \mu_n^2}{e * \left(p * \mu_p + n * \mu_n\right)^2}$$

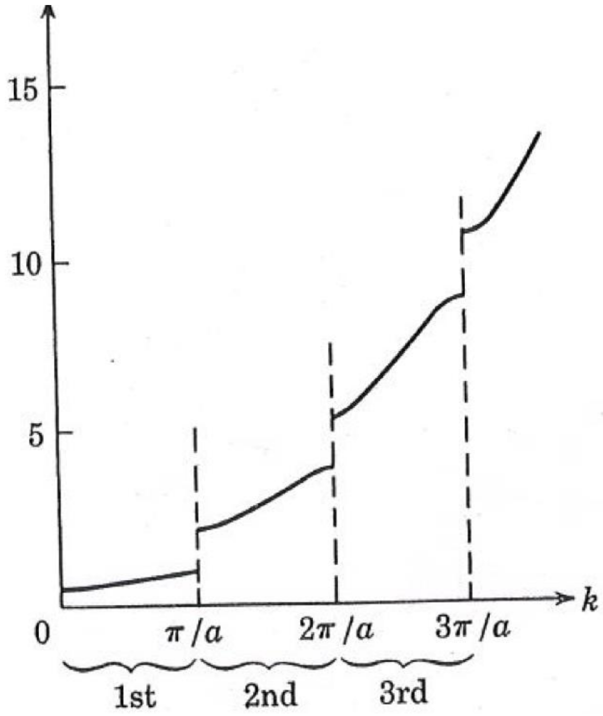
Table 2.4 Hall coefficient end Hall mobility ($\mu_H = |\sigma R_H|$) of selected metals

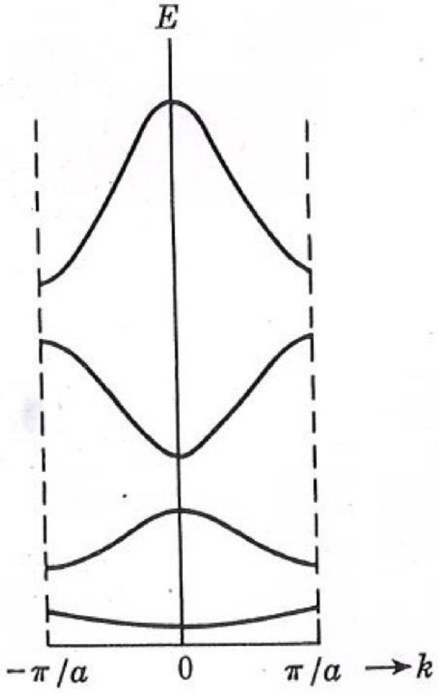
Metal	n [$m^{-3} \times 10^{28}$] ($\times 10^{28}$)	R_H (experimental) [$m^3A^{-1}s^{-1}$] ($\times 10^{-11}$)	μ_H [$m^2V^{-1}s^{-1}$] ($\times 10^{-4}$)
Ag	5.85	-9.0	57
Al	18.06	-3.5	13
Au	5.90	-7.2	31
Be	24.2	+3.4	?
Cu	8.45	-5.5	32
Ga	15.3	-6.3	3.6
In	11.49	-2.4	2.9
Mg	8.60	-9.4	22
Na	2.56	-25	53

In semiconductors, the value and sign of R_H is strongly dependent on the dopant density ("impurities"), R_H can even be zero.



$m^* = \frac{\hbar^2}{(d^2E/dk^2)}$





Curvature of E-k diagram determines effective mass!

Electrons near the top of the valence band behave as if they have negative mass.
The dispersion relation near the top of the valence band is $E = \hbar^2 k^2/(2m^*)$ with *negative* effective mass. So, electrons near the top of the valence band behave like they have negative mass.

Force = mass × acceleration, a negative-effective-mass electron near the top of the valence band would move the opposite direction as a positive-effective-mass electron near the bottom of the conduction band, in response to a given electric or magnetic force.

Positively-charged holes as a shortcut for calculating the total current of an almost-full band. Hole is a positive-charge, positive-mass quasiparticle.

Hole carries a positive charge, and responds to electric and magnetic fields as if it had a positive charge and positive mass.
Because a particle with positive charge and positive mass respond to electric and magnetic fields in the same way as a particle with a negative charge and negative mass.

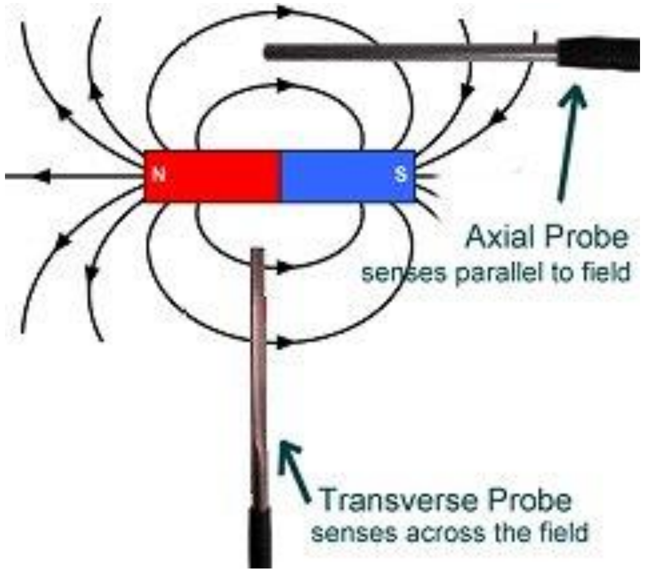
Example: silver ribbon whose cross section is 1.0 cm by 0.20 cm. The ribbon carries a current of 100 A from left to right, and it lies in a uniform magnetic field of magnitude 1.5 T. Using a electron density density value of $n=5.9\times10^{28}$ per cubic meter,
find the Hall potential between the edges of the ribbon.

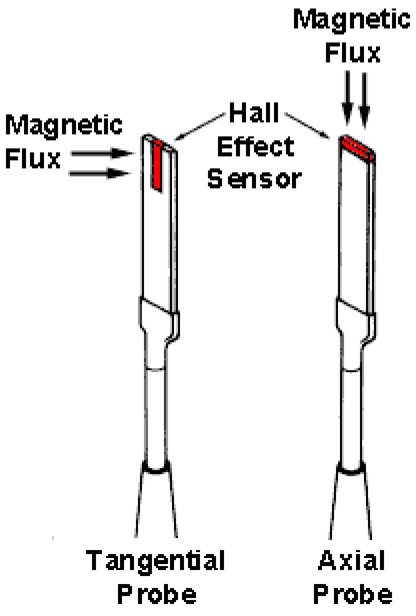
$$V_H = \frac{IBl}{neA} = 7.9\times10^{-6}\ V$$

Applications

- Measurement can tell about charge carrier mobility, concentration
- Conversely, knowing the above allows for sensitive measurement of an external B-field
- Resistant to outside contaminants unlike optical, electromechanical testing

Hall Probes





Hall Effect sensors capable of switching very fast, does not distort like capacitative or inductive sensors (contactless sensing)