Effect of Earth on Capacitance

- In discussion so far effect of earth has been neglected.
- It has been assumed that the conductors are situated in free space.
- Actually, the conductors Run parallel to the ground.
- Earth is assumed to behave like an infinite, perfectly conducting plane.
- Its presence therefore modifies the electric field of the line.
- This causes change in capacitance.

- Consider, a circuit comprising of a single overhead conductor with return path through the earth.
- If the conductor carries a charge of +q coulomb/m, then earth also has the same charge with opposite sign.
- Then, a potential difference exists between the conductor and earth.
- Electrical fluxes from charges on the conductor enter the equipotential surface of earth perpendicularly.
- As the earth's surface is assumed to be a perfect conductor.

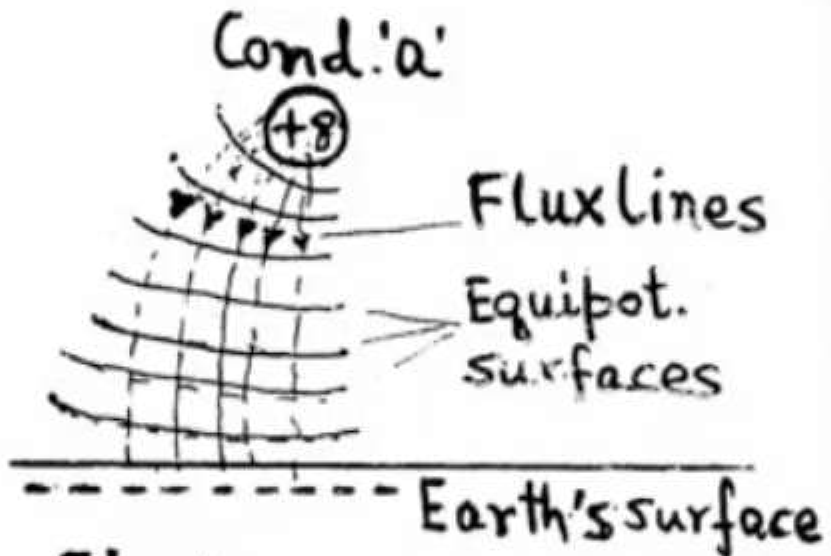
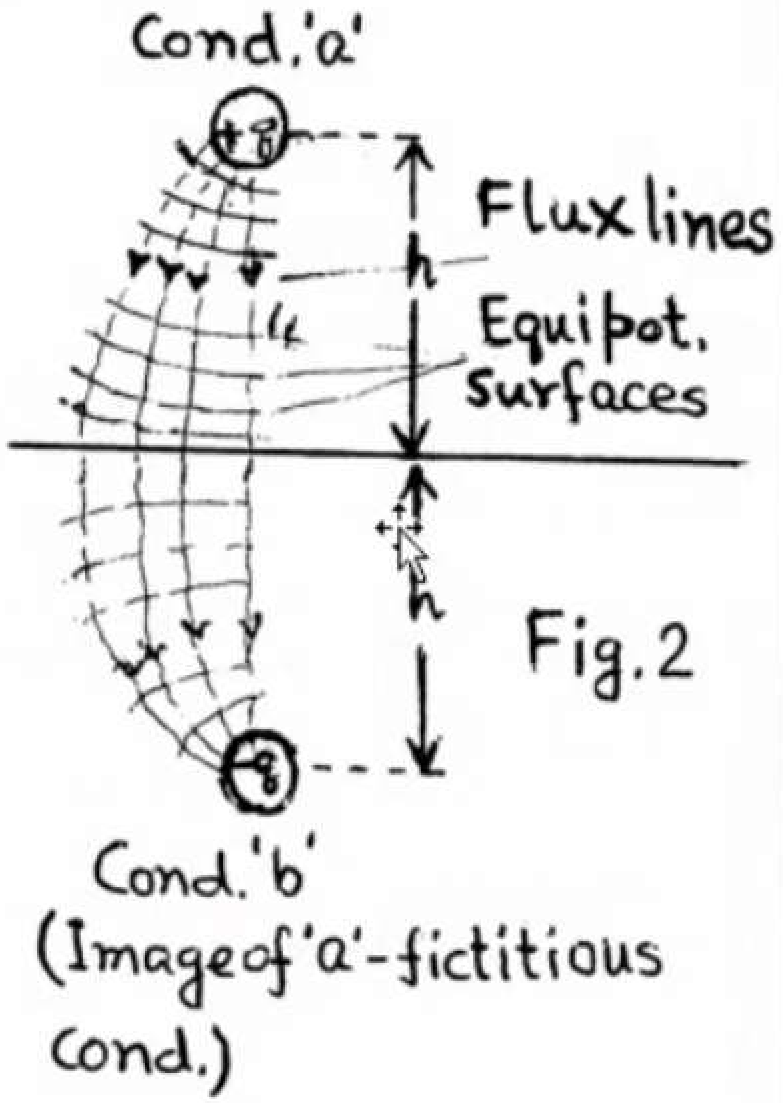


Fig.1

- Assume, a fictitious conductor of the same size and shape as the overhead conductor, lying directly below the original conductor at a distance 2h as shown.
- If earth is removed and a charge of -q coulomb/m is given to the fictitious conductor, then,
- The plane midway between the two conductors is an equipotential surface and occupies the same position as the equipotential surface of the earth.



- Electric flux between the conductor and this equipotential surface is same as that existed between the conductor and the earth.
- So, for capacitance calculation, fig.1 is replaced by fig.2.
- Such a conductor is called the image conductor.
- This process may be extended for more than one conductor.
- Here, are potential difference between conductors a and b (image of a) is,

$$V_{ab} = 2V_{an} = \frac{1}{2\pi k}\left[q_a \ln\frac{D_{ab}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ba}}\right]$$

$$= \frac{q}{2\pi k}\left[\ln\frac{2h}{r} - \ln\frac{r}{2h}\right] = \frac{2q}{2\pi k}\cdot\ln\frac{2h}{r} \quad \ldots\ldots(1)$$

Or, $V_{an} = \frac{q}{2\pi K} \ln \frac{2h}{r}$ Volts ..... (2)

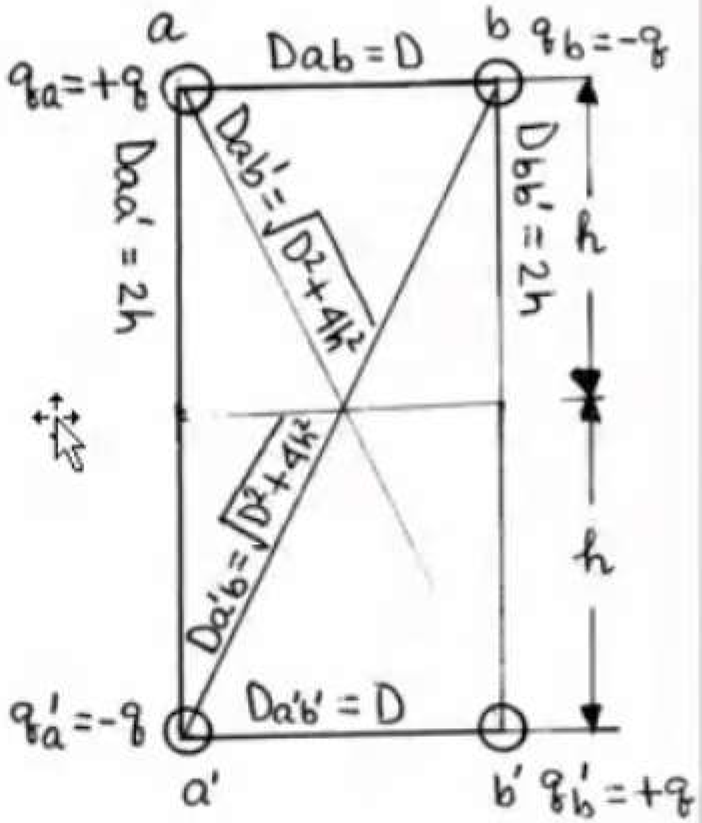
So, Capacitance to ground $= C_{an} = \frac{q}{V_{an}} = \frac{2\pi K}{\ln \frac{2h}{r}}$ F/m ..... (3)

Effect of Earth on a Single Phase Line

Here, $q_a = +q$ coul./m; $q_b = -q$ Coul./m;

$q_a' = -q_a = -q$ coul./m;

$q_b' = -q_b = +q$ Coul/m.



Now,

$$V_{ab} = \frac{1}{2\pi k}\left[q_a \ln\frac{D_{ab}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ba}} + q_a' \ln\frac{D_{a'b}}{D_{a'a}} + q_b' \ln\frac{D_{b'b}}{D_{b'a}}\right]$$

$$= \frac{1}{2\pi k}\left[q \ln\frac{D}{r} - q \ln\frac{r}{D} - q \ln\frac{\sqrt{D^2+4h^2}}{2h} + q \ln\frac{2h}{\sqrt{D^2+4h^2}}\right]$$

$$= \frac{2q}{2\pi k}\left[\ln\frac{D}{r} + \ln\frac{2h}{\sqrt{D^2+4h^2}}\right]$$

$$= \frac{2q}{2\pi k}\left[\ln\frac{D \cdot 2h}{r \cdot \sqrt{D^2+4h^2}}\right]$$

Therefore, line to line capacitance $C_{ab} = \frac{q}{V_{ab}}$

$$= \frac{q}{\frac{2q}{2\pi K}\left[\ln \frac{D.2h}{r.\sqrt{D^2+4h^2}}\right]}$$

$$= \frac{\pi K}{\ln\left(\frac{D}{r}\cdot\frac{1}{\sqrt{D^2/4h^2+1}}\right)} \text{ F/m} \;.....(1)$$

- To neglect effect of earth put $h \to \infty$. Then, $\sqrt{D^2/4h^2+1} \to 1$.

So, $C_{ab} = \frac{\pi K}{\ln \frac{D}{r}}$ F/m -----(2)

$\therefore$ Line to neutral capacitance $= C_n = \frac{q}{\frac{1}{2}V_{ab}}$

$$= \frac{2\pi K}{\ln\left(\frac{D}{r}\cdot\frac{1}{\sqrt{D^2/4h^2+1}}\right)} \text{ F/m} \;.....(3)$$

- Due to presence of $\frac{1}{\sqrt{D^2/4h^2+1}}$ the total value of the denominator reduces.
- Hence, the capacitance value increases with the presence of earth.

Performance of Transmission Lines

Preamble

- Any transmission line has four distributed parameters.
- These parameters together with load current & p.f. determine the electrical 'performance' of the line.
- The term 'performance' basically includes the calculation of sending end voltage, sending end current, sending end p.f. power loss in line, efficiency of transmission, regulation etc.
- Usually the values of receiving end voltage, current & p.f. are known.
- Prior performance calculations are useful in system planning.
- The predominance of one or more of the parameters of a line is governed by its length & conductor configuration.

Preamble – cont.

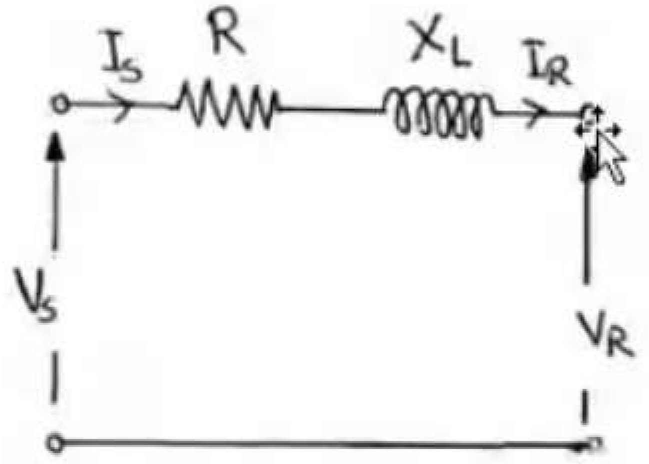
- For overhead lines up to 80 km, 'C' is negligibly small.
- All low voltage overhead lines having length up to 80 km are generally categorized as short lines.
- The lines ranging in length from 80 km to 240 km are termed as medium or moderately long lines.
- For such lines, 'C' is considered to be lumped at one or more points of the line. The leakage conductance 'G' is neglected.
- Long line refers to a line having its length more than 240 km.
- The long line treatment takes all four parameters into account in completely distributed way.

Preamble – cont.

- Classifications on the basis of length is not certainly a perfect criterion to distinguish between short, medium & long lines.
- This classification has been done for desired accuracy.
- Methods used for short & medium lines are approximate.
- Method adopted for long line is rigorous.
- However, the approximate methods are simple without much appreciable error.

Short Transmission Line

- Effects of C & G are neglected.
- Effects of R & L are considered.
- Current (I_S) entering the line is equal to current (I_R) leaving the line.



- Since same current flows through all the sections, R & L are treated as lumped.
- Here,

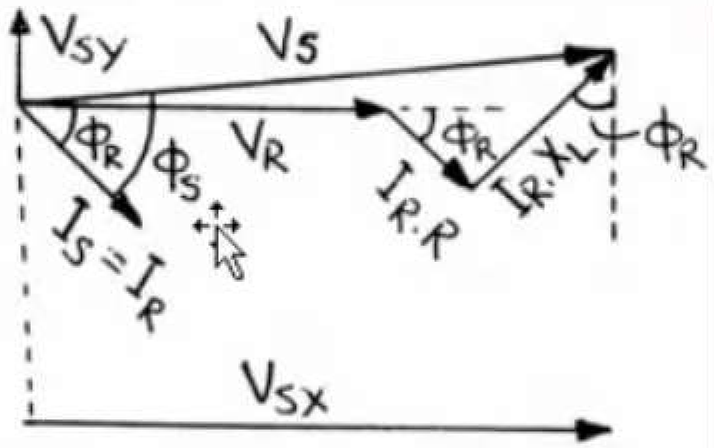
V_S = phase voltage at sending end,
V_R = phase voltage at receiving end,
I_S = phase current at sending end &
I_R = phase current at receiving end.

Cos φ_S = sending end p.f,

Cos φ_R = receiving end p.f,

R = total resistance per phase &

X_L = total inductive reactance per phase.



- Here, I_S & I_R are equal in magnitude but not in phase.

From the equivalent circuit,

$$|I_S| = |I_R| \qquad \text{..........(1)}$$

$$V_S = V_R + (R + j.X_L).I_R$$

$$= V_R + Z . I_R \qquad \text{..........(2)}$$

where, $Z = (R + j.X_L)$

- Hence, if the receiving end conditions are known, sending end voltage may be calculated.

A more approximate method involving scalar quantities is as follows:

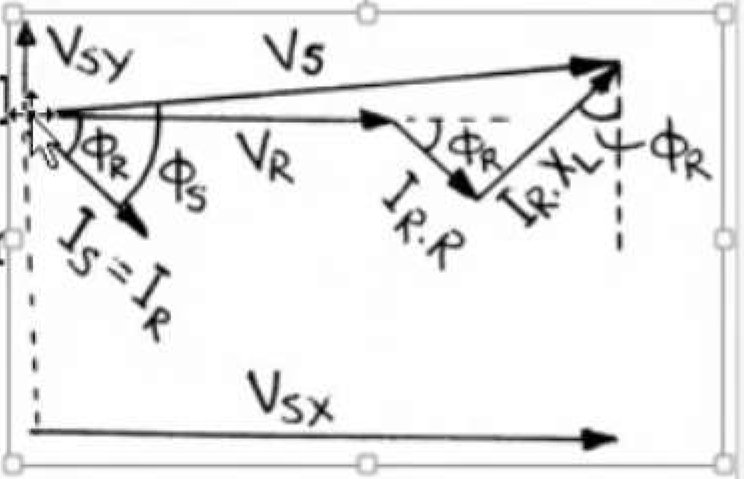
$$V_{SX} = V_R + (I_R .R). \cos \varphi_R + (I_R. X_L).\sin \varphi_R \qquad \text{..........(3)}$$

$$V_{SY} = (I_R . X_L). \cos \varphi_R - (I_R. R).\sin \varphi_R \qquad \text{..........(4)}$$

$$\& \quad V_S^{\,2} = V_{SX}^{\,2} + V_{SY}^{\,2} \qquad \text{..........(5)}$$

However, $(I_R .R)$ & $(I_R. X_L)$ are very much l

Also, the small V_{SY} is in quadrature with m



Hence,

$$V_S \approx V_{SX} = V_R + (I_R .R). \cos \varphi_R + (I_R. X_L).\sin \varphi_R \qquad \text{..........(6)}$$

Voltage regulation of the line is the rise in voltage when full load is removed.

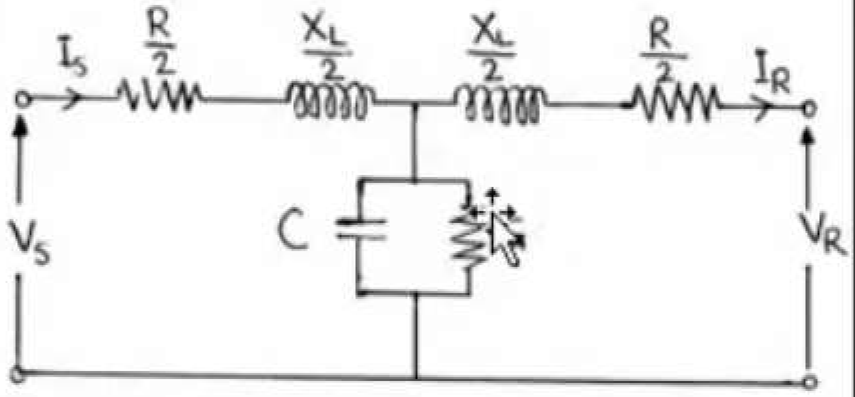
Hence,

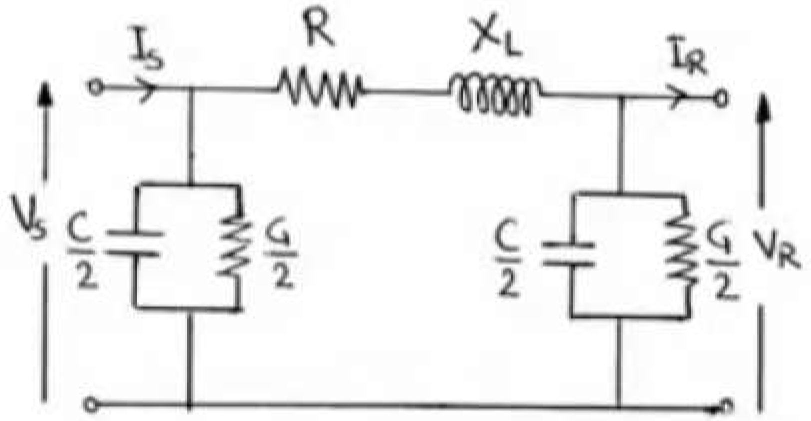
$$\boxed{\%\ \text{Voltage regulation} = \frac{(V_S - V_R)}{V_R} \cdot 100}$$

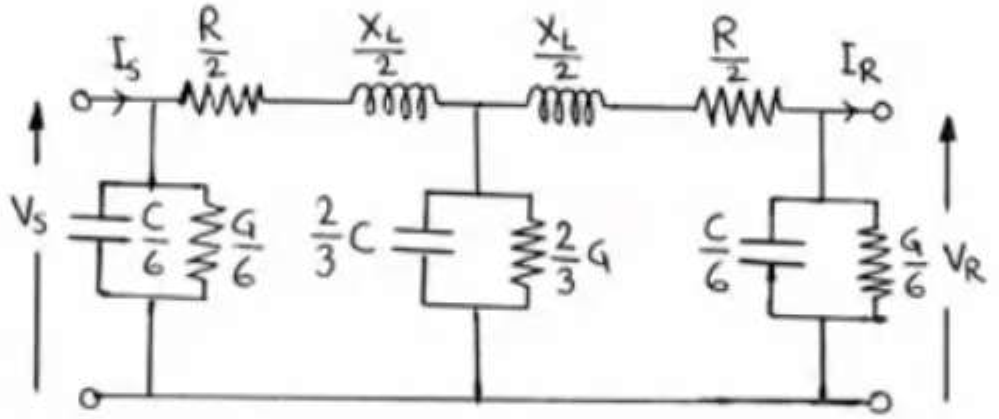
$$= \frac{(I_R\ .R).\ \text{Cos}\ \varphi_R\ + (I_R.\ X_L).\text{Sin}\ \varphi_R}{V_R} \cdot 100 \qquad \text{.........(7)}$$

Moderately Long Transmission Line

- As the length & voltage of transmission line increase – effect of capacitance –hence, charging current becomes significant.
- For medium length lines, voltages up to about 100kV, it is sufficiently accurate to consider total capacitance to be lumped at some particular points.
- Some arrangements are shown.







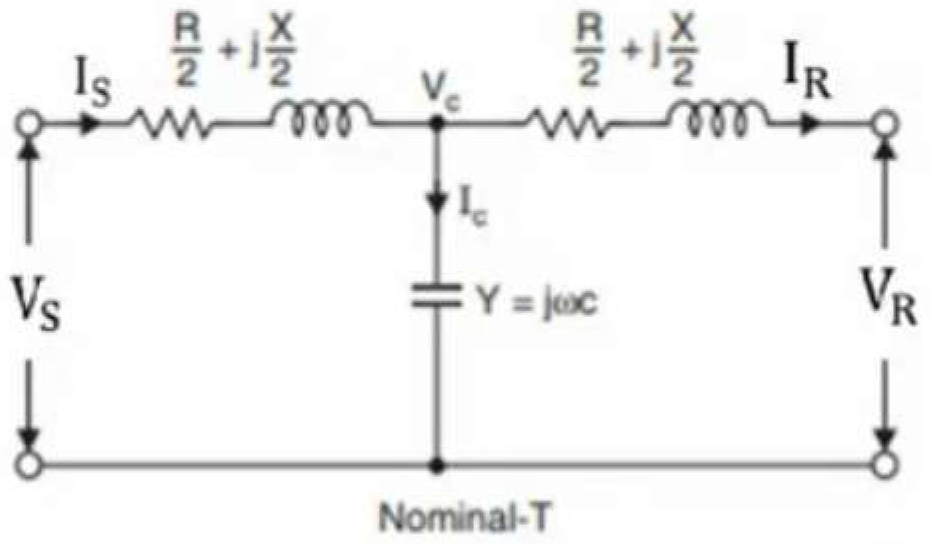
Nominal T – Representation

- Shunt conductance is neglected.

Here,

$V_C = V_R + I_R.\frac{Z}{2}$, where $Z = R + jX_L$

$I_C = V_C.Y$, where $Y = j\,\omega C$

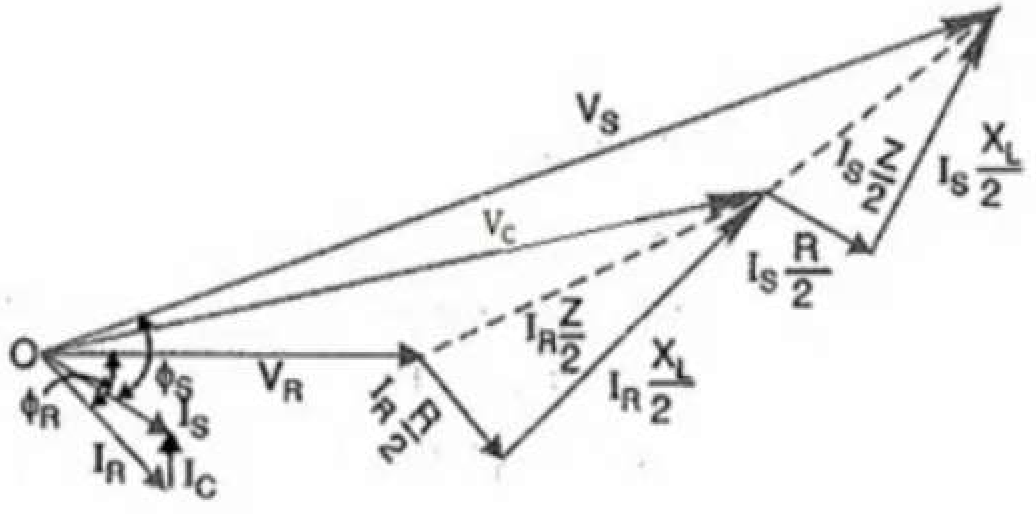


$I_S = I_R + I_C$

$= I_R + V_C.Y$

$= I_R + (V_R + I_R.\frac{Z}{2}).Y$

$= V_R.Y + I_R.(1+\frac{YZ}{2})$.....(1)

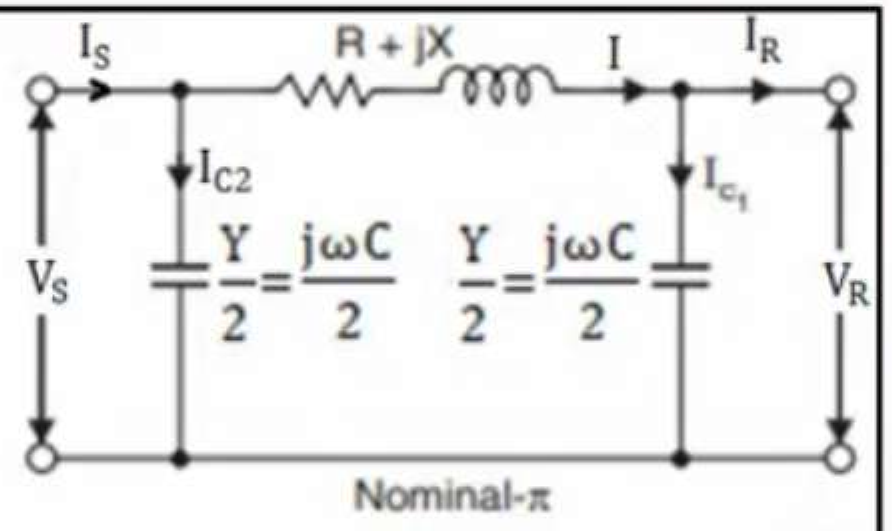


$V_S = V_C + I_S.\frac{Z}{2}$

$= V_R + I_R.\frac{Z}{2} + \{V_R.Y + I_R.(1+\frac{YZ}{2})\}.\frac{Z}{2}$

$= V_R.(1+\frac{YZ}{2}) + I_R.(1+\frac{YZ}{4}).Z$ ..... (2)

Nominal π– Representation



- Shunt conductance is neglected.

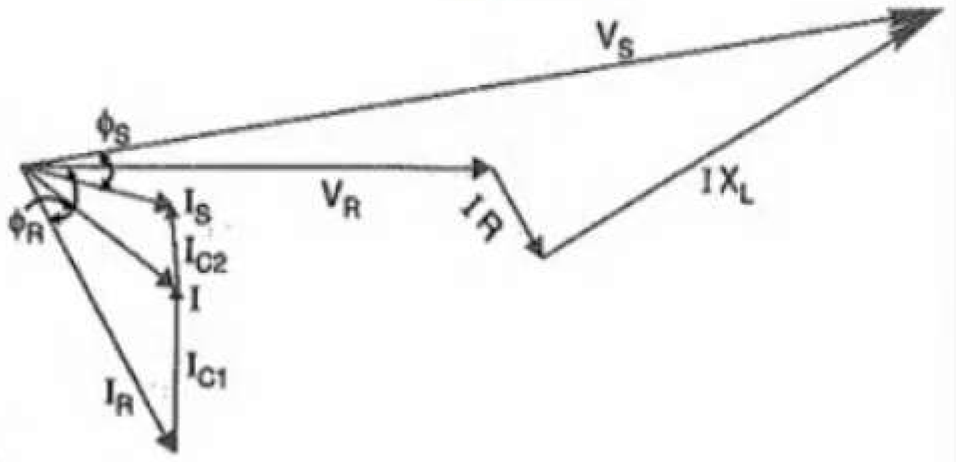
Here,

$$I_{C1} = V_R \cdot \frac{Y}{2}$$

$$I = I_{C1} + I_R = V_R \cdot \frac{Y}{2} + I_R$$

$$V_S = V_R + I.Z = V_R + \left(V_R \cdot \frac{Y}{2} + I_R\right).Z$$

$$= V_R.\left(1 + \frac{YZ}{2}\right) + I_R.Z \quad ..... (1)$$



$$I_{C2} = V_S.\frac{Y}{2} = \frac{Y}{2}.\left\{V_R.\left(1 + \frac{YZ}{2}\right) + I_R.Z\right\}$$

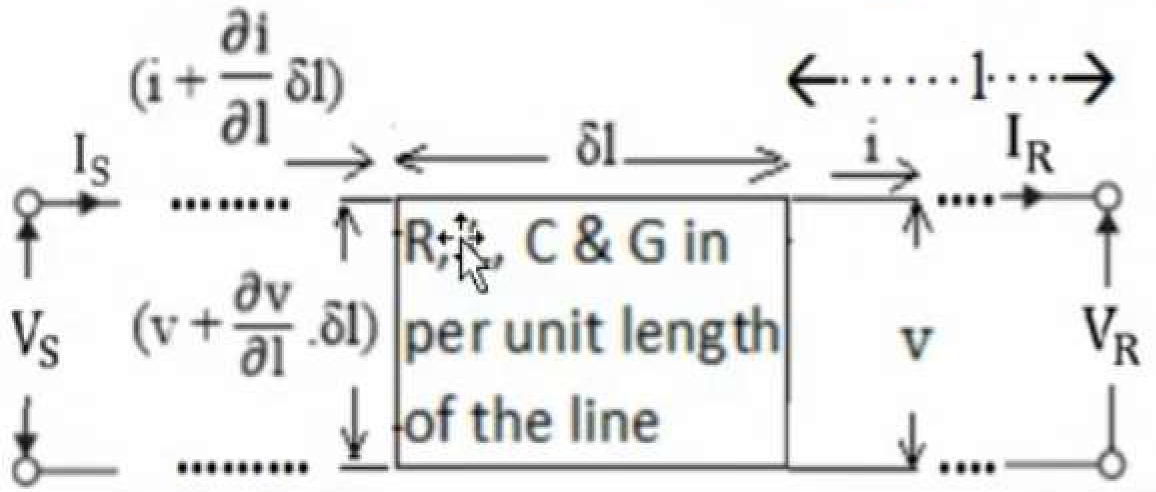
$$= V_R.Y.\left(\frac{1}{2} + \frac{YZ}{4}\right) + I_R.\frac{YZ}{2}$$

$$I_S = I + I_{C2} = \left(V_R \cdot \frac{Y}{2} + I_R\right) + \left\{V_R.Y.\left(\frac{1}{2} + \frac{YZ}{4}\right) + I_R.\frac{YZ}{2}\right\}$$

$$= V_R.Y.\left(1 + \frac{YZ}{4}\right) + I_R.\left(1 + \frac{YZ}{2}\right) \quad ..... (2)$$

Long Transmission Line

- R, L, C & G are considered to be uniformly distributed. R, L, C & G are given per unit length of the line.
- Consider, an elemental distance δl of a line at any distance 'l' from the receiving end.
- Let, current & voltage leaving the section be i & v respectively.
- So, current & voltage entering the section are $(i + \frac{\partial i}{\partial l}\delta l)$ & $(v + \frac{\partial v}{\partial l}.\delta l)$ respectively.
- Electrostatic charge in elemental length δl is $\{v.(C.\delta l)\}$.



- Current leaking through capacitance (C. δl) is $\frac{\partial}{\partial t}$(v. C. δl).
- Again, current leaking through conduction (G. δl) is {v. (G. δl)}.

So,

$$\left(i + \frac{\partial i}{\partial l}\,\delta l\right) - i = v.\,G.\,\delta l + \frac{\partial}{\partial t}(v.\,C.\,\delta l)$$

$$\text{or, } \frac{\partial i}{\partial l} = v.\,G + C.\,\frac{\partial v}{\partial t} = \left(G + C.\,\frac{\partial}{\partial t}\right).v \quad \ldots\ldots (1)$$

- Again, voltage drop in resistance (R. δl) is {i. (R. δl)}.
- Similarly, voltage drop in inductance (L. δl) is (L. δl). $\frac{\partial i}{\partial t}$.

So,

$$\left(v + \frac{\partial v}{\partial l}.\delta l\right) - v = i.\,R.\,\delta l + L.\,\delta l.\,\frac{\partial i}{\partial t}$$

$$\text{or, } \frac{\partial v}{\partial l} = i.\,R + L.\,\frac{\partial i}{\partial t} = \left(R + L.\,\frac{\partial}{\partial t}\right).\,i \quad \ldots\ldots (2)$$

or, $\frac{\partial^2 v}{\partial l^2} = (R + L.\frac{\partial}{\partial t}).\frac{\partial i}{\partial l} = (R + L.\frac{\partial}{\partial t}).(G + C.\frac{\partial}{\partial t}).v$ ..... (3)

- When dealing with sinusoidal functions, the operator $\frac{\partial}{\partial t}$ may be replaced by $j\omega$, where $\omega = 2\pi f$. So,

$$\frac{\partial^2 v}{\partial l^2} = (R + j\omega L.).(G + j\omega C).v = m^2.v \quad ..... (4)$$

where, $m^2 = (R + j\omega L.).(G + j\omega C)$ ..... (5)

Solution of eqn. (4) is of the form: $v = C_1.e^{ml} + C_2.e^{-ml}$ ..... (6)

where, C_1 & C_2 are constants.

From eqn. (6), $\frac{\partial v}{\partial l} = C_1.m.e^{ml} - C_2.m.e^{-ml}$

or, $(R + j\omega L.).i = C_1.m.e^{ml} - C_2.m.e^{-ml}$ {from eqn. (2)}

$$\text{Or, } i = \frac{\sqrt{(R + j\omega L).(G + j\omega C)}}{(R + j\omega L)} . [C_1 . e^{ml} - C_2 . e^{-ml}]$$

$$= \frac{1}{n} . [C_1 . e^{ml} - C_2 . e^{-ml}] \text{ ..... (7)}$$

$$\text{Where, } \sqrt{\frac{(G + j\omega C)}{(R + j\omega L)}} = \frac{1}{n} \text{ ..... (8)}$$

$$\text{So, } \quad n.i = [C_1 . e^{ml} - C_2 . e^{-ml}] \text{ ..... (9)}$$

- Eqns. (6) & (9) give values of voltage & current at any distance 'l' from receiving end of the line, if values of C_1 & C_2 are known.
- At receiving end l=0, $v = V_R$ & $i = I_R$. Hence, from (6) & (9),

$$V_R = C_1 + C_2 \text{ ..... (10)}$$

$$\& \; n.I_R = C_1 - C_2 \text{ ..... (11)}$$

So, $C_1 = \dfrac{V_R + n.I_R}{2}$ ..... (12)

& $C_2 = \dfrac{V_R - n.I_R}{2}$ ..... (13)

So, at any distance 'l' from receiving end, from eqn. (6),

$$v = \frac{V_R + n.I_R}{2}.\, e^{ml} + \frac{V_R - n.I_R}{2}.\, e^{-ml}$$

$$= V_R.\, [\frac{e^{ml} + e^{-ml}}{2}] + n.I_R.\, [\frac{e^{ml} - e^{-ml}}{2}]$$

$= V_R.$ Cosh ml $+ n.I_R.$ Sinh ml ..... (14)

Also, from eqn. (9),

$$n.\, i = \frac{V_R + n.I_R}{2}.\, e^{ml} - \frac{V_R - n.I_R}{2}.\, e^{-ml}$$

$$= V_R.\, [\frac{e^{ml} - e^{-ml}}{2}] + n.I_R.\, [\frac{e^{ml} + e^{-ml}}{2}]$$

$= V_R.$ Sinh ml $+ n.I_R.$ Cosh ml ..... (15)

or, $i = \frac{V_R}{n}.\ \text{Sinh ml} + I_R.\ \text{Cosh ml}$ ..... (16)

If 'l' be the distance from the receiving end to the sending end, then $v = V_s$ & $i = I_s$, the sending end values. Then,

$$\boxed{V_s = V_R.\ \text{Cosh ml} + n.I_R.\ \text{Sinh ml}} \quad ..... (17)$$

$$\boxed{I_s = \frac{V_R}{n}.\ \text{Sinh ml} + I_R.\ \text{Cosh ml}} \quad ..... (18)$$

Generalized Circuit Constants

- Eqns. of the sending end voltage & current for all categories of lines (i.e. short, medium & long) are of same type, namely,

$$\boxed{V_s = A.V_R. + B.I_R} \quad \ldots\ldots (19)$$

$$\boxed{I_s = C.V_R.+ D.I_R} \quad \ldots\ldots (20)$$

- The terms A, B, C & D in eqns. (19) & (20) are known as the generalized circuit constants.
- Their values depend on line parameters & the type of representation chosen.

Generalized Circuit Constant	Short	Medium – T representation	Medium – π representation	Long
A	1	(1+ $YZ/2$)	(1+ $YZ/2$)	Cosh ml
B	Z	(1+ $YZ/4$). Z	Z	n. Sinh ml
C	0	Y	(1+ $YZ/4$). Y	$\frac{1}{n}$. Sinh ml
D	1	(1+ $YZ/2$)	(1+ $YZ/2$)	Cosh ml

- In all the cases A = D &
- A. D – B. C = 1
- The constants are complex numbers.
- A & D are dimension less.
- B has the dimension of impedance (ohm) & C has the dimension of admittance (mho).

Determination of A, B, C & D Constants

- It may be done in two ways.

(i) From the knowledge of line parameters R, L, C & G and the table given in last slide.

(ii) By direct measurement from the actual network.

It is known that,

$$V_s = A.V_R + B.I_R$$
$$I_s = C.V_R + D.I_R$$

- If the receiving end is open circuited & a voltage of V_s' be applied at sending end to give a voltage V_R' at receiving end, then receiving end current $I_R'=0$. If I_S' be the corresponding sending end current, then

$$A = \frac{V_s'}{V_R'} \quad \& \quad C = \frac{I_S'}{V_R'}$$

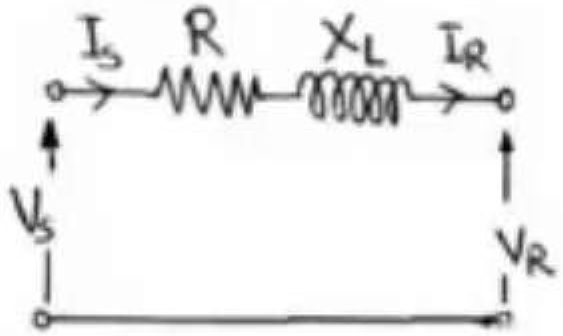
❖ If the receiving end of the line is now short circuited & a voltage of V_s'' be applied at sending end to give sending end & receiving end currents of I_s'' & I_R'' respectively, then $V_R''=0$.

Hence,

$$B = \frac{V_s''}{I_R''} \quad \& \quad D = \frac{I_s''}{I_R''}$$

Example: A single-phase 50 Hz generator supplies an inductive load of 5,000 kW at a power factor of 0.707 lagging by means of an over head transmission line 20 km long. The line resistance and inductance are 0.0195 Ω and 0.63 mH per km. The voltage at the receiving end is required to be kept constant at 10 kV. Find the sending end voltage and voltage regulation of the line.

Soln:



The line constants are

$$R = 0.0195 \times 20 = 0.39\ \Omega$$

and $L = 0.63 \times 10^{-3} \times 20 = 0.0126$ H

$$X = 314.2857 \times 0.0126 = 3.96\ \Omega$$

This is the case of a short line with $I = I_R = I_S$.

So, $|I| = \dfrac{5{,}000}{10 \times 0.707} = 707.2136$ A

Now, $|V_S| \simeq |V_R| + |I|\,(R\cos\phi_R + X\sin\phi_R)$

$= 10{,}000 + 707.2136\,(0.39 \times 0.707 + 3.96 \times 0.707)$ V

$= 12.175$ kV

Voltage regulation $= \dfrac{12.175 - 10}{10} \times 100 = 21.75\%$

Example: Using the nominal-π method, find the sending-end voltage and voltage regulation of a 250 km, three-phase, 50 Hz, transmission line delivering 25 MVA at 0.8 lagging power factor to a balanced load at 132 kV. The line conductors are spaced equilaterally 3 m apart. The conductor resistance is 0.11 ohm/km and its effective diameter is 1.6 cm. Neglect leakance.

Soln: $L = 2.\,10^{-7}.\ln\frac{D}{r'}$ H/m $= 2.\,10^{-7}.\ln\frac{300}{0.7788 \times 0.8}$

$= 1.24$ mH/km

$$C = \frac{2\pi K}{\ln\frac{D}{r}} \text{ F/m} = \frac{0.0556}{\ln\frac{300}{0.8}} = 0.0094\ \mu\text{F/km}$$

$$R = 0.11 \times 250 = 27.5\ \Omega$$

$$X = 2\pi f L = 2\pi \times 50 \times 1.24 \times 10^{-3} \times 250 = 97.4\ \Omega$$

$$Z = R + jX = 27.5 + j97.4 = 101.2\ \angle 74.2^\circ\ \Omega$$

$$Y = j\omega C l = 2\pi \times 50 \times 0.0094 \times 10^{-6} \times 250\ \angle 90^\circ$$

$$= 7.38 \times 10^{-4}\ \angle 90^\circ\ \mho$$

$$I_R = \frac{25\times 1{,}000}{\sqrt{3}\times 132}\angle -36.9^\circ = 109.3\ \angle -36.9^\circ\ \text{A}$$

$$V_R\ \text{(per phase)} = (132/(\sqrt{3})\ \angle 0^\circ = 76.2\ \angle 0^\circ\ \text{kV}$$

$$V_S = \left(1+\frac{1}{2}YZ\right)V_R + ZI_R$$

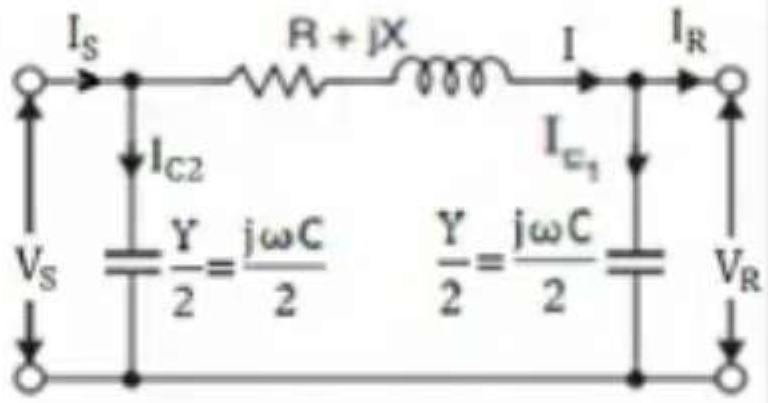
$$= \left(1+\frac{1}{2}\times 7.38\times 10^{-4}\angle 90^\circ \times 101.2\angle 74.2^\circ\right)\times 76.2$$

$$+\ 101.2\ \angle 74.2^\circ \times 109.3\times 10^{-3}\ \angle -36.9^\circ$$

$$= 76.2 + 2.85 < 164.2^\circ + 11.06 < 37.3^\circ$$

$$= 82.26 + j7.48 = 82.6 < 5.2^\circ$$

$$\therefore |V_S|\ \text{(line)} = 82.6\times\sqrt{3} = 143\ \text{kV}$$



Now, $A = 1 + \frac{1}{2} YZ = 1 + 0.0374 \angle 164.2^\circ = 0.964 + j0.01$

$$|V_{R0}| \text{ (line no load)} = \frac{143}{\left|1 + \frac{1}{2} YZ\right|} = \frac{143}{0.964} = 148.3 \text{ kV}$$

$$\therefore \text{Voltage regulation} = \frac{148.3 - 132}{132} \times 100 = 12.3\%$$

Problem: Using nominal T-method find the voltage, current & power factor at the sending end, voltage regulation & efficiency of a 100km, 3ϕ, 50Hz transmission line when delivering 50MVA at 0.8 lagging power factor to a balanced load at 132kV. The line conductors are spaced equilaterally 4.0m apart. The conductor resistance is 0.11Ω/km & effective diameter is 2.5cm. Neglect leakage.

Soln: Total resistance for 100km = 0.11 x 100 = 11 Ω = R (say).

Inductance of the line$= 2.10^{-7}.\ln\dfrac{400}{1.25 \times 0.7788}$ H/m

$=12.03664.10^{-7}$ H/m

Total inductance for 100km = L = $12.03664.10^{-7} \times (100 \times 1000)$

$= 0.1203664$ H

$$\text{Capacitance} = \frac{2\pi K_o}{\ln\frac{400}{1.25}} = \frac{2\pi}{36\pi.10^{9}.\ln\frac{400}{1.25}} = 9.63115.10^{-12}\ \text{F/m}$$

$$\text{Capacitance for 100km} = C = 9.63115.10^{-12}.(100 \times 1000)\ \text{F}$$

$$=9.63115.10^{-7}\ \text{F}$$

$$\text{Receiving end current} = I_R = \frac{50.10^6}{\sqrt{3}.132.10^3}\angle\text{-}\cos^{-1}0.8$$

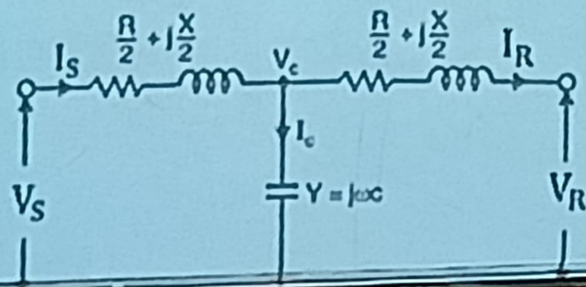
$$= 218.69328\ \angle\text{-}36.86989^o\ \text{A} = 174.95463 - j\ 131.21597\ \text{A}$$

$$\text{Here, } Z = R + j\omega L = 11 + j\ (2\pi.50) \times 0.1203664 = 11 + j\ 37.81423\ \Omega$$

$$= 39.38167\ \angle 73.78051^o\ \Omega$$

$$Y = j\ \omega C = j\ (2\pi.50) \times 9.63115.10^{-7} = 3025.71438.10^{-7}\angle 90^o\ \mho$$

$$V_R = \frac{132.10^3}{\sqrt{3}}\angle 0^o = 76210.23553\ \angle 0^o\ \text{V (Taken as reference)}$$



$$V_C = V_R + I_R \cdot \frac{Z}{2} = 76210.23553 \angle 0^o + 218.69328 \angle{-36.86989^o} \cdot \frac{39.38167 \angle 73.78051^o}{2}$$

$$= 79653.16603 + j\,2586.19964\ V = 79695.13967 \angle 1.85964^o\ V$$

$$I_C = V_C \cdot Y = 79695.13967 \angle 1.85964^o \cdot 3025.71438 \cdot 10^{-7} \angle 90^o$$

$$= 24.11347 \angle 91.85964^o = -0.78251 + j\,24.10077\ A$$

$$I_S = I_R + I_C = 174.95463 - j\,131.21597 + -0.78251 + j\,24.10077$$

$$= 174.17212 - j\,107.11520\ A = 204.47394 \angle -31.59136^o\ A$$

$$V_S = V_C + I_S \cdot \frac{Z}{2}$$

$$= 79653.16603 + j2586.19964 + 204.47394 \angle -31.59136^o \cdot \frac{39.38167 \angle 73.78051^o}{2} V$$

$$= 82636.35215 + j5290.15852 = 82805.50991 \cdot \angle 3.66292^o\ V\ (L-N)$$

Sending end Power factor = $\cos(3.66292^{o} + 31.59136^{o})$

$= 0.81660$ (lagging)

$$\text{Efficiency of transmission} = \frac{|V_R|.\ |I_R|.\cos\varphi_R}{|V_S|.\ |I_S|.\cos\varphi_S} \times 100 = 82.37688\%$$

$$\text{Now, } A = (1 + \frac{YZ}{2})$$

$$= (1 + \frac{3025.71438.10^{-7}\angle 90^{o}.\ 39.38167\angle 73.78051^{o}}{2})$$

$$= 1 + 5.95788.\ 10^{-3}\ \angle 163.78051$$

$$= 1 + 5.95788.\ 10^{-3}\ (-\ 0.96020 + j\ 0.27932)$$

$$= (0.99428 + j\ 1.66414.\ 10^{-3})$$

$$= 0.99428\ \angle 0.09590^{o}$$

$$\%\ \text{Voltage regulation} = \frac{|\frac{V_S}{A}| - |V_R|}{|V_R|} \times 100 = 9.27912\%$$

Electromagnetic Effect

- Let, currents I_a, I_b & I_c be flowing through power line conductors a, b & c respectively.
- They form a circuit, i.e.

$$I_a + I_b + I_c = 0 \quad ..... (1)$$

- Flux linkage of cond. 'd' up to infinity, due to $I_a = \Psi_{ad} = 2.10^{-7}.I_a.\ln\frac{\infty}{D_{ad}}$ wbT/m
- Similarly, flux linkage of conductor 'e' up to infinity, due to $I_a = \Psi_{ae} = 2.10^{-7}.I_a.\ln\frac{\infty}{D_{ae}}$ wbT/m
- So, mutual flux linkage of the loop formed by conductors 'd' & 'e' due to $I_a = \Psi_{de} = \Psi_{ad} - \Psi_{ae} = 2.10^{-7}.I_a.\ln\frac{D_{ae}}{D_{ad}}$ wbT/m

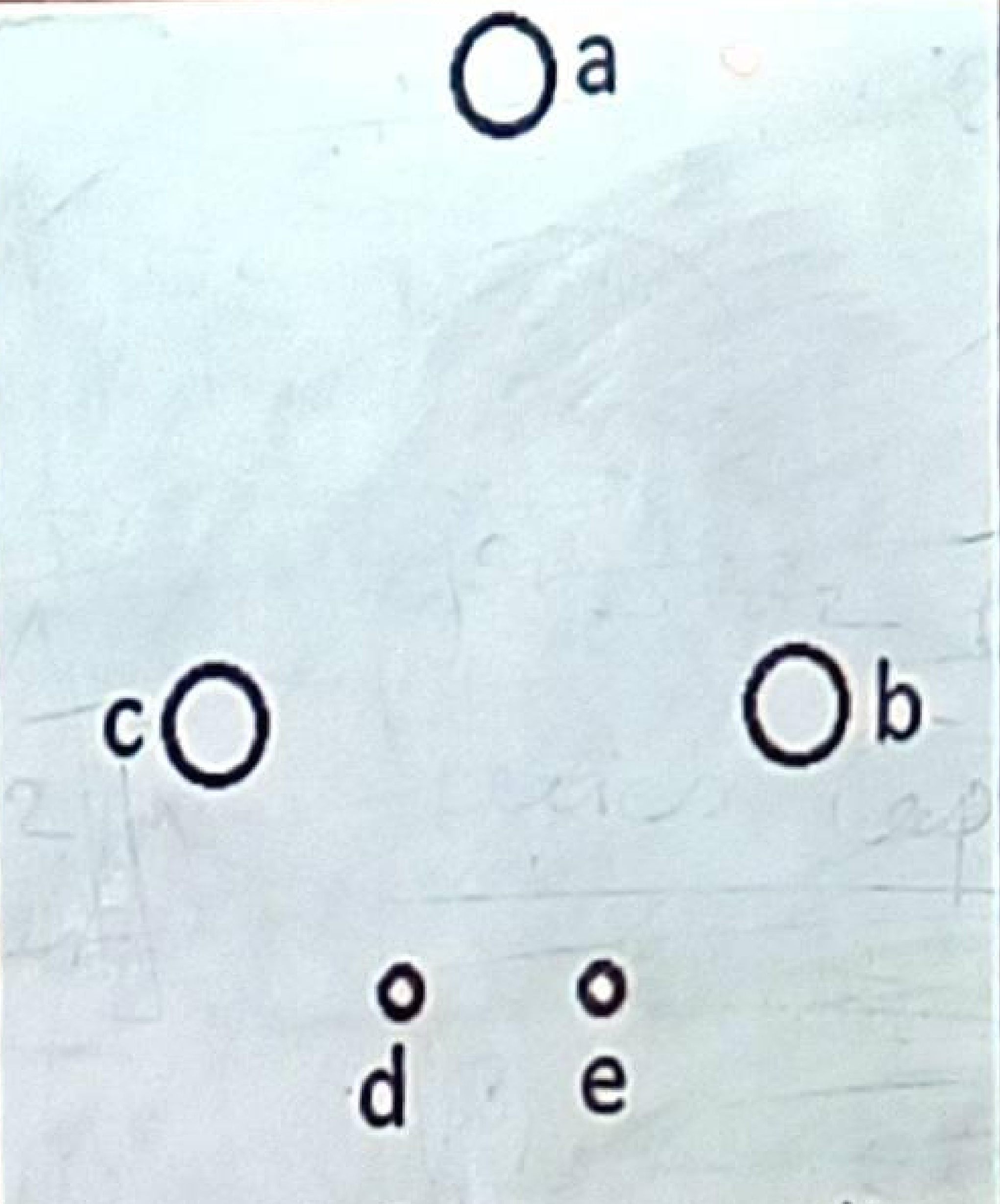


Fig. a, b & c: power line conductors;
d & e: communication line conductors.

- So, mutual inductance between cond. 'a' & the communication circuit consisting of conds. 'd' & 'e' is

$$M_a = \frac{\Psi_{de}}{I_a} = 2.10^{-7}.\ln\frac{D_{ae}}{D_{ad}} \text{ H/m ..... (2)}$$

- Likewise, M_b & M_c are mutual inductances between cond. 'b' & the loop 'de' and between cond. 'c' & the loop 'de' respectively.
- These are given by

$$M_b = 2.10^{-7}.\ln\frac{D_{be}}{D_{bd}} \text{ H/m ..... (3)}$$

$$M_c = 2.10^{-7}.\ln\frac{D_{ce}}{D_{cd}} \text{ H/m ..... (4)}$$

- Here, M_a, M_b & M_c are due to fluxes which have mutual phase displacement of 120^o.
- So, net effect of the magnetic field is

$$\vec{M} = \vec{M}_a + \vec{M}_b + \vec{M}_c \quad ..... (5)$$

- Here $\vec{M}$ is net mutual inductance → phasor sum of three inductances.
- If 'I' be the current in power conductors & 'f' is the frequency, then the induced voltage in the communication line consisting of conductors 'd' & 'e' is

$$V = 2\pi f.\ M.\ I \text{ Volts/m} \quad ..... (6)$$

- There is a partial cancellation of the induced voltages due to power line currents.
- This cancellation is almost complete for balanced 3ϕ line.
- However, greater the distance between two circuits lesser is the mutual effect (see expressions of M_a, M_b & M_c → all distances D_{ae}, D_{ad}, D_{be}, D_{bd}, D_{ce} & D_{cd} tend to become equal).

Electrostatic Effect

- Let, cond. 'a' be carrying a charge of $q_a = +q$ coulomb/m.
- So, potential difference 'a' and its image 'a'' is $V_{aa'} = 2V_{an}$

$$= \frac{1}{2\pi K_0}\left[q_a . \ln\frac{D_{aa'}}{D_{aa}} + q_{a'} . \ln\frac{D_{a'a'}}{D_{a'a}}\right]$$

$$= \frac{q}{2\pi K_0}\left[\ln\frac{2.h_a}{r} - \ln\frac{r}{2.h_a}\right] \text{ ..... (1)}$$

or, $$V_{an} = \frac{q}{2\pi K_0} . \ln\frac{2.h_a}{r} \text{ ...... (2)}$$

or, $$\boxed{\frac{q}{2\pi K_0} = \frac{V_{an}}{\ln\frac{2.h_a}{r}}} \text{ .... (3)}$$

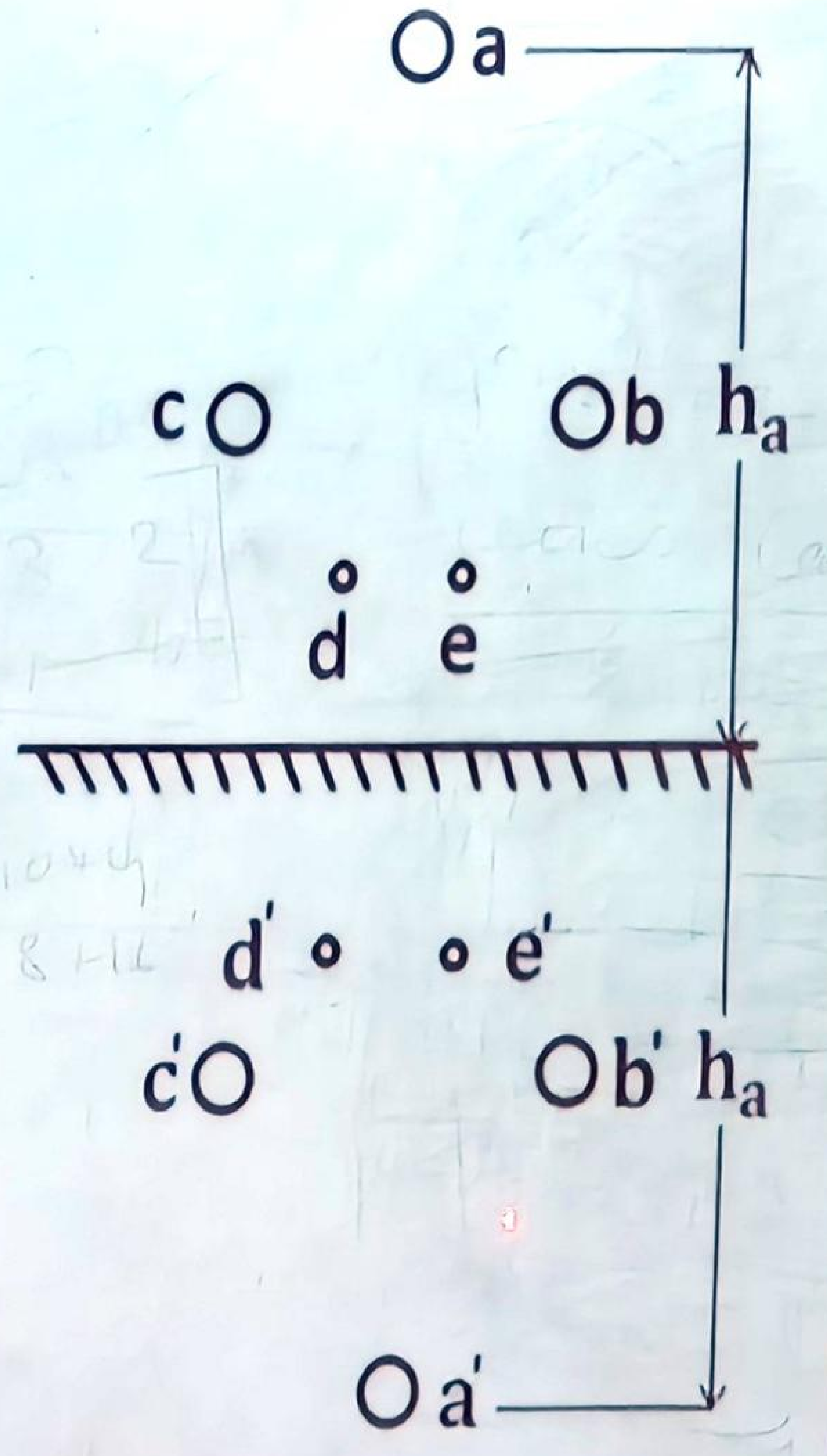


Fig. 3ϕ line, communication Line & their images

- Similarly, potential difference $V_{dd'a}$ between communication line conductors 'd' & its image 'd'' due to a charge '$q_a = +q$' on cond. 'a' & a charge of '$q_{a'} = -q$' on image 'a'' is

$$V_{dd'a} = \frac{1}{2\pi K_o}\left[q_a . \ln\frac{D_{ad'}}{D_{ad}} + q_{a'} . \ln\frac{D_{a'd'}}{D_{a'd}}\right]$$

$$= \frac{q}{2\pi K_o}\left[\ln\frac{D_{ad'}}{D_{ad}} - \ln\frac{D_{a'd'}}{D_{a'd}}\right] = \frac{q}{2\pi K_o}\left[\ln\frac{(2.h_a - D_{ad})}{D_{ad}} - \ln\frac{D_{ad}}{(2.h_a - D_{ad})}\right]$$

$$= \frac{q}{2\pi K_o} . 2. \ln\frac{(2.h_a - D_{ad})}{D_{ad}} \quad ..... (4)$$

- So, potential of cond. 'd' w.r.t. neutral due to charges q_a & $q_{a'}$ is

$$V_{dna} = \frac{q}{2\pi K_o} . \ln\frac{(2.h_a - D_{ad})}{D_{ad}} \quad .....(5)$$

Substituting the value of $\frac{q}{2\pi k_0}$ from eqn. (3),

$$\boxed{V_{dn_a} = \frac{V_{an}}{\ln\frac{2.h_a}{r}} . \ln\frac{(2.h_a - D_{ad})}{D_{ad}} = V_{an} . \frac{\ln\frac{(2.h_a - D_{ad})}{D_{ad}}}{\ln\frac{2.h_a}{r}}} \quad ..... (6)$$

- Similarly, one can obtain potential of cond. 'd' w.r.t. neutral due to charges q_b & $q_{b'}$ as

$$\boxed{V_{dn_b} = V_{bn} . \frac{\ln\frac{(2.h_b - D_{bd})}{D_{bd}}}{\ln\frac{2.h_b}{r}}} \quad ..... (7)$$

- And potential of cond. 'd' w.r.t. neutral due to charges q_c & $q_{c'}$ as

$$\boxed{V_{dn_c} = V_{cn} . \frac{\ln\frac{(2.h_c - D_{cd})}{D_{cd}}}{\ln\frac{2.h_c}{r}}} \quad ..... (8)$$

- So, the potential of cond. 'd' due to all the conds. a, b & c is

$$\boxed{\overrightarrow{V_{d_n}} = \overrightarrow{V_{d_{na}}} + \overrightarrow{V_{d_{nb}}} + \overrightarrow{V_{d_{nc}}}} \quad \text{.... (9)}$$

- Similarly, potential of cond. 'e' may be calculated.

Example: The sending end voltage per phase of a long transmission line is given by the expression

$$V_S = (0.986 \angle 0.32^o).\ V_R + (70.3 \angle 69.2^o).\ I_R.$$

Determine the capacity of a phase modifier to be installed at the receiving end so that when a load of 50MVA is delivered at 132kV and power factor 0.707 lagging, the sending end voltage can also be 132kV.

Soln: It is known that

$$V_S = A.\ V_R. + B.I_R$$

By the problem

$$A = 0.986 \angle 0.32^o$$
$$= (0.986 + j\ 0.0055)\ \&$$
$$B = 70.3 \angle 69.2^o$$
$$= (24.96 + j\ 65.72)$$

Load current magnitude $|I_L| = \dfrac{50,000}{\sqrt{3}\,.132} = 218.6933$ A

Taking receiving end voltage as reference phasor,

$$V_R = \frac{132}{\sqrt{3}} = 76.2102 \angle 0^o \text{ kV/phase} = (76.2102 + j\,0.0) \text{ kV/phase}$$

Load current $I_L = 218.6933 \angle -45^o$ A (as p.f. is 0.707 lagging)

$$= (154.6395 - j\,154.6395) \text{ A}$$

- Problem is to determine the capacity of a phase modifier (What is it?) to be installed at the receiving end.
- Let, modifier current be $j\,I_m$ (assuming losses negligible).

So, receiving end current = $I_R = I_L + j\,I_m$ (as two are in parallel).

$= (154.6395 - j\,154.6395) + j\,I_m = \{154.6395 - j\,(154.6395 - I_m)\}$ A

Then, sending end voltage is

$$V_s = A.V_R. + B.I_R$$

$$=(0.986 + j\,0.0055).(76210.2 + j\,0.0) + (24.96 + j\,65.72).\{154.6395 - j\,(154.6395 - I_m)\}$$

$$= (89152.33 - 65.72.\,I_m) + j\,(6728.21 + 24.96\,I_m)$$

Given that, $|V_s| = |V_R| = 76210.2$ V/phase

Hence,

$$(76210.2)^2 = (89152.33 - 65.72.\,I_m)^2 + (6728.21 + 24.96\,I_m)^2$$

Solving, $I_m = 2093$ A or 212 A

- Although both values of I_m seem to be valid, but they are mathematical results only.
- In practice, higher value is invalid as it lies outside the region of stable operation.

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- So, the accepted value of I_m is 212 A.
- Hence, phase modifier capacity to meet the specification would be

$$\frac{\sqrt{3}.\ 132.\ 212}{1000}\ \text{MVAR} = 48.47\ \text{MVAR}$$

Preamble

- In practice a power lines may be erected in close proximity to telephone lines or communication circuits.
- The communication circuits may be the property of the power company → used for protection or communication link.
- Power lines give rise to electromagnetic & electrostatic fields of sufficient magnitudes.
- These induce currents & voltages respectively in the neighbouring communication lines.
- Induced currents are superimposed on true speech currents and thereby setup distortion.
- Induced voltages raise potential of communication circuits with consequent risk to both equipment & users.

Preamble – Cont.

- In extreme cases, raising of potential may become too high to handle the communication equipment highly dangerous.
- Magnitudes of induced currents & voltages depend on the distance between the two circuits & length of the route over which they run approximately in parallel.
- In case of public telephone lines, the two circuits may run in parallel only for a short distance.
- But, if the communication circuit is the property of the power company, same towers may be used for both circuits.
- Hence, adequate measures must always be taken to reduce the the electromagnetic & electrostatic effects on the communication lines.

Reduction of Interference

- Thorough transposition of both the (i) power line & (ii) communication line conductors.

→ It has the effect of splitting the induced emf into a series of mutually opposed emfs.

- Use of screened cables for communication lines overcomes the trouble due to electrostatic interference.
- Same effect is obtained by the use of an earth wire between the power line & the communication line.
- Further, electromagnetic interference may be reduced by splitting the telephone line into short lengths each being separated from the adjacent section by 1:1 isolating transformer.

→ Such a method can't be used if d.c. signal needs to be sent.

Prob: A load of 30MW is delivered at a distance of 160km at a voltage of 132kV & at a frequency of 50Hz, the p.f. being 0.9. The line conductors which have a radius of 5.5mm are situated at the corners of an equilateral triangle with a side of 3.5m and arranged as shown in the fig. The height of the lowest cond. is 15m from the ground. A telephone line runs on the same supporting towers, the distances between the telephone line & the power line conds. being as shown. Find magnitudes of (i) the electromagnetic & (ii) the electrostatic emf induced in the telephone line.

Soln: Electromagnetic Effect

From the Δabo,

$$(3.5)^2 = D_{ao}^2 + \left(\frac{3.5}{2}\right)^2$$

or, $D_{ao} = 3.0311m$

Now, $D_{ad} = D_{ao} + 3 = 6.0311m$

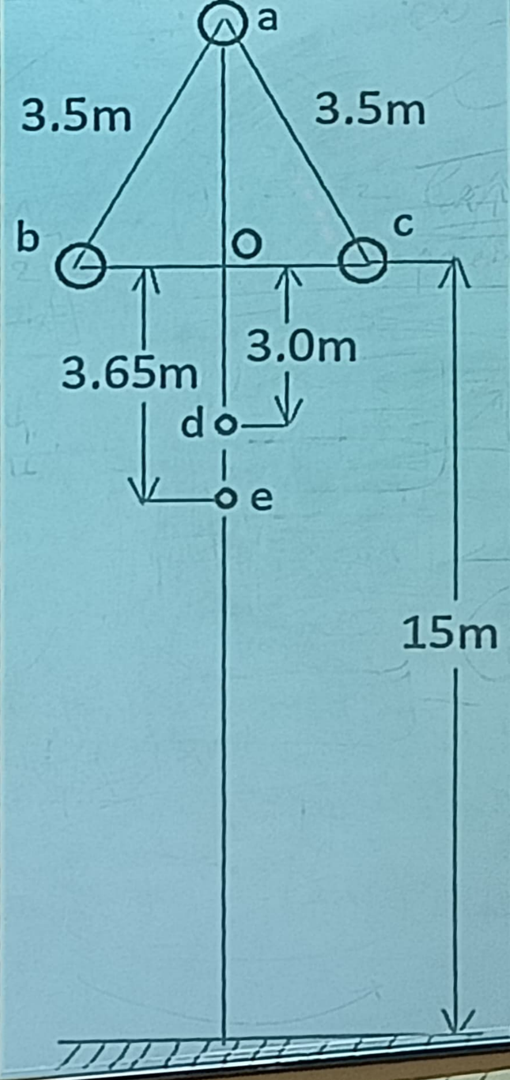
$D_{ae} = D_{ao} + 3.65 = 6.6811m$

Again, $(D_{bd})^2 = (3)^2 + \left(\frac{3.5}{2}\right)^2$

or, $D_{bd} = 3.4731m = D_{cd}$

$$(D_{be})^2 = (3.65)^2 + \left(\frac{3.5}{2}\right)^2$$

or, $D_{be} = 4.0478m = D_{ce}$



$$M_a = 2.10^{-7}.\ln\frac{D_{ae}}{D_{ad}} = 2.10^{-7}.\ln\frac{6.6811}{6.0311} = 0.2047.\,10^{-7}\,H/m$$

$$= 0.2047.\,10^{-4}\,H/km$$

$$M_b = M_c = 2.10^{-7}.\ln\frac{D_{be}}{D_{bd}} = 2.10^{-7}.\ln\frac{4.0478}{3.4731} = 0.3063.\,10^{-7}\,H/m$$

$$= 0.3063.\,10^{-4}\,H/km$$

- Net mutual inductance is equal to phasor sum of M_a, M_b & M_c.
- So, $\vec{M} = \vec{M}_a + \vec{M}_b + \vec{M}_c = M_a + M_b\angle 120^o + M_c\angle 240^o$ (ph seq. a-b-c)

or, $|M|^2 = (M_a + M_b.\text{Cos}120^o + M_c.\text{Cos}240^o)^2 +$

$(M_b.\text{Sin}120^o + M_c.\text{Sin}240^o)^2$

or, $|M| = 0.1016.\,10^{-4}\,H/km$

Now, current $I = \dfrac{30.10^6}{\sqrt{3}.132.\ 10^3.0.9} = 145.7955\ A$

Electromagnetically induced emf $= 2\pi f.\ |M|.I = 0.4654\ V/km$

Total electromagnetically induced emf $= 0.4654.\ 160 = 74.464\ V$

Electrostatic Effect

- Height of cond. 'a' above ground$=15 + D_{ao} = 18.0311m$
- Height of cond. 'b' & 'c' above ground $= 15m$
- So, magnitude of potential of cond. 'd' w.r.t. neutral $= |V_{dn_a}|$

$$= |V_{an}|.\frac{\ln \frac{(2.h_a - D_{ad})}{D_{ad}}}{\ln \frac{2.h_a}{r}} = \frac{132.10^3}{\sqrt{3}.}.\frac{\ln \frac{(2.18.0311 - 6.0311\ )}{6.0311}}{\ln \frac{2.18.0311}{5.5.\ 10^{-3}}}$$

$$= 13920.8142\ V$$

Likewise, magnitude of potential of cond. 'd' w.r.t. neutral due to cond. 'b' = magnitude of potential of cond. 'd' w.r.t. neutral due to cond. 'c' (due to symmetry) = $|V_{dn_b}| = |V_{dn_c}|$

$$=|V_{bn}|.\frac{\ln\frac{(2.h_b - D_{bd})}{D_{bd}}}{\ln\frac{2.h_b}{r}} = \frac{132.10^3}{\sqrt{3}.}.\frac{\ln\frac{(2.15-3.4731)}{3.4731}}{\ln\frac{2.15}{5.5.\,10^{-3}}}$$

$$=18007.8369\text{ V}$$

- So, total potential of cond. 'd' w.r.t. neutral = $\overrightarrow{V_{d_n}}$

$$=\overrightarrow{V_{d_{na}}}+\overrightarrow{V_{d_{nb}}}+\overrightarrow{V_{d_{nc}}}=|V_{dn_a}|+|V_{dn_b}|\angle 120^o+|V_{dn_c}|\angle 240^o$$

$$\text{or, } |V_{dn}| = 4087.0227\text{ V}$$

Similarly, the potential of cond. 'e' can be determined.