ELECTRIC MACHINE

1

Transformer:-

The transformer is a device for transferring electrical energy from one alternating current circuit to another without a change in frequency. A transformer may receive energy at one voltage and deliver it at a higher voltage, it in which case it is called a step-up transformer. When the energy is received at a higher voltage and delivered at a lower voltage, it is called a step-down transformer.

The electric circuit which receives energy from the supply mains, is called primary winding, and the other circuit which delivers electric energy to the load, is called the secondary winding.

Construction:- The simple elements of a transformer consists of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are — some suitable container for the assembled core and windings, a suitable medium for insulating the core and its windings from its container, suitable bushings (either of porcelain, oil-filled or capacitor type) for insulating and bringing out the terminals of windings from the tank.

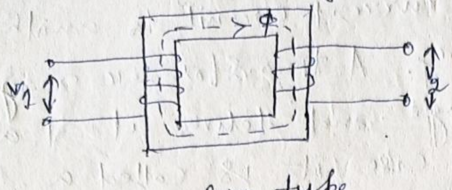
The magnetic core is a stack of thin silicon-steel laminations about 0.35 mm. thick for 50 Hz. transformers. In order to reduce the eddy current losses, these laminations are insulated from one another by thin layers of varnish.

Constructionally the transformers are of two general types:— ① Core type and ② Shell type.

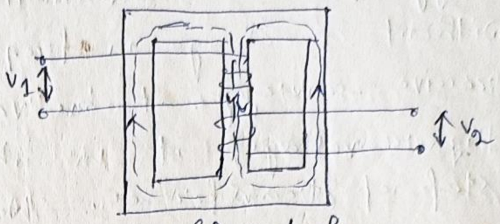
★ What are the standard transmisible voltage level in India?

→ from 66 kV upto 756 kV

In the core type, the windings surround a considerable part of steel core. In the shell type, the steel core surrounds a major part of the windings.



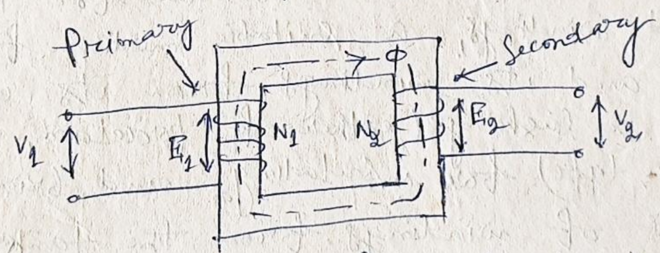
Core type



Shell type

In actual construction, primary and secondary windings are always interleaved to reduce leakage flux.

Transformer Principle:- The transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both sets of coils are on a common magnetic circuit. Electromotive forces are induced by change in flux linkages.

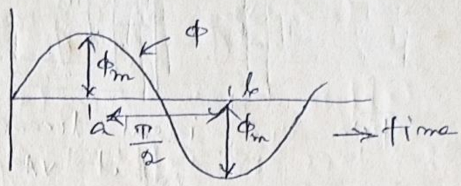


Here, an a.c voltage is applied to the primary winding. As this winding is linked with an iron core, its m.m.f. produces an alternating flux ϕ in the core. This alternating flux links the turns of the secondary winding. As this flux is alternating, it induces in the secondary winding an e.m.f. of the same frequency as the flux. Because of this induced e.m.f., the secondary winding is capable of delivering current and energy. The energy ~~is~~ therefore, is transferred from the primary to the secondary, by means of magnetic flux.

E.m.f. equation of a transformer:-

Let, N_1 = No. of turns in primary

N_2 = No. of turns in secondary.



ϕ_m = Maximum flux in core in webers

$= B_m \times A$

where, B_m = Maximum flux density (Wb/m^2) in the core

A = Area of cross section of the core.

Above figure shows the mutual flux ϕ varying sinusoidally with time. Between points a and b, the total change of flux is $2\phi_m$ ~~wb.~~ webers.

This change of flux occurs in a half cycle or in a time $\frac{T}{2}$ sec., where T is the time period.

∴ Average induced e.m.f. in the primary winding,

$$e_1 = -N_1 \frac{2\phi_m}{T/2} = -N_1 \cdot \frac{2\phi_m}{\frac{1}{2f}}$$

Form factor $= \frac{RMS}{Avg}$

$$= -4 f N_1 \phi_m \text{ Volts.}$$

Since, the form factor for a sine wave is 1.11, r.m.s. induced e.m.f. is, $E_1 = -4.44 f N_1 \phi_m$.

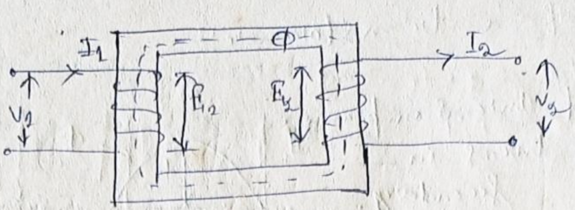
$$\Rightarrow \boxed{E_1 = -4.44 f N_1 B_m A} \text{ Volts.}$$

Similarly, Voltage induced in the secondary winding,

$$\boxed{E_2 = -4.44 f N_2 B_m A}$$

So, $$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

So, Transformation ratio, $$\boxed{K = \frac{E_1}{E_2} = \frac{N_1}{N_2}}$$



For an ideal transformer,
input VA = output VA.

$\therefore V_1 I_1 = V_2 I_2$

$$\therefore \frac{I_2}{I_1} = \frac{V_1}{V_2} = K = \frac{N_1}{N_2} = \frac{E_1}{E_2}.$$

~~So,~~ $\cancel{\frac{V_1}{V_2}}$

Transformer on No-load :-

When an actual transformer is put on load, there is iron loss in the core and copper loss in the windings.

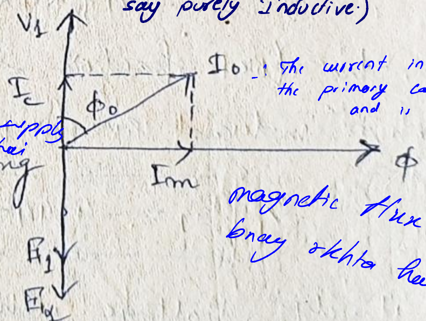
When the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply (i) iron losses in the core i.e, hysteresis loss and eddy current loss and (ii) a very small amount of copper loss in primary winding.

only core loss ↓ Iron loss

Hence no-load primary current, I_0 lags behind V_1 by an angle $\Phi_0 < 90°$. (cuz there is no resistance or we can say purely Inductive)

No-load input power,

$W_0 = V_1 I_0 \cos\Phi_0$.



load supply krta hei

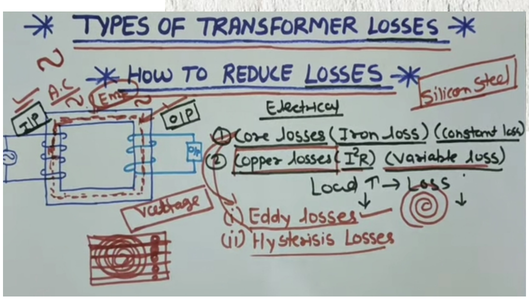
I_0 : the current in the primary called No load current and is midely utilized for two things
i) Magnetize magnetic core
ii) to supply core losses as core material dont hv a finite permeability

magnetic flux bnay rkhta hei

I_c – is called active or working or iron loss component, because it mainly supplies the iron loss plus small quantity of primary Cu-loss.

I_m – is called magnetising component, because its function is to sustain the alternating flux in the core.

I_0 – is very small in comparison to full load primary current. It is about 3% of full load current.



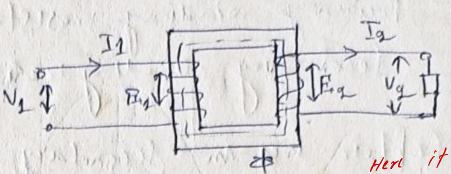
Core losses (Iron loss / constant loss)
⇒ Due to loss in I through core (iron)

Copper losses (I^2R)
⇒ Due to I in the winding made of Cu

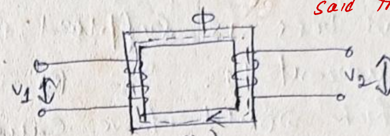
Load ↑ ⇒ Cu loss ↑

Transformer on load :-

When the secondary is loaded, the secondary current I_2 is set up.

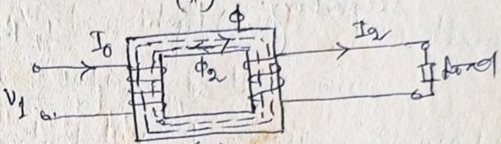


The secondary current sets up its own m.m.f. ($= N_2 I_2$) and hence flux ϕ_2, which is in opposition to main flux ϕ, which is due to I_0.



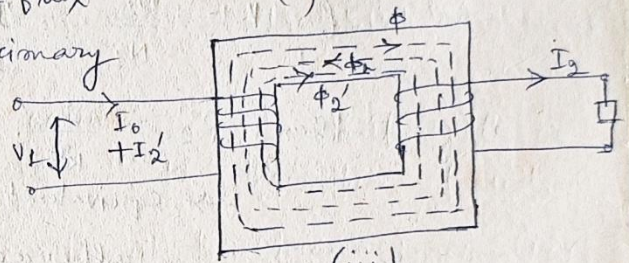
(i)

Here it can be said that when open circuited No current flows through secondary & high impedance



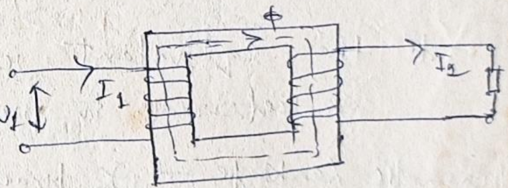
(ii)

The opposing secondary flux ϕ_2 weakens the primary flux ϕ momentarily, hence primary back e.m.f. E_1 tends to be reduced. For a moment V_1 becomes more than E_1 and hence, causes more current to flow in the primary.



(iii)

Let, the additional primary current is I_2'.



(iv)

The additional primary m.m.f. $N_1 I_2'$ sets up its own flux ϕ_2', which is in opposition to ϕ_2 and is equal to ϕ_2 in magnitude. Hence, ϕ_2 and ϕ_2' cancel each other. So, we find that the magnetic effects of secondary current I_2 are immediately neutralised by the additional primary current I_2'.

Hence, whatever the load conditions, the net flux passing through the core is approximately the same as no-load.

Due to constancy of core flux, the core loss is practically constant.

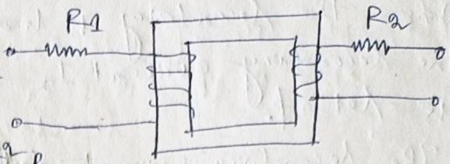
$$N_1 I_2' = N_2 I_2 \Rightarrow I_2' = \frac{N_2}{N_1} \cdot I_2 = K I_2$$

Hence, on load condition, the primary winding has two currents in it — I_0 and I_2'.

Equivalent Resistance :-

Let, R_1 = Primary winding resistance

R_2 = Secondary winding resistance



Copper loss in secondary $= I_2^2 R_2$

This loss is supplied by primary which takes a current I_1. If R_2' is equivalent resistance in primary which would have caused the same loss as R_2 in secondary, then

$$I_1^2 R_2' = I_2^2 R_2$$ heat eqⁿ

or, $$R_2' = \left(\frac{I_2}{I_1}\right)^2 \cdot R_2$$

If $K = \frac{N_2}{N_1} = \frac{I_1}{I_2}$, [Neglecting no-load current I_0]

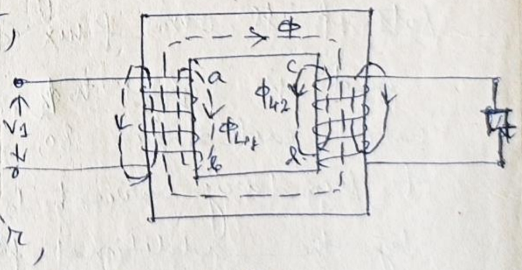
then $$R_2' = \frac{1}{K^2} \cdot R_2$$

→ equivalent secondary resistance as refferred to primary.

The resistance $(R_1 + R_2')$ i.e., $\left(R_1 + \frac{R_2}{K^2}\right)$ is known as the equivalent resistance of the transformer as referred to primary.

Magnetic Leakage :-

In practice, all the flux linked with primary, does not link the secondary, but a part of it, i.e., ϕ_{L1} completes its magnetic circuit by passing through air, rather than around through the core.



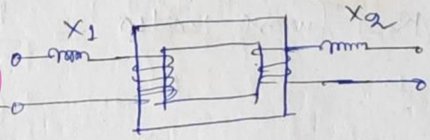
This leakage flux is produced due to primary m.m.f. This flux is known as primary leakage flux and is proportional to the primary ampere turns alone. ϕ_{L1} induces an emf in primary.

Similarly, secondary ampere-turns acting across points c and d, set up leakage flux ϕ_{L2} which is linked with secondary winding alone. ϕ_{L2} induces an e.m.f. e_{L2}

At no-load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible.

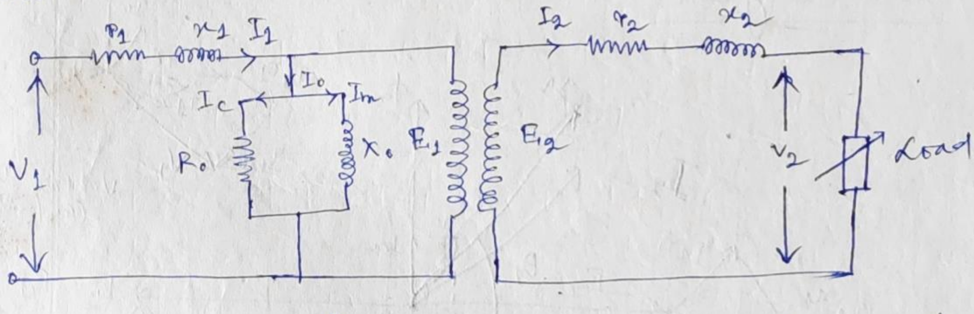
So, a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits, such that voltage drop in each series coil is equal to that produced by leakage flux.

X_1 and X_2 are called primary and secondary leakage reactances.



Equivalent Circuit :-

Equivalent circuit is the electrical representations of the equations that describe the behaviour of an electrical device.



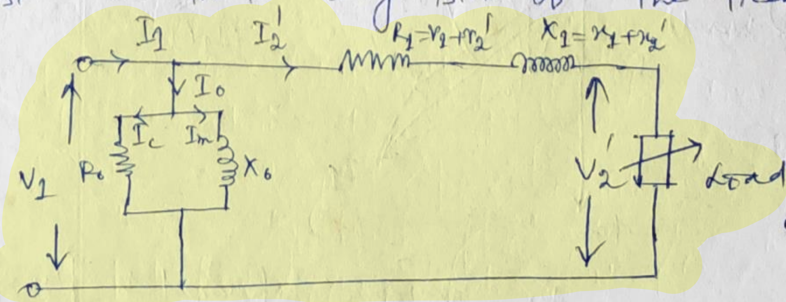
Here, r_1 and x_1 represent the primary resistance and leakage reactances.

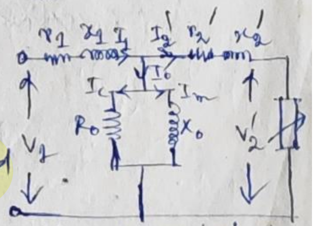
r_2 and x_2 are secondary winding impedances.

R_0 = Core loss component of resistance.

X_0 = reactance corresponding to magnetising component.

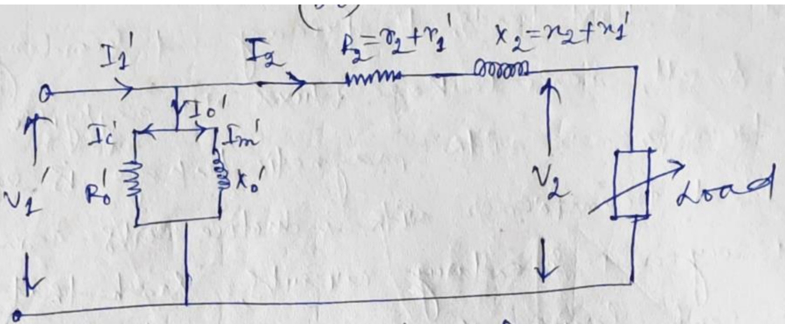
The equivalent circuit can be modified and redrawn by referring the winding impedances to either the primary side or the secondary side of the transformer.





Exact equivalent circuit referred to primary.

Approximate equivalent circuit referred to primary.



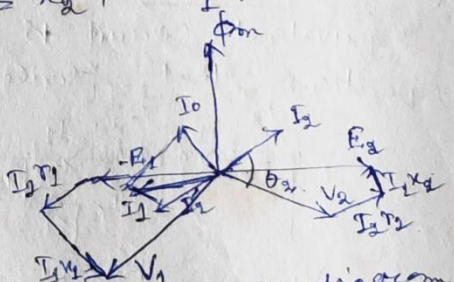
Approximate Equivalent circuit referred to secondary

$$K = \text{Turns ratio} = \frac{N_2}{N_1} = \frac{\text{No. of secondary turns}}{\text{No. of primary turns}}$$

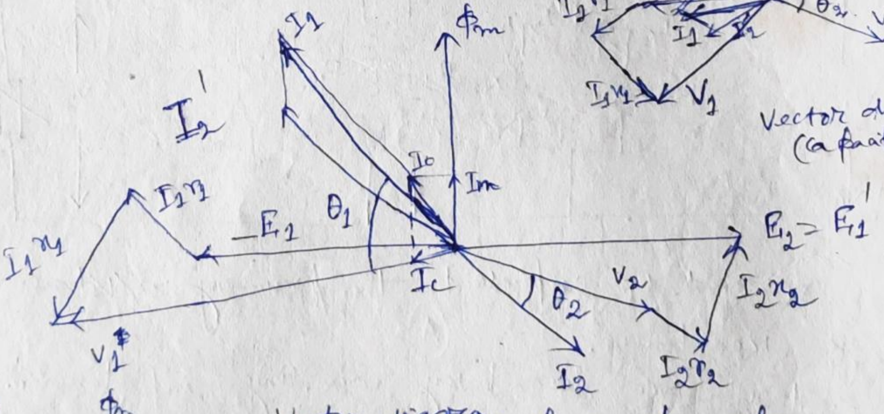
$$R_1 = r_1 + \frac{r_2}{K^2}, \quad R_2 = r_2 + K^2 r_1$$

$$X_1 = x_1 + \frac{x_2}{K^2}, \quad X_2 = x_2 + K^2 x_1.$$

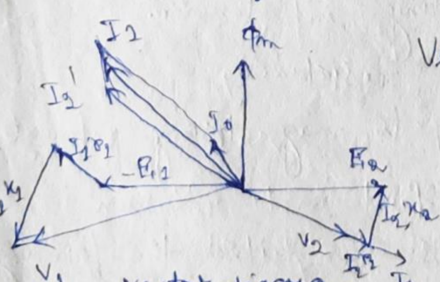
Vector diagram :



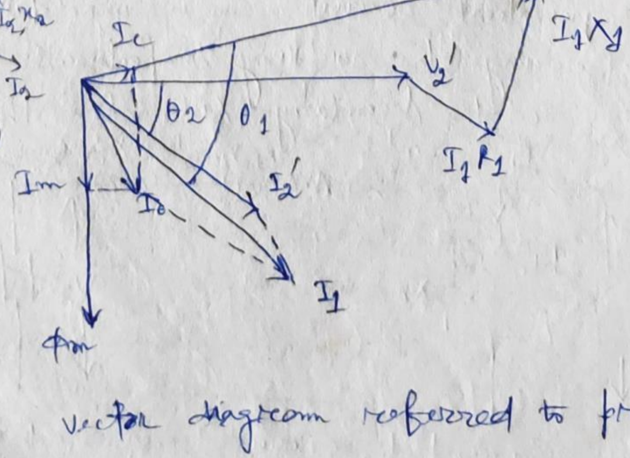
Vector diagram (capacitive load)



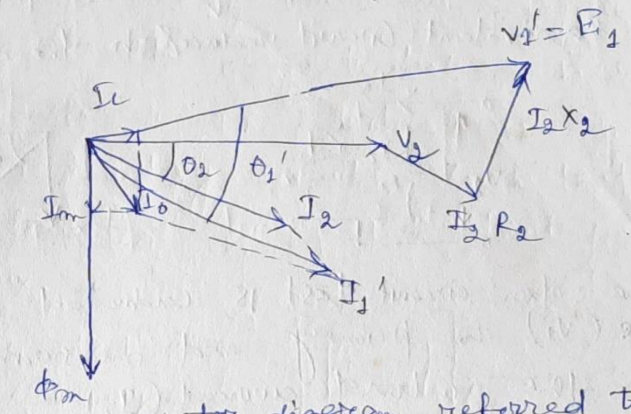
Vector diagram of a transformer (Inductive load)



vector diagram (Resistive load)



vector diagram referred to primary



Vector diagram referred to secondary

① A single phase transformer has 400 primary and 1,000 secondary turns. The net cross sectional area of the core is 60 cm^2. If the primary winding is to be connected to a 50 c/s supply at 500V., calculate the value of maximum flux density in the core and the e.m.f. induced in the ~~core.~~ secondary winding.

Soln.:- No. of secondary turns, $N_2 = 1000$

" " primary " , $N_1 = 400$

$V_1 = 500$ V.

Here, $E_1 = V_1 = 4.44 f. N_1 \Phi_m$

$\therefore 500 = 4.44 \times 50 \times 400 \times \Phi_m$

$\Rightarrow \Phi_m = 0.563 \times 10^{-2}$ wb.

Cross section of the core, $A = 60\ cm^2$

$\therefore$ Maximum flux density, $B_m = \dfrac{\Phi_m}{A}$

$= \dfrac{0.563 \times 10^{-2} \times 10^8}{60} = 9.383$ lines/cm^2.

Voltage per turn $= \dfrac{500}{400} = 1.25$

$\therefore$ secondary voltage $= 1000 \times 1.25 = 1250$ V.

3 Types of Power:

Total Power → VA = Volt-Ampere

Reactive Power → VA $\sin\phi$ → VAr

Active Power → VA $\cos\theta$ → Watt

50 KVA

$I_p = \dfrac{50 \times 10^3}{4400}$

$I_s = \dfrac{50 \times 10^3}{220}$

(2) A 50 KVA, 4400/220 V. transformer has $r_1 = 3.45\,\Omega$, $r_2 = 0.009\,\Omega$. The values of reactances are $x_1 = 5.2\,\Omega$ and $x_2 = 0.015\,\Omega$. Calculate for the transformer –

(i) the equivalent resistance as referred to primary.

(ii) the equivalent resistance as referred to secondary

(iii) equivalent reactance referred to both primary and secondary.

(iv) equivalent impedance both referred to primary and secondary and

(v) total copper loss.

Soln. :– Full-load primary current, $I_1 = \frac{50,000}{4,400}$ A. $= 11.36$ A.

(assuming 100% efficiency)

Full load secondary current, $I_2 = \frac{50,000}{220}$ A. $= 227$ A.

Turns ratio, $K = \frac{220}{4400} = \frac{1}{20}$.

(i) $R_1 = r_1 + \frac{r_2}{K^2} = 3.45 + \frac{0.009}{\left(\frac{1}{20}\right)^2} = 7.05\,\Omega$

(ii) $R_2 = K^2 r_1 + r_2 = \left(\frac{1}{20}\right)^2 \times 3.45 + 0.009 = 0.0176\,\Omega$.

Also, $R_2 = K^2 R_1 = \left(\frac{1}{20}\right)^2 \times 7.05 = 0.0176\,\Omega$ (check).

(iii) $X_1 = x_1 + \frac{x_2}{K^2} = 5.2 + \frac{0.015}{\left(\frac{1}{20}\right)^2} = 11.2\,\Omega$

$X_2 = K^2 x_1 + x_2 = \left(\frac{1}{20}\right)^2 \times 5.2 + 0.015 = 0.028\,\Omega$.

(iv) Also, $X_2 = K^2 X_1 = \left(\frac{1}{20}\right)^2 \times 11.2 = 0.028\,\Omega$ (check)

$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{(7.05)^2 + (11.2)^2} = 13.23\,\Omega$.

$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(0.0176)^2 + (0.028)^2} = 0.0331\,\Omega$.

Also, $Z_2 = K^2 Z_1 = \left(\frac{1}{20}\right)^2 \times 13.23 = 0.0331\,\Omega$ (check).

(v) Copper loss $= I_1^2 r_1 + I_2^2 r_2 = (11.36)^2 \times 3.45 + (227)^2 \times 0.009$
$= 909$ W.

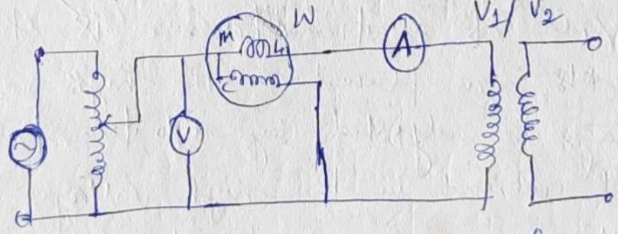
Also, copper loss $= I_1^2 R_1 = (11.36)^2 \times 7.05 = 910$ W.
$= I_2^2 R_2 = (227)^2 \times 0.0176 = 910$ W.

Determination of equivalent circuit parameters:-

The parameters R_0 and X_0 of the equivalent circuit are obtained by conducting open circuit test and the equivalent impedance R_1 and X_1 (or R_2 and X_2) are obtained by conducting short circuit test on transformers.

(1) Open Circuit test or No-load test:-

The purpose of this test is to determine no-load loss or core loss and no-load current I_0 which is helpful in finding R_0 and X_0.

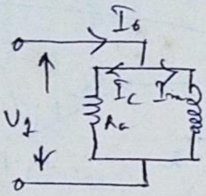


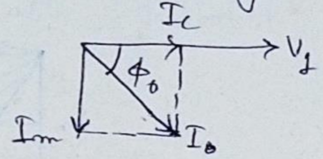
The high voltage side is left open-circuited.
Rated circuit voltage, applied to the primary, i.e, low-voltage side, is varied with the help of a variable ratio auto transformer. When the voltmeter reading is equal to the rated voltage of the L.V. winding, all the three instrument readings are recorded.

As the secondary is open circuited, the load current is zero and hence, the primary draws only no-load current I_0.
The input power given by the wattmeter reading consists of core loss and ohmic loss. As no-load current I_0 is very small about 2% to 6% of full-load current, ohmic loss

in the primary is negligibly small. Hence, the wattmeter reading can be taken as equal to transformer core loss.

Equivalent circuit reduces to the following form —





The open circuit test is conducted by applying rated voltage (V_1) to primary and measuring the input power (W_0) and the no-load current (I_0).

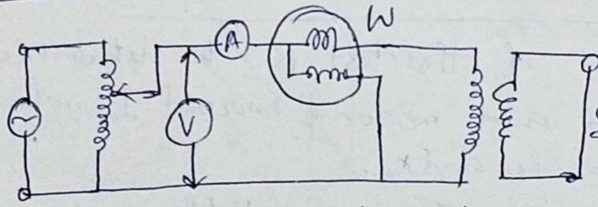
Input power, $W_0 = V_1 I_0 \cos\phi_0$

$$\therefore \cos\phi_0 = \frac{W_0}{V_1 I_0}$$

$$\therefore R_0 = \frac{V_1}{I_0 \cos\phi_0}, \quad X_0 = \frac{V_1}{I_0 \sin\phi_0}$$

$$I_c = I_0 \cos\phi_0, \quad I_m = I_0 \sin\phi_0.$$

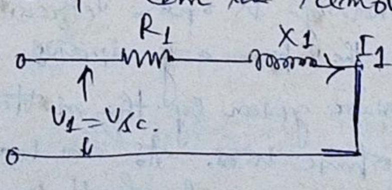
(2) Short circuit test :–



Short circuit

The low voltage side of a transformer is short circuited and the instruments are placed on the high voltage side. The applied voltage is adjusted by auto-transformer, to circulate rated current in the high voltage side.

A primary voltage of 2 to 12% of its rated value is sufficient to circulate rated currents in both primary and secondary windings. As only a reduced voltage is applied, the core loss becomes very much less and may be neglected. Under these conditions, I_0 becomes insignificant and the parallel branch R_0 and X_0 in the equivalent circuit can be removed.



Readings obtained on short circuit test are as given below—

Applied voltage, $V_1 = V_{s.c.}$ volts.

current, $I_1 = I_{sc}$ amp.

Input power, $W_1 = W_{s.c.}$ watts.

$$R_1 = \frac{W_{sc}}{\cancel{I_{sc}}} \quad \frac{W_{s.c.}}{I_{s.c.}^2}$$

$$Z_1 = \frac{V_{s.c.}}{I_{s.c.}}, \quad X_1 = \sqrt{Z_1^2 - R_1^2}.$$

Let us consider a transformer with rating 220/3300V, 33000VA.

For open circuit test on low voltage side, the ranges of voltmeter, ammeter and wattmeter are 220V., 6A. (2 to 6% of rated current of 150A.), and 6A., 220V. respectively. These are the standard ranges for ordinary instruments. So, more accurate readings can be obtained. If the open circuit test is performed on the h.v. side, the instrument ranges are 3300V., 0.4A. and 0.4A., 3300V. which are not within the range of ordinary instruments & Hence results obtained may not be so accurate.

For short-circuit test on the h.v. side, the instrument ranges are 165V. (2 to 12% of rated voltage 3300V.), 10A. (rated current) and 10A., 165V. which are well within the range of ordinary instruments. On the other hand, instrument ranges, for a short circuit test, on l.v. side are 11V., 150A. and ~~11V.,~~ 150A., 11V.. Instruments of such ranges and auto-transformer capable of handling 150A. may not be readily available and results may not be so accurate. For these reasons, open circuit and short circuit tests are conducted on l.v. and h.v. sides respectively. [For O.C., 220V./3300V. 33000VA., 220V., 4% of Full load cl. For S.C., 5% of Full load voltage, rated cl.]

Voltage regulation of a transformer:—

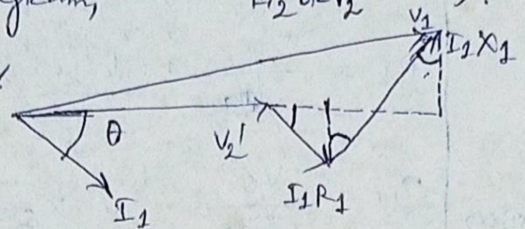
It is defined ~~at~~ as the change in secondary terminal voltage, expressed as a percentage (or per unit) of the secondary rated voltage, when load at a given power factor is reduced to zero, with primary applied voltage held constant.

If V_2 = secondary terminal voltage at any load and E_2 = secondary terminal voltage at no-load.

$\therefore$ Percentage regulation $= \frac{E_2 - V_2}{E_2 \text{ or } V_2} \times 100\%$.

Referring to the vector diagram,

% Regulation $= \frac{V_1 - V_2'}{V_1} \times 100\%$



For a lagging power factor load,

$$V_1 - V_2' = (I_1R_1\cos\theta + I_1X_1\sin\theta) + j(I_1X_1\cos\theta - I_1R_1\sin\theta)$$

Quadrature component $j(I_1X_1\cos\theta - I_1R_1\sin\theta)$ is very much less compared to terminal voltage and may be neglected.

$$\therefore V_1 - V_2' = I_1R_1\cos\theta + I_1X_1\sin\theta.$$

$$\% \text{ Regulation} = \frac{(I_1R_1\cos\theta + I_1X_1\sin\theta)}{V_1} \times 100\%.$$

Taking in to account both lagging and leading power factor loads,

$$\% \text{ Regulation} = \frac{(I_1R_1\cos\theta \pm I_1X_1\sin\theta)}{V_1} \times 100\%$$

Losses in a transformer:- In a static transformer, there are no friction or windage losses. There are mainly two kinds of losses in a transformer.

(1) Core or Iron loss and (2) Ohmic or Copper loss.

(1) Core loss:- Core loss (P_c) consists of hysteresis loss (P_h) and eddy current loss (P_e), i.e., $P_c = P_h + P_e$.

As the core flux in a transformer remains practically constant for all loads, the core loss is practically the same for all loads.

$$P_h = K_h f B_m^x \quad \text{and} \quad P_e = K_e f^2 B_m^2,$$

where, K_h = proportionality constant which depends upon the volume and quality of core material and the units used.

K_e = proportionality constant whose value depends on the volume and resistivity of the core material thickness of laminations and units used.

B_m = maximum flux density in the core

f = frequency.

Value of x (called Steinmetz's constant) varies from 1.5 to 2.5, depending upon the magnetic properties of core material.

② Copper loss :- When a transformer is loaded, ohmic loss (I^2R) occurs in both the primary and secondary winding resistances.

$$\text{Cu-loss} = I_1^2 r_1 + I_2^2 r_2$$
$$= I_1^2 R_1 = I_2^2 R_2.$$

In addition to these, the following losses are also present in a transformer—

(i) Stray load loss :- Leakage fields in a transformer induce eddy currents in conductors, tanks, channels, bolts etc. and these eddy currents give rise to stray load loss.

(ii) Dielectric loss :- This loss occurs in the insulating materials, i.e., in the transformer oil and solid insulation of h.v. transformers.

These two losses are very small and are therefore, neglected.

Efficiency of a transformer :-

The efficiency of a transformer is the ratio of output power to input power.

$$\text{Efficiency } \eta = \frac{\text{output}}{\text{Input}} \times 100\%$$

It is very difficult to measure input and output powers under actual load conditions, hence efficiency is computed from the values of losses obtained from tests.

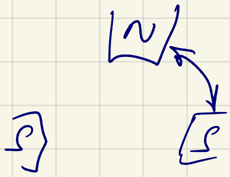
$$\therefore \eta = \frac{\text{output}}{\text{output + losses}} \times 100\%$$

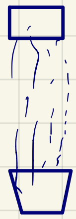
Iron losses P_i are constant at all loads, since ϕ_m is almost constant. Copper losses P_c are proportional to square of the load current.

Transformer out put $= V_2 I_2 \cos\theta_2$, where $\cos\theta_2$ is the load power factor.

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + (P_i + I_2^2 R_2)}$$

P, Z, N, A





$\frac{1}{N}$ min $= \frac{60}{N}$ sec

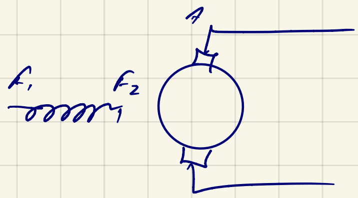
$$\frac{\phi}{t} = \frac{\phi}{\frac{60}{NP}} = \frac{\phi NP}{60}$$

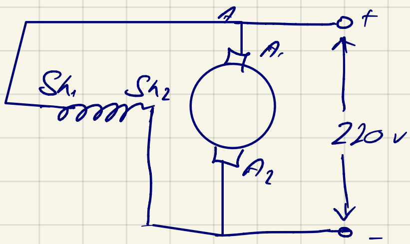
$t = \frac{60}{N} \times \frac{1}{P}$ sec

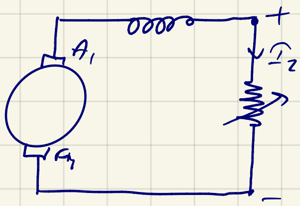
$$\Rightarrow E = \frac{\phi NP}{60} \times \frac{Z}{A} \Rightarrow \boxed{E = \frac{\phi Z N P}{60A}}$$

1. Seperately excited
2. Self-excited

   i) Shunt wound
   ii) Series wound
   iii) Compound wound







Condition for maximum efficiency :-

$$\frac{d\eta}{dI_2} = \frac{\left[V_2 I_2 \cos\theta_2 + (P_i + I_2^2 R_2)\right] V_2\cos\theta_2 - V_2 I_2 \cos\theta_2 \left[V_2\cos\theta_2 + 2I_2R_2\right]}{\left[V_2 I_2 \cos\theta_2 + (P_i + I^2 R_2)\right]^2}$$

Equating numerator to zero,

$$\left[V_2 I_2 \cos\theta_2 + (P_i + I_2^2 R_2)\right] V_2\cos\theta_2 = V_2 I_2 \cos\theta_2 \left[V_2 \cos\theta_2 + 2I_2R_2\right]$$

$$\Rightarrow V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_2 = V_2 I_2 \cos\theta_2 + 2 I_2^2 R_2$$

$$\Rightarrow P_i = I_2^2 R_2 .$$

$\therefore$ Iron loss = Copper loss.

So when iron loss is equal to variable copper loss, the efficiency attains the maximum value.

$I_2 = \sqrt{\frac{P_i}{R_2}}$ = output current corresponding to maximum efficiency.

Ex: In a transformer if the load current is kept constant, find the power factor at which the maximum efficiency occurs.

Here, Copper loss $= I_2^2 R_2$ = constant.

Let, $P_i + I_2^2 R_2 = C$.

$$\therefore \eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + C}$$

$$\therefore \frac{d\eta}{d\theta_2} = \frac{(V_2 I_2 \cos\theta_2 + C).(-V_2 I_2 \sin\theta_2) - V_2 I_2 \cos\theta_2 (-V_2 I_2 \sin\theta_2)}{(V_2 I_2 \cos\theta_2 + C)^2}$$

$= 0.$

$$\therefore (V_2 I_2 \cos\theta_2 + C) V_2 I_2 \sin\theta_2 = V_2^2 I_2^2 \cos\theta_2 \sin\theta_2$$

$$\Rightarrow V_2^2 I_2^2 \sin\theta_2 \cos\theta_2 + C V_2 I_2 \sin\theta_2 = V_2^2 I_2^2 \sin\theta_2 \cos\theta_2$$

$$\Rightarrow \sin\theta_2 = 0. \quad \therefore \cos\theta_2 = 1 = \text{Power factor.}$$

All-day efficiency :- Distribution transformers have their primaries energised all the twenty four hours, although their secondaries supply little or no-load much of the time during the day, except during the house lighting period. It means that core loss occurs during the day ~~for~~ and copper loss occurs only when the transformer is loaded.

Therefore, a better method of assessing the efficiency of a transformer working on variable ~~load~~ is on the energy basis.

The all-day efficiency is defined as the ratio of the total energy output to that of the total energy input over a given period (generally 24-hours).

$$\therefore \eta_{all\text{-}day} = \frac{\text{Output in KWh.}}{\text{Input in KWh.}} \text{ (for 24-hours).}$$

It is also called energy efficiency.

(2) A 5 KVA., 400/200 V., 50 c/s., 1-phase transformer gave the following results —

No-load : 400 V., 1 A., 50 W. (H.V. side)

Short-Circuit : 12 V., 10 A., 40 W. (H.V. side)

Calculate — (i) the components of the no-load current (ii) the efficiency and regulations at full load and power factor of 0.8-lagging.

Soln :— (i) $I_c = I_0 \cos\phi_0 = \frac{W_0}{V_0} = \frac{50}{400} = 0.125$ A.

$I_0 = 1$ A.

$\therefore$ Magnetising component, $I_m = \sqrt{I_0^2 - I_c^2}$

$= \sqrt{1^2 - (0.125)^2} = 0.992$ A.

(ii) Measurements are made on the primary side again, during the short circuit test :

$Z_1 = \frac{12}{10} = 1.2\ \Omega$, $R_1 = \frac{40}{(10)^2} = 0.4\ \Omega$

$\therefore X_1 = \sqrt{(1.2)^2 - (0.4)^2} = 1.13\ \Omega$.

Full-load current on the primary side,

$I_1 = \frac{5000}{400} = 12.5$ A.

$\therefore$ Full-load copper loss, $P_c = I_1^2 R_1$

$= (12.5)^2 \times 0.4 = \cancel{40\text{ W}}\ 62.5$ W

$\therefore \eta_{full-load} = \frac{5000 \times 0.8}{5000 \times 0.8 + 50 + 62.5} \times 100\%$

$= 97.26\%$.

Regulation at full-load and 0.8 lagging power factor,

$\%\ Reg. = \frac{I_1(R_1 \cos\phi + X_1 \sin\phi)}{V_1} \times 100\%$

$= \frac{12.5(0.4 \times 0.8 + 1.13 \times 0.6)}{400} \times 100\% = 3.13\%$.

(4) A 50 kVA, transformer has 5:1 ratio of turns. The secondary full-load current is 200 A. The primary and secondary resistances are respectively 0.55 Ω and 0.023 Ω. If the transformer is designed for maximum efficiency at $\frac{2}{3}$ of full-load, find its efficiency when delivering full load at 0.8 power factor.

Soln.:- Turns ratio, $K = 1/5$.

$$\therefore R_2 = r_1' + r_2 = K^2 r_1 + r_2 = \left(\frac{1}{5}\right)^2 \times 0.55 + 0.023$$

$$= 0.045\ \Omega.$$

Full-load copper loss $= I_2^2 R_2 = (200)^2 \times 0.045$

$= 1800$ W.

$\therefore$ Copper loss at $\frac{2}{3}$ of full-load

$$= \left(\frac{2}{3}\right)^2 \times 1800 = 800\text{ W.}$$

$\therefore$ Iron loss $= 800$ W.

Full-load output at 0.8 p.f. $= 50 \times 0.8 = 40$ kW.

$= 40,000$ W.

Total losses at full-load $= 1800 + 800 = 2,600$ W.

$$\therefore \text{Full-load efficiency} = \frac{40,000}{40,000 + 2,600} = 93.8\%.$$

(5) A single phase, 50-Hz, 200 KVA, 11 kv./230 v. transformer gave the following test results:

No-load: Normal voltage applied; input = 1,600 w.

Short circuit: Rated current flowing with reduced voltage applied = 2,600 w.

Calculate the all-day efficiency if the duty cycle of the transformer is as follows — 160 kw. at 0.8 p.f. for 8-hours, 100 kw. at unity p.f. for 6-hours and no-load for rest of the day.

Soln.:- Energy out put in one day

$= 160 \times 8 + 100 \times 6 + 0 \times 10 = 1880$ KWh.

Energy loss due to core loss in one day

$= 1.6 \times 24 = 38.4$ KWh.

Energy loss due to copper losses

= 8-hours on full load + 6-hours on half full load

$= 8 \times 2.6 + 1 \times 2.6 \times \left(\frac{1}{2}\right)^2 = 24.7$ kWh.

$\therefore$ Total energy input $= 1880 + 38.4 + 24.7$

$= 1943.1$ kWh.

$\therefore$ All-day efficiency $= \dfrac{\text{out put in kWh.}}{\text{Input in kWh.}}$

$= \dfrac{1880}{1943.1} \times 100\% = 96.8\%$.

(6) A 100 KVA, 50-Hz., 440V/11,000V. single phase transformer has an efficiency of 98.5% when supplying full-load current at 0.8 p.f. and an efficiency of 99% when supplying half-load current at unity p.f. Find the iron losses and the copper losses corresponding to full-load current. At what value of load current will the maximum efficiency be attained?

Soln:- Let the copper loss at full-load $= W_c$ KW.

and the iron loss $= W_i$ KW.

$\therefore \dfrac{100 \times 0.8}{100 \times 0.8 + W_c + W_i} = 0.985$ — (1)

and $\dfrac{50 \times 1}{50 \times 1 + \left(\frac{1}{2}\right)^2 W_c + W_i} = 0.990$ — (2)

From (1) $\rightarrow$ $0.985 W_c + 0.985 W_i = 80 - 78.8$

$\Rightarrow \cancel{0.78}\ W_c + W_i = 1.218$ — (3)

From (2) $\rightarrow$ $\dfrac{W_c}{4} + W_i = 0.505$

$\Rightarrow W_c + 4 W_i = 2.02$ — (4)

(4) − (3) $\Rightarrow$ $3 W_i = 0.802$

$\Rightarrow W_i = 0.2673$ KW $= 267.3$ W.

From (3) $\rightarrow$ $W_c = 0.9507$ Kw. $= 950.7$ W.

Let, maximum efficiency occurs at a fraction of x times the full-load.

$\therefore x^2 W_c = W_i \Rightarrow x^2 \times 950.7 = 267.3$

$\Rightarrow x = 0.5302$.

$\therefore$ Maximum efficiency occurs at a load of (0.5302×100) KVA. $= 53.02$ KVA

∴ Full-load current on the primary side

$$= \frac{100 \times 1000}{440} = 227 \text{ A.}$$

∴ Current at maximum efficiency $= (227 \times 0.5302)$ A.

$$= 120.36 \text{ A.}$$

(7) The maximum efficiency of a 500 KVA, 3300V/500V, 50-Hz. single phase transformer is 97%. and occurs at $\frac{3}{4}$ full-load, unity power factor. If the impedance is 10%, calculate the regulation at full-load, power factor 0.8 lagging.

Soln. :- Full-load current referred to primary,

$$I_1 = \frac{500 \times 1000}{3300} = 151.5\,A.$$

$$\text{Efficiency} = \frac{500 \times 0.75 \times 1}{500 \times 0.75 \times 1 + \left(\frac{9}{16} \times P_c + P_i\right)} = 0.97$$

$$\Rightarrow \frac{375}{375 + P_c + P_i} = 0.97$$

$$\therefore P_c + P_i = 11.598 \text{ KW.}$$

For maximum efficiency, $P_c = P_i$

$$\therefore P_c = P_i = \frac{11.598}{2} = 5.799 \text{ KW.}$$

Here $P_i = \frac{9}{16} P_c$

$$\therefore \frac{375}{375 + \frac{18}{16} P_c} = 0.97$$

$$\Rightarrow P_c = 10.31 \text{ KW.}$$

$$\therefore R_1 = \frac{10.31 \times 1000}{(151.5)^2} = 0.449\,\Omega.$$

% impedance = 10%

$$\therefore Z_1 = \frac{3300}{151.5} \times 0.1 = 2.178\,\Omega$$

$$\therefore X_1 = \sqrt{(2.178)^2 - (0.449)^2} = 2.13\,\Omega.$$

∴ Regulation at full load 0.8 p.f. lagging

$$= \frac{I_1 R_1 \cos\theta + I_1 X_1 \sin\theta}{3{,}300} \times 100\%$$

$$= \frac{151.5 \times 0.449 \times 0.8 + 151.5 \times 2.13 \times 0.6}{3{,}300} \times 100\%$$

$$= \frac{54.419 + 193.617}{3{,}300} \times 100\% = 7.52\%$$

(8) A 100-KVA transformer has its maximum efficiency of 0.98 at full-load at unity power factor. During the day, it is loaded as follows—

12-hours – 20 KW at power factor 0.5

6-hours – 45 KW at power factor 0.9

6-hours – 80 KW at power factor 0.8.

Calculate all-day efficiency of the transformer.

Soln:– At maximum efficiency, Core loss P_i = Copper loss P_c at full-load.

$$\therefore 0.98 = \frac{100 \times 1}{100 \times 1 + 2P_c} \Rightarrow P_c = 1.02\,\text{kW}$$

For 12-hours—

Energy output $= 20 \times 12 = 240$ kWh.

Copper loss at $\frac{20}{0.5} = 40$ KVA is $\left(\frac{40}{100}\right)^2 \times 1.02 = 0.1632$ kW.

Energy input $= (\underbrace{20}_{\text{output}} + \underbrace{0.1632}_{\text{copper loss}} + \underbrace{1.02}_{\text{Core loss}}) \times 12 = 254.2$ kWh.

For next 6 hours —

Energy output $= 45 \times 6 = 270$ kWh.

Copper loss at $\frac{45}{0.9} = 50$ KVA is $\left(\frac{50}{100}\right)^2 \times 1.02 = 0.255$ kW.

Energy input $= (45 + 0.255 + 1.02) \times 6 = 277.65$ kWh.

For last 6-hours —

Energy output $= 80 \times 6 = 480$ kWh.

Copper loss at $\frac{80}{0.8} = 100$ KVA (full-load) is 1.02 kW.

∴ Energy input $= (80 + 1.02 + 1.02) \times 6 = 492.24$ kWh.

$$\therefore \text{All-day efficiency} = \frac{240 + 270 + 480}{254.2 + 277.65 + 492.24} = 0.967 = 96.7\%$$

Separation of hysterisis and eddy current losses:-

For a sine flux wave,

Core loss, $P_c = K_h f B_m^x + K_e f^2 B_m^2$.

$\therefore \frac{P_c}{f} = K_h B_m^x + K_e f B_m^2$ —— ①

$\sqrt{2}\pi f N \phi_{max}$. or, $\frac{V}{f} = \sqrt{2}\pi N \phi_{max} = \sqrt{2}\pi N B_m A_i$

Now, $V \simeq E_1 = \cancel{\sqrt{2}\pi f N \phi_{max}} = \cancel{\sqrt{2}\pi N B_m A_i}$

For any transformer, ~~B_m~~ N and A_i are constants.

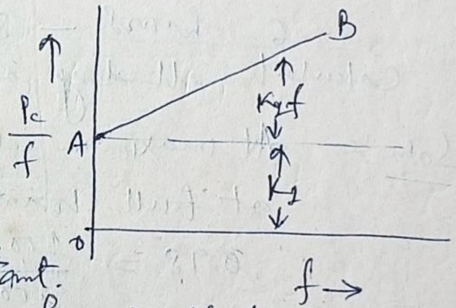
$B_m \propto \frac{V}{f}$.

For a particular value of $\frac{V}{f}$ or of maximum flux-density B_m, equation ① can be written as,

$\frac{P_c}{f} = K_1 + K_2 f$ —— ②

where, $K_1 = K_h B_m^x$ and $K_2 = K_e B_m^2$.

Values of K_1 and K_2 can be determined by performing open circuit test on the transformer. During this test V and f are varied together so as to keep $\frac{V}{f}$ (and therefore B_m) almost constant.



A wattmeter during the o.c. open circuit test, registers the core loss. After P_c, V and f are recorded, $\frac{P_c}{f}$ is plotted against f.

So, $K_1 = OA$ and slope of $AB = K_2$.

From ②, $P_c = K_1 f + K_2 f^2 = P_h + P_e$.

$\therefore P_h = K_1 f$ and $P_e = K_2 f^2$.

Problem:- A transformer has the no-load loss of 55 W. when the primary voltage is 250 V. of frequency 50 Hz. and the no-load loss of 41 W. when the primary voltage is 200 V. of frequency 40 Hz. Determine the hysterisis and eddy current loss at the above conditions.

Soln.:- At no-load, induced voltage = applied voltage.

Here, the ratio $\frac{E_1}{f}$ is identical in the two cases.

$\therefore$ Hysterisis loss, $P_h = K_1 f$

Eddy current loss, $P_e = K_2 f^2$.

Now $\frac{P_c}{f} = K_1 + K_2 f$.

$\therefore \frac{55}{50} = K_1 + K_2 \cdot 50 \Rightarrow K_1 + 50K_2 = 1.1$

and $\frac{41}{40} = K_1 + K_2 \cdot 40 \Rightarrow K_1 + 40K_2 = 1.025$

$\therefore K_1 = 0.725$ and $K_2 = 0.0075$.

$\therefore$ At 50 Hz., $P_h = 0.725 \times 50 = 36.25$ W.

$P_e = \cancel{0.0075 \times 50)}\ 55 - 36.25 = 18.75$ W

At 40 Hz., $P_h = 0.725 \times 40 = 29$ W.

$P_e = 41 - 29 = 12$ W.

Parallel operation of Single phase transformers :-

For supplying a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it.

Conditions for parallel operation are —

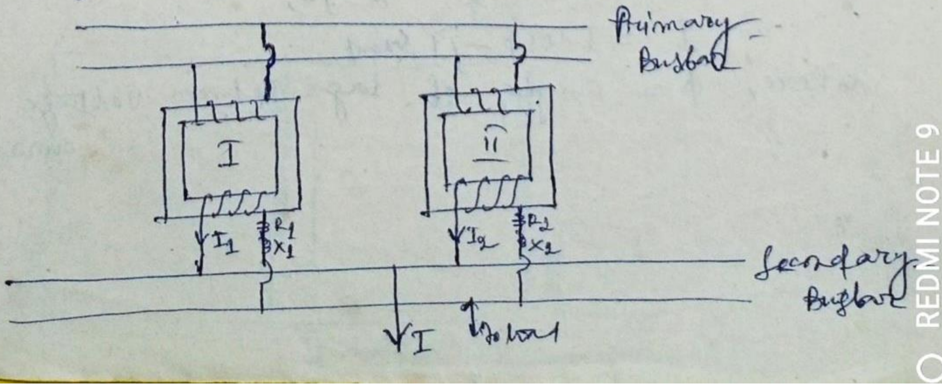
(i) Primary windings of the transformers should be suitable for the supply system voltage and frequency.

(ii) The transformers should be properly connected with regard to polarity.

(iii) The voltage ratings of both primaries and secondaries should be identical.

(iv) The percentage impedances should be equal in magnitude & and have the same $\frac{X}{R}$ - ratio in order to avoid circulating currents and operation at different power - factors.

(v) With transformers having different KVA - ratings, the equivalent impedances should be inversely proportional to the individual KVA - rating if circulating currents are to be avoided.



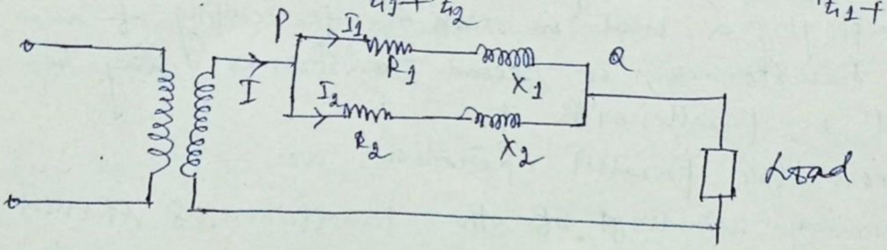
If the transformation ratios are same, the voltages across the secondary terminals will be the same. When these are connected to form a secondary bus, the voltage will not vary with the changing load.

If $\frac{R_1}{X_1} = \frac{R_2}{X_2}$, then I_1 and I_2 will be in phase.

R_1, X_1 and R_2, X_2 — are equivalent resistances, reactances of transformers I and II — respectively.

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} \quad \text{and} \quad I_1 + I_2 = I$$

$$\therefore I_1 = \frac{Z_2}{Z_1 + Z_2} \cdot I \quad \text{and} \quad I_2 = \frac{Z_1}{Z_1 + Z_2} \cdot I$$



$Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$.

Currents, $I_1 = x_1 + jy_1$ and $I_2 = x_2 + jy_2$.

Since, voltage drop across PQ are same,

$$(R_1 + jX_1) I_1 = (R_2 + jX_2) I_2$$

and $I_1 + I_2 = I$.

$$\therefore I_1 = \frac{(R_2 + jX_2) I}{(R_1 + R_2) + j(X_1 + X_2)}$$

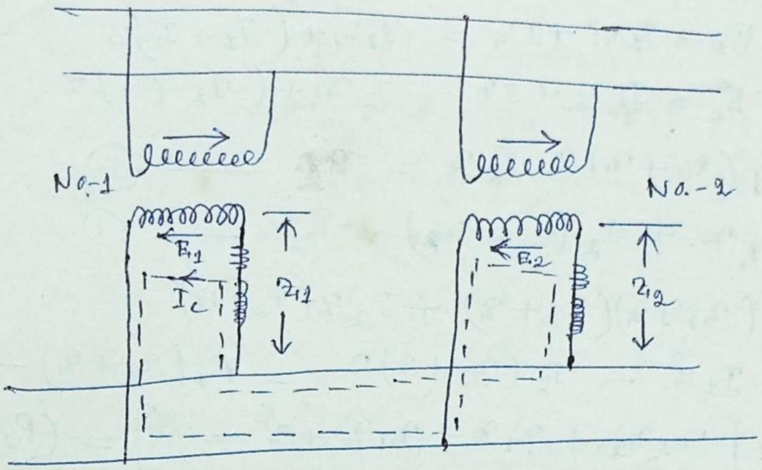
$$I_2 = \frac{(R_1 + jX_1) I}{(R_1 + R_2) + j(X_1 + X_2)}$$

Taking ~~V_2 as~~ secondary terminal voltage V_2 as reference vector, $V_2 = V_2 + j0$,

$$I = I\cos\phi - jI\sin\phi.$$

where, ϕ = angle of lag between voltage and current.

Inequality of turns ratio :-



If the turns-ratios are not equal, the secondary induced e.m.f.s are not equal. The inequality will produce a circulating current which will flow even on no-load condition. On-load, this circulating current will be super-imposed on the load currents.

Let, $E_1 > E_2$.

$\therefore$ Difference in voltage acting on the local circuit,

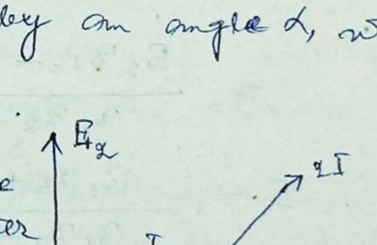
$$E_c = E_1 - E_2.$$

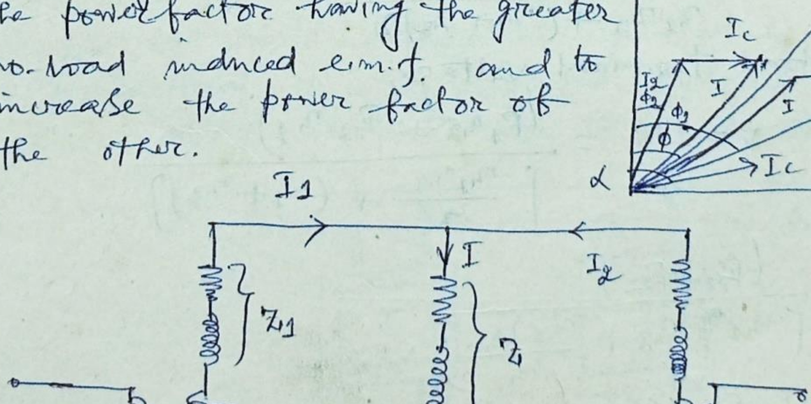
$$\therefore \quad I_c = \frac{E_1 - E_2}{Z_1 + Z_2}.$$

This current will lag E_2 by an angle α, where,

$$\alpha = \tan^{-1}\left(\frac{X_1 + X_2}{R_1 + R_2}\right).$$

It is seen that, the effect of circulating current is to reduce the power factor having the greater no-load induced e.m.f. and to increase the power factor of the other.





Transformer in parallel

Z_1, Z_2 — are transformer impedances
Z — is the load impedance.

$$E_1 = I_1 Z_1 + I Z = I_1 Z_1 + (I_1 + I_2) Z$$
$$E_2 = I_2 Z_2 + I Z = I_2 Z_2 + (I_1 + I_2) Z.$$

$$\therefore \quad I_1 (Z_1 + Z) + I_2 Z = E_1 \quad \text{———} \ (1)$$
$$I_1 Z + I_2 (Z_2 + Z) = E_2 \quad \text{———} \ (2)$$

$$\therefore \quad I_1 (Z_1 + Z)(Z_2 + Z) + I_2 Z (Z_2 + Z)$$
$$- I_1 Z^2 - I_2 (Z_2 + Z) Z = E_1 (Z_2 + Z) - E_2 Z$$
$$\Rightarrow \quad I_1 \left[Z_1 Z_2 + Z_2 Z + Z_1 Z + \cancel{Z^2} - \cancel{Z^2} \right] = (E_1 - E_2) Z + E_1 Z_2$$
$$\Rightarrow \quad I_1 = \frac{E_1 Z_2 + (E_1 - E_2) Z}{Z_1 Z_2 + (Z_1 + Z_2) Z}$$

$$I_1 (Z_1 + Z) Z + I_2 Z^2 - I_1 Z (Z_1 + Z) - I_2 (Z_2 + Z)(Z_1 + Z)$$
$$= E_1 Z - E_2 (Z_1 + Z)$$

$$\Rightarrow \quad I_2 \left[\cancel{Z^2} - Z_1 Z_2 - Z_1 Z - Z_2 Z - \cancel{Z^2} \right] = (E_1 - E_2) Z - E_2 Z_1$$

$$\Rightarrow \quad I_2 = \frac{(E_1 - E_2) Z - E_2 Z_1}{- [Z_1 Z_2 + (Z_1 + Z_2) Z]}$$

$$= \frac{E_2 Z_1 - (E_1 - E_2) Z}{Z_1 Z_2 + (Z_1 + Z_2) Z}.$$

$$\therefore \quad I = \frac{E_1 Z_2 + E_2 Z_1}{Z_1 Z_2 + (Z_1 + Z_2) Z}.$$

Secondary load terminal voltage,

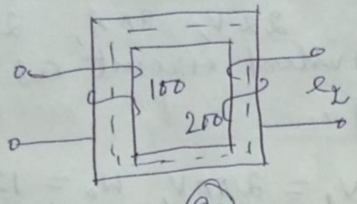
$$V = I Z = \frac{(E_1 Z_2 + E_2 Z_1)}{\left[\frac{Z_1 Z_2}{Z} + (Z_1 + Z_2) \right]}$$

$$I_c = \frac{(E_1 - E_2)}{\left[Z_1 + Z_2 + \frac{Z_1 Z_2}{Z} \right]}.$$

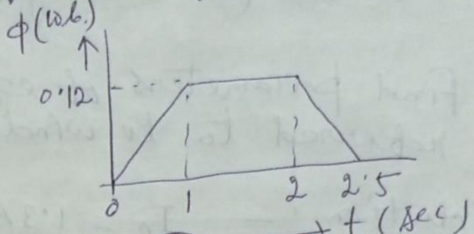
At no-load, Z is infinity,

$$\therefore \quad I_c = \frac{E_1 - E_2}{Z_1 + Z_2}.$$

(*) The nature of mutual flux variation in the core of a transformer is shown in the following figure. Sketch the nature of variation of the induced emf (e_2) in the secondary winding.



(a)



(b)

Solution :- $e_2 = -N_2 \frac{d\phi}{dt}$

For $0 < t < 1$, $\frac{d\phi}{dt} = \frac{0.12 - 0}{1 - 0} = \frac{0.12}{1} = 0.12$

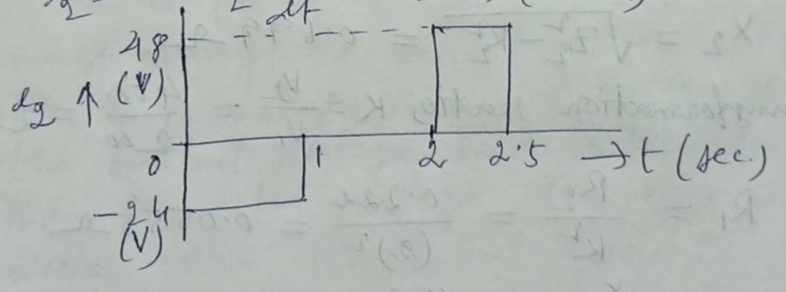
$\therefore e_2 = -N_2 \frac{d\phi}{dt} = -200 \times 0.12 = -24$ V.

For $1 < t < 2$, $\frac{d\phi}{dt} = 0$

$\therefore e_2 = -N_2 \frac{d\phi}{dt} = 0.$

For $2 < t < 2.5$, $\frac{d\phi}{dt} = \frac{0 - 0.12}{2.5 - 2} = \frac{-0.12}{0.5} = -0.24$ V.

$\therefore e_2 = -N_2 \frac{d\phi}{dt} = -200 \times (-0.24) = 48$ V.



(*) A 10 KVA, 200/400V, 50 Hz Single-phase transformer gave the following test results:

oc test (hv winding open) : 200V, 1.3 A, 120 W.

sc test (lv winding short-circuited): 22 V, 30 A, 200 W.

Find parameters of equivalent circuit as referred to lv winding.

Solution :- $I_0 = 1.3\,A$, $V_1 = 200\,V$, $W_0 = 120\,W$.

From oc test :- $\cos\phi_0 = \dfrac{W_0}{V_1 I_0} = \dfrac{120}{200 \times 1.3} = 0.4615$

$\therefore I_c = I_0 \cos\phi_0 = 1.3 \times 0.4615 = 0.6\,A.$

$I_m = I_0 \sin\phi_0 = 1.15\,A.$

$\therefore R_0 = \dfrac{V_1}{I_c} = \dfrac{200}{0.6} = 333.3\,\Omega$

$X_0 = \dfrac{V_1}{I_m} = \dfrac{200}{1.15} = 173.42\,\Omega$

From sc test :- $Z_{02} = \dfrac{V_{sc}}{I_{sc}} = \dfrac{22}{30} = 0.733\,\Omega$

$R_2 = \dfrac{W_{sc}}{I_{sc}^2} = \dfrac{200}{(30)^2} = 0.222\,\Omega$

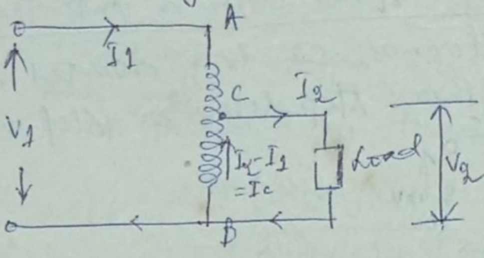
$\therefore X_2 = \sqrt{Z_{02}^2 - R_2^2} = 0.699\,\Omega$

Transformation ratio, $K = \dfrac{V_2}{V_1} = \dfrac{400}{200} = 2$

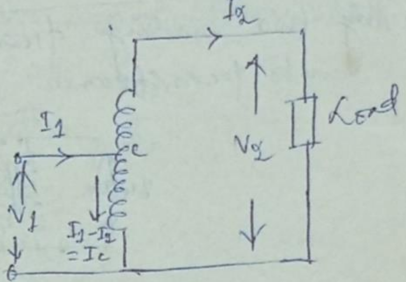
$\therefore R_1 = \dfrac{R_2}{K^2} = \dfrac{0.222}{(2)^2} = 0.0555\,\Omega$

$X_1 = \dfrac{X_2}{K^2} = \dfrac{0.699}{(2)^2} = 0.175\,\Omega$

Auto transformer :- It is a transformer with one winding only, part of this being common to both primary and secondary.



(a)



(b)

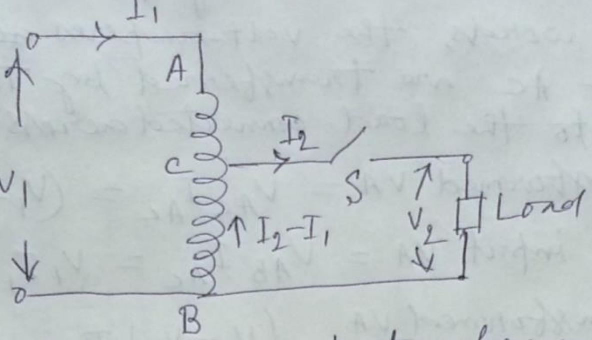
AB – is primary winding having N_1 – turns.

BC – is secondary winding having N_2 – turns.

Negtecting iron losses and no-load current,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K.$$

Auto-transformer



Differences between an auto-transformer and resistive potential divider :-

① A resistive potential divider cannot step up the voltage, where as it is possible in an auto-transformer.

② The potential divider has more losses, and is, therefore, less efficient.

③ In a potential divider, almost entire power to load flows by conduction, where as, in auto-transformer, a part of the power is conducted and the rest is transferred to load by transformer action.

④ In a potential divider, the input current must always be more than the output current, this is not so in an auto-transformer. If the output voltage in auto-transformer is less than the input voltage, the load current is more than the input current.

Transformed and conducted VA :-

In the above figure, $I_{BC} = I_2 - I_1$

current in $AC = I_1$

$\therefore$ mmf of winding $AC = I_1(N_1 - N_2)$

$= I_1 N_1 - I_1 N_2 = I_2 N_2 - I_1 N_2$

$= (I_2 - I_1) N_2 = I_{BC} N_2$

$=$ mmf of winding BC.

It is, therefore, seen that the transformer action takes place between winding AC and winding BC. In other words, the volt-amperes ~~aer~~ across winding AC are transferred by transformer action to the load connected across winding BC.

$\therefore$ Transformed VA $= V_{AC} I_{AC} = (V_1 - V_2) I_1$

Total input VA $= V_{AB} I_{AC} = V_1 I_1 =$ output VA.

$$\therefore \frac{\text{Transformed VA}}{\text{Input VA}} = \frac{(V_1 - V_2) I_1}{V_1 I_1} = \frac{V_1 - V_2}{V_1}$$

$$= 1 - \frac{V_2}{V_1} = 1 - K$$

Out of input volt-amperes $V_{AB} I_{AC}$, only $V_{AC} I_{AC}$ are transformed to the output by transformer action. The rest of the volt-amperes are conducted directly from the input.

$\therefore$ Conducted VA $= V_{AB} I_{AC} - V_{AC} I_{AC}$

$= (V_{AB} - V_{AC}) I_{AC} = V_{BC} I_{AC} = V_2 I_1$

$$\therefore \frac{\text{Conducted VA}}{\text{Input VA}} = \frac{V_2 I_1}{V_1 I_1} = \frac{V_2}{V_1} = K.$$

$$\therefore \frac{\text{Transformed power}}{\text{Input power}} = 1 - K$$

and $$\frac{\text{Conducted power}}{\text{Input power}} = K.$$

Advantages of auto-transformer over two-winding transformer:

1. Saving of Copper

Saving of Cu:-

Volume and hence weight of Cu, is proportional to length and area of cross section of the conductors.

Length of conductor $\propto$ No of turns

Cross section depends on current.

$\therefore$ Weight is proportional to product of current and No of turns.

Weight of Cu in section AC $\propto (N_1 - N_2) I_1$

Weight of Cu in section BC $\propto N_2 (I_2 - I_1)$

$\therefore$ Total weight of Cu in autotransformer

$$\propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1).$$

For a two winding transformer,

total weight of Cu $\propto (N_1 I_1 + N_2 I_2)$

$$\therefore \frac{\text{Weight of Cu in autotransformer}}{\text{weight of Cu in ordinary transformer}} = \frac{(N_1 - N_2) I_1 + N_2 (I_2 - I_1)}{N_1 I_1 + N_2 I_2}$$

$$\Rightarrow \frac{N_1 I_1 + N_2 I_2 - 2 N_2 I_1}{N_1 I_1 + N_2 I_2} = 1 - \frac{2 N_2}{N_1 + N_2 \cdot \frac{I_2}{I_1}}$$

$$\Rightarrow 1 - \frac{2 \cdot \frac{N_2}{N_1}}{1 + \frac{N_2}{N_1} \cdot \frac{I_2}{I_1}} = 1 - \frac{2K}{2} = 1 - K.$$

So, there is a saving of Cu in auto-transformer. These are used for transformation ratios around unity and not for very wide ranges.

2. Owing to the reduction in conductor and core materials, the ohmic losses in conductor and the core loss are reduced. Therefore, an auto-transformer has higher efficiency than a two-winding transformer of the same output.

3. Reduction in the conductor material means lower value of ohmic resistance. A part of the winding being common, leakage flux and, therefore, leakage reactance is less. In other words, an auto-transformer has lower value of leakage impedance and has superior voltage regulation than a tw0-winding transformer of the same output.

Disadvantages:

1. If the ratio of transformation, k differ far from unity, the economic advantages of auto-transformer over two-winding transformer decrease.

2. The main disadvantage of an auto-transformer is due to the direct electrical connection between the low tension and high tension sides. If primary is supplied at high voltage, then an open circuit in the common winding BC would result in the appearance of dangerously high voltage on the lv side. This high voltage may be detrimental to the load and the persons working there. Thus, a suitable protection must be provided against such an occurrence.

Uses:

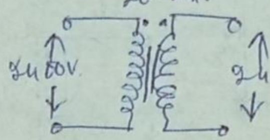
Single-phase and three-phase auto-transformers are mainly employed_

(i) for interconnecting power systems having voltage ratios, not differing far from unity.

(ii) for obtaining variable output voltages. When used as variable ratio auto-transformers, these are known by their trade names, such as, variac, dimmerstat, autostat etc.

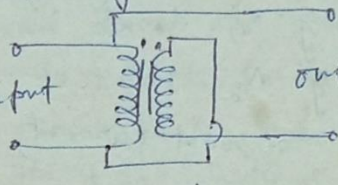
Conversion of 2-winding transformer in to Auto-transformer:-

Any two winding transformer can be converted into an auto transformer either step down or step-up.

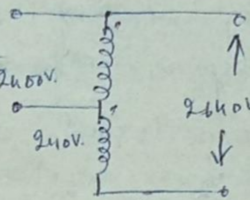


(a)

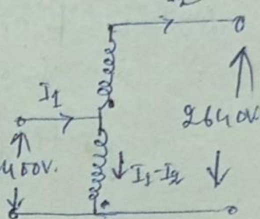
Additive polarity:-



(i)



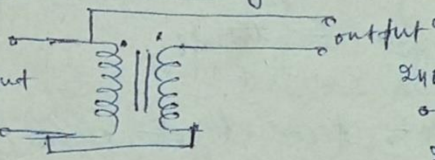
(ii)



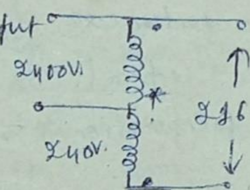
(iii)

Because of additive polarity $V_2 = 2400 + 240 = 2640V.$
and $V_1 = 2400V.$

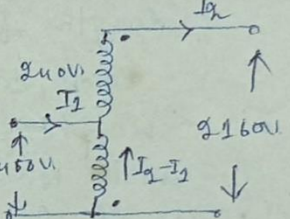
Subtractive Polarity:-



(i)



(ii)



(iii)

In this case, transformer acts as step down transformer.

Problem:- ① Two single phase transformers rated for 25-KVA. and 50-KVA. respectively have their primaries connected in parallel across an 1100 V. supply and have a primary to secondary ~~turns~~ voltage ratio 5:1. Their secondaries are also connected in parallel and supply a common load of 300Amps. at 0.8 p.f. lag. Referred to the secondary, the equivalent resistances are 0.044 and 0.011 ohms respectively and the equivalent reactances are 0.072 and 0.042 ohms respectively.

What current does each transformer supply to the load? Express each current as a fraction of the full-load current of the transformer.

Soln:- It is seen that the ratio of R/X is not equal but the transformation ratio is equal. Hence, both the secondary voltages are 220V.

Current supplied by transformer-I,

$$I_1 = \frac{I(R_2+jX_2)}{[(R_1+R_2)+j(X_1+X_2)]}$$

$$= \frac{300(0.011+j0.042)}{(0.055+j0.114)}$$

$$= \frac{300(0.011+j0.042)(0.055-j0.114)}{(0.055+j0.114)(0.055-j0.114)}$$

$$= 101+j21.6 = 103.3 \text{ Amps.}$$

Similarly, $$I_2 = \frac{300(R_1+jX_1)}{[(R_1+R_2)+j(X_1+X_2)]}$$

$$= \frac{300(0.044+j0.072)}{(0.055+j0.114)} = 199-j17.8$$

$$= 199.1 \text{ A.}$$

Full load current of transformer-I $= \frac{25,000}{220} = 113.6$ Amps

Full load current of transformer-II $= \frac{50,000}{220} = 227.2$ Amps

$\therefore$ Transformer-I supplies $= \frac{103.3}{113.6} = 90.9\%$ full-load current

Transformer-II supplies $= \frac{199.1}{227.2} = 87.6\%$ full load current.

(2) Two single phase transformers in parallel supply a load of 500 Amps. at 0.8 p.f. lagging and at 400V. Their equivalent impedances referred to secondary winding are $(2+j3)$ ohms. and $(2.5+j5)$ ohms respectively. Compute the current and KVA supplied by each transformer.

Soln:- Current supplied by transformer-I

$$= \frac{500(2.5+j5)}{4.5+j8} = 304.6 \text{ A.}$$

current by transformer-II $= \dfrac{500(2+j3)}{4.5+j8} = 196.4$ A.

∴ KVA. supplied by transformer-I

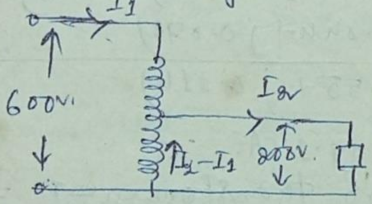
$= \dfrac{304.6 \times 400}{1000} = 121.84$ KVA.

KVA. supplied by transformer-II.

$= \dfrac{196.4 \times 400}{1000} = 78.56$ KVA.

(3) A 200/400V., 20-KVA., 50-Hz. transformer is connected as an auto-transformer to work of 600/200 V. supplies. Determine the KVA. rating of the auto-transformer. With a load of 20-KVA. 0.8 p.f. lagging connected to the 200V. terminals, find the currents in the load and in the two winding sections.

Soln.:- The two windings must be connected in series with the proper polarity so that 600V. can be applied across the total windings.



With 20-KVA. load, the load current,

$$I_2 = \frac{20 \times 1000}{200} = 100 \text{ A.}$$

$$I_1 = \frac{20 \times 1000}{600} = 33.33 \text{ A.}$$

∴ Current in 400V. section, $I_1 = 33.33$ A.

" " 200 V. " $\cancel{I_2} = I_2 - I_1$

$= 66.66$ A.

400 V. winding can carry a maximum current of 50A.

$\therefore I_1 = 50$ A.

200 V. winding can carry a maximum current of 100 A.

$\therefore I_2 - I_1 = 100$ A.

$\therefore I_2 = 150$ A.

∴ KVA. rating of the auto-transformer

$= V_1 I_1 = 600 \times 50 = 30$ KVA. $= V_2 I_2 = 200 \times 150 = 30$ KVA.

Three phase transformer connections and vector groups:-

A Bank of three transformers or a 3-phase transformer may have its primary and secondary windings connected in star, delta or zigzag. The choice of particular connections depends upon the service conditions. The commonly employed connections are star, delta, zig-zag and these are designated by the symbols Y, D and Z respectively. Zigzag connection is also called inter-connected star or "interstar".

Polyphase (three phase) transformers are allotted symbols giving the type of phase connection and the angle of advance turned through in passing from the vector representing the h.v. emf to that representing the l.v. emf at the corresponding terminal. The angle may be indicated by a clock face hour figure, the h.v. vector being the 12 o'clock (zero) and the corresponding l.v. vector being represented by the hour hand. Thus, "Yd11" represents a (h.v. star and l.v. delta connected) three-phase transformer with the l.v. emf vector at 11 o'clock position i.e., +30° in advance of the h.v. emf which is at 12 o'clock position.

Depending on the phase displacement of the voltages of h.v. and l.v. sides, the transformers are classified into groups called 'Vector groups'. Transformers with the same phase displacement between the h.v. and l.v. sides are classified into one group. For satisfactory parallel operations of transformers, they should belong to the same vector group. For example, a star-star connected three-phase transformer (or bank of three single-phase transformers) can be connected in parallel with another 3-phase transformer (or bank of three single-phase transformers) whose windings are either star-star or delta-delta connected. A star-star connected transformer cannot be connected in parallel with another star-delta connected transformer as this may result in short-circuiting of the secondary side.

The groups into which all possible three-phase transformer connections are classified as—

group-1 : Zero degree phase displacement (Yy0, Dd0)

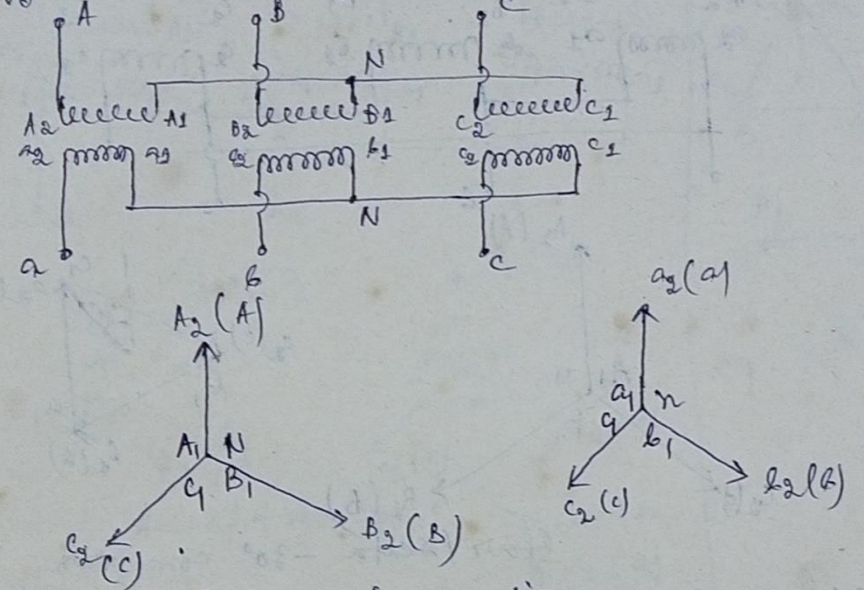
group-2 : 180° phase displacement (Yy6, Dd6)

group-3 : 30° lag phase displacement (Dy1, Yd1)

group-4 : 30° lead phase displacement (Dy11, Yd11).

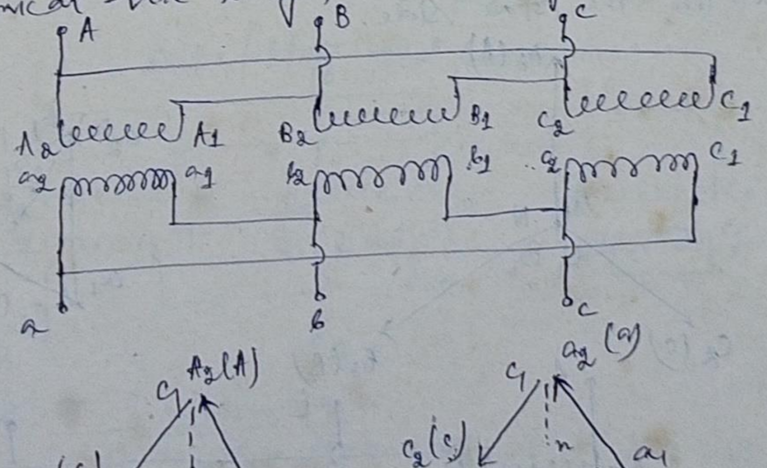
Three-phase transformer connections :-

① **Star/-Star (Y/Y) - connection** :- This connection is most economical for small high voltage transformers, because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only $\frac{1}{\sqrt{3}}$ of line voltage).



Star/Star 0° connection

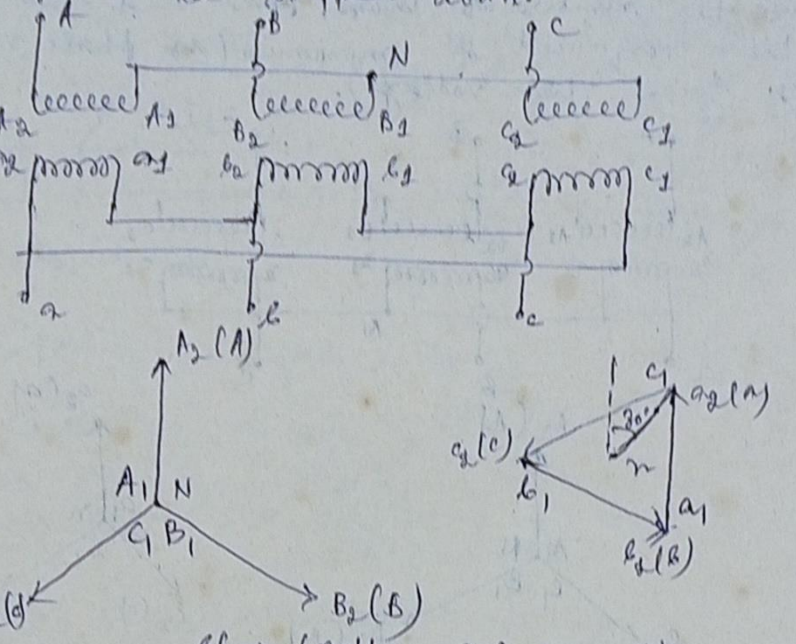
② **Delta/Delta (Δ/Δ) - connection** :- This connection is economical for large, low voltage transformers.



Delta/Delta 0° connection

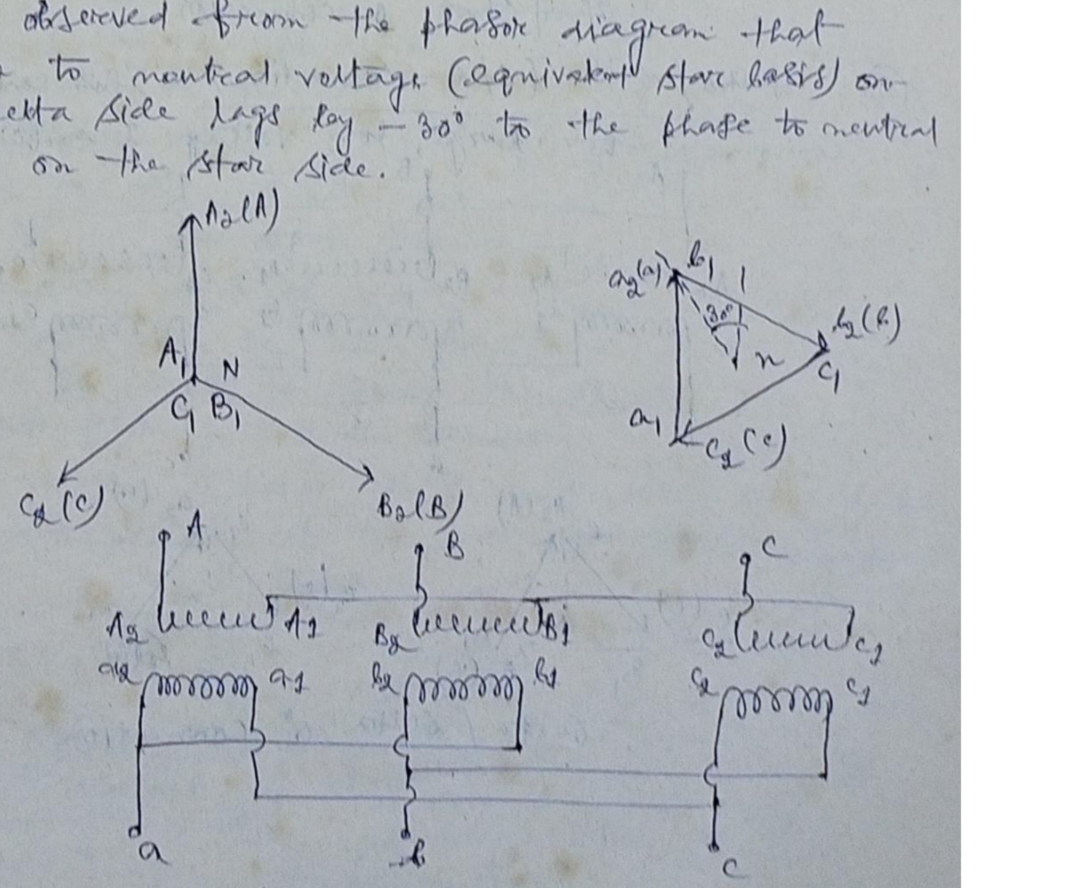
(3) Star/Delta (Y/Δ) – Connection:-

The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down.



Star/delta –30° connection.

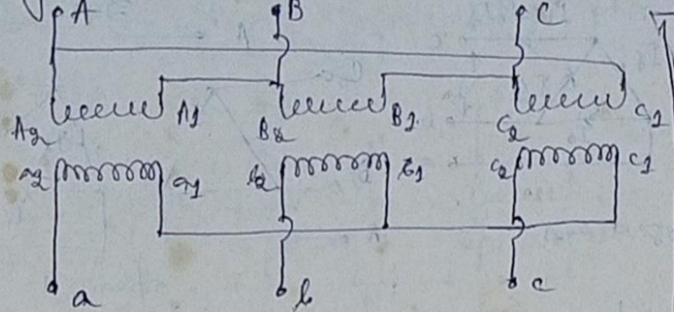
It is observed from the phasor diagram that phase a to neutral voltage (equivalent star basis) on the delta side lags by –30° to the phase to neutral voltage on the star side.

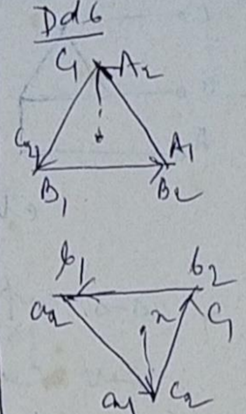


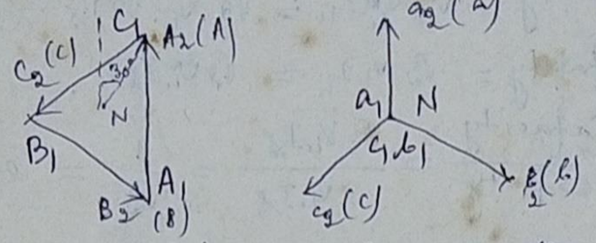
Star/delta +30° – connection

(4) Delta / Star (Δ - Y) – Connection :–

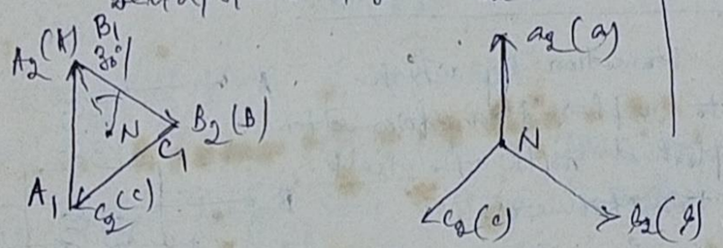
This connection is generally employed where it is necessary to step-up the voltage, such as at the beginning of high tension transmission system.



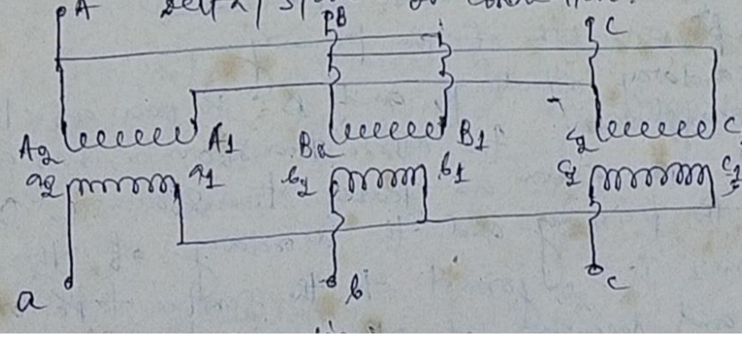




delta/star +30° – connection

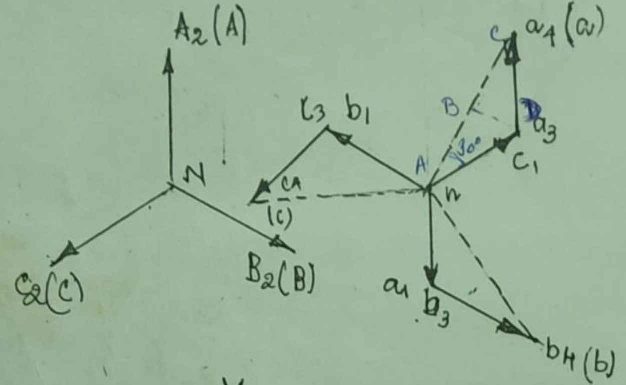


delta / star –30° connection



Zigzag/star (Yz1 or Yz11) :-

The winding of each phase on the star side is divided into two equal halves, as shown.



Yz1 Connection.

Say, AC = V

$AB = \frac{V}{2}$

$AD \cos 30^\circ = AB$

$\Rightarrow AD = \frac{2}{\sqrt{3}} AB$

$= 1.15\, AB$

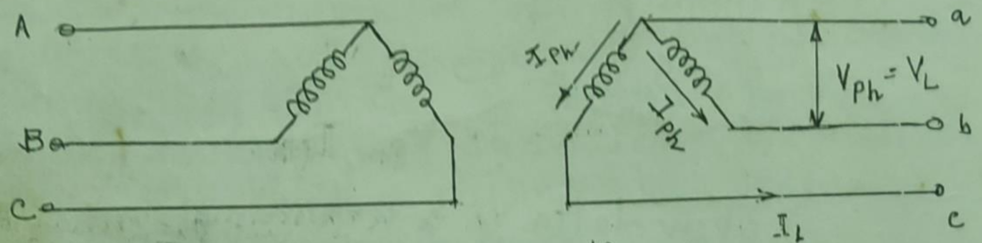
$\therefore$ ~~$a_1a_2 = 115\%$~~

$a_1 a_n = 15\%$ more than normal a_1a_2 of star connection.

Zigzag or inter star connection is primarily used to suppress the third harmonic emfs between the line and neutral. Since the phase voltages are composed on the zigzag side of two half-voltages with a phase difference of 60°, 15% more turns are required for a given total voltage per phase compared with a normal phase connection, which may necessitate an increase in the frame size over that normally used for the rating. The zigzag/star connection has been employed where delta connections were mechanically weak (on account of large no of turns and small copper sections) in h.v. transformers. Inter-star connections are also used for rectifier circuits.

Open-delta or V-connection:-

In a delta/delta connection if one of the transformers is disconnected, the resulting connection is known as open-delta or V-connection. In a open-delta or V-connection, the power can still be supplied, though at a reduced level.



V-connection:

KVA delivered by open delta :-

let V_{ph} = rated phase voltage, in each of the three transformer secondaries.

I_{ph} = rated phase current in each of the three transformer secondaries.

When all the three transformers are connected in closed delta, then

line voltage, V_L = Phase voltage, V_{ph}

line current, $I_L = \sqrt{3}\, I_{ph}$.

∴ VA rating of the bank of three transformers in delta

$$S_{delta} = \sqrt{3}\, V_L I_L = \sqrt{3}\, V_{ph} (\sqrt{3}\, I_{ph}) = 3\, V_{ph} I_{ph}.$$

when one transformer is entirely removed from the closed delta,

then for an open-delta,

line voltage, V_L = Phase voltage, V_{ph} as before.

line current, I_L = Phase current, I_{ph}.

∴ VA rating of the open delta connection,

$$S_{open-delta} = \sqrt{3}\, V_L\, I_L$$
$$= \sqrt{3}\, V_{ph}\, I_{ph}.$$

$$\therefore \frac{\text{open delta VA or KVA rating, } S_{open-delta}}{\text{closed delta VA or KVA rating, } S_{delta}}$$

$$= \frac{\sqrt{3}\, V_{ph}\, I_{ph}}{3\, V_{ph}\, I_{ph}} = \frac{1}{\sqrt{3}}$$

$$\frac{S_{open-delta}}{S_{delta}} = \frac{1}{\sqrt{3}} = 0.58$$

Thus the open delta connection has a VA or KVA rating of $\frac{1}{\sqrt{3}} = 0.58$ of the rating of the normal delta/delta connection.

It may be noted that open-delta operates at a lower KVA capacity $(= \sqrt{3}\, V_P\, I_{ph} \times 10^{-3})$ compared with the sum of the individual transformer KVA capacities $(= 2\, V_{ph}\, I_{ph} \times 10^{-3})$.

The ratio $\frac{\text{Actual available KVA}}{\text{Sum of the KVA ratings of the transformers installed}}$,

is called the utilisation, or rating, factor for a particular type of connections.

For open-delta connection, the utilisation factor is $\frac{\sqrt{3}\, V_{ph} I_{ph}}{2 V_{ph} I_{ph}} = 0.866$.

and for a closed delta, the utilisation factor is unity.

Uses :- The open delta connection is used on transmission ~~or~~ lines or distribution systems, which have been recently put into service. In doing so, the cost of one transformer unit is saved and provision is also made for further raising the system KVA capacity in future. The capacity of the open delta should be sufficient enough to meet the growth of load, at least for some years more. When the load demand exceeds the installed KVA capacity in open delta, the third transformer is added to form the closed delta, thereby augmenting the capacity of the system from $\sqrt{3} V_{ph} I_{ph}$ to $3 V_{ph} I_{ph}$. Its further use is to maintain the continuity of supply to 3-phase loads, though at reduced level, in the case of damage to one transformer.

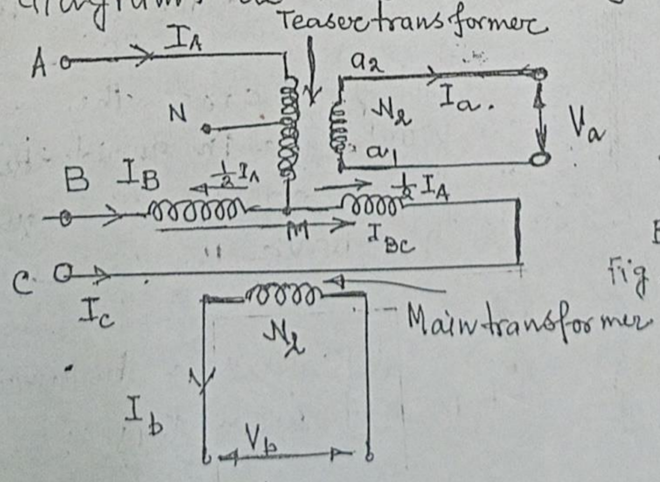
Three./Two-phase ~~conver~~ conversion (Scott Connection):.

At present three-phase energy system are by far the most common, but for certain specialized applications two-phase supplies are essential. For example two-phase energy system is required to feed power to

(i) single-phase arc furnaces.
(ii) low voltage single-phase rural supplies
(iii) electrified tracks in electric traction
(iv) two phase control motors

Charles F. Scott converted ~~three~~ two-phase to ~~two~~ three-phases at Niagara Falls and this arrange-ment used by him is known as Scott-connection, in his honour.

Scott connection is used to obtain three to two-phase transformation, and vice-versa. The underlying principle is based on the three-phase balanced voltage ~~phasor~~ triangle diagram, as shown in Fig. (b)



(a)

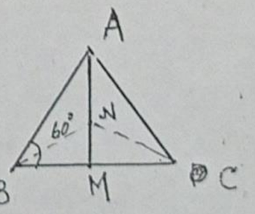


Fig. (b)

(c)

In which, it can be seen that (a) the perpendicular from the vertex A on BC at a point M gives, $BM = MC$.

(b) $AM = AB \sin 60^\circ = \frac{\sqrt{3}}{2} AB = 0.866 AB$

Thus, if a two single-phase transformers I and II in Fig (a) are so chosen that

(a) Transformer I (known as main transformer) has N_1 turns in the primary with a mid-point tap at M.

(b) Transformer II (known as teaser transformer) has $0.866\, N_1$ turns in the primary and

(c) both transformers have equal turns N_2 in the secondary, and the primaries are connected as shown in Fig (a), applications of balanced three-phase voltage across A, B and C will result in a

(a) induced counter voltage in BC and AM are in quadrature with each other.

(b) counter voltage in $AM = 0.866$ times that in BC.

~~Thus~~. That is, the voltage across the secondaries windings would be in quadrature with each other with their magnitudes equal to each other (since they have the same no. of turns).

In other words, two-phase balanced outputs will be obtained from this connection.

The neutral point on the 3-phase side, if required, could be located at the point N. Which divides the primary winding of the tertiary transformer in the ratio 1:2

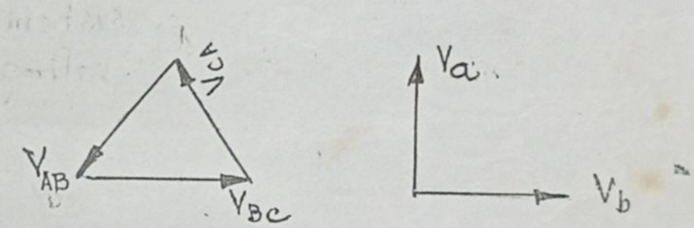
[Since $AN = V/\sqrt{3}$ and $AM = \frac{\sqrt{3}}{2}V$

$$\therefore \quad MN = AM - AN = \frac{\sqrt{3}}{2}V - \frac{V}{\sqrt{3}}$$

$$= \frac{3-2}{2\times\sqrt{3}}V = \frac{1}{2\sqrt{3}}V$$

$\cancel{\therefore AN : MN =}$

$$\therefore \quad MN : AN = \frac{V}{2\sqrt{3}} : \frac{V}{\sqrt{3}} = \frac{1}{2} : 1 = 1:2 \;]$$



Voltage-phasor diagram.

Load Analysis :-

If the Secondary load currents are $\bar{I}_a$ and $\bar{I}_b$, the currents on the 3-phase side can be found ~~out~~ by balancing primary and secondary ampere turns.

For the teaser transformer :-

$$\bar{I}_A \times \frac{\sqrt{3}}{2} N_1 = \bar{I}_a N_2$$

$$\text{or} \quad \bar{I}_A = 1.15\, \bar{I}_a \frac{N_2}{N_1} = 1.15\, \bar{I}_a \quad \left(\text{for } N_1/N_2 = 1\right)$$

For the main transformer :-

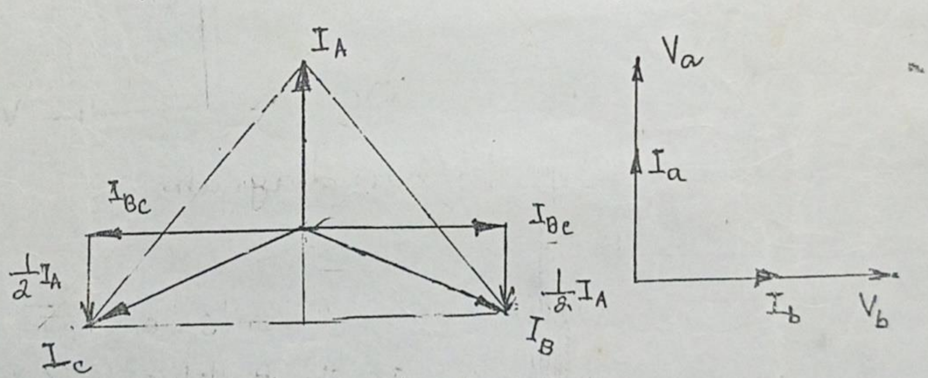
$$\overline{I}_{BC} \times N_1 = \overline{I}_b \times N_2$$

$$\overline{I}_{BC} = \overline{I}_b \times \frac{N_2}{N_1} = \overline{I}_b \quad \left(\text{for } \frac{N_2}{N_1} = 1\right)$$

$$\therefore \quad \overline{I}_B = \overline{I}_{BC} - \frac{1}{2}\overline{I}_A$$

$$\overline{I}_C = -\overline{I}_{BC} - \frac{1}{2}\overline{I}_A$$

The corresponding phasor diagram for balanced Secondary side load of unity p.f. is drawn in Fig. : from which it is obvious that the currents drawn from the 3-phase system are balanced and cophasal with star voltages.



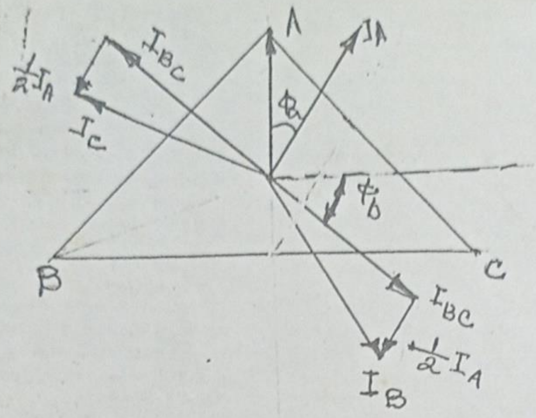
For a balanced load, $I_a = I_b$

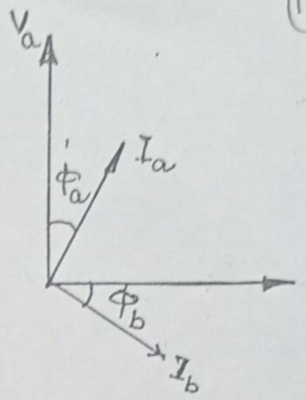
$$\therefore \quad I_A = 1.15\, I_a,$$

$$I_B = \sqrt{(I_{BC})^2 + \left(\frac{1}{2} I_A\right)^2} = \sqrt{I_a^2 + \left(\frac{1}{2} \times 1.15\, I_a\right)^2}$$

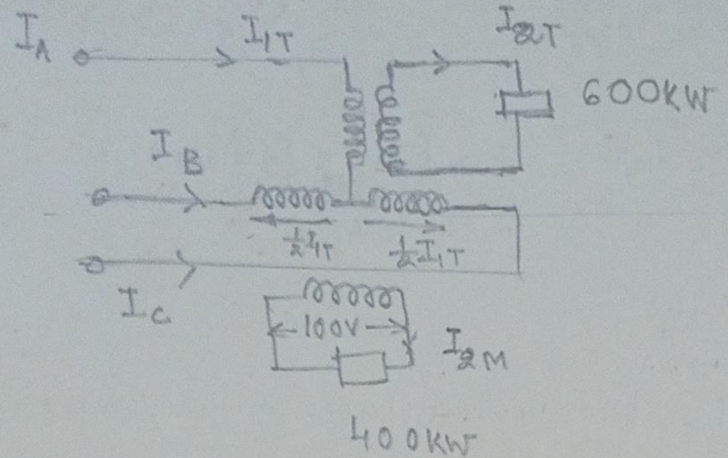
$$= 1.15\, I_a.$$

Also, $I_C = 1.15\, I_a$


A
I_A
I_{BC}
$\frac{1}{2}I_A$
I_C
ϕ_a
ϕ_b
B
C
I_{BC}
$\frac{1}{2}I_A$
I_B


V_a
I_a
ϕ_a
ϕ_b
I_b

Two 100-V single phase transformers take loads of 600 KW ~~an~~, 400 KW respectively at unity p.f and are supplied from 6.6 kV; 3-phase mains through scott-connection. Calculate the currents in the 3-phase lines. Neglect transformer losses.



Secondary teaser current, $I_{2T} = \frac{600\times10^3}{100} A = 6000A$

Secondary main current, $I_{2M} = \frac{400\times10^5}{100} A = 4,000A$.

Now the main primary current is given by the equation $I_{1M} \times T_1 = I_{2M} T_2$

or $I_{1M} = \frac{T_2}{T_1} \times I_{2M} = \frac{V_2}{V_1} \times I_{2M} = \frac{100}{6600} \times 4000 A$

$= 60.6A$

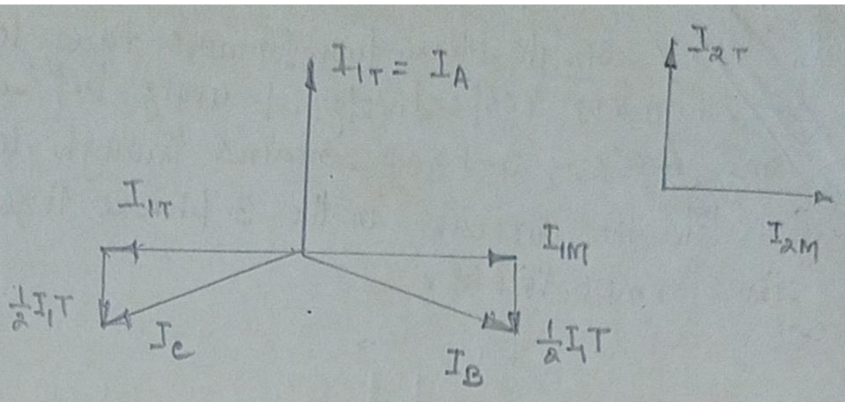
∴ Similarly teaser primary current is given by $I_{1T} \times 0.866 \times T_1 = I_{2T} T_2$

or $I_{1T} = \frac{T_2}{T_1} \times \frac{1}{0.866} \times I_{2T}$

$= \frac{V_2}{V_1} \times \frac{1}{0.866} \times I_{2T} = \frac{100}{6600} \times \frac{1}{0.866} \times 600$

$= 104.97 A$

Now ~~b~~ currents in the three phase lines are obtained from the phasor diagram as shown.



$$I_B = \sqrt{I_{1M}^2 + \left(\frac{1}{2} I_{1T}\right)^2} = \sqrt{60.6^2 + \left(\frac{1}{2} \times 104.97\right)^2} \text{ A}$$

$$= 80.16 \text{ A}$$

$I_B = I_c = 80.16 A$

$I_A = I_1 T = 104.97 A$.

So, the three phase line currents are 80.16 A, 80.16 A and 104.97 A.

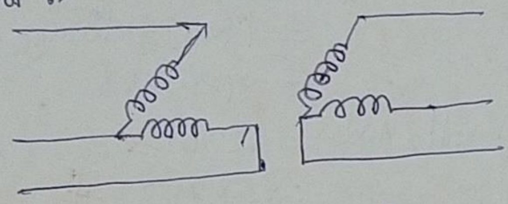
The primary and secondary windings of two transformers, each rated 250 KVA, 11/2KV and 50HZ, are connected in open delta.

Find

(a) the KVA load that can be supplied from this connection.

(b) currents on the h.v side of a delta connected 3-phase load 250 KVA, 0.8 pf lag, 2 kV is connected to the LV side r connection.

Solⁿ:-



(a) KVA rating of each transformer

$= 250\,KVA$

KVA rating of closed delta connection

$= 3 \times 250\ KVA = 750\ KVA$.

Then the KVA load that can be supplied from open delta connection

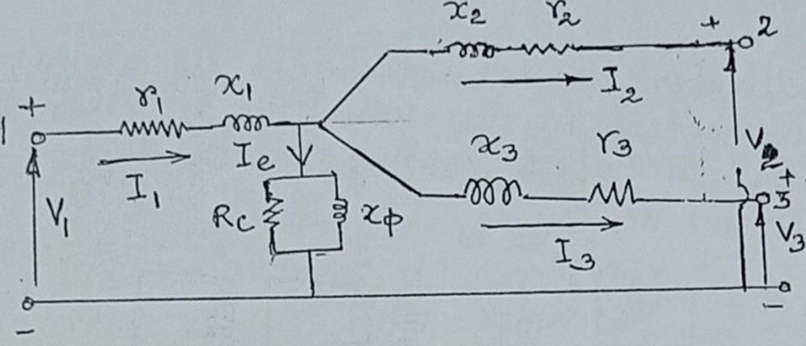
$= \frac{1}{\sqrt{3}} \times$ closed delta KVA rating.

$= \frac{1}{\sqrt{3}} \times 750\ KVA = 433\,KVA$.

(b) currents on h.v. side of a delta connected 3-phase load $= \frac{250 \times 10^3}{11 \times 10^3} = 22.73\ A$.

Equivalent circuit of a 3-Winding transformer

Star equivalent Circuit of a 3-winding transformer is shown in Fig.



r_1, r_2, r_3 = resistances of the windings 1, 2 & 3 respectively.

x_1, x_2, x_3 = equivalent leakage reactances of the windings, 1, 2 & 3 respectively.

x_ϕ = magnetising reactance.

R_e = core resistance.

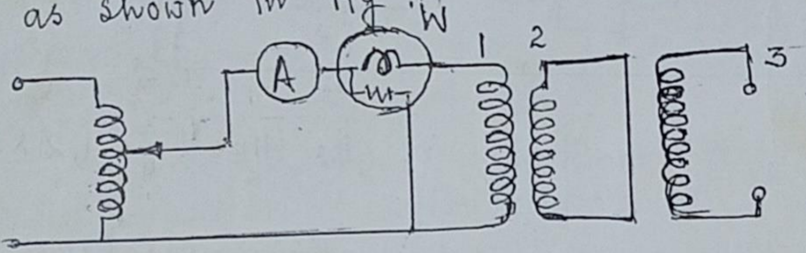
Z_1, Z_2 and Z_3 = equivalent leakage impedances of the windings 1, 2 & 3 respectively.

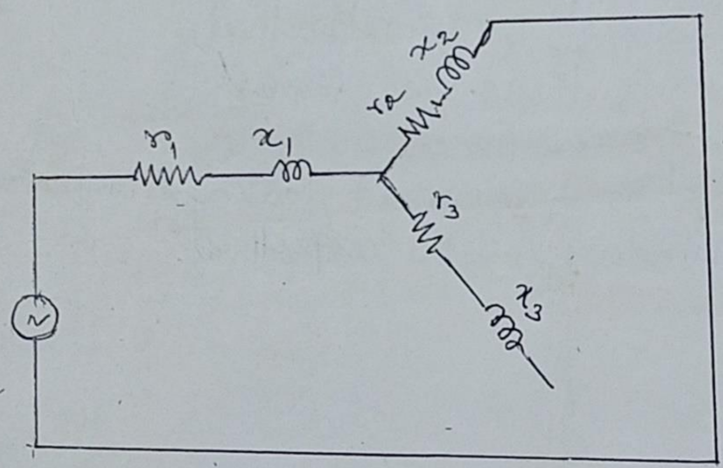
Determination of equivalent circuit parameters:-

(a) Short-Circuit test:-

Three-short circuit tests are to be performed in order to determine the equivalent leakage impedances.

1. A Wattmeter, a voltmeter and a ammeter are connected in the winding 1, winding 2 is short circuited keeping the winding 3 open as shown in Fig.





Let the voltmeter, Wattmeter and ammeter readings are V_1, W_1 and I_1 respectively.

Then the magnitude of ~~Short~~ equivalent or short circuit Impedance Z_{12} of the winding 1 and 2 is given by

$$Z_{12} = \frac{V_1}{I_1}$$

equivalent resistance $r_{12} = \frac{W_1}{I_1^2}$

and equivalent leakage reactance,

$$x_{12} = \sqrt{Z_{12}^2 - r_{12}^2}$$

From the above fig.

$$Z_{12} = r_{12} + j x_{12} = Z_1 + Z_2.$$

$$r_{12} = r_1 + r_2.$$

where $Z_1 = r_1 + j x_1$, $Z_2 = r_2 + j x_2$.

Similarly other tests are performed with the following connections :-

2. Instruments are ~~a~~ connected in the winding 1, winding 3 is shorted circuited and winding 2 is open circuited. Then the short circuit impedance Z_{13} of the windings 1 and 3 can be determined as before from the voltmeter, wattmeter and ammeter readings,

$$Z_{13} = r_{13} + j x_{13} = Z_1 + Z_3$$

$r_{13} = r_1 + r_3$; where $Z_1 = r_1 + j x_1$ and $Z_3 = r_3 + j x_3$

3. Instruments are connected in the winding 2, ~~winding~~ winding 3 is short-circuited and winding 1 is open circuited. Then the short circuit impedance Z_{23} of the winding 2 and 3 can be determined as before from the voltmeter, wattmeter and ammeter readings.

$$Z_{23} = r_{23} + j x_{23} = Z_2 + Z_3$$

$$r_{23} = r_2 + r_3,$$

$\therefore\ Z_{12} = r_{12} + j\,x_{12} = Z_1 + Z_2$

$Z_{13} = r_{13} + j\,x_{13} = Z_1 + Z_3$

$Z_{23} = r_{23} + j\,x_{23} = Z_2 + Z_3.$

Solving the above equations, we get,

$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$

$Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$

$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$

Again, $r_{12} = r_1 + r_2$

$r_{13} = r_1 + r_3$

$r_{23} = r_2 + r_3$

Solving,

$r_1 = \frac{1}{2}(r_{12} + r_{13} - r_{23})$

$r_2 = \frac{1}{2}(r_{12} + r_{23} - r_{13})$

$r_3 = \frac{1}{2}(r_{13} + r_{23} - r_{12})$

Parallel operation of three-phase transformers:

The need for parallel operation of three-phase transformers arises more frequently, since the generation, transmission and distribution of power is almost always three-phase.

The various conditions that must be fulfilled for successful parallel operation of three-phase transformers are as follows:

(a) The line voltage ratios of the transformers must be the same.

(*b*) The transformers should have equal per unit leakage impedances.

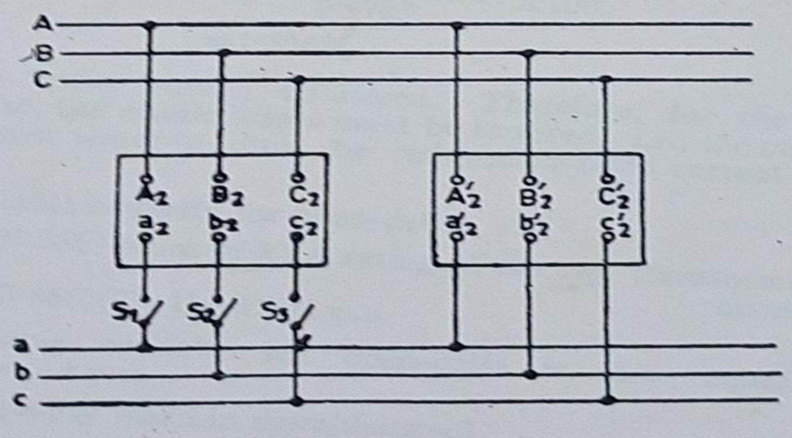
(*c*) The ratio of equivalent leakage reactance to equivalent resistance should be same for all the transformers.

(*d*) The transformers should have the same polarity.

The above four conditions are also applicable for the successful parallel operation of single-phase transformers. In addition to these four conditions, two more *essential* conditions that must be fulfilled for the parallel operation of three-phase transformers are as follows :

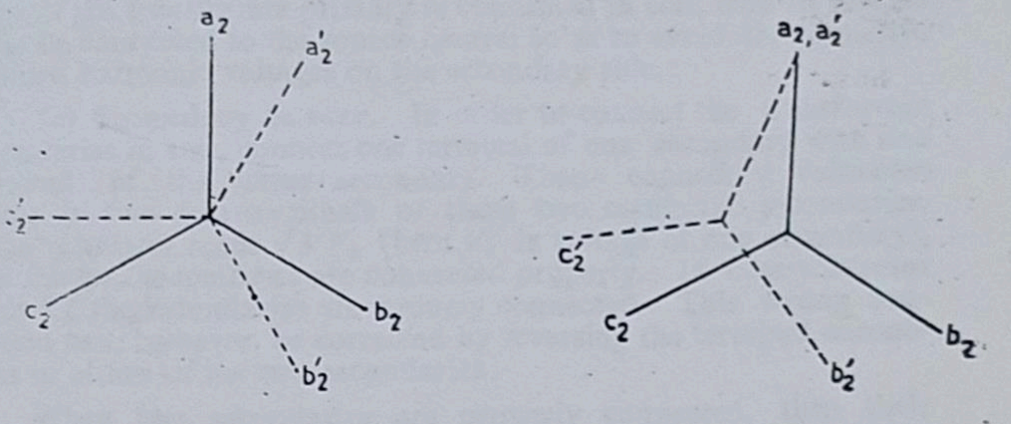
(*e*) **Relative-phase displacement.** The relative-phase displacement between the secondary line voltage of all the transformers must be zero ; *i.e.* the transformers to be connected in parallel, must belong to the same group number. For example, *Yy*0 and *Dd*0 belong to group number 1, these can, therefore, be operated in parallel.

What would happen if transformers of different group numbers are connected in parallel ? In order to examine this, consider



(*a*)

two, 3-phase transformers of different group numbers connected to the same source of supply as shown in Fig. 8·19 (*a*). The secondary line voltages of these two transformers are not in phase as shown in Fig. 8·19 (*b*). In this figure, the phasors joining a_2, a_2', ; b_2, b_2' and c_2, c_2' represent the voltages across switches S_1, S_2 and S_3 respectively. Any one of the three switches can be closed without any danger. For example, if S_1 is closed, there will be no circulating current because the secondary circuit is not complete. The effect of closing the switch S_1 is to bring a_2 and a_2' together so that these overlap each other as shown in Fig. 8·19 (*c*). Now the voltage across switches S_2 and S_3 will be equal to the phasors joining b_2b_2' and c_2c_2' respectively in Fig. 8·19 (*c*). After closing S_1, if S_3 is also closed, voltage b_2b_2' would send a large circulating current in phases A and B which may be damaged. Hence, for successful parallel operation, the voltage across switches S_1, S_2 and

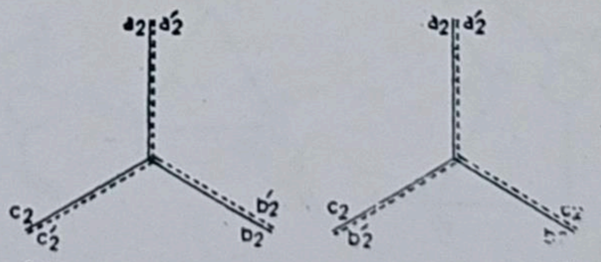


(b) (c)

Fig. 8·19. Parallel operation of three-phase transformers with different group numbers.

S_3 (or across S_2 and S_3 if S_1 is closed), should be zero. In other words, it is essential that the transformers belong to the same group number so that the relative-phase displacement between the secondary line voltages is zero. However, transformers of group numbers 3 and 4 can be successfully operated in parallel as explained in Example 8·1.

(*f*) **Phase-sequence.** If the secondary line voltages are of the same phase sequence as shown in Fig. 8·20 (*a*), then the voltage across switches S_1, S_2 and S_3 of Fig. 8·19 (*a*) would be zero and the parallel operation is possible. However, an improper phase sequence as shown in Fig. 8·20 (*b*), would give zero voltage across switch S_1 and line voltages across switch S_2 and S_3. Consequently the parallel operation is not possible. Hence, it is essential that the secondary line voltages are of proper phase sequence.



(*a*) (*b*)

Fig. 8·20. Pertaining to the phase sequence of three-phase transformers.

The computations, pertaining to the parallel operation of three-phase transformers under balanced conditions, can be carried out by reducing a 3-phase problem to an equivalent single-phase problem. For this purpose, equivalent circuits and various relations, derived for the parallel operation of single-phase transformers, can be used.

(NF)

Tertiary Winding :-

In addition to the primary and secondary windings, the transformers are sometimes provided with third winding, called the tertiary or stabilizing winding. It is usually delta connected.

The various functions of ~~the~~ a tertiary winding are ~~as~~ as follows:-

1. To supply the substation auxiliaries at a voltage different from those of the primary and secondary windings.

2. Static capacitors or synchronous condensers may be connected to the tertiary winding for reactive power injection into the system for voltage control.

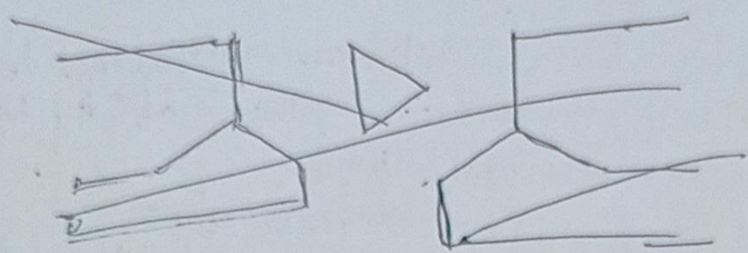
3. A delta-connected tertiary reduces the impedance offered to the zero sequence currents thereby allowing a larger earth-fault current to flow for proper operation of protective equipment. Further, it limits voltage ~~impedance~~ imbalance when the load is unbalanced. It also permits the third harmonic current to flow thereby reducing third harmonic voltages.

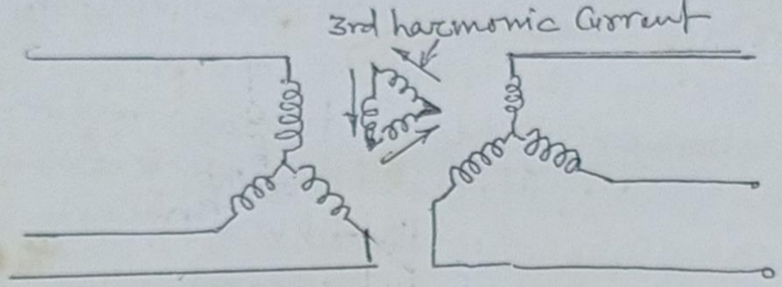
4. Three windings may be used for interconnecting three transmission lines at different voltages.

5. Tertiary can serve the purpose of measuring voltage of an HV testing transformer.

When used for the purpose as stated in (iii) (3) above the tertiary winding is called a stabilizing winding.

~~Stabilization by tertiary winding~~:-





The provision of a delta connected tertiary winding in star/star transformers permits the flow of third harmonic currents in it, therefore, the flux and emfs become almost sine wave as shown in Fig.

two points have already been discussed.

1.14. Tap-Changers on Transformers

The modern equipments, utilising electrical energy, are designed to operate satisfactorily at one voltage level. It is, therefore, of paramount importance to keep the consumers' terminal voltage within the prescribed limits. The transformer output voltage and hence the consumers' terminal voltage can be controlled by providing taps either on the primary or on the secondary.

The principle of regulating the secondary output voltage is based on changing the number of turns in the primary or secondary. Let V_1, N_1 and V_2, N_2 be the primary and secondary quantities. If N_1 is decreased, emf per turn on primary $\left(=\frac{V_1}{N_1}\right)$ increases, therefore, secondary output voltage $(V_1/N_1)\ N_2$ increases. On the other hand, if N_2 is increased keeping N_1 constant, the secondary output voltage $(V_1/N_1)\ N_2$ also increases. In other words, decreasing primary turns N_1 has the same effect as that of increasing the secondary turns N_2.

The taps which help in altering the turns ratio, may be placed on the primary or secondary side. The choice between the two sides should be based on maintaining the voltage per turn constant, as far as possible. If primary voltage per turn decreases, the core flux decreases and this results in poor utilisation of the core, though core losses are reduced. On the other hand, if primary voltage per turn increases, the core flux increases and this results in magnetic saturation of the core, more core losses, increased magnetizing current and pronounced third harmonic. In transformers at the generating stations, the primary voltage can be kept almost constant, consequently the taps should be provided on the secondary side. If transformer is energised from a variable voltage source, as at the receiving end of a transmission line, the taps should be provided on the primary side.

Other factors, described below, may also be taken into consideration, while deciding upon the side to be provided with taps.

(*i*) Transformers with large turns ratio, are tapped on the h.v. side, since this enables a smoother control of the output voltage. On the other hand, taps on the l.v. winding, vary output voltage in large steps, which is usually undesirable.

(*ii*) Tap-changing gear on the h.v. side will have to handle low currents, though more insulation will have to be provided.

(*iii*) It is difficult to tap the l.v. winding, since it is placed next to the core due to insulation considerations. The h.v. winding, placed outside the l.v. winding, is easily accessible and can, therefore, be tapped without any difficulty.

A consideration of the foregoing points can help in deciding upon the side of the transformer to be tapped.

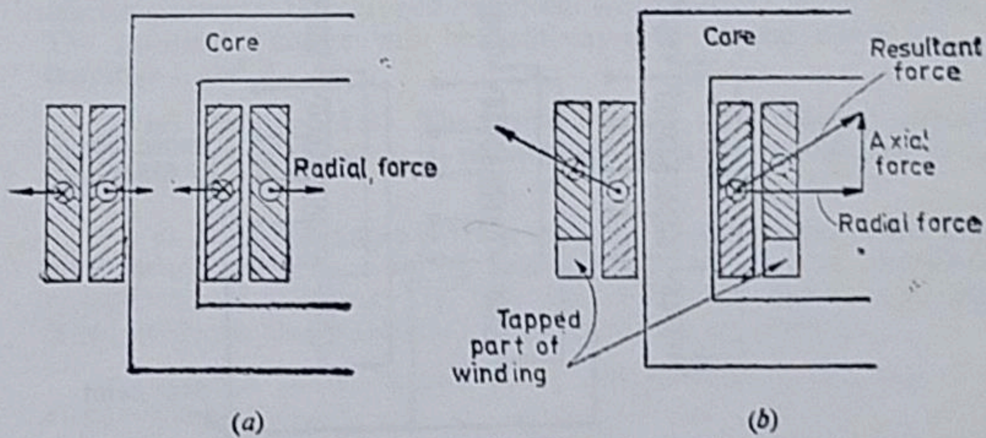


Fig. 1·43. (*a*) Radial force, (*b*) Effect of providing tapped coils at the end of a winding.

A further question arises about whether the transformer winding should be tapped at the end or in the middle. In order to investigate this, refer to Fig. 1·43 (*a*), where the currents in the primary and secondary coils must flow in oposite directions. These currents interact with leakage flux in between the two coils and produce a radial force, repelling each other. This radial force tends to compress the inner coil on to the core and burst the outer coil away from the core. The repelling force may be regarded as acting along the line joining the centres of gravity of the primary and secondary coils.

Suppose the winding is tapped at one end. When some of the turns are cut out by tap changer as shown in Fig. 1·43 (*b*) ; axial forces, in addition to radial forces, are also developed. Under short circuit conditions, the axial forces tending to compress the coils axially, are very large. In order to obviate this, the physical position of the tapped coils should be in the middle of winding, so that no axial forces arise after some turns are cut out. Electrically the tap-changer is connected where the voltage to neutral is minimum. For example, in a star-connected transformer, the tapped end of the windings are connected to form the star point, though physically the tapped coils are placed in the middle of the winding, see Fig. 1·44. This however is not possible in case of delta-connected transformers, where it is electrically essential to provide the tapped coils in the middle so that the tap-changing gear is far removed from the line and lightning surges.

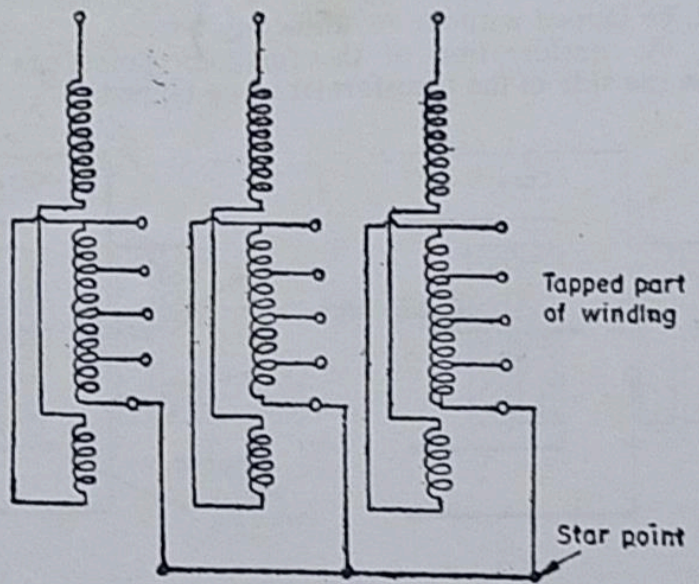


Fig. 1·44. Star connection of the tapped coils.

If the tap-changer is designed to operate with the transformer out of circuit, it is then called *off load* (or no-load) *tap-*

changer. A tap-changer designed to operate with the transformer in circuit, is called *on-load tap-changer.*

1·14·1. No-Load (or off-load) tap changer. This tap changer is used for seasonal voltage variations. An elementary form of no-load tap changer is illustrated in Fig. 1·45. It has six studs marked from one to six. The winding is tapped at six points, equal to the number of studs. The tapping leads are connected to six correspondingly marked stationary studs arranged in circle. The face plate carrying the six studs, can be mounted anywhere on the transformer, say on the yoke or on any other convenient place. The rotatable arm R can be rotated by means of handwheel, from outside the tank.

If the winding is tapped at 2·5% intervals, then with the rotatable arm R ;

(*i*) at studs 1, 2 ; full winding is in circuit ;

(*ii*) at studs 2, 3 ; 97·5% of the winding is in circuit ;

(*iii*) at studs 3, 4 ; 95% of the winding is in circuit ;

(*iv*) at studs 4, 5 ; 92·5% of the winding is in circuit ; and

(*v*) at studs 5, 6 ; 90% of the winding is in circuit.

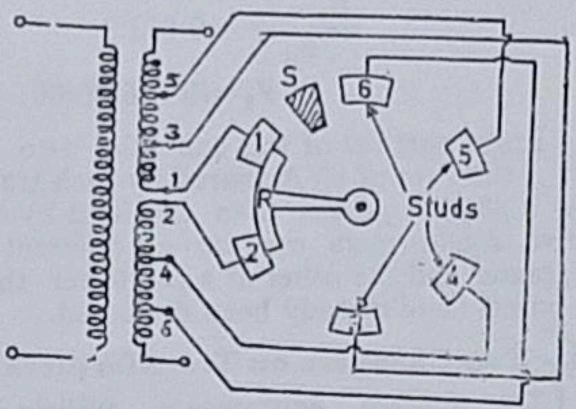


Fig. 1·45. No-load tap changer.

Stop S fixes the final position and prevents the arm R from being rotated clockwise. In the absence of stop S, the arm R may come in contact with studs 1 and 6. In such a case, only the lower part of the winding is cut out of circuit and this is undesirable from mechanical-stress considerations.

The tap-changing must be carried out only after the transformer is disconnected from the supply. Suppose arm R is at studs 1 and 2. For bringing arm R at studs 2 and 3, the transformer is first de-energised and then the arm R is rotated to bridge studs 2 and 3. After this, transformer is switched on to the supply and now 97·5% of the winding remains in circuit.

1·14·2. On-load tap-changer. This tap-changer is used for daily or short period voltage alterations. The output voltage can be regulated with the changer, without any supply interruptions. During the operation of an on-load tap changer ;

(i) the main circuit should not be opened otherwise dangerous sparking will occur and

(ii) no part of the tapped winding should get short-circuited.

One form of elementary on load tap-changer is illustrated in Fig. 1·46 (a). The centre tapped reactor C prevents the tapped

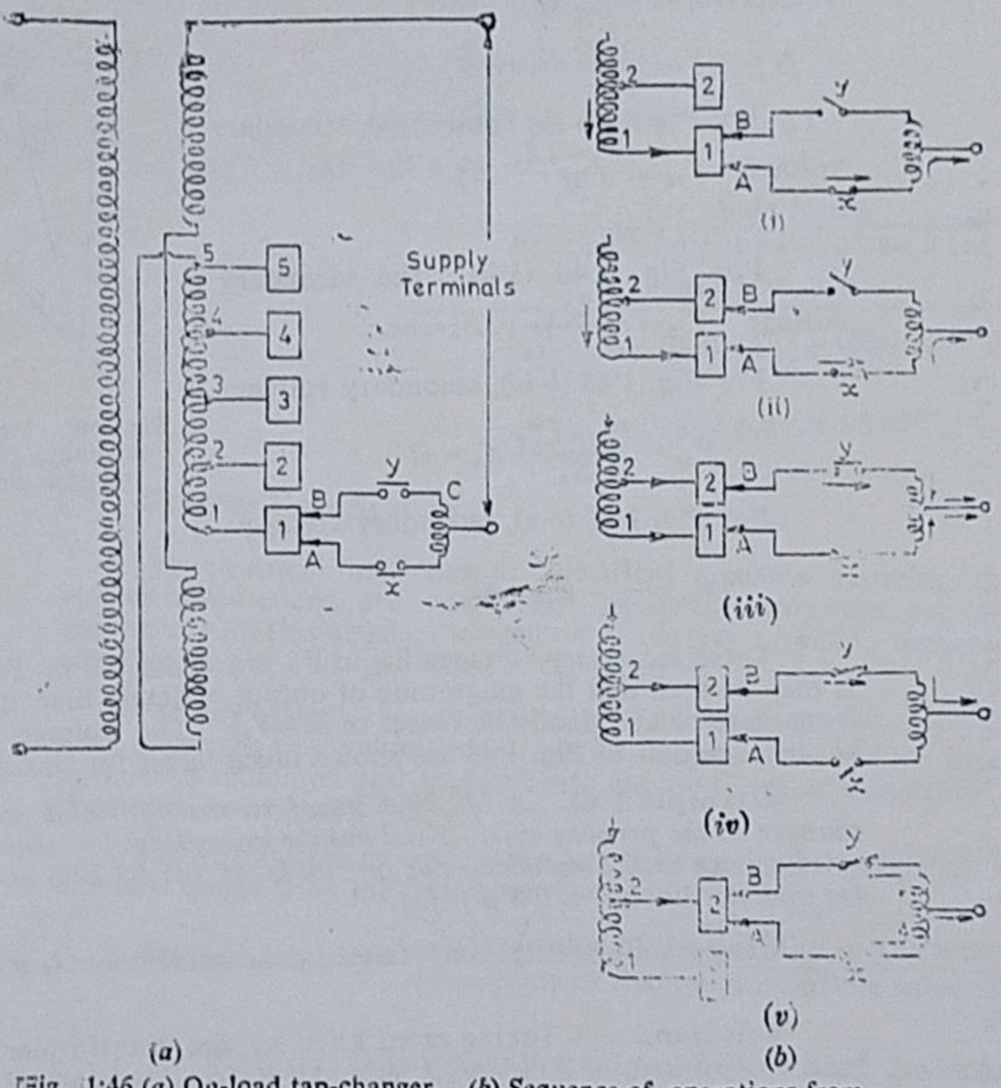


Fig. 1·46 (a) On-load tap-changer. (b) Sequence of operations from tapping 1 to tapping 2.

winding from getting short-circuited. The transformer tappings are connected to the correspondingly marked segments 1 to 5. Two movable fingers A and B, connected to centre-tapped reactor *via* switches x and y, make contact with any one of the segments under normal operation.

In Fig. 1·46 (*a*), both the fingers are in contact with segment 1 and full winding is in circuit. Switches x, y are closed. One half of the total current flows through x, lower half of the reactor and then to the external circuit. The other half of the total current flows through y, upper half of the reactor and then to the external circuit. It is seen that currents in the upper and lower halves of the reactor flow in opposite directions. Since the whole reactor is wound in the same direction, the m.m.f. produced by one-half is opposite to the m.m.f. produced by the second half. These m.m.fs. are equal and the net m.m.f. is practically zero ; therefore, the reactor is almost non-inductive and the impedance offered by it is very small. Consequently the voltage drop in the centre-tapped reactor is negligible.

When a change in voltage is required, the fingers A and B can be brought to segment 2, by adopting the following sequence of operations :

(*i*) *Open switch y, Fig. 1·46 (b-i).* The entire current must now flow through the lower half of the reactor. It, therefore, becomes highly inductive and there is a large voltage drop. It should be noted that the reactor must be designed to handle full load current, momentarily.

(*ii*) The finger B carries no current and can, therefore, be moved to segment 2, without any sparking [Fig. 1·46 (*b-ii*)].

(*iii*) *Close switch y, Fig. 1·46 (b-iii).* The transformer winding between taps 1 and 2 gets connected across the reactor. Since the impedance offered by the reactor is high for a current flowing in only one direction, the local circulating current flowing through the reactor and tapped winding is quite small. In this manner, the reactor prevents the tapped winding from getting short-circuited. The terminal voltage will be mid-way between the potentials of tappings 1 and 2.

(*iv*) *Open switch x.* The entire current starts flowing through the upper half of the reactor, manifested by a large voltage drop, Fig. 1·46 (*b-iv*).

(*v*) Move the finger A from segment 1 to segment 2 and then close switch x. The winding between taps 1 and 2 is, therefore, completely out of circuit, Fig. 1·46 (*b-v*). If further change in voltage is required, the above sequence of operations is repeated.

For large power transformers, the switches x and y may be circuit-breakers.

Another form of on-load tap-changer, also provided with a centre-tapped reactor, is illustrated in Fig. 1·47. The function of the reactor is again to prevent the short-circuit of the tapped winding. The switches 1, 2, .., 5 are connected to the correspondingly marked taps.

1·11. Testing of Transformers

A wide variety of tests can be performed on a transformer and in this article, only a few of them are dealt with. The tests to be described here, are simple, can be performed easily in the laboratory and are helpful in gaining a better physical insight into the transformer behaviour.

(a) *Polarity test.* On the primary side of a two-winding transformer, one terminal is positive with respect to the other terminal at any one instant. At the same instant, one terminal of the secondary winding is positive with respect to the other terminal. These relative polarities of the primary and secondary terminals at any instant must be known if the transformers are to be operated in parallel or are to be used in a polyphase circuit.

When viewed from the h.v. side, the terminals are marked A_1 and A_2, the former, *i.e.* A_1 being on the extreme right, as per IS—2026. Terminals A_1 and A_2 are marked plus and minus arbitrarily in Fig. 1·26. Now terminal A_1 is connected to one end of the secondary winding and a voltmeter is connected between A_2 and other end of the secondary winding. A voltage of suitable value is now applied to the h.v. winding. Let E_1 and E_2 be the e m.fs.

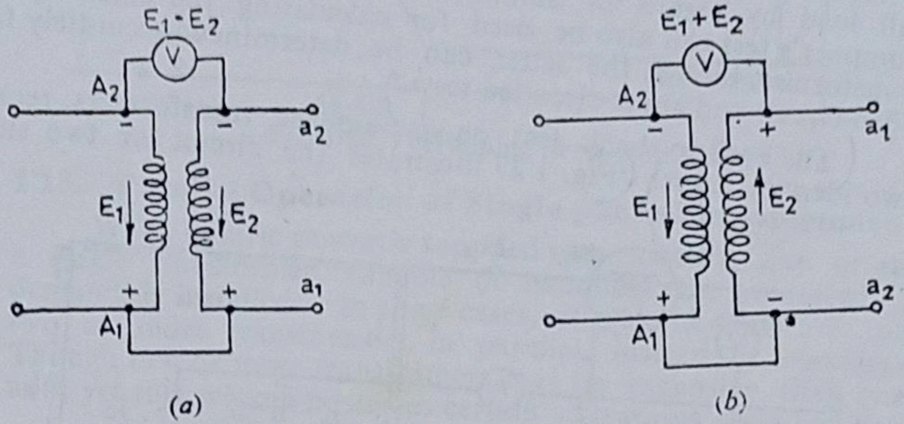


Fig. 1·26. Polarity test on a two winding transformer.

induced on h.v. and l.v. sides respectively. If the voltmeter reading is equal to E_1-E_2, then secondary terminal connected to A_1 is positive and is marked a_1, the l.v. terminal connected to A_2 through the voltmeter is negative and is marked a_2 as shown in Fig. 1·26 (*a*). If voltmeter reading is equal to E_1+E_2, then the terminals connected to A_1 and A_2 are negative and positive and are marked a_2 and a_1 respectively as shown in Fig. 1·26 (*b*). The subscript numbers 1, 2 on the h.v. and l.v. windings are so arranged that when A_2 is negative with respect to A_1, a_2 is also negative with respect to a_1 at the same instant. In other words, if the instantaneous emf is directed from A_2 to A_1 in h.v. winding, it is at the same time directed from a_2 to a_1 in the l.v. winding.

When the voltmeter reads the difference E_1-E_2, the transformer is said to possess a subtractive polarity and when voltmeter reads E_1+E_2, the transformer has additive polarity. In subtractive polarity, the voltage between A_2 and a_2 (or A_1 and a_1) is reduced. The leads connected to these terminals and the two windings are therefore, not subjected to high voltage stress. In additive polarity the two windings and the leads connected to A_1, A_2, a_1 and a_2 are subjected to high voltage stresses. On account of these reasons, subtractive polarity is preferable to additive polarity.

(*b*) *Open circuit and* (*c*) *Short circuit tests.* These two tests have already been described in detail in Art. 1·7.

(*d*) *Load test* (*Back to back or Sumpner's test*). A load test on a transformer is necessary if its maximum temperature rise is to be determined. A small transformer can be put on full load by means of a suitable load impedance. But for large transformers, full load test is difficult, since it involves considerable waste of energy and a suitable load, capable of absorbing full load power, is not easily available. However, large transformers can be put on full load by means of Sumpner's or back to back test. The Sumpner's test can also be used for calculating the efficiency of a transformer, though the latter can be determined accurately from open-circuit and short-circuited tests.

The back to back test on single-phase transformers, requires two identical units. Fig. 1·27 illustrates the circuit for two single-

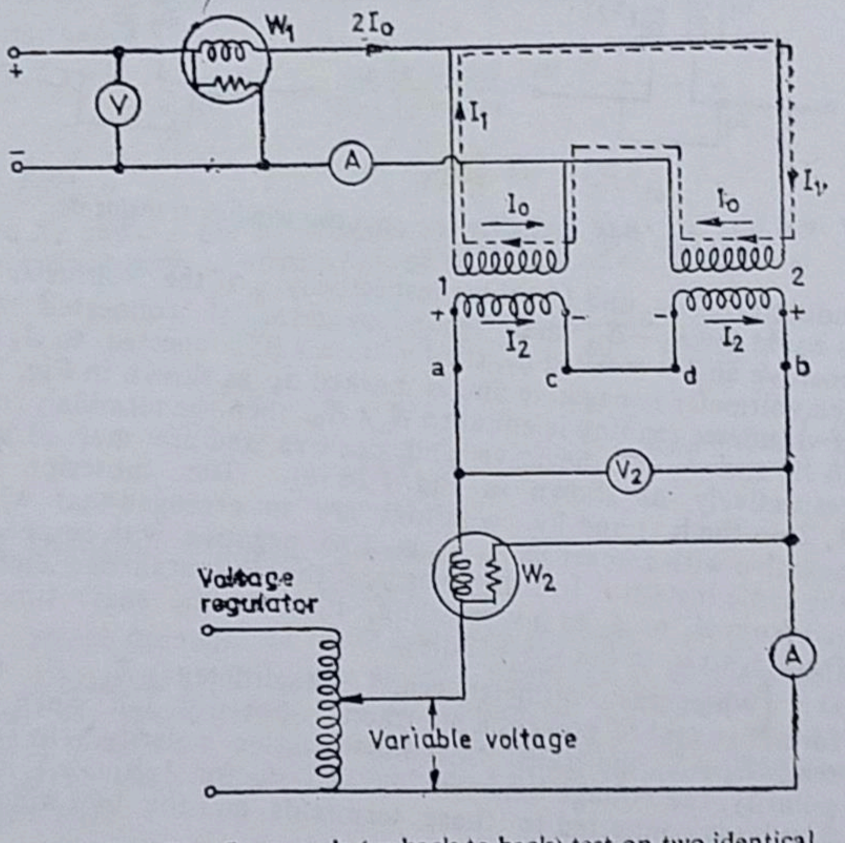


Fig. 1·27. Sumpner's (or back to back) test on two identical single-phase transformers.

phase transformers, where two primaries connected in parallel, are energised at rated voltage and rated frequency. With secondaries open, the wattmeter W_1 records the core losses of both the transformers. The two secondaries are connected in series with their polarities in phase opposition, which can be checked by means of a voltmeter. The range of this voltmeter connected across terminals ab, Fig. 1·27, should be double the rated voltage of either transformer secondary. Zero voltmeter reading ($V_{ab}=0$) indicates the secondaries are connected in opposition. Now, if the terminals ab are short-circuited, the current in the secondary would be zero because $V_{ab}=0$ and the wattmeter reading W_1 remains unaltered. In case the voltmeter reads the sum of the two secondary voltages, the secondaries are in the same phase. In order to bring them in phase opposition, terminals ad should be joined together to result in zero voltage across terminals bc.

In Fig. 1·27, it is assumed that voltage across ab is zero and the two secondaries are in phase opposition. Now a voltage is injected in the secondary circuit by means of a voltage regulator, fed from the source connected to the primaries or from a separate source. The injected voltage is adjusted till rated current flows in the two series-connected secondaries. By transformer action, primary windings also carry rated current. Note that the full load current in the primaries, completes its path through the main bus bars (shown dotted) and, therefore, the reading of wattmeter W_1 remains unaffected. It may be seen that the reading of voltmeter V_2 is equal to the sum of leakage impedance drops in both the transformers. The low-injected voltage has given rise to full load currents in primary and secondary windings, therefore, the full load ohmic losses of both the transformers are measured by the wattmeter W_2 (Fig. 1·27). If P_c and P_{sc} are the core and ohmic losses in each transformer, then the reading of wattmeter $W_1=2P_c$ and that of wattmeter $W_2=2P_{sc}$. The efficiency can now be determined by using Eq. (1·52) or Eq. (1·55).

It is seen from above that in Sumpner's test, even though the transformers are not supplying any load current, yet full iron-loss occurs in their cores and full copper-loss occurs in their windings. Net power input to the two transformers is $(2P_c+2P_{sc})$. If temperature rise of the two transformers is to be measured, then the two transformers are kept under rated loss conditions for several hours till maximum stable temperature is reached.

If $2I_0$ is the no load current, then for the assumed directions of I_0 and I_2, the primary current of transformer 1 is less (difference of $\bar{I}_1$ and $\bar{I}_0$) than the primary current of transformer 2 (sum of $\bar{I}_1$ and $\bar{I}_0$). Therefore, the two transformers do not operate under identical conditions—one may have slightly less temperature than the other.

Cooling :- The various methods of cooling of transformer are given below—

(i) AN — Air Natural. The transformer core and coils are open all round to air and are cooled by natural circulation without any ~~dev~~ additional device.

(ii) AB — Air Blast. Cooling is improved by using air blast instead of natural circulation.

(iii) ON — Oil Natural. The heat developed in the transformer is passed to tank walls through oil where it is dissipated by natural circulation of air.

(iv) OB — Oil Blast. Improved cooling of ON-type transformer is achieved by blasting air over the outside of the tank.

(v) OFN — Oil Forced Natural. Forced circulation of oil is done by a pump while cooling is done by natural circulation of air.

(vi) OFB — Oil Forced (Air) Blast. The cooling of OFN-type transformers is improved by employing air-blast over the coolers.

(vii) OW — Oil water cooled. Oil is cooled by circulation of water over the cooling tubes.

(viii) OFW — Oil Forced Water cooled. Similar to OFB with the difference that oil is cooled by water instead of ~~at~~ air-blast.

In addition, there are several mixed cooling methods viz., ON/OB, ON/OFN, ON/OFB, ON/OFW, ON/OB/OFB, ON/OW/OFN etc..

7.9 HARMONICS IN TRANSFORMERS

In addition to the operation of transformers on the sinusoidal supplies, the harmonic behaviour becomes important as the size and rating of the transformer increases. Harmonic signals are those having frequency other than the fundamental frequency. The effects of the harmonic currents are:

1. Additional copper losses due to harmonic currents
2. Additional core losses due to Increased Eddy current loss
3. Increased electro-magnetic interference with nearby communication circuits.

On the other hand, the harmonic voltages in a transformer cause

1. Increased dielectric stress on insulation
2. Electro-static interference with nearby communication circuits.
3. Resonance between winding reactance and transmission line capacitance.

In present times, a greater awareness is generated by the problems of harmonic voltages and currents produced by non-linear loads like the power electronic converters. In addition to these external effects, non-linear nature of transformer core produces distortions in voltage and currents and increase the power loss. Study of harmonics is thus of great practical significance for operation of transformers.

7.9.1 Excitation Phenomena in Transformers

Excitation refers to the magnetization phenomenon that takes place in the transformer core and deals with the current, voltage, flux, and EMF related to the magnetization process. In an ideal transformer, the iron core is considered to be operating below the saturation level so that the permeability and reluctance are constants, i.e., the B-H characteristic is a straight line. This property of linearity of the magnetic core is utilized in developing the transformer equivalent circuit and corresponding phasor diagram. The effect of saturation and resulting magnetic non-linearity cannot be incorporated in the electrical equivalent circuit of a transformer. For such a linearized magnetization curve, the flux is proportional to the magnetization current and their waveforms are identical. Flux and exciting current is simply related by the equation:

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} = \frac{T_p I_\phi}{S}$$

where T_p is the primary winding number of turns, I_ϕ the magnetizing current, and S the reluctance of core. If the supply voltage V_1 is purely sinusoidal, then the induced EMF E_1 must be sinusoidal so that it can correctly oppose the supply voltage (neglecting winding voltage drops).

Mathematically, the relation between induced EMF and flux is:

$$e = -T\frac{d\phi}{dt}$$

Thus, the flux should also be sinusoidal (rather co-sinusoidal) when the EMF is sinusoidal.

When the flux is of sinusoidal nature, the magnetizing current is also sinusoidal for a linear magnetic material.

Thus, in an ideal situation, when the primary supply voltage is sinusoidal, the exciting current (magnetizing current), flux in the core, and induced EMF are all pure sinusoids. This situation can be graphically described as below:

Considering linear magnetization characteristics, the B-H curve of the core magnetic material is shown in Figure 7.77.

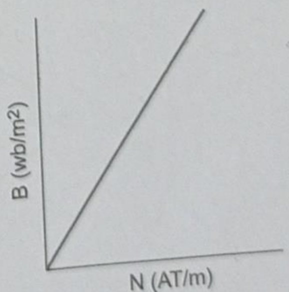


Fig. 7.77 Linear magnetization characteristics (*B-H* curve) of transformer core

Note that the Y-axis variable flux density (B) is proportional to the flux ϕ ($\phi = B \times \text{Area}$) and the X-axis variable magnetizing force (H) is proportional to the magnetizing current I_ϕ ($I_\phi = H \times L/T_p$). Thus, axes variables of Figure 7.77 can be replaced by ϕ and I_ϕ in place of B and H, respectively, so that the magnetizing current I_ϕ values can be directly read from the graph.

For that purpose let us redraw the B-H curve of Figure 7.77 in the form of a ϕ-I_ϕ graph as shown in Figure 7.78. In Figure 7.78, only one-half cycle of the input voltage (sine wave) has been considered. This will make the flux wave also to look sinusoidal as shown in Figure 7.78.

Consider the point a on the flux wave. Draw a horizontal line from a that meets the *magnetization* curve at 1. The flux value at this instant is A-1. Corresponding value of the magnetizing current as read from the magnetization curve is o′-A. This condition occurs at a time t_1 which is at a distance o-t_1 from the origin of the flux wave. At the same instant of time o′-t_1', the value of magnetizing current is t_1'-a' (= o′-A) which is required to set up the flux A-1. The point a' is obtained as the point of intersection of a horizontal line drawn from t_1' and extending the vertical line from 1-A downwards.

Consider another point b on the flux wave. Draw a horizontal line from b that meets the *magnetization* curve at 2. The flux value at this condition is B-2. Corresponding value of the magnetizing current as read from the magnetization curve is o′-B. This condition occurs at a time t_2 which is at a distance o-t_2 from the origin of the flux wave. At the same instant of time o′-t_2', the value of magnetizing current is t_2'-b′ (= o′-B) which is required to set up the flux B-2. The point b′ is obtained as the point of intersection of a horizontal line drawn from t_2' and extending the vertical line from 2-B downwards.

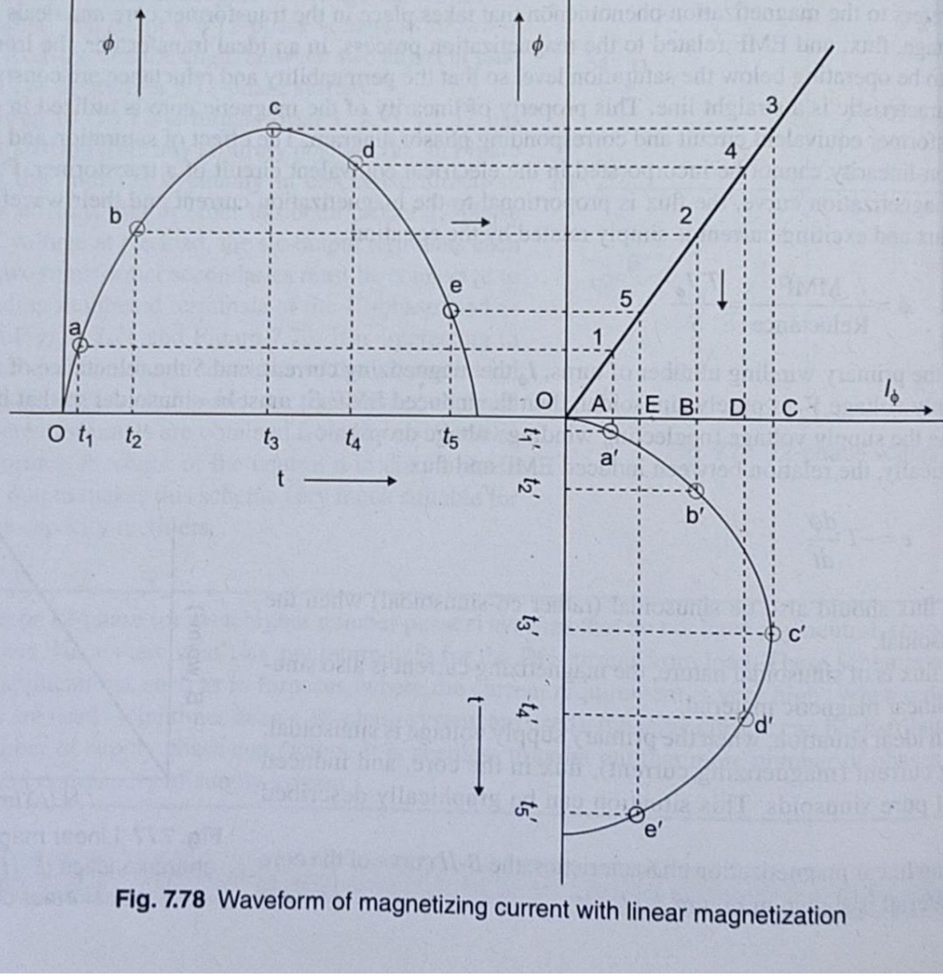


Fig. 7.78 Waveform of magnetizing current with linear magnetization

Similarly, travelling along the sinusoidal flux wave, the points c, d, and e can be located and the corresponding magnetizing current values are obtained at c′, d′, and e′, respectively. Joining the points a′, b′, c′, d′, e′ sequentially. A curve joining all these points represents the waveform of magnetizing current. As seen in Figure 7.78 that the magnetizing current waveform is also sinusoidal.

Next, consider the magnetization characteristic is non-linear, but without any Hysteresis. Sometimes for meeting the requirement of material saving, transformers are operated with high value of flux such that the core material can go into saturation. Figure 7.79 shows such a situation when the flux wave peak has reached at the point 3 on the magnetization curve which is well up to saturation. When a sinusoidal voltage is applied to the primary, the flux wave is also sinusoidal (rather co-sinusoidal), and nature of the magnetization current can be found out in the same manner as was in Figure 7.78 as described by Figure 7.79 below.

The magnetization current waveform is traced out by joining the points a′, b′, c′, d′, e′ sequentially corresponding to the points a, b , c, d, and e on the flux wave. As seen in Figure 7.79, even with sinusoidal input supply voltage and sinusoidal flux, the magnetization current wave does not remain sinusoidal; rather it becomes peaky in nature. The peaky waveform, however, is symmetrical about its peak value. It is also to be noted that the peak values of the voltage, flux, and the magnetizing current takes place at the same time instant. *Remember that when the input supply voltage is sinusoidal, the induced EMF and hence the flux is always sinusoidal, even with a non-sinusoidal magnetizing current.*

Fourier analysis of the non-sinusoidal magnetizing current will show that it contains the fundamental signal along with higher order odd harmonics, predominantly the third harmonic as shown in Figure 7.80.

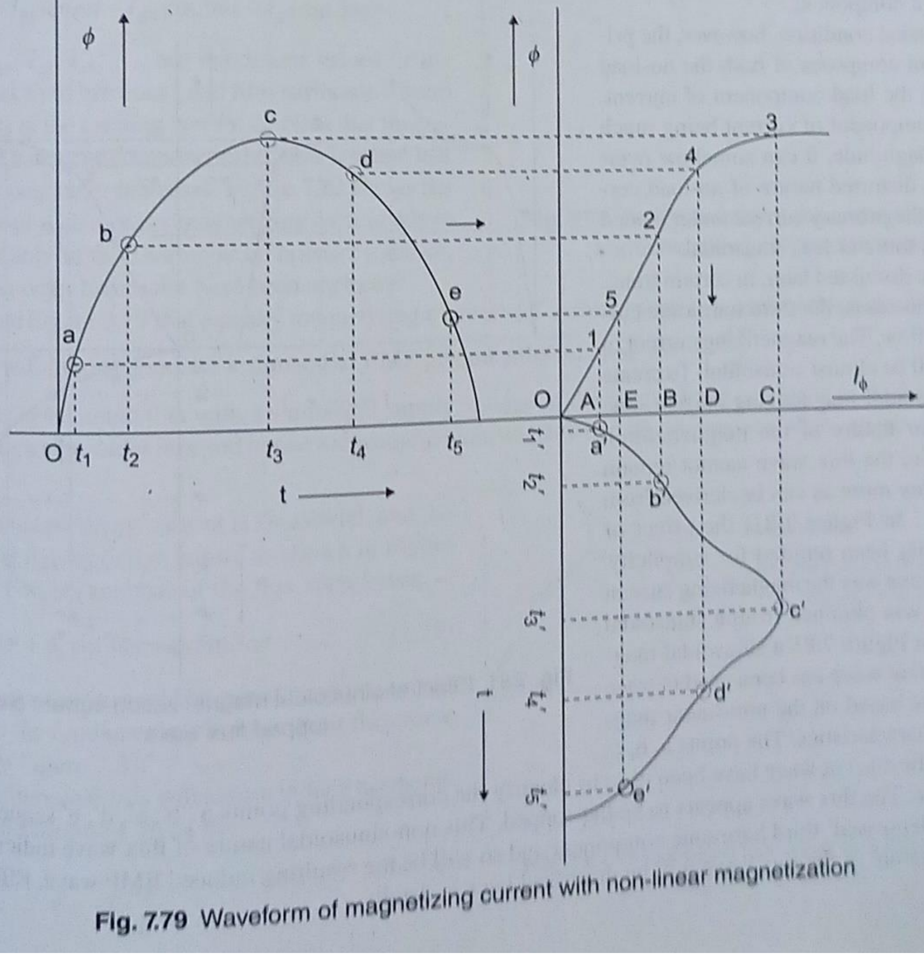


Fig. 7.79 Waveform of magnetizing current with non-linear magnetization

As seen from Figure 7.80, the fundamental component of magnetizing current I_ϕ is in the same phase with the flux ϕ. Since ϕ is 90° out of phase with the supply voltage V_1, the power loss due to this fundamental component of magnetizing current ($I_{\phi f}$) is zero ($V_1 \times I_{\phi f}\cos 90° = 0$). Similarly, all odd harmonic, including the third harmonic components of the magnetizing current has a time phase difference of n 90° with the supply voltage V_1, where n is the order of the harmonic. Thus, power associated with the voltage V_1 and fundamental as well as all odd harmonic components of the magnetizing current is zero. Thus, even if there is saturation in the core magnetic material, without Hysteresis, there is no associated power loss. The effect of saturation is only to distort the magnetizing current.

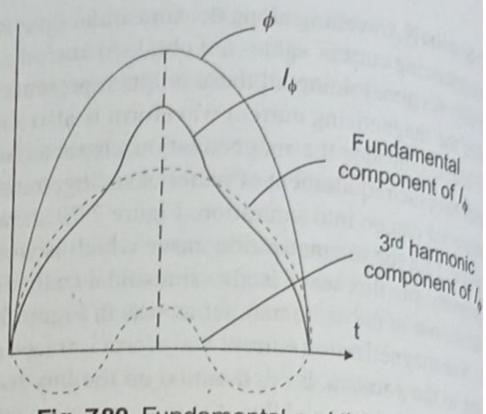


Fig. 7.80 Fundamental and third harmonic components of I_ϕ

For more detail, go to online resources.

The no-load current in the above case is found to contain third, fifth, and seventh and other higher odd-harmonic components, and their magnitudes increase rapidly as the transformer operation is pushed more into saturation. Magnitude of the third harmonic component is predominant; amounting may be up to as high as 10% of the fundamental component.

Under loaded condition, however, the primary current composes of both the no-load current and the load component of current. The load component of current being much higher in magnitude, it can somehow overshadow the distorted nature of no-load current so that the primary current under loaded condition is more or less sinusoidal.

As will be discussed later, in certain transformer connections, the third harmonic current cannot flow. The magnetizing current in that case will be almost sinusoidal. To create such a sinusoidal magnetizing current, due to non-linear nature of the magnetization characteristic, the flux wave cannot remain sinusoidal any more as can be derived from Figure 7.81. In Figure 7.81, the effect of Hysteresis has been omitted for simplicity. Like in the same way the magnetizing current wave shape was obtained from a sinusoidal flux wave, in Figure 7.81 a sinusoidal magnetizing current wave has been used to trace the flux wave based on the non-linear magnetization characteristics. The points a, b, c, d, and e on the current wave have been used to identify the corresponding points a′, b′, c′, d′, e′ sequentially on the flux wave. The flux wave appears to be flat-topped. This non-sinusoidal nature of flux wave indicates that it contains a 'depressed' third harmonic component and so will be the resulting induced EMF wave. Effect of this flat-topped nature of the flux wave will be discussed later in details.

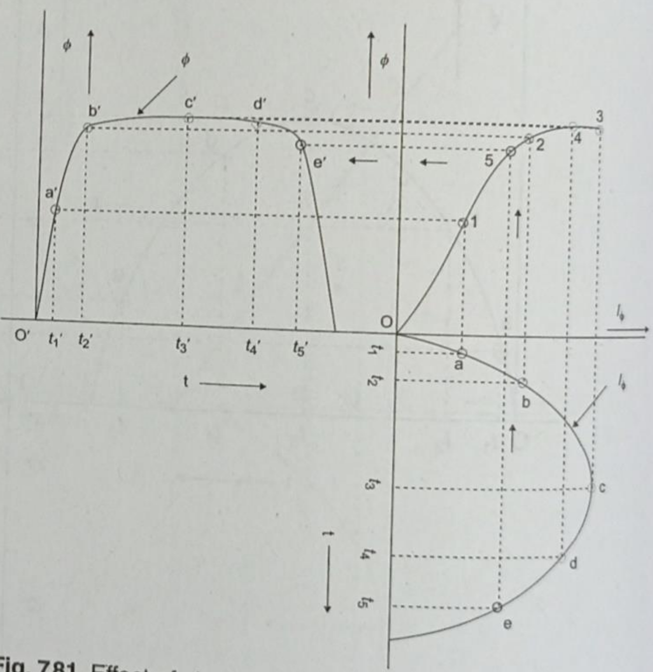


Fig. 7.81 Effect of sinusoidal magnetization current creating flat-topped flux wave

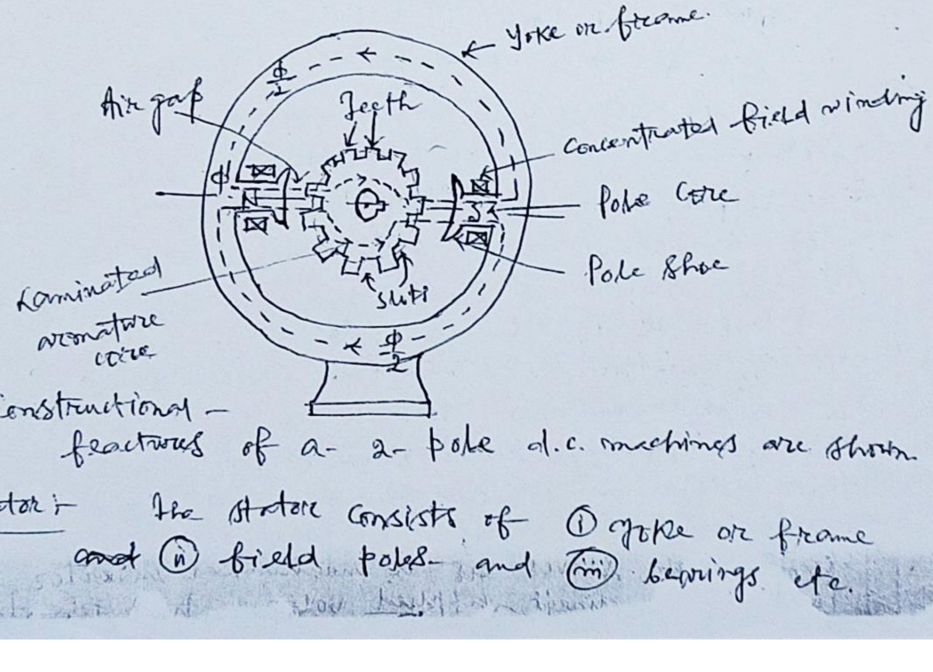
D.C. Machines :-

An electric machine is an electro-mechanical device or a dynamo-electric machine or more briefly dynamo which converts mechanical energy into electrical energy or viceversa. The machine which converts mechanical energy in to electrical energy is known as generator, while the machine converting electrical energy in to mechanical energy is called a motor.

D.C. machine is a highly versatile energy conversion device. It can meet the demand of loads requiring high starting torques, accelerating and decelerating torques. At the same-time, d.c. machine is easily adaptable for drives requiring wide range speed control and quick reversals.

Construction:- The armature of an electric machine is the part in which e.m.f. is generated in the case of a generator or the part in which the working current interacts to develop mechanical power in the case of a motor. The field member is the part that produces the magnetic field.

In a d.c. machine, the field winding is on the stator and the armature winding is on the rotor.



Constructional feactures of a 2-pole d.c. machines are shown.

Stator :- The stator consists of (i) yoke or frame and (ii) field poles and (iii) bearings etc.

The yoke or frame of a d.c. machine serves two functions. It forms a portion of the magnetic circuit and supports the poles. Also, it acts as a mechanical support for the entire machine.
Poles carry field windings and when the winding carries a current, the pole becomes an electromagnet and establishes the magnetic field in the machine.

Pole core is usually of smaller cross section than the pole shoe due to the following reasons—

(a) The reduced cross section of the pole core requires less copper for the field winding.

(b) The large pole shoe area increases the flux per pole entering the armature, due to the reduction in air-gap reluctance.

(c) Pole shoes provide mechanical strength and support to the field winding.

Rotor:— The armature of a d.c. machine is the revolving member under the poles and contains the conductors enclosed in slots. As the armature conductors go through N and S poles alternatively, the voltage induced in the armature coils is an alternating one and has a frequency,

$$f = \frac{PN}{120}$$

where, P = No. of poles

N = Speed of revolution in r.p.m.

Armature slots in a d.c. machine are normally rectangular in shape and the conductors, usually of copper, are placed in them in two layers. The armature and the poles are usually made of silicon steel laminations to reduce hysterisis and eddy current losses in the machine.

In addition to the field and armature windings, a d.c. machine must have a commutator to serve as a mechanical rectifier for converting the alternating e.m.f generated in the armature winding to direct e.m.f at brush terminals. For a d.c. motor, the commutator serves as a mechanical inverter to invert the direct applied voltage to alternating voltage

Commutator-
is a group of wedge shaped copper segments, insulated from eachother by thin mica sheets.

Direction of induced emf. — Fleming's right hand rules.

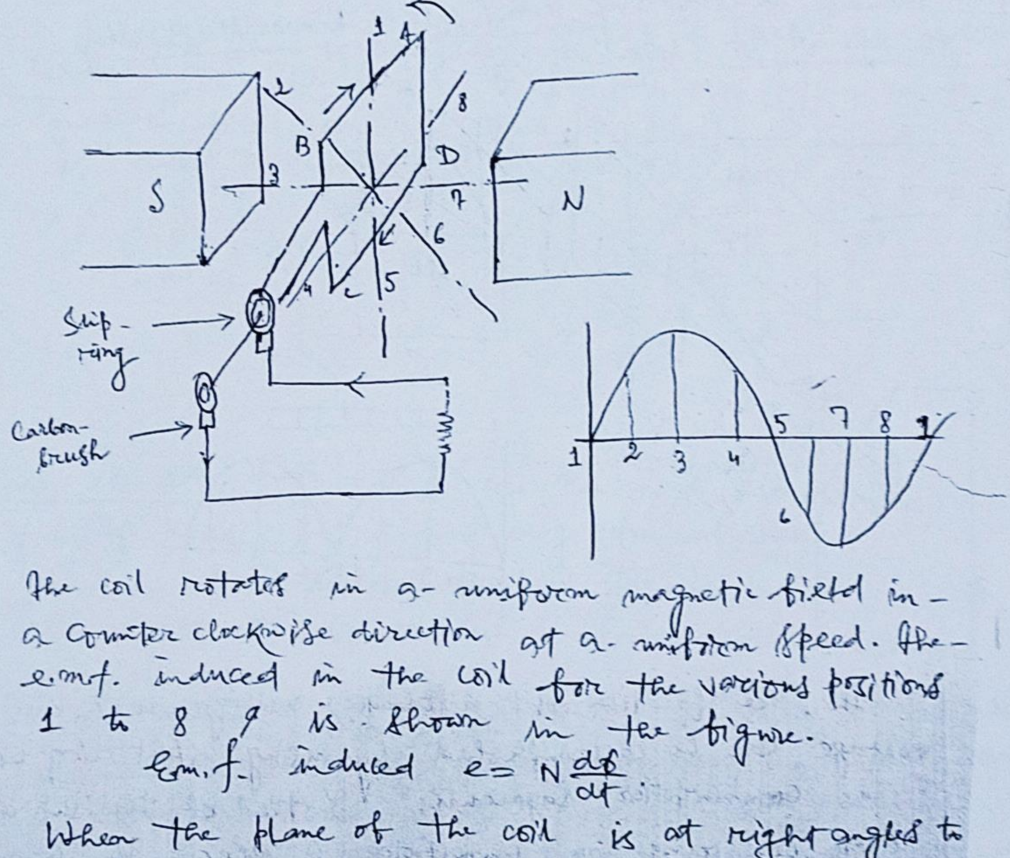
If the forefinger points along the line of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of induced emf.

The emf. induced e by a conductor of length l-metre cutting a flux density B wb./m² at a velocity v m/sec, is given by

$$e = Blv \text{ volts,}$$

provided that B, l and v are mutually perpendicular.

Emf. generated by rotation of a-coil:-



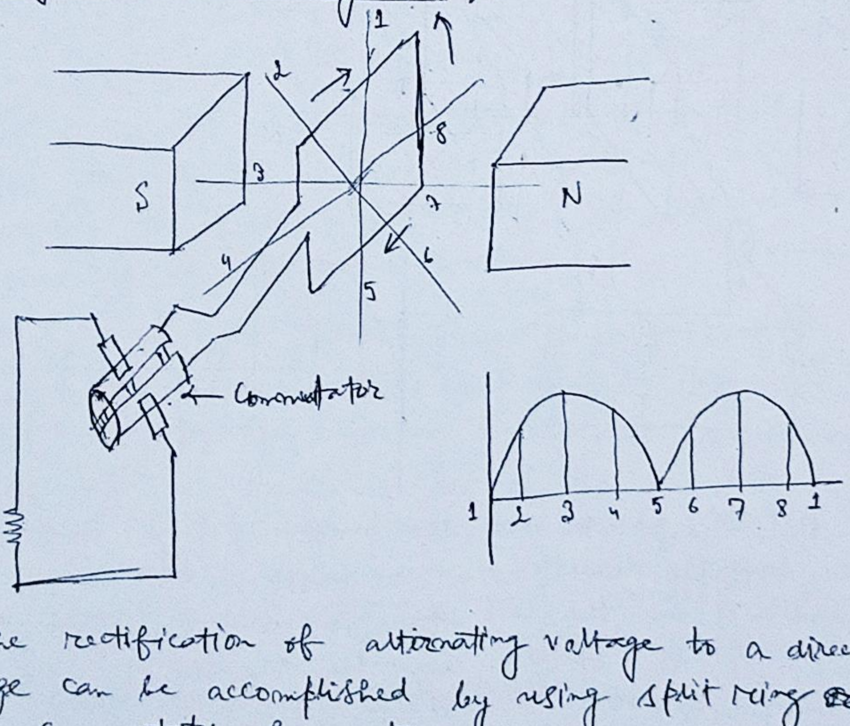
The coil rotates in a uniform magnetic field in a counter clockwise direction at a uniform speed. The emf. induced in the coil for the various positions 1 to 8, 9 is shown in the figure.

Emf. induced $e = N\frac{d\phi}{dt}$

When the plane of the coil is at right angles to lines of flux i.e, when it is in position 1, then flux linked with the coil is maximum but rate of change of flux linkages is minimum. Because, in this position, the coil sides AB and CD slides and they move parallel (...) to them. Hence, there is no induced e.m.f. in the coil.

At position 3, coil plane is horizontal, i.e., parallel to the lines of flux. The flux linked with the coil is minimum, but rate of change of flux linkage is maximum. Hence, maximum e.m.f. is induced in the coil. It is seen that the e.m.f. induced is an alternating e.m.f. and is varying sinusoidally. This alternating e.m.f. can be impressed on an external circuit by means of two slip-rings. Each ring is continuous and is insulated from the other ring and from the shaft. A metal or carbon brush rests on each ring and conducts the current from the coil to the external circuit.

<u>Conversion of alternating e.m.f. to unidirectional voltage using commutator segments</u> :-



The rectification of alternating voltage to a direct voltage can be accomplished by using split ring i.e., commutator segments. Instead of two slip rings only one ring or commutator is used. The commutator is split in to two segments (each segment insulated from the other) and the ends of coil are connected to the segments produced.

make contact alternatively to a particular segment moving under a given pole flux and hence, the voltage in the external circuit becomes unidirectional. Even though the voltage becomes unidirectional, it is not of a constant magnitude. This can be achieved by connecting more no. of coils and hence, with more no. of commutator segments.

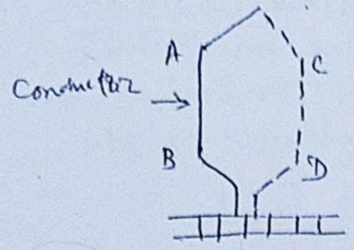
Armature Windings:-

Pole Pitch:- It may be defined as-

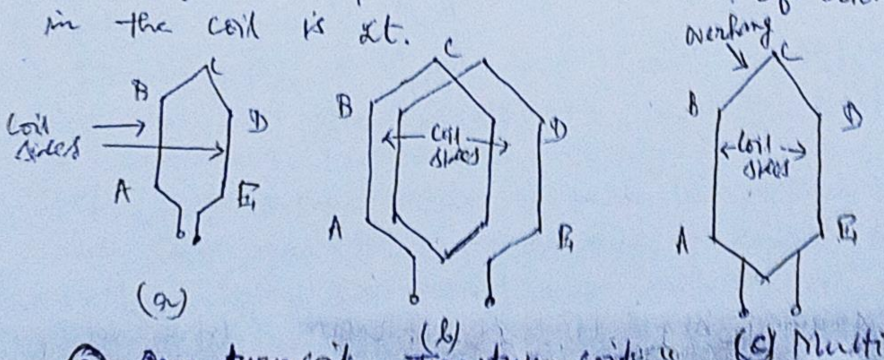
(i) the peripheral distance between two adjacent poles, i.e., the periphery of the armature divided by the no. of poles.

(ii) It is equal to the no. of armature conductors or armature slots per pole. If there are 48 conductors and 4 poles, then pole pitch $= \frac{48}{4} = 12$.
Pole pitch is always equal to 180° electrical.

Conductor:- The length of a wire lying in the magnetic field and in which an e.m.f. is induced, is called a conductor. For example, length AB or CD.



Coil, coil side, Turn:- A coil can have one or more no. of turns, but it has only two coil sides.
The no. of conductors per coil side is t if t is the no. of turns per coil and the no. of conductors in the coil is 2t.

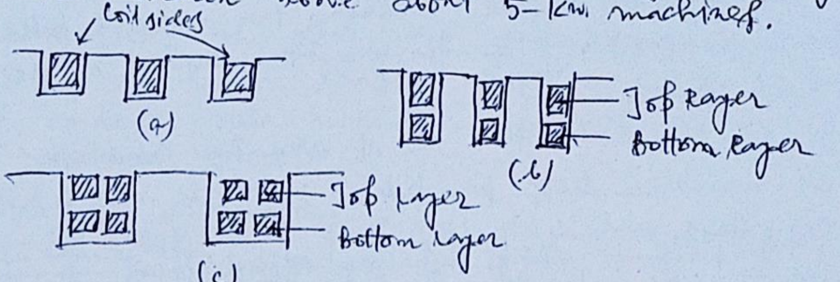


(a) One turn coil (b) Two turn coils (c) Multi-turn coil

BCD - is called the end connection or overhang.

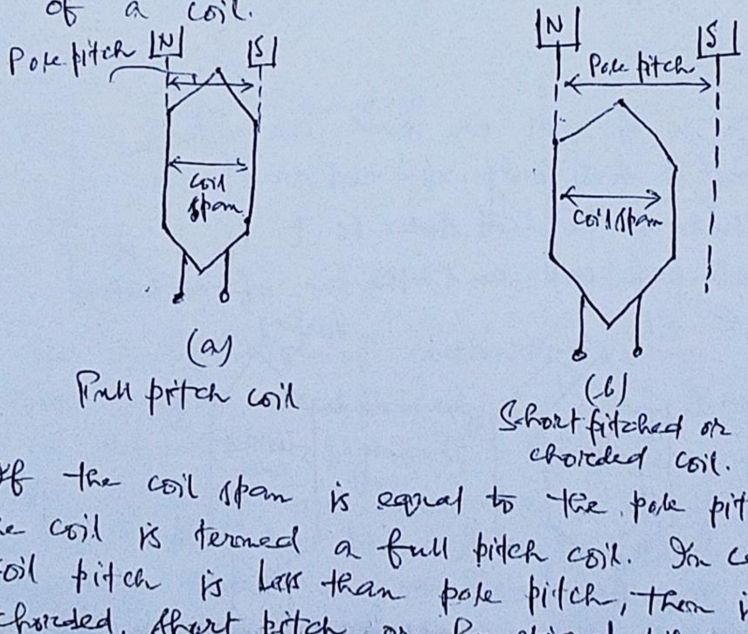
Single layer and Double layer winding :-

If the winding is so designed that one coil side occupies the total slot area, then it is called a single-layer-winding. In case, the slot contains even no. (may be 2, 4, 6 etc) of coil sides in two layers, the winding is referred to as a two-layer-winding. Single layer winding is used only in small a.c. machines, where as double layer winding is more common above about 5-kw. machines.



(a) one coil side per slot (b) two coil sides per slot
(c) 4- coil sides per slot.

Coil span or Coil Pitch :- The distance between the two coil sides of a coil is called coil-span or coil-pitch. It is usually measured in terms of armature slots or armature conductors between the two sides of a coil.



(a) Full pitch coil

(b) Short pitched or chorded coil.

If the coil span is equal to the pole pitch, then the coil is termed a full pitch coil. In case, the coil pitch is less than pole pitch, then it is called chorded, short pitch or fractional pitch coil.

If there are S-slots and P-poles, then

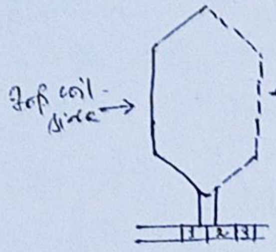
Pole pitch $= \frac{S}{P}$ slots per pole.

If coil pitch $= \frac{S}{P}$, it results in full-pitch winding.

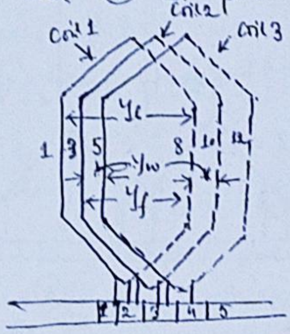
In case, coil pitch $< \frac{S}{P}$, it results in chorded, short-pitched or fractional pitch winding. The coil-pitch is rarely greater than pole pitch.

Closed windings:- Closed armature windings are always double layer windings. Each coil in double layer winding has its one coil side in top layer and its other coil side in the bottom layer. Top coil side is shown with a solid line and the bottom coil side by a dotted line.

The simple closed windings are of two types- (i) Simplex lap winding and (ii) Simplex wave winding.



(a)

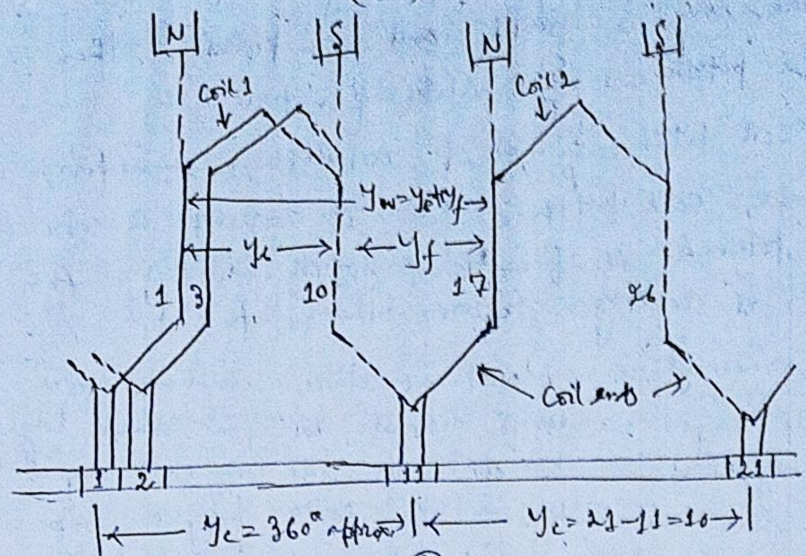


(c)

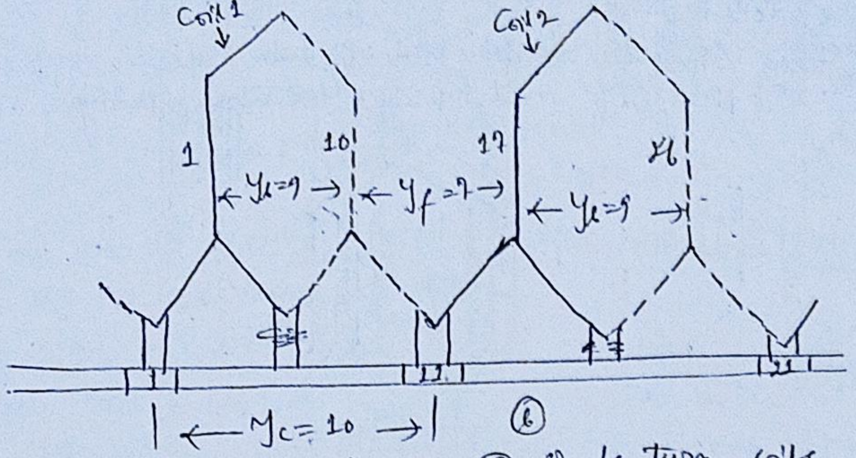
Lap coil connections - (a) single multi-turn lap coil
(b) three multi-turn lap coils.

In simplex lap winding (or lap winding), the two coil-ends of a coil are connected to the two adjacent commutator segments as shown in fig. (a). Note that two coil ends, one from top coil side and the other from bottom coil side are connected to adjacent commutator segments.

From fig. (c), it is seen that bottom coil side of coil 1 and top coil side of coil 2 are connected to segment 2; bottom coil side of coil 2 and top coil side of coil 3 are connected to segment 3 and so on. In other words, for simplex lap winding, each commutator segment has two coil ends connected to it, one coil end is from the bottom coil side of one coil and the other from the top coil side of next coil.



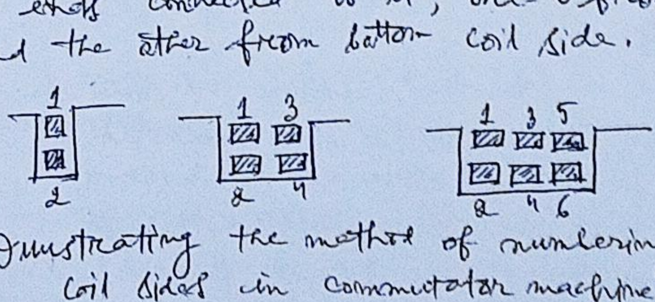
(a)



(b)

Simplex wave winding — (a) single turn coils
(b) multi turn coils.

In simplex wave winding, the two coil ends of a coil are bent in opposite directions and connected to commutator segments which are approximately two pole pitches (i.e., 360° electrical) apart. In wave winding also, each commutator segment has two coil ends connected to it, one from top coil side and the other from bottom coil side.



Illustrating the method of numbering coil sides in commutator machine.

Back pitch (y_b) :- The distance between the top and bottom coil sides of one coil, measured at the back of the armature (or measured at the other side of the commutator) is called ~~the~~ back pitch.

In fig. (a) for lap coil, for coil 1, the top coil side is numbered 1 and the bottom coil side is numbered 8. Therefore, back pitch for coil 1 is $8-1=7$. Similarly, for other coils $y_b = 10-3 = 7 = 12-5$.

In figure for wave winding, $y_b = 10-1 = 9$ for coil 1. Similarly, for coil 2, $y_b = 26-17 = 9$. So, y_b is always odd.

Front pitch (y_f) :- The distance between the two coil sides connected to the same commutator segment, is called front pitch.

In fig. for lap winding, front pitch $y_f = 8-3 = 5$. For segment 3, $y_f = 10-5 = 5$.

In. fig. for wave winding, $y_f = 17-10 = 7$.

So, front pitch, y_f is always odd.

Winding pitch (y_w) :- The distance between the two consequtive and similar top or bottom coil sides, as the winding progress, is called the winding pitch. It is expressed in terms of coil sides.

In fig. for lap winding the consequtive and similar top coil sides are numbered 1, 3, 5 or similar bottom coil sides are numbered 8, 10, 12 etc. So winding pitch $y_w = 3-1 = 5-3 = 10-8 = 12-10 = 2$. For simplex lap winding, $y_w = y_b - y_f$.

In fig for wave winding, winding pitch $y_w = 17-1 = 26-10 = 16$. For simplex wave winding, $y_w = y_b + y_f$. y_w is always even.

Commutator pitch (y_c) :- The distance between the two commutator segments, to which the two sides of one coil are joined, is called the commutator pitch. It is always expressed in terms of commutator segments.

For simplex lap winding the two ends of coil 1 are joined to segments 2 and 1. So, $Y_c = 2 - 1 = 1$.
For simplex wave winding, the two ends of coil 1 are joined to segments 11 and 1. So $Y_c = 11 - 1 = 10$.

as being a *simplex* (single) or *multiplex* ...

In the simplex lap winding there are as many parallel paths or circuits through the winding as there are field poles on the machine.

Double and triple windings are used on armature designed for supply of large currents at low voltage. The sole purpose of such a winding is to increase the number of parallel paths enabling the armature to carry a large total current, at the same time reducing the conductor current to improve commutation conditions. A double or duplex winding consists of two similar simplex windings placed in alternate slots on the armature and connected to alternate commutator segments. Each winding carries half the armature current. Likewise, a triple or triplex winding has three similar windings occupying every third slot and connected to every third commutator segment. Hence in duplex lap winding the number of parallel circuits is twice the number of poles and in triplex lap winding, three times the number of poles. For this reason the lap winding is sometimes called the *multiple or parallel winding* and is suited for machines that operate at relatively low voltages but with high current outputs.

Important Point Regarding Lap Winding. 1. The coil or back pitch Y_b must be approximately equal to the pole pitch *i.e.* $Y_b \approx \frac{Z}{P}$ where Z is the number of conductors on armature and P is the number of poles.

2. The back pitch Y_b should be either lesser or greater than front pitch Y_f by $2m$ where m is the multiplicity of the winding.

$$i.e.\ Y_b = Y_f \pm 2m$$

where, $m = 1$ for simplex winding
$m = 2$ for duplex winding
$m = 3$ for triplex winding and so on.

When Y_b is greater than Y_f the winding progresses from left to right and so known as *progressive winding*. When Y_b is lesser than Y_f the winding progresses from right to left and, therefore, such a winding is known as *retrogressive winding*.

3. The back pitch and front pitch must be odd.

4. The average pitch is given by

$$Y_{av} = \frac{Y_b + Y_f}{2}$$

and should be equal to pole pitch *i.e.* $\frac{Z}{P}$

5. The resultant pitch Y_R is always even, being the difference of two odd numbers and is equal to $2m$ where m is the multiplicity of the winding.

i.e. Resultant pitch, $Y_R = 2$ for simplex lap winding
$Y_R = 4$ for duplex lap winding
and $Y_R = 6$ for triplex lap winding.

6. The commutator pitch, $Y_c = m$ *i.e.*, Y_c is equal to 1, 2, 3, 4 etc., respectively for simplex, duplex, triplex, quadruplex etc. lap windings.

7. Number of parallel paths in lap winding = mP *i.e.* number of parallel paths is equal to P, 2P, 3P, 4P etc. respectively, for simplex, duplex, triplex, quadruplex, etc. lap windings.

Example 4.1. Draw the developed winding diagram of progressive lap winding for 4 poles, 24 slots with one coil side per slot, single layer showing there in position of the poles, direction of motion, direction of induced emfs and position of brushes.

Solution: Developed winding diagram is obtained by imagining the armature surface removed and so laid out flat that the slots and conductors can be viewed without the necessity of turning round the armature in order to trace out the armature winding. Such a developed winding diagram is shown in fig. 4.22.

Number of poles, P = 4

Number of coil sides, Z = 24

$$\text{Average pitch, } Y_{av} = \frac{Y_b + Y_f}{2} = \frac{Z}{P} = \frac{24}{4} = 6$$

$$\text{or } Y_b + Y_f = 12 \quad \text{...(i)}$$

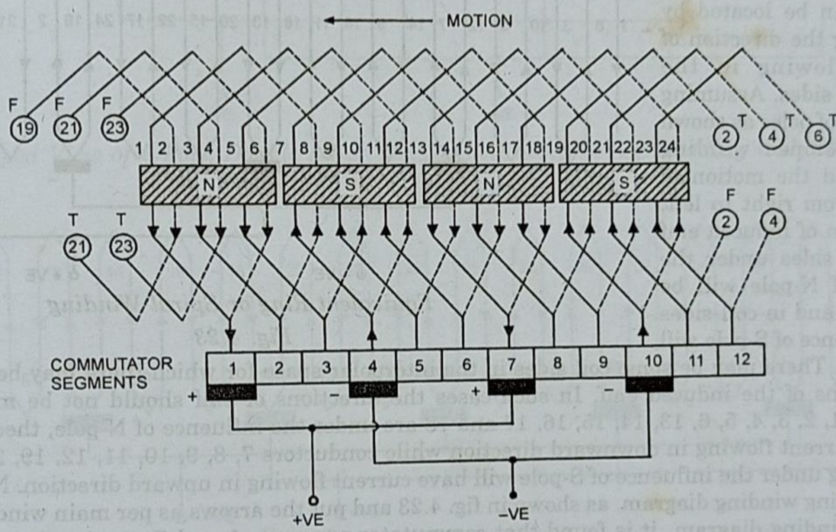
and for progressive simplex lap winding $Y_b = Y_f + 2$ ...(ii)

Solving equations (i) and (ii) we get $Y_b = 7$ and $Y_f = 5$

Drawing of Winding: In order to draw the winding diagram, first of all draw the coil sides and number them. Now to make connections start from any coil side, say with first coil side.

In order to get the coil side to which the 1st coil side is to be connected at the back, add back pitch to it. So first coil side will be connected to the 1 + 7 = 8th coil side at the back by means of end connections.

In order to get the coil side to which 8th coil side is to be connected on the front or commutator side of the winding, deduct front pitch from it, and, therefore, 8th coil side will be connected to the 8 – 5 = 3rd coil at the front or commutator end.



Developed View of 4-Pole, Single Layer, Progressive Simplex Lap Winding With 24 Coil Sides

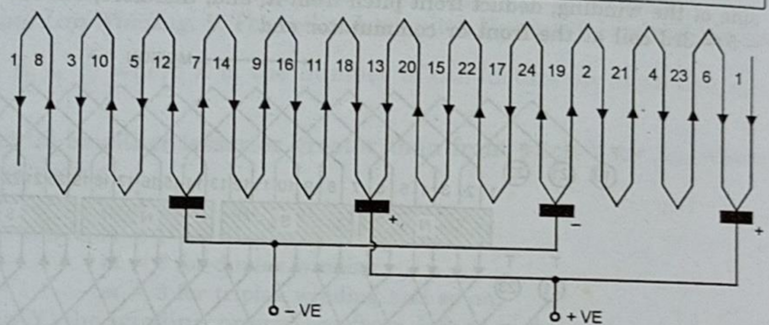
Fig. 4.22

Now repeat the process, according to which 3rd coil side will be connected to 10th coil side at the back and 10th coil side will be connected to 5th coil side at the next commutator segment.

Thus a table can be made indicating the way in which various coil sides will be connected. The coil sides are connected according to table of the end connections given below.

At Back End		At Front End		
Coil Side No.	*Connected to Coil Side No.*	*Coil Side No.*	*Connected to Coil Side No.*	*Through Segment No.*
1	8	8	3	2
3	10	10	5	3
5	12	12	7	4
7	14	14	9	5
9	16	16	11	6
11	18	18	13	7
13	20	20	15	8
15	22	22	17	9
17	24	24	19	10
19	2	2	21	11
21	4	4	23	12
23	6	6	1	1

Position of Brushes. Brush positions can be located by determining the direction of currents flowing in the various coil sides. Assuming the position of poles as shown in the developed winding diagram and the motion of coil sides from right to left, the direction of induced emf in the coil sides under the influence of N-pole will be downwards and in coil sides under influence of S-pole will be upwards. There may be some coil sides in the interpolar space for which there may be doubt about the directions of the induced emf. In such cases the directions of emf should not be marked. Since conductors 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17 and 18 are under the influence of N-pole, these conductors will have current flowing in downward direction while conductors 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23 and 24 being under the influence of S-pole will have current flowing in upward direction. Now draw the equivalent ring winding diagram, as shown in fig. 4.23 and put the arrows as per main winding diagram. From the winding diagram it is found that commutator segments 1 and 7 are meeting point of emfs and currents are flowing outwards from the conductors, so + ve brushes be fixed at these segments. Similarly commutator segments 4 and 10 are separating points of emfs and currents are flowing inwards so –ve brushes be fixed at these segments.



Equivalent Ring or Spiral Winding

Fig. 4.23

The brushes of same polarity are connected together and, therefore, the armature winding is divided in four paths in parallel.