Estimating Cosmological Parameters from Type Ia Supernovae: A Flat ΛCDM Approach

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Contents

1. Introduction:

In this project, I explored how Type Ia supernovae can be used to determine some of the most important parameters in cosmology: the Hubble constant (H_0), the matter density parameter (Ω_m), and the age of the universe. Type Ia supernovae are considered "standard candles" because their intrinsic luminosity is well understood, allowing us to estimate their distances accurately. Using this property and observational data from the Pantheon+SH0ES dataset, I constructed and tested a flat ΛCDM cosmological model and evaluated how well it fits the data.

2. Methodology and Approach:

In this project, I employed a structured computational approach using Python. The required libraries included NumPy for numerical calculations, Pandas for data manipulation, Matplotlib for visualization, SciPy for statistical analysis and integration, and Astropy for handling astrophysical constants and units.

Initially, I downloaded and carefully cleaned the Pantheon+SH0ES dataset, which involved extracting relevant data columns—redshifts (zHD), distance moduli (MU_SH0ES), and their uncertainties (MU_SH0ES_ERR_DIAG). The cleaning process included converting columns to numeric types and removing any incomplete entries to maintain data integrity.

After preparing the data, I constructed the theoretical cosmological framework based on the flat ΛCDM model. This included defining the dimensionless Hubble parameter E(z), calculating the comoving distance through numerical integration using `quad`, and determining the luminosity distance $d_L(z) = (1 + z)D_C$. I then used this distance to calculate the theoretical distance modulus μ(z) through the relation $\mu = 5\log_{10}(d_L/Mpc) + 25$.

To fit the model, I used `curve_fit` from SciPy, performing a non-linear least squares optimization to estimate the best-fit values of H_0 and Ω_m. I also tested an alternative fitting approach by fixing Ω_m at a range of values from 0.1 to 0.5 and re-fitting H_0, which allowed me to observe how both H_0 and the inferred age of the universe changed with different matter densities.

To verify the quality of the fit, I analyzed the residuals (μ_obs − μ_model) across all redshifts. I also performed segmented fitting by dividing the dataset into low-redshift (z <

0.1) and high-redshift ($z \geq 0.1$) subsets. For each subset, I repeated the fixed-Ω_m fits and computed the corresponding H_0 and age of the universe. These comparisons helped me understand how redshift influences parameter estimates and allowed for a more nuanced interpretation of cosmological trends.

Finally, I visualized the Hubble diagram and the residuals to assess how well the model fit the data. I also created tables summarizing the fitted H_0 and universe age for various fixed Ω_m values, both for the full dataset and for the low/high redshift subsets, revealing important trends and dependencies.

3. **My Responses:**

a) **What value of the Hubble constant ($H0$) did you obtain from the full dataset?**

I obtained $H_0 = 72.97 \pm 0.26$ km/s/Mpc when fitting the entire Pantheon+SH0ES dataset. This value was derived by simultaneously fitting for both H_0 and Ω_m using the theoretical model described above.

b) **How does your estimated $H0$ compare with the Planck18 measurement of the same?**

The Planck18 result gives $H_0 \approx 67.4$ km/s/Mpc, which is significantly lower than my result. This difference is part of the well-known "Hubble tension," which refers to the disagreement between early-universe measurements (like Planck CMB data) and late-universe measurements (like supernovae and Cepheids). My result is closer to the SH0ES measurement, which supports a higher value for H_0.

c) **What is the age of the Universe based on your value of H_0? (Assume Ωm = 0.3) How does it change for different values of Ωm?**

Using $H_0 = 72.97$ km/s/Mpc and $\Omega_m = 0.351$ (the best-fit value), I calculated the age of the universe to be approximately 12.36 Gyr. I then tested several fixed Ω_m values ranging from 0.1 to 0.5. For low Ω_m values (e.g., 0.1), the universe appears older (≈16.44 Gyr), and for higher Ω_m values (e.g., 0.5), it appears younger (≈11.37 Gyr).

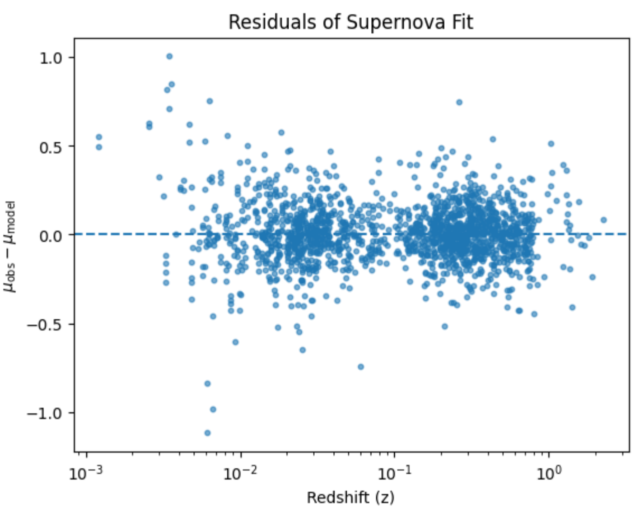
```
Ωm = 0.10 -> H0 = 76.03 ± 0.18 km/s/Mpc, Age = 16.44 Gyr
Ωm = 0.15 -> H0 = 75.35 ± 0.18 km/s/Mpc, Age = 15.03 Gyr
Ωm = 0.20 -> H0 = 74.71 ± 0.17 km/s/Mpc, Age = 14.08 Gyr
Ωm = 0.25 -> H0 = 74.10 ± 0.17 km/s/Mpc, Age = 13.38 Gyr
Ωm = 0.30 -> H0 = 73.53 ± 0.17 km/s/Mpc, Age = 12.82 Gyr
Ωm = 0.35 -> H0 = 72.98 ± 0.17 km/s/Mpc, Age = 12.37 Gyr
Ωm = 0.40 -> H0 = 72.46 ± 0.17 km/s/Mpc, Age = 11.98 Gyr
Ωm = 0.45 -> H0 = 71.96 ± 0.17 km/s/Mpc, Age = 11.65 Gyr
Ωm = 0.50 -> H0 = 71.48 ± 0.17 km/s/Mpc, Age = 11.37 Gyr
```

This shows that the age of the universe is quite sensitive to the matter density parameter. The image from my python notebook showing how the values of H_0 and age of universe are changing by varying the value of Ω_m.

d) Discuss the difference in *H*0 values obtained from the low-z and high-z samples. What could this imply?

When I split the dataset into low-z (z < 0.1) and high-z (z ≥ 0.1), I found $H_0 = 73.01 \pm 0.28$ km/s/Mpc for low-z and $H_0 = 73.85 \pm 0.22$ km/s/Mpc for high-z. Although the difference is within error bars, it may suggest subtle redshift-dependent effects or evolution in the supernova sample. It also reflects how measurements at different epochs or scales can influence the inferred value of H_0.

e) Plot the residuals and comment on any trends or anomalies you observe.



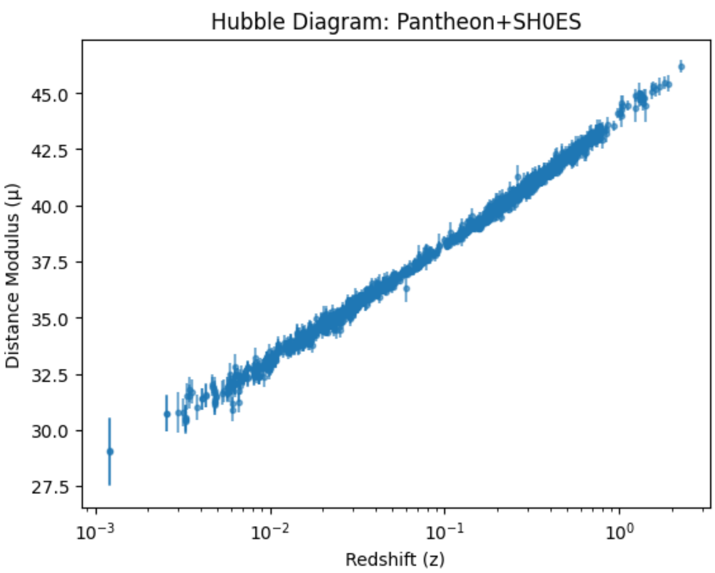
I plotted the residuals (μ_obs − μ_model) against redshift on a logarithmic scale. The scatter plot showed that the residuals are evenly distributed around zero across the entire redshift range, with no significant trends or deviations. There was slightly more scatter at lower redshifts, which is expected due to observational uncertainties and peculiar velocities, but overall the distribution appeared symmetric and unbiased. This supports the conclusion that the flat ΛCDM model is a good fit for the data. The absence of systematic trends in the residuals indicates that the theoretical model accurately captures the observed redshift-distance relationship.

f) What assumptions were made in the cosmological model, and how might relaxing them affect your results?

In my analysis, I assumed the standard flat ΛCDM model. This includes several key assumptions: (1) the universe is spatially flat ($\Omega_k = 0$), (2) dark energy is represented as a cosmological constant (with equation-of-state parameter $w = -1$), and (3) the matter content of the universe is composed of cold dark matter and baryons, with no contribution from radiation or curvature at late times. These assumptions simplify the equations and reduce the number of free parameters, which helps in fitting the data robustly. However, they may also mask underlying complexities. For example, allowing Ω_k to vary (i.e., a non-flat universe) could shift the inferred values of H_0 and Ω_m. Similarly, if dark energy is not a cosmological constant but evolves with time ($w \neq -1$), this could also alter the shape of the redshift-distance relation and change the best-fit parameters. Additionally, alternative cosmologies—such as dynamical dark energy models, modified gravity theories, or models with early dark energy—may better reconcile the current Hubble tension. By relaxing the flatness or fixed-w assumptions, we open up the parameter space, which could potentially lead to a model that fits both early- and late-universe data more consistently.

g) Based on the redshift-distance relation, what can we infer about the expansion history of the Universe?

The redshift-distance relation as visualized in the Hubble diagram clearly shows a logarithmic and monotonic increase in distance modulus (μ) with increasing redshift (z), forming a smooth upward curve. This indicates that supernovae farther away appear dimmer not only because they are more distant but also due to the accelerated expansion of the universe. The gentle upward curvature confirms that at lower redshifts the expansion was slower, and as we move to higher redshifts,



the expansion becomes more pronounced. This is strong evidence for an accelerating universe, which aligns with the presence of dark energy. The shape of this curve validates the ΛCDM model and implies that the universe's expansion has changed over time — it was decelerating in the past due to matter domination and has been accelerating recently due to dark energy.

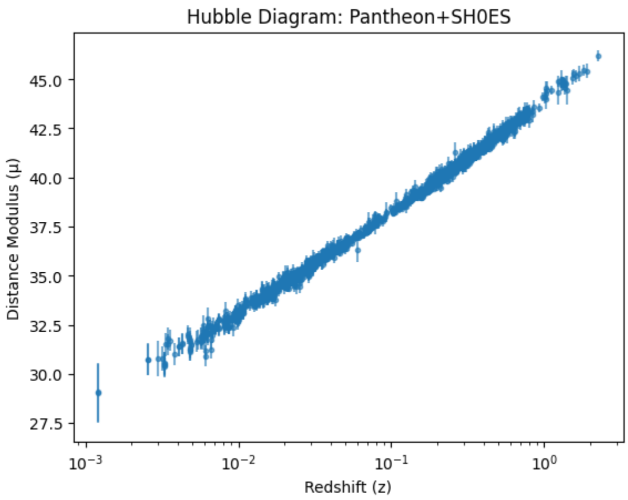
4. Observations and Interpretations:

Based on the results from all stages of the analysis, I made several key observations:

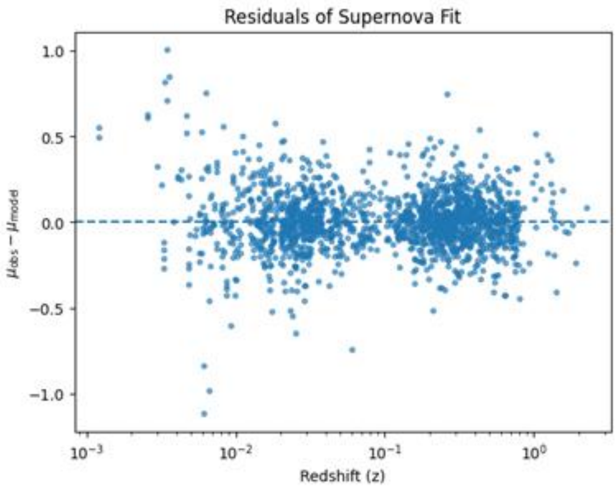
- The best-fit values of H_0 from the full dataset, the fixed Ω_m analysis, and the redshift-split subsets are all in excellent agreement with recent late-universe measurements (e.g., SH0ES), but remain significantly higher than the Planck18 result. This reinforces the ongoing Hubble tension in cosmology.
- The age of the universe showed a clear inverse relationship with Ω_m. As Ω_m increased, the calculated age decreased, ranging from about 16.4 Gyr (for $\Omega_m = 0.1$) to about 11.4 Gyr (for $\Omega_m = 0.5$). This was consistent both in the full dataset analysis and in the redshift-split subsets. This trend highlights the sensitivity of cosmological conclusions to the assumed matter density.
- The values of H_0 derived separately from low-z and high-z subsets showed slight but systematic differences. For example, at $\Omega_m = 0.3$, $H_{0\_low} = 73.01$ km/s/Mpc and $H_{0\_high} = 73.85$ km/s/Mpc. This could hint at redshift evolution or population effects in the supernova sample, or subtle systematic biases.
- The residuals plot showed excellent agreement between the theoretical model and observational data. The scatter was symmetrical and centered around zero, especially for mid- and high-redshift regions. The increased spread at very low redshifts is expected due to local peculiar velocities and observational noise.

These findings together confirm the validity of the flat ΛCDM model while also demonstrating that even within such a successful model, small variations in input assumptions or sample segmentation can yield slightly different outcomes.

5. Graphical Visualization:



I plotted distance modulus (μ) against redshift (z) on a logarithmic x-axis. The plot shows a clear increasing trend consistent with cosmic expansion and matched well with the theoretical model.



The residuals between observed and modeled distance moduli were plotted against redshift. These were well-centered around zero and showed no strong trends, confirming that the model fits the data robustly.

```
Ωm = 0.10 -> H0 = 76.03 ± 0.18 km/s/Mpc, Age = 16.44 Gyr
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Ωm = 0.50 -> H0 = 71.48 ± 0.17 km/s/Mpc, Age = 11.37 Gyr
```

This table shows how H_0 and the age of the universe change as Ω_m is varied from 0.1 to 0.5 using the full dataset. It visually demonstrates how increasing Ω_m leads to a younger universe and slightly lower H_0 values.

Ω_m	H_0_low ± err	Age_low (Gyr)	H_0_high ± err	Age_high (Gyr)
0.10	73.39 ± 0.28	17.03	77.67 ± 0.23	16.09
0.15	73.29 ± 0.28	15.45	76.62 ± 0.23	14.78
0.20	73.20 ± 0.28	14.37	75.64 ± 0.22	13.91
0.25	73.10 ± 0.28	13.56	74.72 ± 0.22	13.27
0.30	73.01 ± 0.28	12.91	73.85 ± 0.22	12.77
0.35	72.91 ± 0.28	12.38	73.03 ± 0.22	12.36
0.40	72.82 ± 0.28	11.92	72.24 ± 0.21	12.02
0.45	72.73 ± 0.28	11.53	71.50 ± 0.21	11.73
0.50	72.63 ± 0.28	11.19	70.79 ± 0.21	11.48

This second table compares H_0 and the age of the universe for low-z and high-z subsets across the same Ω_m range. It highlights how the two subsets yield slightly different values and reinforces the redshift-dependence of the results.

6. Conclusion:

This project allowed me to engage deeply with real astronomical data and apply fundamental cosmological models to interpret it. Through the step-by-step modeling and analysis of the Pantheon+SH0ES supernova dataset, I was able to estimate the Hubble constant, explore the effects of the matter density parameter, and compute the corresponding age of the universe. I also examined how model assumptions and data segmentation influence the results, which gave me a more critical understanding of both the power and limitations of cosmological inference.

Overall, the project not only solidified my technical skills in data analysis and scientific computing but also highlighted the nuances behind contemporary debates like the Hubble tension. It was interesting to see how small changes in assumptions or methodology can lead to differences in conclusions. This hands-on experience has strengthened my appreciation for the interplay between theoretical physics and observational data in modern astrophysics.

7. Python Notebook (Appendix)

🔭 Assignment: Measuring Cosmological Parameters Using Type Ia Supernovae

In this assignment, you'll analyze observational data from the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant H_0 and estimate the age of the universe. You will:

- Plot the Hubble diagram (distance modulus vs. redshift)
- Fit a cosmological model to derive H_0 and Ω_m
- Estimate the age of the universe
- Analyze residuals to assess the model
- Explore the effect of fixing Ω_m
- Compare low-z and high-z results

Let's get started!

📦 Getting Started: Setup and Libraries

Before we dive into the analysis, we need to import the necessary Python libraries:

- `numpy`, `pandas` — for numerical operations and data handling
- `matplotlib` — for plotting graphs
- `scipy.optimize.curve_fit` and `scipy.integrate.quad` — for fitting cosmological models and integrating equations
- `astropy.constants` and `astropy.units` — for physical constants and unit conversions

Make sure these libraries are installed in your environment. If not, you can install them using:

```
pip install numpy pandas matplotlib scipy astropy
```

In [ ]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.integrate import quad
from astropy.constants import c
from astropy import units as u
```

📂 Load the Pantheon+SH0ES Dataset

We now load the observational supernova data from the Pantheon+SH0ES sample. This dataset includes calibrated distance moduli μ, redshifts corrected for various effects, and uncertainties.

Instructions:

- Make sure the data file is downloaded from Pantheon dataset and available locally.
- We use `delim_whitespace=True` because the file is space-delimited rather than comma-separated.
- Commented rows (starting with `#`) are automatically skipped.

We will extract:

- `zHD`: Hubble diagram redshift

- `MU_SH0ES` : Distance modulus using SH0ES calibration
- `MU_SH0ES_ERR_DIAG` : Associated uncertainty

In [ ]:
```
# Path to the Pantheon+SH0ES data file
file_path = 'Pantheon+SH0ES.dat'

# Load the file
df = pd.read_csv(file_path, delim_whitespace=True, comment='#', header=None)
```

```
/tmp/ipython-input-4-3529426103.py:5: FutureWarning: The 'delim_whitespace' keyword in pd.re
ad_csv is deprecated and will be removed in a future version. Use ``sep='\s+'`` instead
  df = pd.read_csv(file_path, delim_whitespace=True, comment='#', header=None)
```

In [ ]:
```
df.head()
```

Out[ ]:

	0	1	2	3	4	5	6	7	8	
0	CID	IDSURVEY	zHD	zHDERR	zCMB	zCMBERR	zHEL	zHELERR	m_b_corr	m_b_corr_err
1	2011fe	51	0.00122	0.00084	0.00122	2e-05	0.00082	2e-05	9.74571	1.
2	2011fe	56	0.00122	0.00084	0.00122	2e-05	0.00082	2e-05	9.80286	1.
3	2012cg	51	0.00256	0.00084	0.00256	2e-05	0.00144	2e-05	11.4703	0.7
4	2012cg	56	0.00256	0.00084	0.00256	2e-05	0.00144	2e-05	11.4919	0.7

5 rows × 47 columns

🧾 Preview Dataset Columns

Before diving into the analysis, let's take a quick look at the column names in the dataset. This helps us verify the data loaded correctly and identify the relevant columns we'll use for cosmological modeling.

In [ ]:
```
df.columns = df.iloc[0]
```

In [ ]:
```
df = df.drop(index=0).reset_index(drop=True)
```

In [ ]:
```
df.columns
```

Out[ ]:
```
Index(['CID', 'IDSURVEY', 'zHD', 'zHDERR', 'zCMB', 'zCMBERR', 'zHEL',
       'zHELERR', 'm_b_corr', 'm_b_corr_err_DIAG', 'MU_SH0ES',
       'MU_SH0ES_ERR_DIAG', 'CEPH_DIST', 'IS_CALIBRATOR', 'USED_IN_SH0ES_HF',
       'c', 'cERR', 'x1', 'x1ERR', 'mB', 'mBERR', 'x0', 'x0ERR', 'COV_x1_c',
       'COV_x1_x0', 'COV_c_x0', 'RA', 'DEC', 'HOST_RA', 'HOST_DEC',
       'HOST_ANGSEP', 'VPEC', 'VPECERR', 'MWEBV', 'HOST_LOGMASS',
       'HOST_LOGMASS_ERR', 'PKMJD', 'PKMJDERR', 'NDOF', 'FITCHI2', 'FITPROB',
       'm_b_corr_err_RAW', 'm_b_corr_err_VPEC', 'biasCor_m_b',
       'biasCorErr_m_b', 'biasCor_m_b_COVSCALE', 'biasCor_m_b_COVADD'],
      dtype='object', name=0)
```

🧹 Clean and Extract Relevant Data

To ensure reliable fitting, we remove any rows that have missing values in key columns:

- `zHD` : redshift for the Hubble diagram
- `MU_SH0ES` : distance modulus
- `MU_SH0ES_ERR_DIAG` : uncertainty in the distance modulus

We then extract these cleaned columns as NumPy arrays to prepare for analysis and modeling.

In [ ]:
```
# Convert relevant columns to numeric, coercing invalid entries to NaN
df['zHD'] = pd.to_numeric(df['zHD'], errors='coerce')
df['MU_SH0ES'] = pd.to_numeric(df['MU_SH0ES'], errors='coerce')
df['MU_SH0ES_ERR_DIAG'] = pd.to_numeric(df['MU_SH0ES_ERR_DIAG'], errors='coerce')
```

In [ ]:
```
# Drop any rows missing values in our key columns
df_clean = df.dropna(subset=['zHD', 'MU_SH0ES', 'MU_SH0ES_ERR_DIAG'])
```

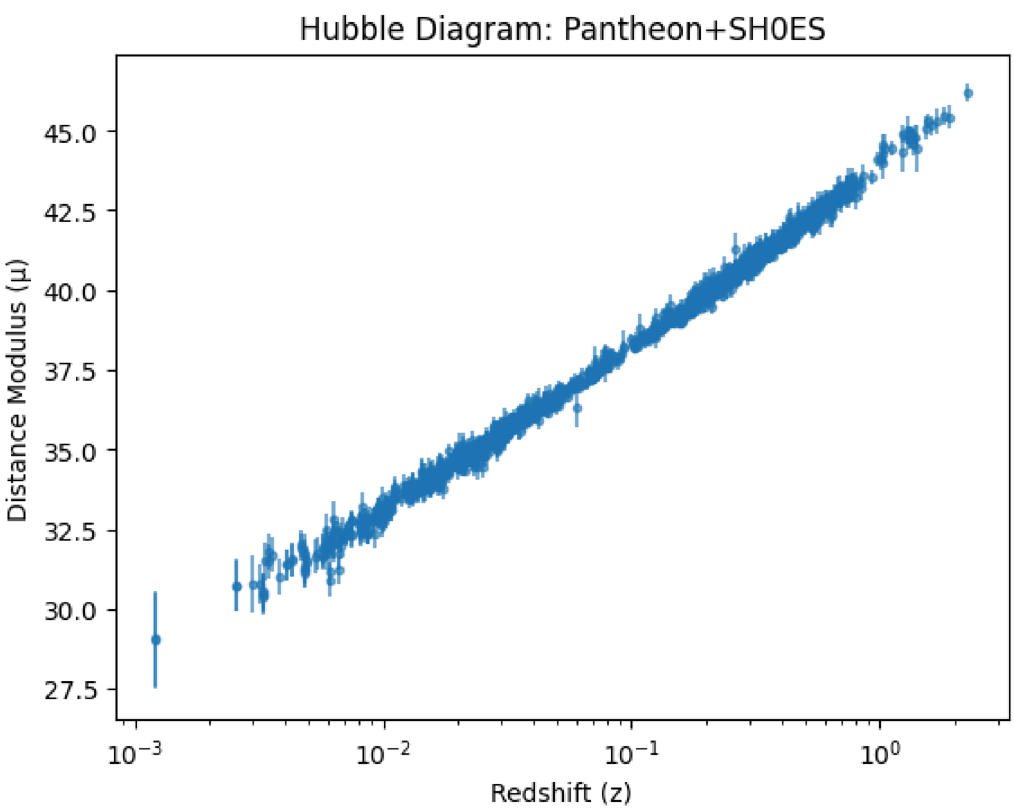
In [ ]:
```
# Extract NumPy arrays for modeling
z = df_clean['zHD'].values
mu = df_clean['MU_SH0ES'].values
mu_err = df_clean['MU_SH0ES_ERR_DIAG'].values
```

📈 Plot the Hubble Diagram

Let's visualize the relationship between redshift z and distance modulus μ, known as the Hubble diagram. This plot is a cornerstone of observational cosmology—it allows us to compare supernova observations with theoretical predictions based on different cosmological models.

We use a logarithmic scale on the redshift axis to clearly display both nearby and distant supernovae.

In [ ]:
```
plt.figure()
plt.errorbar(z, mu, yerr=mu_err, fmt='.', alpha=0.6)
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram: Pantheon+SH0ES')
plt.show()
```



🧮 Define the Cosmological Model

We now define the theoretical framework based on the flat $\Lambda\mathrm{CDM}$ model (read about the model in wikipedia if needed). This involves:

- The dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$$

- The distance modulus is:

$$\mu(z) = 5\log_{10}(d_L/\mathrm{Mpc}) + 25$$

- And the corresponding luminosity distance :

$$d_L(z) = (1+z)\cdot\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

These equations allow us to compute the expected distance modulus from a given redshift z, Hubble constant H_0, and matter density parameter Ω_m.

In [ ]:
```
# Define the E(z) for flat LCDM
# Dimensionless Hubble parameter for flat ΛCDM
def E(z, Omega_m):
    return np.sqrt(Omega_m * (1 + z)**3 + (1.0 - Omega_m))
```

In [ ]:
```
# Luminosity distance in Mpc, try using scipy quad to integrate.
def luminosity_distance(z, H0, Omega_m):

    if not hasattr(H0, 'unit'):   # Ensure H0 is a Quantity with units km/s/Mpc
        H0 = H0 * (u.km / u.s / u.Mpc)

    c_km_s = c.to(u.km / u.s)  # Speed of light in km/s
    factor = (c_km_s / H0).to(u.Mpc)  # Prefactor c/H0 has units of Mpc

    def integrand(zp):
        return 1.0 / E(zp, Omega_m)

    # Integrate for comoving distance
    if np.isscalar(z):
        Dc = factor * quad(integrand, 0, z)[0]
    else:
        Dc = np.array([quad(integrand, 0, zi)[0] for zi in z]) * factor

    D_L = (1 + z) * Dc # Luminosity distance D_L = (1+z) * D_C
    return D_L.to(u.Mpc)
```

In [ ]:
```
# Theoretical distance modulus
def mu_theory(z, H0, Omega_m):
    D_L = luminosity_distance(z, H0, Omega_m)
    return 5 * np.log10(D_L.value) + 25   # Convert to dimensionless for log10
```

🔧 Fit the Model to Supernova Data

We now perform a non-linear least squares fit to the supernova data using our theoretical model for $\mu(z)$. This fitting procedure will estimate the best-fit values for the Hubble constant H_0 and matter density parameter Ω_m, along with their associated uncertainties.

We'll use:

- `curve_fit` from `scipy.optimize` for the fitting.
- The observed distance modulus (\mu), redshift (z), and measurement errors.

The initial guess is:

- $H_0 = 70\,\mathrm{km/s/Mpc}$
- $\Omega_m = 0.3$

In [ ]:
```
# Initial guess: H0 = 70 km/s/Mpc, Omega_m = 0.3
p0 = [70, 0.3]
```

In [ ]:
```
# Fit mu_theory(z, H0, Omega_m) to the data
popt, pcov = curve_fit(
    mu_theory,
    z, mu,
    sigma=mu_err,
    p0=p0,
    absolute_sigma=True,
    maxfev=5000
)
```

In [ ]:
```
# Extract best-fit parameters and their 1σ uncertainties
H0_fit, Omega_m_fit = popt
H0_err, Omega_m_err = np.sqrt(np.diag(pcov))

print(f"Fitted H0 = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc")
print(f"Fitted Omega_m = {Omega_m_fit:.3f} ± {Omega_m_err:.3f}")
```

```
Fitted H0 = 72.97 ± 0.26 km/s/Mpc
Fitted Omega_m = 0.351 ± 0.019
```

⏳ Estimate the Age of the Universe

Now that we have the best-fit values of H_0 and Ω_m, we can estimate the age of the universe. This is done by integrating the inverse of the Hubble parameter over redshift:

$$t_0 = \int_0^\infty \frac{1}{(1+z)H(z)}\,dz$$

We convert H_0 to SI units and express the result in gigayears (Gyr). This provides an independent check on our cosmological model by comparing the estimated age to values from other probes like Planck CMB measurements.

In [ ]:
```
def age_of_universe(H0, Omega_m):
    H0_si = (H0 * u.km / u.s / u.Mpc).to(1 / u.s)  # Convert H0 [km/s/Mpc] to s⁻¹
    integral = quad(lambda z: 1.0 / ((1 + z) * E(z, Omega_m)), 0, np.inf, limit=200)[0]
    t0_sec = integral / H0_si.value  # Age in seconds = integral / H0_si
    return (t0_sec * u.s).to(u.Gyr).value # Convert to gigayears
```

In [ ]:
```
t0 = age_of_universe(H0_fit, Omega_m_fit)
print(f"Estimated age of Universe: {t0:.2f} Gyr")
```

```
Estimated age of Universe: 12.36 Gyr
```

📊 Analyze Residuals

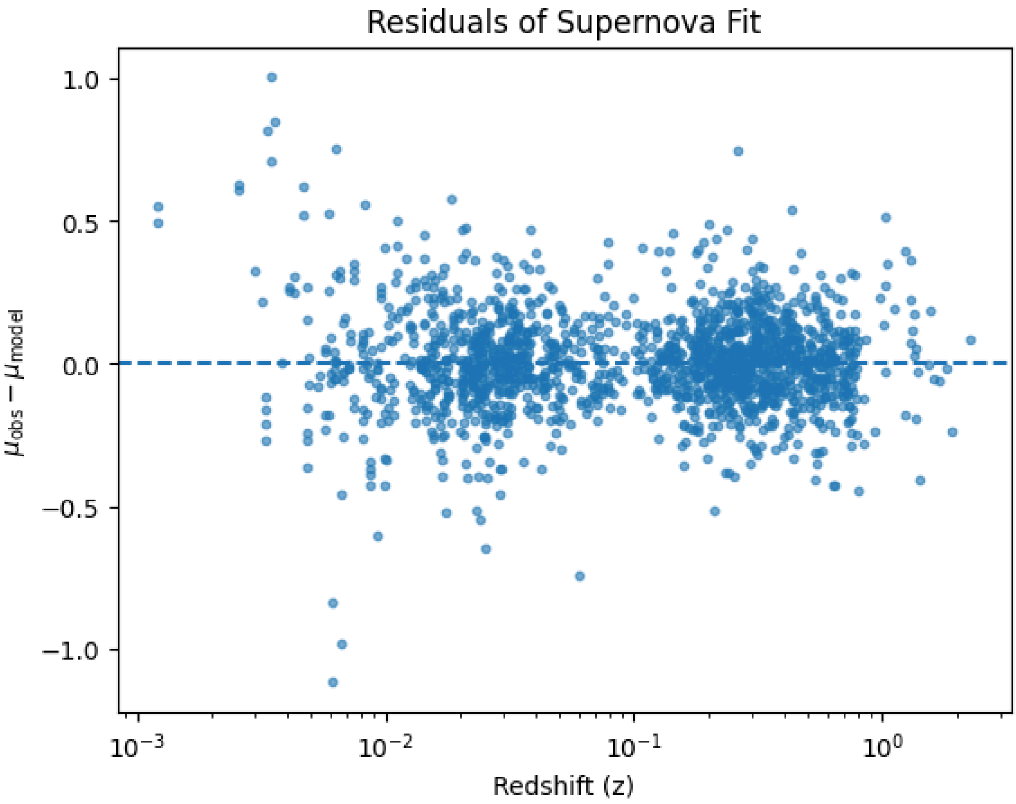
To evaluate how well our cosmological model fits the data, we compute the residuals:

$$\text{Residual} = \mu_{\text{obs}} - \mu_{\text{model}}$$

Plotting these residuals against redshift helps identify any systematic trends, biases, or outliers. A good model fit should show residuals scattered randomly around zero without any significant structure.

In [ ]:
```
mu_pred = mu_theory(z, H0_fit * u.km/u.s/u.Mpc, Omega_m_fit)
residuals = mu - mu_pred
```

In [ ]:
```
# Plot residuals vs. redshift
plt.figure()
plt.scatter(z, residuals, s=10, alpha=0.6)
plt.xscale('log')
plt.axhline(0, linestyle='--')
plt.xlabel('Redshift (z)')
plt.ylabel(r'$\mu_{\rm obs} - \mu_{\rm model}$')
plt.title('Residuals of Supernova Fit')
plt.show()
```



🔧 Fit with Fixed Matter Density

To reduce parameter degeneracy, let's fix $\Omega_m = 0.3$ and fit only for the Hubble constant H_0.

In [ ]:
```
# Fix Omega_m = 0.3, fit only H0
def mu_fixed_Om(z, H0):
    return mu_theory(z, H0, Omega_m=0.3)
```

In [ ]:
```
# Perform the fit
popt, pcov = curve_fit(
    mu_fixed_Om,
    z, mu,
    sigma=mu_err,
    p0=[70],
    absolute_sigma=True,
    maxfev=5000
)
```

In [ ]:
```
# Extract result
H0_fixed, = popt
H0_fixed_err, = np.sqrt(np.diag(pcov))
```

```python
print(f"Fixed Ωm=0.3 → H0 = {H0_fixed:.2f} ± {H0_fixed_err:.2f} km/s/Mpc")
```

```
Fixed Ωm=0.3 → H0 = 73.53 ± 0.17 km/s/Mpc
```

Checking with other fixed values of Ω_m

In [ ]:
```python
omega_values = [0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5]
results = []
```

In [ ]:
```python
for Om in omega_values:
    # Define model with Ωₘ fixed
    def mu_fixed(z, H0):
        return mu_theory(z, H0, Omega_m=Om)

    # Fit for H0
    popt, pcov = curve_fit(
        mu_fixed,
        z, mu,
        sigma=mu_err,
        p0=[70],
        absolute_sigma=True,
        maxfev=5000
    )
    H0_fit = popt[0]
    H0_err = np.sqrt(pcov[0,0])

    # Compute age of universe for this (H0, Om)
    t0 = age_of_universe(H0_fit, Om)

    results.append((Om, H0_fit, H0_err, t0))
    print(f"Ωₘ = {Om:.2f} -> H₀ = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc, Age = {t0:.2f} Gyr"
```

```
Ωₘ = 0.10 -> H₀ = 76.03 ± 0.18 km/s/Mpc, Age = 16.44 Gyr
Ωₘ = 0.15 -> H₀ = 75.35 ± 0.18 km/s/Mpc, Age = 15.03 Gyr
Ωₘ = 0.20 -> H₀ = 74.71 ± 0.17 km/s/Mpc, Age = 14.08 Gyr
Ωₘ = 0.25 -> H₀ = 74.10 ± 0.17 km/s/Mpc, Age = 13.38 Gyr
Ωₘ = 0.30 -> H₀ = 73.53 ± 0.17 km/s/Mpc, Age = 12.82 Gyr
Ωₘ = 0.35 -> H₀ = 72.98 ± 0.17 km/s/Mpc, Age = 12.37 Gyr
Ωₘ = 0.40 -> H₀ = 72.46 ± 0.17 km/s/Mpc, Age = 11.98 Gyr
Ωₘ = 0.45 -> H₀ = 71.96 ± 0.17 km/s/Mpc, Age = 11.65 Gyr
Ωₘ = 0.50 -> H₀ = 71.48 ± 0.17 km/s/Mpc, Age = 11.37 Gyr
```

🔍 Compare Low-z and High-z Subsamples

Finally, we examine whether the inferred value of H_0 changes with redshift by splitting the dataset into:

- **Low-z** supernovae ($z < 0.1$)
- **High-z** supernovae ($z \geq 0.1$)

We then fit each subset separately (keeping $\Omega_m = 0.3$) to explore any potential tension or trend with redshift.

In [ ]:
```python
# Redshift split threshold
z_split = 0.1

# Masks for low-z and high-z samples
mask_low  = z <  z_split
mask_high = z >= z_split
```

In [ ]:
```python
# Fit low-z subsample
popt_low, pcov_low = curve_fit(
    mu_fixed_Om,
```

```
    z[mask_low],
    mu[mask_low],
    sigma=mu_err[mask_low],
    p0=[70],
    absolute_sigma=True,
    maxfev=5000
)

H0_low     = popt_low[0]
H0_low_err = np.sqrt(pcov_low[0,0])
```

In [ ]:

```
# Fit high-z subsample
popt_high, pcov_high = curve_fit(
    mu_fixed_Om,
    z[mask_high],
    mu[mask_high],
    sigma=mu_err[mask_high],
    p0=[70],
    absolute_sigma=True,
    maxfev=5000
)

H0_high     = popt_high[0]
H0_high_err = np.sqrt(pcov_high[0,0])
```

In [32]:

```
t0_low  = age_of_universe(H0_low,  0.3)
t0_high = age_of_universe(H0_high, 0.3)

print(f"Low-z sample → H₀ = {H0_low:.2f} km/s/Mpc, Age = {t0_low:.2f} Gyr")
print(f"High-z sample → H₀ = {H0_high:.2f} km/s/Mpc, Age = {t0_high:.2f} Gyr")
```

```
Low-z sample → H₀ = 73.01 km/s/Mpc, Age = 12.91 Gyr
High-z sample → H₀ = 73.85 km/s/Mpc, Age = 12.77 Gyr
```

In [33]:

```
omega_values = [0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5]
z_split = 0.1
mask_low  = z <  z_split
mask_high = z >= z_split
```

In [41]:

```
print("Ωₘ   |  H₀_low ± err   | Age_low (Gyr) |  H₀_high ± err | Age_high (Gyr)")
print("-----|----------------|---------------|----------------|--------------")

for Om in omega_values:
    # Fit H₀ for low-z with Ωₘ fixed
    def mu_low(z_vals, H0):
        return mu_theory(z_vals, H0, Omega_m=Om)
    p_low, cov_low = curve_fit(mu_low,
                               z[mask_low], mu[mask_low],
                               sigma=mu_err[mask_low],
                               p0=[70],
                               absolute_sigma=True,
                               maxfev=5000)
    H0_low, H0_low_err = p_low[0], np.sqrt(cov_low[0,0])
    t0_low = age_of_universe(H0_low, Om)

    # Fit H₀ for high-z with Ωₘ fixed
    def mu_high(z_vals, H0):
        return mu_theory(z_vals, H0, Omega_m=Om)
    p_high, cov_high = curve_fit(mu_high,
                                 z[mask_high], mu[mask_high],
                                 sigma=mu_err[mask_high],
                                 p0=[70],
                                 absolute_sigma=True,
                                 maxfev=5000)
    H0_high, H0_high_err = p_high[0], np.sqrt(cov_high[0,0])
```

```
    t0_high = age_of_universe(H0_high, Om)
    print(f"{Om:>4.2f} | {H0_low:6.2f} ± {H0_low_err:<5.2f} | {t0_low:13.2f} | "f"{H0_high:
```

```
Ωm    |  H₀_low ± err   | Age_low (Gyr) |  H₀_high ± err | Age_high (Gyr)
-----|----------------|---------------|----------------|--------------
0.10 |  73.39 ± 0.28  |         17.03 |  77.67 ± 0.23  |         16.09
0.15 |  73.29 ± 0.28  |         15.45 |  76.62 ± 0.23  |         14.78
0.20 |  73.20 ± 0.28  |         14.37 |  75.64 ± 0.22  |         13.91
0.25 |  73.10 ± 0.28  |         13.56 |  74.72 ± 0.22  |         13.27
0.30 |  73.01 ± 0.28  |         12.91 |  73.85 ± 0.22  |         12.77
0.35 |  72.91 ± 0.28  |         12.38 |  73.03 ± 0.22  |         12.36
0.40 |  72.82 ± 0.28  |         11.92 |  72.24 ± 0.21  |         12.02
0.45 |  72.73 ± 0.28  |         11.53 |  71.50 ± 0.21  |         11.73
0.50 |  72.63 ± 0.28  |         11.19 |  70.79 ± 0.21  |         11.48
```

You can check your results and potential reasons for different values from accepted constant using this paper by authors of the Pantheon+ dataset

You can find more about the dataset in the paper too