

Week 2 Graded assignment (May 2023)

Mathematics for Data Science - 1

1 Instructions:

- There are some questions that have functions with discrete-valued domains (such as day, month, year, etc). For simplicity, we treat them as continuous functions.
- For NAT type question, enter only one right answer even if you get multiple answers for that particular question.

- **Notations:**

- \mathbb{R} = Set of real numbers
 - \mathbb{Q} = Set of rational numbers
 - \mathbb{Z} = Set of integers
 - \mathbb{N} = Set of natural numbers
- The set of natural numbers includes 0.

1. Find the y - coordinate of the point of intersection of straight lines represented by (1) and (2), given the following equations:

$$ax + by + c = E \quad (1)$$

$$bx + cy + d^2 = F \quad (2)$$

Given that

$$E = F = 0$$

Arithmetic mean of a and b is c

Geometric mean of a and b is d

Choose the set of correct option(s).

Note:

Arithmetic mean of m and n is $\frac{m+n}{2}$

Geometric mean of m and n is \sqrt{mn}

Answer: Option a

[MCQ:1 Marks]

- $\left(\frac{2a^2b-ab-b^2}{2b^2-a^2-ab} \right)$
- $\left(\frac{a^2}{a-b} - 1 \right)$
- $\left(\frac{2b^2b-ab-b^2}{2b^2-b^2-ab} \right)$
- $\left(\frac{b^2}{a-b} - 1 \right)$

Soluⁿ

c is the arithmetic mean of a and b

$$\text{so } c = \frac{a+b}{2}$$

d is the geometric mean of a and b

Once we will solve the equation $\text{so } d = \sqrt{ab}$ and ① and ② we get value of n and y

so , Let multiply by b both sides of equation (1) and
multiply by a both sides of equation (2)

we get,

$$abu + b^2y + bc = 0 \quad \text{--- } ③$$

$$abu + acy + ad^2 = 0 \quad \text{--- } ④$$

Subtracting both sides of the equation (3) from equation
both sides of the equation (4) we get,

$$\begin{aligned} & (4c - b^2)y + ad^2 - bc = 0 \\ \Rightarrow y &= \frac{bc - ad^2}{4c - b^2} = \frac{b\left(\frac{a+b}{2}\right) - a(\sqrt{ab})^2}{a\left(\frac{a+b}{2}\right) - b^2} \\ &= \frac{ab + b^2 - 2a^2b}{a^2 + ab - 2b^2} \\ y &= \frac{2a^2b - ab - b^2}{2b^2 - a^2 - ab}. \end{aligned}$$

Hence, y -co-ordinate of the intersection point
is $\frac{2a^2b - ab - b^2}{2b^2 - a^2 - ab}$.

2. A mobile company wants to launch its new model in collaboration with a network provider named **Astron** to attract more customers. The following are two options the mobile company gives to buy the mobile.

Option 1 - Mobile and 1-year Astron Network costs 34000 rupees (Network offers unlimited calls for one year)

Option 2 - Only Mobile costs 22000 rupees

Now Lalith wants to buy this new model mobile. Lalith needs only 200 minutes per month. There are two network providers in the country.

Network Provider	Fixed Charge (Per month)	Per minute Charge
Astron	100	2
Proton	200	0.5

Answer the following questions:

- (a) What is the best option for Lalith to buy the mobile?

Choose the set of correct option(s).

Answer: Option c

[MCQ:1 Marks]

- Buy only a mobile and take 1 year Astron.
- Buy mobile along with 1-year Astron network offered by the mobile company
- Buy only mobile and take 1 year Proton network
- Buy only mobile and use Astron network for 6 months and Proton network for 6 months.

- (b) How much will he save per year if he chooses the best option to buy the mobile compared to the collaborated offer given by the company?

Answer: 8400 [NAT:1 Marks]

(a) *We have two cases and 3rd case we will see later.* [NAT:1 Marks]
Case ① If Lalith uses option ① for one year
then he has to pay ₹ 34000

Case ② If Lalith uses option ② for one year then
he has to pay, mobile costs + mobile
recharge.

So lets form two equation according to the table
as fixed charge and rate is given.

So for Astron $\rightarrow y = 2x + 100$

and for proton $y = 0.5x + 200$

where x is the number of minutes using

by a person and y is the cost per month.

So for 200 min in a month recharge cost when Lalith uses Aptron network is $2 \times 200 + 100 = 500$

And so for one year recharge cost $= 12 \times 500 = 6000$

Similarly for proton, recharge cost for 12 month for Lalith is $12 \times (0.5 \times 200 + 200) = 3600$

So cost ① with proton network is $22000 + 3600 = 25600$

and cost ② with Aptron network is $22000 + 6000 = 28000$

Ques ③ if Lalith uses option ② with 6 month proton and 6 month Aptron network then, for 12 month Lalith has to pay $22000 + 6(2 \times 200 + 100) + 6 \times (0.5 \times 200 + 200)$
 $= 22000 + 3000 + 1800$
 $= 26800$

Hence option ③ is true.

(b) If Lalith uses best option to buy mobile then have to pay for one year (with mobile recharge) is 25600 ₹

And with collaborated offer is 34000 ₹

$$\begin{aligned} \text{So same money} &= 34000 - 25600 \\ &= 8400 \text{ ₹} \end{aligned}$$

3. State Government wants to connect the state road to the national highway from a town. There are 3 possible locations in the town A,B and C to connect to the National Highway whose locations are given by coordinates $(3, 8), (5, 7), (6, 9)$. The National Highway connects the 2 points $(2, 1), (10, 7)$ and You, being the contractor, have the freedom to select any one of the 3 possible locations in the town.

Hint Always select the shortest path to construct the road.

Note: 1 unit = 100 meter

Answer the following questions:

(a) What point will you select to build the road?

- A
- B
- C
- None

Answer: Point B

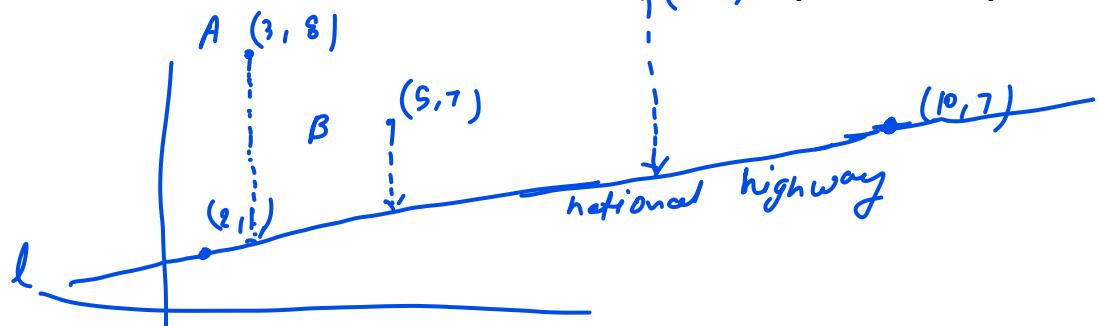
[MCQ:1 Marks]

(b) What is the minimum length of road in meter required to construct to connect to the National Highway?

Answer: 300 m

[NAT:1 Marks]

(q)



Let's find the equation of national highway.

$$y - 1 = \frac{7 - 1}{10 - 2} (x - 2)$$

$$\Rightarrow y - 1 = \frac{6}{8} (x - 2) \Rightarrow 8y - 8 = 6x - 12 \\ \Rightarrow 6x - 8y - 4 = 0$$

Now let's find the distance of the points A, B, C from the line l (national highway).

$$\text{for } A, d_1 = \frac{|6 \times 3 - 8 \times 8 - 4|}{\sqrt{6^2 + 8^2}} = \frac{|18 - 64 - 4|}{10} = 5$$

$$\text{for } B, d_2 = \frac{|6 \times 5 - 8 \times 7 - 4|}{\sqrt{6^2 + 8^2}} = \frac{|30 - 56 - 4|}{10} = 3$$

$$\text{for } C, d_3 = \frac{|6 \times 6 - 8 \times 9 - 4|}{\sqrt{6^2 + 8^2}} = \frac{|36 - 72 - 4|}{10} = 4$$

Hence option B is correct.

(b) The minimum length = 300 meter.

4. The total expenses of mess consists of fixed cost and the variable cost, Variable cost is proportional to the number of inmates of the mess, The total expenses are 16000 rupees when 12 members in the mess, and 20000 rupees when 20 members in the mess, find the Fixed cost of the mess. **Answer:** 10000 rupees [NAT:1 Marks]

Solu

If 12 members are in the mess then total expenses = 16000

Similarly, if 20 member are in the mess, then total expenses = 20000

We can think it as point $(12, 16000)$ and $(20, 20000)$

Let's find the equation of a line pass through these two points:

$$y - 16000 = \frac{4000}{8} (x - 12)$$

$$\Rightarrow y - 16000 = 500(x - 12)$$

$$\Rightarrow y - 16000 = 500x - 6000$$

$$\Rightarrow y = 500x + 10000$$

Hence fixed cost = 10000 ₹

5. A line perpendicular to the line segment joining A (1, 0) and B (2, 3), divides it at C in the ratio of 1:5 internally. Then the equation of line is

- $3x + 9y - 8 = 0$
- $3x + 9y + 8 = 0$
- $x + 3y - 8 = 0$
- $3x + 9y - 16 = 0$

Given ratio $m_1:m_2 = 1:5$

Answer: A

[MCQ:1 Marks]

By using Section formula:-

$$\text{point } C(x,y) = \left(\frac{2+5}{6}, \frac{3+0}{6} \right)$$

$$(x,y) = \left(\frac{7}{6}, \frac{1}{2} \right)$$

Let's substitute $\left(\frac{7}{6}, \frac{1}{2} \right)$ in equation $3x + 9y - 8 = 0$

$$\frac{7}{2} + \frac{9}{2} - 8 = 0$$

Hence respective line is $3x + 9y - 8 = 0$

This question can be solved by different method
also (using slope point form).

6. A bird is flying along the straight line $2y - 6x = 20$. In the same plane, an aeroplane starts to fly in a straight line and passes through the point $(4, 12)$. Consider the point where aeroplane starts to fly as origin. If the bird and plane collides then enter the answer as 1 and if not then 0.

Note: Bird and aeroplane can be considered to be of negligible size.

(Answer: 0)

[Marks: 1]

Soluⁿ: Slope of the equation of the line

$2y - 6x = 20$ is 3. (We can get by changing
equation in slope intercept form)

Slope of the line which passes through the
origin the point $(4, 12) = \frac{12}{4} = 3$

Since both lines have the same slope hence
both lines are parallel.

That means arrow will never hit the bird.

7. A surveyor needs to determine the area of a land show in Fig below. The coordinates of the four vertices of the land are as follows: A (8, 13) B (3, 10) C (4, 4) D (16, 5)

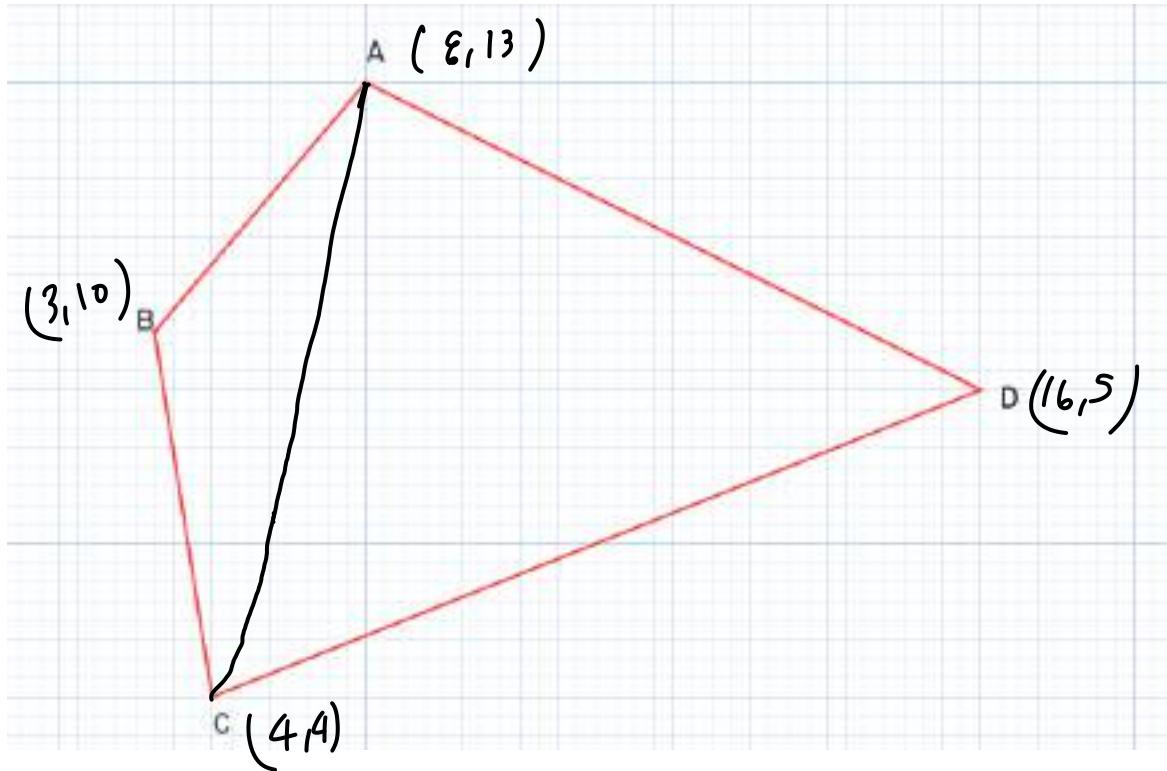


Fig:Survey Area

Answer: 68.5 Sq Units

[NAT:1 Marks]

Let's find the find the area of the triangles ABC and triangle ADC

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} | 8(10-4) + 3(4-13) + 4(13-10) | \\ &= \frac{1}{2} | 48 - 27 + 12 | = \frac{33}{2} = 16.5 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ADC &= \frac{1}{2} | 8(5-4) + 16(4-13) + 4(13-5) | \\ &= \frac{1}{2} | 8 - 144 + 32 | \\ &= 52 \end{aligned}$$

$$\text{Total area} = 52 + 16.5 = 68.5 \text{ sq units.}$$

8. A fitness trainer is analyzing the weight loss progress of his best client over a period of 6 months, to use it as for marketing. He recorded the weight of the client at the beginning and end of each month. Using straight line fitting, he came up with an equation $W = -8t + 98$, where W = Weight in Kg t = time in months. Now you want to check whether this equation is correct or not so you collected the data from the gym the data is given in table below.

Time (months)	Weight (Kgs)
0	98
1	90
2	82
3	74
4	66
5	57
6	49

Answer the following questions

- (a) Equation that fitness trainer came up with $W = -8t + 98$ is well fitted to data.

(Equation is said to be well fitted to data if the SSE is less than 5) True/False

Answer: True

[MCQ:1 Marks]

- (b) You were impressed by the performance of the fitness trainer, so you want to get trained under him, you assumed that the rate of weight loss (weight loss per month) will be same as the case of the best client mentioned in the question, Considering your assumption is true, How many days are required for you to loss weight from 100 kg to 72 kg.

Note: 1 month has 30 days

Answer: 105 days

[NAT:1 Marks]

(a) We have $W = -8t + 98$

$$\begin{aligned} SSE &= \sum (98 - 98)^2 + (90 + 8 - 98)^2 + (82 + 16 - 98)^2 \\ &\quad + (74 + 24 - 98)^2 + (66 + 32 - 98)^2 \\ &\quad + (57 + 40 - 98)^2 + (49 + 48 - 98)^2 \end{aligned}$$

$$= \sum 0 + 0 + 0 + 0 + 0 + 1 + 1$$

$$= 2 < 5$$

Hence equation is well fitted.

(b)

We have

$$W = -8t + 98$$

so weight loss per month = 8 kg

Total weight loss from 100 kg to 72 kg
 $= 100 - 72 = 28 \text{ kg}$

So number of days to lose the
weight 28 kg = $\frac{28}{8} \times 30^{15} = 105 \text{ day}$

9. The equation used to measure the C_d (discharge coefficient) of the venturimeter value in the lab is:

$$Q = \frac{c_d}{\sqrt{1 - \beta^4}} \sqrt{2g\Delta h} \quad (3)$$

c_d – discharge coefficient

$\beta = \frac{d}{D}$

A1 – Pipe section

A2 – Restriction area

Δh – head losses

Q – flow rate

Ramesh plotted the graph between Q on the y-axis and $\sqrt{\Delta h}$ on the x-axis and claimed that $\frac{c_d}{\sqrt{1 - \beta^4}} \sqrt{2g}$ is the slope of the obtained straight line.

Suresh plotted the graph between $\sqrt{\Delta h}$ on the y-axis and Q on the x-axis and claimed that $\frac{\sqrt{1 - \beta^4}}{c_d \sqrt{2g}}$ is the slope of the straight line.

Who is/are actually correct?

- Only Ramesh
- Only Suresh
- Both Ramesh and Suresh
- None of them are correct

Answer: C

[MCQ:1 Marks]

Soln:

$$Q = \frac{c_d}{\sqrt{1 - \beta^4}} \sqrt{2g \Delta h}$$

$$\Rightarrow Q = \frac{c_d}{\sqrt{1 - \beta^4}} \sqrt{2h} \sqrt{\Delta h}$$

Comparing with slope intercept form $y = mx + c$

$$\Rightarrow \text{slope } m = \frac{c_d}{\sqrt{1 - \beta^4}} \sqrt{2h}$$

gain we have $Q = \frac{C_d}{\sqrt{1-\beta^4}} \sqrt{2g} \sqrt{\Delta h}$

$$\Rightarrow \sqrt{\Delta h} = \frac{\sqrt{1-\beta^4}}{C_d \sqrt{2g}} Q$$

Same as above, Slope $m = \frac{\sqrt{1-\beta^4}}{C_d \sqrt{2g}}$

Hence option C is true.

10. A rock is thrown in a pond, and creates circular ripples whose radius increases at a rate of 0.2 meter per second. What will be the value of $\frac{A}{\pi}$, where A is the area (in square meter) of the circle after 20 seconds?

Hint: The area of a circle = πr^2 , where r is the radius of the circle. (1 marks)

$$\text{radius} = 0.2t$$

$$\begin{aligned}\text{So after 20 sec, radius} &= 0.2 \times 20 \\ &= 4 \text{ m.}\end{aligned}$$

$$\text{So area } A = \pi 4^2$$

$$\Rightarrow \frac{A}{\pi} = 16$$