Natural Language Processing MEMMs and CRFs

Felipe Bravo-Marquez

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- Maximum-entropy Markov models (MEMMs) make use of log-linear models for sequence labeling tasks.
- In the early NLP literature, logistic regression was often called maximum entropy classification [?].
- Hence, MEMMs will look very similar to the multi-class softmax models seen in the lecture about linear models.
- The goal of MEMMs is model the following conditional distribution:

$$P(s_1, s_2 \ldots, s_m | x_1, \ldots, x_m)$$

Where each x_j for j = 1...m is the j-th input symbol (for example the j-th word in a sentence), and each s_j for j = 1...m is the j-th tag.¹

¹These slides are based on lecture notes of Michael Collins http://www.cs.columbia.edu/~mcollins/. The notation and terminology has been adapted to be consistent with the other material.

- We use S to denote the set of possible tags.
- We assume that S is a finite set.
- For example, in part-of-speech tagging of English, S would be the set of all
 possible parts of speech in English (noun, verb, determiner, preposition, etc.).
- Given a sequence of words x_1, \ldots, x_m , there are k^m possible part-of-speech sequences s_1, \ldots, s_m , where k = |S| is the number of possible parts of speech.
- We want to estimate a distribution over these k^m possible sequences.

In a first step, MEMMs use the following decomposition:

$$P(s_{1}, s_{2}..., s_{m}|x_{1},..., x_{m}) = \prod_{i=1}^{m} P(s_{i}|s_{1}..., s_{i-1}, x_{1},..., x_{m})$$

$$= \prod_{i=1}^{m} P(s_{i}|s_{i-1}, x_{1},..., x_{m})$$
(1)

- The first equality is exact (it follows by the chain rule of conditional probabilities).
- The second equality follows from an independence assumption, namely that for all i,

$$P(s_i|s_1...,s_{i-1},x_1,...,x_m) = P(s_i|s_{i-1},x_1,...,x_m)$$

- Hence we are making an first order Markov assumption similar to the Markov assumption in HMMs.
- It is also possible to have second-order Markov models here.
- The tag in the *i*-th position depends only on the tag in the (i-1)-th position.
- Having made these independence assumptions, we then model each term using a log-linear model:

$$P(s_{i}|s_{i-1},x_{1},...,x_{m}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s_{i}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s'))}$$
(2)

Here $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)$ is a feature vector where:

- x_1, \dots, x_m is the entire sentence being tagged.
- *i* is the position to be tagged (can take any value from 1 to *m*)
- s is the previous tag value (can take any value in S).
- s' is the new tag value (can take any value in S)

Example of Features used in Part-of-Speech Tagging

- 1. $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[1]} = 1$ if $s_i = \mathsf{ADVERB}$ and word x_i ends in "-ly"; 0 otherwise. If the weight $\vec{w}_{[1]}$ associated with this feature is large and positive, then this feature is essentially saying that we prefer labelings where words ending in -ly get labeled as ADVERB.
- 2. $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[2]} = 1$ if $i = 1, s_i = VERB$, and $x_m = ?$; 0 otherwise. If the weight $\vec{w}_{[2]}$ associated with this feature is large and positive, then labelings that assign VERB to the first word in a question (e.g., "Is this a sentence beginning with a verb?") are preferred.
- 3. $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[3]} = 1$ if $s_{i-1} = \text{ADJECTIVE}$ and $l_i = \text{NOUN}$; 0 otherwise. Again, a positive weight for this feature means that adjectives tend to be followed by nouns.
- 4. $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[4]} = 1$ if s_{i-1} = PREPOSITION and s_i = PREPOSITION. A negative weight $\vec{w}_{[4]}$ for this function would mean that prepositions don't tend to follow prepositions.

²Source: https://blog.echen.me/2012/01/03/ introduction-to-conditional-random-fields/

Example of Features used in Part-of-Speech Tagging

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MEMMs and Multi-class Softmax

- Notice that the log-linear model from above is very similar to the multi-class softmax model presented in the lecture about linear models.
- A general log-linear model has the following form:

$$P(y|x; \vec{w}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x, y))}{\sum_{y' \in Y} \exp(\vec{w} \cdot \vec{\phi}(x, y'))}$$

A multi-class softmax model has the following form:

$$\hat{\vec{y}} = \operatorname{softmax}(\vec{x} \cdot W + \vec{b})
\hat{\vec{y}}_{[i]} = \frac{e^{(\vec{x} \cdot W + \vec{b})_{[i]}}}{\sum_{j} e^{(\vec{x} \cdot W + \vec{b})_{[j]}}}$$
(3)

- Difference 1: in the log-linear model we have a fixed parameter vector \vec{w} instead of having multiple vectors (one column of W for each class value).
- Difference 2: the feature vector of the log-linear model $\vec{\phi}(x,y)$ includes information of the label y, whereas the input vector \vec{x} of the softmax model is independent of y.
- Log-linear models allow using features that consider the interaction between x and y (e.g., x end in "ly" and y is an ADVERB).

Questions?

Thanks for your Attention!

References I