2400033348 Tutorial-4 M.Shabarinath

1) Compute the first and second order derivites for $f(x) = e^{xy} + 3x^2 - 5y^3$ and Verify ~~f(x) = f~~ $f(xy) = f(yx)$?

A) $f = e^{xy} + 3x^2 - 5y^3$

$f(x) = \frac{d}{dx}(e^{xy} + 3x^2 - 5y^3)$

$= e^{xy}\frac{d}{dx}(xy) + \frac{d}{dx}3x^2 - 0$

$= e^{xy} 1 \cdot y + 6x.$

$f(y) = \frac{d}{dy}(e^{xy} + 3x^2 - 5y^3)$

$= e^{xy}\frac{d}{dy}(e^{xy}) + 0 - \frac{d}{dy}(5y^3)$

$= e^{xy} 1 \cdot \cancel{y} x - 15y^2$

$f(x)x = \frac{d}{dx}f(x) = \frac{d}{dx}(e^{xy}y + 6x)$

$= \frac{d}{dx}e^{xy}y + \frac{d}{dx}6x$

$= y \cdot y e^{xy} + 6$

$= y^2 e^{xy} + 6$

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$$f(y)\,y = \frac{d}{dy}\left(e^{xy}x - 15y^2\right)$$

$$= x\frac{d}{dy}e^{xy} - \frac{d}{dx}\left(15y^2\right)$$

$$= x\,e^{xy}x - 30y$$

$$= x^2 e^{xy} - 30y$$

$$f_{xy} = \frac{d}{dy}f(x)$$

$$= \frac{d}{dy}\left(\underset{U\,V}{ye^{xy}} + 6x\right) \qquad UV = (UV' + VU')$$

$$= y\frac{d}{dy}e^{xy} + e^{xy}\frac{d}{dy}y + 0$$

$$= y\cdot e^{xy}\frac{d}{dy}(xy) + e^{xy}$$

$$= y\cdot e^{xy}\cdot x + e^{xy}$$

$$= e^{xy}(xy+1)$$

$$f_{yx} = \frac{d}{dx}f(y)$$

$$= \frac{d}{dx}\left(\underset{U}{e^{xy}}\underset{V}{x} - 15y^2\right)$$

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$= e^{xy}\frac{d}{dx}x + x\frac{d}{dx}e^{xy} - 0$

$= e^{xy} + x \cdot e^{xy}\frac{d}{dx}(xy)$

$= e^{xy} + x \cdot e^{xy} \cdot y$

$= e^{xy}(1+xy)$

$\boxed{f_{xy} = f_{yx}}$

(2) find first order and second order for $f(x) = x^2y^3 + \sin x \cos y$

(1) $f(x) = \frac{\partial}{\partial x}(x^2y^3) + \sin x \cos y$

$= \frac{\partial}{\partial x}(x^2y^3) + \frac{\partial}{\partial x}\sin x \cos y$

$= 2xy^3 + \cos x \cos y$

$f(y) = \frac{\partial}{\partial y}(x^2y^3) + \sin x \cos y$

$= x^2 \cdot 3y^2 + \sin x - \sin y$

$= x^2 \cdot 3y^2 + \underset{(-)}{\sin x} \sin y$

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$$f(x)(x) = \frac{d}{dx}(f(x))$$

$$= \frac{\partial}{dx}(2xy^3 + \cos x \cos y)$$

$$= \frac{d}{dx}2xy^3 + \frac{d}{dx}(\cos x \cos y)$$

$$= 2y^3 - \sin x \cos y$$

$$f(y)(y) = \frac{\partial}{dy} f(y)$$

$$= \frac{\partial}{dy}(x^2 3y^2 - \sin x - \sin y)$$

$$= x^2 \frac{d}{dy} 3y^2 - \sin x \frac{d}{dy} \sin y.$$

$$= x^2 6y - \sin x \cos y$$

$$f_{xy} = \frac{d}{dy} f(x)$$

$$= \frac{d}{dy}(2xy^3 + \cos x \cos y)$$

$$= 2x \frac{d}{dy} y^3 + \cos x \frac{d}{dy} \cos y.$$

$$= 2x 3y^2 + \cos x - \sin y$$

$$= 3y^2 2x - \cos x \sin y$$

$$f_{yx} = \frac{d}{dx} f(y)$$

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$$= \frac{d}{dx}\left(x^2 3y^2 + \sin x \sin y\right)$$

$$= \frac{d}{dx} x^2 \cdot 3y^2 + \frac{d}{dx} \sin x \cdot \sin y$$

$$= 2x\, 3y^2 + \cos x \sin y$$

③ Verify $\dfrac{\partial^2 U}{\partial x \partial y} = \dfrac{\partial^2 U}{\partial y \partial x}$ where $U = \tan^{-1}\left(\frac{x}{y}\right)$

① $U = \tan^{-1}\left(\frac{x}{y}\right)$.

$$U(x) = \frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{y}\right)\right)$$

$$\tan^{-1} f = \frac{f'}{1+f^2}$$

$$= \frac{\frac{d}{dx}\left(\frac{x}{y}\right)}{1+\left(\frac{x}{y}\right)^2}$$

$$= \frac{\frac{1}{y}}{\frac{y^2+x^2}{y^2}} = \frac{y}{y^2+x^2} =$$

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$$U(y) = \frac{d}{dy}\left(\tan^{-1}\left(\frac{x}{y}\right)\right)$$

$$= \frac{\frac{d}{dy}\left(\frac{x}{y}\right)}{1+\frac{x^2}{y^2}}$$

$$\frac{d}{dx}\left(\frac{1}{y}\right) = \frac{-1}{y^{-2}}$$

$$= \frac{x}{y^2+x^2} = \frac{-x}{x^2+y^2}.$$

$$\cancel{U_{xx} = \frac{\partial}{\partial x} U_x}$$

$$\cancel{= \frac{\partial}{\partial x}\frac{y}{x^2+y^2}}$$

$$\cancel{= y\frac{\partial}{\partial x}\frac{1}{x^2+y^2}}$$

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}\right)$$

$$= \frac{d}{dx}\left(\frac{-x}{x^2+y^2}\right)$$

$$= \frac{(x^2+y^2)\frac{d}{dx}(-x) - (-x)\frac{d}{dx}(x^2+y^2)}{(x^2+y^2)^2}.$$

$$\left(\frac{U}{V}\right)' = \frac{VU' - UV'}{V^2}$$

$$= \frac{(x^2+y^2)(-1) + x(2x+0)}{(x^2+y^2)^2}.$$

$$= \frac{-x^2-y^2+2x^2}{x^2+y^2} \cdot \frac{x^2-y^2}{(x^2+y^2)}$$

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$$\cancel{U_{yy} = \frac{d}{dy} U_y}$$

$$\frac{\partial^2 U}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{-x}{x^2+y^2}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{y}{y^2+x^2}\right)$$

$$= \frac{(x^2+y^2)\frac{\partial}{\partial y}(y) - y\frac{\partial}{\partial y}(y^2+x^2)}{(y^2+x^2)^2}$$

$$= \frac{x^2+y^2 - y(0+2y)}{(y^2+x^2)^2}$$

$$= \frac{x^2-y^2}{x^2+y^2} = \frac{\partial^2 U}{\partial x \partial y}$$

Total derivative

- If a function f is in term of x, y and x, y in terms of t then the U

$U \to x, y \to t$

$$\frac{\partial U}{dt} = \frac{\partial U}{\partial x}\frac{dx}{dt} + \frac{\partial U}{dy}\frac{dy}{\partial t}$$

$U \to x, y, z \to t$

$$\frac{dU}{dt} = \frac{\partial U}{\partial x}\frac{dx}{dt} + \frac{\partial U}{dy}\frac{dy}{\partial t} + \frac{\partial U}{\partial z}\frac{\partial z}{\partial t}$$

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(4) Given $U = e^x \cos y$, $x = t^2+1$, $y = 2t$ then find the total derivative $\frac{du}{dt}$?

(A) $U = e^x \cos y \quad x = t^2+1 \quad y = 2t$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$= \frac{\partial}{\partial x}(e^x \cos y)\frac{dx}{dt} + \frac{d}{dy}(e^x \cos y)\frac{dy}{dt}$$

$$= e^x \cos y \frac{d}{dt}(t^2+1) + e^x - \sin y \frac{d}{dt} 2t$$

$$= e^x \cos y \; 2t + 0 + e^x - \sin y \; 2$$

$$= e^{t^2+1} \cos 2t \cdot 2t + e^{t^2+1} - \sin(2t) \cdot 2.$$

(5) Given $U = \log(x+y+z)$ $x = e^t$, $y = \sin t$ $z = \cos t$ then find the total derivite $\frac{dU}{dt}$?

(A) $$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

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$$= \frac{d}{dx}(\log(x+y+z) \cdot \frac{d}{dt}(e^t)$$

$$= \frac{\frac{d}{dx}\cancel{\log}(x+y+z)}{x+y+z} = \frac{1}{x+y+z} \cdot e^t$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\log(x+y+z) \cdot \frac{d}{dt}\sin t$$

$$= \frac{\cancel{\log}\frac{d}{dy}(x+y+z)}{x+y+z} = \frac{1}{x+y+z} \cdot \frac{d}{dt}\sin t$$

$$= \frac{1}{x+y+z}\cos t$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\log(x+y+z) \cdot \frac{d}{dt}\cos t$$

$$= \frac{1}{x+y+z} \cdot \frac{d}{dt}\cos t$$

$$= \frac{1}{x+y+z} - \sin t$$

$$\frac{du}{dt} = \frac{1}{x+y+z}(e^t + \cos^t - \sin t)$$

$$= \frac{1}{e^t + \sin t + \cos t}(e^t + \cos^t - \sin t)$$

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Jacobian (J)

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$$J\left(\frac{u\,v}{x\,y}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$J\left(\frac{u\,v\,w}{x\,y\,z}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

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(6) find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ of

~~$u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$~~

(1) $u = x^2 - 2y \quad v = 5x + 7y$

A $J\left(\frac{u\,v}{x,y}\right) = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$

$$= \begin{vmatrix} \frac{\partial}{\partial x}(x^2-2y) & \frac{\partial}{\partial y}(x^2-2y) \\ \frac{\partial}{\partial x}(5x+7y) & \frac{\partial}{\partial y}(5x+7y) \end{vmatrix}$$

$$= \begin{vmatrix} 2x-0 & 0-2 \\ 5+0 & 0+7 \end{vmatrix} = 14x+10$$

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(II) $U = x(1-y) \quad V = xy$

$$J\left(\frac{UV}{xy}\right) = \begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial}{\partial x}(x(1-y)) & \frac{\partial}{\partial y} x(1-y) \\ \frac{\partial}{\partial x} xy & \frac{\partial}{\partial y} xy \end{vmatrix}$$

$$= \begin{vmatrix} 1-y & -x \\ y & x \end{vmatrix}$$

$= x - xy + y/x$

$= x,$

(7) find the Jacobian of $\frac{\partial(U,V,W)}{\partial(x,y,z)}$ 24000332

of $U = \frac{yz}{x}$ $V = \frac{xz}{y}$ $W = \frac{xy}{z}$?

(A) Jacobian of U = $\begin{vmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix}$

$$= \begin{vmatrix} \frac{\partial}{\partial x}\frac{yz}{x} & \frac{\partial}{\partial y}\left(\frac{yz}{x}\right) & \frac{\partial}{\partial z}\left(\frac{yz}{x}\right) \\ \frac{\partial}{\partial x}\left(\frac{xz}{y}\right) & \frac{\partial}{\partial y}\left(\frac{xz}{y}\right) & \frac{\partial}{\partial z}\left(\frac{xz}{y}\right) \\ \frac{\partial}{\partial x}\left(\frac{xy}{z}\right) & \frac{\partial}{\partial y}\left(\frac{xy}{z}\right) & \frac{\partial}{\partial z}\left(\frac{xy}{z}\right) \end{vmatrix}$$

$$= \begin{vmatrix} \frac{-yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ \frac{z}{y} & \frac{-xz}{y^2} & \frac{x}{y} \\ \frac{y}{z} & \frac{x}{z} & \frac{-xy}{z^2} \end{vmatrix}$$

$= y\cancel{z\left(xz(xy)\right)}$

$$= \frac{1}{x^2y^2z^2} \begin{vmatrix} -yz & 2x & yx \\ 2y & -xz & xy \\ yz & xz & -xy \end{vmatrix}$$

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$$= \frac{\cancel{yz}\ \cancel{2x}\ \cancel{yx}}{\cancel{x^2y^2z^2}} \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= -1(1-1) - 1(-1-1) + 1(1+1)$$

$$= 0 + 2 + 2 = 4.$$

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Tutorial-5

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(1) Expand the problem using taylors series in Powers of $f(x,y) = x^3 + 2xy + y^3$ in Powers of $(x+1), (y+2)$

$x-a \quad y-b$

$(x+1) \quad (y+2)$

$a = -1 \quad b = -2$

(A) $f = x^3 + 2xy + y^3$

$f_x = 3x^2 + 2y$

$f_y = 0 + 2x + 3y^2$

$f_{xx} = 6x$

$f_{yy} = +6y$

$f_{xy} = \frac{\partial}{\partial y} f(x) = \frac{\partial}{\partial y}(\cancel{3x^2+2y}) = 2$

$3x^2 + 2y.$

$f(-1,-2) = (-1)^3 + 2(-1)(-2) + (-2)^3 = -5$

$f_x(-1,-2) = 3 - 4 = -1$

$f_y(-1,-2) = -2 + 12 = 10$

$f_{xy}(-1,-2) = (-1,-2) = 2$

$f_{yy}(-1,-2) = -12.$

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$$f(x,y) = f(a,b) + \frac{1}{1!}(x-a)f_x(a,b) + (y-b)f_y(a,b) + \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \ldots$$

$$= -5 + \frac{1}{1!}\left[(x+1)(-1) + (y+2)(10)\right] + \frac{1}{2!}\left[(x+1)^2(-6) + 2(x+1)(y+2)2 + (y+2)^2(-12)\right] + \ldots$$

② Applying Taylor's Series expansion expand the function $f(x,y) = e^x \sin y$ at $(-1, \pi/4)$ upto the terms of second degree.

Ⓐ $(-1, \pi/4) = (a,b)$

$f = e^x \sin y$

$f_x = e^x \sin y$

$f_y = e^x \cancel{\sin} \cos y$

$f_{xx} = \frac{\partial}{\partial x} f(x) = e^x \sin y$

$f_{xy} = \frac{\partial}{\partial x} f(x) = e^x \cos y$

$f_{yy} = \ldots$

$f(-1, \pi/4) = e^{-1}\sin\pi/4 = \frac{1}{e\sqrt{2}}$

$f_x(-1, \pi/4) = e^{-1}\sin\pi/4 = \frac{1}{e\sqrt{2}}$

$f_y(-1, \pi/4) = e^{-1}\cos\pi/4 = \frac{1}{e\sqrt{2}}$

$f_{xy} = e^{-1}\cancel{\sin}\cos\pi/4 = \frac{1}{e\sqrt{2}}$

$f_{yy} = -\frac{1}{\ldots}$

$$f_{xx} = \frac{1}{e\sqrt{2}}$$

$$f(x,y) = f(a,b) + \frac{1}{1!}(x-a) f_x(a,b) + (y-b) f_y(a,b)$$
$$+ \frac{1}{2!}(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b)$$
$$+ (y-b)^2 f_{yy}(a,b) + \cdots$$

$$f(x,y) = \frac{1}{e\sqrt{2}} + \frac{1}{1!}(x+1)\frac{1}{e\sqrt{2}} + (y-\pi/4)\frac{1}{e\sqrt{2}} +$$
$$\frac{1}{2!}(x+1)^2 \frac{1}{e\sqrt{2}} + 2(x+1)(y-\pi/4)\frac{1}{e\sqrt{2}} +$$
$$(y-\pi/4)^2 \frac{-1}{e\sqrt{2}} + \cdots$$

$$= \frac{1}{e\sqrt{2}}\left[1 + \frac{1}{1!}(x+1) + (y-\pi/4) + \frac{1}{2!}(x+1)^2 +\right.$$
$$2(x+1)(y-\pi/4) - (y-\pi/4)^2 + \cdots$$

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(3) Examine the maximum and minimum for the function $f(x,y) = x^3+y^3-63(x+y)+12xy$?

(A) $f(x,y) = x^3+y^3-63(x+y)+12xy$

$\cancel{r = f_x}$

$f(x) = 3x^2 - 63 + 12y$

$r = f_{xx} = 6x - 0$

$\cancel{t = f_{yy}}$ $f(y) = 3y^2 - 63 + 12x$

$t = f_{yy} = 6y - 0$

$s = f_{xy} \Rightarrow \frac{d}{dy} f(x)$

$= \frac{\partial}{\partial y} 3x^2 - 63 + 12y$

$= 12$

$\cancel{rt - s^2} =$ for stationary values Solve

$f(x) = 0 \quad f_y = 0$

$rt - s^2$ $\quad 3x^2 - 63 + 12y = 0 \quad -(1)$

$3y^2 - 63 + 12x = 0 \quad -(2)$

(1) - (2)

$x^2 - y^2 + 4(y - x) = 0$

$(x-y)(x+y)-4(x-y)=0$

$(x-y)(x+y-4)=0$

$(x-y=0), \; x+y-4=0$

$x=y$ (or) $y=-x+4$

if $x=y$ from (1)

$x^2-21+4x=0$

$x=\frac{-4\pm10}{2}=-7, 3$

if $x=-7$, $y=-7 \Rightarrow (-7,-7)$

if $x=3$ $y=3 \Rightarrow (3,3)$

if $y=-x+4. \to$ from (1)

~~from~~ $x^2-21+4(-x+4)=0$

$x^2-4x-5=0$

$x=\frac{4\pm6}{2}=-1, 5$

if $x=-1$ & $y=-x+4=5$.

$\Rightarrow (-1,5)$

if $\{x = 5 \quad y = -5+4 = -1 \Rightarrow (5,-1)$

Stationary Points: $P_1(-7,-7)$, $P_2(3,3)$ $P_3(-1,5)$ $P_4(5,-1)$

$rt - s^2 = 6x\,6y - 12^2 = 36xy - 144$

$rt - s^2 / P_1 = 36(-7)(-7) - 144 > 0$

$r = 6x$ $\quad r < 0$

$= -42$

r is max at $(-7,-7)$.

$f(-7,-7) =$

$rt - s^2 / P_2 = 36(3)(3) - 144 > 0$

$\Rightarrow r/P_2 = 6(3) > 0$

f has minimom at $(3,3)$

$f(3,3) =$

$rt - s^2 / P_3 = 36(-1)(5) - 144 < 0$

$rt - s^2 / P_4 = 36(1)(-5 - 144 < 0$] Sattel Point.

(4) The Sum of three numbers is Constant. Prove that these provide is maximum when they are equal.

(A) Let x, y, z be $\Rightarrow$ $x+y+z=C = \phi$

Say $f = xyz$ is max.

$F = f + \lambda\phi$

$f = xyz + (x+y+z)\lambda = 0$

for stationary Value.

$f(x)=0 \Rightarrow yz+\lambda=0 \Rightarrow -\lambda = yz$ — (1)

$f(y)=0 \Rightarrow xz+\lambda=0 \Rightarrow -\lambda = xz$ — (2)

$f(z)=0 \Rightarrow xy+\lambda=0 \Rightarrow -\lambda = xy$ — (3)

(1) = (2) $\Rightarrow$ $y\cancel{z} = x\cancel{z}$

$y = x$ — (4)

$P(x,y,z) = [C/3, C/3,$

(1) = (3) $\Rightarrow$ $\cancel{y}z = x\cancel{y}$

$z = x$ — (5)

$x = y = z.$

$\phi = 0$ $\quad x + x + x = C$

$3x = C$

$[x = C/3]$

⑤ Evaluate minimum Values of $x^2+y^2+z^2$.

is $ax+by+cz=p$?

① $f = x^2+y^2+z^2$

$\phi = ax+by+cz-p$

by Lagrangian. $F = f+\lambda\phi$

$= x^2+y^2+z^2+(ax+by+cz-p)\lambda$

for stationary values solve

$f(x)=0 \Rightarrow 2x+\lambda a=0 \Rightarrow -\lambda = 2x/a$ — ①

$f(y)=0 \Rightarrow 2y+\lambda b=0 \Rightarrow -\lambda = 2y/b$ — ②

$f(z)=0 \Rightarrow 2z+\lambda c=0 \Rightarrow -\lambda = 2z/c$ — ③

①=② $\Rightarrow \frac{\cancel{2}x}{a} = \frac{\cancel{2}y}{b} = y = \frac{bx}{a}$.

①=③ $\Rightarrow \frac{\cancel{2}x}{a} = \frac{\cancel{2}z}{c} = \frac{xc}{a} = z$

$ax+by+cz=p$.

$ax + b\left(\frac{bx}{a}\right) + c\left(\frac{c}{a}\right)x = p$.

$\left(\frac{a^2+b^2+c^2}{a}\right)x = p$.

$$\boxed{x = \frac{pa}{a^2+b^2+c^2}}$$

$y = \frac{pb}{a^2+b^2+c^2}$ $\quad z = \frac{pc}{a^2+b^2+c^2}$

minimom of $= x^2+y^2+z^2$

$$= \frac{p^2a^2}{(a^2+b^2+c^2)^2} + \frac{p^2b^2}{(a^2+b^2+c^2)^2} + \frac{p^2c^2}{(a^2+b^2+c^2)^2}$$

$$= \frac{p^2}{a^2+b^2+c^2}$$

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⑥ A rectangle box open at the top is to have a Voluem of 32 Cobic find the dimesions of both.

Ⓐ Let x, y, z be the l, b, h.

$V = xyz = 32$

$S = 2xy + 2yz + xz$

$f = f + \phi\lambda$

$= S + \phi V\lambda$

$= 2xy + 2yz + xz + (xyz - 32)\lambda$

$f_x = 0 = 2y + z + (yz\lambda)$

$\lambda = 2y + z + (yz)\lambda$

$-\lambda = \frac{2y + z}{yz} = \frac{2}{z} + \frac{1}{y}$ — (1)

$f_y = 0 \Rightarrow 2x + 2z + \lambda xz = 0$

$-\lambda = \frac{2x + 2z}{xz} = \frac{2}{z} + 2/x$ — (2)

$f_z = 0 \quad 2y + x + \lambda(xy) = 0$

$\Rightarrow -\lambda = \frac{2}{x} + \frac{1}{y}$ — (3)

(1) = (2) $\quad \cancel{\frac{2}{z}} + \frac{1}{y} = \cancel{\frac{2}{z}} + 2/x$

$$\boxed{y = \frac{x}{2}}$$

(1) = (3) $\quad \frac{2}{z} + \cancel{\frac{1}{y}} = \frac{2}{x} + \cancel{\frac{1}{y}}$

$$\boxed{z = x}$$

but given $xyz = 32$

$x \cdot x \cdot x = 32.$

$x^3 = 64$

$\boxed{x = 4}$

$\boxed{y = x/2 = 2}$ $\boxed{z = x = 4}$

$P(4,2,4) = 2(4)(2) + 2(2)(4) + 4(4)$

$= 16 + 16 + 16$

$= 48$

(7) The temperature ~~at~~ T at any point (x,y,z) in space is $T = Kxyz^2$ find the higest temp on the surface of unit sphere of $x^2+y^2+z^2=1$?

(A) $f = K\,xyz^2$

$\phi = x^2+y^2+z^2 = 1$

$F = f + \phi$

$F = Kxyz^2 + \lambda(x^2+y^2+z^2-1)$

$f(x) = Kyz^2 + \lambda 2x$

$-\lambda = \dfrac{Kyz^2}{2x}$ — (1)

$f(y) = Kxz^2 + \lambda$

$-\lambda = \dfrac{Kxz^2}{2y}$ — (2)

$f(z) = Ky x + \lambda$

$\lambda = \frac{Ky\, x}{2z}$ (3)

~~(1) = (2)~~ $\cancel{Ky z^2 = Ky x \; Kx z^2}$

$\boxed{\cancel{x = z^2}}$ $\boxed{y = x}$

~~(1) = (3)~~ $\cancel{Ky z^2 = Ky x}$

$z^2 = x$

$\cancel{x \cdot x \cdot \sqrt{x} \quad z^2 \cdot z^2} \cdot \cancel{x \cdot x \cdot \sqrt{x}}$

(1) = (2) $= \frac{Ky\, z^2}{2x} = \frac{Kx\, z^2}{y}$

$y^2 = x^2 \Rightarrow y = x.$

(1) = (3) $\frac{Ky\, z^2}{2x} = \frac{Kxy\, 2z}{2z} \Rightarrow z^2 = 2x^2.$

but $x^2 + y^2 + z^2 = 1$

$x^2 + x^2 + 2x^2 = 1$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$\boxed{x = \frac{1}{2}}$ $\boxed{y = \frac{1}{2}}$ $z^2 = 2\left(\frac{1}{4}\right)^2$

$\boxed{z = \frac{1}{\sqrt{2}}}$

Tutorial - 6

(1) Solve the D.E $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} - 15y$

(A) $\frac{d}{dx} = D$

$D^3y - 9D^2y + 23Dy \; 15y$

$(D^3 - 9D^2 + 23D - 15)y = 0$

$f(D)y = 0.$

Sol is $y = y_c + y_p$

To Solve y_c

A.E $\Rightarrow f(m) = 0$

$m^3 - 9m^2 + 23m - 15 = 0$

if $m = 1$

$$\begin{array}{c|cccc} & 1 & -9 & 23 & -15 \\ & 0 & 1 & -8 & 15 \\ \hline & 1 & -8 & 15 & \boxed{0} \end{array}$$

$(m-1)(m^2 - 8m + 15) = 0$

$(m-1)(m^2 - 5m - 3m + 15) = 0$

$(m-1)\; m(m-5)-3(m-5)]]=0$

$m = \cancel{1,1,5}\; 1,3,5.$

$y = y_c$

$$\boxed{= C_1 e^{x} + C_2 e^{3x} + C_3 e^{5x}}$$

② Solve D.E $y''' + 4y'' + 4y' = 0$

① $\frac{d}{dx} = D$

$D^3 y + 4D^2 y + 4Dy = 0$

$(D^3 + 4D^2 + 4D)y = 0$

$f(D)y = 0$

T sol is $y = y_c + y_p$

To solve $y_c \Rightarrow f(m) = 0$

$= m^3 + 4m^2 + 4m = 0$

$= m(m^2 + 4m + 4) = 0$

$= m((m+2)^2 = 0$

$m = 0,\; m = -2, -2$

$y_c = C_1 e^{ax} + (C_2 + C_3 x) e^{bx}$

$$\boxed{Y_{c.} = C_1 e^{0x} + (C_2 + C_3 x) e^{-2x}}$$

③ Ⓐ Determine the solution of the initial Value problem $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 2.$ given the $y(0)=0$, $y^1(0) = 1$?

Ⓐ Given $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 2.$

$$D^2y - 8Dy + 15y = 2$$

$$(D^2 - 8D + 15)y = 2$$

$$f(D)y = \phi.$$

to Solve $Y = Y_c + Y_p.$

$Y_c \Rightarrow A.E \Rightarrow f(m) = 0.$

$$= m^2 - 8m + 15 = 0$$

$$= m(m-5) - 3(m-5)$$

$$= m = 3, 5.$$

$$\boxed{Y_c = C_1 e^{3x} + C_2 e^{5x}}$$

To find $y_p = \frac{1}{f(D)} \phi$

$$= \frac{1}{D^2 - 8D + 15} 2.$$

$$= \frac{1}{D^2 - 8D + 15} 2 \cdot e^{ax}$$

$a = 0$. replace a with D

$$= \frac{1}{a^2 - 8a + 15}$$

$$\boxed{y_p = 2/15}$$

$$y = C_1 e^{3x} + C_2 e^{5x} + 2/15$$

$$y_1 = C_1 \cdot 3e^{3x} + C_2 5 \cdot e^{5x}$$

$y(0) = 0$

$y'(0) = 1$

$$0 = C_1 e^{0} + C_2 e^{5(0)} + 2/15$$

$$1 = C_1 \cdot 3e^0 + C_2 5 \cdot e^0$$

$$\boxed{-2/15 = C_1 + C_2} \quad (1)$$

$$\boxed{1 = 3C_1 + 5C_2} \quad (2)$$

~~$C_1 + C_2 = -2/15$~~

~~$C_1 = \frac{-2}{15} - C_2$~~

~~$1 = 3\left(\frac{-2}{15} - C_2\right) + 5C_2$~~

~~$1 = \frac{-2 - 5C_2}{5} + 5C_2$~~

$(2) - 3(1)$

$$1 = 3C_1 + 5C_2$$

$$6/15 = 3C_1 + 3C_2$$

$$1 + 2/5 = 2C_2$$

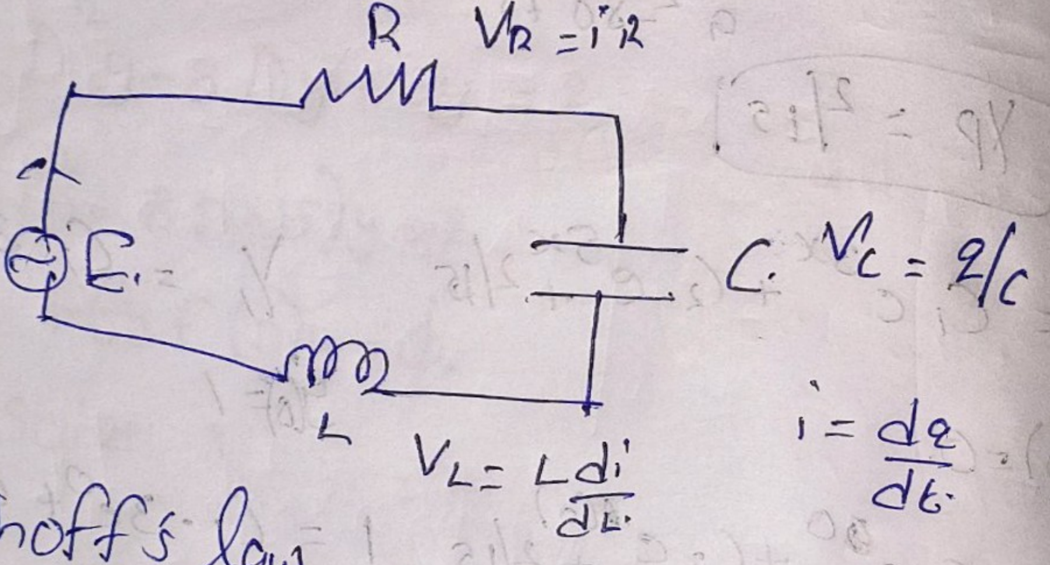
$$\boxed{C_2 = 7/10}$$

$C_1 = 2/15 - C_2$

$= 2/15 - \frac{7}{10} = -5/6$

Determine the Charge of the Capacitor in LRC Series Circuit when indutance is 1H. resistance is 4 Ω. Capacitance is 0.25 f, $E(t) = 0V$, $q(0) = 5C$ and $i(0) = 0$.

(A)



$i = \frac{dq}{dt}$

Kirchoff's law

Sum of Voltages ~~across~~ drops in Closed loop = applied electromotive force

$V_R + V_C + V_L = E$

$$\boxed{iR + \frac{q}{C} + L\frac{di}{dt} = E}$$

$$R\frac{dq}{dt}+\frac{q}{C}+L\cdot\frac{d}{dt}\left(\frac{dq}{dt}\right)=f$$

$$L\cdot\frac{d^2q}{dt^2}+\frac{R\cdot dq}{dt}+\frac{q}{C}=f$$

Given

$L=1$ $R=4$ $C=0.25$, $e(t)=0$, $q(0)=5$ $i(0)=0$

$$1\cdot\frac{d^2q}{dt^2}+4\cdot\frac{dq}{dt}+\frac{q}{0.25}=0$$

$D^2q+4D+4q=0$

$(D^2+4D+4)q=0$

$f(D)q=0$

$q=q_c+q_p$

to find $q_c=f(m)=0$

$=m^2+4m+4=0$

$=(m+2)^2$

$m=-2,-2$

$$\boxed{q=(C_1+C_2t)e^{-2x}}$$

$q(0)=5$, $t=0$, $q=5$

$5=(C_1+C_2(0))e^{-2(0)}$

$$\boxed{5=C_1}$$

$i=0$

$\frac{dq}{dt}(0)=0$

$q'=0$, $t=0$

$\cancel{q=C_1e^{-2x}}$

$q'=(C_1+C_2t)(-2e^{-2t})+e^{-2t}(0+C_2)$

$q'=0$ $t=0$

$0 = (C_1 + C_2 0)(-2)e^0) + C_2 e^0.$

$$\boxed{0 = -2C_1 + C_2}$$

$$\boxed{C_1 = 5}$$

$0 = -2(5) + C_2$

$$\boxed{C_2 = 10}.$$