$$\lim_{x\to -3}\left[\frac{\sqrt{8+x^2}-3}{x+1}\right] = \frac{\sqrt{8+(-3)^2}-3}{-3+1}$$

Kamonewa Tshegameno
Direct substitution, because of the marks ,its only 2 marks
please beware of the marks

2. $$\lim_{x\to \frac{\pi}{4}}\left[\frac{1-\cos x}{x}\right]$$

Kamonewa Tshegameno
I have not finished sloving this one, because i think its direct substitution

3. $$\lim_{x\to 4}\left[\frac{x^2-4x}{2-\sqrt{x}}\right] = \lim_{x\to 4}\left[\frac{x(x-4)}{2-\sqrt{x}}\right] = \lim_{x\to 4}\left[\frac{-x\cancel{(2-\sqrt{x})}(2+\sqrt{x})}{\cancel{(2-\sqrt{x})}}\right]$$

$$= \lim_{x\to 4}\left[-x(2+\sqrt{x})\right]$$

$$= -4(2+\sqrt{4})$$

$$= -16$$

4. $$\lim_{x\to 2}\left[\frac{x^4-16}{x^3-8}\right] = \lim_{x\to 2}\left[\frac{(x^2-4)(x^2+4)}{(x-2)(x^2+2x+4)}\right]$$

Kamonewa Tshegameno
Difference of two squares and difference of cubes

$$= \lim_{x\to 2}\left[\frac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{(x-2)}(x^2+2x+4)}\right]$$

$$= \lim_{x\to 2}\left[\frac{(x+2)(x^2+4)}{(x^2+2x+4)}\right]$$

$$= \frac{(2+2)(4+4)}{(4+4+4)}$$

$$= \frac{8}{3}$$

1.2 $$T(x) = \begin{cases} 4+ax, & \text{if } x<1 \\ 4, & \text{if } x=1 \\ x^2+1, & \text{if } x>1 \end{cases}$$

$$\lim_{x\to 1} T(x) = F(1)$$

(i) $F(1) = 4$

(ii) $\lim_{x\to 1^+} x^2+1 = 1^2+1 = 2$

$$\lim_{x\to 1^-} 4+ax = 4+a(1) = 4+a$$

$$\lim_{x\to 1^+} T(x) = \lim_{x\to 1^-} T(x)$$

$$2 = 4+a$$

$$a = -2$$

1.3 $$\lim_{x\to 3}(2x-1) = 5$$

Given $\varepsilon>0$, We seek $\delta>0$ such that

$|(2x-1)-5|<\varepsilon$ Whenever $0<|x-3|<\delta$

Finding δ: $|(2x-1)-5|<\varepsilon$

$\Rightarrow |(2x-6)|<\varepsilon$

$\Rightarrow |2(x-3)|<\varepsilon$

$\Rightarrow 2|x-3|<\varepsilon$

$\Rightarrow |x-3|<\frac{\varepsilon}{2}$

Proof: Give $\varepsilon>0$, Choose $\delta=\frac{\varepsilon}{2}$ Suppose that $0<|x-3|<\varepsilon$

$|x-3|<\varepsilon$

$\Rightarrow |x-3|<\varepsilon/2$

$\Rightarrow 2|x-3|<\varepsilon$

$\Rightarrow |2x-6|<\varepsilon$

$\Rightarrow |(2x-1)-5|<\varepsilon$