This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

A Decomposition Based Evolutionary Algorithm for Many Objective Optimization

Md. Asafuddoula, Tapabrata Ray and Ruhul Sarker

***Abstract*—Decomposition based evolutionary algorithms have been quite successful in solving optimization problems involving two and three objectives. Recently there have been some attempts to exploit the strengths of decomposition based approaches to deal with many objective optimization problems. Performance of such approaches are largely dependent on three key factors i.e., (a) means of reference point generation (b) schemes to simultaneously deal with convergence and diversity and finally (c) methods to associate solutions to reference directions. In this paper, we introduce a decomposition based evolutionary algorithm, wherein, uniformly distributed reference points are generated via systematic sampling, balance between convergence and diversity is maintained using two independent distance measures and a simple preemptive distance comparison scheme is used for association. In order to deal with constraints, an adaptive epsilon formulation is used. The performance of the algorithm is evaluated using standard benchmark problems i.e., DTLZ1-DTLZ4 for 3, 5, 8, 10 and 15 objectives, WFG1-WFG9, the car side impact problem, the water resource management problem and the constrained ten-objective general aviation aircraft (GAA) design problem. Results of problems involving redundant objectives and disconnected Pareto fronts are also included in this study to illustrate the capability of the algorithm. The study clearly highlights that the proposed algorithm is better or at par with recent reference direction based approaches for many objective optimization.**

***Index Terms*—many-objective optimization, decomposition, evolutionary algorithm, adaptive epsilon constraint-handling**

I. Introduction

Many-objective optimization refers to optimization problems where the number of objectives is large, in general greater than four [1]. There is significant amount of literature discussing the challenges involved in solving them and interested readers may refer to [1] for further details. The main difficulty arises from the inability of the non-dominance-based schemes to generate sufficient selection pressure to drive the solutions to the Pareto front. Therefore, the commonly used dominance based methods for multi-objective optimization, such as NSGA-II [2] or SPEA2 [3] do not offer satisfactory results. There have been a number of attempts to modify the underlying selection pressure through the use of secondary metrics such as substitute distance measures [4] [5], average rank domination [6], fuzzy dominance [7], ϵ-dominance [8], [9], adaptive ϵ-ranking [10] etc. without a great success. In all the above approaches, while the diversity and/or the convergence of the population improved during the course of evolution, there is no control to ensure that the final non-dominated set of solutions spans the entire Pareto surface uniformly.

There are also radically different approaches to deal with many objective optimization, such as attempts to identify the reduced set of objectives [11], [12] or corners of the Pareto front [13] and subsequently solving the problem using these reduced set of objectives. Other attempts include interactive use of decision makers preferences [14], use of reference directions via decomposition as in Cellular Multi-objective Genetic Algorithms (C-MOGA) [15] and in Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [16], use of *Vector-Angle-Distance-Scaling* scheme in Multiple Single Objective Pareto Sampling (MSOPS) [17], [18] or solution of the problem as a hypervolume maximization problem [19]. While some progress has been made along these lines, the limiting factors include the inability to obtain solutions close to Pareto set for an accurate identification of redundant objectives, decision making burden associated with preference elicitation, problems associated with scalarization in decomposition based schemes and the computational complexity of hypervolume computation.

Decomposition based evolutionary algorithms (MOEA/D [16] and C-MOGA [15]) are members of yet another class of algorithms, where a multi-objective optimization problem is explicitly decomposed into a series of scalar optimization problems. MOEA/D has been quite successful in solving optimization problems involving two and three objectives and there is significant interest in developing it further to deal with many objective optimization problems. Notable works in the area include the development of surface evolutionary algorithm (SEA) [20], many-objective evolutionary algorithm based on generalized decomposition (MAEA-gD) [21]–[23], approximation model guided selection (AMS) [24], NSGA-III [25] and recent works of the authors i.e., decomposition based EAs [26] and quantum inspired many objective EAs [27].

Fundamentally, in all such approaches one needs to generate a set of uniformly distributed reference directions and adopt a method of scalarization. In the context of many objective optimization, the first issue relates to the design of a computationally efficient scheme to generate W uniform reference directions for a M objective optimization problem, where M is typically more than four and W is often chosen to be the same as the population size. The second issue relates to scalarization, which essentially assigns the $fittest$

Manuscript received July 09, 2013; revised February 19, 2014 and 09 June, 2014; accepted 30 June, 2014;

Md. Asafuddoula, Tapabrata Ray and Ruhul Sarker are with School Engineering and Information Technology, University of New South Wales, Canberra, Australia

Copyright (c) 2014 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

individual to each reference direction. The notion of $fittest$ is derived using a trade-off between convergence and diversity measured with respect to any given reference direction. One of the early attempts to generate uniformly distributed reference directions appear in the works of Hughes [18]. The method was not computationally efficient for problems with more than six objectives and often resulted in a large number of reference directions that in turn required a huge population size. More recently, computationally efficient and scalable sampling schemes have been used in the context of many-objective optimization. A systematic sampling [28] scheme was used in NSGA-III [25], whereas an uniform sampling scheme was used in MOEA/D [29].

The second issue related to scalarization has been addressed via two fundamental means i.e., through a systematic association and niche preservation mechanism as in NSGA-III [25] or through the use of a penalty function (i.e., an aggregation of the projected distance along a reference direction and the perpendicular distance from a point to a given reference direction) within the framework of MOEA/D. The performance of the penalty function based approach is dependent on the penalty parameter, whereas the association and the niche preservation process require a careful implementation to address a number of possibilities.

In this paper, we introduce a decomposition based evolutionary algorithm for many-objective optimization. The reference directions are generated using systematic sampling, wherein the points are systematically generated on a hyperplane with unit intercepts in each objective axis. The process of reference point generation is the same as adopted in NSGA-III [25]. The association of solutions to reference directions are based on two independent distance measures. The distance along the reference direction controls convergence, whereas the perpendicular distance from the solution to the reference direction controls the diversity. The proposed algorithm utilizes a simple prioritized distance comparison scheme to maintain this balance and control association. In order to improve the efficiency of the algorithm, a steady state form is adopted in contrast to a generational model used in NSGA-III [25]. Furthermore, to deal with constraints, an adaptive epsilon level based scheme is adopted which had outstanding performance on recent constrained optimization benchmarks [30], [31]. The study also illustrates the performance of the algorithm on the problems involving redundant objectives and disconnected Pareto fronts.

A background is presented in Section II and the details of the proposed algorithm are presented in Section III. The performance of the proposed algorithm on benchmark problems (DTLZ1-DTLZ4 for 3, 5, 8, 10, 15 objectives) and (WFG1-WFG9 for 3, 5, 10 and 15) are presented and compared with MOEA/D-PBI and NSGA-III in Section IV, and the performance on degenerate problems (DTLZ5-(I,M)) is also presented in Section IV. In addition to the above set of mathematical benchmarks, the performance of the algorithm is also presented and compared using a number of engineering design problems (car side impact, water resource management and the constrained ten-objective general aviation aircraft (GAA) design). The final section summarizes the contributions and future directions for further improvement.

II. Background

As identified earlier, native non-dominance based multi-objective optimization algorithms are incapable of dealing with problems involving many objectives [1]. Modified non-dominance based methods such as ϵ-dominance [8], grid-dominance [32], dominance using preferred goals [33] and use of secondary ranking schemes for diversity management [34], [35] have been suggested recently for such classes of problems. Other prominent directions for many-objective optimization include the use of reference directions [25], [36] and the use of indicators [17]. Reference direction based approaches require generation of uniformly distributed reference directions, appropriate means of scaling and aggregation, whereas indicator based approaches rely on the use of hypervolume computation which is known to be computationally expensive. Table I provides a list of prominent many objective optimization algorithms along with their features (i.e., existing difficulties associated with the underlying schemes and the scope of the reported applications).

In the context of optimization problems involving two and three objectives, reference based schemes using decomposition e.g., MOEA/D had outstanding success [16], [38]. While a number of variants of MOEA/D have been introduced to solve bi- and tri-objective optimization problems, there are only a handful of reports that extend the capability to deal with many objective optimization problems [22], [29]. MOEA/D utilizes a decomposition approach, wherein solutions along uniformly distributed directions ($W's$) are sought. During the course of search, recombination and replacement is performed among the solutions restricted to a neighborhood (T) i.e., solutions associated with the neighboring reference directions. For every individual direction, a penalized distance measure is used of the form $d_1 + \theta d_2$, where d_1 denotes the distance along the reference direction and d_2 denotes the distance from the solution to the foot of the perpendicular drawn to the reference direction (Figure 1). The effect of user defined parameters (e.g. neighborhood size (T) and penalty parameter θ) and difficulties associated in managing objectives in different orders of magnitude appear in [37].

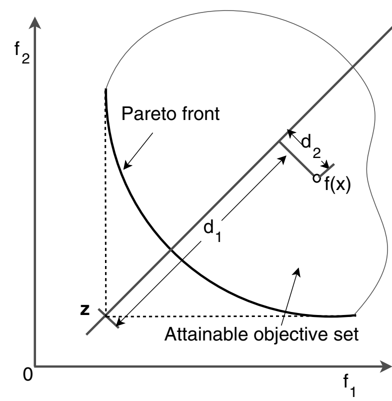


Fig. 1. Illustration of distance measures (i.e., d_1 and d_2) in the context of minimization with respect to a reference direction.

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE I
PREVIOUS STUDIES ON MANY-OBJECTIVE OPTIMIZATION

Reference Work	Approaches	Existing challenges	Illustration using problems
Adra and Fleming [34]	Modified Pareto-dominance using diversity control	Appropriate design of activation/deactivation and mutation adaptation strategies. Knowledge about targets/extremities.	Unconstrained many-objective problems up to 20 objectives (DTLZ1 and DTLZ2)
Wang *et al.* [33]	Modified Pareto dominance using predefined goals	Use of known ideal and Nadir points, user defined/preferred goals	Unconstrained many-objective problems up to 10 objectives
Yang *et al.* [32]	Modified Pareto dominance using grid based measures	User defined grid sizes and choice between various fitness assignment schemes	Unconstrained many-objective problems up to 10 objectives
Hughes [36]	Reference direction based multiple single objective Pareto sampling	Appropriate scalable target vector generation mechanism and aggregation scheme	Constrained many-objective problems up to 6 objectives
Deb and Jain [25], [37]	Reference direction based non-dominated sorting	Elaborate association and niching schemes	Unconstrained many-objective problems up to 15 objectives and constrained engineering design problems up to 5 objectives
Tan *et al.* [29]	Reference direction based decomposition	Neighborhood parameter settings, scaling of objectives	Unconstrained many-objective problems up to 5 objectives
Wagner *et al.* [17]	Indicator based evolutionary algorithm	Computation of hypervolume	Unconstrained many-objective problems up to 6 objectives

In order to alleviate the above problems associated with MOEA/D, a decomposition based evolutionary algorithm with epsilon level comparison (DBEA-Eps [26]) was introduced by the authors to deal with many objective optimization problems. DBEA-Eps relied on the use of an adaptive epsilon level comparison to avoid aggregation, whereas a scaling based on axis intercepts similar to the scheme introduced in [37] was used to deal with objectives in different orders of magnitude. The proposed improved decomposition based evolutionary algorithm (I-DBEA) presented in this paper is an extension of the authors previous work on DBEA-Eps [26]. While DBEA-Eps [26] was successful in solving a range of many objective optimization problems, the performance was dependent on the choice of a number of parameters and several adaptive rules. This extension is clearly focused on elimination of such parameters and adaptive rules. The differences are summarized below for a greater clarity.

- In DBEA-Eps, the original concept of neighborhood similar to MOEA/D was used to select parents for recombination with a given mating probability δ. In the proposed algorithm, we have eliminated both these parameters (i.e., neighborhood size (T) and mating probability (δ)). The entire population is considered as a neighborhood and a first encounter replacement strategy has been adopted in I-DBEA. Results presented later in the paper illustrates the benefits of such a scheme in the context of many objective optimization as opposed to restricted neighborhood models aided by similar parent recombination [39] which could be beneficial for certain benchmark problems [40].
- Every solution in DBEA-Eps, had two distance measures associated with it i.e., distance along a reference direction d_1 and distance perpendicular to the reference direction d_2. Comparisons between solutions were based on an adaptive epsilon level of d_2. In the proposed algorithm, a simple precedence rule is used, where d_2 has a precedence over d_1.
- Scaling is an important aspect in any decomposition based scheme. In DBEA-Eps, a hyperplane was constructed using M extreme non-dominated solutions which in turn provided the lengths of the axis intercepts. In the proposed algorithm, such intercepts are computed by constructing a plane using M extreme points identified via *corner-sort* [13].

III. PROPOSED ALGORITHM

A many-objective optimization problem can be defined as follows:

$$\underset{(\mathbf{x})}{\text{Minimize}}(f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), ...f_M(\mathbf{x})), \mathbf{x} \in \Omega$$

Subject to

$$\begin{aligned} g_j(\mathbf{x}) &\geq 0, j = 1, 2, ...p \\ h_k(\mathbf{x}) &= 0, k = 1, 2, ...q \end{aligned} \quad (1)$$

where $f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), ...f_M(\mathbf{x})$ are the M objective functions, p is the number of inequalities and q is the number of equalities.

The algorithm is presented below and the individual components related to (a) generation of reference points (b) normalization and computation of distances (c) method of recombination (d) selection/replacement (e) means of constraint handling are discussed in subsequent subsections.

A. Generation of reference points

A structured set of reference points is generated spanning a hyperplane with unit intercepts in each objective axis using normal boundary intersection method (NBI) [28]. The approach generates W points on the hyperplane with a uniform spacing of $\delta = 1/s$. The process of generation of the reference points is illustrated using a 3-objective optimization problem ($M = 3$) with an assumed spacing of $\delta = 0.2$ ($s = 5$) in

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

Algorithm 1 I-DBEA

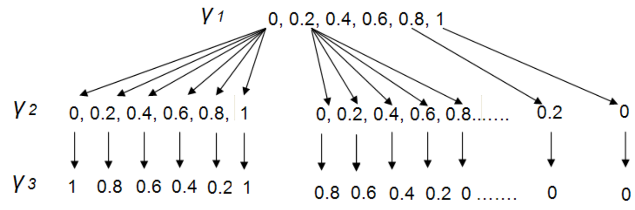
Input: Gen_{max} maximum number of generations, W the number of reference points

Generate the reference points using NBI
Initialize the population P; $|P| = W$ and assign each individual of P to an unique reference direction randomly
Evaluate the initial population and compute the ideal point, identify the corners and compute intercepts
Scale the individuals of the population
Assign the $2M$ corner solutions to a corner set S
while $(gen \leq Gen_{max})$ **do**
  for i=1:W **do**
    Select P_i as the base parent
    I=Select its partner randomly from W
    Create a child via recombination as C_i
    Evaluate C_i and compute the distances (d_1 and d_2) using all reference directions
    Insert C_i within S and compute corner set S using *corner-sort*
    Replace the parent P_l with C_i using *single-first encounter strategy*, where l denotes the index of the first parent satisfying the condition of replacement
    Update the ideal point (**z**), the intercepts and re-scale the population
  end for
end while

Figure 2. The process results in the generation of 21 reference points (computed using Equation 2).

$$W = {}^{(M+s-1)}C_s \tag{2}$$

The distribution of the reference points is presented in Figure 3. The reference directions are formed by constructing a straight line from the origin to each of these reference points. The population size of the algorithm is set to the number of reference points. The initial population consists of W individuals generated randomly within the variable bounds. Such solutions are thereafter assigned randomly to reference directions during the phase of initialization.



(a)

γ_1	γ_2	$(1-\gamma_1-\gamma_2)$
0	0	1.0000
0	0.2000	0.8000
0	0.4000	0.6000
0	0.6000	0.4000
0	0.8000	0.2000
0	1.0000	0
0.2000	0	0.8000
0.2000	0.2000	0.6000
0.2000	0.4000	0.4000
0.2000	0.6000	0.2000
0.2000	0.8000	0
0.4000	0	0.6000
0.4000	0.2000	0.4000
0.4000	0.4000	0.2000
0.4000	0.6000	0
0.6000	0	0.4000
0.6000	0.2000	0.2000
0.6000	0.4000	0
0.8000	0	0.2000
0.8000	0.2000	0
1.0000	0	0

(b)

Fig. 2. (a) the reference points are generated computing η_rs recursively (b) the table shows the combination of all η_rs in each column

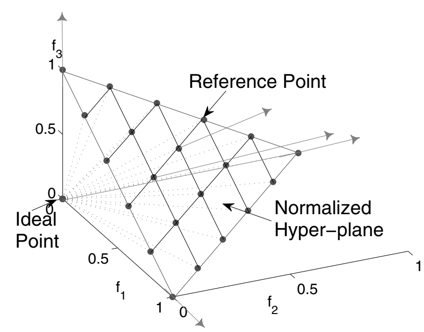


Fig. 3. A set of reference points in a normalized hyper-plane for $M = 3$ and $p = 5$.

B. Normalization and computation of distances

In a decomposition based method, the $fitness$ of a solution with respect to a given direction can be judged using two measures d_1 and d_2 as formulated in Equations 3 and 4. The first measure d_1 is the Euclidean distance between origin and the foot of the normal drawn from the solution to the reference direction, while the second measure d_2 is the length of the normal. These two measures are depicted in Figure 4.

Mathematically, d_1 and d_2 are computed as follows:

$$d_1 = \mathbf{w}^T \mathbf{f}'(\mathbf{x}) \tag{3}$$
$$d_2 = \|\mathbf{f}'(\mathbf{x}) - \mathbf{w}^T \mathbf{f}'(\mathbf{x})\mathbf{w}\| \tag{4}$$

where **w** is a unit vector along any given reference direction. It is clear that a value of $d_2 = 0$ ensures the solutions are perfectly aligned along the required reference direction ensuring perfect diversity, while a smaller value of d_1 indicates superior convergence. These two measures are subsequently used to control diversity and convergence of the algorithm.

Since these are Euclidean distances, an appropriate normalization is necessary to deal with objectives in different orders of magnitude. In DBEA-Eps and NSGA-III, the normalization is based on intercepts calculated using M extreme points of the non-dominated set. In I-DBEA, M solutions are identified using a *corner-sort* ranking [13] procedure. In *corner-sort*, the top M solutions are the minimum in each objective, while the following M solutions are the minimum based on L_2 norm

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

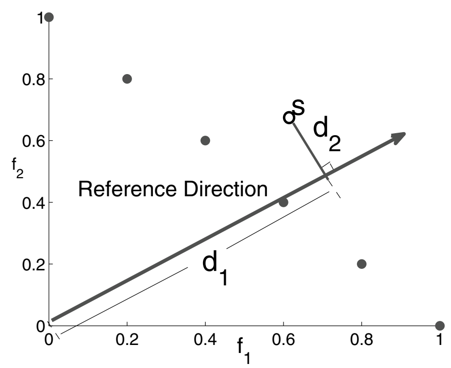


Fig. 4. Distance measures with respect to a reference direction.

of all but one objectives. From the set of $2M$ solutions, the maximum in each objective is identified and corresponding solutions which have led to the maximum value is selected and referred as extreme points $\mathbf{z}^e$. Such extreme points are used to create the hyperplane and compute the intercepts. In the event the number of such extreme points are less than M, the maximum value of the objective is used as the intercept value. The ideal point of a population is denoted by $\mathbf{z} = (f_1^{min}, f_2^{min}, ..., f_M^{min})$. The intercepts of the hyperplane along the objective axes are denoted by $a_1, a_2, ..., a_M$. The generic equation of a plane through these points can be represented using the following equation

$$C_1 f_1 + C_2 f_2 + ... + C_M f_M = 1 \tag{5}$$

where, C_1, C_2,....,C_M are the unit normal of the plane. The intercepts of the plane with the axis are given by $a_1 = 1/C_1$, $a_2 = 1/C_2$,...., and $a_M = 1/C_M$.

In the event, the number of such solutions are less than M or any of the a_j's are negative, a_j's are set to f_j^{max}. Every solution in the population is subsequently scaled as follows:

$$f'_j(\mathbf{x}) = \frac{f_j(\mathbf{x}) - z_j}{a_j - z_j}, \quad \forall j = 1, 2, ...M \tag{6}$$

C. Method of recombination

In the recombination process, two child solutions are generated using simulated binary crossover (SBX) operator [41] and polynomial mutation. The first child is considered as an individual attempting to replace any parent in the population.

D. Selection/replacement

In the steady state form, if a child solution is non-dominated with respect to the individuals in the population, it attempts to enter the population via a replacement. The child solution competes with all solutions in the population in a random order until it makes a successful replacement or has competed with all individuals. If we denote the distances as $\{d_{1_r}, d_{2_r}\}$ for a r^{th} solution in the population and $\{d_{1_c}, d_{2_c}\}$ denotes the distances for the child solution along r^{th} reference direction, a child is considered winner if d_{2_c} is less than d_{2_r}. In the event the d_{2_c} is equal to d_{2_r}, the child is considered a winner if d_{1_c} is less than d_{1_r}. The simple precedence of d_2 over d_1 eliminates the need for a complex epsilon based scheme adopted in DBEA-Eps.

E. Constraint handling

The constraint handling approach used in this work is based on epsilon level comparison and has been reported earlier in [30]. The feasibility ratio (FR) of a population refers to the ratio of the number of feasible solutions in the population to the number of solutions (W). The allowable violation is calculated as follows:

$$CV = \sum_{i=1}^{p} max(g_i, 0) + \sum_{i=1}^{q} max(|h_i - \epsilon|, 0) \tag{7}$$

$$CV_{mean} = \frac{1}{W}\sum_{j=1}^{W}(CV_j) \tag{8}$$

$$\text{Allowable violation}(\epsilon_{CV}) = CV_{mean} * FR \tag{9}$$

An epsilon level comparison using this allowable violation measure is used to compare two solutions. If two solutions have their constraint violation value less than this epsilon level, the solutions are compared based on their objective values i.e., via d_1 and d_2 measures. Such a constraint handling scheme performed better than *feasibility first* schemes on recent constrained optimization benchmarks [30].

IV. Experimental Results

In this section, we present experimental results of the proposed improved decomposition based evolutionary algorithm (I-DBEA). Our experimental results are compared with reported results of DBEA-Eps in [26] and those of NSGA-III and MOEA/D-PBI in [25] for DTLZ1-DTLZ4 problems with 3, 5, 8, 10 and 15 objectives and further compared with the results obtained from MOEA/D-PBI in [16] for WFG problems with 3, 5, 10 and 15 objectives. The results of I-DBEA and DBEA-Eps are also analyzed on three other constrained engineering design optimization problems.

The population sizes used in this study are the same as those adopted in [25]. The reference points are generated following Equation 2. For M=3, s is chosen as 12 resulting in 91 reference points, while for M=5, s is set to 6 resulting in 210 reference points (Table II). For M greater than 8, the reference points are generated via a two-layer sampling scheme with two values of s i.e., one for each layer as outlined in [25] (Table II). These settings have been used to make consistent comparisons between I-DBEA and the recently proposed reference direction based NSGA-III [25].

It is important to highlight that the two-layer sampling scheme [25] results in redundant extreme points (i.e., along each objective axis). While such a scheme may provide benefit to NSGA-III, it is not required for I-DBEA as it intrinsically identifies extreme points via *corner-sort*.

Parameters for I-DBEA include a probability of crossover is set to 1 and the probability of mutation is set to $p_m = 1/D$, where D is the dimensionality of the problem. Parameters for

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE II
NUMBER OF REFERENCE POINTS/DIRECTIONS/POPULATION USED IN THE STUDY.

No. of Obj. (M)	Sampling size (s) in each axis	Popsize/Ref. dirn. (W)
3	s=12	91
5	s=6	210
8	s=3, s=2	156
10	s=3, s=2	275
15	s=2, s=1	135

DBEA-Eps include a neighborhood size of 20 and the probability of selecting a parent from its neighborhood (T) is set to 0.9. The distribution index of crossover is set to η_c=30 and the distribution index of mutation is set to η_m=20 as in [25]. Parameters for MOEA/D-PBI include a neighborhood size of 20, probability of selecting a parent from its neighborhood (T) is set to 0.9, the distribution index of crossover is set to η_c=20, the distribution index of mutation is set to η_m=20, maximum number of solutions replaced by a child solution $\eta_r = T$, and a penalty parameter $\theta = 5$ as in [25].

The performance of MOEA/D-PBI could have been affected by the choice of the above parameters. The value of penalty parameter $\theta = 5$ used for MOEA/D-PBI in this study may not the most appropriate value for many-objective problems although the same has been used in [16] and [25] across all test problems. The influence of the penalty parameter on the performance of MOEA/D-PBI was examined in [40], where it was identified that totally different specifications of the penalty parameter values are needed in MOEA/D-PBI for two-objective and ten-objective problems.

To assess the performance, we have selected inverted generational distance (IGD) [42] [16] and Hypervolume (HV) [43] as the performance metric. The IGD metric is calculated with respect to a given reference set (generated using uniformly distributed sampling points on a unit intercept hyperplane via systematic sampling) normalized with the theoretical ideal and nadir points for the DTLZ problems. For other problems (i.e., WFG and all other engineering design optimization problems), the reference set was constructed by creating a nondominated set from all final solutions accumulated across all runs of all algorithms. The reference sets for all the problems used in this study are now available from http://seit.unsw.adfa.edu.au/.../Reference_set.rar. The exact hypervolume [43] was computed for 3 to 8 objective problems, whereas an approximate hypervolume was computed using Monte Carlo simulation [44] for 10 to 15 objective problems.

A. Performance on unconstrained DTLZ problems

In this comparison, we have reported the best, median and worst IGD results obtained using 30 independent runs for DTLZ1-DTLZ4. The test instances of DTLZ problems are briefly discussed in below:

- DTLZ1

$$
\begin{aligned}
&Minimize\\
&f_1(\mathbf{x}) = \frac{1}{2}x_1x_2.................x_{M-1}(1+g(x_M))\\
&f_2(\mathbf{x}) = \frac{1}{2}x_1x_2.........(1-x_{M-1})(1+g(x_M))\\
&\vdots\\
&f_{M-1}(\mathbf{x}) = \frac{1}{2}x_1(1-x_2)(1+g(x_M))\\
&f_M(\mathbf{x}) = \frac{1}{2}(1-x_1)(1+g(x_M))
\end{aligned}
\tag{10}
$$

where, $g(x_M) = 100(|K| + \sum_{i=M}^{M+K-1}(x_i - 0.5)^2 - cos(20\pi(x_i - 0.5)))$ and $\mathbf{x} = (x_1, x_2, ..., x_n)^T \in [0,1]^{M+K-1}$.

The Pareto-optimal solution corresponds to $\mathbf{x}^* = 0.5$ and the objective function values lie on the linear hyper-plane: $\sum_{i=1}^{M} f_i^*$=0.5. A value of $K = 5$ is used in this study.

- DTLZ2 - DTLZ4

$$
\begin{aligned}
&Minimize\\
&f_1(\mathbf{x}) = cos(\frac{x_1^\alpha \pi}{2})...cos(\frac{x_{M-1}^\alpha \pi}{2})(1+g(x_M))\\
&f_2(\mathbf{x}) = cos(\frac{x_1^\alpha \pi}{2})...sin(\frac{x_{M-1}^\alpha \pi}{2})(1+g(x_M))\\
&\vdots\\
&f_M(\mathbf{x}) = sin(\frac{x_1^\alpha \pi}{2})(1+g(x_M))
\end{aligned}
\tag{11}
$$

where,
$g(x_M) = \sum_{i=1}^{M+K-1}(x_i - 0.5)^2$
and $\mathbf{x} = (x_1, x_2, ..., x_n)^T \in [0,1]^{M+K-1}$.

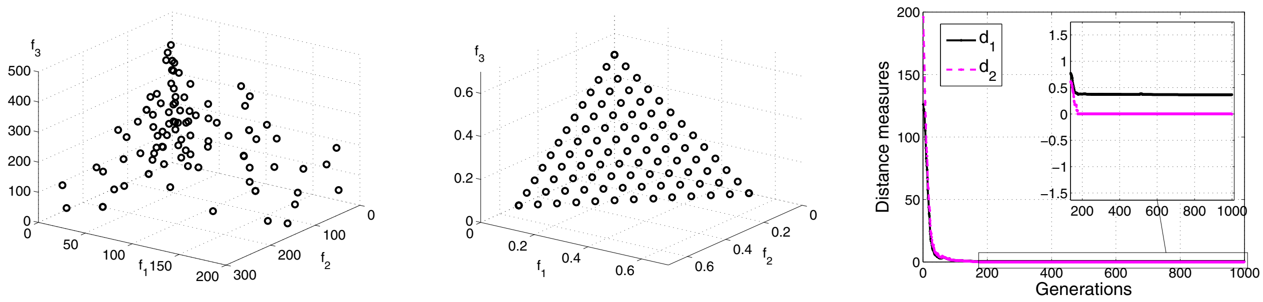
The Pareto-optimal solution corresponds to $\mathbf{x}^* = 0.5$ and the objective function values lie on the linear hyper-plane: $\sum_{i=1}^{M}(f_i^*)^2$=1. A value of $K = 10$ and $\alpha = 1$ are suggested for the DTLZ2 and DTLZ3 problems, while an $\alpha = 100$ is suggested for DTLZ4. In the above problem, the total number of variables is $n = M + K - 1$.

1) I-DBEA evolution process: In order to observe the process of evolution, we computed the average performance of the population i.e., average of the d_1 and d_2 values of the individuals for DTLZ1 (3 objectives) using 91 reference points and the population was allowed to evolve over 400 generations. One can observe from Figure 5, that the average d_2 converges to near zero (i.e., near perfect alignment to the reference directions), while the average d_1 measure stabilizes at around 0.5 indicating convergence to the Pareto front.

The association mechanism (i.e., association of the solutions to each reference direction) for the same problem is presented in Figure 6 using a small number of reference points. The figure shows the associations in generation 1, 500 and 1000 using 15 reference points. One can observe that although initially the association is random, the solutions automatically get associated to the closest reference directions during the course of evolution via the pressure induced by d_2. This alleviates the need of an extensive niching and association operation as encountered in NSGA-III [45].

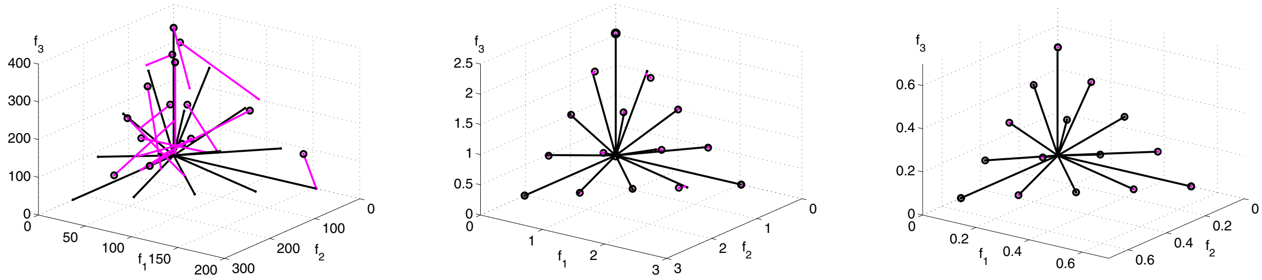
1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation



(a) (b) (c)

Fig. 5. (a) initial population of DTLZ1 test problem for $M = 3$ (b) final Pareto-front of DTLZ1 test problem (c) convergence of distance measure over generations



(a) (b) (c)

Fig. 6. (a) initial population of DTLZ1 test problem for $M = 3$ and $W = 15$ (b) population at generation 500 (c) population at final generation 1000

2) Comparison of results: The results of I-DBEA, DBEA-Eps, NSGA-III and MOEA/D-PBI are presented in Table III with the population sizes and number of generations listed in Table II and Table III. One can notice that I-DBEA obtained the best IGD values in 15 instances out of 20 (see Table III).

We investigate the effects of two key strategies on the performance of I-DBEA i.e., (a) use of all individuals in the population for recombination and replacement as opposed to use of neighboring solutions only as in MOEA/D-PBI (b) use of a single first replacement strategy as opposed to replacement up to 2 individuals in the neighborhood. The performance of I-DBEA based on IGD with neighborhood sizes of 20, W/2 and W (entire population) and with replacement up to 2 individuals are presented in (Table IV). One can clearly observe that I-DBEA with neighborhood size of W (i.e., the entire population) and a single first replacement strategy offers the best performance in terms of IGD. The rate of convergence of the mean IGD values (based on 30 independent runs) of various forms for 8 and 10 objective DTLZ1 and DTLZ2 problems are presented in Figures 7 and 8 .

One can observe that I-DBEA has a better rate of convergence in the earlier stages of evolution. Since the offspring solutions generated with I-DBEA have the opportunity to replace any individual in the population, they are likely to contribute towards better IGD values unlike forms with restricted neighborhoods. While the use of restricted neighborhood might be beneficial from a recombination point of view (based on previously reported results of MOEA/D-PBI for bi- and tri-objective optimization problems [16], [38]), use of neighborhood sizes of $T = 20$ or $T = W/2$ did not seem to offer benefit possibly due to the sparse nature of direction vectors used in the many objective optimization problems.

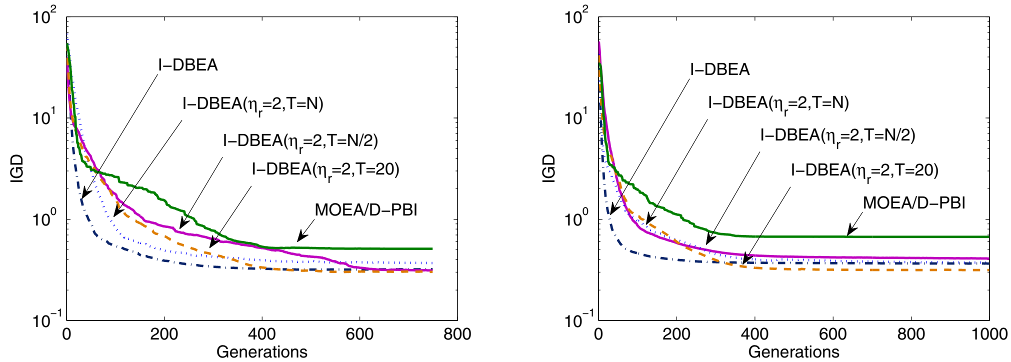
For completeness, we include the results based on HV for I-DBEA, DBEA-Eps and MOEA/D-PBI. NSGA-III has been excluded as the results of NSGA-III are not available. Out of 20 instances, I-DBEA (i.e., $\eta_r = 1$, $T = N$) obtained a better HV metric in 13 instances based on best, 7 instances based on median and 8 instances based on worst results respectively (Tables V and VI), whereas DBEA-Eps scored better in 1 instance based on worst and MOEA/D-PBI scored 2 instances based on best, 3 instances based on median and 4 instances based on worst.

B. Performance on unconstrained WFG problems

Next, the WFG test problems are considered which involve non-separable variables. These problems have different features i.e., disconnected, convex, concave, degenerate and linear Pareto-optimal front. The first test problem named WFG1 has a mixed Pareto-optimal front. For WFG2, the Pareto-optimal front is convex and disconnected and for WFG3 the Pareto front is linear and degenerate. The rest of the problems WFG4-WFG9 have concave Pareto front. In this study, we have used the toolkit [43] to observe the performance of our proposed algorithm. As reported in [43], the number of decision variables is set to 24 which is close to the decision variables used in the DTLZ problems. The complete settings of the values of distance parameter and position parameter are listed in Table VII.

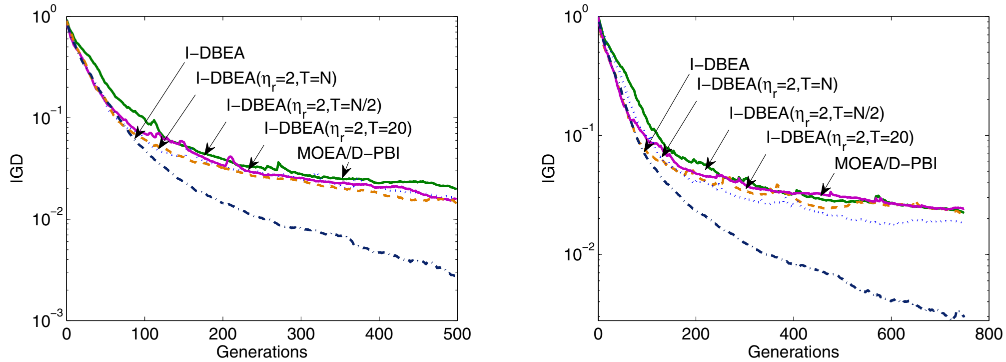
1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation



(a) (b)

Fig. 7. Variation of the mean IGD metric with different variants of I-DBEA and MOEA/D-PBI for (a) 8-objective (b) 10-objective DTLZ1 problem, where η_r represents the maximum number of parents that can be replaced by a child solution and T represents the size of neighborhood used for parent selection and replacement.



(a) (b)

Fig. 8. Variation of the mean IGD metric with different variants of I-DBEA and MOEA/D-PBI for (a) 8-objective (b) 10-objective DTLZ2 problem, where η_r represents the maximum number of parents that can be replaced by a child solution and T represents the size of neighborhood used for parent selection and replacement.

TABLE III
COMPARISON ON IGD (BEST, MEDIAN AND WORST) VALUES ON M-OBJECTIVE DTLZ1, DTLZ2, DTLZ3 AND DTLZ4 PROBLEMS. BEST PERFORMANCE IS SHOWN IN BOLD.

DTLZ	Obj.(MaxGen)	I-DBEA	DBEA-Eps	NSGA-III	MOEA/D-PBI
1	3 (400)	(1.075e-03, 1.043e-02, 6.502e-01)	(**8.771e-05**, 9.521e-03, 5.854e-01)	(4.880e-04, **1.308e-03**, 4.880e-03)	(4.095e-04, 1.495e-03, **4.743e-03**)
	5 (600)	(9.433e-04, 5.993e-04, 5.481e-01)	(**1.771e-05**, **5.116e-04**, 5.854e-01)	(5.116e-04, 9.799e-04, 1.979e-03)	(3.179e-04, 6.372e-04, **1.635e-03**)
	8 (750)	(8.570e-04, **2.421e-04**, **1.864e-03**)	(**4.387e-05**, 3.581e-04, 1.981e-03)	(2.044e-03, 3.979e-03, 8.721e-03)	(3.914e-03, 6.106e-03, 8.537e-03)
	10 (1000)	(**3.510e-04**, 5.178e-03, 1.112e-02)	(7.691e-04, **1.504e-03**, **2.700e-03**)	(2.215e-03, 3.462e-03, 6.869e-03)	(3.872e-03, 5.073e-03, 6.130e-03)
	15 (1500)	(**1.325e-03**, **2.329e-03**, 3.356e-03)	(1.696e-03, 2.606e-03, **2.686e-03**)	(2.649e-03, 5.063e-03, 1.123e-02)	(1.236e-02, 1.431e-02, 1.692e-02)
2	3 (250)	(6.372e-04, 1.243e-03, 8.402e-03)	(2.040e-02, 4.138e-02, 6.417e-02)	(1.262e-03, 1.357e-03, 2.114e-03)	(**5.432e-04**, **6.406e-04**, **8.008e-04**)
	5 (350)	(**1.118e-03**, 2.097e-03, 6.165e-03)	(1.199e-03, 3.024e-03, 2.272e-02)	(4.254e-03, 4.982e-03, 5.862e-03)	(1.219e-03, **1.437e-03**, **1.727e-03**)
	8 (500)	(**2.218e-03**, 3.185e-03, 9.694e-03)	(1.172e-03, **2.899e-03**, 6.915e-03)	(1.371e-02, 1.571e-02, 1.811e-02)	(3.097e-03, 3.763e-03, **5.198e-03**)
	10 (750)	(**2.173e-03**, 3.025e-03, **3.122e-03**)	(3.656e-03, 3.657e-03, 3.657e-03)	(1.350e-02, 1.528e-02, 1.697e-02)	(2.474e-03, **2.778e-03**, 3.235e-03)
	15 (1000)	(**4.238e-03**, **4.251e-03**, **4.267e-03**)	(5.160e-03, 5.960e-03, 5.960e-03)	(1.360e-02, 1.726e-02, 2.114e-02)	(5.254e-03, 6.005e-03, 9.409e-03)
3	3 (1000)	(**1.420e-04**, 5.701e-04, **3.590e-02**)	(4.171e-04, **4.278e-04**, 4.753e-01)	(9.751e-04, 4.007e-03, 6.665e-03)	(9.773e-04, 3.426e-03, 9.113e-03)
	5 (1000)	(**4.984e-04**, **4.076e-03**, **1.075e-01**)	(1.102e-03, 9.171e-02, 5.713e-01)	(3.086e-03, 5.960e-03, 1.196e-02)	(1.129e-03, 2.213e-03, 6.147e-03)
	8 (1000)	(**1.126e-03**, **2.666e-03**, **6.983e-02**)	(5.523e-02, 7.821e-02, 5.951e-01)	(1.244e-02, 2.375e-02, 9.649e-02)	(6.459e-03, 1.948e-02, 1.123e-00)
	10 (1500)	(**4.438e-03**, 5.320e-03, **5.396e-03**)	(5.773e-03, **4.137e-03**, 7.853e-01)	(8.849e-03, 1.188e-02, 2.083e-02)	(2.791e-03, 4.319e-03, 1.010e-00)
	15 (2000)	(8.612e-03, **8.681e-03**, **8.724e-03**)	(8.785e-03, 9.135e-03, 5.137e-01)	(1.401e-02, 2.145e-02, 4.195e-02)	(**4.360e-03**, 1.664e-02, 1.260e-00)
4	3 (600)	(**9.858e-05**, **2.300e-04**, 8.700e-01)	(2.175e-04, 3.578e-03, 9.154e-01)	(2.915e-04, 5.970e-04, **4.286e-01**)	(2.929e-01, 4.280e-01, 5.234e-01)
	5 (1000)	(**1.354e-04**, **2.857e-04**, **9.894e-03**)	(2.753e-04, 2.121e-03, 5.157e-01)	(9.849e-04, 1.255e-03, 1.721e-03)	(1.080e-01, 5.787e-01, 7.348e-01)
	8 (1250)	(**1.761e-04**, **3.319e-04**, **1.764e-02**)	(3.771e-03, 9.155e-03, 5.951e-01)	(5.079e-03, 7.054e-03, 6.051e-01)	(5.298e-01, 8.816e-01, 9.723e-01)
	10 (2000)	(**1.716e-04**, **1.716e-04**, **1.716e-04**)	(3.771e-03, 4.125e-03, 5.754e-01)	(5.694e-03, 6.337e-03, 1.076e-01)	(3.966e-01, 9.203e-01, 1.077e-00)
	15 (3000)	(**1.716e-04**, **1.716e-04**, **2.796e-04**)	(6.173e-03, 7.323e-03, 5.753e-01)	(7.110e-03, 3.431e-01, 1.073e-00)	(5.890e-01, 1.133e-00, 1.249e-00)

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE IV
EFFECTS OF NEIGHBORHOOD SIZE ON IGD (BEST, MEDIAN AND WORST) VALUES ON M-OBJECTIVE DTLZ1, DTLZ2, DTLZ3 AND DTLZ4 PROBLEMS. BEST PERFORMANCE IS SHOWN IN BOLD.

DTLZ	Obj.(MaxGen)	I-DBEA(η_r=2,T=W)	I-DBEA(η_r=2,T=W/2)	I-DBEA(η_r=2,T=20)
	3 (400)	(1.973e-03, 3.257e-03, 3.158e-02)	(2.015e-04, 1.991e-03, 3.36e+00)	(2.398e-03, 2.884e-03, 1.857e-02)
	5 (600)	(1.973e-03, 3.757e-03, 3.358e-02)	(3.265e-03, 3.307e-03, 3.331e-03)	(3.698e-04, 3.235e-03, 3.306e-03)
1	8 (750)	(**2.627e-03**, 3.306e-03, 8.567e-03)	(3.510e-03, 3.631e-03, 3.707e-03)	(2.884e-03, **2.957e-03**, **3.683e-03**)
	10 (1000)	(**2.800e-03**, **2.968e-03**, 3.951e-03)	(3.774e-03, 4.013e-03, 4.150e-03)	(3.087e-03, 3.157e-03, **3.166e-03**)
	15 (1500)	(**3.032e-03**, **3.643e-03**, **4.707e-03**)	(5.043e-03, 5.326e-03, 5.685e-03)	(4.765e-03, 5.301e-03, 6.024e-03)
	3 (250)	(5.841e-03, 6.067e-03, 6.255e-03)	(**4.819e-04**, **4.861e-04**, **4.925e-04**)	(4.836e-04, 4.934e-04, 5.018e-04)
	5 (350)	(**5.546e-04**, **5.581e-04**, **5.638e-04**)	(1.631e-03, 1.643e-03, 1.653e-03)	(1.389e-03, 1.398e-03, 1.405e-03)
2	8 (500)	(**1.545e-03**, **1.583e-03**, **1.590e-03**)	(2.661e-03, 2.784e-03, 2.841e-03)	(2.156e-03, 2.315e-03, 2.378e-03)
	10 (750)	(**1.808e-03**, **1.931e-03**, **1.984e-03**)	(4.585e-03, 5.072e-03, 5.491e-03)	(2.482e-03, 2.560e-03, 2.590e-03)
	15 (1000)	(**2.085e-03**, **2.148e-03**, **2.210e-03**)	(5.614e-03, 6.438e-03, 6.978e-03)	(5.243e-03, 5.971e-03, 6.629e-03)
	3 (1000)	(6.470e-03, 6.980e-03, **8.036e-03**)	(**5.961e-04**, 2.294e-03, 4.301e-02)	(8.352e-04, **1.146e-03**, 2.762e-02)
	5 (1000)	(6.795e-04, **1.051e-03**, 3.768e-02)	(1.391e-03, 1.714e-03, 1.503e-02)	(1.520e-03, 1.582e-03, 4.446e-03)
3	8 (1000)	(**1.754e-03**, **2.019e-03**, 3.812e-02)	(2.643e-03, 2.820e-03, 2.989e-03)	(2.182e-03, 2.265e-03, **2.524e-03**)
	10 (1500)	(**1.928e-03**, **2.135e-03**, 1.383e-02)	(2.747e-03, 2.999e-03, 3.078e-03)	(2.599e-03, 2.806e-03, **2.951e-03**)
	15 (2000)	(**2.155e-03**, **2.234e-03**, **2.444e-03**)	(2.572e-03, 2.770e-03, 4.025e-03)	(6.324e-03, 6.778e-03, 7.743e-03)
	3 (600)	(5.014e-03, 5.665e-03, 6.122e-03)	(**4.221e-04**, **4.257e-04**, 5.317e-03)	(4.477e-04, 4.644e-04, **5.071e-04**)
	5 (1000)	(**2.729e-04**, **5.115e-04**, 5.319e-03)	(7.501e-04, 8.156e-04, **3.684e-03**)	(6.533e-04, 8.894e-04, 3.740e-03)
4	8 (1250)	(**4.647e-04**, **4.661e-04**, 3.592e-03)	(8.889e-04, 1.410e-03, 2.946e-03)	(8.713e-04, 1.045e-03, **2.724e-03**)
	10 (2000)	(**1.033e-03**, 1.159e-03, 2.788e-03)	(1.206e-03, 1.379e-03, **1.398e-03**)	(1.036e-03, **1.145e-03**, 1.409e-03)
	15 (3000)	(1.123e-03, 1.219e-03, **1.222e-03**)	(**9.142e-04**, **1.035e-03**, 2.019e-03)	(1.345e-03, 1.406e-03, 2.268e-03)

*η_r = maximum number of parents that can be replaced by a child solution, T = size of neighborhood used for parent selection and replacement.

TABLE V
HV (BEST, MEDIAN AND WORST) VALUES OBTAINED FOR MOEA/D-PBI ON DTLZ1, DTLZ2, DTLZ3 AND DTLZ4 PROBLEMS. BEST PERFORMANCE IS SHOWN IN BOLD.

DTLZ	Obj.(MaxGen)	I-DBEA	DBEA-Eps	MOEA/D-PBI
	3 (400)	(8.091e-02, 8.076e-02, 8.019e-02)	(9.826e-02, 9.532e-02, **9.218e-02**)	(**9.891e-02**, **9.586e-02**, 9.148e-02)
	5 (600)	(**3.081e-02**, 3.077e-02, **3.074e-02**)	(2.916e-02, 2.991e-02, 2.016e-02)	(3.105e-02, **3.092e-02**, 2.028e-02)
1	8 (750)	(6.732e-03, 6.722e-03, 6.691e-03)	(6.676e-03, 6.751e-03, 5.082e-03)	(6.762e-03, 6.760e-03, 5.103e-03)
	10 (1000)	(**2.891e-03**, 2.883e-03, 2.882e-03)	(2.837e-03, 2.876e-03, 2.875e-03)	(2.888e-03, **2.887e-03**, **2.886e-03**)
	15 (1500)	(**3.779e-01**, **3.779e-01**, **3.779e-01**)	(3.752e-01, 3.778e-01, 3.773e-01)	(3.778e-01, 3.778e-01, 3.778e-01)
	3 (250)	(4.781e-01, **4.756e-01**, **4.725e-01**)	(4.752e-01 4.751e-01 4.701e-01)	(**4.790e-01**, 4.730e-01, 4.694e-01)
	5 (350)	(**2.828e+00**, 2.801e+00, 2.792e+00)	(2.800e+00, 2.806e+00, 2.806e+00)	(2.820e+00, 2.812e+00, **2.807e+00**)
2	8 (500)	(**5.758e+01**, 5.754e+01, **5.754e+01**)	(5.695e+01, 5.755e+01, 5.752e+01)	(5.757e+01, 5.754e+01, 5.751e+01)
	10 (750)	(**3.597e+02**, 3.557e+02, 3.545e+02)	(3.495e+02, 3.492e+02, 3.491e+02)	(3.501e+02, 3.501e+02, 3.501e+02)
	15 (1000)	(**5.432e+05**, **5.415e+05**, **5.410e+05**)	(5.362e+05, 5.369e+05, 5.361e+05)	(5.373e+05, 5.373e+05, 5.371e+05)
	3 (1000)	(4.019e-01, **2.607e-01**, 1.230e-05)	(3.792e-01, 3.240e-01, 5.918e-07)	(3.806e-01, 3.232e-01, 0.000e+00)
	5 (1000)	(8.316e-01, 7.510e-01, 0.000e+00)	8.348e-01, 8.029e-01, 2.855e-01	(8.345e-01, 8.037e-01, **2.780e-01**)
3	8 (1000)	(**5.191e+02**, **5.189e+02**, **5.169e+02**)	(5.159e+02, 5.169e+02, 5.160e+02)	(5.170e+02, 5.169e+02, 5.168e+02)
	10 (1500)	(**5.805e+02**, **5.797e+02**, **5.789e+02**)	(5.775e+02, 5.775e+02, 5.769e+02)	(5.786e+02, 5.786e+02, 5.785e+02)
	15 (2000)	(**1.931e+06**, 1.926e+06, 1.924e+06)	(1.912e+06, 1.924e+06, 1.923e+06)	(1.924e+06, 1.924e+06, 1.923e+06)
	3 (600)	(1.087e+00, 1.081e+00, 7.977e-01)	(1.079e+00, 1.080e+00, 1.076e+00)	(1.085e+00, 1.080e+00, 1.074e+00)
	5 (1000)	(8.302e+00, 8.246e+00, 8.105e+00)	(8.251e+00, 8.252e+00, 7.915e+00)	(8.257e+00, 8.245e+00, 7.939e+00)
4	8 (1250)	(**6.341e+01**, 6.259e+01, 6.243e+01)	(6.259e+01, 6.251e+01, 6.231e+01)	(6.262e+01, 6.260e+01, 6.234e+01)
	10 (2000)	(**1.350e+07**, **1.333e+02**, **1.323e+02**)	(1.302e+02, 1.313e+02, 1.313e+02)	(1.313e+02, 1.313e+02, 1.313e+02)
	15 (3000)	(**5.696e+02**, 5.685e+02, 5.684e+02)	(5.691e+02, 5.600e+02, 5.596e+02)	(5.684e+02, 5.684e+02, 5.683e+02)

Next we look at the performance measure for the individual test problems. Table VIII shows the best, median and worst IGD and HV measures for all numbers of objectives.

Two algorithms (i.e., I-DBEA and MOEA/D-PBI [16]) are used to compare the results with a maximum number of generations of 5000. The IGD metric is computed for both the algorithms on the targeted set of non-dominated solutions obtained from all runs of the algorithms on each problem. Out of 32 instances, I-DBEA obtained better results in 23 instances based on best, whereas MOEAD-PBI obtained better results in 9 instances based on best. In most of the cases involving higher number of objectives i.e., 10 and 15, the performance of I-DBEA is noticeably better. The hypervolume metric is also considered and computed for comparison with the identified reference points from the targeted set of non-dominated solutions obtained from all runs of the algorithms on each problem. The reference points are given in Table XV for further comparison. For WFG1 problem with 3, 5, 10 and 15 objectives, I-DBEA outperformed MOEA/D-PBI. For WFG2, MOEA/D-PBI was better than I-DBEA for 3 and 5

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE VI
EFFECTS OF NEIGHBORHOOD SIZE ON HV (BEST, MEDIAN AND WORST) VALUES ON M-OBJECTIVE DTLZ1, DTLZ2, DTLZ3 AND DTLZ4 PROBLEMS. BEST PERFORMANCE IS SHOWN IN BOLD.

DTLZ	Obj.(MaxGen)	I-DBEA(η_r=2,T=W)	I-DBEA(η_r=2,T=W/2)	I-DBEA(η_r=2,T=20)
1	3 (400)	(8.192e-02, 8.091e-02, 7.902e-02)	(9.091e-02, 9.086e-02, 9.015e-02)	(9.811e-02, 9.546e-02, 9.138e-02)
	5 (600)	(3.074e-02, 3.090e-02, 3.071e-02)	(**3.081e-02**, 3.077e-02, 3.065e-02)	(3.106e-02, 3.091e-02, 2.015e-02)
	8 (750)	(**6.769e-03**, 6.726e-03, 6.541e-03)	(6.727e-03, 6.715e-03, **6.697e-03**)	(6.746e-03, **6.762e-03**, 5.091e-03)
	10 (1000)	(2.879e-03, 2.853e-03, 2.878e-03)	(2.884e-03, 2.884e-03, 2.883e-03)	(2.887e-03, 2.886e-03, 2.886e-03)
	15 (1500)	(3.779e-01, 3.774e-01, 3.672e-01)	(3.778e-01, 3.778e-01, 3.778e-01)	(3.778e-01, 3.778e-01, 3.778e-01)
2	3 (250)	(4.675e-01, 4.716e-01, 4.610e-01)	(4.562e-01, 4.529e-01, 4.505e-01)	(4.764e-01, 4.738e-01, 4.701e-01)
	5 (350)	(2.827e+00, 2.802e+00, 2.650e+00)	(2.809e+00, 2.804e+00, 2.795e+00)	(2.820e+00, **2.812e+00**, 2.805e+00)
	8 (500)	(5.757e+01, **5.757e+01**, 5.746e+01)	(5.757e+01, 5.755e+01, 5.753e+01)	(5.755e+01, 5.754e+01, 5.751e+01)
	10 (750)	(3.579e+02, **3.561e+02**, **3.547e+02**)	(3.504e+02, 3.503e+02, 3.502e+02)	(3.501e+02, 3.501e+02, 3.501e+02)
	15 (1000)	(5.376e+05, 5.385e+05, 5.365e+05)	(5.373e+05, 5.374e+05, 5.374e+05)	(5.373e+05, 5.373e+05, 5.371e+05)
3	3 (1000)	(**4.099e-01**, 1.607e-01, **1.170e-05**)	(3.688e-01, 5.893e-02, 0.000e+00)	(3.813e-01, 3.247e-01, 0.000e+00)
	5 (1000)	(**8.386e-01**, 7.519e-01, 0.000e+00)	(8.304e-01, **8.039e-01**, 8.559e-03)	(8.357e-01, 8.030e-01, 2.765e-01)
	8 (1000)	(5.189e+02, 5.179e+02, 5.158e+02)	(5.170e+02, 5.169e+02, 5.168e+02)	(5.170e+02, 5.169e+02, 5.168e+02)
	10 (1500)	(5.796e+02, 5.787e+02, 5.782e+02)	(5.786e+02, 5.786e+02, 5.786e+02)	(5.786e+02, 5.786e+02, 5.785e+02)
	15 (2000)	(1.928e+06, **1.927e+07**, **1.925e+06**)	(1.925e+06, 1.924e+06, 1.924e+06)	(1.924e+06, 1.924e+06, 1.923e+06)
4	3 (600)	(1.088e+00, 1.081e+00, 7.860e-01)	(1.086e+00, 1.083e+00, 1.074e+00)	(1.082e+00, 1.080e+00, **1.075e+00**)
	5 (1000)	(**8.317e+00**, 8.241e+00, **8.215e+00**)	(8.259e+00, 8.247e+00, 8.096e+00)	(8.261e+00, **8.248e+00**, 7.935e+00)
	8 (1250)	(6.261e+01, 6.258e+01, 6.254e+01)	(6.259e+01, **6.259e+01**, **6.257e+01**)	(6.263e+01, 6.261e+01, 6.231e+01)
	10 (2000)	(1.313e+02, 1.313e+02, 1.313e+02)	(1.343e+02, 1.323e+02, 1.313e+02)	(1.313e+02, 1.313e+02, 1.313e+02)
	15 (3000)	(5.685e+02, 5.685e+02, 5.680e+02)	(5.795e+02, 5.684e+02, 5.684e+02)	(5.696e+02, **5.694e+02**, **5.693e+02**)

*η_r = maximum number of parents that can be replaced by a child solution, T = size of neighborhood used for parent selection and replacement.

TABLE VII
NUMBER OF DISTANCE PARAMETERS AND THE POSITION PARAMETERS USED TO COMBINE THE DECISION VARIABLES IN THE WFG TEST FUNCTIONS DEPENDING ON THE NUMBER OF OBJECTIVES.

	Number of Objectives			
	3	5	10	15
Distance parameters	20	20	17	6
Position parameters	4	4	7	18
Decision variables	24	24	24	24

objective problems while I-DBEA was better for problems involving 10 and 15 objectives. For the remaining 6 test problems, I-DBEA reaches the best performance with best and median results for objectives 3, 5, 10 and 15.

By inspecting the IGD and HV values of WFG problems (Table VIII), one can observe that I-DBEA tend to perform well for problems with higher number of objectives.

C. Performance on degenerate problems

1) DTLZ5-(I, M): After demonstrating the performance of the proposed I-DBEA on commonly studied benchmark problems, its performance is illustrated using DTLZ5-(I,M) [11] test problems. For DTLZ5-(I,M), I denotes the actual dimensionality of the Pareto front, while M denotes the original number of objectives of the problem. The details of the problems can be obtained from the above cited references.

In this study, the dimensionality analysis is carried out using the final population obtained by I-DBEA along the lines suggested in [13]. If an objective is redundant, its omission from the reference set (F_R) should not result in a significant change in the number of non-dominated solutions. For quantifying the change in the number of non-dominated solutions, a parameter R is defined as a ratio of number of non-dominated solutions in the reference set F_R to the number of non-dominated solutions in F_R after discarding objective f_m (i.e., $(F_R \setminus f_m)$). The high value of R represents the omitted objective is redundant, whereas the low value of R represents the objective is relevant. A value of R of 0.9 has been used in this study. The results are summarized in Table IX. It is clear the I-DBEA is able to accurately solve degenerate problems involving many objectives.

TABLE IX
RESULTS OBTAINED FOR DTLZ-(I, M) TEST PROBLEMS

Test problem	Reduced set of objectives	Success rate
DTLZ5-(2,3)	f_2, f_3	30/30
DTLZ5-(2,5)	f_4, f_5	30/30
DTLZ5-(2,8)	f_7, f_8	30/30
DTLZ5-(2,10)	f_9, f_{10}	30/30
DTLZ5-(2,15)	f_{14}, f_{15}	30/30
DTLZ5-(3,3)	f_1, f_2, f_3	30/30
DTLZ5-(3,5)	f_3, f_4, f_5	30/30
DTLZ5-(3,8)	f_6, f_7, f_8	30/30
DTLZ5-(3,10)	f_8, f_9, f_{10}	30/30
DTLZ5-(3,15)	f_{13}, f_{14}, f_{15}	30/30

V. CONSTRAINED ENGINEERING DESIGN PROBLEMS

Since the performance of the proposed algorithm was competitive on unconstrained test problems, we investigated its performance on three constrained engineering design optimization problems i.e., the three-objective car-side-impact problem [37] with ten inequality constraints, five-objective water resource management problem [46] with seven inequality constraints and finally the ten objective general aviation

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE VIII
IGD AND HV (BEST, MEDIAN AND WORST) VALUES OBTAINED USING I-DBEA AND MOEA/D-PBI ON WFG1-WFG2, WFG4-WFG9 PROBLEMS. BEST PERFORMANCE IS SHOWN IN BOLD.

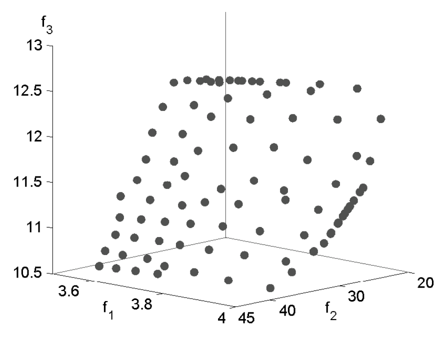
WFG	Objective	I-DBEA	MOEA/D-PBI	I-DBEA	MOEA/D-PBI
		IGD		Hypervolume	
1	3	(**1.221e-01**, **1.625e-01**, 8.420e-01)	(1.876e-01, 2.079e-01, **4.723e-01**)	(**4.378e+01**, **4.353e+01**, 3.643e+00)	(3.909e+01, 3.821e+01, **1.223e+01**)
	5	(3.440e-01, 3.846e-01, 4.572e-01)	(**3.215e-01**, **3.604e-01**, **3.778e-01**)	(**2.484e+03**, **2.458e+03**, 7.513e+02)	(2.386e+03, 2.136e+03, **1.396e+03**)
	10	(**4.522e-01**, **5.014e-01**, **1.028e+00**)	(7.122e-01, 8.019e-01, 1.425e+00)	(**7.598e-01**, **7.071e-01**, **6.393e-01**)	(2.106e-01, 1.955e-01, 1.953e-01)
	15	(**8.235e-01**, **8.625e-01**, 1.572e+00)	(9.115e-01, 9.603e-01, **1.150e+00**)	(**9.987e-01**, **9.210e-01**, 2.154e-01)	(9.906e-01, 9.030e-01, **9.030e-01**)
2	3	(**1.156e-01**, **2.014e-01**, 1.183e+00)	(3.936e-01, 4.665e-01, **1.145e+00**)	(**4.423e+01**, **4.283e+01**, **2.083e+01**)	(4.382e+01, 4.220e+01, 1.620e+01)
	5	(5.024e-01, 7.501e-01, 1.080e+00)	(**2.798e-01**, **2.935e-01**, **6.932e-01**)	(3.605e+03, 3.149e+03, 4.402e+02)	(**3.618e+03**, **3.152e+03**, **2.344e+03**)
	10	(**2.175e-01**, **2.261e-01**, **5.892e-01**)	(5.916e-01, 6.665e-01, 1.422e+00)	(**9.972e-01**, **9.924e-01**, 8.153e-01)	(9.932e-01, 9.232e-01, **9.232e-01**)
	15	(**8.012e-01**, 8.505e-01, **1.343e+00**)	(8.195e-01, **8.236e-01**, 1.773e+00)	(**9.983e-01**, **9.204e-01**, **8.514e-01**)	(9.913e-01, 9.037e-01, 7.035e-01)
4	3	(**8.750e-02**, **9.136e-02**, **4.648e-01**)	(1.621e-01, 1.682e-01, 4.779e-01)	(**1.967e+01**, **1.948e+01**, 1.237e+01)	(1.769e+01, 1.756e+01, **1.656e+01**)
	5	(**5.395e-01**, **5.537e-01**, **7.919e-01**)	(5.548e-01, 5.702e-01, 1.141e+00)	(**2.557e+03**, **2.540e+03**, **1.750e+03**)	(2.168e+03, 2.153e+03, 1.184e+03)
	10	(3.862e-01, 3.922e-01, **6.370e-01**)	(**3.822e-01**, **3.682e-01**, 1.124e+00)	(**8.256e-01**, **8.130e-01**, 4.761e-01)	(7.581e-01, 7.438e-01, **7.438e-01**)
	15	(**7.121e-01**, **7.376e-01**, **7.929e-01**)	(7.552e-01, 7.702e-01, 1.476e+00)	(**9.621e-01**, **8.334e-01**, 1.320e-01)	(9.608e-01, 8.203e-01, **8.203e-01**)
5	3	(**7.018e-02**, **7.595e-02**, 1.060e+00)	(1.360e-01, 1.442e-01, **6.497e-01**)	(**1.998e+01**, **1.990e+01**, **1.490e+01**)	(1.967e+01, 1.952e+01, 8.523e+00)
	5	(6.541e-01, 6.622e-01, 1.628e+00)	(**3.113e-01**, **3.166e-01**, **1.200e+00**)	(**2.577e+03**, **2.570e+03**, 8.099e+02)	(2.450e+03, 2.423e+03, **2.229e+03**)
	10	(**4.521e-01**, **4.553e-01**, 1.087e+00)	(6.310e-01, 6.452e-01, **8.169e-01**)	(**8.881e-01**, **8.526e-01**, 7.219e-01)	(8.136e-01, 7.943e-01, **7.943e-01**)
	15	(**3.988e-01**, **3.996e-01**, 1.291e+00)	(4.113e-01, 4.567e-01, **1.044e+00**)	(**8.874e-01**, **8.180e-01**, 3.154e-01)	(8.842e-01, 8.015e-01, **8.015e-01**)
6	3	(**1.208e-01**, **1.350e-01**, **4.578e-01**)	(1.555e-01, 1.695e-01, 8.544e-01)	(**1.999e+01**, **1.912e+01**, **1.112e+01**)	(1.952e+01, 1.741e+01, 3.409e+00)
	5	(**4.941e-01**, **4.990e-01**, **1.134e+00**)	(5.123e-01, 5.232e-01, 1.190e+00)	(**2.568e+03**, **2.540e+03**, 2.384e+02)	(2.419e+03, 2.281e+03, **2.095e+03**)
	10	(**3.158e-01**, **3.296e-01**, **8.121e-01**)	(4.658e-01, 4.795e-01, 1.423e+00)	(**8.012e-01**, **8.002e-01**, **8.002e-01**)	(7.627e-01, 7.602e-01, 7.602e-01)
	15	(5.816e-01, **5.891e-01**, 1.376e+00)	(**5.738e-01**, 5.989e-01, **9.782e-01**)	(**9.834e-01**, **9.238e-01**, 7.140e-01)	(8.598e-01, 8.593e-01, **8.593e-01**)
7	3	(**7.891e-02**, **8.057e-02**, **4.493e-01**)	(1.067e-01, 1.125e-01, 5.643e-01)	(1.961e+01, 1.947e+01, 3.473e+00)	(**1.981e+01**, **1.978e+01**, **1.278e+01**)
	5	(5.166e-01, 5.217e-01, 1.459e+00)	(**4.614e-01**, **4.724e-01**, **1.237e+00**)	(2.565e+03, 2.559e+03, **1.759e+03**)	(**2.570e+03**, **2.569e+03**, 1.071e+03)
	10	(2.896e-01, 3.125e-01, **6.652e-01**)	(**2.793e-01**, **2.813e-01**, 7.628e-01)	(**7.915e-01**, **7.823e-01**, 5.880e-01)	(7.434e-01, 7.275e-01, **7.275e-01**)
	15	(**4.798e-01**, **4.821e-01**, **7.490e-01**)	(4.871e-01, 5.023e-01, 1.435e+00)	(**9.621e-01**, **9.287e-01**, 4.284e-01)	(9.611e-01, 9.183e-01, **9.183e-01**)
8	3	(**1.627e-01**, **1.775e-01**, 5.040e-01)	(1.759e-01, 1.820e-01, **2.053e-01**)	(1.974e+01, 1.925e+01, **1.025e+01**)	(**2.007e+01**, **1.940e+01**, 9.399e+00)
	5	(6.881e-01, 6.903e-01, **7.124e-01**)	(**2.942e-01**, **2.958e-01**, 1.179e+00)	(2.729e+03, **2.708e+03**, **1.539e+03**)	(**2.775e+03**, 2.080e+03, 1.395e+03)
	10	(3.152e-01, 3.821e-01, **5.636e-01**)	(**2.917e-01**, **3.021e-01**, 1.089e+00)	(**8.183e-01**, **7.720e-01**, 5.132e-01)	(6.951e-01, 6.658e-01, **6.658e-01**)
	15	(**3.802e-01**, **3.808e-01**, 8.404e-01)	(3.912e-01, 3.958e-01, **7.859e-01**)	(**8.305e-01**, **7.210e-01**, 2.563e-01)	(7.860e-01, 6.664e-01, **6.664e-01**)
9	3	(**9.288e-02**, **1.198e-01**, **4.753e-01**)	(1.757e-01, 2.076e-01, 5.301e-01)	(**1.898e+01**, **1.799e+01**, **1.699e+01**)	(1.620e+01, 1.619e+01, 1.224e+01)
	5	(**5.262e-01**, **5.390e-01**, **1.036e+00**)	(7.041e-01, 7.471e-01, 1.059e+00)	(**2.709e+03**, **2.651e+03**, **1.059e+03**)	(2.527e+03, 2.478e+03, 1.009e+03)
	10	(**2.141e-01**, **3.076e-01**, **4.462e-01**)	(3.351e-01, 3.676e-01, 9.415e-01)	(**8.109e-01**, **7.033e-01**, 6.519e-01)	(7.101e-01, 6.928e-01, **6.928e-01**)
	15	(**7.058e-01**, **7.060e-01**, **7.611e-01**)	(7.141e-01, 7.471e-01, 9.484e-01)	(**9.247e-01**, **6.666e-01**, 4.564e-01)	(8.551e-01, 5.894e-01, **5.894e-01**)

aircraft (GAA) design problem [9] having a single inequality constraint.

A. Car side impact problem

The problem aims to minimize the weight of a car, the pubic force experienced by a passenger and the average velocity of the V-Pillar responsible for bearing the impact load subject to the constraints involving limiting values of abdomen load, pubic force, velocity of V-Pillar, rib deflection etc [37]. Although the problem does not involve too many objectives, it is used to observe the performance of I-DBEA on a constrained optimization problem. The problem is solved using 91 reference points and the population was allowed to evolve over 500 generations. Figures 9 and 10 show the final non-dominated solutions obtained from I-DBEA and DBEA-Eps. One can see from Figures 9 and 10 that I-DBEA achieved better alignment than DBEA-Eps.

We have computed the IGD using the targeted reference set of 1041 non-dominated solutions found from all the runs considering all the algorithms and HV is computed by normalizing the solutions using the ideal point of (i.e.,[23.586, 3.5852, 10.611]) and the extreme point of (i.e.,[42.768, 4, 12.453]) the reference set. I-DBEA also outperforms DBEA-Eps on both IGD and HV metrics based on all three aspects i.e., best, median and worst (Table X).

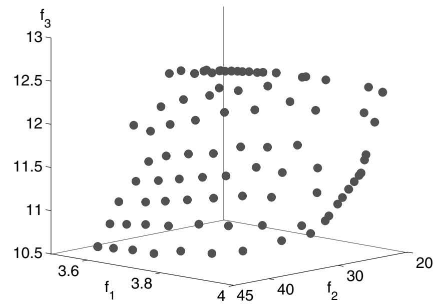


(a)

Fig. 9. Solutions obtained using I-DBEA on three-objective car side impact problem

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation



(a)

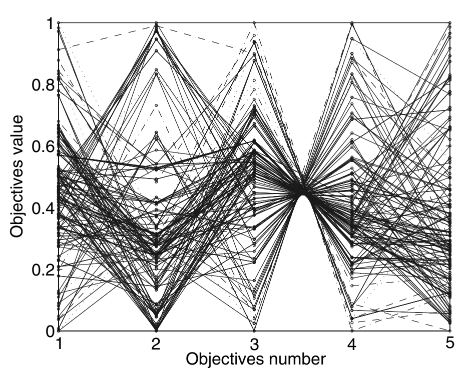
Fig. 10. Solutions obtained using DBEA-Eps on three-objective car side impact problem

TABLE X
IGD AND HV (BEST, MEDIAN AND WORST) VALUES OBTAINED USING I-DBEA AND DBEA-EPS FOR THE CAR SIDE IMPACT PROBLEM

Algo.	FE	IGD
I-DBEA DBEA-Eps	45500	(**1.493e-1**, **1.501e-1**, **5.732e-1**) (1.529e-1, 1.805e-1, 5.956e-1)
Algo.	FE	Hypervolume
I-DBEA DBEA-Eps	45500	(**7.091e+00**, **6.781e+00**, **3.573e+00**) (7.014e+00, 6.780e+00, 2.013e+00)

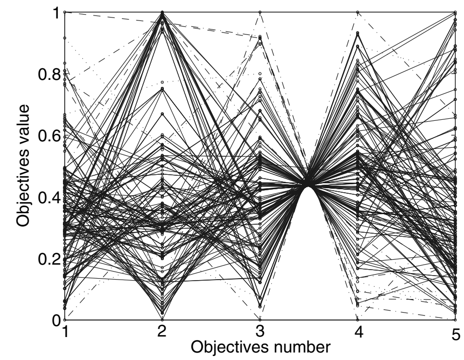
B. Water resource management problem

This is a five objective problem having seven constraints taken from the literature [46]. The problem is solved using 210 reference points and the population was allowed to evolve over 1000 generations. IGD value is computed using the targeted reference set of 2429 solutions reported in [47] and HV is computed by normalizing the solutions using the ideal point of (i.e.,[63840,56.4, 2.8535e+05, 1.8375e+05,7.2222]) and the extreme point of (i.e.,[73451, 1350, 2.8535e+06, 6.5753e+06, 24779]) the reference set. The normalized parallel coordinate plots generated using I-DBEA and DBEA-Eps are presented in Figures 11 and 12.



(a)

Fig. 11. Solutions obtained using I-DBEA on five-objective water resource management problem



(a)

Fig. 12. Solutions obtained using DBEA-Eps on five-objective water resource management problem

The result from I-DBEA is shown in the upper most plot and the result from DBEA-Eps is at lower. While the results appear similar, I-DBEA obtained a better distribution for objectives 1, 2 and 3. The IGD and HV metrics obtained from both the algorithms are given in Table XI. One can see in terms of IGD, DBEA-Eps has a better result based on best only, while I-DBEA outperforms DBEA-Eps in all other measures (IGD and HV).

TABLE XI
IGD AND HV (BEST, MEDIAN AND WORST) VALUES OBTAINED USING I-DBEA AND DBEA-EPS FOR THE WATER RESOURCE MANAGEMENT PROBLEM

Algo.	FE	IGD
I-DBEA DBEA-Eps	210000	(3.312e-2, **3.339e-2**, **1.123e-1**) (**3.291e-2**, 3.375e-2, 1.986e-1)
Algo.	FE	Hypervolume
I-DBEA DBEA-Eps	210000	(**2.554e-1**, **2.467e-1**, **1.962e-2**) (2.513e-1, 2.397e-1, 1.678e-2)

C. General aviation aircraft (GAA) design problem

This problem was first introduced by Simpson *et al.* [48] and has been recently solved using an evolutionary algorithm [9]. The problem involves 9 design variables i.e., cruise speed, aspect ratio, sweep angle, propeller diameter, wing loading, engine activity factor, seat width, tail length/ diameter ratio and taper ratio and the aim is to minimize the takeoff noise, empty weight, direct operating cost, ride roughness, fuel weight, purchase price, product family dissimilarity and maximize the flight range, lift/ drag ratio and cruise speed. Previous studies encountered difficulties in obtaining feasible solutions due to tight constraints [48].

In this example, we have used 275 reference points and the population was allowed to evolve over 5000 generations. A reference set of 530 non-dominated solutions obtained from ϵ-MOEA and Borg-MOEA is used to compute the IGD metric. The results of the proposed algorithm are compared with five other algorithms i.e., DBEA-Eps [30], ϵ-MOEA [9],

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

Borg-MOEA [9], MOEA/D-DE [38] and ϵ-NSGA-II [9]. We have also computed the hypervolume using the ideal point of (i.e.,[73.251, 1881.5, 59.114, 1.7977, 359.92, 41879, -2580.2, -16.823, -204.02, 0.26847]) and the extreme point of (i.e.,[74.036, 2011.5, 79.993, 2, 483.13, 44590, -2000, -14.408, -189.3, 1.9844]) obtained from the reference set. The performance of the algorithms are compared using the hypervolume in Table XII and IGD in Table XIII. One can observe that the proposed algorithms (I-DBEA and DBEA-Eps) perform marginally better than others for this problem.

TABLE XII
HV VALUES BASED ON 50 INDEPENDENT RUNS

Algorithm	FE	Hypervolume			
		Best	Mean	Worst	Std
I-DBEA	50,000	**0.02995**	**0.01726**	**0.00699**	0.05121
DBEA-Eps		0.02899	0.01715	0.00689	0.04561
ϵ-MOEA		0.02032	0.01032	0.00259	0.04125
Borg-MOEA		0.02245	0.01013	0.00424	0.02327
MOEA/D-DE		0.00092	0.00087	0.00045	0.00145
ϵ-NSGA-II		0.01636	0.01005	0.00236	**0.05232**

TABLE XIII
IGD VALUES BASED ON 50 INDEPENDENT RUNS

Algorithm	FE	IGD			
		Best	Mean	Worst	Std
I-DBEA	50,000	0.63150	0.80217	0.83101	0.09613
DBEA-Eps		**0.62070**	**0.80123**	**0.82430**	0.09210
ϵ-MOEA		0.98312	0.99123	0.99678	0.10312
Borg-MOEA		0.98211	0.99113	0.99337	0.02321
MOEA/D-DE		0.99117	0.99587	0.99723	**0.02145**
ϵ-NSGA-II		0.98571	0.98872	0.99131	0.72123

Figure 13 shows the parallel coordinate plot. The figure clearly shows that I-DBEA is able to find a widely distributed set of non-dominated points for 10-objective GAA design problem.

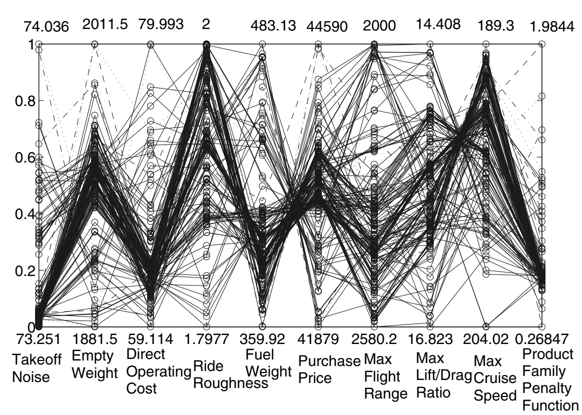
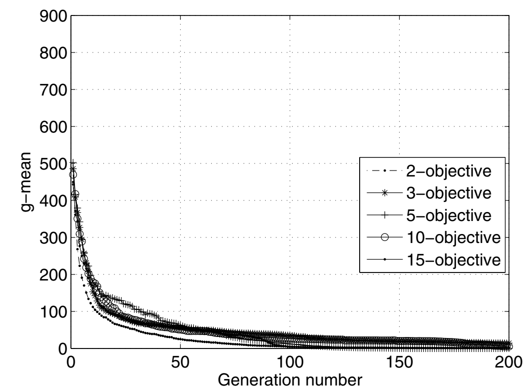


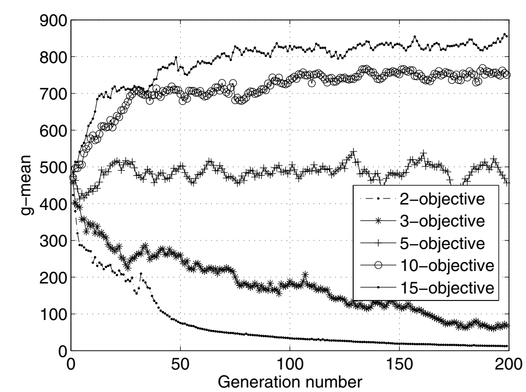
Fig. 13. Parallel coordinate plot of the final set of solutions obtained from the median run of I-DBEA based on HV

VI. SUMMARY OF OVERALL PERFORMANCE

Firstly, we introduced a decomposition based approach for many objective optimization which offers competitive performance when compared with other recently proposed similar algorithms. To start with, we compare the performance of I-DBEA and NSGA-II for DTLZ1 problems with 3, 5, 8, 10 and 15 objectives. The mean value of g (i.e., a multi-modal function) as shown in Figure 14 clearly highlights that a native nondominance scheme is ineffective in solving problems involving many objectives. Similar observations have also been reported in [36].



(a)



(b)

Fig. 14. Convergence behavior illustrated using DTLZ1 problem (a) decomposition based algorithm (e.g. I-DBEA) (b) non-dominance based algorithm (e.g. NSGA-II)

Secondly, the performance of I-DBEA is compared with various state of the art approaches using four widely studied benchmark problems [DTLZ1-DTLZ4, DTLZ5-(I, M), WFG1-WFG2, WFG4-WFG9], and three engineering design problems (car side impact, water resource and General aviation aircraft (GAA) design problem). The results clearly indicate the superiority of the proposed algorithm over existing algorithms for many-objective optimization. Among the 20 test instances of four DTLZ problems, I-DBEA obtained better results in 15 instances. In the context of WFG problems, once again the performance of I-DBEA is clearly better than MOEA/D-PBI. The ability to deal with concave, convex, mixed and degenerate problems with up to 15 objectives is showcased

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

using the examples. Since the solutions obtained using I-DBEA is often of better quality, identification of redundant objectives is likely to more accurate as demonstrated using DTLZ5-(I, M) problems.

The success of the epsilon based constraint handling scheme is highlighted using the GAA problem which is known to pose problems to existing algorithms due to tight constraints.

I-DBEA attempts to identify solutions along a set of uniformly distributed reference directions. Although a set of equally spaced reference directions are used in our approach and other reference direction based [16] and grid based approaches [32], it cannot ensure that these points will be located uniformly especially for problems involving irregular shaped Pareto fronts. In the context of disconnected Pareto fronts, there may not be a solution along a reference direction. The behavior of the approach is illustrated using a modified bi-objective ZDT3 [16] problem. The original problem has the limits of the Pareto front with f_1 between 0 and 0.852 and f_2 between -0.773 and 1. The problem is modified as below to ensure positive limits of f_2.

- ZDT3

$$Minimize$$
$$f_1(\mathbf{x}) = x_1$$
$$f_2(\mathbf{x}) = g(\mathbf{x}) \left[1 - \sqrt{\frac{f_1(\mathbf{x})}{g(\mathbf{x})}} - \frac{f_1(\mathbf{x})}{g(\mathbf{x})} sin(10\pi x_1) \right] + 1 \tag{12}$$

where, $g(\mathbf{x}) = 1 + \frac{9(\sum_{i=2}^{n} x_i)}{(n-1)}$ and $\mathbf{x} = (x_1, x_2, ..., x_m)^T \in [0,1]$. Its PF is disconnected and the value of n is 10.

The performance of I-DBEA and MOEA/D-PBI is computed using 30 independent runs. A population size of 21 is evolved over 1500 generations. The performance is measured by computing the non-dominated (ND) solutions obtained by the individual algorithms. Table XIV shows the number of ND solutions obtained by I-DBEA and MOEA/D-PBI.

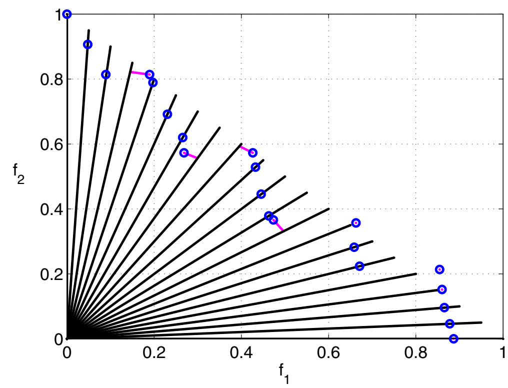
TABLE XIV
NUMBER OF NON-DOMINATED (ND) SOLUTIONS AND THEIR (STD) ACROSS MULTIPLE RUNS

Algorithm	Population size	ND solutions (Std)
I-DBEA	21	19 (3.6056)
MOEA/D-PBI	21	17 (4.1032)

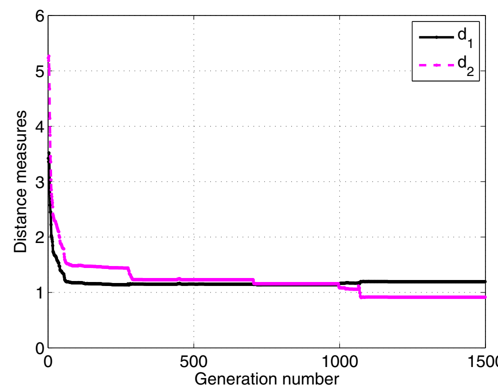
Figure 15 shows the alignment of the solutions to the reference directions. It is clear that some of the solutions have d_2 values significantly greater zero indicating presence of disconnected front. Figure 16 shows the final non-dominated solutions achieved by both the algorithms.

VII. CONCLUSION AND FUTURE WORK

In this paper, a decomposition based evolutionary algorithm is introduced and its performance is demonstrated using



(a)



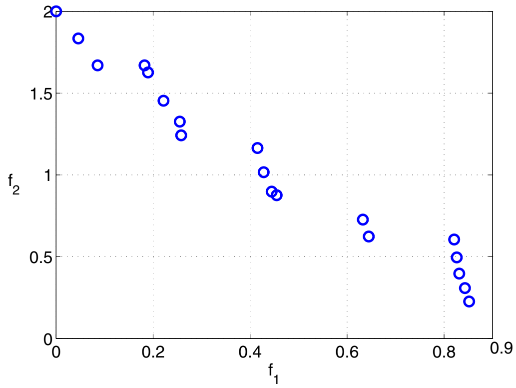
(b)

Fig. 15. Association and convergence plots of I-DBEA for the ZDT3 problem (a) association and (b) convergence

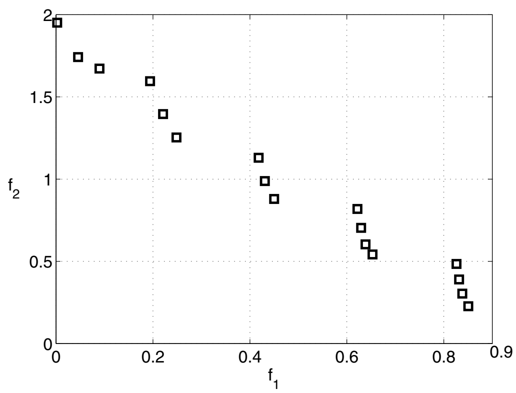
unconstrained and constrained many objective optimization problems. The approach utilizes reference directions to guide the search, wherein the reference directions are generated using a systematic sampling scheme as introduced by Das and Dennis [28]. The algorithm is designed using a steady state form. In an attempt to alleviate the problems associated with scalarization (commonly encountered in the context of reference direction based methods), the balance between diversity and convergence is maintained using a simple preemptive distance comparison scheme. Such a process also eliminates the need for a detailed association and niching operation as employed in NSGA-III. In order to deal with constraints, an epsilon level comparison is used which is known to be more effective than methods employing *feasibility first* principles. The performance of the algorithm is evaluated using standard benchmark problems i.e., DTLZ1-DTLZ4 for 3, 5, 8, 10 and 15 objectives, WFG1-WFG9, the car side impact problem, the water resource management problem and the constrained ten-objective general aviation aircraft (GAA) design problem. Results of problems involving redundant objectives and disconnected Pareto fronts are also included in this study to illustrate the capability of the algorithm. The results indicate that the proposed algorithm is able to deal with unconstrained and constrained many-objective optimization problems better

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation



(a)



(b)

Fig. 16. Non-dominated solutions obtained for ZDT3 using (a) I-DBEA and (b) MOEA/D-PBI

or at par with existing state of the art algorithms such as NSGA-III and MOEA/D-PBI.

In the current form, the population size is set to be the same as the number of reference directions. It is clear that for problems with a large number of reference directions, evolving a large population is not practical and there is a potential to develop archive based schemes. Another direct use of this algorithm would be for robust multi-objective optimization problems, where one can formulate the problem as a many objective optimization problem and obtain solutions with various levels of robustness simultaneously. These directions are currently being explored by the authors.

Appendix

The reference points used in this study are provided below in Table XV for completeness. The code can be obtained from the authors.

Acknowledgment

The first author would like to acknowledge the support of Publication Fellowship offered by UNSW Canberra and the second author would like to acknowledge the support of Future Fellowship offered by the Australian Research Council.

References

[1] H. Ishibuchi, N. Tsukamoto, and Y. Nojima, "Evolutionary many-objective optimization: A short review," in *Proc. IEEE World Congress Computational Intelligence*, 2008, pp. 2419–2426.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[3] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength pareto evolutionary algorithm for multi-objective optimisation," in *Evolutionary Methods for Design*. Optimisation and Control with Application to Industrial Problems, 2002, pp. 95–100.

[4] M. Koppen and K. Yoshida, "Substitute distance assignment in NSGA-II for handling many-objective optimization problems," in *EMO*, S. Obayashi, Ed., vol. 4403. Springer, Heidelberg: LNCS, 2007, pp. 727–741.

[5] H. K. Singh, A. Isaacs, T. Ray, and W. Smith, "A study on the performance of substitute distance based approaches for evolutionary many objective optimization," in *SEAL*, X. Li, Ed., vol. 5361. Springer-Verlag Berlin Heidelberg: LNCS, 2008, pp. 401–410.

[6] S. Kachroudi and M. Grossard, "Average rank domination relation for NSGA-II and SMPSO algorithms for many-objective optimization," in *Proc. Second World Congress Nature and Biologically Inspired Computing (NaBIC)*, 2010, pp. 19–24.

[7] G. Wang and J. Wu, "A new fuzzy dominance GA applied to solve many-objective optimization problem," in *Proc. Second Int. Conf. Innovative Computing, Information and Control ICICIC '07*, 2007, pp. 617–621.

[8] X. Zou, Y. Chen, M. Liu, and L. Kang, "A new evolutionary algorithm for solving many-objective optimization problems," *IEEE Transactions On Systems, Man, and Cybernetics-Part B*, vol. 38, no. 5, pp. 1402–1412, 2008.

[9] D. Hadka, P. M. Reed, and T. W. Simpson, "Diagnostic assessment of the Borg MOEA for many-objective product family design problems," in *IEEE World Congress on Computational Intelligence*, Brisbane, Australia, June 2012, pp. 10–15.

[10] H. Aguirre and K. Tanaka, "Adaptive ϵ-ranking on MNK-landscapes," in *Proc. IEEE symposium Computational intelligence in miulti-criteria decision-making (MCDM) '09*, 2009, pp. 104–111.

[11] D. K. Saxena and K. Deb, "Non-linear dimensionality reduction procedures for certain large-dimensional multi-objective optimization problems: Employing correntropy and a novel maximum variance unfolding," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, Eds. Springer Berlin Heidelberg, 2007, vol. 4403, pp. 772–787.

[12] ——, "Dimensionality reduction of objectives and constraints in multi-objective optimization problems: A system design perspective," in *IEEE Congress on Evolutionary Computation*, 2008, pp. 3204–3211.

[13] H. K. Singh, A. Isaacs, and T. Ray, "A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems," *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 4, pp. 539–556, 2011.

[14] K. Deb, A. Sinha, P. J. Korhonen, and J. Wallenius, "An interactive evolutionary multi-objective optimization method based on progressively approximated value functions," *IEEE Transactions on Evolutionary Computation*, vol. 14, pp. 723–739, 2010.

[15] T. Murata, H. Ishibuchi, and M. Gen, "Specification of genetic search directions in cellular multi-objective genetic algorithms," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, E. Zitzler, Ed., vol. 1993. Springer-Verlag Berlin Heidelberg, March 7-9 2001, pp. 82–95.

[16] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.

[17] T. Wagner, N. Beume, and B. Naujoks, "Pareto-, aggregation-,and indicator-based methods in many -objective optimization," in *EMO*, S. Obayashi, Ed., vol. 4403. Springer, Heidelberg: LNCS, 2007, pp. 742–756.

[18] E. J. Hughes, "MSOPS-II: A general-purpose many-objective optimiser," in *Proc. IEEE Congress Evolutionary Computation*, 2007, pp. 3944–3951.

[19] N. Beume, B. Naujoks, and M. Emmerich, "SMS-EMOA: Multiobjective selection based on dominated hypervolume," *European Journal of Operational Research*, vol. 181, pp. 1653–1669, 2006.

[20] H. J. Moen, N. B. Hansen, H. Hovland, and J. Trresen, "Many-objective optimization using taxi-cab surface evolutionary algorithm," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, R. Purshouse, Ed., vol. 7811. Springer Berlin Heidelberg, March 19-22 2013, pp. 128–142.

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TEVC.2014.2339823, IEEE Transactions on Evolutionary Computation

TABLE XV
REFERENCE POINT USED FOR HYPERVOLUME CALCULATION OF WFG PROBLEMS

Test Problem	Obj.	Reference point
WFG1	3	[2.6765 , 4.6893 , 6.7021]
	5	[2.5659 , 4.4052 , 6.4816 , 8.5614 , 8.7174]
	10	[2.6955, 4.6709, 4.5965, 6.2138, 7.6552, 9.7142, 12.6239, 15.2759, 18.6304, 20.8089]
	15	[2.7322, 4.6650, 2.0958, 8.5328, 10.6251, 12.1879, 14.4389, 16.7062, 18.6898, 20.7244, 22.7111, 24.7145, 26.7299, 28.7244, 30.7421]
WFG2	3	[2.0002 , 3.9953 , 5.9927]
	5	[1.9747 , 3.9441 , 5.9695 , 7.8723 , 9.9405]
	10	[1.6819, 3.4599, 4.7745, 6.9021, 8.2840, 8.5491, 11.6828, 12.9936, 14.3702, 19.5332]
	15	[1.4033, 1.011, 5.9989, 7.9998, 9.9894, 12,13.797, 15.946, 18.001, 19.981, 21.982, 23.969, 25.998, 28, 29.999]
WFG4	3	[2.0033 , 4.0030 , 6.0052]
	5	[2.0182 , 4.0172 , 6.0099 , 8.0152 , 10.018]
	10	[2.1136, 4.0950, 6.0876, 8.1385, 10.1373, 12.1196, 14.1150, 16.1095, 18.1099, 20.0927]
	15	[2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]
WFG5	3	[2.0500 , 4.0502 , 6.0501]
	5	[2.0508 , 4.0509 , 6.0509 , 8.0509 , 10.051]
	10	[2.3387, 4.1593, 6.0954, 8.0870, 10.1113, 12.0202, 14.0653, 16.0654, 18.0691, 20.0500]
	15	[2.05, 4.05, 6.05, 8.05, 10.05, 12.05, 14.05, 16.05, 18.05, 20.05, 22.05, 24.05, 26.05, 28.05, 30.05]
WFG6	3	[2.0274 , 4.0274 , 6.0273]
	5	[2.0235 , 4.0246 , 6.0219 , 8.0222 , 10.022]
	10	[2.1250, 4.1250, 6.1250, 8.1250, 10.1250, 12.1250, 14.1250, 16.1250, 18.0300, 20.1250]
	15	[2.0363, 4.0495, 6.0495, 8.0388, 10.05, 12.01, 14.005, 16.004, 18.004, 20.002, 22.002, 24.007, 26.001, 28, 30.002]
WFG7	3	[2.0017, 4.0018, 6.0016]
	5	[2.0035 , 4.0034 , 6.0037 , 8.0036 , 10.003]
	10	[2.1222, 4.0978, 6.1236, 8.1505, 10.1239, 12.1146, 14.1325, 16.1385, 18.0839, 20.0747]
	15	[2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]
WFG8	3	[2.2070 , 4.0114 , 6.0130]
	5	[2.204 , 4.1084 , 6.0227 , 8.0217 , 10.023]
	10	[2.4587, 4.3840, 6.4188, 8.2408, 10.1233, 12.0873, 14.0761, 16.0573, 18.0470, 20.0399]
	15	[2.8006, 4.8003, 6.801, 8.7923, 10.602, 12.4, 14.201, 16.001, 18, 20, 22, 24, 26, 28, 30]
WFG9	3	[2.0107 , 4.0072 , 6.0085]
	5	[2.1136 , 4.0936 , 6.1016 , 8.0977 , 10.071]
	10	[2.1762, 4.1437, 6.1694, 8.1460, 10.1525, 12.1521, 14.1659, 16.1448, 18.1400, 20.1421]
	15	[2.0604, 4.0911, 6.0555, 8.0515, 10.049, 12.051, 14.042, 16.009, 18.044, 20.01, 22.013, 24.012, 26.009, 28.009, 30.005]

[21] I. Giagkiozis, R. C. Purshouse, and P. J. Fleming, "Generalized decomposition," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, R. Purshouse, Ed., vol. 7811. Springer Berlin Heidelberg, March 19-22 2013, pp. 428–442.

[22] S. Jiang, Z. Cai, J. Zhang, and Y.-S. Ong, "Multi-objective optimization by decomposition with pareto-adaptive weight vectors," in *Seventh International Conference on Natural Computation*, 2011, pp. 1260–1264.

[23] F. Gu, H.-L. Liu, and K. C. Tan, "A multiobjective evolutionary algorithm using dynamic weight design method," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 5(B), pp. 3677–3688, 2012.

[24] A. Zhou, Q. Zhang, and G. Zhang, "Approximation model guided selection for evolutionary multiobjective optimization," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, R. Purshouse, Ed., vol. 7811. Springer Berlin Heidelberg, March 19-22 2013, pp. 398–412.

[25] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, Part I: Solving problems with box constraints," *IEEE Transactions on Evolutionary Computation*, 2013, available online.

[26] M. Asafuddoula, T. Ray, and R. Sarker, "A decomposition based evolutionary algorithm for many objective optimization with systematic sampling and adaptive epsilon control," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, R. Purshouse, Ed., vol. 7811. Springer Berlin Heidelberg, March 19-22 2013, pp. 413–427.

[27] T. Ray, M. Asafuddoula, and Isaacs, "A steady state decomposition based quantum genetic algorithm for many objective optimization," in *Proc. IEEE Congress Evolutionary Computation*, 2013, pp. 2817–2824.

[28] I. Das. and J. E. Dennis, "Normal-bounday intersection: A new method for generating Pareto optimal points in multicriteria optimization problems," *SIAM J. Optim.*, vol. 8, no. 3, pp. 631–657, August 1998.

[29] Y. Y. Tan, Y. C. Jiao, H. Lib, and X. K. Wang, "MOEA/D + uniform design: A new version of MOEA/D for optimization problems with many objectives," *Computers & Operations Research*, vol. 40, pp. 1648–1660, January 2013.

[30] M. Asafuddoula, T. Ray, R. Sarker, and K. Alam, "An adaptive constraint handling approach embedded MOEA/D," in *IEEE World Congress on Computational Intelligence*, 10-15 June 2012, pp. 1–8.

[31] T. Takahama and S. Sakai, "Constrained optimization by applying the α constrained method to the nonlinear simplex method with mutations," *IEEE Transactions on Evolutionary Computation*, vol. 9, no. 5, pp. 437–451, 2005.

[32] S. Yang, M. Li, X. Liu, and J. Zheng, "A grid-based evolutionary algorithm for many-objective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 5, pp. 721–736, 2013.

[33] R. Wang, R. C. Purshouse, and P. J. Fleming, "Preference-inspired co-evolutionary algorithms for many-objective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 4, pp. 474–494, 2013.

[34] S. F. Adra and P. J. Fleming, "Diversity management in evolutionary many-objective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 2, pp. 183–195, 2011.

[35] S. Gee, X. Qiu, and K. C. Tan, "A novel diversity maintenance scheme for evolutionary multi-objective optimization," in *Intelligent Data Engineering and Automated Learning IDEAL 2013*, ser. Lecture Notes in Computer Science, H. Yin, K. Tang, Y. Gao, F. Klawonn, M. Lee, T. Weise, B. Li, and X. Yao, Eds. Springer Berlin Heidelberg, 2013, vol. 8206, pp. 270–277.

[36] E. J. Hughes, "Evolutionary many-objective optimisation: many once or one many?" in *Proc. IEEE Congress Evolutionary Computation*, vol. 1, 2005, pp. 222–227.

[37] H. Jain and K. Deb, "An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, Part II: Handling constraints and extending to an adaptive approach," *IEEE Transactions on Evolutionary Computation*, 2013, available online.

[38] H. Li and Q. Zhang, "Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284–302, 2009.

[39] H. Sato, H. Aguirre, and K. Tanaka, "Variable space diversity, crossover and mutation in moea solving many-objective knapsack problems," *Annals of Mathematics and Artificial Intelligence*, vol. 68, pp. 197–224, 2013.

[40] H. Ishibuchi, N. Akedo, and Y. Nojima, "Behavior of multi-objective evolutionary algorithms on many-objective knapsack problems," *IEEE Transactions on Evolutionary Computation*, 2013, available online.

[41] K. Deb and R. B. Agarwal, "Simulated binary crossover for continuous search space," *Complex Systems*, vol. 9, no. 2, pp. 115–148, 1995.

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

[42] Z. Lili and Z. Wenhua, "Research on performance measures of multi-objective optimization evolutionary algorithms," in *Proc. 3rd Int. Conf. Intelligent System and Knowledge Engineering ISKE*, vol. 1, 2008, pp. 502–507.

[43] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 5, pp. 477–506, 2006.

[44] J. Bader, K. Deb, and E. Zitzler, "Faster hypervolume-based search using monte carlo sampling," in *Multiple Criteria Decision Making for Sustainable Energy and Transportation Systems*, ser. Lecture Notes in Economics and Mathematical Systems, M. Ehrgott, B. Naujoks, T. J. Stewart, and J. Wallenius, Eds. Springer Berlin Heidelberg, 2010, vol. 634, pp. 313–326.

[45] K. Deb and H. Jain, "Handling many-objective problems using an improved NSGA-II procedure," in *IEEE World Congress on Computational Intelligence*, Brisbane, Australia, June 2012, pp. 10–15.

[46] T. Ray, K. Tai, and K. C. Seow, "An evolutionary algorithm for multiobjective optimization," *Engineering Optimization*, vol. 33, no. 3, pp. 399–424, 2001.

[47] J. Durillo and A. Nebro, "jMetal: a java framework for multi-objective optimization," *Advances in Engineering Software*, vol. 42, pp. 760–771, 2011.

[48] T. W. Simpson, W. Chen, J. K. Allen, and F. Mistree, "Conceptual design of a family of products through the use of the robust concept exploration method," *AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, vol. 2, pp. 1535–1545, 1996.

Md Asafuddoula received the B.Sc. degree in Computer Science and Engineering from Chittagong University of Engineering and Technology at Chittagong, Bangladesh, in 2008 and the Ph.D. degree in Computer Science in University of New South Wales at Canberra, Australia, in 2014. He is currently a Research Associate with the School of Engineering and Information Technology, University of New South Wales at Canberra, Australia. His current research interests include evolutionary optimization, constraint handling, many-objective optimization, robust design optimization and multi-fidelity based optimization.

Tapabrata Ray is currently an Associate Professor with the School of Engineering and Information Technology, University of New South Wales at Canberra, Australia. He is a Future Fellow of the Australian Research Council and currently leads the Multidisciplinary Design Optimization Group at UNSW Canberra. His current research interests include evolutionary optimization, surrogate-assisted optimization, robust design optimization, and shape and topology optimization

Ruhul Sarker received his Ph.D. in 1991 from Dalhousie University, Halifax, Canada and is currently an Associate Professor in the School of Engineering and IT, University of New South Wales, Canberra, Australia. His main research interests are Configuration of Evolutionary Algorithms, Evolutionary Optimization, and Applied Operations Research. He is the lead author of the book Optimization Modelling: A Practical Approach published in 2007. He has published more than 250 refereed articles in the international journals, edited books, and conference proceedings. He is currently an associate editor of Memetic Computing Journal, and Flexible Service and Manufacturing Journal. A/Prof. Sarker is a member of IEEE and INFORMS.

1089-778X (c) 2013 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.