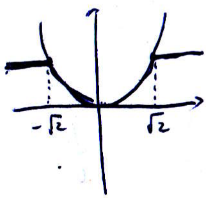
(3) 设 $x+y^2=\int_0^{y-x}\cos^2 t dt$，求 $\frac{dy}{dx}$。

$1+2y\cdot y'=\cos^2(y-x)\cdot(y'-1)$

$\therefore y'=\dfrac{\cos^2(y-x)+1}{\cos^2(y-x)-2y}$

(4) $\int_{-3}^{2}\min\{2,x^2\}dx$



$=\left(\int_{-3}^{-\sqrt{2}}+\int_{\sqrt{2}}^{2}\right)2dx+\int_{-\sqrt{2}}^{\sqrt{2}}x^2dx$

$=2[(2-\sqrt{2})+(3-\sqrt{2})]+2\int_0^{\sqrt{2}}x^2dx$

$=2\cdot(5-2\sqrt{2})+2\cdot\frac{1}{3}\cdot2\sqrt{2}$

$=10-\frac{8}{3}\sqrt{2}$

(5) $\int_0^1\frac{1}{x+\sqrt{1-x^2}}dx$

$\xlongequal{令x=\sin t}\int_0^{\frac{\pi}{2}}\frac{\cos t}{\sin t+\cos t}dt=\frac{\pi}{4}$

(6) $\int_0^1 x^2\sqrt{1-x^2}dx$

$\xlongequal{令x=\sin t}\int_0^{\frac{\pi}{2}}\sin^2 t\cos^2 t\,dt=\int_0^{\frac{\pi}{2}}\frac{1}{4}\sin^2 2t\,dt$

$=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{1-\cos4t}{2}dt=\frac{1}{8}\cdot\frac{\pi}{2}=\frac{\pi}{16}$

(7) $\int_{\frac{3}{4}}^{1}\frac{1}{\sqrt{1-x}-1}dx$

令$\sqrt{1-x}=t$ $\therefore x=1-t^2$ $dx=-2tdt$

$\int_{\frac{1}{2}}^{0}\frac{1}{t-1}(-2t)dt=2\int_0^{\frac{1}{2}}\frac{t}{t-1}dt=2\int_0^{\frac{1}{2}}\left(1+\frac{1}{t-1}\right)dt$

$=1+2\ln|t-1|\Big|_0^{\frac{1}{2}}=1-2\ln2$

(8) $\int_0^{\pi}x^2\sin^2 xdx=\int_0^{\pi}x^2\frac{1-\cos2x}{2}dx$

其中 $\int_0^{\pi}x^2\cos2x\,dx=\frac{1}{2}\int_0^{\pi}x^2d\sin2x=\frac{1}{2}\left(x^2\sin2x\Big|_0^{\pi}-\int_0^{\pi}2x\sin2x\,dx\right)$

$=-\int_0^{\pi}x\sin2x\,dx=\frac{1}{2}\int_0^{\pi}x\,d\cos2x=\frac{1}{2}\left(x\cos2x\Big|_0^{\pi}-\int_0^{\pi}\cos2x\,dx\right)$

$=\frac{\pi}{2}+\frac{1}{4}\sin2x\Big|_0^{\pi}=\frac{\pi}{2}$

$\therefore$ 原式 $=\int_0^{\pi}\frac{1}{2}x^2dx-\frac{1}{2}\int_0^{\pi}x^2\cos2x$

$=\frac{\pi^3}{6}-\frac{\pi}{4}$