

5.8 总习题

(4) 下列广义积分收敛的是

(C)

1、填空题

$$(1) \int_{\frac{1}{2}}^1 \ln \frac{1-x}{1+x} dx = 0$$

$$(2) \lim_{x \rightarrow 0} \frac{\int_0^x \cos^3 t dt}{\sin x} = 1$$

$$(3) \text{ 设 } F(x) = \int_1^x \frac{\ln t}{1+t^2} dt, \text{ 则 } F(x) - F\left(\frac{1}{x}\right) = 0$$

$$(4) \lim_{x \rightarrow +\infty} \left(\frac{x+c}{x-c} \right)^x = \int_{-\infty}^c te^{2t} dt, \text{ 求 } c = \frac{5}{2}$$

2、选择题

$$(1) \text{ 设 } f(x) = \int_0^{\sqrt{1+x-1}} \ln(1+t) dt, g(x) = \int_0^2 \arcsin t dt, \text{ 则 } x \rightarrow 0 \text{ 时}$$

(D)

(A) $f(x)$ 是 $g(x)$ 的高阶无穷小(C) $f(x)$ 是 $g(x)$ 的低阶无穷小

$$(2) \text{ 已知 } f(x) = \lim_{n \rightarrow +\infty} \frac{1-x^{2n}}{1+x^{2n}} x, \text{ 则 } \int_0^2 f(x) dx =$$

(A)

(D) 2

$$(3) \text{ 设 } I = \int_0^s f(tx) dx, \text{ 其中 } f(x) \text{ 连续, } t > 0, s > 0, \text{ 则 } I \text{ 的值}$$

(A) 依赖于 s, t (C) 依赖于 s, t, x

$$\int_0^s tx = u, I = \int_0^s f(u) \cdot \frac{1}{t} du = \int_0^s f(u) du$$

$$(A) \int_e^{+\infty} \frac{\ln x}{x} dx$$

$$(B) \int_e^{+\infty} \frac{1}{x \ln x} dx$$

$$(C) \int_e^{+\infty} \frac{1}{x \ln^2 x} dx$$

$$(D) \int_e^{+\infty} \frac{1}{x \sqrt{\ln x}} dx$$

$$F\left(\frac{1}{x}\right) = \int_1^{\frac{1}{x}} \frac{\ln t}{1+t^2} dt \stackrel{\text{令 } u = \frac{1}{t}}{=} \int_1^x \frac{\ln \frac{1}{u}}{1+\frac{1}{u^2}} \cdot \left(-\frac{1}{u^2}\right) du$$

$$= \int_1^x \frac{\ln u}{1+u^2} du = F(x)$$

山曲线 $y = \sin^2 x (0 \leq x \leq \pi)$ 与 x 轴围成的平面图形绕 x 轴旋

转而成的旋转体体积是

(B)

$$(A) \frac{4}{3} \quad (B) \frac{4}{3} \pi \quad (C) \frac{2}{3} \pi^2 \quad (D) \frac{2}{3} \pi$$

3、计算题

$$(1) \lim_{x \rightarrow 0} \frac{x - \int_0^x e^{t^2} dt}{x^2 \sin 2x}$$

$$= \lim_{x \rightarrow 0} \frac{x - \int_0^x e^{t^2} dt}{2x^3} = \lim_{x \rightarrow 0} \frac{1 - e^{x^2}}{6x^2} = \lim_{x \rightarrow 0} \frac{-x^2}{6x^2} = -\frac{1}{6}$$

$$\int_0^{x^2} [\int_0^u \sin t^2 dt] du$$

$$(2) \lim_{x \rightarrow 0} \frac{x^8 \int_0^x \sin t^2 dt \cdot 2x}{8x^7}$$

$$= \lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt \cdot 2x}{8x^6} = \lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{4x^6}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x^6 \cdot 2x}{24x^5} = \frac{1}{12}$$