(2) $\int_0^1 a^x e^x dx \, (a \neq \frac{1}{e})$

$= \int_0^1 (ae)^x dx$

$= \frac{(ae)^x}{\ln ae}\Big|_0^1 = \frac{ae-1}{\ln a+1}$

(3) $\int_0^{2\pi} |\sin x| dx$

$= \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx$

$= -\cos x\Big|_0^{\pi} + \cos x\Big|_{\pi}^{2\pi}$

$= -(-1-1)+(1+1) = 4$

(4) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^2 x \cos^2 x} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\frac{1}{4}\sin^2 2x} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4\csc^2 2x \, dx$

$= -2\cot 2x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{4\sqrt{3}}{3}$

6、设 $f(x) = \begin{cases} \frac{1}{2}\sin x, & 0 \le x \le \pi \\ 0, & x < 0 \text{或} x > \pi \end{cases}$，求 $F(x) = \int_0^x f(t)dt$ 在 $(-\infty,+\infty)$ 内的表达式。

$x<0$ 时：$\int_0^x f(t)dt = \int_0^x 0 dt = 0$

$0 \le x \le \pi$ 时：$\int_0^x f(t)dt = \int_0^x \frac{1}{2}\sin t dt = -\frac{1}{2}\cos t\Big|_0^x = \frac{1}{2}(1-\cos x)$

$x>\pi$ 时：$\int_0^x f(t)dt = \int_0^{\pi} \frac{1}{2}\sin t dt + \int_{\pi}^x 0 dt$

$= -\frac{1}{2}\cos t\Big|_0^{\pi} + 0 = 1$

$\therefore F(x) = \int_0^x f(t)dt = \begin{cases} 0 & x<0 \\ \frac{1}{2}(1-\cos x) & 0 \le x \le \pi \\ 1 & x>\pi \end{cases}$

7、求 a、$b\,(a>0)$ 的值，使 $\lim\limits_{x\to 0} \frac{1}{bx-\sin x}\int_0^x \frac{t^2}{\sqrt{a+t}}dt = 1$ 。

$\lim\limits_{x\to 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+t}}dt}{bx-\sin x} \overset{\text{“}\frac{0}{0}\text{”}}{=\!=} \lim\limits_{x\to 0} \frac{\frac{x^2}{\sqrt{a+x}}}{b-\cos x}$　$(\therefore b=1)$

$= \lim\limits_{x\to 0} \frac{\frac{x^2}{\sqrt{a+x}}}{1-\cos x} = \lim\limits_{x\to 0} \frac{\frac{x^2}{\sqrt{a+x}}}{\frac{1}{2}x^2} = \lim\limits_{x\to 0}\frac{2}{\sqrt{a+x}} = 1$

$\therefore a=4$