(14) $\int \frac{1-\ln x}{(x-\ln x)^2}dx$

$(1-\ln x = x-\ln x - x + 1)$

原式 $= \int \frac{x-\ln x - x(1-\frac{1}{x})}{(x-\ln x)^2}dx$

$= \frac{x}{x-\ln x} + C$

(15) $\int \frac{\sin x\cos x}{\sin x+\cos x}dx$

$= \frac{1}{2}\int \frac{2\sin x\cos x+1-1}{\sin x+\cos x}dx = \frac{1}{2}\int\frac{(\sin x+\cos x)^2}{\sin x+\cos x} - \frac{1}{2}\int\frac{1}{\sqrt{2}\sin(x+\frac{\pi}{4})}dx$

$= \frac{1}{2}(\sin x-\cos x) - \frac{1}{2\sqrt{2}}\ln|\csc(x+\frac{\pi}{4})-\cot(x+\frac{\pi}{4})| + C$

(16) $\int \frac{1}{\sin^3 x\cos^3 x}dx = \int\frac{\sin^2 x+\cos^2 x}{\sin^3 x\cos^3 x}dx = \int(\frac{1}{\sin x\cos^3 x}+\frac{1}{\sin^3 x\cos x})dx$

$= \int(\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}+\frac{\sin^2 x+\cos^2 x}{\sin^3 x\cos x})dx = \int(\frac{\sin x}{\cos^3 x}+\frac{2}{\sin x\cos x}+\frac{\cos x}{\sin^3 x})dx$

$= -\int\frac{d\cos x}{\cos^3 x}+\int\frac{2\sec^2 x\,dx}{\tan x}+\int\frac{d\sin x}{\sin^3 x} = \frac{1}{2\cos^2 x}+2\ln|\tan x| - \frac{1}{2\sin^2 x}+C$

(17) $\int \frac{\arctan x}{x^2(1+x^2)}dx = \int\frac{\arctan x}{x^2}dx - \int\frac{\arctan x}{1+x^2}dx$

其中 $\int\frac{\arctan x}{x^2}dx = -\int\arctan x\, d\frac{1}{x} = -(\frac{1}{x}\arctan x - \int\frac{1}{(1+x^2)x}dx)$

$= -\frac{1}{x}\arctan x + \int\frac{1+x^2-x^2}{(1+x^2)x}dx = -\frac{1}{x}\arctan x + \int\frac{1}{x}dx - \int\frac{x}{1+x^2}dx$

$\int\frac{\arctan x}{1+x^2}dx = \int\arctan x\, d\arctan x = \frac{1}{2}\arctan^2 x + C$

$\therefore$ 原式 $= -\frac{1}{x}\arctan x + \ln|x| - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}\arctan^2 x + C$

4、已知 $f(\ln x) = \frac{\ln(1+x)}{x}$，求 $\int f(x)dx$。

令 $\ln x = u$，则 $f(u) = \frac{\ln(1+u)}{e^u}$

$\therefore \int f(x)dx = \int\frac{\ln(1+e^x)}{e^x}dx \xlongequal[\text{则 } dx=\frac{1}{t}dt]{\text{令 } e^x=t} \int\frac{\ln(1+t)}{t^2}dt$

$= \int\ln(1+t)\,d(-\frac{1}{t}) = -(\frac{1}{t}\ln(1+t) - \int\frac{1}{(1+t)t}dt)$

$= -\frac{1}{t}\ln(1+t) + \int(\frac{1}{t}-\frac{1}{t+1})dt = -\frac{1}{t}\ln(1+t)+\ln t - \ln(1+t) + C$

$= -\frac{\ln(1+e^x)}{e^x} + x - \ln(1+e^x) + C$

5、已知 $f(x) = \begin{cases} x+1 & x\le 1 \\ 2x & x>1 \end{cases}$，求 $\int f(x)dx$

$\because f(x)$ 在 $(-\infty,+\infty)$ 上连续，$\therefore$ 必存在原函数 $F(x)$

$F(x) = \begin{cases} \frac{1}{2}x^2+x+C_1 & x\le 1 \\ x^2+C_2 & x>1 \end{cases}$　又 $\because F(x)$ 处处连续，$\therefore$ 有

$F(1^+)=F(1^-)$　即 $\frac{1}{2}+1+C_1 = 1+C_2$　$\therefore C_2 = C_1+\frac{1}{2}$.

令 $C=C_1$，则 $\int f(x)dx = \begin{cases} \frac{1}{2}x^2+x+C & x\le 1 \\ x^2+\frac{1}{2}+C & x>1 \end{cases}$

6、已知 $f(x)$ 的一个原函数为 $\ln(x+\sqrt{1+x^2})$，求 $\int xf''(x)dx$。

$\int xf''(x)dx = \int x\,df'(x)$

$= xf'(x) - \int f'(x)dx$

$= xf'(x) - f(x) + C$

又 $f(x) = [\ln(x+\sqrt{1+x^2})]' = \frac{1}{\sqrt{1+x^2}}$

$\therefore f'(x) = \frac{-x}{\sqrt{(1+x^2)^3}}$

$\therefore \int xf''(x)dx = -\frac{x}{\sqrt{(1+x^2)^3}} - \frac{1}{\sqrt{1+x^2}} + C$