

1. Eigenvalue is a number λ that has relation with a matrix A and a vector \vec{v} : $A\vec{v} = \lambda\vec{v}$

\vec{v} is called eigenvector in this case.

In PCA eigenvectors represent direction, eigenvalue describes variance.

2. Rotation is one of several linear transformations. Others are scaling, reflection and etc.

3. a) $m = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

b) $u \cdot v = (2 \cdot 1) + (3 \cdot 4) = 2 + 12 = 14$

c) $|u| = \sqrt{1+0} = 1$ $|v| = \sqrt{1} = 1$

$$\cos \theta = \frac{u \cdot v}{|u| \cdot |v|} = \frac{(1 \cdot 0) + (0 \cdot 1)}{1 \cdot 1} = 0$$

$$\theta = 90^\circ$$

d) $\det(C) = (3 \cdot 2) - (0 \cdot 0) = 6$

e) $D = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ $D - \lambda = \begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix}$

$$\det(D - \lambda) = (1-\lambda)(3-\lambda) - (2 \cdot 0) = 3 - 4\lambda + \lambda^2$$

$$3 - 4\lambda + \lambda^2 = 0$$

$$\boxed{\lambda_1 = 3 \quad \lambda_2 = 1}$$

~~Part 2~~

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1. Bayes' Theorem gives us a formula by which we can calculate probability of an event 'B' in relation to already happened event 'A'. It can be used in any problem involving several probabilities. For example, in data of medical association to calculate relation between smoking and being ill of some sickness.

2. Mean - is mathematical average of a set

Median - represents center of a set (ordered)

Mode - represents the most popular value of a set

The choice of usage is determined by data's properties. If data has extreme values median is better than mean, and so on.

3. a) $P_{x=4} = \frac{2}{6}$

b) $\{HT, TH\}$ $P = \frac{2}{4} = \frac{1}{2}$
 $\{HH, HT, TH, TT\}$

c) 4 kings in 52 cards $P = \frac{4}{52} = \frac{1}{13}$

d) $\{3, 5, 7, 8, 12, 13, 14, 18, 21\}$

a) Mean = $\frac{3+5+7+8+12+13+14+18+21}{9} = \frac{101}{9} = 11.22$

b) Median = 12 (middle)

c) Var = $\frac{\sum (x_i - \text{mean})^2}{n} \Rightarrow \frac{\sum (x_i - \text{mean})^2}{n} = \frac{(3-11.22)^2 + (5-11.22)^2 + \dots}{9} = 31.9506$

e) $N = 400$ $P_0 = \frac{150}{400} = \frac{15}{40} = \frac{3}{8}$

f) $P = \frac{200}{400} = 0.5$ $P' = 0.6$ $z_{0.9} = 1.645$

$E = z \sqrt{\frac{P \cdot P'}{n}} = 1.645 \sqrt{\frac{0.5 \cdot 0.6}{500}} = 0.036$ $[0.4 \pm 0.036, 0.4 \pm 0.036] = [0.364, 0.436]$

g) $\overline{4 \cdot 3 \cdot 2 \cdot 1} = 4! = 24$

Part 3

1. Gradients show direction of change and its rate. It is used to minimize losses during evaluations.

2. Derivative is a slope of a tangent line at any point of a function.

In other word it represent the smallest line of a function and draws it to be a tangent of set function. It show rate of change of a function.

3. 1) (A) $(-\infty, -3) \cup (-3, \infty)$ $y = \frac{1}{x+3}$

2) $f(x) = (x-2)^2 - 1$ is a parabola function ($f(x) = x^2$) that is moved 2 spaces to right and one space bottom.

3) (A) $y = 2x$ $x=0 \Rightarrow y=0$ $x=3 \Rightarrow y=6$

6) $z = x^3 + 4xy + y^2 - 7$ $\frac{dz}{dx} = 3x^2 + 4y + 0 + 0 = 3x^2 + 4y$

(A)

7) $f(x,y) = e^{xy}$ $\frac{df}{dy} = x e^{xy}$

(A)