Date . . . No

2.9

a $A \times (H \setminus K) = (A \times H) \setminus (A \times K)$

VT = ~~$\{(x,y) \mid x \in A \wedge y \in (H \setminus K)\}$~~

$VT = \{(x,y) \mid A \times (H \setminus K)\}$

tồn tại $(x,y) \in A \times (H \setminus K)$

$\Rightarrow$ ~~(x,y)~~ $x \in A \wedge y \in (H \setminus K)$

$\Rightarrow$ $x \in A \wedge (y \in H \wedge y \notin K)$

$\Rightarrow \begin{cases} x \in A \wedge y \in H \\ x \in A \wedge y \notin K \end{cases}$

$\Rightarrow \begin{cases} (x,y) \in A \times H \\ (x,y) \notin A \times K \end{cases}$

$(x,y) \in A \times K$
$x \in A$
$y \notin K$
$\therefore (x,y) \notin A \times K$

$\Rightarrow (x,y) \in A \times H \wedge (x,y) \notin A \times K$

$\Rightarrow (x,y) \in (A \times H) \setminus (A \times K)$

$\therefore (x,y) \in A \times (H \setminus K) \Rightarrow \therefore (x,y) \in (A \times H) \setminus (A \times K)$

$\therefore A \times (H \setminus K) \subseteq (A \times H) \setminus (A \times K)$ (1)

$VP = \{(p,q) \mid (A \times H) \setminus (A \times K)\}$

$\Rightarrow \begin{cases} (p,q) \in A \times H \\ (p,q) \notin A \times K \end{cases} \Rightarrow \begin{cases} p \in A \wedge q \in H \\ q \notin K \end{cases}$

$p \notin A \vee q \in K$
$p \in A$
$\therefore q \notin K$

Date . . . . . No

$\Rightarrow \begin{cases} p \in A \wedge q \in H \wedge q \notin K \end{cases}$

$\Rightarrow \quad p \in A \wedge q \in (H \setminus K)$

$\Rightarrow \quad (p,q) \in A \times (H \setminus K)$

$\because (p,q) \in (A \times H) \setminus (A \times K) \Rightarrow \because (p,q) \in A \times (H \setminus K)$

$\Rightarrow \quad \because (A \times H) \setminus (A \times K) \subseteq A \times (H \setminus K)$ (2)

(1) và (2) $\Rightarrow A \times (H \setminus K) = (A \times H) \setminus (A \times K)$

b. $[(A \times H) \setminus (B \times K)] = [(A \setminus B) \times H] \cup [A \times (H \setminus K)]$

$VT = \{(x,y) \mid [(A \times H) \setminus (B \times K)]\}$

tồn tại $(x,y) \in [(A \times H) \setminus (B \times K)]$

$\Rightarrow \begin{cases} (x,y) \in A \times H \\ (x,y) \notin B \times K \end{cases} \Rightarrow \begin{cases} x \in A \wedge y \in H \\ x \notin B \vee y \notin K \end{cases}$

$\Rightarrow \begin{cases} \begin{cases} x \in A \wedge y \notin K \\ x \in A \wedge x \notin B \end{cases} \\ y \in H \end{cases} \Rightarrow \begin{cases} x \in (A \setminus B) \\ y \in H \end{cases}$

$\Rightarrow \begin{cases} x \in A \wedge y \notin K \wedge y \in H \\ x \in (A \setminus B) \wedge y \in H \end{cases} \Leftrightarrow \begin{cases} (x,y) \in A \times (H \setminus K) \\ (x,y) \in (A \setminus B) \times H \end{cases}$

$\because (x,y) \in VT \Rightarrow \because (x,y) \in VP$

$\therefore VT \subseteq VP$ (1)

$VP = \{(p,q) \mid [(A \setminus B) \times H] \cup [A \times (H \setminus K)]\}$

tồn tại $(p,q) \in VP$

$\Rightarrow \begin{cases} p \in A \wedge p \notin B \wedge q \in H \\ p \in A \wedge q \in H \wedge q \notin K \end{cases} \Rightarrow \begin{cases} (p,q) \in (A \times H) \\ (p,q) \notin (B \times H) \\ (p,q) \in (A \times H) \\ (p,q) \notin (A \times K) \end{cases}$

Date . . . No

$\Rightarrow \begin{cases} (p,q) \in (A \times H) \\ p \notin B \text{ or } q \notin K \end{cases} \Leftrightarrow \begin{cases} (p,q) \in (A \times H) \\ (p,q) \notin (B \times K) \end{cases}$

$\Rightarrow (p,q) \in (A \times H) \setminus (B \times K)$

$\therefore (p,q) \in [(A \setminus B) \times H] \cup [A \times (H \setminus K)] \therefore (p,q) \in (A \times H) \setminus (B \times K)$

$\therefore VP \subseteq VT$ (2)

(1) và (2) $\Rightarrow VP = VT$

c/ $(A \times H) \cap (B \times K) = (A \cap B) \times (H \cap K)$

$\{(x;y) \mid (A \times H) \cap (B \times K)\}$

tồn tại $(x,y) \in (A \times H) \cap (B \times K)$

$\Rightarrow \begin{cases} x \in A \wedge y \in H \\ x \in B \wedge y \in K \end{cases} \Rightarrow \begin{cases} x \in A \wedge x \in B \\ y \in H \wedge y \in K \end{cases}$

$\Rightarrow \begin{cases} x \in (A \cap B) \\ y \in (H \cap K) \end{cases} \Rightarrow (x,y) \in (A \cap B) \times (H \cap K)$

$\therefore (x,y) \in (A \times H) \cap (B \times K) \Rightarrow \therefore (x,y) \in (A \cap B) \times (H \cap K)$

$\therefore (A \times H) \cap (B \times K) \subseteq (A \cap B) \times (H \cap K)$ (1)

$\{(p,q) \mid (A \cap B) \times (H \cap K)\}$

tồn tại $(p,q) \in VP$

$\Rightarrow p \in A \wedge p \in B \wedge q \in H \wedge q \in K$

$\Rightarrow \begin{cases} (p,q) \in A \times H \\ (p,q) \in B \times K \end{cases} \Rightarrow (p,q) \in (A \times H) \cap (B \times K)$

$\Rightarrow \therefore (p,q) \in VP \Rightarrow \therefore (p,q) \in VT$

$\Rightarrow \therefore VP \subseteq VT$ (2)

(1) và (2) $\Rightarrow VP = VT$

Date . . . No

d [(A×H) ∪ (B×K)] ⊂ [(A∪B) × (H∪K)]

VT ⇒ có (x,y) ∈ [(A×H) ∪ (B×K)]

⇒ { (x,y) ∈ (A×H)
  { (x,y) ∈ (B×K)

Ta có x ∈ A

• ∀ (x,y) ∈ A×H, (x,y) ∈ (A∪B) × (H∪K)

vì x ∈ A ⊆ A∪B

y ∈ H ⊆ H∪K

• ∀ (x,y) ∈ B×K, (x,y) ∈ (A∪B) × (H∪K)

vì x ∈ B ⊆ A∪B

y ∈ K ⊆ H∪K

⇒ ∴ (x,y) ∈ VT ⇒ ∴ (x,y) ∈ VP

⇒ VT ⊆ VP

VP ⇒ có (p,q) ∈ (A∪B) × (H∪K)

⇒ p ∈ A ∧ q ∈ K

⇒ (p,q) ∈ (A×K)

⇒ (p,q) ∉ (A×H) ∪ (B×K)

⇒ Không có dấu đẳng thức xảy ra

Date . . No

4/ $(A \setminus B) \times (H \setminus K) \subset (A \times H) \setminus (B \times K)$

VT $\Rightarrow$ có $(x,y) \in (A \setminus B) \times (H \setminus K)$

$\Rightarrow x \in A \setminus B \wedge y \in H \setminus K$

$\Rightarrow \begin{cases} x \in A \setminus B \subseteq A, x \notin B \\ y \in H \setminus K \subseteq H, y \notin K \end{cases}$

$\therefore (x,y) \in VT \Rightarrow \therefore (x,y) \in (A \times H) \setminus (B \times K)$

$\Rightarrow VT \subseteq VP$

VP có $(p,q) \in (A \times H) \setminus (B \times K)$

$\Rightarrow p \in A \cap B \wedge q \in H \wedge q \notin K$

$\Rightarrow \therefore (p,q) \notin (A \setminus B) \times (H \setminus K)$

$\Rightarrow$ Không có dấu đẳng thức xảy ra