CHAPTER

8

LOGICAL CIRCUIT DESIGN

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REMIND

- Boolean algebra

Read chapter 11 Computer Organization and Architecture 10th edition, William Stallings

PREREQUITES

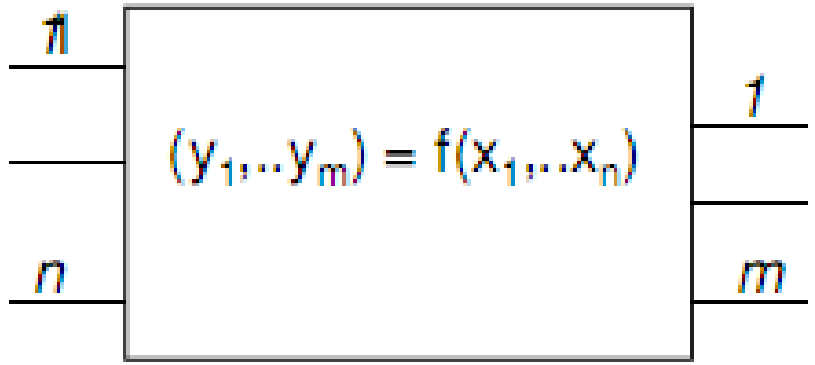
- Install Logisims tool already

What will you learn?

- Combinational circuit
- How to design a combinational circuit?
- Application of combinational circuit
- Basic ALU design
- Sequential circuit
- Application of sequential circuit

Combinational circuit

- A combinational circuit is an interconnected set of gates whose *output at any time is a function only of the input at that time*
- Consists of n binary input, m binary output
- Can be defined in three ways:
  - True table
  - Graphic symbol
  - Boolean equation



Implement of Boolean functions

Sum of Product:

The SOP form expresses that the output is 1 if any of the input combinations that produce 1 is true

$$f = u_1 + u_2 + \ldots + u_n \qquad With\, u_j = x_1 . x_2 \ldots x_i$$

Sum of Product (SOP)

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$$f = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}z$$

$\bar{x}\bar{y}z$

$\bar{x}y\bar{z}$

$x\bar{y}z$

Implement of Boolean functions

Product of Sum:

The POS form expresses that the output is 1 if all the input combinations that produce 1 is true

$$f = u_1.u_2.\ldots.u_n \qquad With\, u_j = x_1 + x_2 + \ldots + x_i$$

Product of Sum (POS)

x	y	z	$f = \bar{g}$	g
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$g = \bar{x}.y.\bar{z} + x\bar{y}\bar{z}$$

$$f = \bar{g} = (x + \bar{y} + z)(\bar{x} + y + z)$$

Simplify the Boolean function

- Using simplification methods:
  - Algebraic Simplification
  - Karnaugh map

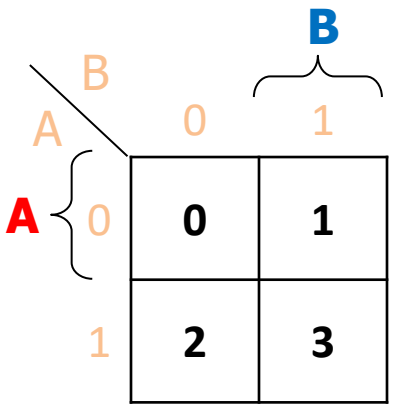
Simplify the Boolean function

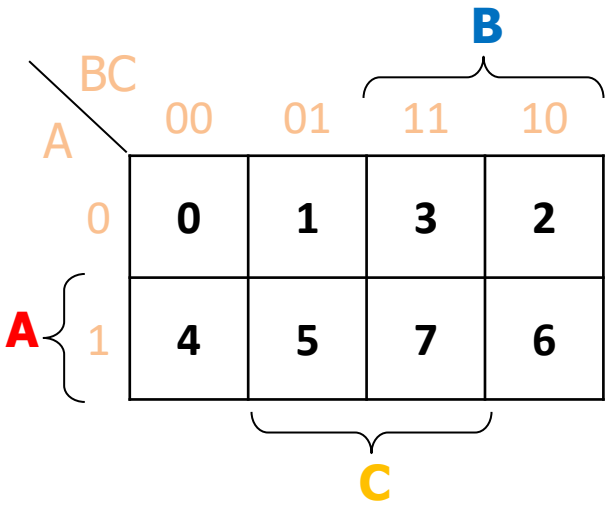
Refer to Basic Identities of Boolean Algebra

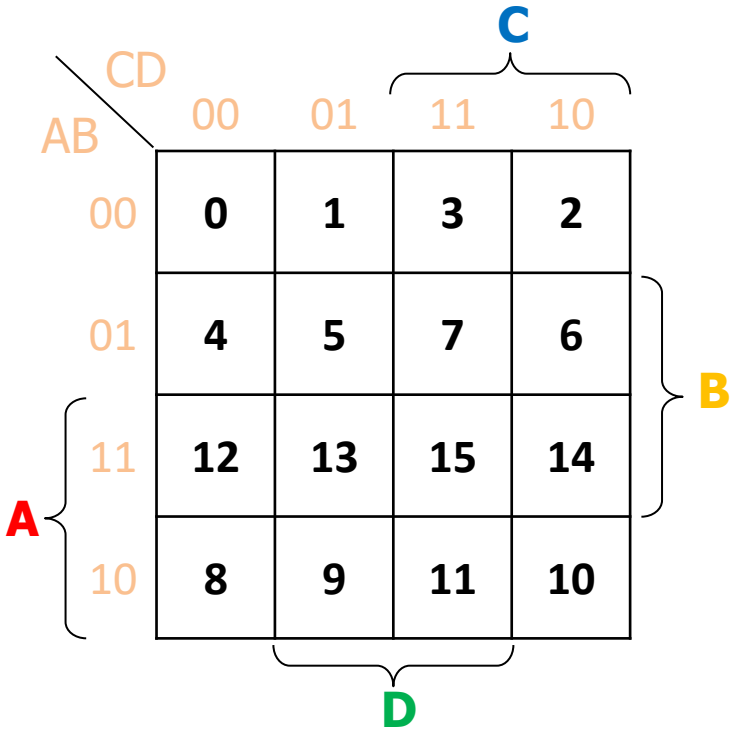
Basic Postulates		
$A \cdot B = B \cdot A$	$A + B = B + A$	Commutative Laws
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$	Distributive Laws
$1 \cdot A = A$	$0 + A = A$	Identity Elements
$A \cdot \overline{A} = 0$	$A + \overline{A} = 1$	Inverse Elements
Other Identities		
$0 \cdot A = 0$	$1 + A = 1$	
$A \cdot A = A$	$A + A = A$	
$A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$	Associative Laws
$\overline{A \cdot B} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A} \cdot \overline{B}$	DeMorgan's Theorem

Simplify the Boolean function

- Basic types of Karnaugh map

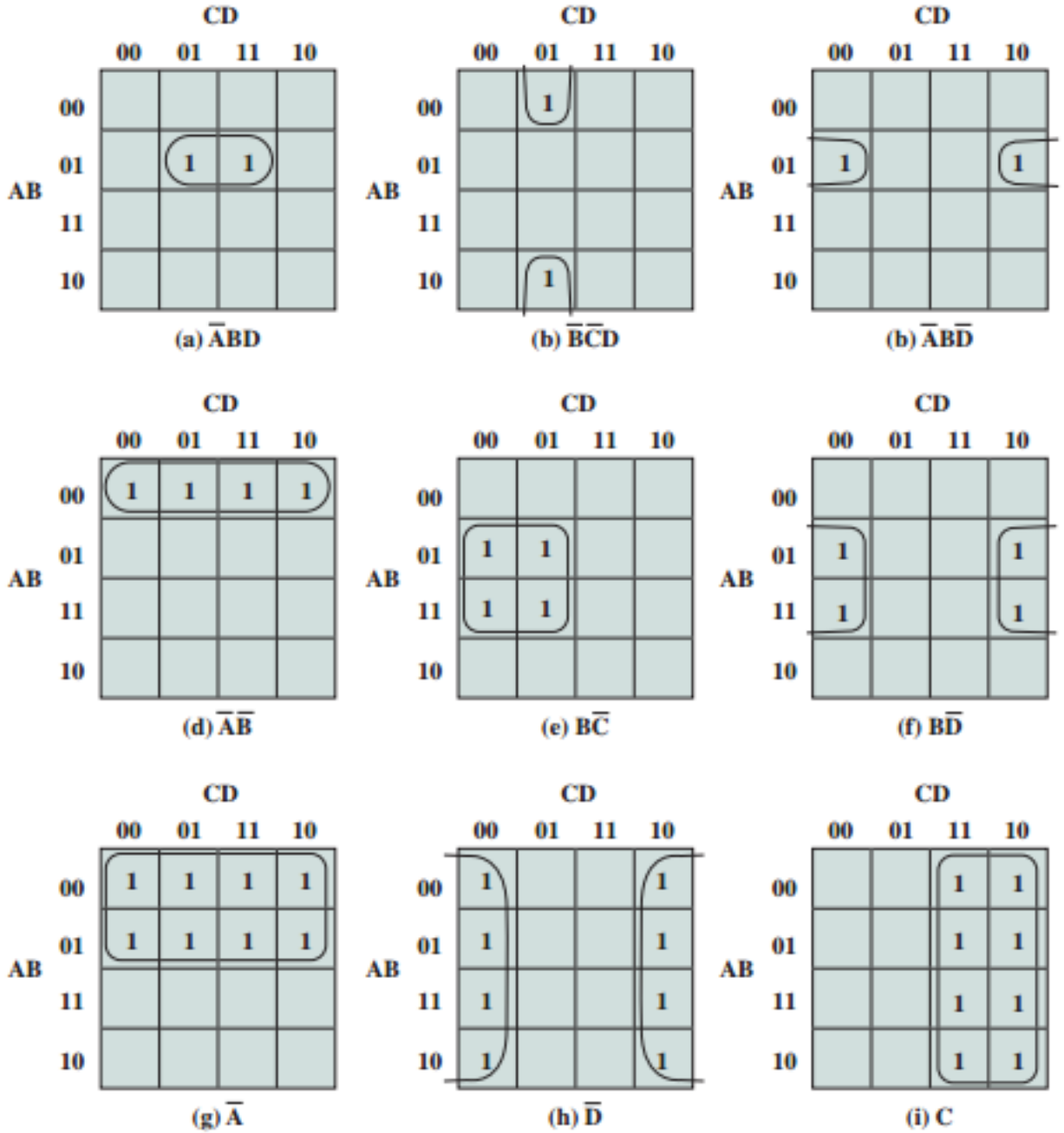






Simplify the Boolean function

- Any two squares that are adjacent differ in only one of the variables
- The top square of a column is adjacent to the bottom square, and the leftmost square of a row is adjacent to the rightmost square
- 4 cells located in 4 corners of the map are also considered adjacent cells



Design a combinational circuit

3 steps:

- Step 1: Construct the true table
- Step 2: Identify the Boolean function
- Step 3: Draw the logical circuit and test

Design a combinational circuit

Example:

Design a 3-input, 1-output combination circuit, so that the logic value of the output is the majority of the inputs.

Design a combinational circuit

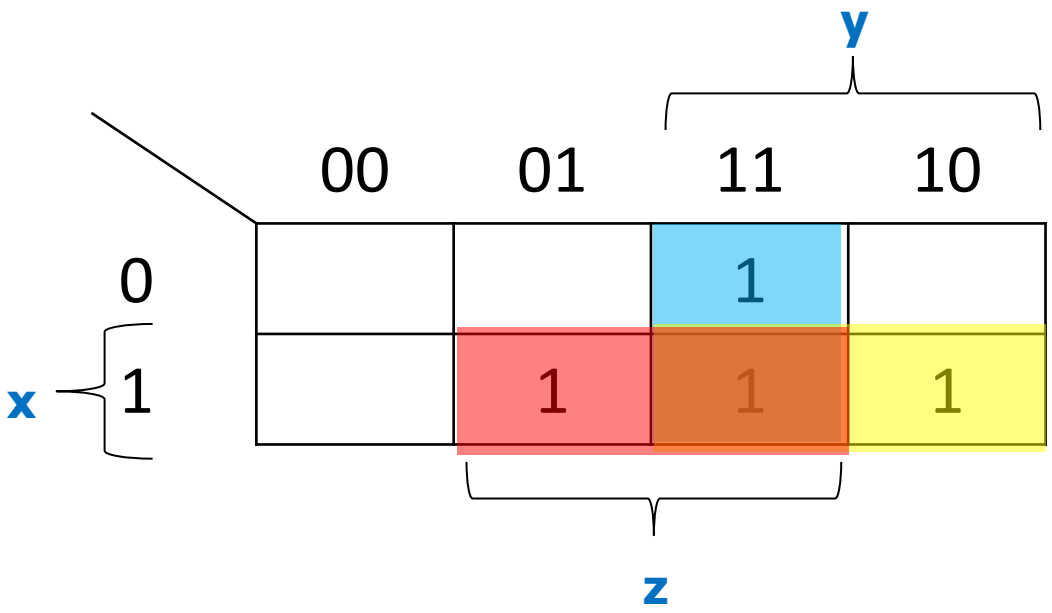
x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Step 1: The true table

$$f(x, y, z) = \sum(3, 5, 6, 7)$$

Design a combinational circuit

$$f(x, y, z) = \sum(3, 5, 6, 7)$$

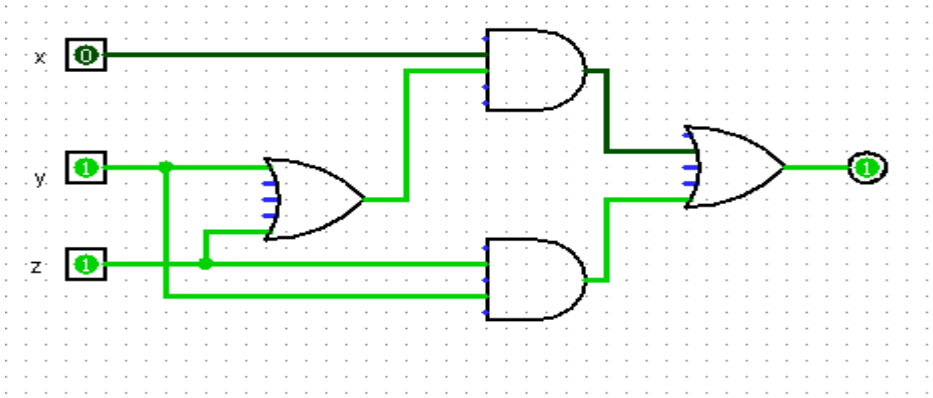


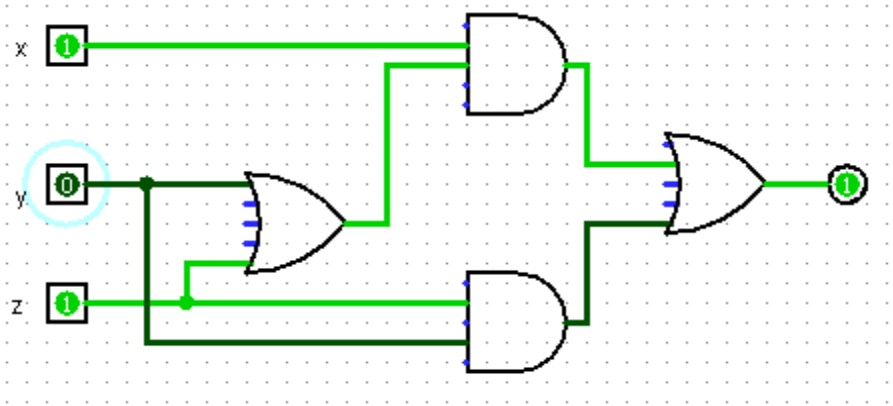
Step 2: Boolean Algebra function

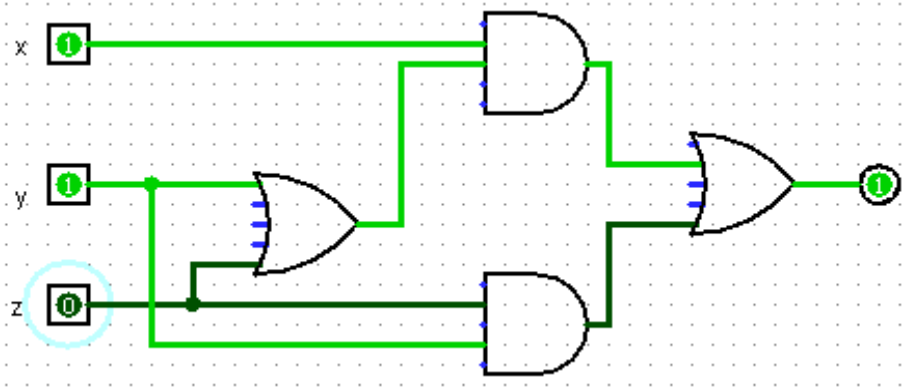
$$f = xz + xy + yz = x.(y + z) + yz$$

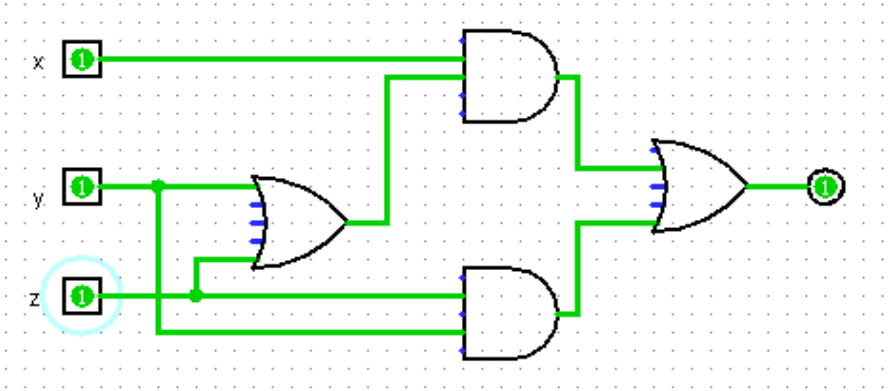
Design a combinational circuit

- Step 3: Drawing the logic circuit and testing





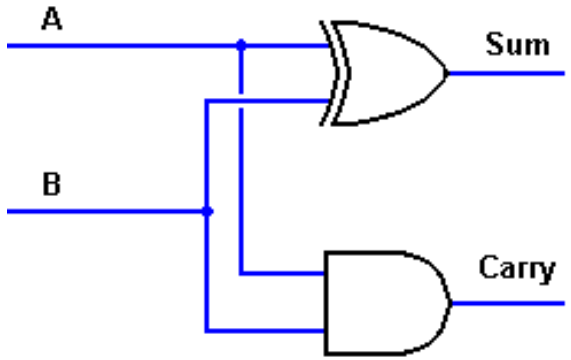




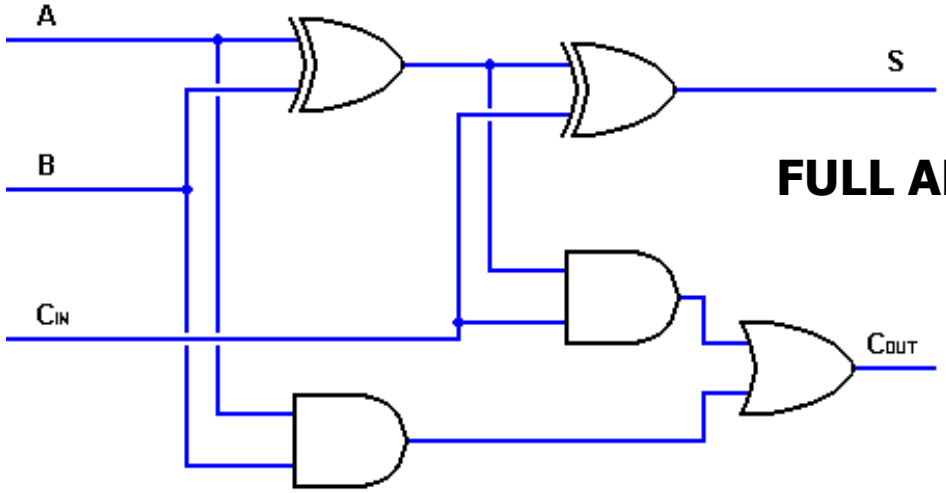
Application of combinational circuit

- Adder/ Subtractor
- Encode/ Decode
- Multiplexer/ Demultiplexer
- ALU
- ...

1-bit Adder



HALF ADDER



FULL ADDER

$S = F(A, B) = \Sigma(1, 2)$

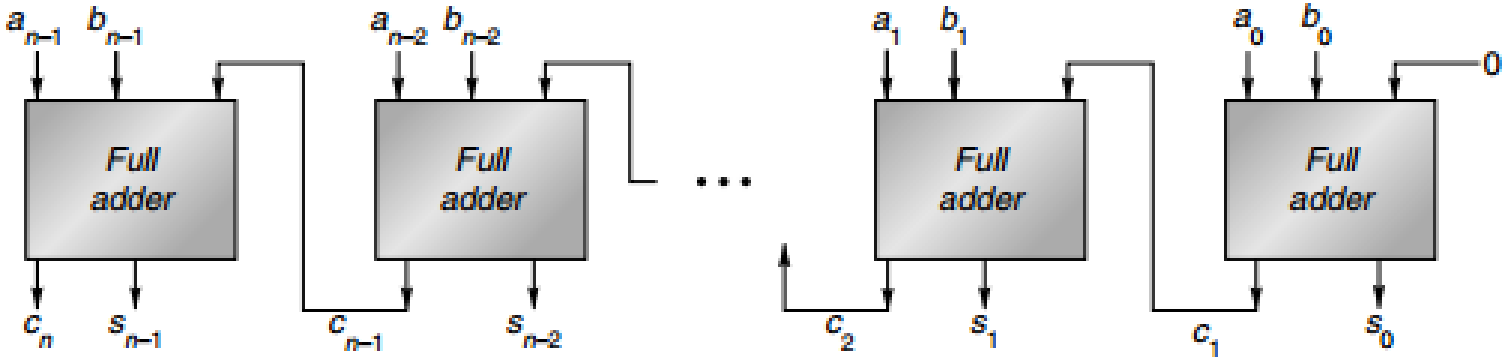
$C_{carr} = F(A, B) = \Sigma(0, 3)$

$S = F(A, B, C_{in}) = \Sigma(1, 2, 4, 7)$

$C_{carr} = F(A, B, C_{in}) = \Sigma(3, 5, 6, 7$

n-bit Full Adder

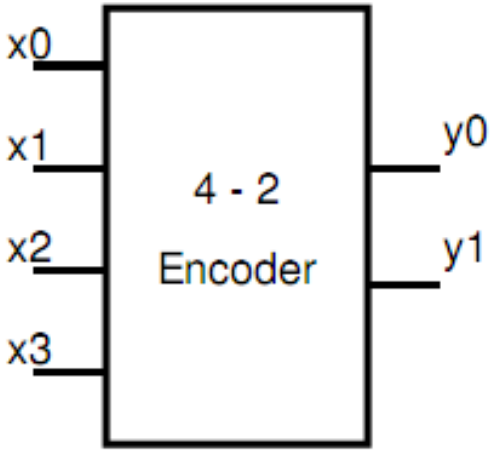
- The carry-out of one full adder is connected to the carry-in of the adder for the next most-significant bit
- The carries ripple from the least-significant bit (on the right) to the most-significant bit (on the left)



The ripple-carry adder, consists of n-bit full adders

Encoder

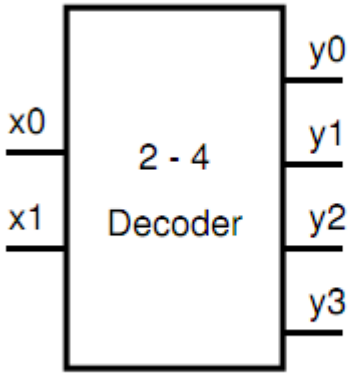
- Consists of 2^n input, n output
- Only one of input has the 1's value at the time
- If the k^{th} input has 1's value, the output will perform a value equal to k



x0	x1	x2	x3	y1	y0
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

Decoder

- Consists of n input, 2^n output
- Only one of which is asserted at any time
- If the inputs form a binary pattern with the value k, then the output = 1 is the k^{th} output



x1	x0	y0	y1	y2	y3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

$y0 = \overline{x1}.\overline{x0}$

$y1 = \overline{x1}.x0$

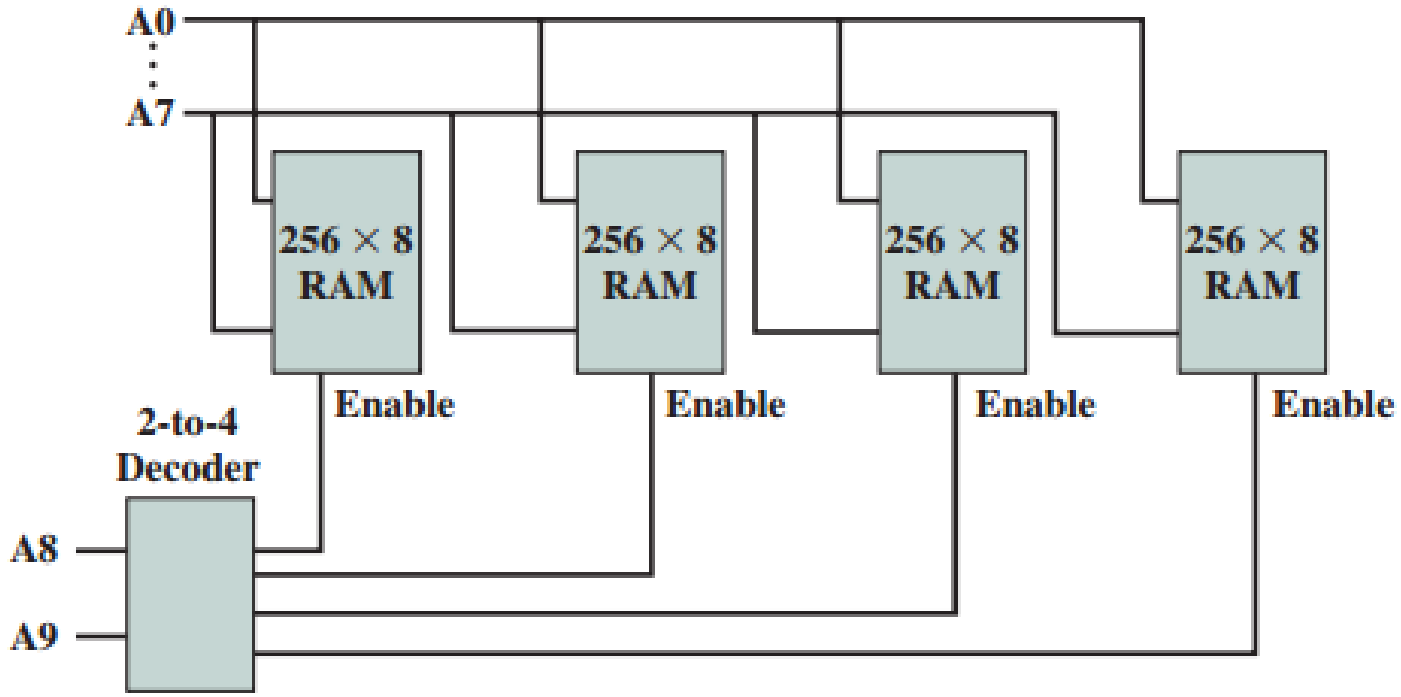
$y2 = x1.\overline{x0}$

$y3 = x1.x0$

Decoder

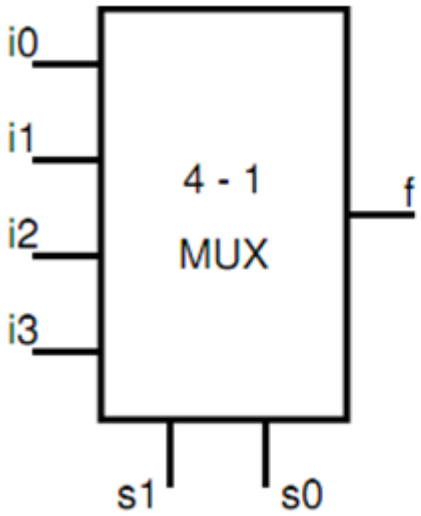
Ex: Construct a 1K-byte memory using four 256 * 8-bit RAM chips. A single unified address space, which can be broken down as follows:

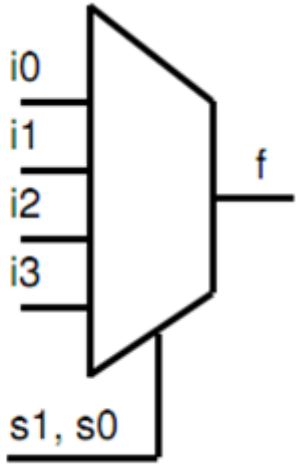
Address	Chip
0000–00FF	0
0100–01FF	1
0200–02FF	2
0300–03FF	3



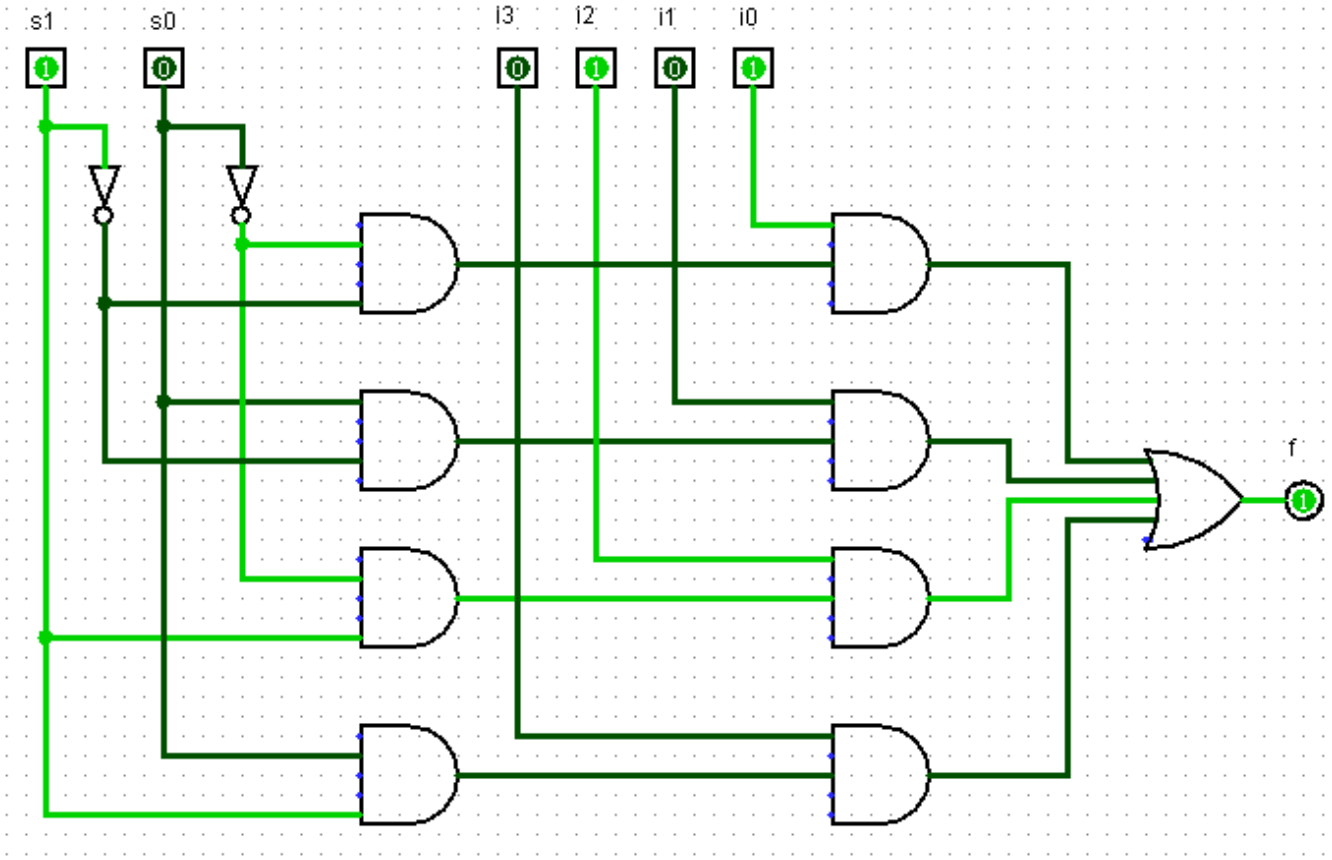
Multiplexer

- Selects one of the 2^n inputs to be only one output based on the n control input
- Ex: MUX 4-1 has 4 inputs, 1 output, and 2 control input





s1	s0	f
0	0	i_0
0	1	i_1
1	0	i_2
1	1	i_3

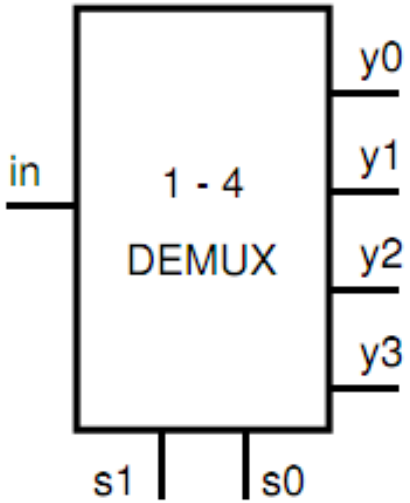


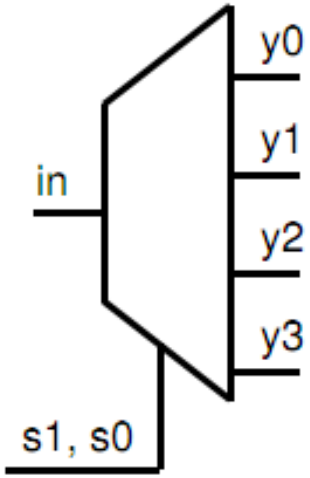
Multiplexer

The logic circuit of MUX 4-1

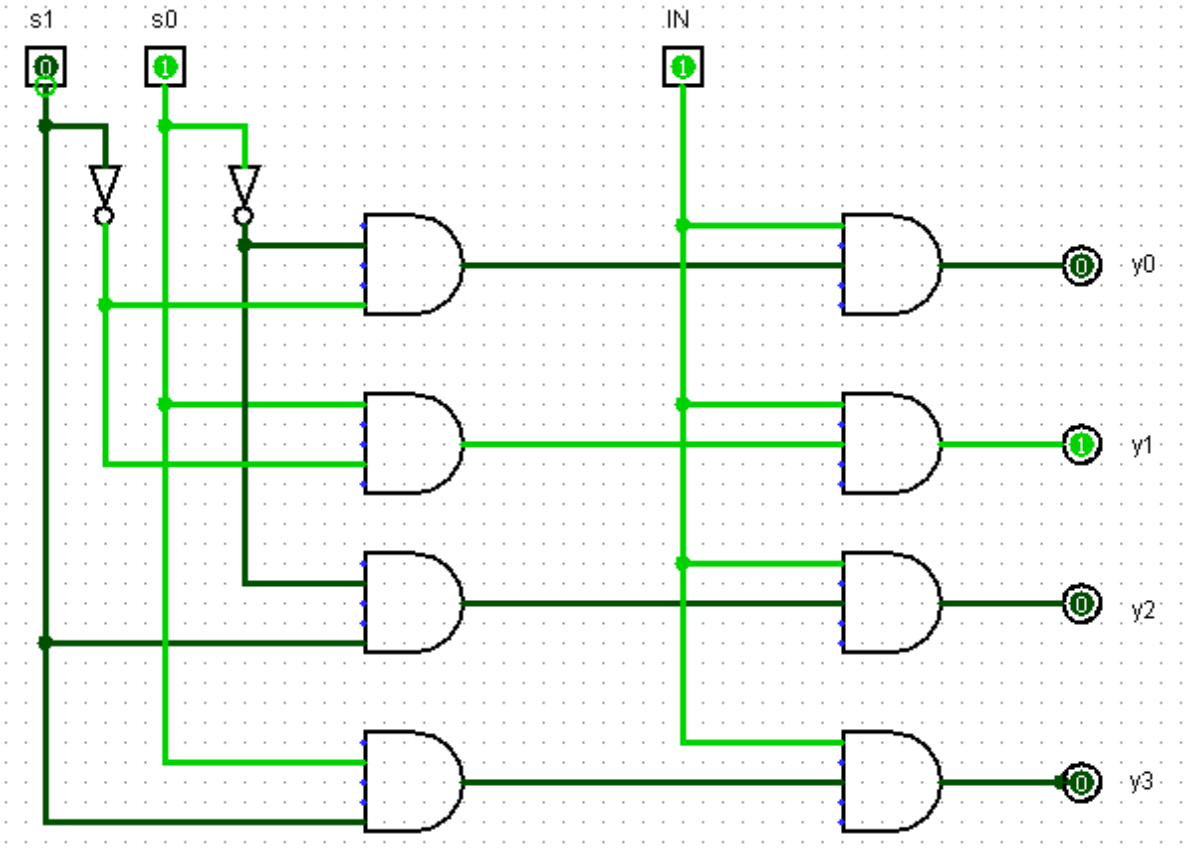
Demultiplexer

- Selects one of the 2^n output from only one input based on the n control input
- Ex: DEMUX 1-4 has 1 input, 4 output and 2 control input





s1	s0	y0	y1	y2	y3
0	0	in	0	0	0
0	1	0	in	0	0
1	0	0	0	in	0
1	1	0	0	0	in

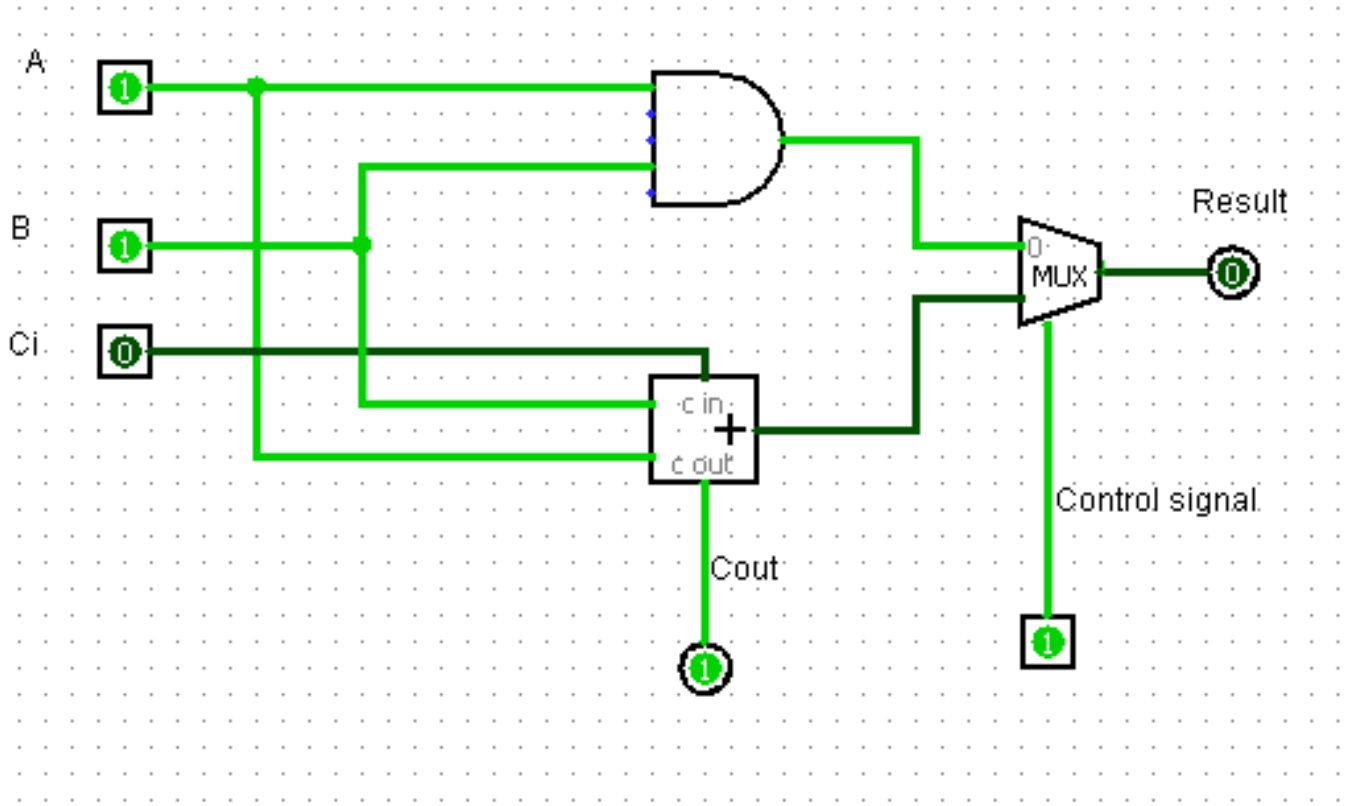


Demultiplexer

The logic circuit of DEMUX 1-4

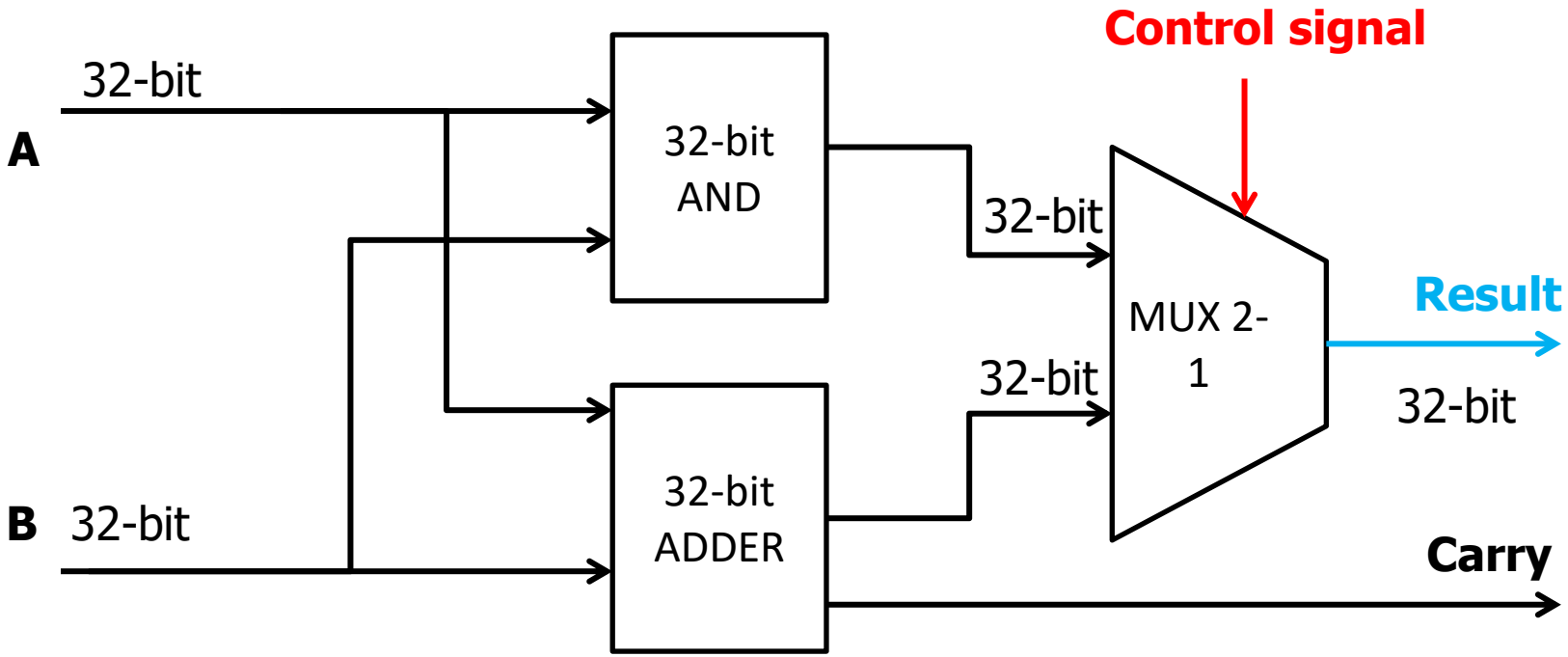
ALU

- The 1-bit ALU with 2 functions: and, add



ALU

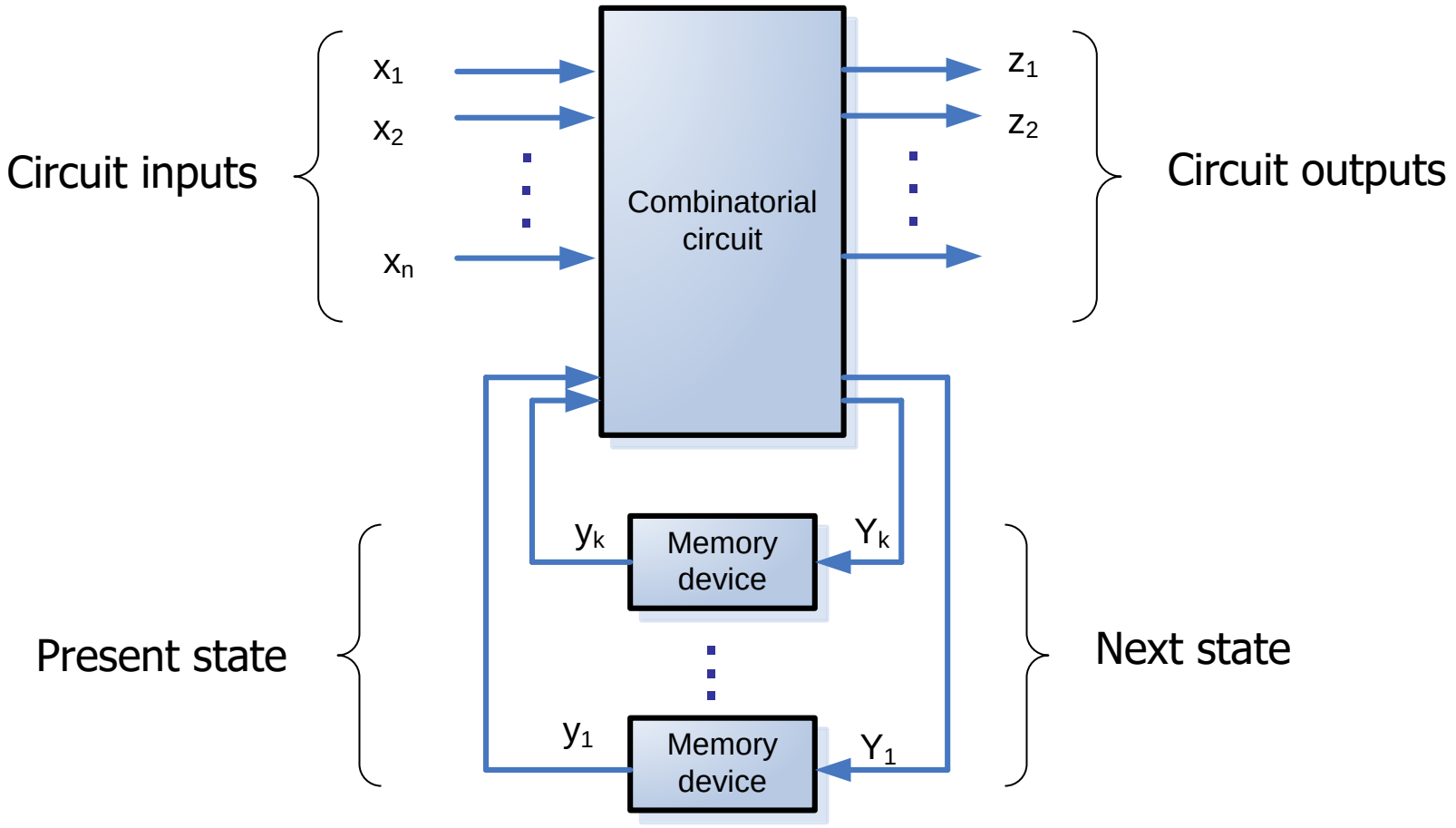
- The 32-bits ALU with 2 functions: and, add



Sequential circuit

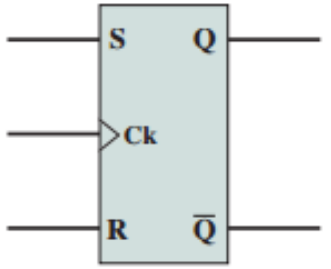
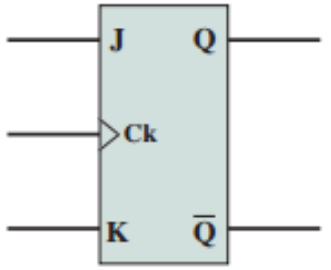
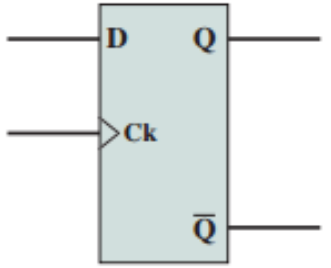
- A sequential circuit is an interconnected set of gates and the clock pulse
- The current output of a sequential circuit depends not only on the current input, but also on the history of inputs
- Sequential circuit is capable of "remembering past states"

Sequential circuit



Sequential circuit

- Basic sequential circuit is flip-flops
- Some types of flip-fops circuit is shown in the next table

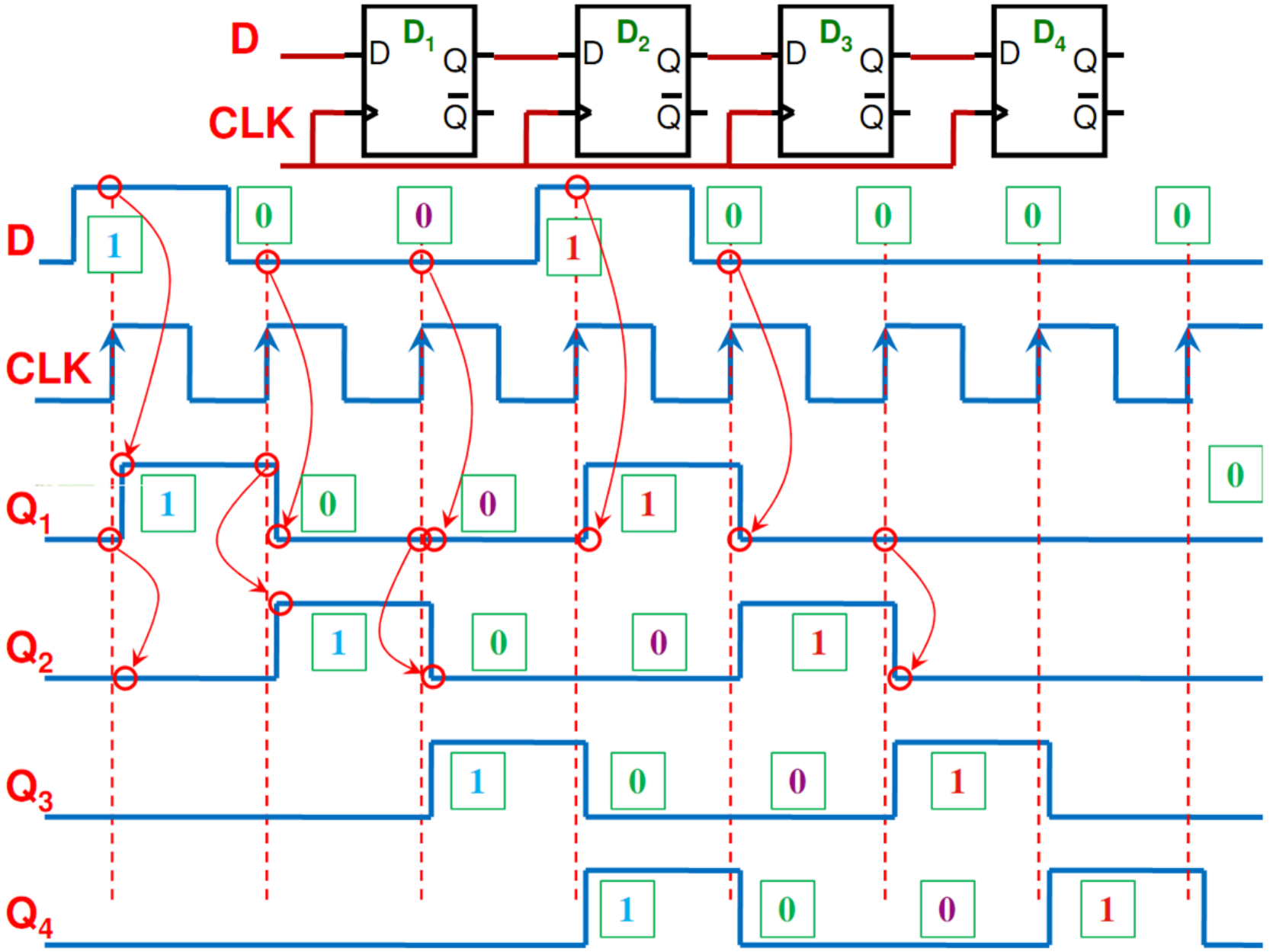
Name	Graphical Symbol	Truth Table
S–R		S R Q_{n+1} 0 0 Q_n 0 1 0 1 0 1 1 1 –
J–K		J K Q_{n+1} 0 0 Q_n 0 1 0 1 0 1 1 1 $\overline{Q_n}$
D		D Q_{n+1} 0 0 1 1

Application of sequential circuit

- Register
- Counter
- ...

Register

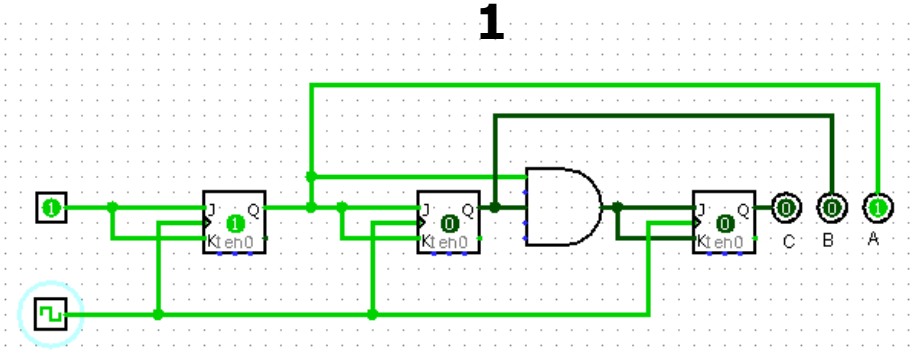
4-bit shift register



Counter

- A synchronous counter

1



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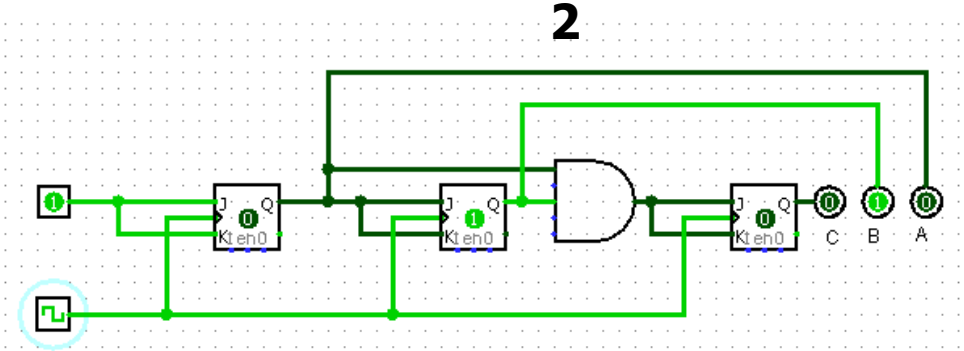

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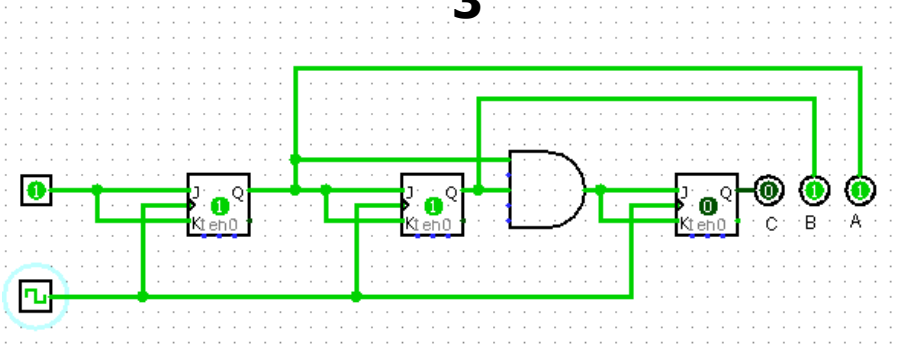

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