Differential Drive Kinematics

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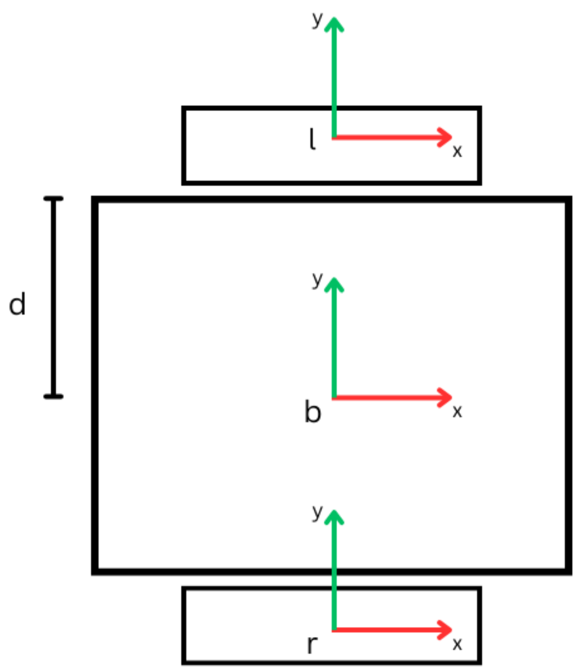

Figure 1: Differential Drive Robot

Define the transforms to the wheels in the body frame:

$$T_{bl}(0,0,d) \quad T_{br}(0,0,-d) \tag{1}$$

Define the adjoint matrices between the body and the wheels

$$A_{bl} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{br} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

Invert the adjoint matrices:

$$A_{lb} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{rb} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

Body twist in the wheel frames can be found using:

$$V_i = A_{ib} V_b \tag{4}$$

Define the body twist in the wheel frames:

$$V_l = A_{lb} V_b = \begin{bmatrix} \dot{\theta} \\ v_{xl} \\ v_{yl} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ v_x - d\dot{\theta} \\ v_y \end{bmatrix} \tag{5}$$

$$V_r = A_{rb} V_b = \begin{bmatrix} \dot{\theta} \\ v_{xr} \\ v_{yr} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ v_x + d\dot{\theta} \\ v_y \end{bmatrix} \tag{6}$$

Since the differential drive robot has conventional wheels, we can relate the wheel rotational velocity to the wheel velocity using:

$$\begin{bmatrix} v_{xi} \\ v_{yi} \end{bmatrix} = \begin{bmatrix} r\dot{\phi} \\ 0 \end{bmatrix} \tag{7}$$

Substitute equation (7) in equations (5) and (6) to obtain the Inverse Kinematics equations for the differential drive robot:

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_l \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ v_x - d\dot{\theta} \\ v_y \end{bmatrix} \Rightarrow \dot{\phi}_l = \frac{v_x - d\dot{\theta}}{r} \tag{8}$$

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_r \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ v_x + d\dot{\theta} \\ v_y \end{bmatrix} \Rightarrow \dot{\phi}_r = \frac{v_x + d\dot{\theta}}{r} \tag{9}$$

$$\begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -d & 1 & 0 \\ d & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} \tag{10}$$

The forward kinematics of the differential drive robot is defined as (From Equation 13.34 of the Modern Robotics Textbook [1]):

$$V_b = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = r \begin{bmatrix} -1/(2d) & 1/(2d) \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix} \tag{11}$$

References

[1] Kevin M. Lynch and Frank C. Park. *Modern Robotics: Mechanics, Planning, and Control*. Cambridge University Press, USA, 1st edition, 2017.