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## Franklin's 13 virtues and Puritan ethics

Temperance Silence  
Order Resolution  
Frugality Industry  
Sincerity Justice

**(9) Moderation : Avoid extremes; forbear resenting injuries so much as you think they deserve.**

中庸：为人处事不要极端偏激，对待他人的一时发泄，应加以宽容，不要斤斤计较、怀恨记仇。

“邓宁-克鲁格效应”：如果你很蠢，你就发现不了自己的蠢，因为发现自己的蠢需要相当高的智力。

**(10) Cleanliness: Tolerate no uncleanness in body, cloths, or habitation.**

清洁：注重仪表，身体、衣服和住所都应当力求清洁。



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# Compiling and Running of Program

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## § 5 Bottom-up Parsing

### 5.1 Overview of Bottom-up Parsing

### 5.2 Finite Automata for LR(0) Parsing

### 5.3 LR(0) Parsing

### 5.4 LR(1) Parsing

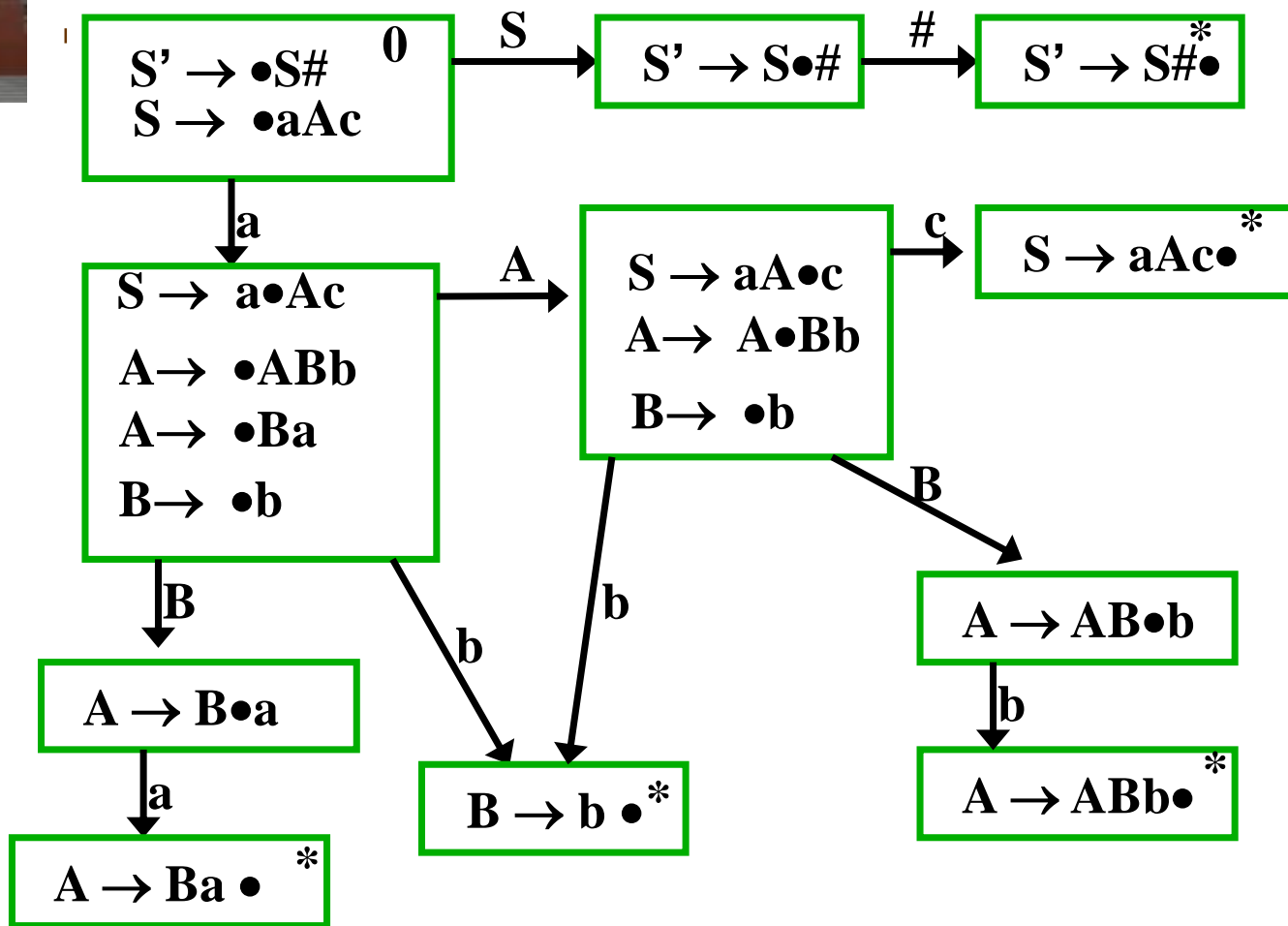
### 5.5 LALR(1) Parser Generator (YACC)



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# The process to construct LR(0) Automata

$V_T = \{a, b, c\}$   
 $V_N = \{S, A, B\}$   
 $S = S$   
 $P:$   
 $\{$   $S' \rightarrow S$   
 $S \rightarrow aAc$   
 $A \rightarrow ABb$   
 $A \rightarrow Ba$   
 $B \rightarrow b$   
 $\}$



可归约活前缀集合为  $\{S, aAc, aABb, aBa, aAb, ab\}$



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- **Conclusion:**
  - 一个CFG的LR(0)自动机接受的是该CFG的所有 可归约规范活前缀;



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## 5.3 LR(0) Parsing

- **LR(0) automata related concepts**
- **LR(0) grammar**
- **LR(0) Parsing Method**
  - LR(0) Parsing Table
  - LR(0) Parsing Engine(驱动程序)
- **LR(0) Parsing Process**



## LR(0) automata related concepts

- 移入项目:  $A \rightarrow \alpha \bullet a \beta, a \in V_T$
- 归约项目:  $A \rightarrow \alpha \bullet,$
- 接受项目:  $S' \rightarrow S \bullet \#, (S' \rightarrow S \# \text{是增广产生式})$
- 待约项目:  $A \rightarrow \alpha \bullet B \beta, B \in V_N$
- 移入状态: 包含移入项目的状态 (项目集)
- 归约状态: 包含归约项目的状态 (项目集)
- 冲突状态:
  - reduce – reduce conflict, 不同的归约项目
  - Shift – reduce conflict, 同时移入项目和归约项目



## LR(0) grammar

- Given a CFG  $G$
- $LR_0$  is LR(0) automata for  $G$
- If there is no conflict in any states of  $LR_0$ ,  $G$  is called LR(0) grammar;
  - 任意状态或者是移入状态, 或者是归约状态
  - 如果是归约状态, 一定存在一个唯一的归约项目, 该归约项目对应一个产生式 $p$ , 因此, 该归约状态称为 $p$ -归约状态



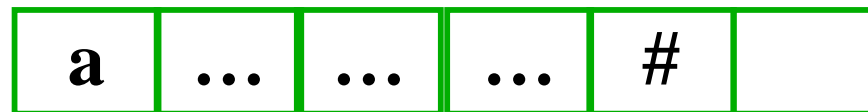


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规范活  
前缀

# LR(0) Parsing Method

LR分析表



Input

LR驱动程序LR(0) 驱动程序

- shift (移入) : 移入型规范活前缀
- reduce (归约) : 可归约规范活前缀

状态栈

action表

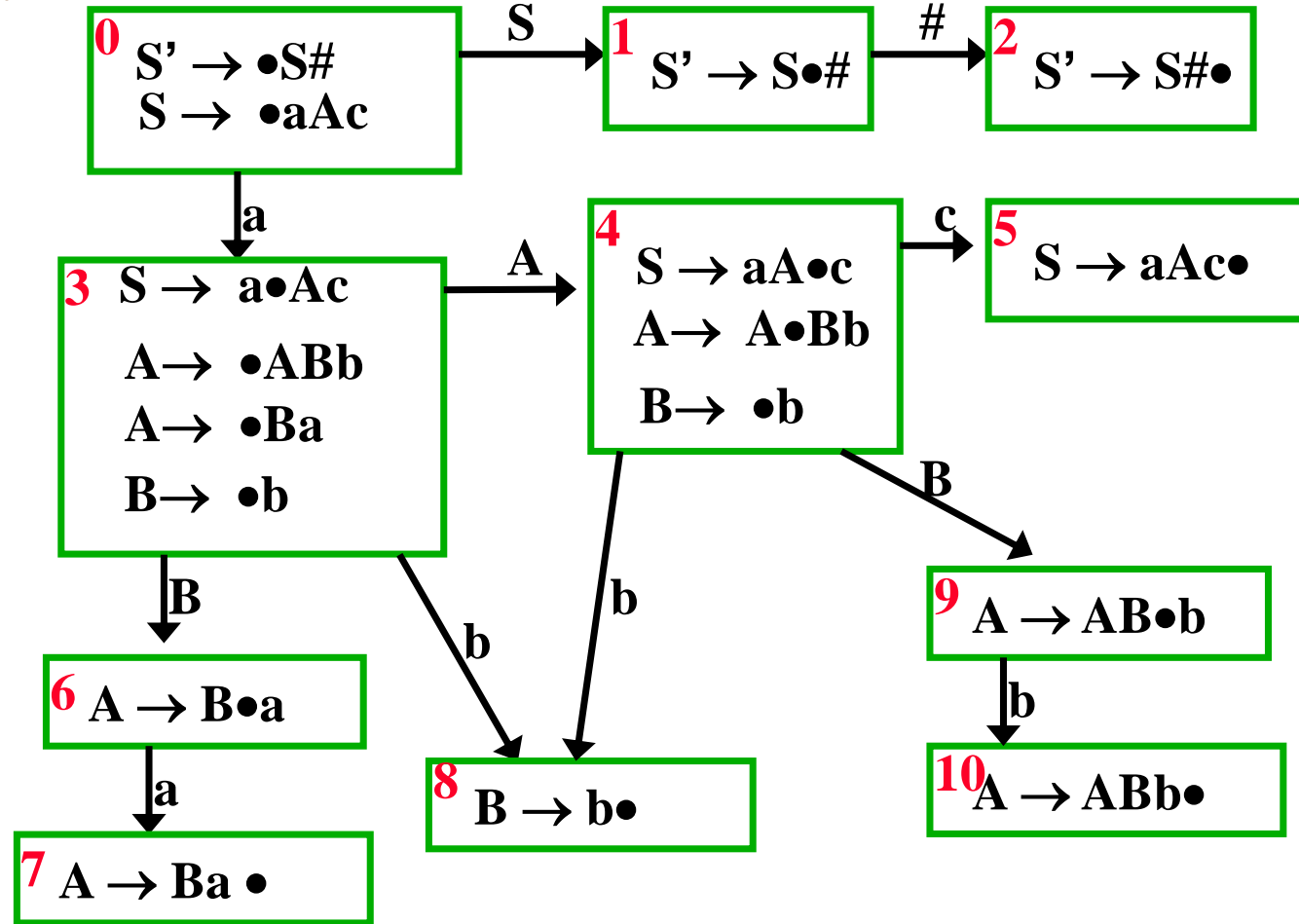
goto表




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# LR(0) Grammar

$V_T = \{a, b, c\}$   
 $V_N = \{S, A, B\}$   
 $S = S$   
**P:**  
 { (1)  $S \rightarrow aAc$   
 (2)  $A \rightarrow ABb$   
 (3)  $A \rightarrow Ba$   
 (4)  $B \rightarrow b$   
 }

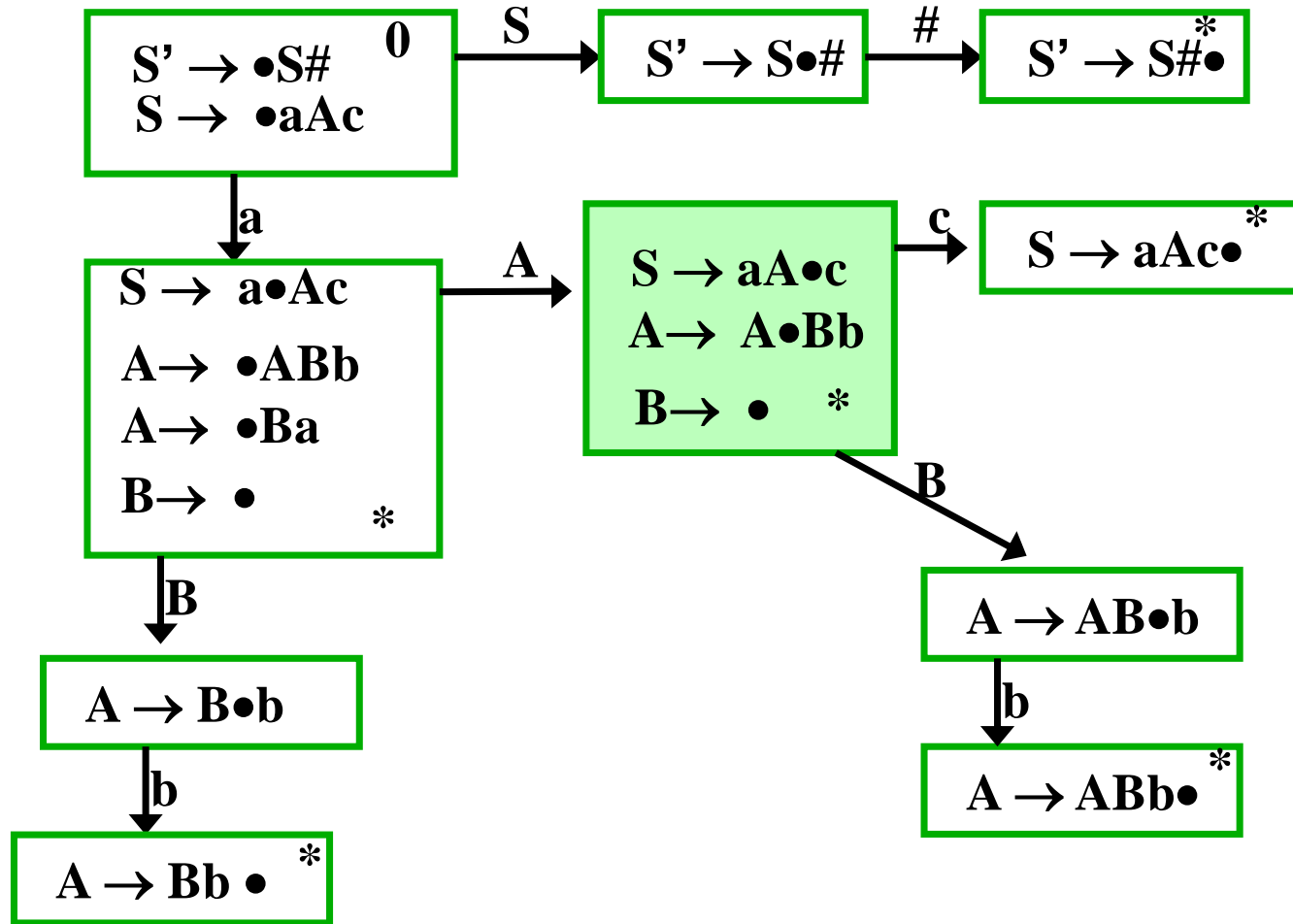




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$V_T = \{a, b, c\}$   
 $V_N = \{S, A, B\}$   
 $S = S$   
**P:**  
 {  $S \rightarrow aAc$   
 $A \rightarrow ABb$   
 $A \rightarrow Ba$   
 $B \rightarrow \varepsilon$   
 }

## ≠LR(0) Grammar





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# LR(0) Parsing Table

- action表
- goto表



# LR(0) Parsing Table

- **action表**

状 态 \ 终极符	$a_1$	...	#
$S_1$	<b>action(<math>S_i, a</math>) = <math>S_j</math>, 如果<math>S_i</math>到<math>S_j</math>有<math>a</math>输出边</b> <b>action(<math>S_i, a</math>) = <math>R_p</math>, 如果<math>S_i</math>是<math>p</math>-归约状态</b> <b>action(<math>S_i, \#</math>) = accept, 如果<math>S_i</math>是接受状态</b> <b>action(<math>S_i, a</math>) = error, 其他情形</b>		
...			
$S_n$			



# LR(0) Parsing Table

- goto表

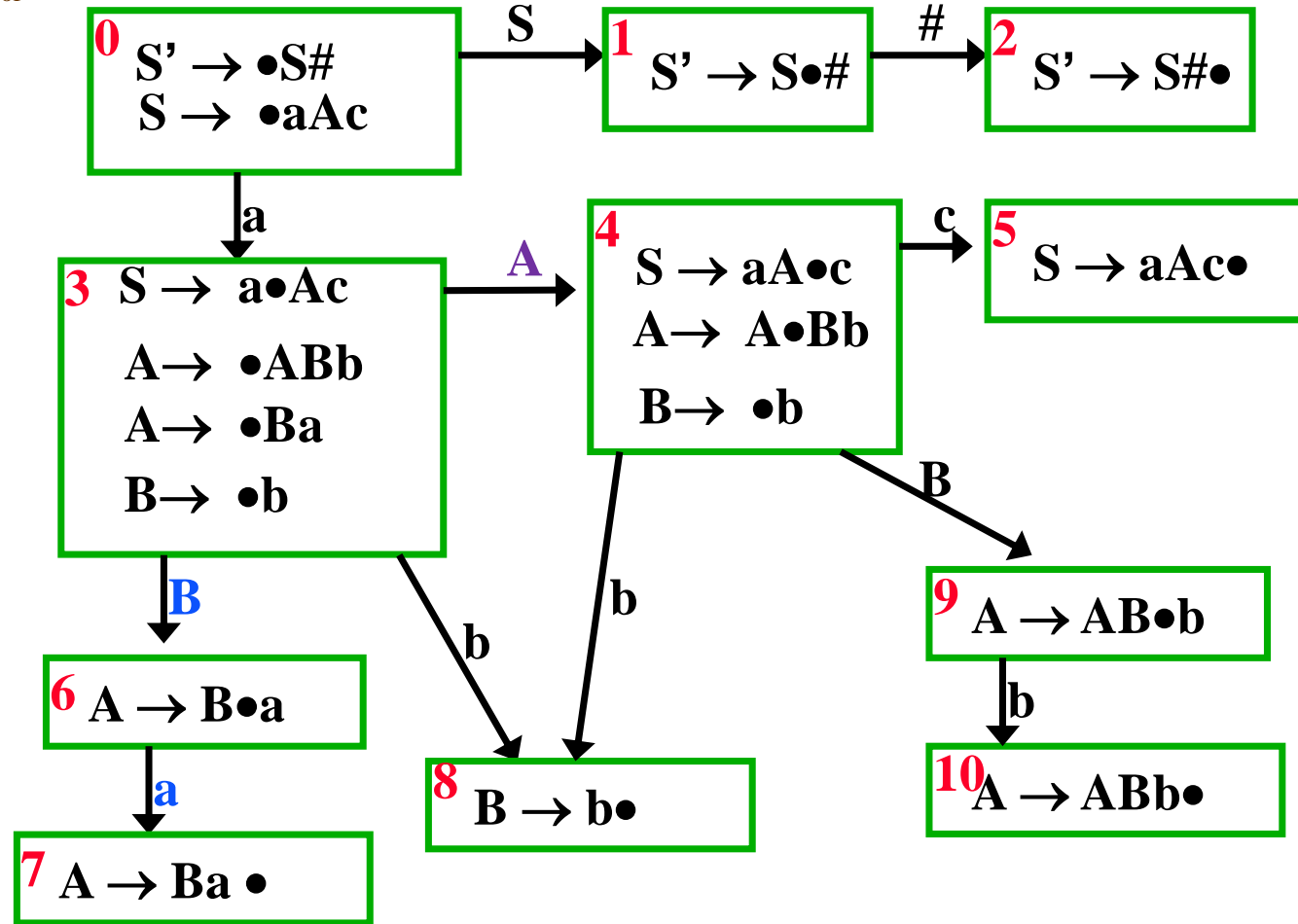
非终极符 状 态	$A_1$	...	
$S_1$	<div> <p><math>\text{goto}(S_i, A) = S_j</math>, 如果<math>S_i</math>到<math>S_j</math>有<math>A</math>输出边</p> <p><math>\text{goto}(S_i, A) = \text{error}</math>, 如果<math>S_i</math>没有<math>A</math>输出边</p> </div>		
...			
$S_n$			



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## Example(Parsing Table)

$V_T = \{a, b, c\}$   
 $V_N = \{S, A, B\}$   
 $S = S$   
**P:**  
 { (1)  $S \rightarrow aAc$   
 (2)  $A \rightarrow ABb$   
 (3)  $A \rightarrow Ba$   
 (4)  $B \rightarrow b$   
 }



# 分析表

	a	b	c	#
0	S3			
1				accept
2				
3		S8		
4		S8	S5	
5	R1	R1	R1	R1
6	S7			
7	R3	R3	R3	R3
8	R4	R4	R4	R4
9		S10		
10	R2	R2	R2	R2

	S	A	B
0	1		
1			
2			
3		4	6
4			9
5			
6			
7			
8			
9			
10			

- (0)  $S' \rightarrow S$
- (1)  $S \rightarrow aAc$
- (2)  $A \rightarrow ABb$
- (3)  $A \rightarrow Ba$
- (4)  $B \rightarrow b$

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# LR(0) Parsing Engine

- **Notations**

- **S0: start state**
- **Stack:状态栈**
- **Stack(top):栈顶元素**
- **P:产生式**
- **| P |:产生式P右部符号个数;**
- **P<sub>A</sub>:产生式P左部非终极符;**
- **Push(S):把状态S压入stack;**
- **Pop(n):从stack弹出n个栈顶元素;**

## ■ 计算思维的典型方法

- 知识与控制的分离
- 自动化

## LR(0) Parsing Engine

- 初始化: `push(S0); a = readOne();`
- L: Switch `action(stack(top), a)`
  - Case error: `error();`
  - Case accept: `return true;`
  - Case  $S_i$ : `push(Si), a=readOne(); goto L;`
  - Case  $R_P$ : `pop(|P|);`  
`push(goto(stack(top),  $P_A$  ));`  
`goto L;`



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**P:**

(0)  $S' \rightarrow S$

(1)  $S \rightarrow aAc$

(2)  $A \rightarrow ABb$

(3)  $A \rightarrow Ba$

(4)  $B \rightarrow b$

a

b

a

c

# LR(0) Parsing Process

状态栈	输入流	分析动作
0	abac#	S3
03	bac#	S8
038	ac#	R4, Goto(3, B)=6
036	ac#	S7
0367	c#	R3, Goto(3, A)=4
034	c#	S5
0345	#	R1, Goto(0, S)=1
01	#	Accept



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# Assignment

$V_T = \{a, b, c, d\}$

$V_N = \{Z, A, B\}$

$S = Z$

P:

$\{Z \rightarrow ABd$

$A \rightarrow a$

$B \rightarrow d$

$B \rightarrow c$

$B \rightarrow bB$

$\}$

(1) 构造LR(0) 分析表

(2) 给出abcd#的分析过程;



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## § 5 Bottom-up Parsing

### 5.1 Overview of Bottom-up Parsing

### 5.2 Finite Automata for LR(0) Parsing

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### 5.4 LR(1) Parsing

### 5.5 LALR(1) Parser Generator (YACC)



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$$V_T = \{a, b, =\}$$

$$V_N = \{S, L, R\}$$

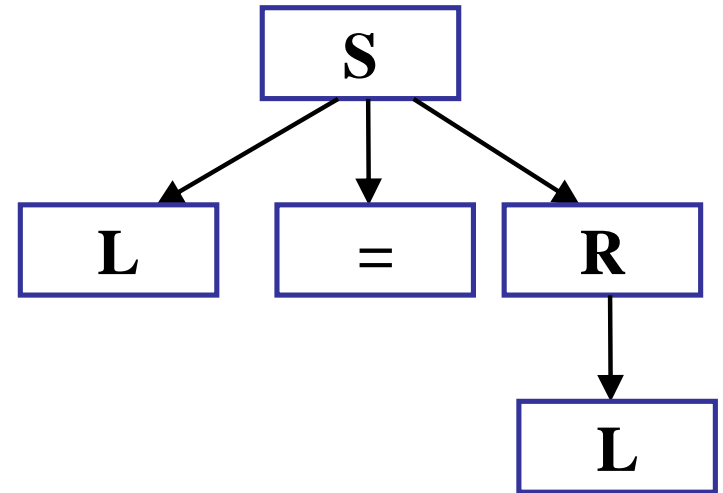
$$S = S$$

**P:**

{(1)  $S \rightarrow L = R$   
 (2)  $S \rightarrow R$   
 (3)  $L \rightarrow aR$   
 (4)  $L \rightarrow b$   
 (5)  $R \rightarrow L$   
 }

- For the same non-terminal symbol, if it is in different position, its follow set might be different;

3	
$S \rightarrow L \bullet = R$	
$R \rightarrow L \bullet$	





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# One Solution

- **LR(1) Parsing**

- **Basic idea:**

- 对于非终极符的每个不同出现求其 **后继终极符**, 称为 **展望符**;
    - 对于一个非终极符的一个出现的所有后继终极符构成的集合称为 **展望符集**;

- **Steps**

- **Construct LR(1) automata**
      - LR(1) item
    - **Generate LR(1) parsing table (action & goto)**



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## 5.4 LR(1) Parsing

- **Limitations of LR(0) Parsing**
- **LR(1) Automata**
- **LR(1) Parsing Table**
- **LR(1) Grammar**
- **LR(1) Parsing Process**





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# LR(1) Automata

- **LR(1) item**
  - **Two parts** :  $(A \rightarrow \alpha \bullet \beta, \{a, \dots\})$ 
    - (1) LR(0) item:  $A \rightarrow \alpha \bullet \beta$
    - (2) 展望符集:  $\{a, \dots\}$
  - **Example**
    - $S \rightarrow L \bullet = R, \{\#\}$
    - $A \rightarrow \alpha \bullet, \{a, b\}$
  - **展望符集的作用**:
    - 对于移入型项目, 不起作用, 但是需要保存;
    - 对于归约型项目, 表示只有当下一个输入符是其中一个展望符时, 才可以进行归约动作;



# LR(1) Automata

- **LR(1) 项目集合 关于符号 $X$ 的投影**
  - **IS** is a set of LR(1) items;
  - **X** is a symbol;
  - **IS<sub>(X)</sub>** represents the projection of IS with respect to X:
  - $\text{IS}_{(X)} = \{(S \rightarrow \alpha X \bullet \beta, ss) \mid (S \rightarrow \alpha \bullet X \beta, ss) \in \text{IS},$   
 $X \in V_T \cup V_N\}$
- **IS =  $\{(A \rightarrow A \bullet B b, \{a, b\}), (B \rightarrow a \bullet, \#), (B \rightarrow b \bullet B, \{b\})\}$**
- **X = B**
- **IS<sub>(B)</sub> =  $\{(A \rightarrow AB \bullet b, \{a, b\}), (B \rightarrow b B \bullet, \{b\})\}$**



# LR(1) Automata

- LR(1)项目集合的**闭包**

- IS is a set of LR(1) items;
- CLOSURE(IS)是一个LR(1)项目集合, 按照下面的步骤计算:
  - [1] 初始,  $\text{CLOSURE}(\text{IS}) = \text{IS}$ ;
  - [2] 对于CLOSURE(IS)没有处理的LR(1)项目,  
如果其形式为  $(B \rightarrow \beta \bullet A \pi, ss)$ ,  
而且A的全部产生式是  $\{A \rightarrow \alpha_1, \dots, A \rightarrow \alpha_n\}$   
则增加如下LR(1)项目到CLOSURE(IS)  
 $\{(A \rightarrow \bullet \alpha_1, ss'), \dots, (A \rightarrow \bullet \alpha_n, ss')\}$ ,  
其中  $ss' = \text{first}(\pi)$ , 如果符号串 $\pi$ 不导出空;  
 $ss' = (\text{first}(\pi) - \{\epsilon\}) \cup ss$ , 如果符号串 $\pi$ 导出空;
  - [3] 重复[2]直到 CLOSURE(IS)收敛;



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# LR(1)自动机

- goto函数
  - IS is a set of LR(1) items;
  - X is a symbol;
  - $\text{goto}(\text{IS}, X) = \text{CLOSURE}(\text{IS}_{(X)})$

$V_T = \{a, b, =\}$

$V_N = \{S, L, R\}$

$S = S$

P:

{(1)  $S \rightarrow L = R$

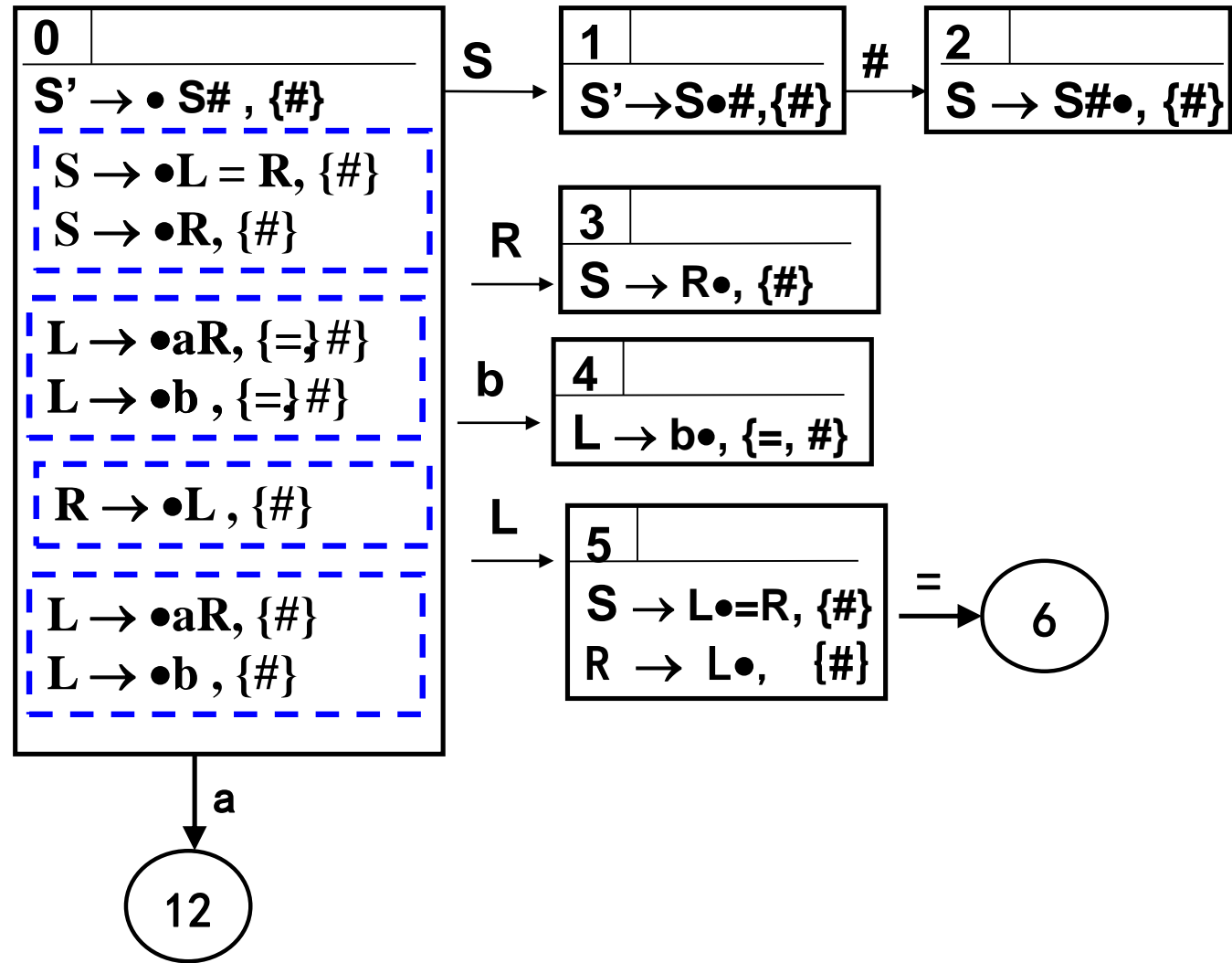
(2)  $S \rightarrow R$

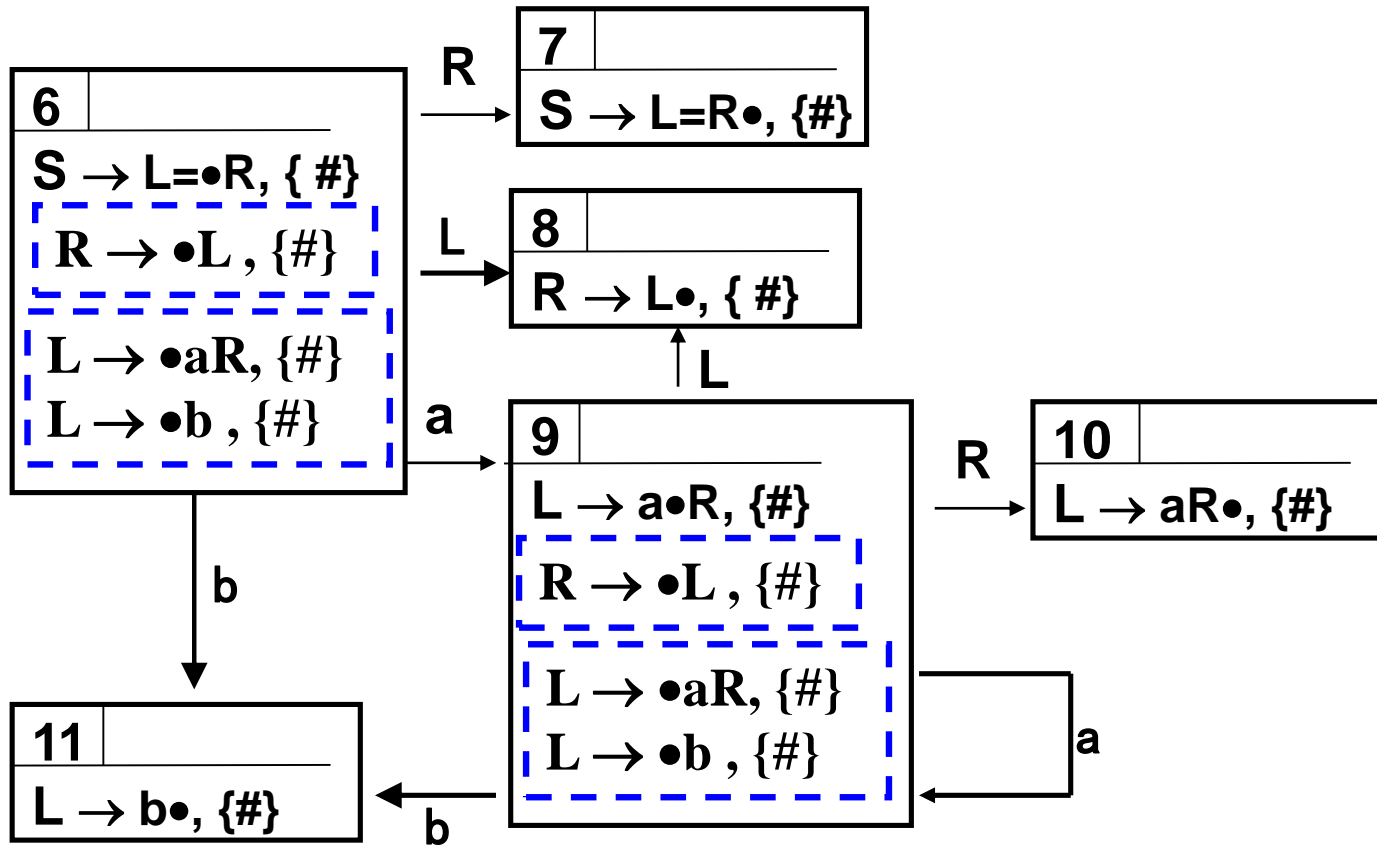
(3)  $L \rightarrow aR$

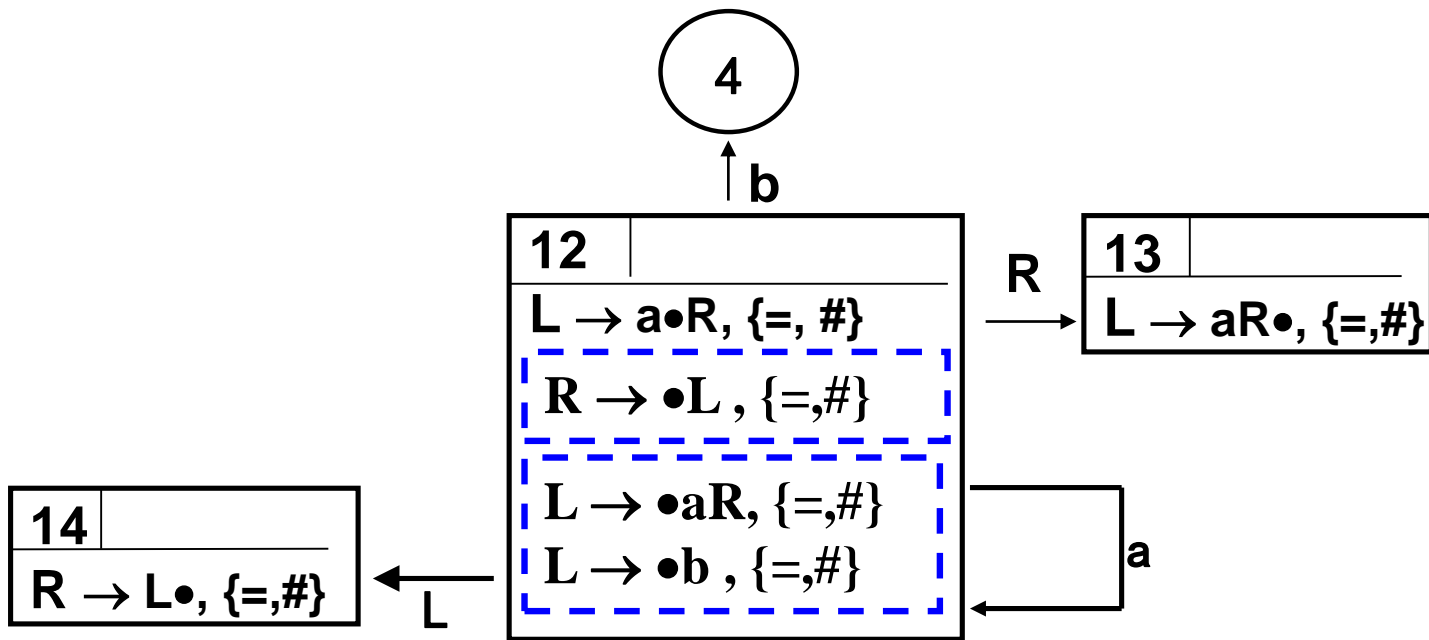
(4)  $L \rightarrow b$

(5)  $R \rightarrow L$

}





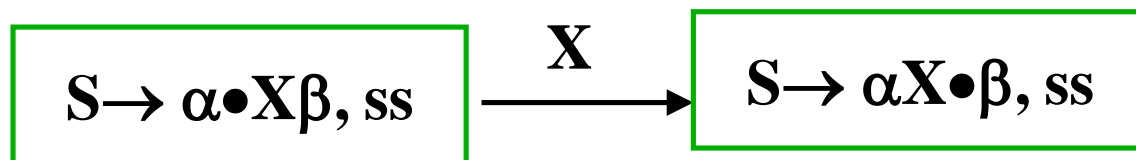




## 如何计算展望符集?

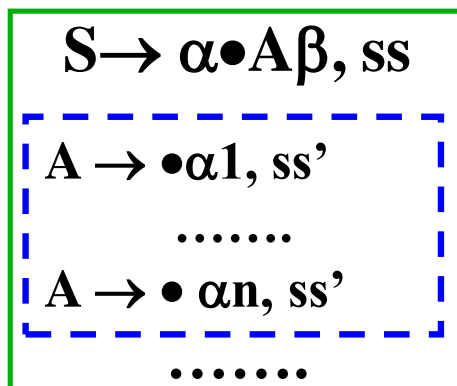
- 投影得到的项目

- 继承



- 闭包新产生的项目

- 扩展



$ss' = \text{first}(\beta)$ , 如果 $\beta$ 不导出空;

$ss' = (\text{first}(\beta) - \{\epsilon\}) \cup ss$ , 如果 $\beta$ 导出空;





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## Process of Constructing LR(1) Automata

- [1] 增广产生式  $S' \rightarrow S\#$
- [2]  $\Sigma = V_T \cup V_N \cup \{\#\}$
- [3]  $S_0 = \text{CLOSURE}(S' \rightarrow \bullet S)$
- [4]  $ISS = \{S_0\}$
- [5] 对于ISS中的每一个项目集合IS, 和每个符号  $X \in \Sigma$ ,  
计算  $IS' = \text{goto}(IS, X)$ ,  
如果  $IS'$  不为空, 则 建立  $IS \xrightarrow{X} IS'$ ,  
如果  $IS'$  不为空且  $IS'$  不属于ISS, 则把  $IS'$  加入ISS;
- [6] 重复[5]直到ISS收敛;



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# LR(1) Parsing Table

- action表
- goto表



# LR(1) Parsing Table

## • action表

终极符 状态	$a_1$	...	#
$S_1$	<div> <p>(1) <math>\text{action}(S_i, a) = S_j</math>, 如果<math>S_i</math>到<math>S_j</math>有<math>a</math>输出边</p> <p>(2) <math>\text{action}(S_i, \mathbf{a}) = R_p</math>, 如果<math>S_i</math>中包含这样LR(1)项目,  <math>(A \rightarrow \alpha \bullet, \mathbf{ss})</math>, 其中<math>A \rightarrow \alpha</math>是产生式<math>P</math>, 且<math>\mathbf{a} \in \mathbf{ss}</math>;</p> <p>(3) <math>\text{action}(S_i, \#) = \text{accept}</math>, 如果<math>S_i</math>是接受状态</p> <p>(4) <math>\text{action}(S_i, a) = \text{error}</math>, 其他情形</p> </div>		
...			
$S_n$			



# LR(1) Parsing Table

- goto表

非终极符 状 态	$A_1$	...	$A_n$
$S_1$	<div> <p>goto (<math>S_i, A</math>) = <math>S_j</math>, 如果<math>S_i</math>到<math>S_j</math>有<math>A</math>输出边</p> <p>goto (<math>S_i, A</math>) = error, 如果<math>S_i</math>没有<math>A</math>输出边</p> </div>		
...			
$S_n$			



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- (1)  $S \rightarrow L = R$
- (2)  $S \rightarrow R$
- (3)  $L \rightarrow aR$
- (4)  $L \rightarrow b$
- (5)  $R \rightarrow L$

# LR(1) Parsing Table

	Action table				Goto table		
	a	b	=	#	S	L	R
<b>0</b>	<b>S12</b>	<b>S4</b>			<b>1</b>	<b>5</b>	<b>3</b>
<b>1</b>				<b>Accept</b>			
<b>2</b>							
<b>3</b>				<b>R2</b>			
<b>4</b>			<b>R4</b>	<b>R4</b>			
<b>5</b>			<b>S6</b>	<b>R5</b>			
<b>6</b>	<b>S9</b>	<b>S11</b>				<b>8</b>	<b>7</b>
<b>7</b>				<b>R1</b>			



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- (1)  $S \rightarrow L = R$
- (2)  $S \rightarrow R$
- (3)  $L \rightarrow aR$
- (4)  $L \rightarrow b$
- (5)  $R \rightarrow L$

## LR(1) Parsing Table (cond.)

	Action table					Goto table		
	a	b	=	#		S	L	R
8				R5				
9	S9	S11						10
10				R3				
11				R4				
12	S12	S4					14	13
13			R3	R3				
14			R4	R4				
15								



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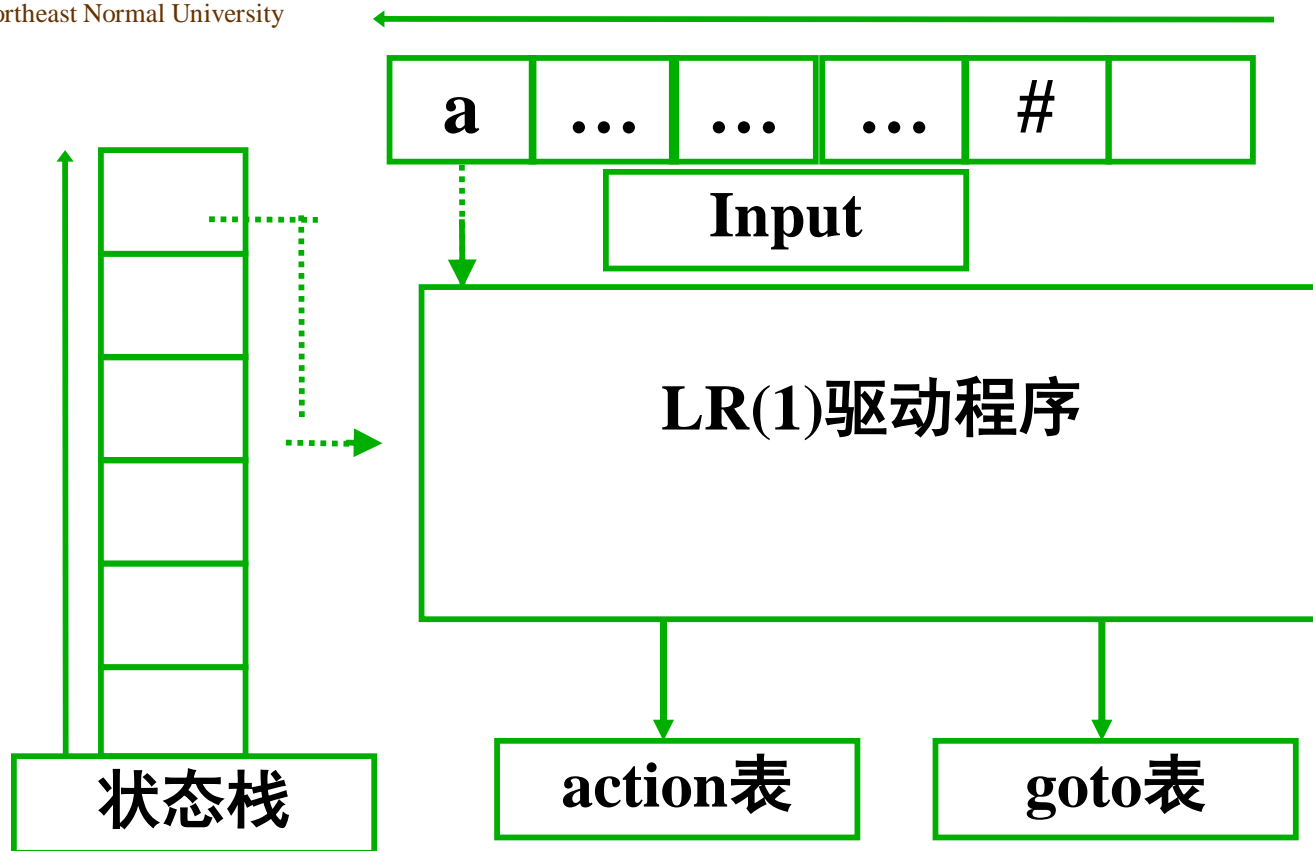
# LR(1) Grammar

- Given a CFG  $G$
- $LR_1$  is LR(1) automata for  $G$
- $A_1$  is *action table* for  $G$
- If there is only one action for one state and one terminal symbol in  $A_1$ , then  $G$  is called LR(1) Grammar;
  - Shift
  - Reduce
  - Accept
  - Error



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# LR(1) Parsing Method





## ■ 计算思维的典型方法

- 知识与控制的分离
- 自动化

## LR(1) Parsing Engine

- 初始化: `push(S0); a = readOne();`
- L: Switch `action(stack(top), a)`
  - Case error: `error();`
  - Case accept: **`return true;`**
  - Case Si: **`push(Si), a=readOne(); goto L;`**
  - Case R<sub>P</sub>: **`pop(|P|);`**  
**`push(goto(stack(top), PA ));`**  
**`goto L;`**

How to generate  
parse tree during  
LR parsing?



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# The Process of LR(1) Parsing

$V_T = \{a, b, =\}$

$V_N = \{S, L, R\}$

$S = S$

P:

{(1)  $S \rightarrow L = R$

(2)  $S \rightarrow R$

(3)  $L \rightarrow aR$

(4)  $L \rightarrow b$

(5)  $R \rightarrow L$

}

状态栈	输入流	分析动作
0	b=b#	S4
04	=b#	R4, Goto(0,L)=5
05	=b#	S6
056	b#	S11
056(11)	#	R4, Goto(6, L)=8
0568	#	R5 , Goto(6, R)=7
0567	#	R1, , Goto(0, S)=1
01	#	Accept



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## 5.5 LALR(1) Parser Generator (YACC)

- **Widely used for automatically generating parser for a programming language;**
- **It is difficult to solve conflicts if the grammar is not LALR(1) grammar;**
- **Please find out the general process of using YACC to generate a parser?**



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## 5.5 LALR(1) Parser Generator (YACC)

- YACC——Yet Another Compiler Compiler
  - Stephen C. Johnson. YACC: Yet Another Compiler-Compiler. *Unix Programmer's Manual*/Vol 2b, 1979.
  - LALR(1) 分析
  - GNU Bison: 基本兼容Yacc, 与flex一起使用
  - Berkeley Yacc
- The Lex & Yacc Page
  - <http://dinosaur.compilertools.net/>



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# Assignment(1)

$$V_T = \{0, 1\}$$

$$V_N = \{S\}$$

$$S = S$$

**P:**

$$\{(1) S \rightarrow 0S1$$

$$(2) S \rightarrow 1S0$$

$$(3) S \rightarrow 10$$

**}**

**Please check whether  
this grammar is LR(1)  
Grammar?  
Please give the process  
of how you produce the  
result.**



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# Assignment(2)

(1)构造LR(1)自动机;  
(2)构造LR(1)分析表;

$V_T = \{ \{, \}, :, id, +, (, ), =, [, ] \}$

$V_N = \{ B, SL, S, V, E, T \}$

$S = B$

P:

{(1)  $B \rightarrow \{ SL \}$       (2)  $SL \rightarrow SL; S$       (3)  $SL \rightarrow S$   
(4)  $S \rightarrow B$               (5)  $S \rightarrow V = E$       (6)  $V \rightarrow id$   
(7)  $V \rightarrow id[E]$           (8)  $E \rightarrow E + T$       (9)  $E \rightarrow T$   
(10)  $T \rightarrow V$               (11)  $T \rightarrow (E)$   
}



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# Answer to Assignments

*Compiling and Running of Program*



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# Assignment(1)

$$V_T = \{0, 1\}$$

$$V_N = \{S\}$$

$$S = S$$

**P:**

$$\{(1) S \rightarrow 0S1$$

$$(2) S \rightarrow 1S0$$

$$(3) S \rightarrow 10$$

**}**

**Please check whether  
this grammar is LR(1)  
Grammar?  
Please give the process  
of how you produce the  
result.**



$V_T = \{0, 1\}$

$V_N = \{S\}$

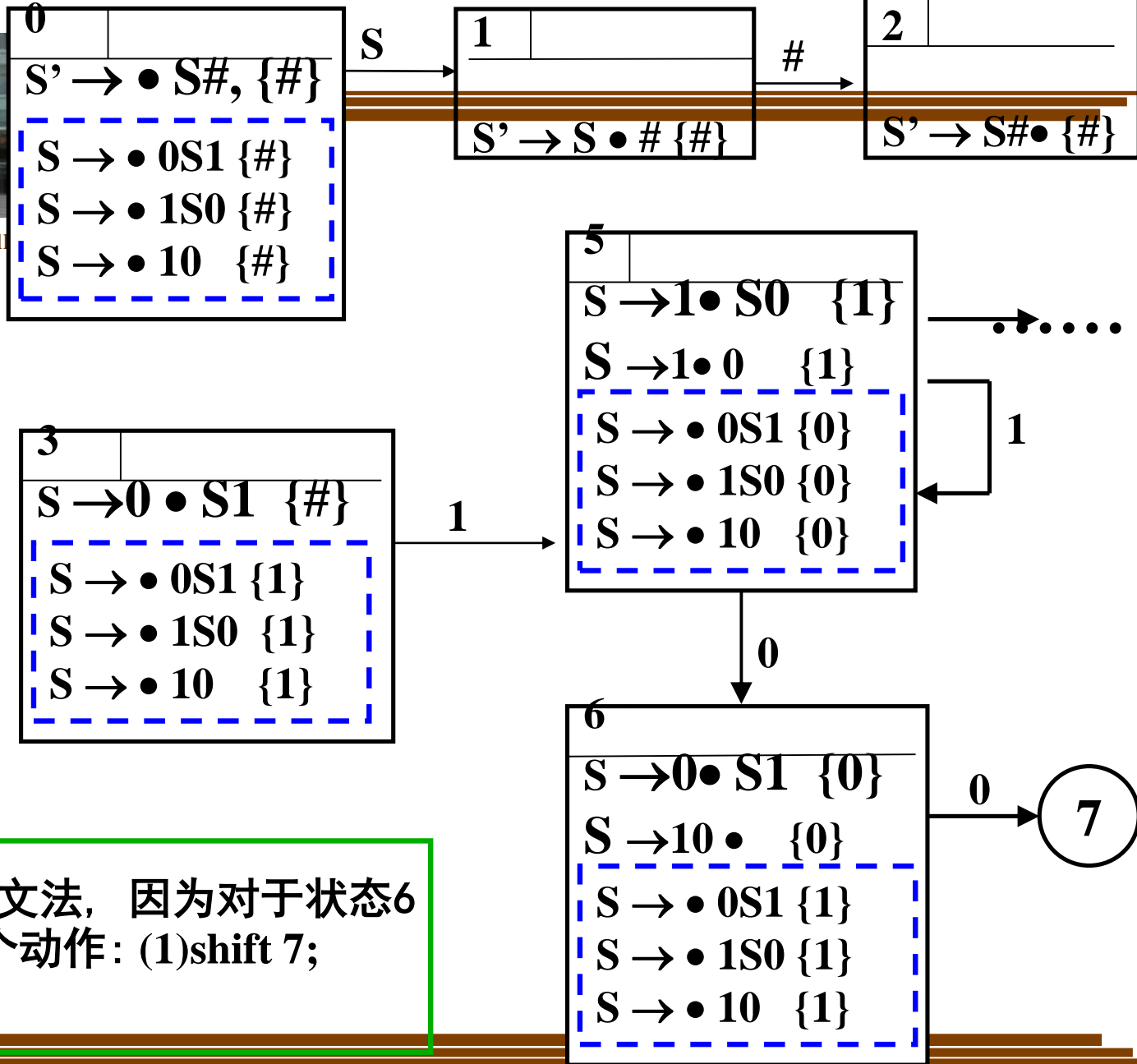
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$S = S$

P:

{(1)  $S \rightarrow 0S1$   
(2)  $S \rightarrow 1S0$   
(3)  $S \rightarrow 10$   
}

该文法不是LR(1)文法, 因为对于状态6和终极符0, 有两个动作: (1)shift 7;  
(2) Reduce 3;





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# Assignment(2)

- (1) 构造LR(1) 自动机;
- (2) 构造LR(1) 分析表;

$V_T = \{ \{, \}, :, id, +, (, ), =, [, ] \}$

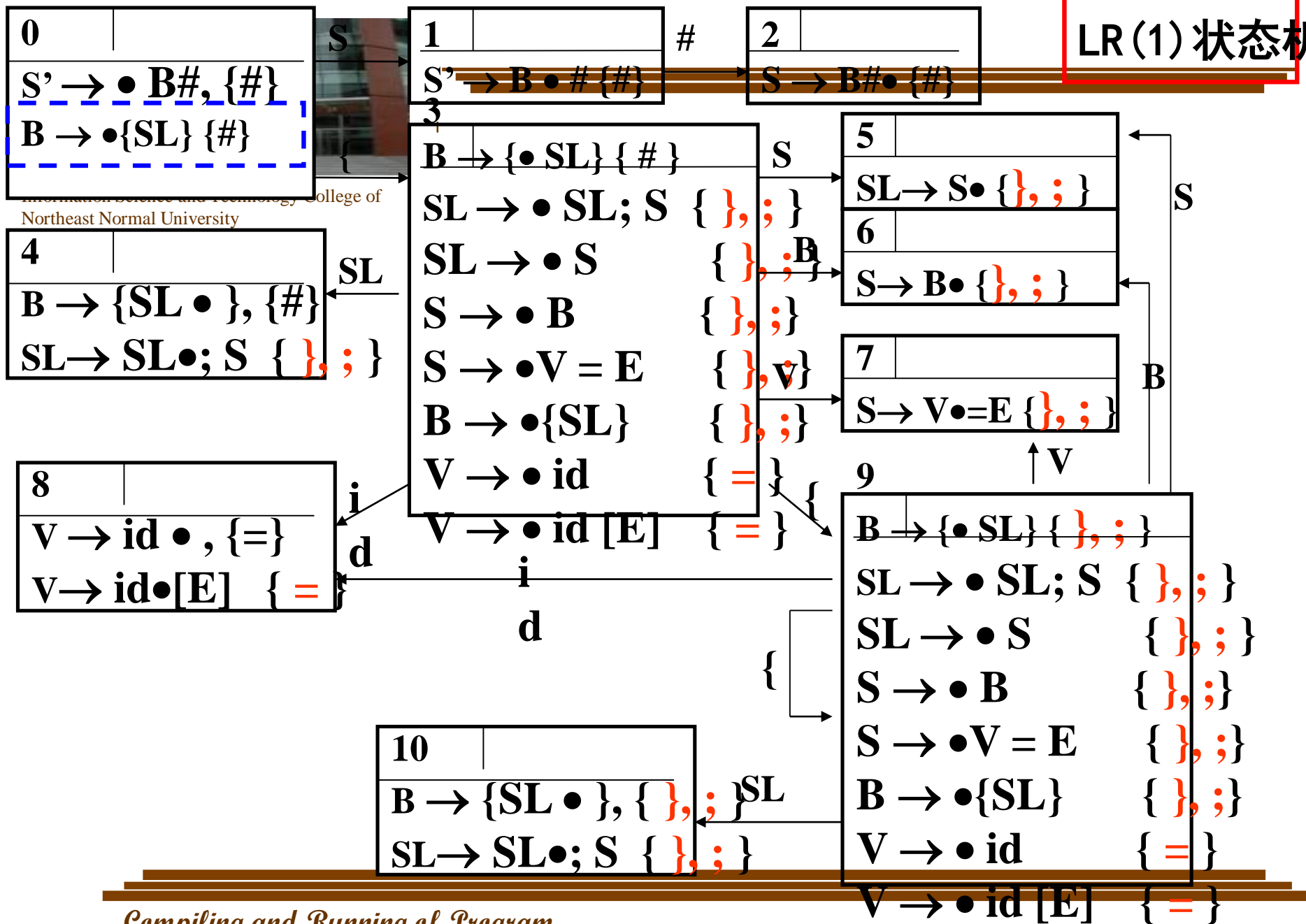
$V_N = \{ B, SL, S, V, E, T \}$

$S = B$

P:

- |                                |                            |                        |
|--------------------------------|----------------------------|------------------------|
| $\{(1) B \rightarrow \{ SL \}$ | $(2) SL \rightarrow SL; S$ | $(3) SL \rightarrow S$ |
| $(4) S \rightarrow B$          | $(5) S \rightarrow V = E$  | $(6) V \rightarrow id$ |
| $(7) V \rightarrow id[E]$      | $(8) E \rightarrow E + T$  | $(9) E \rightarrow T$  |
| $(10) T \rightarrow V$         | $(11) T \rightarrow (E)$   |                        |
| }                              |                            |                        |

# LR(1) 状态机



8	
$V \rightarrow id \bullet, \{=\}$	
$V \rightarrow id \bullet [E] \quad \{=\}$	

11	
$V \rightarrow id [\bullet E] \quad \{=\} E \rightarrow$	
$E \rightarrow \bullet E + T \quad \{[,+\}$	
$E \rightarrow \bullet T \quad \{[,+\}$	
$T \rightarrow \bullet V \quad \{[,+\}$	
$T \rightarrow \bullet (E) \quad \{[,+\}$	
$V \rightarrow \bullet id \quad \{[,+\}$	
$V \rightarrow \bullet id [E] \quad \{[,+\}$	

12	
$V \rightarrow id \bullet, \{[,+\}$	
$V \rightarrow id \bullet [E] \quad \{[,+\}$	

13	
$V \rightarrow id [E \bullet] \quad \{=\}$	

14	
$T \rightarrow T \bullet \{[,+\}$	

15	
$T \rightarrow V \bullet \{[,+\}$	

15	
$T \rightarrow (\bullet E) \quad \{[,=\}$	
$E \rightarrow \bullet E + T \quad \{),+\}$	
$E \rightarrow \bullet T \quad \{),+\}$	
$T \rightarrow \bullet V \quad \{),+\}$	
$T \rightarrow \bullet (E) \quad \{),+\}$	
$V \rightarrow \bullet id \quad \{),+\}$	
$V \rightarrow \bullet id [E] \quad \{),+\}$	



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7

$S \rightarrow V \bullet = E \{ \}, ; \}$

=

16

$S \rightarrow V = \bullet E \{ \}, ; \}$

$E \rightarrow \bullet E + T \{ \}, ;, + \}$

$E \rightarrow \bullet T \{ \}, ;, + \}$

$T \rightarrow \bullet V \{ \}, ;, + \}$

$T \rightarrow \bullet (E) \{ \}, ;, + \}$

$V \rightarrow \bullet id \{ \}, ;, + \}$

$V \rightarrow \bullet id [E] \{ \}, ;, + \}$