

# OptForce Tutorial

**Author:** Sebastián N. Mendoza, Center for Mathematical Modeling, University of Chile.  
snmendoz@uc.cl

**Reviewers(s):** Chiam Yu Ng (Costas D. Maranas group), Lin Wang (Costas D. Maranas group)

## INTRODUCTION:

In this tutorial we will run optForce. For a detailed description of the procedure, please see [1]. Briefly, the problem is to find a set of interventions of size "K" such that when these interventions are applied to a wild-type strain, the mutant created will produce a particular target of interest in a higher rate than the wild-type strain. The interventions could be knockouts (lead to zero the flux for a particular reaction), upregulations (increase the flux for a particular reaction) and downregulations (decrease the flux for a particular reaction).

For example, imagine that we would like to increase the production of succinate in Escherichia coli. Which are the interventions needed to increase the production of succinate? We will approach this problem in this tutorial and we will see how each of the steps of OptForce are solved.

## MATERIALS

### EQUIPMENT

1. MATLAB
2. A solver for Mixed Integer Linear Programming (MILP) problems. For example, Gurobi.

### EQUIPMENT SETUP

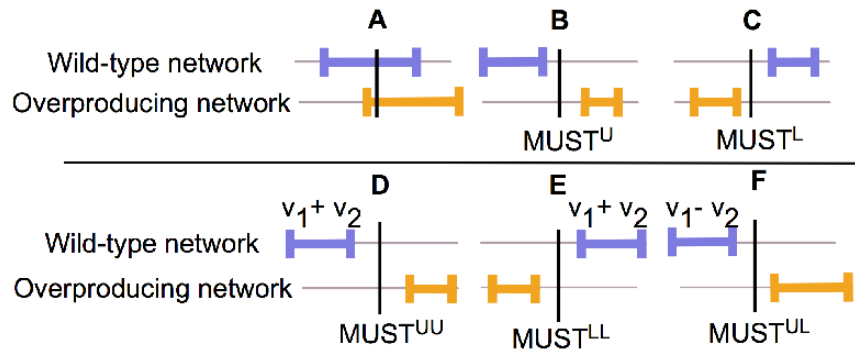
Use `changeCobraSolver` to choose the solver for MILP problems.

## PROCEDURE

The procedure consists on the following steps

- 1) Maximize specific growth rate and product formation.
- 2) Define constraints for both wild-type and mutant strain:
- 3) Perform flux variability analysis for both wild-type and mutant strain.
- 4) Find must sets, i.e, reactions that MUST increase or decrease their flux in order to achieve the phenotype in the mutant strain.

**Figure 1.**



5) Find the interventions needed that will ensure a increased production of the target of interest

Now, we will approach each step in detail.

## STEP 1: Maximize specific growth rate and product formation

First, we load the model. This model comprises only 90 reactions, which describe the central metabolism of *E. coli* [2].

Then, we change the objective function to maximize biomass ("R75"). We also change the lower bounds, so *E. coli* will be able to consume glucose, oxygen, sulfate, ammonium, citrate and glycerol.

```
changeCobraSolver('gurobi', 'ALL');
```

```
> Gurobi interface added to MATLAB path.
> Solver for LP problems has been set to gurobi.

> Gurobi interface added to MATLAB path.
> Solver for MILP problems has been set to gurobi.

> Gurobi interface added to MATLAB path.
> Solver for QP problems has been set to gurobi.

> Gurobi interface added to MATLAB path.
> Solver for MIQP problems has been set to gurobi.
> Solver gurobi not supported for problems of type NLP. Currently used: matlab
```

```
pathTutorial = which('tutorial_optForce.mlx');
pathstr = fileparts(pathTutorial);
cd(pathstr)

load('AntCore.mat');
model.c(strcmp(model.rxns, 'R75')) = 1;
model = changeRxnBounds(model, 'EX_gluc', -100, 'l');
model = changeRxnBounds(model, 'EX_o2', -100, 'l');
model = changeRxnBounds(model, 'EX_so4', -100, 'l');
model = changeRxnBounds(model, 'EX_nh3', -100, 'l');
model = changeRxnBounds(model, 'EX_cit', -100, 'l');
model = changeRxnBounds(model, 'EX_glyc', -100, 'l');
```

Then, we calculate the maximum specific growth rate and the maximum production rate for succinate

```
growthRate = optimizeCbModel(model);
fprintf('The maximum growth rate is %1.2f', growthRate.f);
```

The maximum growth rate is 14.36

```
model = changeObjective(model, 'EX_suc');
maxSucc = optimizeCbModel(model);
fprintf('The maximum production rate of succinate is %1.2f', maxSucc.f);
```

The maximum production rate of succinate is 155.56

**TIP:** The biomass reaction is usually set to 1%-10% of maximum theoretical biomass yield when running the following steps, to prevent solutions with not biomass formation

1. maximizing product formation
2. finding MUST sets of second order
3. finding FORCE sets

## STEP 2: Define constraints for both wild-type and mutant strain

**TIMING:** This step should take a few days or weeks, depending on the information available for your species.

**CRITICAL STEP:** This is a manual task, so you should search for information in articles or even perform your own experiments. You can also make assumptions for describing the phenotypes of both strains which will make this task a little faster but make sure to have two strains different enough, because you should be able to find differences in reactions ranges.

First, we load the model. This model comprises only 90 reactions, which describe the central metabolism of *E. coli* [2].

Then, we change the objective function to maximize biomass ("R75"). We also change the lower bounds, so *E. coli* will be able to consume glucose, oxygen, sulfate, ammonium, citrate and glycerol.

We define constraints for each strain

```
constrWT = struct('rxnList', {'R75'}, 'rxnValues', 14, 'rxnBoundType', 'b');
constrMT = struct('rxnList', {'R75', 'EX_suc'}, 'rxnValues', [0, 155.55], 'rxnBoundType', 'b');
```

## Step 3: Flux Variability Analysis

**TIMING:** This task should take from a few seconds to a few hours depending on the size of your reconstruction

We run the FVA analysis for both strains

```
[minFluxesW, maxFluxesW, minFluxesM, maxFluxesM, ~, ~] = FVA0ptForce(model, constrWT, constrMT);
disp([minFluxesW, maxFluxesW, minFluxesM, maxFluxesM]);
```

|          |          |          |          |
|----------|----------|----------|----------|
| -90.1251 | 97.1300  | 44.4313  | 100.0000 |
| 0        | 86.0700  | 44.4375  | 100.0000 |
| 0        | 86.0700  | 44.4375  | 100.0000 |
| -56.1567 | 86.0700  | -44.4500 | 11.1143  |
| 21.3033  | 163.5300 | 55.5500  | 111.1143 |
| -3.0777  | 154.8640 | 55.5500  | 111.1143 |
| 0        | 151.5086 | 0        | 55.5625  |

|           |          |           |          |
|-----------|----------|-----------|----------|
| 0         | 187.2551 | 0         | 55.5687  |
| 0         | 169.5163 | 0         | 0.0187   |
| -10.0660  | 102.9449 | 0         | 0.0125   |
| 10.0660   | 66.5714  | 0         | 0.0063   |
| -10.0660  | 102.9449 | 0         | 0.0125   |
| -48.9454  | 7.5600   | -0.0063   | 0        |
| -53.9994  | 2.5060   | -0.0063   | 0        |
| -53.9994  | 2.5060   | -0.0063   | 0        |
| -2.5060   | 53.9994  | 0         | 0.0063   |
| 0         | 86.0700  | 0         | 55.5625  |
| 0         | 86.0700  | 0         | 55.5625  |
| 9.7020    | 114.6466 | 55.5500   | 55.5625  |
| 0         | 56.5564  | 55.5500   | 55.5571  |
| 16.0264   | 145.2048 | 155.5500  | 155.5563 |
| 16.0264   | 145.2048 | 155.5500  | 155.5563 |
| 0.9344    | 130.1128 | 155.5500  | 155.5562 |
| -5.6736   | 123.5048 | 155.5500  | 155.5563 |
| 0         | 118.0576 | 0         | 0.0062   |
| 5.1940    | 123.2516 | 0         | 0.0062   |
| -98.1150  | 123.2516 | -55.5625  | 0.0062   |
| 0         | 151.5086 | 0         | 55.5625  |
| 0         | 151.5086 | 0         | 55.5625  |
| 0         | 254.5400 | 55.5500   | 777.7875 |
| 0         | 253.2493 | 0         | 722.2375 |
| -7.1960   | 94.6056  | 0         | 0.0125   |
| 0         | 84.8467  | 88.8750   | 88.9000  |
| 0         | 84.8467  | 88.8750   | 88.9000  |
| 0         | 175.1064 | 188.8500  | 188.9000 |
| 0         | 175.1064 | 188.8500  | 188.9000 |
| 91.4130   | 107.1280 | 0         | 0        |
| 9.4500    | 9.4500   | 0         | 0        |
| 2.9400    | 2.9400   | 0         | 0        |
| 3.9340    | 3.9340   | 0         | 0        |
| 25.4520   | 56.8820  | 0         | 0        |
| 3.2060    | 3.2060   | 0         | 0        |
| 6.8320    | 6.8320   | 0         | 0        |
| 0         | 15.7150  | 0         | 0        |
| -6.8880   | 8.8270   | 0         | 0        |
| 0.6790    | 16.3940  | 0         | 0        |
| 0         | 31.4300  | 0         | 0        |
| 3.2620    | 3.2620   | 0         | 0        |
| 4.5640    | 4.5640   | 0         | 0        |
| 4.5640    | 4.5640   | 0         | 0        |
| 7.2380    | 38.6680  | 0         | 0        |
| 2.0440    | 2.0440   | 0         | 0        |
| 5.6280    | 5.6280   | 0         | 0        |
| 5.9920    | 5.9920   | 0         | 0        |
| 3.8640    | 3.8640   | 0         | 0        |
| 2.4640    | 2.4640   | 0         | 0        |
| 1.8340    | 1.8340   | 0         | 0        |
| 0.7560    | 0.7560   | 0         | 0        |
| 1.2600    | 1.2600   | 0         | 0        |
| 2.0440    | 2.0440   | 0         | 0        |
| 1.2600    | 1.2600   | 0         | 0        |
| 79.7324   | 200.0000 | 199.9500  | 200.0000 |
| 0         | 118.0576 | 0         | 0.0062   |
| -39.5563  | 353.9124 | -22.2500  | 33.3500  |
| 0         | 253.2493 | 0         | 722.2375 |
| 40.6268   | 100.0000 | 99.9875   | 100.0000 |
| 15.0890   | 100.0000 | 99.9929   | 100.0000 |
| -100.0000 | 84.8467  | -100.0000 | -99.9500 |
| 0         | 175.1064 | 188.8500  | 188.9000 |
| 0         | 101.8016 | 0         | 0.0125   |
| 134.9718  | 407.3274 | 311.1000  | 311.1187 |
| 62.1267   | 100.0000 | 99.9750   | 100.0000 |
| 97.4820   | 97.4820  | 0         | 0        |
| 3.2620    | 3.2620   | 0         | 0        |
| 14.0000   | 14.0000  | 0         | 0        |

|           |          |           |          |
|-----------|----------|-----------|----------|
| 0         | 175.1064 | 188.8500  | 188.9000 |
| 134.9718  | 407.3274 | 311.1000  | 311.1187 |
| 0         | 101.8016 | 0         | 0.0125   |
| 0         | 253.2493 | 0         | 722.2375 |
| -100.0000 | -40.6268 | -100.0000 | -99.9875 |
| -100.0000 | -15.0890 | -100.0000 | -99.9929 |
| -100.0000 | 84.8467  | -100.0000 | -99.9500 |
| -97.4820  | -97.4820 | 0         | 0        |
| -100.0000 | -62.1267 | -100.0000 | -99.9750 |
| -3.2620   | -3.2620  | 0         | 0        |
| 0         | 105.4230 | 155.5500  | 155.5500 |
| 0         | 105.4230 | 155.5500  | 155.5500 |
| 11.6200   | 11.6200  | 0         | 0        |
| 5.0540    | 5.0540   | 0         | 0        |
| 5.9920    | 5.9920   | 0         | 0        |

---

Now, the run the next step of OptForce.

## Step 4: Find Must Sets

**TIMING:** This task should take from a few seconds to a few hours depending on the size of your reconstruction

First, we define an ID for this run. Each time you run the functions associated to the optForce procedure, some folders can be generated to store inputs used in that run. Outputs are stored as well. These folder will be located inside the folder defined by your run ID. Thus, if your runID is "TestOptForce", the structure of the folders will be the following:

```

├─ CurrentFolder
|   └─ TestOptForce
|       └─ Inputs
|       └─ Outputs

```

To avoid the generation of inputs and outputs folders, set `keepInputs = 0`, `printExcel = 0` and `printText = 0`.

Also, a report of the run is generated each time you run the functions associated to the optForce procedure. So, the idea is to give a different `runID` each time you run the functions, so you will be able to see the report (inputs used, outputs generated, errors in the run) for each run.

We define then our `runID`

```
runID = 'TestOptForceM';
```

Now, only functions to find first and second order must sets are supported in this third step. As depicted in **Figure 1**, the first order must sets are MUSTU and MUSTL; and second order must sets are MUSTUU, MUSTLL and MUSTUL

### A) Finding first order must sets

We define constraints

```
constrOpt = struct('rxnList', {'EX_glc', 'R75', 'EX_suc'}, 'values', [-100, 0, 155.5]);
```

We then run the functions `findMustL` and `findMustU` that will allow us to find `mustU` and `mustL` sets, respectively.

#### i) MustL Set:

```
[mustLSet, pos_mustL] = findMustL(model, minFluxesW, maxFluxesW, 'constrOpt', constrOpt, ...  
                                'runID', runID, 'outputFolder', 'OutputsFindMustL', ...  
                                'outputFileName', 'MustL', 'printExcel', 1, 'printText', 1,  
                                'printReport', 1, 'keepInputs', 1, 'verbose', 0);
```

Optimize a model with 710 rows, 798 columns and 2715 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 564 rows and 482 columns

Presolve time: 0.02s

Presolved: 146 rows, 316 columns, 957 nonzeros

Variable types: 273 continuous, 43 integer (43 binary)

Root relaxation: objective 9.748200e+01, 161 iterations, 0.00 seconds

| Nodes |        |   | Current Node |       |        | Objective Bounds |          |       | Work    |      |
|-------|--------|---|--------------|-------|--------|------------------|----------|-------|---------|------|
| Expl  | Unexpl |   | Obj          | Depth | IntInf | Incumbent        | BestBd   | Gap   | It/Node | Time |
| *     | 0      | 0 |              |       | 0      | 97.4820000       | 97.48200 | 0.00% | -       | 0s   |

Explored 0 nodes (161 simplex iterations) in 0.06 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 97.482

Pool objective bound 97.482

Optimal solution found (tolerance 1.00e-12)

Best objective 9.748200000003e+01, best bound 9.748200000003e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2710 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 564 rows and 483 columns

Presolve time: 0.02s

Presolved: 146 rows, 315 columns, 954 nonzeros

Variable types: 273 continuous, 42 integer (42 binary)

Root relaxation: objective 9.141300e+01, 175 iterations, 0.00 seconds

| Nodes |        |   | Current Node |       |        | Objective Bounds |          |       | Work    |      |
|-------|--------|---|--------------|-------|--------|------------------|----------|-------|---------|------|
| Expl  | Unexpl |   | Obj          | Depth | IntInf | Incumbent        | BestBd   | Gap   | It/Node | Time |
| *     | 0      | 0 |              |       | 0      | 91.4130000       | 91.41300 | 0.00% | -       | 0s   |

Explored 0 nodes (175 simplex iterations) in 0.03 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 91.413

Pool objective bound 91.413

Optimal solution found (tolerance 1.00e-12)

Best objective 9.141300000002e+01, best bound 9.141300000002e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2705 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 564 rows and 484 columns

Presolve time: 0.02s

Presolved: 146 rows, 314 columns, 951 nonzeros

Variable types: 273 continuous, 41 integer (41 binary)

Root relaxation: objective 2.545200e+01, 174 iterations, 0.00 seconds

| Nodes |        |   | Current Node |       |        | Objective Bounds |          |       | Work    |      |
|-------|--------|---|--------------|-------|--------|------------------|----------|-------|---------|------|
| Expl  | Unexpl |   | Obj          | Depth | IntInf | Incumbent        | BestBd   | Gap   | It/Node | Time |
| *     | 0      | 0 |              |       | 0      | 25.4520000       | 25.45200 | 0.00% | -       | 0s   |

Explored 0 nodes (174 simplex iterations) in 0.04 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 25.452

Pool objective bound 25.452

Optimal solution found (tolerance 1.00e-12)

Best objective 2.545200000001e+01, best bound 2.545200000001e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2700 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 564 rows and 485 columns

Presolve time: 0.02s

Presolved: 146 rows, 313 columns, 948 nonzeros

Variable types: 273 continuous, 40 integer (40 binary)

Root relaxation: objective 1.162000e+01, 160 iterations, 0.00 seconds

| Nodes |        |   | Current Node |       |        | Objective Bounds |          |       | Work    |      |
|-------|--------|---|--------------|-------|--------|------------------|----------|-------|---------|------|
| Expl  | Unexpl |   | Obj          | Depth | IntInf | Incumbent        | BestBd   | Gap   | It/Node | Time |
| *     | 0      | 0 |              |       | 0      | 11.6200000       | 11.62000 | 0.00% | -       | 0s   |

Explored 0 nodes (160 simplex iterations) in 0.04 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 11.62

Pool objective bound 11.62

Optimal solution found (tolerance 1.00e-12)

Best objective 1.162000000003e+01, best bound 1.162000000003e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2695 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 564 rows and 486 columns

Presolve time: 0.02s

Presolved: 146 rows, 312 columns, 945 nonzeros

Variable types: 273 continuous, 39 integer (39 binary)

Root relaxation: objective 1.000350e+01, 186 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |          |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd   | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 10.0035000       | 10.00350 | 0.00% | -       | 0s   |

Explored 0 nodes (186 simplex iterations) in 0.07 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 10.0035  
Pool objective bound 10.0035

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.000350000000e+01, best bound 1.000350000000e+01, gap 0.0000%  
Optimize a model with 710 rows, 798 columns and 2690 nonzeros  
Variable types: 708 continuous, 90 integer (90 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [5e-01, 1e+03]  
Presolve removed 567 rows and 488 columns  
Presolve time: 0.02s  
Presolved: 143 rows, 310 columns, 933 nonzeros  
Variable types: 272 continuous, 38 integer (38 binary)

Root relaxation: objective 9.450000e+00, 170 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |         |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|---------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd  | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 9.4500000        | 9.45000 | 0.00% | -       | 0s   |

Explored 0 nodes (170 simplex iterations) in 0.04 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 9.45  
Pool objective bound 9.45

Optimal solution found (tolerance 1.00e-12)  
Best objective 9.450000000029e+00, best bound 9.450000000029e+00, gap 0.0000%  
Optimize a model with 710 rows, 798 columns and 2685 nonzeros  
Variable types: 708 continuous, 90 integer (90 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [5e-01, 1e+03]  
Presolve removed 567 rows and 489 columns  
Presolve time: 0.01s  
Presolved: 143 rows, 309 columns, 930 nonzeros  
Variable types: 272 continuous, 37 integer (37 binary)

Root relaxation: objective 7.238000e+00, 166 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |         |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|---------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd  | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 7.2380000        | 7.23800 | 0.00% | -       | 0s   |

Explored 0 nodes (166 simplex iterations) in 0.04 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 7.238  
Pool objective bound 7.238

Optimal solution found (tolerance 1.00e-12)  
Best objective 7.238000000012e+00, best bound 7.238000000012e+00, gap 0.0000%  
Optimize a model with 710 rows, 798 columns and 2680 nonzeros



Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 567 rows and 490 columns

Presolve time: 0.02s

Presolved: 143 rows, 308 columns, 927 nonzeros

Variable types: 272 continuous, 36 integer (36 binary)

Root relaxation: objective 6.832000e+00, 157 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |         |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|---------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd  | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 6.8320000        | 6.83200 | 0.00% | -       | 0s   |

Explored 0 nodes (157 simplex iterations) in 0.05 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 6.832

Pool objective bound 6.832

Optimal solution found (tolerance 1.00e-12)

Best objective 6.832000000029e+00, best bound 6.832000000029e+00, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2675 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 567 rows and 491 columns

Presolve time: 0.02s

Presolved: 143 rows, 307 columns, 924 nonzeros

Variable types: 272 continuous, 35 integer (35 binary)

Root relaxation: objective 5.992000e+00, 184 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |         |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|---------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd  | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 5.9920000        | 5.99200 | 0.00% | -       | 0s   |

Explored 0 nodes (184 simplex iterations) in 0.06 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 5.992

Pool objective bound 5.992

Optimal solution found (tolerance 1.00e-12)

Best objective 5.992000000029e+00, best bound 5.992000000029e+00, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2670 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 567 rows and 492 columns

Presolve time: 0.02s

Presolved: 143 rows, 306 columns, 921 nonzeros

Variable types: 272 continuous, 34 integer (34 binary)

Root relaxation: objective 5.992000e+00, 157 iterations, 0.00 seconds

| Nodes |  | Current Node |  |  | Objective Bounds |  |  | Work |  |
|-------|--|--------------|--|--|------------------|--|--|------|--|
|-------|--|--------------|--|--|------------------|--|--|------|--|

| Expl | Unexpl | Obj | Depth | IntInf | Incumbent | BestBd  | Gap   | It/Node | Time |
|------|--------|-----|-------|--------|-----------|---------|-------|---------|------|
| *    | 0      | 0   |       | 0      | 5.9920000 | 5.99200 | 0.00% | -       | 0s   |

Note that the folder "TestOptForceM" was created. Inside this folder, two additional folders were created: "InputsMustL" and "OutputsMustL". In the inputs folder you will find all the inputs required to run the the function `findMustL`. Additionally, in the outputs folder you will find the `mustL` set found, which were saved in two files (.xls and .txt). Furthermore, a report which summarize all the inputs and outputs used during your running was generated. The name of the report will be in this format "report-Day-Month-Year-Hour-Minutes". So, you can maintain a chronological order of your experiments.

We display the reactions that belongs to the `mustL` set

```
disp(mustLSet)
```

```
'R11'
'R26'
'R37'
'R38'
'R39'
'R40'
'R41'
'R42'
'R43'
'R46'
'R48'
'R49'
'R50'
'R51'
'R52'
'R53'
'R54'
'R55'
'R56'
'R57'
'R58'
'R59'
'R60'
'R61'
'R73'
'R74'
'PSEUD0pyr_1'
'PSEUD0pep_1'
'PSEUD0co2_1'
```

## ii) MustU set:

```
[mustUSet, pos_mustU] = findMustU(model, minFluxesW, maxFluxesW, 'constrOpt', constrOpt, ...
                                'runID', runID, 'outputFolder', 'OutputsFindMustU', 'outputF
                                'MustU', 'printExcel', 1, 'printText', 1, ...
                                'printReport', 1, 'keepInputs', 1, 'verbose', 0);
```

```
Optimize a model with 710 rows, 798 columns and 2769 nonzeros
Variable types: 708 continuous, 90 integer (90 binary)
Coefficient statistics:
  Matrix range      [5e-02, 1e+03]
  Objective range   [1e+00, 1e+00]
  Bounds range      [1e+00, 1e+03]
  RHS range         [5e-01, 1e+03]
Presolve removed 473 rows and 451 columns
```

Presolve time: 0.03s  
 Presolved: 237 rows, 347 columns, 1238 nonzeros  
 Variable types: 299 continuous, 48 integer (48 binary)

Root relaxation: objective 1.063553e+02, 182 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533 | -     | -       | 0s   |
| *     | 0      | 0            |       | 0      | 97.4820000       | 97.48200  | 0.00% | -       | 0s   |

Cutting planes:  
 Gomory: 1  
 Implied bound: 1

Explored 0 nodes (204 simplex iterations) in 0.06 seconds  
 Thread count was 4 (of 4 available processors)

Solution count 1: 97.482  
 Pool objective bound 97.482

Optimal solution found (tolerance 1.00e-12)  
 Best objective 9.748199999997e+01, best bound 9.748199999997e+01, gap 0.00000%  
 Optimize a model with 710 rows, 798 columns and 2764 nonzeros  
 Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]  
 Objective range [1e+00, 1e+00]  
 Bounds range [1e+00, 1e+03]  
 RHS range [5e-01, 1e+03]

Presolve removed 473 rows and 452 columns

Presolve time: 0.02s

Presolved: 237 rows, 346 columns, 1235 nonzeros

Variable types: 299 continuous, 47 integer (47 binary)

Root relaxation: objective 1.063553e+02, 194 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533 | -     | -       | 0s   |
| *     | 0      | 0            |       | 0      | 50.0770000       | 50.07700  | 0.00% | -       | 0s   |

Cutting planes:  
 Gomory: 1  
 Implied bound: 1

Explored 0 nodes (225 simplex iterations) in 0.06 seconds  
 Thread count was 4 (of 4 available processors)

Solution count 1: 50.077  
 Pool objective bound 50.077

Optimal solution found (tolerance 1.00e-12)  
 Best objective 5.007699999997e+01, best bound 5.007699999997e+01, gap 0.00000%  
 Optimize a model with 710 rows, 798 columns and 2759 nonzeros  
 Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]  
 Objective range [1e+00, 1e+00]  
 Bounds range [1e+00, 1e+03]  
 RHS range [5e-01, 1e+03]

Presolve removed 473 rows and 453 columns

Presolve time: 0.03s

Presolved: 237 rows, 345 columns, 1231 nonzeros

Variable types: 299 continuous, 46 integer (46 binary)

Root relaxation: objective 1.063553e+02, 191 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533 | -     | -       | 0s   |
| *     | 0      | 0            |       | 0      | 31.9951818       | 31.99518  | 0.00% | -       | 0s   |

Cutting planes:

Gomory: 1

Implied bound: 1

Explored 0 nodes (235 simplex iterations) in 0.09 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 31.9952

Pool objective bound 31.9952

Optimal solution found (tolerance 1.00e-12)

Best objective 3.199518181815e+01, best bound 3.199518181815e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2754 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 477 rows and 455 columns

Presolve time: 0.04s

Presolved: 233 rows, 343 columns, 1216 nonzeros

Variable types: 298 continuous, 45 integer (45 binary)

Root relaxation: objective 1.063553e+02, 186 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533 | -     | -       | 0s   |
| *     | 0      | 0            |       | 0      | 25.3871818       | 25.38718  | 0.00% | -       | 0s   |

Cutting planes:

Gomory: 1

Implied bound: 1

Explored 0 nodes (207 simplex iterations) in 0.08 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 25.3872

Pool objective bound 25.3872

Optimal solution found (tolerance 1.00e-12)

Best objective 2.538718181814e+01, best bound 2.538718181814e+01, gap 0.0000%

Optimize a model with 710 rows, 798 columns and 2749 nonzeros

Variable types: 708 continuous, 90 integer (90 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [5e-01, 1e+03]

Presolve removed 481 rows and 457 columns

Presolve time: 0.02s

Presolved: 229 rows, 341 columns, 1201 nonzeros

Variable types: 297 continuous, 44 integer (44 binary)

Root relaxation: objective 1.063553e+02, 197 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |        |     | Work    |      |
|-------|--------|--------------|-------|--------|------------------|--------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd | Gap | It/Node | Time |

|   |   |   |           |   |   |            |           |       |   |    |
|---|---|---|-----------|---|---|------------|-----------|-------|---|----|
|   | 0 | 0 | 106.35533 | 0 | 2 | -          | 106.35533 | -     | - | 0s |
|   | 0 | 0 | 13.39362  | 0 | 2 | -          | 13.39362  | -     | - | 0s |
| * | 0 | 0 |           | 0 |   | 13.3936250 | 13.39363  | 0.00% | - | 0s |

Cutting planes:  
 Gomory: 1  
 Implied bound: 1

Explored 0 nodes (417 simplex iterations) in 0.07 seconds  
 Thread count was 4 (of 4 available processors)

Solution count 1: 13.3936  
 Pool objective bound 13.3936

Optimal solution found (tolerance 1.00e-12)  
 Best objective 1.339362500000e+01, best bound 1.339362500000e+01, gap 0.0000%  
 Optimize a model with 710 rows, 798 columns and 2744 nonzeros  
 Variable types: 708 continuous, 90 integer (90 binary)  
 Coefficient statistics:  
   Matrix range [5e-02, 1e+03]  
   Objective range [1e+00, 1e+00]  
   Bounds range [1e+00, 1e+03]  
   RHS range [5e-01, 1e+03]  
 Presolve removed 484 rows and 459 columns  
 Presolve time: 0.02s  
 Presolved: 226 rows, 339 columns, 1189 nonzeros  
 Variable types: 296 continuous, 43 integer (43 binary)

Root relaxation: objective 1.063553e+02, 164 iterations, 0.00 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |            |           | Work    |      |    |
|-------|--------|--------------|-----------|--------|------------------|------------|-----------|---------|------|----|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd     | Gap       | It/Node | Time |    |
|       | 0      | 0            | 106.35533 | 0      | 2                | -          | 106.35533 | -       | -    | 0s |
| *     | 0      | 0            |           | 0      |                  | 13.3936250 | 13.39362  | 0.00%   | -    | 0s |

Cutting planes:  
 Gomory: 1  
 Implied bound: 1

Explored 0 nodes (201 simplex iterations) in 0.06 seconds  
 Thread count was 4 (of 4 available processors)

Solution count 1: 13.3936  
 Pool objective bound 13.3936

Optimal solution found (tolerance 1.00e-12)  
 Best objective 1.339362499996e+01, best bound 1.339362499996e+01, gap 0.0000%  
 Optimize a model with 710 rows, 798 columns and 2739 nonzeros  
 Variable types: 708 continuous, 90 integer (90 binary)  
 Coefficient statistics:  
   Matrix range [5e-02, 1e+03]  
   Objective range [1e+00, 1e+00]  
   Bounds range [1e+00, 1e+03]  
   RHS range [5e-01, 1e+03]  
 Presolve removed 487 rows and 461 columns  
 Presolve time: 0.02s  
 Presolved: 223 rows, 337 columns, 1177 nonzeros  
 Variable types: 295 continuous, 42 integer (42 binary)

Root relaxation: objective 1.063553e+02, 176 iterations, 0.00 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |            |           | Work    |      |    |
|-------|--------|--------------|-----------|--------|------------------|------------|-----------|---------|------|----|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd     | Gap       | It/Node | Time |    |
|       | 0      | 0            | 106.35533 | 0      | 2                | -          | 106.35533 | -       | -    | 0s |
| *     | 0      | 0            |           | 0      |                  | 13.3936250 | 13.39362  | 0.00%   | -    | 0s |

Cutting planes:  
Gomory: 1  
Implied bound: 1

Explored 0 nodes (207 simplex iterations) in 0.07 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 13.3936  
Pool objective bound 13.3936

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.339362499997e+01, best bound 1.339362499997e+01, gap 0.0000%  
Optimize a model with 710 rows, 798 columns and 2734 nonzeros  
Variable types: 708 continuous, 90 integer (90 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [5e-01, 1e+03]  
Presolve removed 497 rows and 468 columns  
Presolve time: 0.02s  
Presolved: 213 rows, 330 columns, 1138 nonzeros  
Variable types: 291 continuous, 39 integer (39 binary)

Root relaxation: objective 1.063553e+02, 192 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533 | -     | -       | 0s   |
| 0     | 0      | 29.81968     | 0     | 3      | -                | 29.81968  | -     | -       | 0s   |
| *     | 0      | 0            | 0     |        | 13.3936250       | 13.39363  | 0.00% | -       | 0s   |

Cutting planes:  
Gomory: 1  
Implied bound: 2  
Flow cover: 1

Explored 0 nodes (232 simplex iterations) in 0.08 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 13.3936  
Pool objective bound 13.3936

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.339362500000e+01, best bound 1.339362500000e+01, gap 0.0000%  
Optimize a model with 710 rows, 798 columns and 2729 nonzeros  
Variable types: 708 continuous, 90 integer (90 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [5e-01, 1e+03]  
Presolve removed 493 rows and 465 columns  
Presolve time: 0.02s  
Presolved: 217 rows, 333 columns, 1153 nonzeros  
Variable types: 293 continuous, 40 integer (40 binary)

Root relaxation: objective 1.063553e+02, 177 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |        |     | Work    |      |
|-------|--------|--------------|-------|--------|------------------|--------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd | Gap | It/Node | Time |

Note that the folders "InputsMustU" and "OutputsFindMustU" were created. These folders contain the inputs and outputs of findMustU, respectively.

We display the reactions that belongs to the mustU set

```
disp(mustUSet)
```

```
'R21'  
'R22'  
'R23'  
'R24'  
'R33'  
'R34'  
'R35'  
'R36'  
'R69'  
'EX_pdo'  
'EX_nh3'  
'EX_so4'  
'SUct'
```

## B) Finding second order must sets

First, we define the reactions that will be excluded from the analysis. It is suggested to include in this list the reactions found in the previous step as well as exchange reactions

```
constrOpt = struct('rxnList', {'EX_gluc', 'R75', 'EX_suc'}, 'values', [-100, 0, 155.5]);  
exchangeRxns = model.rxns(cellfun(@isempty, strfind(model.rxns, 'EX_')) == 0);  
excludedRxns = unique([mustUSet; mustLSet; exchangeRxns]);
```

Now, we run the functions for finding second order must sets

### i) MustUU:

```
[mustUU, pos_mustUU, mustUU_linear, pos_mustUU_linear] = ...  
    findMustUU(model, minFluxesW, maxFluxesW, 'constrOpt', constrOpt, ...  
                'excludedRxns', excludedRxns, 'runID', runID, ...  
                'outputFolder', 'OutputsFindMustUU', 'outputFileName', 'MustUU', ...  
                'printExcel', 1, 'printText', 1, 'printReport', 1, 'keepInputs', 1, ...  
                'verbose', 1);
```

Optimize a model with 1165 rows, 980 columns and 4128 nonzeros

Variable types: 800 continuous, 180 integer (180 binary)

Coefficient statistics:

Matrix range [5e-02, 2e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e-01, 2e+03]

Presolve removed 799 rows and 575 columns

Presolve time: 0.03s

Presolved: 366 rows, 405 columns, 1668 nonzeros

Variable types: 327 continuous, 78 integer (78 binary)

Root relaxation: objective 2.127107e+02, 268 iterations, 0.01 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 212.7106667      | 212.71067 | 0.00% | -       | 0s   |

Explored 0 nodes (397 simplex iterations) in 0.08 seconds

Thread count was 4 (of 4 available processors)

Solution count 1: 212.711

Pool objective bound 212.711

Optimal solution found (tolerance 1.00e-12)  
Best objective 2.127106666667e+02, best bound 2.127106666667e+02, gap 0.0000%  
Optimize a model with 1167 rows, 980 columns and 4132 nonzeros  
Variable types: 800 continuous, 180 integer (180 binary)  
Coefficient statistics:  
Matrix range [5e-02, 2e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e-01, 2e+03]  
Presolve removed 802 rows and 575 columns  
Presolve time: 0.02s  
Presolved: 365 rows, 405 columns, 1668 nonzeros  
Variable types: 327 continuous, 78 integer (78 binary)

Root relaxation: objective 1.585013e+02, 246 iterations, 0.01 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-------|--------|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |
| *     | 0      | 0            |       | 0      | 158.5013333      | 158.50133 | 0.00% | -       | 0s   |

Explored 0 nodes (307 simplex iterations) in 0.07 seconds  
Thread count was 4 (of 4 available processors)

Solution count 1: 158.501  
Pool objective bound 158.501

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.585013333333e+02, best bound 1.585013333333e+02, gap 0.0000%  
Optimize a model with 1169 rows, 980 columns and 4136 nonzeros  
Variable types: 800 continuous, 180 integer (180 binary)  
Coefficient statistics:  
Matrix range [5e-02, 2e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e-01, 2e+03]  
Presolve removed 803 rows and 575 columns  
Presolve time: 0.03s  
Presolved: 366 rows, 405 columns, 1674 nonzeros  
Variable types: 327 continuous, 78 integer (78 binary)

Root relaxation: objective 1.237373e+02, 247 iterations, 0.01 seconds

| Nodes |        | Current Node |            |        | Objective Bounds |              |     | Work    |      |
|-------|--------|--------------|------------|--------|------------------|--------------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth      | IntInf | Incumbent        | BestBd       | Gap | It/Node | Time |
|       | 0      | 0            | 123.73733  | 0      | 4                | - 123.73733  | -   | -       | 0s   |
|       | 0      | 0            | infeasible | 0      |                  | - infeasible | -   | -       | 0s   |

Cutting planes:  
Gomory: 2  
MIR: 1  
Flow cover: 1

Explored 0 nodes (303 simplex iterations) in 0.09 seconds  
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible  
Best objective -, best bound -, gap -  
a MustUU set was found  
MustUU set was printed in MustUU.txt  
MustUU set was also printed in MustUU\_Info.txt



Note that the folders "InputsMustUU" and "OutputsFindMustUU" were created. These folders contain the inputs and outputs of `findMustUU`, respectively.

We display the reactions that belongs to the `mustUU` set

```
disp(mustUU);
```

```
'R30'    'R65'
'R31'    'R65'
```

## ii) MustLL:

```
[mustLL, pos_mustLL, mustLL_linear, pos_mustLL_linear] = ...
    findMustLL(model, minFluxesW, maxFluxesW, 'constrOpt', constrOpt, ...
        'excludedRxns', excludedRxns, 'runID', runID, ...
        'outputFolder', 'OutputsFindMustLL', 'outputFileName', 'MustLL', ...
        'printExcel', 1, 'printText', 1, 'printReport', 1, 'keepInputs', 1, ...
        'verbose', 1);
```

Optimize a model with 1165 rows, 980 columns and 4074 nonzeros

Variable types: 800 continuous, 180 integer (180 binary)

Coefficient statistics:

Matrix range [5e-02, 2e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e-01, 2e+03]

Presolve removed 799 rows and 578 columns

Presolve time: 0.02s

Presolved: 366 rows, 402 columns, 1633 nonzeros

Variable types: 324 continuous, 78 integer (78 binary)

Root relaxation: infeasible, 235 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |            |     | Work    |      |
|-------|--------|--------------|-------|--------|------------------|------------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd     | Gap | It/Node | Time |
| 0     | 0      | infeasible   | 0     |        | -                | infeasible | -   | -       | 0s   |

Explored 0 nodes (235 simplex iterations) in 0.04 seconds

Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible

Best objective -, best bound -, gap -

a MustLL set was not found

No mustLL set was not found. Therefore, no plain text file was generated

Note that the folders "InputsMustLL" and "OutputsFindMustLL" were created. These folders contain the inputs and outputs of `findMustLL`, respectively.

We display the reactions that belongs to the `mustLL` set. In this case, `mustLL` is an empty array because no reaction was found in the `mustLL` set.

```
disp(mustLL);
```

## iii) MustUL:

```
[mustUL, pos_mustUL, mustUL_linear, pos_mustUL_linear] = ...
    findMustUL(model, minFluxesW, maxFluxesW, 'constrOpt', constrOpt, ...
        'excludedRxns', excludedRxns, 'runID', runID, ...
        'outputFolder', 'OutputsFindMustUL', 'outputFileName', 'MustUL', ...
        'printExcel', 1, 'printText', 1, 'printReport', 1, 'keepInputs', 1, ...
        'verbose', 1);
```

Optimize a model with 1165 rows, 980 columns and 4101 nonzeros

Variable types: 800 continuous, 180 integer (180 binary)

Coefficient statistics:

Matrix range [5e-02, 2e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e-01, 2e+03]

Presolve removed 799 rows and 578 columns

Presolve time: 0.02s

Presolved: 366 rows, 402 columns, 1649 nonzeros

Variable types: 324 continuous, 78 integer (78 binary)

Root relaxation: objective 1.063553e+02, 243 iterations, 0.00 seconds

| Nodes |        | Current Node |       |        | Objective Bounds |            |     | Work    |      |
|-------|--------|--------------|-------|--------|------------------|------------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth | IntInf | Incumbent        | BestBd     | Gap | It/Node | Time |
| 0     | 0      | 106.35533    | 0     | 2      | -                | 106.35533  | -   | -       | 0s   |
| 0     | 0      | infeasible   | 0     |        | -                | infeasible | -   | -       | 0s   |

Cutting planes:

Gomory: 1

Flow cover: 1

Explored 0 nodes (249 simplex iterations) in 0.07 seconds

Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible

Best objective -, best bound -, gap -

a MustUL set was not found

No mustUL set was not found. Therefore, no plain text file was generated

Note that the folders "InputsMustUL" and "OutputsFindMustUL" were created. These folders contain the inputs and outputs of `findMustUL`, respectively.

We display the reactions that belongs to the `mustUL` set. In this case, `mustUL` is an empty array because no reaction was found in the `mustUL` set.

```
disp(mustUL);
```

**TROUBLESHOOTING 1:** "I didn't find any reaction in my must sets"

**TROUBLESHOOTING 2:** "I got an error when running the `findMustX` functions (X = L or U or LL or UL or UU depending on the case)"

## Step 5: OptForce

**TIMING:** This task should take from a few seconds to a few hours depending on the size of your reconstruction

We define constraints and we define `K` the number of interventions allowed, `nSets` the maximum number of sets to find, and `targetRxn` the reaction producing the metabolite of interest (in this case, succinate).

Additionally, we define the `mustU` set as the union of the reactions that must be upregulated in both first and second order must sets; and `mustL` set as the union of the reactions that must be downregulated in both first and second order must sets .

```
mustU = unique(union(mustUSet, mustUU));
mustL = unique(union(mustLSet, mustLL));
targetRxn = 'EX_suc';
biomassRxn = 'R75';
k = 1;
nSets = 1;
constrOpt = struct('rxnList', {'EX_gluc', 'R75'}, 'values', [-100, 0]);

[optForceSets, posOptForceSets, typeRegOptForceSets, flux_optForceSets] = optForce(model, targetRxn, minFluxesW, 'k', k, 'nSets', nSets, 'runID', runID, 'outputFile', outputFile, 'printReport', printReport);
```

Optimize a model with 2062 rows, 1248 columns and 6306 nonzeros  
Variable types: 978 continuous, 270 integer (270 binary)

Coefficient statistics:

```
Matrix range      [5e-02, 1e+03]
Objective range   [1e+00, 1e+00]
Bounds range      [1e+00, 1e+03]
RHS range         [1e+00, 1e+03]
```

Presolve removed 1216 rows and 437 columns

Presolve time: 0.03s

Presolved: 846 rows, 811 columns, 3005 nonzeros

Variable types: 678 continuous, 133 integer (133 binary)

Root relaxation: objective 1.555556e+02, 655 iterations, 0.02 seconds

| Nodes |        |   | Current Node |       |        | Objective Bounds |           |       | Work    |      |  |
|-------|--------|---|--------------|-------|--------|------------------|-----------|-------|---------|------|--|
| Expl  | Unexpl |   | Obj          | Depth | IntInf | Incumbent        | BestBd    | Gap   | It/Node | Time |  |
|       | 0      | 0 | 155.55556    | 0     | 2      | -                | 155.55556 | -     | -       | 0s   |  |
| H     | 0      | 0 |              |       |        | 0.0000000        | 155.55556 | -     | -       | 0s   |  |
|       | 0      | 0 | 155.55556    | 0     | 2      | 0.000000         | 155.55556 | -     | -       | 0s   |  |
|       | 0      | 2 | 155.55556    | 0     | 2      | 0.000000         | 155.55556 | -     | -       | 0s   |  |
| *     | 4      | 2 |              | 2     |        | 155.5500000      | 155.55556 | 0.00% | 30.8    | 0s   |  |

Explored 60 nodes (3605 simplex iterations) in 0.23 seconds

Thread count was 4 (of 4 available processors)

Solution count 2: 155.55 8.73123e-09

Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)

Best objective 1.5555000000087e+02, best bound 1.5555000000087e+02, gap 0.0000%  
set n 1 was found

optForce found 1 sets

Sets found by optForce were printed in OptForce.txt

Note that the folders "InputsOptForce" and "OutputsOptForce" were created. These folders contain the inputs and outputs of `optForce`, respectively.

We display the reactions found by `optForce`

```
disp(optForceSets)
```

```
'SUCt'
```

The reaction found was "SUCt", i.e. a transporter for succinate (a very intuitive solution).

Next, we will increase `k` and we will exclude "SUCt" from upregulations to found non-intuitive solutions.

**TIP:** Sometimes the product is at the end of a long linear pathway. In that case, the recommendation is to also exclude most reactions on the linear pathway. Essential reactions and reactions not associated with any gene should also be excluded.

We will only search for the 20 best solutions, but you can try with a higher number.

We will change the `runID` to save this second result (`K = 2`) in a different folder than the previous result (`K = 1`)

```
k = 2;
nSets = 20;
runID = 'TestOptForceM2';
excludedRxns = struct('rxnList', {'SUCt'}, 'typeReg','U');
[optForceSets, posOptForceSets, typeRegOptForceSets, flux_optForceSets] = ...
    optForce(model, targetRxn, biomassRxn, mustU, mustL, ...
        minFluxesW, maxFluxesW, minFluxesM, maxFluxesM, ...
        'k', k, 'nSets', nSets, 'constrOpt', constrOpt, ...
        'excludedRxns', excludedRxns, ...
        'runID', runID, 'outputFolder', 'OutputsOptForce', ...
        'outputFileName', 'OptForce', 'printExcel', 1, 'printText', 1, ...
        'printReport', 1, 'keepInputs', 1, 'verbose', 1);
```

Optimize a model with 2062 rows, 1248 columns and 6306 nonzeros

Variable types: 978 continuous, 270 integer (270 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e+00, 1e+03]

Presolve removed 1176 rows and 439 columns

Presolve time: 0.03s

Presolved: 886 rows, 809 columns, 3082 nonzeros

Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |           | Gap       | Work    |      |
|-------|--------|--------------|-----------|--------|------------------|-----------|-----------|---------|------|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd    |           | It/Node | Time |
|       | 0      | 0            | 155.55556 | 0      | 4                | -         | 155.55556 | -       | 0s   |
| H     | 0      | 0            |           |        |                  | 0.0000000 | 155.55556 | -       | 0s   |
|       | 0      | 0            | 155.55556 | 0      | 4                | 0.000000  | 155.55556 | -       | 0s   |
|       | 0      | 2            | 155.55556 | 0      | 4                | 0.000000  | 155.55556 | -       | 0s   |
| *     | 695    | 78           |           | 39     | 0.0000000        | 155.55556 | -         | 33.3    | 1s   |
| *     | 1059   | 86           |           | 18     | 139.9900000      | 155.55556 | 11.1%     | 32.9    | 2s   |
| *     | 1125   | 62           |           | 24     | 155.5500000      | 155.55556 | 0.00%     | 31.9    | 2s   |

Cutting planes:

Cover: 1

Inf proof: 2

Explored 1427 nodes (42791 simplex iterations) in 2.58 seconds

Thread count was 4 (of 4 available processors)

Solution count 4: 155.55 139.99 8.84756e-09 8.73115e-09

Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)

Best objective 1.555500000087e+02, best bound 1.555500000087e+02, gap 0.0000%  
set n 1 was found

Optimize a model with 2063 rows, 1248 columns and 6308 nonzeros

Variable types: 978 continuous, 270 integer (270 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e+00, 1e+03]

Presolve removed 1176 rows and 439 columns

Presolve time: 0.03s

Presolved: 887 rows, 809 columns, 3084 nonzeros

Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |             |           | Work    |         |
|-------|--------|--------------|-----------|--------|------------------|-------------|-----------|---------|---------|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd      | Gap       | It/Node | Time    |
|       | 0      | 0            | 155.55556 | 0      | 2                | -           | 155.55556 | -       | 0s      |
| H     | 0      | 0            |           |        |                  | 0.0000000   | 155.55556 | -       | 0s      |
|       | 0      | 0            | 155.55556 | 0      | 7                | 0.00000     | 155.55556 | -       | 0s      |
|       | 0      | 0            | 155.55556 | 0      | 2                | 0.00000     | 155.55556 | -       | 0s      |
|       | 0      | 2            | 155.55556 | 0      | 2                | 0.00000     | 155.55556 | -       | 0s      |
| *     | 8      | 6            |           | 3      |                  | 155.5500000 | 155.55556 | 0.00%   | 28.9 0s |

Cutting planes:

Gomory: 2

Cover: 5

Explored 259 nodes (6681 simplex iterations) in 0.54 seconds

Thread count was 4 (of 4 available processors)

Solution count 2: 155.55 8.73115e-09

Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)

Best objective 1.555500000087e+02, best bound 1.555500000087e+02, gap 0.0000%  
set n 2 was found

Optimize a model with 2064 rows, 1248 columns and 6310 nonzeros

Variable types: 978 continuous, 270 integer (270 binary)

Coefficient statistics:

Matrix range [5e-02, 1e+03]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+03]

RHS range [1e+00, 1e+03]

Presolve removed 1176 rows and 439 columns

Presolve time: 0.04s

Presolved: 888 rows, 809 columns, 3086 nonzeros

Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.03 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |             |           | Work    |         |
|-------|--------|--------------|-----------|--------|------------------|-------------|-----------|---------|---------|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd      | Gap       | It/Node | Time    |
|       | 0      | 0            | 155.55556 | 0      | 4                | -           | 155.55556 | -       | 0s      |
| H     | 0      | 0            |           |        |                  | 0.0000000   | 155.55556 | -       | 0s      |
|       | 0      | 0            | 155.55556 | 0      | 4                | 0.00000     | 155.55556 | -       | 0s      |
|       | 0      | 2            | 155.55556 | 0      | 4                | 0.00000     | 155.55556 | -       | 0s      |
| *     | 89     | 17           |           | 27     |                  | 155.5500000 | 155.55556 | 0.00%   | 25.5 0s |

Cutting planes:  
Cover: 6  
Implied bound: 1  
Inf proof: 2

Explored 382 nodes (9575 simplex iterations) in 0.69 seconds  
Thread count was 4 (of 4 available processors)

Solution count 2: 155.55 8.73115e-09  
Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.555500000087e+02, best bound 1.555500000087e+02, gap 0.0000%  
set n 3 was found  
Optimize a model with 2065 rows, 1248 columns and 6312 nonzeros  
Variable types: 978 continuous, 270 integer (270 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e+00, 1e+03]  
Presolve removed 1176 rows and 439 columns  
Presolve time: 0.03s  
Presolved: 889 rows, 809 columns, 3088 nonzeros  
Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes |        | Current Node |           |        |   | Objective Bounds |           |       | Work    |      |
|-------|--------|--------------|-----------|--------|---|------------------|-----------|-------|---------|------|
| Expl  | Unexpl | Obj          | Depth     | IntInf |   | Incumbent        | BestBd    | Gap   | It/Node | Time |
|       | 0      | 0            | 155.55556 | 0      | 2 | -                | 155.55556 | -     | -       | 0s   |
| H     | 0      | 0            |           |        |   | 0.0000000        | 155.55556 | -     | -       | 0s   |
|       | 0      | 2            | 155.55556 | 0      | 2 | 0.00000          | 155.55556 | -     | -       | 0s   |
| *     | 14     | 4            |           | 6      |   | 155.5437500      | 155.55556 | 0.01% | 29.4    | 0s   |
| *     | 154    | 20           |           | 22     |   | 155.5500000      | 155.55556 | 0.00% | 15.9    | 0s   |

Cutting planes:  
Cover: 4

Explored 276 nodes (6690 simplex iterations) in 0.52 seconds  
Thread count was 4 (of 4 available processors)

Solution count 3: 155.55 155.544 8.73115e-09  
Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.555500000087e+02, best bound 1.555500000087e+02, gap 0.0000%  
set n 4 was found  
Optimize a model with 2066 rows, 1248 columns and 6314 nonzeros  
Variable types: 978 continuous, 270 integer (270 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e+00, 1e+03]  
Presolve removed 1176 rows and 439 columns  
Presolve time: 0.03s  
Presolved: 890 rows, 809 columns, 3090 nonzeros  
Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes |        | Current Node |           |        |   | Objective Bounds |           |     | Work    |      |
|-------|--------|--------------|-----------|--------|---|------------------|-----------|-----|---------|------|
| Expl  | Unexpl | Obj          | Depth     | IntInf |   | Incumbent        | BestBd    | Gap | It/Node | Time |
|       | 0      | 0            | 155.55556 | 0      | 2 | -                | 155.55556 | -   | -       | 0s   |
| H     | 0      | 0            |           |        |   | 0.0000000        | 155.55556 | -   | -       | 0s   |

|   |    |   |           |   |             |           |           |      |   |    |
|---|----|---|-----------|---|-------------|-----------|-----------|------|---|----|
|   | 0  | 0 | 155.55556 | 0 | 2           | 0.00000   | 155.55556 | -    | - | 0s |
|   | 0  | 2 | 155.55556 | 0 | 2           | 0.00000   | 155.55556 | -    | - | 0s |
| * | 13 | 8 |           | 5 | 155.5437500 | 155.55556 | 0.01%     | 33.3 |   | 0s |
| * | 23 | 6 |           | 8 | 155.5500000 | 155.55556 | 0.00%     | 36.4 |   | 0s |

Cutting planes:  
Cover: 2

Explored 301 nodes (7779 simplex iterations) in 0.62 seconds  
Thread count was 4 (of 4 available processors)

Solution count 3: 155.55 155.544 8.71086e-09  
Pool objective bound 155.55

Optimal solution found (tolerance 1.00e-12)  
Best objective 1.555500000087e+02, best bound 1.555500000087e+02, gap 0.0000%  
set n 5 was found  
Optimize a model with 2067 rows, 1248 columns and 6316 nonzeros  
Variable types: 978 continuous, 270 integer (270 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e+00, 1e+03]  
Presolve removed 1176 rows and 439 columns  
Presolve time: 0.03s  
Presolved: 891 rows, 809 columns, 3092 nonzeros  
Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes |        | Current Node |           |        | Objective Bounds |           |           | Work    |      |    |
|-------|--------|--------------|-----------|--------|------------------|-----------|-----------|---------|------|----|
| Expl  | Unexpl | Obj          | Depth     | IntInf | Incumbent        | BestBd    | Gap       | It/Node | Time |    |
|       | 0      | 0            | 155.55556 | 0      | 3                | -         | 155.55556 | -       | -    | 0s |
| H     | 0      | 0            |           |        |                  | 0.0000000 | 155.55556 | -       | -    | 0s |
|       | 0      | 0            | 155.55556 | 0      | 3                | 0.00000   | 155.55556 | -       | -    | 0s |
|       | 0      | 2            | 155.55556 | 0      | 2                | 0.00000   | 155.55556 | -       | -    | 0s |
| *     | 64     | 21           |           | 26     | 155.5500000      | 155.55556 | 0.00%     | 14.2    |      | 0s |
| *     | 248    | 0            |           | 8      | 155.5500000      | 155.55556 | 0.00%     | 14.0    |      | 0s |

Cutting planes:  
Inf proof: 1

Explored 252 nodes (6319 simplex iterations) in 0.47 seconds  
Thread count was 4 (of 4 available processors)

Solution count 3: 155.55 155.55 8.73115e-09  
Pool objective bound 155.55

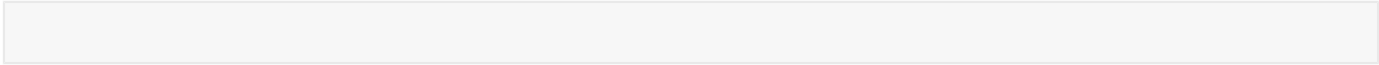
Optimal solution found (tolerance 1.00e-12)  
Best objective 1.555500000244e+02, best bound 1.555500000244e+02, gap 0.0000%  
set n 6 was found  
Optimize a model with 2068 rows, 1248 columns and 6318 nonzeros  
Variable types: 978 continuous, 270 integer (270 binary)  
Coefficient statistics:  
Matrix range [5e-02, 1e+03]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+03]  
RHS range [1e+00, 1e+03]  
Presolve removed 1176 rows and 439 columns  
Presolve time: 0.03s  
Presolved: 892 rows, 809 columns, 3094 nonzeros  
Variable types: 677 continuous, 132 integer (132 binary)

Root relaxation: objective 1.555556e+02, 578 iterations, 0.02 seconds

| Nodes | Current Node | Objective Bounds | Work |
|-------|--------------|------------------|------|
|-------|--------------|------------------|------|

|   | Expl | Unexpl | Obj       | Depth | IntInf | Incumbent   | BestBd    | Gap   | It/Node | Time |
|---|------|--------|-----------|-------|--------|-------------|-----------|-------|---------|------|
|   | 0    | 0      | 155.55556 | 0     | 2      | -           | 155.55556 | -     | -       | 0s   |
| H | 0    | 0      |           |       |        | 0.0000000   | 155.55556 | -     | -       | 0s   |
|   | 0    | 0      | 155.55556 | 0     | 2      | 0.000000    | 155.55556 | -     | -       | 0s   |
|   | 0    | 2      | 155.55556 | 0     | 2      | 0.000000    | 155.55556 | -     | -       | 0s   |
| * | 116  | 4      |           | 32    |        | 14.2857143  | 155.55556 | 989%  | 21.1    | 0s   |
| * | 156  | 7      |           | 4     |        | 155.5500000 | 155.55556 | 0.00% | 20.1    | 0s   |

Cutting planes:  
 Cover: 4



Note that the folders "InputsOptForce" and "OutputsOptForce" were created inside TestOptForce2. These folders contain the inputs and outputs of `optForce`, respectively.

We display the reactions found by `optForce`

```
disp(optForceSets)
```

|       |       |
|-------|-------|
| 'R22' | 'R25' |
| 'R24' | 'R25' |
| 'R24' | 'R26' |
| 'R22' | 'R26' |
| 'R23' | 'R25' |
| 'R24' | 'R63' |
| 'R21' | 'R63' |
| 'R21' | 'R26' |
| 'R23' | 'R26' |
| 'R22' | 'R63' |
| 'R23' | 'R63' |
| 'R21' | 'R25' |
| 'R24' | 'R26' |
| 'R22' | 'R26' |
| 'R23' | 'R26' |
| 'R21' | 'R26' |
| 'R23' | 'R4'  |
| 'R21' | 'R4'  |
| 'R24' | 'R4'  |
| 'R22' | 'R4'  |

### TIMING

1. STEP 1 ~ 1-2 seconds
2. STEP 2: ~ 2-5 seconds
3. STEP 3: ~ 10-20 seconds
4. STEP 4: ~ 10-20 seconds

### TROUBLESHOOTING

1) problem: "I didn't find any reaction in my must sets"  
 possible reason: the wild-type or mutant strain is not constrained enough.



solution: add more constraints to your strains until you find differences in your reaction ranges. If you don't find any differences, it is better to change the approach and use another algorithm.

2) problem: "I got an error when running the `findMust` functions"

possible reason: inputs are not defined well or solver is not defined

solution: verify your inputs, use `changeCobraSolver`, verify that the global variable `CBT_MILP_SOLVER` is not empty. It should contain the identifier for a MILP solver.

## ANTICIPATED RESULTS

In this tutorial some folders will be created inside the folder called "runID" to store inputs and outputs of the `optForce` functions (`findMustU.m`, `findMustL.m`, `findMustUU.m`, `findMustLL.m`, `findMustUL.m`, `optForce.m`)

In this case `runID = 'TestOptForce'`, so inside this folder the following folders will be created:

```
├── CurrentFolder
│   ├── TestOptForceM
│   │   ├── InputsFindMustL
│   │   ├── OutputsFindMustL
│   │   ├── InputsFindMustU
│   │   ├── OutputsFindMustU
│   │   ├── InputsFindMustLL
│   │   ├── OutputsFindMustLL
│   │   ├── InputsFindMustUU
│   │   ├── OutputsFindMustUU
│   │   ├── InputsFindMustUL
│   │   ├── OutputsFindMustUL
│   │   ├── InputsOptForce
│   │   └── OutputsOptForce
```

The input folders contain inputs (.mat files) for running the functions to solve each one of the bilevel problems. Output folders contain results of the algorithms (.xls and .txt files) as well as a report (.txt) summarizing the outcomes of the steps performed during the execution of the `optForce` functions.

The `optForce` algorithm will find sets of reactions that should increase the production of your target. The first sets found should be the best ones because the production rate will be the highest. The last ones should be the worse because the production rate will be slower. Be aware that some sets could not guarantee a minimum production rate for your target, so you always have to check the minimum production rate. You can do this using the function `testOptForceSol.m`. Some sets could allow a higher growth rate than others, so keep in mind this too when deciding which set is better.

## Acknowledgments

I would to thanks to the research group of Costas D. Maranas who provided the GAMS functions to solve this example. In particular I would like to thank to Chiam Yu Ng who kindly provides examples for using GAMS.

## References

- [1] Ranganathan S, Suthers PF, Maranas CD (2010) OptForce: An Optimization Procedure for Identifying All Genetic Manipulations Leading to Targeted Overproductions. PLOS Computational Biology 6(4): e1000744. <https://doi.org/10.1371/journal.pcbi.1000744>.
- [2] Maciek R. Antoniewicz, David F. Kraynie, Lisa A. Laffend, Joanna González-Lergier, Joanne K. Kelleher, Gregory Stephanopoulos, Metabolic flux analysis in a nonstationary system: Fed-batch fermentation of a high yielding strain of E. coli producing 1,3-propanediol, Metabolic Engineering, Volume 9, Issue 3, May 2007, Pages 277-292, ISSN 1096-7176, <https://doi.org/10.1016/j.ymben.2007.01.003>.