

Relaxed Flux Balance Analysis: Recon 3

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Reviewer:

Introduction

We consider a biochemical network of m molecular species and n biochemical reactions. The biochemical network is mathematically represented by a stoichiometric matrix $S \in \mathbb{R}^{m \times n}$. In standard notation, flux balance analysis (FBA) is the linear optimisation problem

$$\begin{aligned} \min_{v} \quad & p(v) = c^T v \\ \text{s.t.} \quad & Sv = b, \\ & l \leq v \leq u, \end{aligned}$$

where $c \in \mathbb{R}^n$ is a parameter vector that linearly combines one or more reaction fluxes to form what is termed the objective function, and where a $b_i < 0$, or $b_i > 0$, represents some fixed output, or input, of the i th molecular species.

Every FBA solution must satisfy the constraints, independent of any objective chosen to optimise over the set of constraints. It may occur that the constraints on the FBA problem are not all simultaneously feasible, i.e., the system of inequalities is infeasible. This situation might be caused by an incorrectly specified reaction bound or the absence of a reaction from the stoichiometric matrix, such that a nonzero $b \notin \mathcal{R}(S)$. To resolve the infeasibility, we consider a cardinality optimisation problem that seeks to minimise the number of bounds to relax, the number of fixed outputs to relax, or a combination of all three, in order to render the problem feasible. The cardinality optimisation problem, termed relaxed flux balance analysis, is

$$\begin{aligned} \min_{v, r, \alpha} \quad & \lambda |r|_0 + \alpha |l| |p|_0 + \alpha |q|_0 \\ \text{s.t.} \quad & Sv + r = b \\ & l \leq p + r \leq u \\ & p, q, r \geq 0 \end{aligned}$$

where $P, q \in \mathbb{R}^n$ denote the relaxations of the lower and upper bounds on reaction rates of the reaction rates vector v , and where $r \in \mathbb{R}^n$ denotes a relaxation of the mass balance constraint. Non-negative scalar parameters λ and α can be used to trade-off between relaxation of mass balance or bound constraints. A non-negative vector parameter k can be used to prioritise relaxation of one mass balance constraint over another, e.g. to avoid relaxation of a mass balance constraint on a metabolite that is not desired to be exchanged across the boundary of the system. A non-negative vector parameter γ may be used to prioritise relaxation of bounds on some reactions rather than others, e.g., relaxation of bounds on exchange reactions rather than internal reactions. The optimal choice of parameters depends heavily on the biochemical context. A relaxation of the minimum number of constraints is desirable because ideally one should be able to justify the choice of bounds or choice of metabolites to be exchanged across the boundary of the system by recourse to experimental literature. This task is magnified by the number of constraints proposed to be relaxed.

PROCEDURE: RelaxedFBA applied to Recon 3.0

Timing: 20 seconds (computation), minutes - days (integration)

Recon 3D ([Recon3D](#)) is the latest, most comprehensive, manually curated, genome-scale reconstruction of human metabolism. Recon3D is a reconstruction which currently encompasses ~3300 open reading frames, ~8000 unique metabolites, as well as ~13000 biochemical and transport reactions distributed over nine cellular compartments: cytoplasm [c], lysosome [l], nucleus [n], mitochondrion [m], mitochondrial intermembrane space [i], peroxisome [p], extracellular space [e], Golgi apparatus [g], and endoplasmic reticulum [r] ([Recon3D](#)). Recon3D is a flux balance analysis model and the largest stoichiometrically and flux consistent subset of Recon3D. That is, no internal reaction in Recon3D model is mass imbalanced and furthermore, every internal and every external reaction is admits a non-zero steady state flux. In this example, we take Recon3D model, set the lower bound on the biomass reaction to require the synthesis of biomass yet close all of the external reactions in the model. The resulting model is therefore infeasible, that is, no steady state flux vector satisfies the steady state constraints and the bound constraints for the resulting flux balance analysis problem, irrespective of the objective coefficients, so we use relaxed flux balance analysis to identify the minimal set of external reaction bounds that are required to be relaxed in order to make biomass synthesis feasible.

Load Recon3D model, unless it is already loaded into the workspace.

```
gload('Recon3D')

%Load the model if recon3d is available replace the model name.
modelFilename = 'Recon3D_model.mat';
modelDirectory = getcwd(fullfile(modelFilename)); %Ask up the folder for the distributed Models.
modelFilename = fullfile(modelDirectory, modelFilename); % Set the full path. Necessary to be sure, that the right model is loaded
model = readModel(modelFilename);
modelDirig = model;
```

Identify the exchange reactions and biomass reaction(s) heuristically and close (a subset) of them

```
model = findBiomass(model, size(model.L, 1), 1);
```

```
Found biomass reaction: biomass_reaction
Found biomass reactions: biomass_maintenance
Found biomass reactions: biomass_maintenance_noTriv
ATP demand reaction is not considered an exchange reaction by default. It should be mass balanced:
DR_atp_c_426[x] + atp[x] -> h[x] + atp[x] + pi[x]
```

```
if ~any(model.biomassReactions)
    error('could not heuristically identify a biomass reaction')
end
```

Add a linear objective coefficient corresponding to the biomass reaction

```
model.biomassReactionsDirig(model.L_rows('biomass_reaction'));
model.c(model.biomassReactionsDirig)=1;
```

Check that biomass production is feasible

```
PMASolution = optimizeCBModel(model,"max");
if PMASolution.état == 1
    disp('Relaxed model is feasible');
    biomassProductionRate=PMASolution.v(model.biomassRxn);
    fprintf(' %g\n', biomassProductionRate, " is the biomass production rate")
else
    disp('Relaxed model is infeasible');
end
```

Relaxed model is feasible

753.330 is the biomass production rate

Remove superfluous biomass reactions and display the size of the reduced model

```
model = removeReactions(model,{'biomass_maintenance','biomass_maintenance_out/in'});
[n,e] = size(model.R);
fprintf('Nb of nbcs is %d\n','Nb of e is %d\n',n,e); fprintf('Nb of nbcs/nbcs is %d,%d\n',n,e);
```

Nb of nbcs

5000 5000 nbcs..

First close all exchange reactions, except the biomass reaction

```
model.setExchange(0)/crap(model.rxn,'biomass_reaction')=0;
model.lb(-model.setExchange)=0;
model.ub(-model.setExchange)=0;
```

Now force the biomass reaction to be active

```
model.lb(model.biomassRxn) = 1;
model.ub(model.biomassRxn) = 50;
```

Check if the model is feasible

```
PMASolution = optimizeCBModel(model,"max", 0, true);
if PMASolution.état == 1
    disp('Model is feasible. Nothing to do. ');
    return
else
    disp('Model is infeasible');
end
```

Model is infeasible

Relaxed flux balance analysis is implemented with the function relaxedFBA

```
% [solution] = relaxedFBA(model, relaxOptions)
```

The inputs are a COBRA model and an optional parameter vector

```
% INPUTS:
% model: COBRA model structure
% relaxOptions: Structure containing the relaxation options:
% + internalRelax:
% * 0 = do not allow to relax bounds on internal reactions
% * 1 = do not allow to relax bounds on internal reactions with finite bounds
% * 2 = allow to relax bounds on all internal reactions
%
% + exchangeRelax:
% * 0 = do not allow to relax bounds on exchange reactions
% * 1 = do not allow to relax bounds on exchange reactions of the type [0,0]
% * 2 = allow to relax bounds on all exchange reactions
%
% + steadyStateRelax:
% * 0 = do not allow to relax the steady state constraint See = b
% * 1 = allow to relax the steady state constraint See = 0
%
% + unblockedReactions - n x 1 vector indicating the reactions to be unblocked (optional)
% + unblockedReactions(i) = 1 : impose v(i) to be positive
% + unblockedReactions(i) = -1 : impose v(i) to be negative
% + unblockedReactions(i) = 0 : do not add any constraint
%
% + excludedReactions - n x 1 Bool vector indicating the reactions to be excluded from relaxation (optional)
% + excludedReactions(i) = false : allow to relax bounds on reaction i
% + excludedReactions(i) = true : do not allow to relax bounds on reaction i
%
% + excludedMetabolites - n x 1 Bool vector indicating the metabolites to be excluded from relaxation (optional)
% + excludedMetabolites(i) = false : allow to relax steady state constraint on metabolite i
% + excludedMetabolites(i) = true : do not allow to relax steady state constraint on metabolite i
%
% + lambda - trade-off parameter of relaxation on steady state constraint
% + alpha - trade-off parameter of relaxation on bounds
%
% Note, excludedReactions and excludedMetabolites override all other relaxation options.
```

Do not allow to relax bounds on any internal reaction

```
relaxOptim.internalRelax = 0;
```

Allow to relax bounds on all exchange reactions

```
relaxOptim.exchangeRelax = 2;
```

Do not allow to relax the steady state constraint $S^*v = 0$

```
relaxOptim.steadyStateRelax = 0;
```

Set the tolerance to distinguish between zero and non-zero flux

```
thresh = getGabraSolverParams('LP', 'thresh');  
relaxOptim.epsilon = thresh/1000; %=100
```

Set the trade-off parameter for relaxation of bounds (advanced user). A larger value of gamma will

```
relaxOptim.gamma = 10;
```

Set the trade-off parameter for relaxation on steady state constraint (advanced user)

```
relaxOptim.lambda = 10;
```

Call the relaxedFBA function, deal the solution, and set small values to zero

```
Tic;  
solution = relaxedFBA(model,relaxOptim);  
TimeTaken=Toc;  
[v,r,p,q] = deal(solution.v,solution.r,solution.p,solution.q);  
if 0  
    p([relaxOptim.epsilon) = 0;%lower bound relaxation  
    q([relaxOptim.epsilon) = 0;%upper bound relaxation  
    r([relaxOptim.epsilon) = 0;%steady state constraint relaxation  
end
```

The output is a solution structure with a 'stat' field reporting the solver status and a set of fields matching the relaxation of constraints given in the mathematical formulation of the relaxed flux balance problem above.

```
% OUTPUT:  
% solution: Structure containing the following fields:  
%     + stat - status  
%     + I = Solution Found  
%     + 0 = Infeasible  
%     + -1 = Invalid Input  
%     + r - relaxation on steady state constraints  $S^*v = 0$   
%     + p - relaxation on lower bound of reactions  
%     + q - relaxation on upper bound of reactions  
%     + v - reaction rate
```

Summarise the proposed relaxation solution

```
if solution.stat == 1  
    disp('relaxOptim.epsilon');  
  
    fprintf(' %s', ['Relaxed flux balance analysis problem solved in ' num2str(TimeTaken) ' seconds.']);  
    fprintf(' %s', num2str(r), ' steady state constraints relaxed');  
  
    fprintf(' %s', num2str(p-disgcutoff) & -abs(q)-disgcutoff & model.NIntNonZero), ' internal only lower bounds relaxed');  
    fprintf(' %s', num2str(q)-disgcutoff & -abs(p)-disgcutoff & model.NIntNonZero), ' internal only upper bounds relaxed');  
    fprintf(' %s', num2str(p)-disgcutoff & abs(q)-disgcutoff & model.NIntNonZero), ' internal lower and upper bounds relaxed');  
  
    fprintf(' %s', num2str(p)-disgcutoff & -abs(q)-disgcutoff & -model.NIntNonZero), ' external only lower bounds relaxed');  
    fprintf(' %s', num2str(q)-disgcutoff & -abs(p)-disgcutoff & -model.NIntNonZero), ' external only upper bounds relaxed');  
    fprintf(' %s', num2str(p)-disgcutoff & abs(q)-disgcutoff & -model.NIntNonZero), ' external lower and upper bounds relaxed');  
  
    fprintf(' %s', num2str(p)-disgcutoff | abs(q)-disgcutoff & -model.NIntNonZero), ' external lower or upper bounds relaxed');  
  
    minLB = min(min(model.lb),-max(model.lb));  
    minUB = min(-max(model.ub),min(model.lb));  
    IntNonFiniteBound = ((model.ub < minUB) & (model.lb > minLB));  
    fprintf(' %s', num2str(abs(p)-disgcutoff & IntNonFiniteBound), ' finite lower bounds relaxed');  
    fprintf(' %s', num2str(abs(q)-disgcutoff & IntNonFiniteBound), ' finite upper bounds relaxed');  
  
    extNonZero = ((model.ub == 0) & (model.lb == 0));  
    fprintf(' %s', num2str(p)-disgcutoff & extNonZero), ' lower bounds relaxed on fixed reactions (lb=ub=0)');  
    fprintf(' %s', num2str(q)-disgcutoff & extNonZero), ' upper bounds relaxed on fixed reactions (lb=ub=0)');  
  
else  
    disp('relaxedFBA problem infeasible, check relaxation fields');  
end
```

Relaxed flux balance analysis problem solved in 07.6492 seconds.

- 0 steady state constraints relaxed
- 0 internal only lower bounds relaxed
- 0 internal only upper bounds relaxed
- 0 internal lower and upper bounds relaxed
- 187 external only lower bounds relaxed
- 198 external only upper bounds relaxed
- 187 external lower and upper bounds relaxed
- 1182 external lower or upper bounds relaxed
- 684 flexible lower bounds relaxed
- 685 flexible upper bounds relaxed
- 683 lower bounds relaxed on fixed reactions (Unscaled)
- 685 upper bounds relaxed on fixed reactions (Unscaled)

TROUBLESHOOTING

Given an infeasible problem,

$$Sv = b,$$

$$l \leq v \leq u,$$

the relaxed flux balance analysis problem

$$\min_{v,v'} \quad \lambda \|v\|_0 + \tau \|p\|_0 + \eta \|q\|_0,$$

$$\text{s.t.} \quad Sv + r = b$$

$$l - p \leq v \leq u + q$$

$$p, q, r \geq 0$$

will always find a solution. However, relaxedFBA offers the user the option to disallow relaxation of some of the constraints. If too many constraints are not allowed to be relaxed, then relaxedFBA will report an infeasible problem. The fields of `relaxOption` should be reviewed. For example, if relaxation of steady state constraints is not allowed, yet `b` is nonzero and not in the range of the stoichiometric matrix, then the relaxedFBA problem will be infeasible. To allow the relaxation of the steady state constraint, `Sv'=0`, then use

```
%relaxOption.steadyStateRelax = 1;
```

If relaxedFBA does return a solution, but it is not biochemically realistic, then again review the fields of `relaxOption`, to allow or disallow relaxation of certain constraints. For example, to specifically disallow relaxation of the bounds on reaction with model name abbreviation 'myReaction', use

```
%relaxOption.excludeReactionsToRelax{0,1};
%relaxOption.excludeReactionsIf(strcmp(modelName,'myReaction'))==1;
```

To specifically disallow relaxation of the steady state constraint on a molecule species with model name abbreviation 'myMetabolite', then use:

```
%relaxOption.excludeMetaboliteRelax{false,0,1};
%relaxOption.excludeMetaboliteIf(strcmp(modelName,'myMetabolite'))==1;
```

Even if the set of relaxations are properly set, in a boolean sense, tweaking of the OCA card trade-off parameters can help narrow down to a biochemically realistic solution, by trading between the biochemical literature and the numerical results from relaxedFBA after tweaking the parameters. This flexibility is provided for the expert user. See `relaxFBA_capped.1.m`. A standard set of advanced parameters are:

```
%relaxOption.maxIteration = 1000; %max number of iterations of the capped.1 problem
%relaxOption.gamma0 = 0; %trade-off parameter of 10 part of v
%relaxOption.gamma1 = 0; %trade-off parameter of 11 part of v
%relaxOption.lambda0 = 10; %trade-off parameter of 10 part of r
%relaxOption.lambda1 = 0; %trade-off parameter of 11 part of r
%relaxOption.alpha0 = 10; %trade-off parameter of 10 part of p and q
%relaxOption.alpha1 = 0; %trade-off parameter of 11 part of p and q
%relaxOption.theta = 1; %parameter of capped.1 approximation
```

ANTICIPATED RESULTS

relaxedFBA will return a set of steady state constraints, lower bounds, and upper bounds, that are required to be relaxed to ensure that the FBA problem is feasible. It is necessary to analyse the solution biochemically, to see if it makes sense to relax the suggested constraints. The following code will report a summary of the results.

```
if solution.stat == 1
    printf(0);
    lineChangeFlag=0;
    if 1
        displayCutoffLower=relaxOption.epsilon;
        displayCutoffUpper=inf;
    else
        %useful for numerical debugging
        displayCutoffLower=-10;
        displayCutoffUpper=10;
    end
    if any(r)
        fprintf("\n\nv", "Steady state constraints relaxed");
        for i=1:n
            if abs(r(i))/displayCutoffLower && abs(r(i))/displayCutoffUpper
                fprintf("\n\n", modelNames{i});
            end
        end
    else
        fprintf("\n\nv", "No steady state constraints relaxed");
    end
    if any(p)
```

```

fprintf("No %s", "Lower bounds relaxed");
for j=2:n
    if abs(qj)>diagCutoffLower && abs(pj)>diagCutoffUpper && qj>=0
        rowAddList=model.row(i);
        formula = printRowFormula(model, rowAddList, printFlag, lineChangeFlag);
        fprintf("\nq(1%):",pj),formula(i));
    end
end
else
    fprintf("\nNo %s", "No lower bounds relaxed");
end
if any(q)
    fprintf("\nNo %s", "Upper bounds relaxed");
    for j=2:n
        if abs(qj)>diagCutoffLower && abs(qj)>diagCutoffUpper && qj>=0
            rowAddList=model.row(i);
            formula = printRowFormula(model, rowAddList, printFlag, lineChangeFlag);
            fprintf("\nq(1%):",qj),formula(i));
        end
    end
else
    fprintf("\nNo %s", "No upper bounds relaxed");
end
end
end

```

No steady state constraints relaxed

Lower bounds relaxed

```

1000 delp(x) ~>
1000 delp(x) ~>
1000 delp(x) ~>
1000 dlp(x) ~>
1000 dlp(x) ~>
1000 eilamp(r) ~>
1000 gpi_sig(r) ~>
1000 newZmpasgail_prev_h(r) ~>
1000 for_gly_dla_x_dlp(1) ~>
1000 lbfctf7tu(x) ~>
1000 dmp(x) ~>
1000 haddictlucose(x) ~>
1000 Tdt(x) ~>
1000 Tdt(x) ~>
1000 wdp(x) ~>
1000 edwp(x) ~>
1000 edvt(x) ~>
1000 xla_2(x) ~>
1000 equbal(x) ~>
1000 auehd(x) ~>
1000 awk_1(x) ~>
1000 wip(x) ~>
1000 kllfcur(x) ~>
1000 kinepl(x) ~>
1000 chofate(x) ~>
1000 chofate(x) ~>
1000 chio(x) ~>
1000 cnp(x) ~>
1000 cnp_h(x) ~>
1000 cns(x) ~>
1000 cns(x) ~>
1000 cnp_g(x) ~>
1000 cnp_g(x) ~>
1000 dng_h(x) ~>
1000 dnm(x) ~>
1000 dntag(x) ~>
8.10 micrct(x) ~>
1000 eulrues(x) ~>
1000 gbside_h(x) ~>
1000 gbside(x) ~>
1000 glyofa(x) ~>
1000 glyp2(x) ~>
1000 glyp4(x) ~>
1000 glyof(x) ~>
1000 gly(x) ~>
1000 h2o2(x) ~>
1000 ha(x) ~>
1000 hdm(x) ~>
1000 li(x) ~>
1000 ldp(x) ~>
1000 lmp(x) ~>
1000 kvi(x) ~>
1000 loydic(x) ~>
1000 loushd(x) ~>
1000 lousd(x) ~>
1000 lola(x) ~>
1000 lola(x) ~>

```

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

```

1000 address[x] ~v
1000 getarea[x] ~v
1000 retint1[x] ~v
1000 s1sum[x] ~v
1000 lang_ho[x] ~v
1000 lbf[x] ~v
1000 lmbdu[x] ~v
1000 dclru[x] ~v
1000 dthubelen1ansize[x] ~v
1000 thubelen1size[x] ~v
1000 out7ab1[x] ~v
1000 out7ab11[x] ~v
1000 out7sum[x] ~v
1000 7b1d1chul[x] ~v
1000 glia[x] ~v
1000 acas[x] ~v
1000 adn[x] ~v
1000 ahg[x] ~v
1000 aw_1[x] ~v
1000 awp_1[x] ~v
1000 ax2[x] ~v
1000 dnd_2[x] ~v
1000 dxyt[x] ~v
1000 fw2[x] ~v
1000 gal[x] ~v
1000 gla_1[x] ~v
1000 glyb[x] ~v
1000 glypxu[x] ~v
1000 gla_1[x] ~v
1000 lna[x] ~v
1000 k[x] ~v
1000 lni_1[x] ~v
1000 lnu_1[x] ~v
1000 nal_1[x] ~v
1000 nri_1[x] ~v
1000 na2[x] ~v
1000 pi[x] ~v
1000 pnu_1[x] ~v
1000 smr_1[x] ~v
1000 snu[x] ~v
1000 urnu[x] ~v
1000 ur1[x] ~v
1000 val_1[x] ~v
1000 pnia_M[x] ~v
1000 gly[x] ~v
1000 cya_1[x] ~v
1000 nla_1[x] ~v
1000 lbr_1[x] ~v
1000 gla_1[x] ~v
1000 ghu_1[x] ~v
1000 lye_1[x] ~v
1000 fur[x] ~v
1000 shd[x] ~v
1000 ai[x] ~v
1000 acpau[x] ~v
1000 cil[x] ~v
1000 d1ik[x] ~v
1000 fru[x] ~v
1000 gal1[x] ~v
1000 glir[x] ~v
1000 gluar[x] ~v
1000 glyv[x] ~v
1000 huan[x] ~v
1000 nal1[x] ~v
1000 plv[x] ~v
1000 spnd[x] ~v
1000 lyp_1[x] ~v
1000 uru[x] ~v
1000 xnu[x] ~v
1000 ppu[x] ~v
1000 pyv[x] ~v
1000 lru[x] ~v
1000 ndr[x] ~v
1000 aciaid[x] ~v
1000 quk[x] ~v
1000 dthub1[x] ~v
1000 lnuv[x] ~v
1000 phpyr[x] ~v
1000 CLNCH[x] ~v
1000 CLNCH[x] ~v

```

[illegible]

