检索与推荐中的排序和索引问题

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中国科学院自动化研究所





大纲

- > 背景介绍
- > 排序与索引
- > 近似近邻搜索
- > 总结与展望



多媒体内容索引与排序



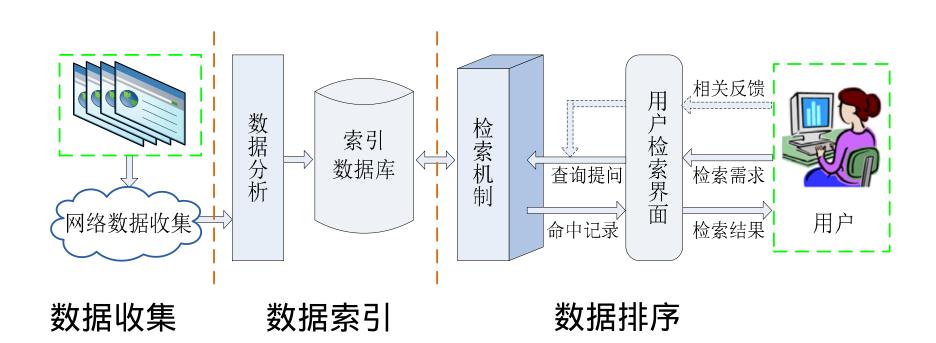
浩如烟海 无从下手

唯有索引 方得真经





网络多媒体检索的基本流程





Performance Evaluation

- Measures for top-N recommendation
 - NDCG(Normalized Discounted Cumulative Gain)

$$\mathrm{DCG_p} = rel_1 + \sum_{i=2}^p \frac{rel_i}{\log_2(i)}$$
 定义不唯一
$$\mathrm{DCG_p} = \frac{DCG_p}{IDCG_p}$$

$$\mathrm{Ideal\ DCG}$$

$$\mathrm{Ideal\ DCG}$$



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多媒体内容索引

- 排序准则 = 动态相关性 +静态相关性
 - 动态相关性:与查询词的相关性
 - TF, IDF
 - Title, Body, Anchor, URL
 - Proximity
 - ...

TF-IDF

$$w_{i,j} = tf_{i,j} \times \log\left(\frac{N}{df_i}\right)$$

 tf_{ij} = number of occurrences of i in j df_i = number of documents containing iN = total number of documents

- 静态相关性: 查询对象集的质量(权威性)
 - PageRank
 - PageQuality, Spam

— ...



与查询词的相关性

- 查询词在文档中出现的位置:前>后
- 出现在Tag "Title and Description"
- 出现在Tag "Keyword"
 - 注意:Spam 问题
 - If a query term appears in this tag,
 - It must appear somewhere in the body, otherwise penalize
 - Query term should not appear more than twice in this tag, otherwise penalize
- TF-IDF
- 文件大小:
 - < 40K is preferred, very large file is penalized</p>
- •



PageRank

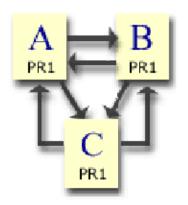
• PageRank的基本公式

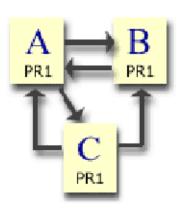
$$r(p) = \alpha \times \sum_{q:(q,p)\in \varepsilon} \frac{r(q)}{\omega(q)} + (1-\alpha) \times \frac{1}{N}$$

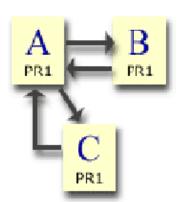
- q-网页 p的后向链接, i.e., q 指向 p
- $-\omega(q)$ 网页 q的前向链接数目.
- -r(q) 网页 q的PageRank.
- N 整个网络中网页的总数



PageRank







Page A = 1

Page B = 1

Page C = 1

Page A = 1.425

Page B = 1

Page C = 0.575

Page A = 1.4594

Page B = 0.7702

Page C = 0.7702



PageRank





优点:与查询无关的静态算法;

缺点:与主题无关,旧网页比新网页排名高

S. Brin, L. Page: The Anatomy of a Large-scale Hypertextual Web Search Engine Computer Networks and ISDN Systems. WWW 1998



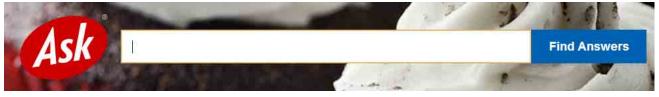
HITS



基本假设:

✓一个好的 "Authority"页面会被很多好的 "Hub"页面指向;

✓ 一个好的 "Hub"页面会指向很多好的 "Authority"页面;



优点:自然语言、社交网络取得很好效果;

缺点: 计算复杂、主题漂移、易作弊



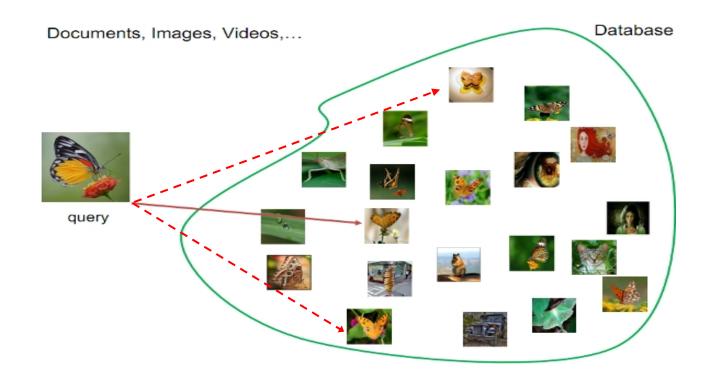
大纲

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近似近邻搜索

- 近似近邻:检索精度和检索时间的平衡
- 对大多数应用,近似近邻足够满足需求





■ How to index data?

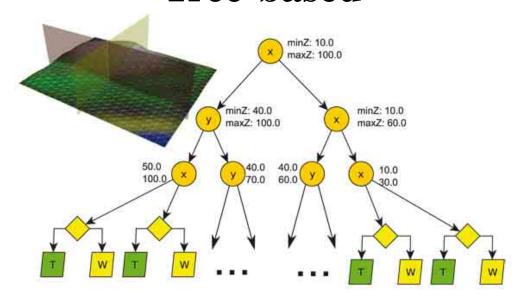
TF-IDF

$$w_{i,j} = tf_{i,j} \times \log\left(\frac{N}{df_i}\right)$$

 $tf_{i,j}$ = number of occurrences of i in j df_i = number of documents containing iN = total number of documents

Document analysis

Tree-based

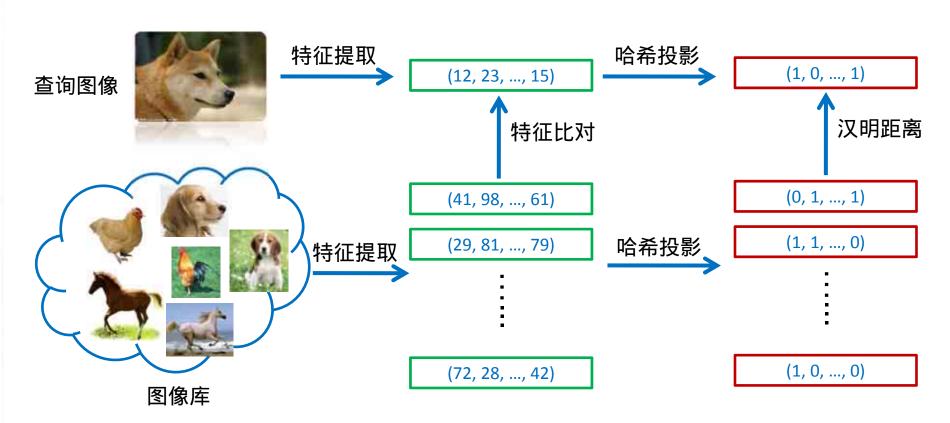


Curse of dimensionality

Pictures from web

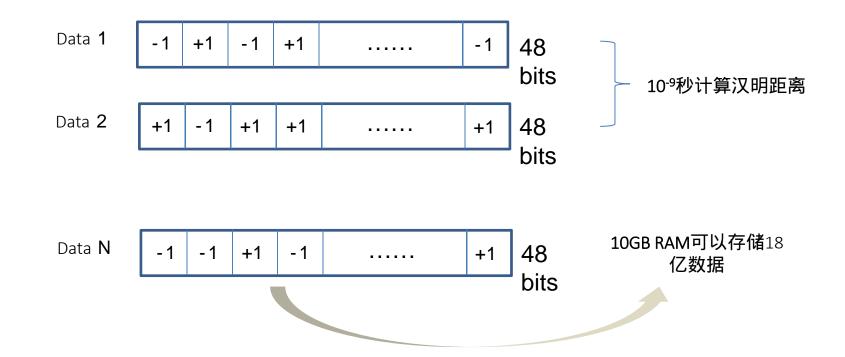


What is hashing?





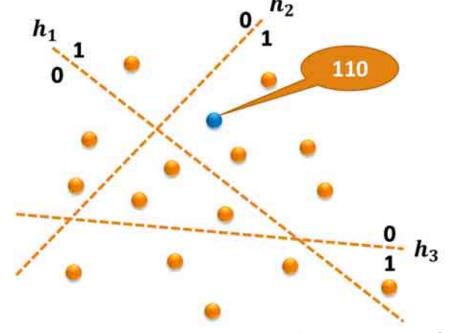
- 时间上高效:基于XOR操作的快速计算
- 存储上高效:基于位存储的紧致表达





- Locality Sensitive Hashing (LSH)
 - ✓ Date-independent, unsupervised
 - Map the data points with random projections

Hash Function: B = sgn(XW)



P.Indyk and R. Motwani. Approximate nearest neighbors: towards removing the curse of dimensionality. STOC'98





REPORT

A neural algorithm for a fundamental computing problem

Sanjoy Dasgupta¹, Charles F. Stevens^{2,3}, Saket Navlakha^{4,*}

+ See all authors and affiliations

Science 10 Nov 2017: Vol. 358, Issue 6364, pp. 793-796 DOI: 10.1126/science.aam9868

研究发现果蝇的嗅觉环路对相近的嗅觉产生相近的神经活动模式,可将一种味觉习得的行为应用于接触相似味觉时。研究者将其中的三种全新计算策略应用于计算领域,提出局部敏感哈希算法,该方法可有效改善近似检索的计算表现



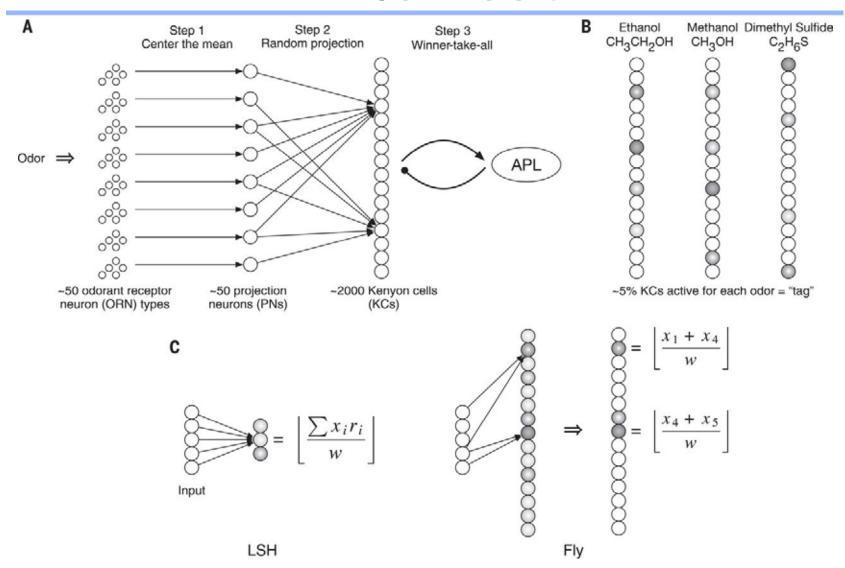


Fig. 1 Mapping between the fly olfactory circuit and locality-sensitive hashing (LSH).



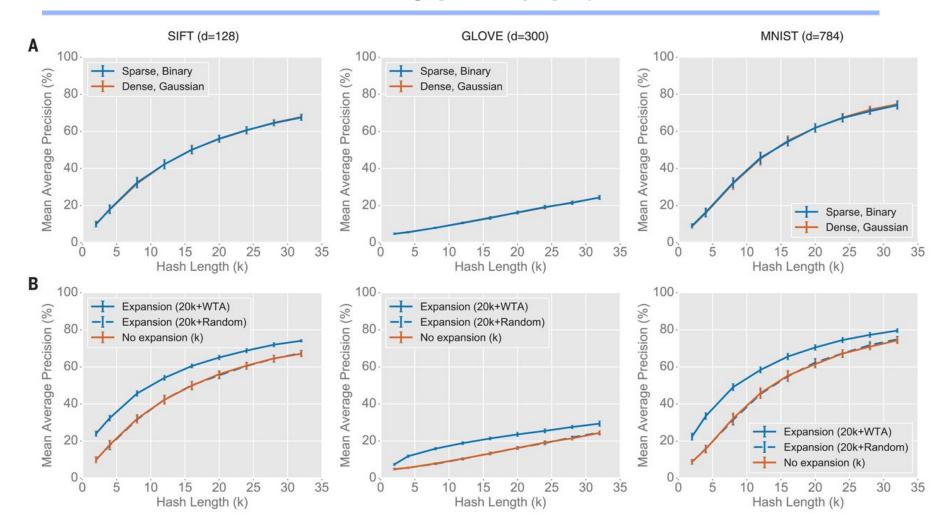


Fig. 2 Empirical comparison of different random projection types and tag-selection methods.



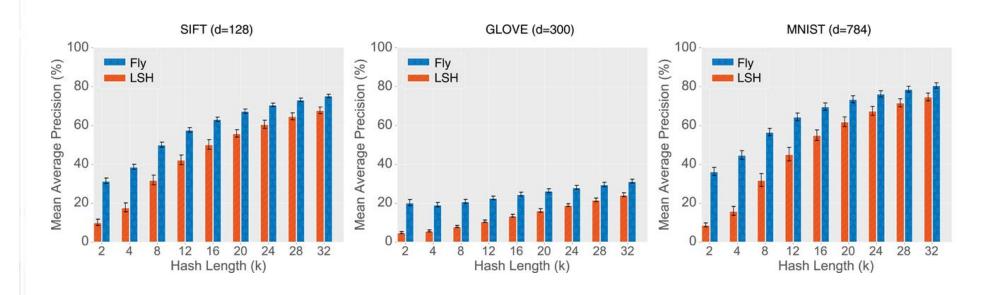
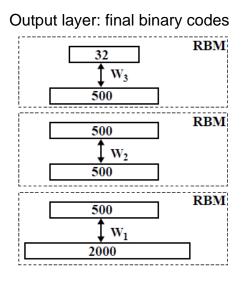
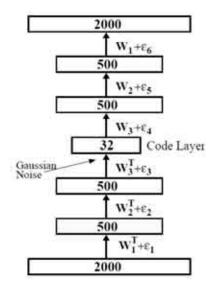


Fig. 3 Overall comparison between the fly algorithm and LSH.



- Restricted Boltzmann Machines (RBMs)
 - Date-dependent, unsupervised/supervised
 - Learn the binary codes layer by layer with deep networks







- Spectral Hashing (SH)
 - Date-dependent, unsupervised
 - Design compact binary codes based on spectral graph partitioning

$$\begin{aligned} & minimize : \sum_{ij} W_{ij} \| y_i - y_j \|^2 \\ & subject \ to : y_i \in \{-1,1\}^k \\ & \sum_i y_i = 0 \\ & \frac{1}{n} \sum_i y_i y_i^T = I \end{aligned} \qquad \begin{aligned} & minimize : trace(Y^T(D-W)Y) \\ & subject \ to : Y(i,j) \in \{-1,1\} \\ & Y^T 1 = 0 \\ & Y^T Y = I \end{aligned}$$

Y. Weiss, A.B. Torralba, and R. Fergus. Spectral hashing. NIPS 2008



- Semi-Supervised Hashing (SSH)
 - Date-dependent, semi-supervised
 - Combines the empirical loss over the labeled data with other desirable constraints over both labeled and unlabeled data.

$$\max J(W) = \operatorname{tr} (W^T X_l S X_l^T W) + \eta * \operatorname{tr} (W^T X X^T W)$$

- A sequential learning scheme (SPLH) is also developed for hash function leaning.
- J. Wang, S. Kumar, and S.-F. Chang. Semi-supervised hashing for large scale search. TPAMI 2012.

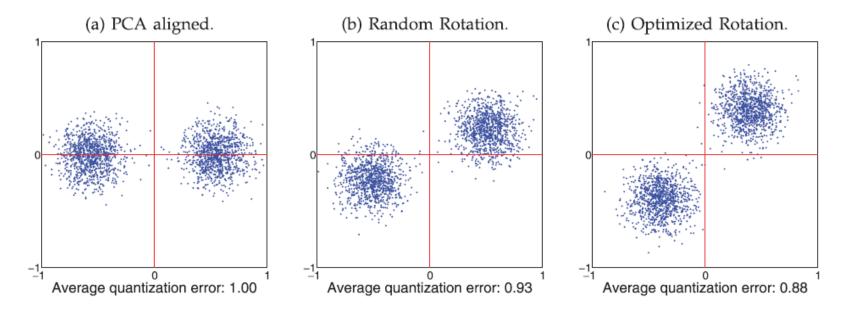


26

基于哈希的表示

• 迭代量化 (Iterative Quantization, ITQ)

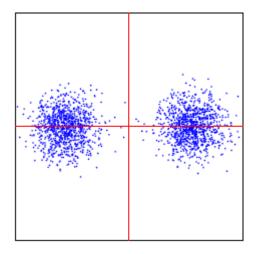
$$min||B - XWR||_F^2$$
, $s.t.R^TR = I$



Yunchao Gong, et.al.Iterative Quantization: A Procrustean Approach to Learning Binary Codes for Large-scale Large Retreival. CVPR 2011, IEEE TPAMI 2013.



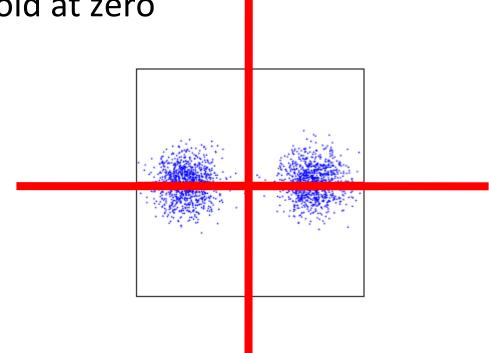
- Baseline scheme:
 - Find PCA embedding of the data
 - For a c-bit code, take top c PCA directions and threshold at zero





- Baseline scheme:
 - Find PCA embedding of the data

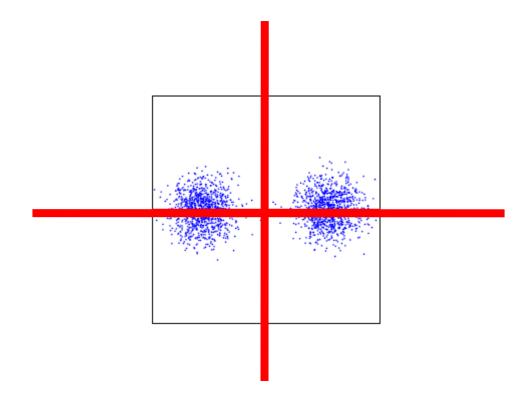
For a *c*-bit code, take top *c* PCA directions and threshold at zero





Problem with PCA

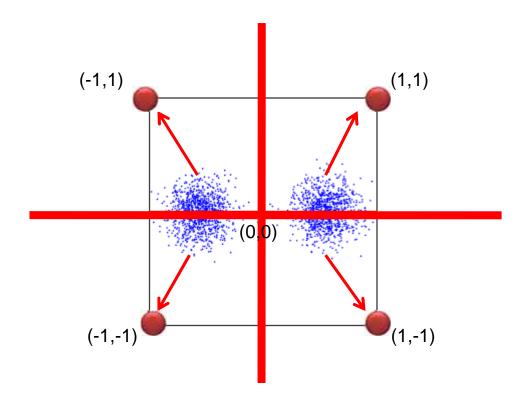
 Variance of different dimensions is not balanced





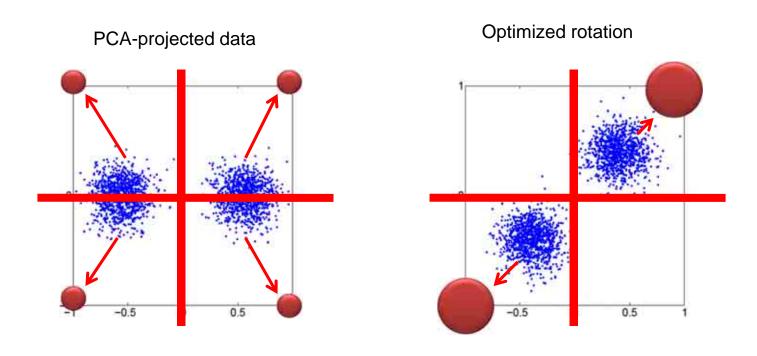
Binary coding as quantization

 Key idea: similarity-preserving codes should have low quantization error





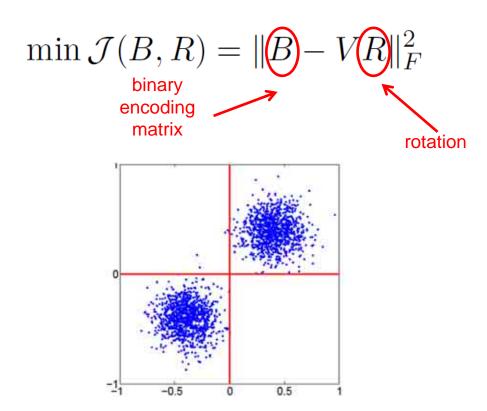
 Rotate the PCA-projected data to minimize quantization error





$$\min \mathcal{J}(B,R) = \|B - VR\|_F^2$$
 PCA-projected data







$$\min \mathcal{J}(B, R) = ||B - VR||_F^2$$
subject to $B \in \{-1, 1\}^{n \times c}, R^T R = I$

- B is an n x c matrix where each row is the binary string encoding a data point
- Vis a matrix of PCA-projected data
- -R is a $c \times c$ rotation matrix

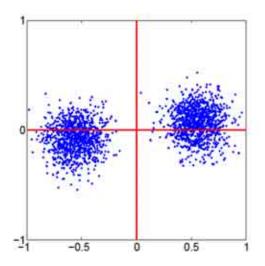


$$\min \mathcal{J}(B, R) = ||B - VR||_F^2$$
subject to $B \in \{-1, 1\}^{n \times c}, R^T R = I$

- Alternating minimization:
 - Initialize R to a random rotation
 - Fix R, solve for B
 - Fix B, solve for R
 - Iterate until convergence



Initialization

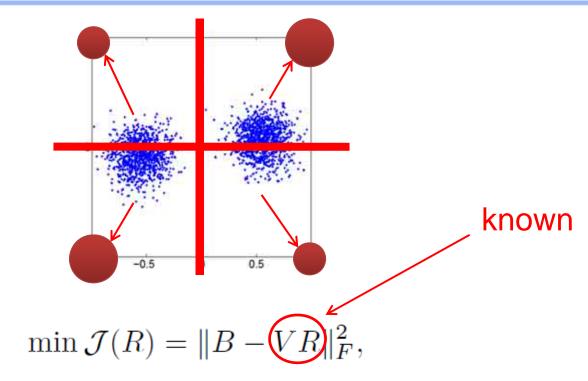


randomly rotated data

$$\min \mathcal{J}(R) = \|B - VR\|_F^2,$$



Fix *R*, find *B*

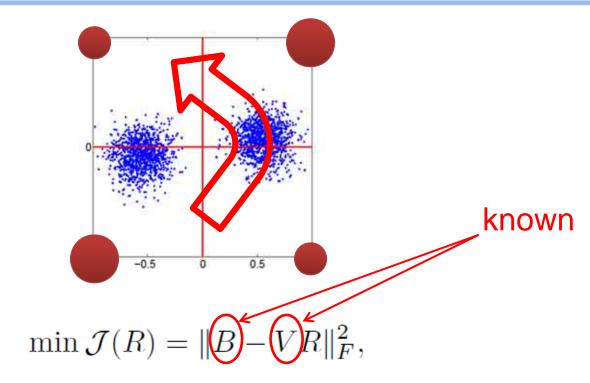


 Optimal B is given by thresholding rotated coordinates:

$$T = VR$$
 $\mathcal{J}(B) = \sum_{i=1}^{n} \sum_{j=1}^{c} B_{i,j} T_{i,j}.$ $B_{ij} = \begin{cases} 1, & \text{if } T_{i,j} \ge 0; \\ -1, & \text{otherwise.} \end{cases}$



Fix B, find R



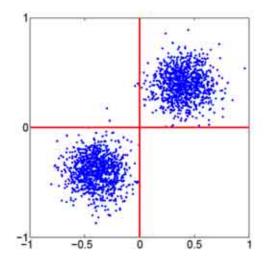
 Optimal R is found by solving the orthogonal Procrustes problem:

$$B^T V = S \Omega \hat{S}^T \qquad R = \hat{S} S^T.$$

P. Schonemann. A generalized solution of the orthogonal Procrustes problem. Psychometrika, 31, 1966.



Iterate until convergence

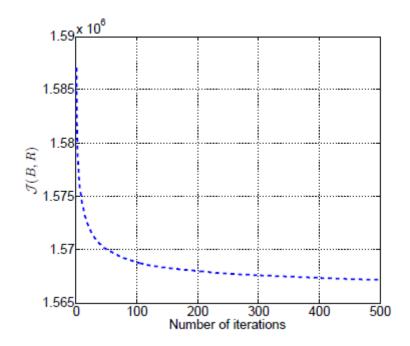


Locally optimal rotation



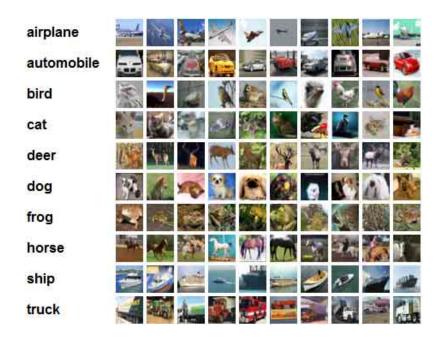
Iterate until convergence

• Behavior of the objective function:



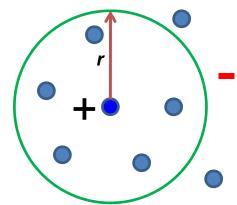


- CIFAR dataset: ~60,000 images, 11 categories
 - "Tiny images" converted to 320-dimensional gist feature vectors

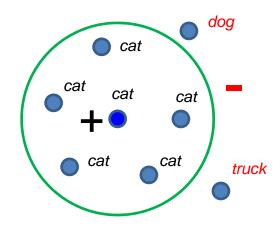




- Euclidean neighbor retrieval
 - Ground truth neighborhood radius defined by average distance to 50th nearest neighbor
 - Performance measured by area under the recall-precision curve (mAP)

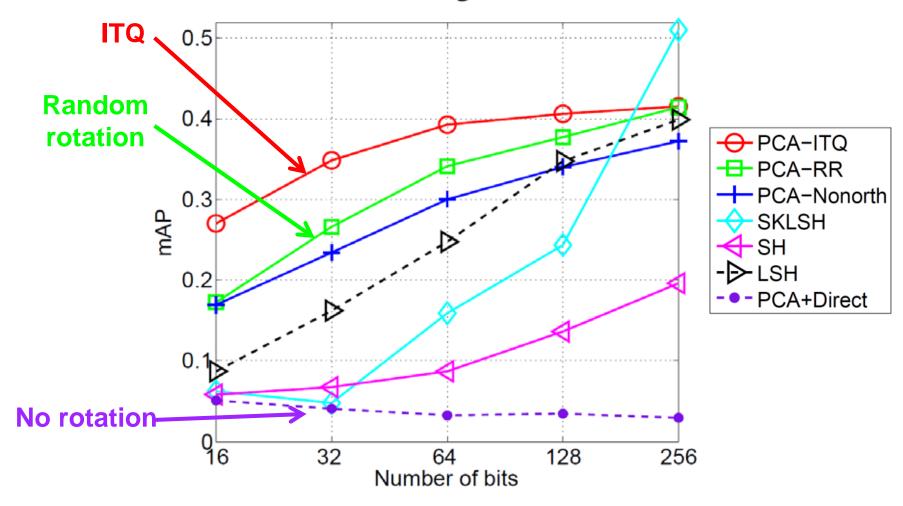


- Semantic neighbor retrieval
 - Ground truth defined by class label
 - Performance measured by average precision of top 500 retrieved matches

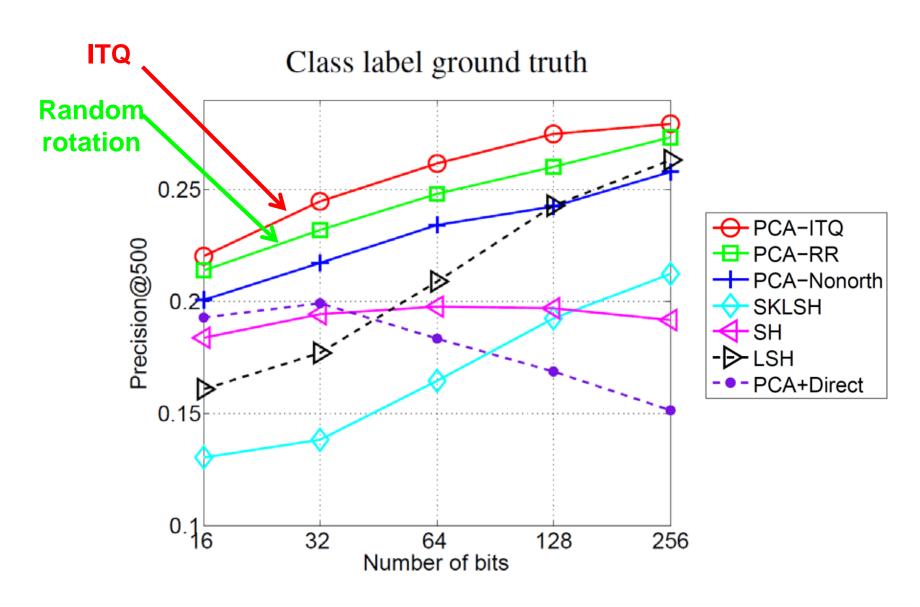




Euclidean ground truth









在实际检索系统中 数据是流式的、不 断更新的 在线更新

大规模

对于超大规模数据, 很难将其载入内存 进行哈希函数学习



全部数据 = 之前数据 + 新数据

- 综合所有新老数据,重新学习
- 超高的时间和计算代价

在线学习: Passive-Aggressive (PA) 算法 (JMLR'06)

$$W^t = \arg\min_W \frac{1}{2} \|W - W^{t-1}\|_F^2 + C\xi$$
 s.t.
$$L(W; \langle (x_i^t, x_j^t), s_{ij} \rangle) \leq \xi \text{ and } \xi \geq 0$$

Online Hashing (IJCAI'13)



- 矩阵素描(sketching or coreset)
 - ✓一个比原始矩阵小的多的矩阵
 - ✓保留原始矩阵的大多数特性

$$\forall x, \|x\| = 1$$
 $\|P^T x\|^2 - \|Q^T x\|^2 \le \varepsilon \|P\|_F^2$



Frequent Directions (FD)

Lemma 1. (Liberty [21]) Apply Algorithm 1 to matrix P to obtain a sketch Q with prescribed l, then

$$\forall x, \|x\| = 1$$
 $0 \le \|P^T x\|^2 - \|Q^T x\|^2 \le \frac{2}{l} \|P\|_F^2$

or

$$0 \le \|PP^T - QQ^T\|_2 \le \frac{2}{l} \|P\|_F^2$$

Edo Liberty, "Simple and Deterministic Matrix Sketching". SIGKDD 2013 (Best paper award)



广泛意义上的PCA Hashing:

$$\max_{W \in \mathbb{R}^{d \times r}} tr(W^T (X - \mu)(X - \mu)^T W)$$
 s.t.
$$W^T W = I_r$$

其中 μ 是全体数据的均值向量.

能否为矩阵 $X - \mu$ 在线维护一个素描矩阵 Y , 以至于

$$YY^T \approx (X - \mu)(X - \mu)^T$$



■均值漂移问题:

在流式问题中,数据在不断更新,因此数据的均值 μ 也会不断变化.

给每个数据块加一个虚拟的样本:

■ 对于流式数据 $X_t = [D_1, D_2, ..., D_t]$, 重新设计数据矩阵 E_t :

$$E_{t} = [\mathcal{D}_{1} - \overline{\mathcal{D}_{1}}, \qquad \mathcal{D}_{2} - \overline{\mathcal{D}_{2}}, \sqrt{\frac{n_{1}m_{2}}{n_{1} + m_{2}}} (\overline{\mathcal{D}_{2}} - \mu_{1}), \cdots,$$

$$\mathcal{D}_{i} - \overline{\mathcal{D}_{i}}, \sqrt{\frac{n_{i-1}m_{i}}{n_{i-1} + m_{i}}} (\overline{\mathcal{D}_{i}} - \mu_{i-1}), \cdots,$$

$$\mathcal{D}_{t} - \overline{\mathcal{D}_{t}}, \sqrt{\frac{n_{t-1}m_{t}}{n_{t-1} + m_{t}}} (\overline{\mathcal{D}_{t}} - \mu_{t-1})]$$



基于以上的设计,在任意适合 t,可以证明:

$$E_t E_t^T = cov(X_t)$$

通过为新设计的矩阵 E_t 在线维护素描矩阵 Y, 我们可以基于这个新的小矩阵 Y学习哈希函数.

Algorithm 2 Zero Mean Sketching

Input: Streaming data chunk $\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_k$, All zeros matrix Y of size $d \times l$.

- 1: Sketch $\mathcal{D}_1 \overline{\mathcal{D}_1}$ into Y with Algorithm 1
- 2: $n \leftarrow m_1$ and $\mu \leftarrow \overline{\mathcal{D}_1}$
- 3: **for** i = 2: k, **do**
- 4: Sketch $[\mathcal{D}_i \overline{\mathcal{D}_i}, \sqrt{\frac{nm_i}{n+m_i}}(\overline{\mathcal{D}_i} \mu)]$ into Y
- 5: $\mu \leftarrow \frac{n\mu}{n+m_i} + \frac{m_i \overline{D_i}}{n+m_i}$ [update the data mean]
- 6: $n \leftarrow n + m_i$ [update the data size]
- 7: end for



对比实验

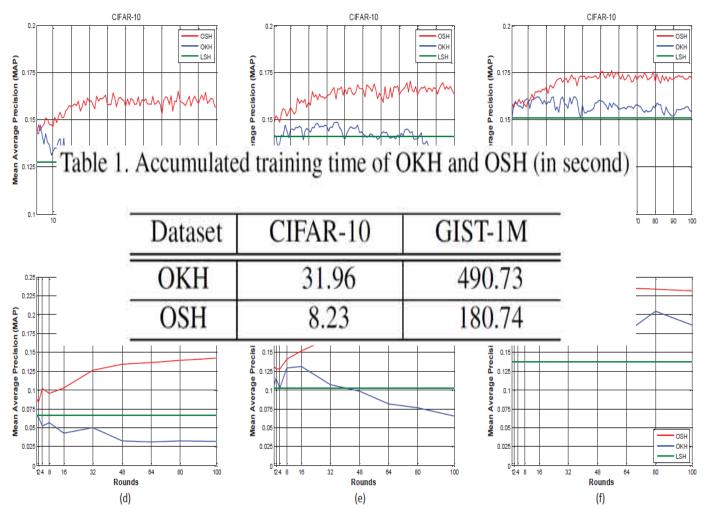
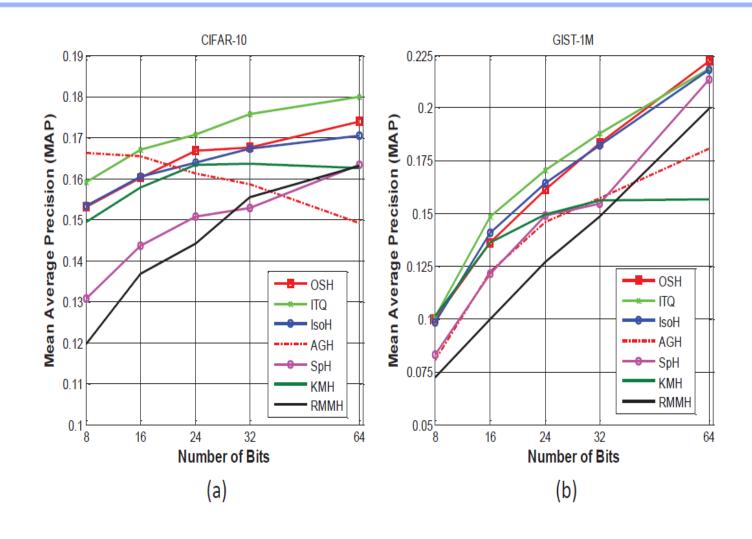


Figure 3. (a)(b)(c) Mean average precision (MAP) on CIFAR-10 dataset at each round with 16, 32, 64 bits. (d)(e)(f) MAP on GIST-1M dataset at each round with 16, 32, 64 bits. (Best viewed in color)

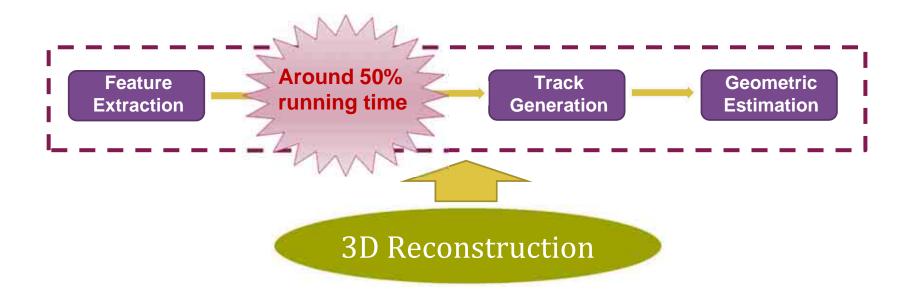


对比实验



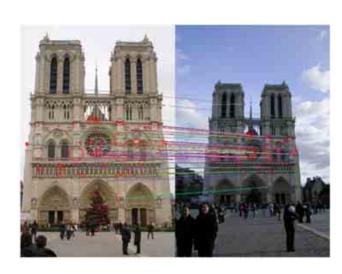


□ 三维重建主要包括以下四步:



# Images	# Cores	Match Time	Reconstruction Time	Largest Component
150,000	496	13 Hours	8 Hours	2,106





特征匹配

- > 图像搜索
- > 三维重建
- > 图像分类
- > 目标识别

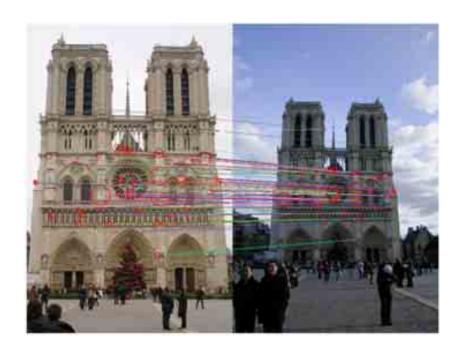
典型应用

- ➤ 匹配复杂度太高: *O*(N*(N-1) * M²)
- ➤ 硬件加速成本高:GPU、并行



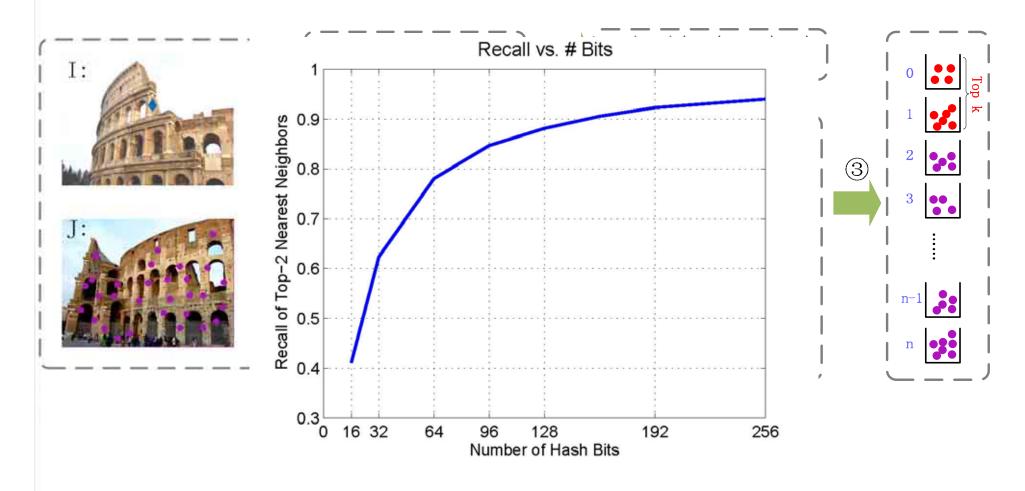
□ 现有特征匹配方法可以分为三大类:

- Point matching
- Line matching
- Region matching



Point matching is searching in essence!





Jian Cheng et al., "Fast and Accurate Image Matching with Cascade Hashing for 3D Reconstruction". CVPR 2014





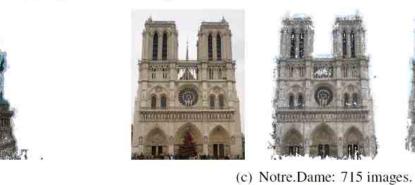




(a) Taj.Mahal: 189 images.











(b) Statue.of.Liberty: 674 images.





(d) Colosseum: 1357 images.





Method	Taj.Mahal			Statue.of.Liberty			
Wiethod	T_{Match}	Speed-Up	Points	T_{Match}	Speed-Up	Points	
Bruto	52575c	1.00~	12/038	30608c	1.00~	\$8001	
KI		_	*		Ļ	3674	
比brute force方法快上百倍							
CasH	CasH 3						
比通用的k-d tree方法快10-40倍							
Me	ענונדע	-u tre			- 1 010		
171	T_{Match}	Speed-Up	Points	T_{Match}	Speed-Up	Points	
Brute	396729s	1.00×	358121	12307s	1.00×	540308	
KDTree	60663s	6.54×	347056	2430s	5.06×	445774	
LDAHash	13136s	30.20×	413348	851s	14.46×	492040	
CasHash-8Bit	2266s	$175.08 \times$	484960	222s	55.44×	393408	
CasHash-10Bit	1354s	293.01×	400673	196s	62.79×	512508	



通过测试确信它在不同 数据集上都非常有效

OpenMVG (open Multiple View Geometry)



I'm actually testing it to ensure it works well on different datasets and that I have a code that works as good as your original version.

https://github.com/openMVG/openMVG/issues/194



Theia Vision Library

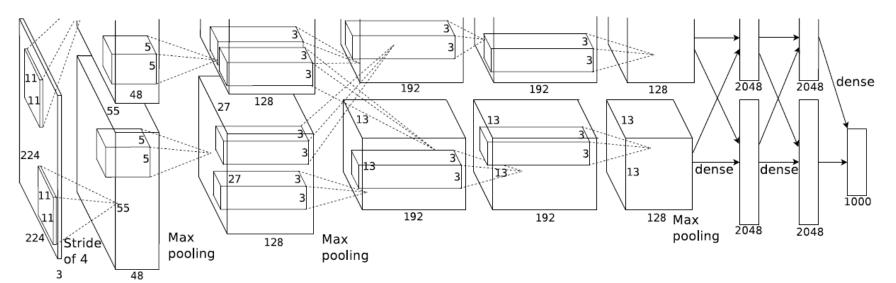
这是非常有用的见解 和足够简单的想法

it's a really useful insight and is a simple enough idea.

https://github.com/kip622/Theia



- 卷积神经网络(convolutional neural network)
 - -特征表征能力更强



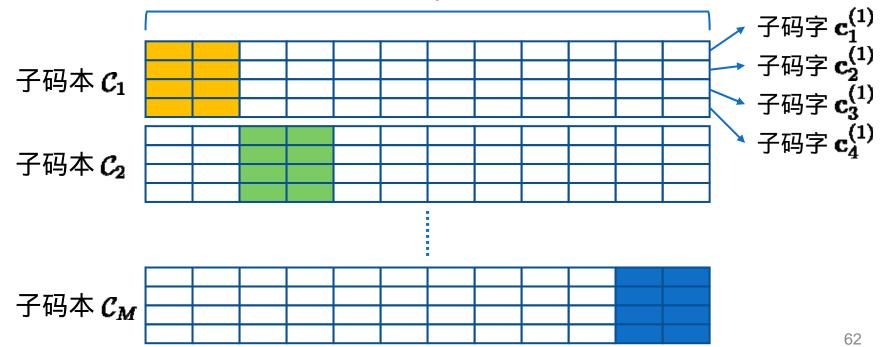


Hash—>PQ

- 乘积量化 (product quantization, PQ)
 - □ 将样本特征量化为M个子码字之和的形式

$$q\left(\mathbf{x}\right) = \sum_{m=1}^{M} q_m\left(\mathbf{x}\right), \ q_m\left(\mathbf{x}\right) \in \mathcal{C}_m$$

D维





Hash—>PQ

- 乘积量化
 - □ 优化目标:最小化量化误差

$$\min_{c_1, \dots, c_M} \sum_{\mathbf{x}} \left\| \mathbf{x} - q(\mathbf{x}) \right\|_2^2$$

□可等价为分别优化各个子空间下的子码本

$$\min_{\mathcal{C}_{m}} \sum_{\mathbf{x}} \left\| u_{m} \left(\mathbf{x} \right) - q_{m} \left(\mathbf{x} \right) \right\|_{2}^{2}$$

- □ 求解方法: k均值聚类
- 时间复杂度: O(TkND)

难以应用于大规模数据集



Fully-connected Layer:

$$T(c_t) = \langle W_{c_t}, S \rangle$$

Convoluational Layer:

$$T_{p_t}(c_t) = \sum_{(p_k, p_s)} \langle W_{c_t, p_k}, S_{p_s} \rangle$$



For the fully-connected layer, we split the weighting vector W_{c_t} and layer input S into M sub-vectors, each of $C'_s = C_s/M$ dimensions:

$$T(c_t) = \langle W_{c_t}, S \rangle$$

$$= \sum_{m=1}^{M} \langle W_{c_t}^{(m)}, S^{(m)} \rangle$$

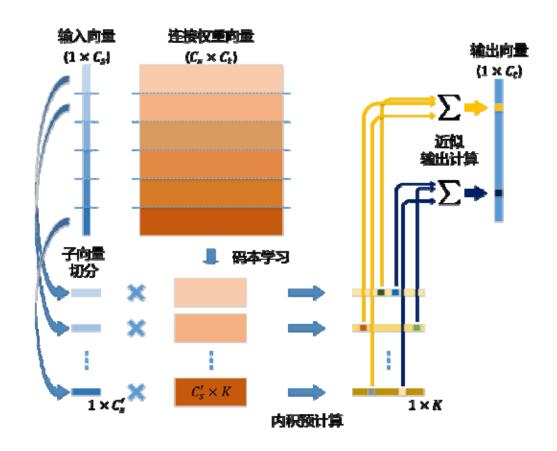
We quantize each sub-vector $W_{c_t}^{(m)}$ with:

$$W_{c_t}^{(m)} \approx D^{(m)} B_{c_t}^{(m)}$$

where $W_{c_t}^{(m)} \in \mathbb{R}^{C_s' \times 1}$, $D^{(m)} \in \mathbb{R}^{C_s' \times K}$, and $B_{c_t}^{(m)} \in \{0, 1\}^{K \times 1}$

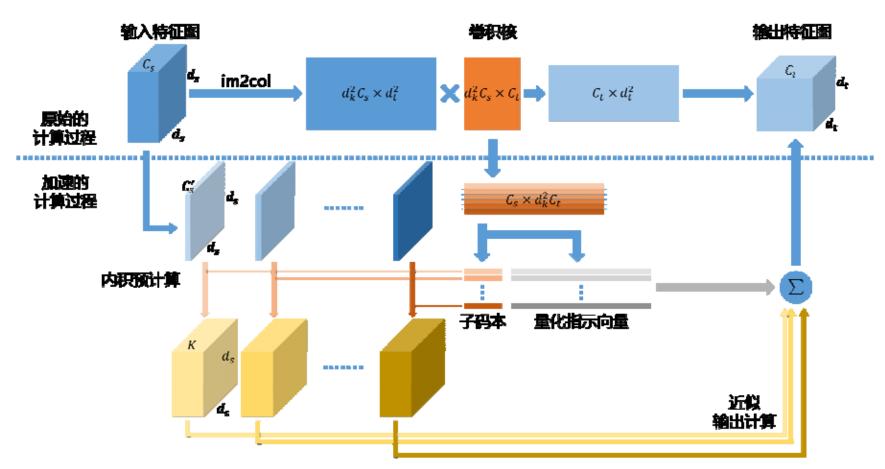


• 全连接层的参数量化与加速计算





• 卷积层的参数量化与加速计算





- 参数量化的求解方法
 - 最小化网络参数的量化误差

$$\min_{\mathbf{D}, \mathbf{B}} \ \left\| \mathbf{W} - \tilde{\mathbf{W}} \right\|_F^2$$
 s.t. $\tilde{\mathbf{W}} = g\left(\mathbf{D}, \mathbf{B}\right)$

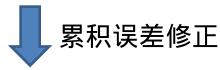
- 最小化网络输出的近似误差

$$\min_{\mathbf{D}, \mathbf{B}} \| f(\mathbf{X}; \mathbf{W}) - f(\mathbf{X}; \tilde{\mathbf{W}}) \|_F^2$$
s.t. $\tilde{\mathbf{W}} = g(\mathbf{D}, \mathbf{B})$



- 累积误差问题
 - 网络中浅层的参数被量化后,会改变深层的输入 $\underset{F}{\text{min}} \| f(\mathbf{X}; \mathbf{W}) f(\mathbf{X}; \tilde{\mathbf{W}}) \|_{F}$

s.t.
$$\tilde{\mathbf{W}} = g(\mathbf{D}, \mathbf{B})$$



$$\min_{\mathbf{D}, \mathbf{B}} \| f(\mathbf{X}; \mathbf{W}) - f(\tilde{\mathbf{X}}; \tilde{\mathbf{W}}) \|_F^2$$
s.t. $\tilde{\mathbf{W}} = g(\mathbf{D}, \mathbf{B})$



• 分类性能对比

- 数据集: MNIST

方法	三月	层网络	五层网络		
<i>/14</i>	压缩倍数	分类错误率	压缩倍数	分类错误率	
原始网络	-	1.35%	-	1.12%	
RER [33]	8.0×	2.19%	8.0×	1.24%	
LRD [165]	8.0×	1.89%	8.0×	1.77%	
DK [50]	8.0×	1.71%	8.0×	1.26%	
NN-ES [48]	8.0×	1.69%	8.0×	1.35%	
HashNets [48]	8.0×	1.43%	8.0×	1.22%	
Q-CNN ¹	10.9×	1.42%	13.0×	1.34%	
Q - CNN^2	10.9×	1.39%	13.0×	1.19%	



• 分类性能对比

- 数据集:ILSVRC-12

网络	方法	超参		加速倍数	压缩倍数	T1 八米烘!! 並 ^	m r 八米烘2p岁 x
M给 		卷积层	全连接层	加壓恒剱	压细恒数	Top-1 分类错误率↑	Top-5 分类错误率↑
	BC [43]	-	-	2.00×	$32.00 \times$	21.20%	19.20%
AlexNet	BWN [46]	-	-	$2.00 \times$	$32.00\times$	2.80%	3.20%
	DC [38]	-	-	-	$9.00/35.00\times$	0.00%	-0.03%
	Q-CNN ²	8/128	3/32	$4.05 \times$	15.10×	1.38%	0.84%
		8/128	4/32	$4.15 \times$	$18.32 \times$	1.46%	0.97%
CaffeNet	Q-CNN ²	8/128	3/32	4.04×	15.10×	1.43%	0.99%
		8/128	4/32	$4.16 \times$	$18.32 \times$	1.54%	1.12%
CNN-S	Q-CNN ²	8/128	3/32	5.69×	15.93×	1.48%	0.81%
		8/128	4/32	$5.78 \times$	$19.57\times$	1.64%	0.85%
VGG-16	DC [38]	-	-	-	$13.00/49.00 \times$	-0.33%	-0.41%
	Q-CNN ²	8/128	3/32	4.92×	16.06×	1.02%	0.38%
		8/128	4/32	4.94×	19.60×	1.13%	0.45%



• 移动设备上的运算效率

- 数据集:ILSVRC-12

网络	方法	运行时间	硬盘存储	内存占用	Top-5 分类错误率
AlexNet	原始网络	2.93s	232.56MB	264.74MB	19.74%
	Q-CNN ²	0.95s	12.60MB	74.65MB	20.70%
CNN-S	原始网络	10.58s	392.57MB	468.90MB	15.82%
	Q-CNN ²	2.61s	20.13MB	129.49MB	16.68%



大纲

-)背景介绍
- > 排序与索引
- > 近似近邻搜索
- > 总结与展望



总结与展望

• 排序与索引是检索和推荐的本质问题

• 正在走向成熟,但依然方兴未艾

• 与其它学科交叉融合



Thanks! Q&A

