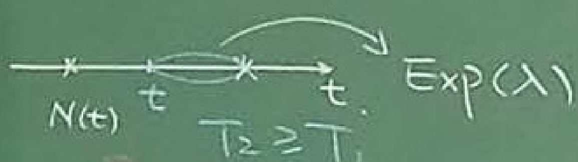
$$= \frac{P(N(s)=1)\,P(N(t)-N(s)=0)}{P(N(t)=1)} = \frac{\lambda s \exp(-\lambda s)\exp(-\lambda(t-s))}{\lambda t \exp(-\lambda t)}.$$

$$= \frac{s}{t} \qquad f_T(t\,|\,N(t)=1) = \frac{d}{ds} F_T(s\,|\,N(t)=1) = \frac{1}{t}, \quad (0 \le s \le t)$$

$$F_{T_1,T_2}(s_1, s_2\,|\,N(t)=2) = P(T_1 \le s_1, T_2 \le s_2\,|\,N(t)=2)$$

$$= P(N(s_1)=2, N(s_2)-N(s_1)=0, N(t)-N(s_2)=0\,|\,N(t)=2)$$
$$+ P(N(s_1)=1, N(s_2)-N(s_1)=1, N(t)-N(s_2)=0\,|\,N(t)=2)$$

$N(t).\quad S_1, S_2, \cdots, S_n.\quad S_n - S_{n-1} = T_n \sim Exp(\lambda).$



$$S_{N(t)+1} - S_{N(t)} \Rightarrow E(S_{N(t)+1} - S_{N(t)}\,|\,N(t)=n)$$

$N(t)=1.\quad F_T(s\,|\,N(t)=1) = P(T \le s\,|\,N(t)=1)$

$$= P(N(s)=1, N(t)-N(s)=0\,|\,N(t)=1) = \frac{P(N(s)=1, N(t)-N(s)=0, N(t)=1)}{P(N(t)=1)}$$

$$f_{T_1,T_2,\cdots,T_n}(s_1,s_2,\cdots,s_n \mid N(t)=n)$$

$$= \lim_{\substack{\Delta s_k \to 0 \\ k=1,\cdots,n}} \frac{1}{\Delta s_1\cdots\Delta s_n} P(s_1<T_1\le s_1+\Delta s_1,\cdots, s_n<T_n\le s_n+\Delta s_n \mid N(t)=n)$$

$$N(t)-N(\Delta s_1)-N(\Delta s_2)-\cdots-N(\Delta s_n)=0$$

$$= \lim_{\substack{\Delta s_k \to 0 \\ k=1,\cdots,n}} \frac{1}{\Delta s_1\cdots\Delta s_n} P(N(s_1+\Delta s_1)-N(s_1)=1,\cdots, N(s_n+\Delta s_n)-N(s_n)=1 \mid N(t)=n)$$

$$= \lim_{\substack{\Delta s_k \to 0 \\ k=1,\cdots,n}} \frac{1}{\Delta s_1\cdots\Delta s_n} \frac{\prod_{k=1}^{n} P(N(s_k+\Delta s_k)-N(s_k)=1)\, P(N(t-\Delta s_1-\cdots-\Delta s_n)=0)}{P(N(t)=n)}$$

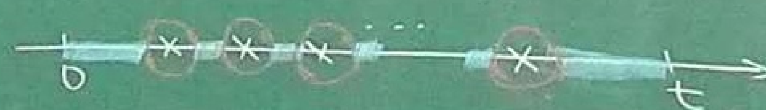
$$= \frac{P(N(s_1)=1)\,P(N(s_2)-N(s_1)=1)\,P(N(t)-N(s_2)=0)}{P(N(t)=2)} + \frac{P(N(s_1)=2)\,P(N(t)-N(s_1)=0)}{P(N(t)=2)}$$

$$(0\le s_1\le s_2\le t)$$

$$= \frac{\lambda s_1\,\lambda(s_2-s_1)\exp(-\lambda t)}{\frac{(\lambda t)^2}{2}\exp(-\lambda t)} + \frac{\frac{(\lambda s_1)^2}{2}\exp(-\lambda t)}{\frac{(\lambda t)^2}{2}\exp(-\lambda t)} = \frac{2(s_2-s_1)\cdot s_1}{t^2} + \frac{s_1^2}{t^2} = \frac{2s_2s_1-s_1^2}{t^2}$$

$$f_{T_1,T_2}(s_1,s_2 \mid N(t)=2) = \frac{\partial^2}{\partial s_1\partial s_2} F_{T_1,T_2}(s_1,s_2 \mid N(t)=2) = \frac{2}{t^2},\quad 0\le s_1\le s_2\le t$$

$N(t)=n$. "$\boxed{\text{Micro}}$cell"



$$= \lim_{\substack{\Delta s_k \to 0 \\ k=1,\cdots,n}} \frac{1}{\Delta s_1 \Delta s_2 \cdots \Delta s_n} \frac{\prod_{k=1}^{n} \left(\lambda \Delta s_k \exp(-\lambda \Delta s_k)\right) \exp\left(-\lambda\left(t - \sum_{k=1}^{n} \Delta s_k\right)\right)}{\frac{(\lambda t)^n}{n!} \exp(-\lambda t)}.$$

$$\int_0^{s_3} s_2 \, ds_2 = \frac{s_3^2}{2}$$

$$\int_0^{s_4} \frac{s_3^2}{2} \, ds_3 = \frac{s_4^3}{3!}$$

$$= \frac{n!}{t^n}, \quad 0 \le s_1 \le s_2 \le \cdots \le s_n \le t.$$

$$\int \cdots \int_{0 \le s_1 \le \cdots \le s_n \le t} \frac{n!}{t^n} \, ds_1 \cdots ds_n = \frac{n!}{t^n} \underline{\int_0^t \cdots \int_0^{s_3} \int_0^{s_2} \boxed{} \, ds_1 \, ds_2 \cdots ds_n} = \frac{t^n}{n!}$$

$$= \frac{n!}{t^n} \int_0^t \cdots \int_{s_{n-2}}^t \int_{s_{n-1}}^t ds_n \, ds_{n-1} \cdots ds_1$$

$$= \frac{s}{t}$$

$$\ldots = 1) = \frac{d}{ds} F_{T_1}(s \mid N(t$$

$$F_{T_1, T_2}(s \ldots = P(T_1 \le s_1, T_2 \ldots$$

$$= P(\ldots) = 0, \; N(t) = \ldots$$

$$+ P \ldots$$

$$f_{X_{(n)}}(x) = \frac{d}{dx} F_{X_{(n)}}(x) = n\left(F_X(x)\right)^{n-1} f_X(x).$$

$$F_{X_{(1)}}(x) = P(X_{(1)} \le x) = P(\min(X_1, \cdots, X_n) \le x)$$

$$= 1 - P(\min(X_1, \cdots, X_n) > x) = 1 - P(X_1 > x, \cdots, X_n > x)$$

$$= 1 - \left(P(X_1 > x)\right)^n = 1 - \left(1 - F_X(x)\right)^n \quad f_{X_{(1)}}(x) = n\left(1 - F_X(x)\right)^{n-1} f_X(x)$$

$$F_{X_{(k)}}(x) = P(X_{(k)} \le x). \quad f_{X_{(k)}}(x) = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(F_{X_{(k)}}(x + \Delta x) - F_{X_{(k)}}(x)\right)$$

$X_1, X_2, \cdots, X_n$, i.i.d. $f_X(x)$. $F_X(x)$

$$X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}.$$

$$X_{(1)} = \min(X_1, \cdots, X_n)$$

$$\vdots$$

$$X_{(n)} = \max(X_1, \cdots, X_n)$$

$$F_{X_{(n)}}(x) = P(X_{(n)} \le x).$$

$$= P(\max(X_1, \cdots, X_n) \le x)$$

$$= P(X_1 \le x, \cdots, X_n \le x)$$

$$= \prod_{k=1}^{n} P(X_k \le x) = \left(F_X(x)\right)^n$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} P(x < Z_{(k)} \le x + \Delta x).$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} P(Z_{(1)}, \cdots, Z_{(k-1)} \le x,\ x < Z_{(k)} \le x + \Delta x,\ Z_{(k+1)}, \cdots, Z_{(n)} > x+\Delta x)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \boxed{\left(F_Z(x)\right)^{k-1}} \cdot \left(1 - F_Z(x+\Delta x)\right)^{n-k} \left(F_Z(x+\Delta x) - F_Z(x)\right)$$

$$= \binom{n-k+1}{n-k}\binom{n}{k-1}\left(F_Z(x)\right)^{k-1}\left(1 - F_Z(x)\right)^{n-k} f_Z(x)$$

$$f_{Z_{(m)}, Z_{(k)}}(x_m, x_k),\quad m < k,\quad C = \binom{n}{m-1}\binom{n-m+1}{1}\binom{n-m}{k-m-1}\binom{n-k+1}{1}.$$

$P(x_m < Z \le x_k)$

$$= C \left(F_Z(x_m)\right)^{m-1} f_Z(x_m)\left(F_Z(x_k) - F_Z(x_m)\right)^{k-m-1} f_Z(x_k)\left(1 - F_Z(x_k)\right)^{n-k}.$$

$$f_{Z_{(1)}, \cdots, Z_{(n)}}(x_1, x_2, \cdots, x_n) = \binom{n}{1}\binom{n-1}{1}\cdots\binom{2}{1} f_Z(x_1) f_Z(x_2) \cdots f_Z(x_n)$$

$$= n! \prod_{k=1}^{n} f_Z(x_k), \quad x_1 \le x_2 \le \cdots \le x_n.$$

$f_{X_{(m)},X_{(k)}}(x_m,x_k)$. $m<k$. $C=\binom{n}{m-1}\binom{n-m+1}{1}\binom{n-m}{k-m-1}\binom{n-k+1}{1}$.

$$= C\,(F_X(x_m))^{m-1} f_X(x_m)\,(F_X(x_k)-F_X(x_m))^{k-m-1} f_X(x_k)\,(1-F_X(x_k))^{n-k}.$$

$$f_{X_{(1)},\cdots,X_{(n)}}(x_1,x_2,\cdots,x_n)=\binom{n}{1}\binom{n-1}{1}\cdots\binom{2}{1}f_X(x_1)f_X(x_2)\cdots f_X(x_n)$$

$$n!\boxed{\int\cdots\int_{x_1\le\cdots\le x_n}\prod_{k=1}^{n}f_X(x_k)\,dx_1\cdots dx_n}=\frac{n!}{n!}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\prod_{k=1}^{n}f_X(x_k)\,dx_1\cdots dx_n=1.$$

$P($ … $x<X_{(k)}\le x+\Delta x,\ X_{(k+1)},\cdots,X_{(n)}>x+\Delta x)$

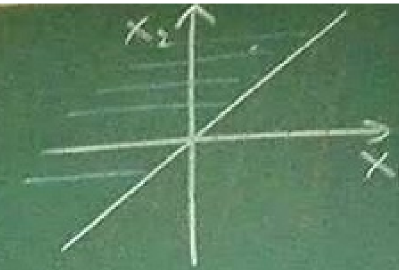
… $(1-F_X(x+\Delta x))^{n-k}\,(F_X(x+\Delta x)-F_X(x))$

$=$ … $(1-F_X(x))^{n-k}\,f_X(x)$

$m-1$ $k-m-1$ $n-k$

x ($X_{(m)}$) x ($X_{(k)}$)

$$\begin{cases} \{(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n \mid x_1 \leq x_2 \leq \cdots \leq x_n\}. \\ \{(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n \mid x_n \leq x_{n-1} \leq \cdots \leq x_1\}. \\ \{(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n \mid x_{\sigma(1)} \leq x_{\sigma(n-1)}, \leq \cdots \leq x_{\sigma(1)}\}. \ \forall \sigma. \end{cases}$$



$$g(x_1, x_2) = g(x_2, x_1) \qquad \int_{-\infty}^{+\infty}\int_{-\infty}^{x_2} g(x_1, x_2)\,dx_1 dx_2$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{x_2} g(x_2, x_1)\,dx_1 dx_2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{x_1} g(x_1, x_2)\,dx_2 dx_1.$$

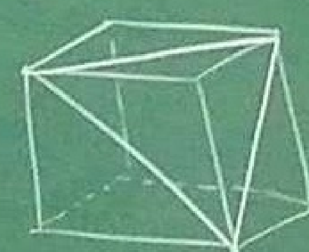
$$n! \int \cdots \int_{x_1 \leq \cdots \leq x_n} f_X(x_1) \cdots f_X(x_n)\,dx_1 \cdots dx_n.$$

X_1, X_2, X_3.
X_2, X_1, X_3
X_1, X_3, X_2
$\cdots$

$$= n! \int_0^t \cdots \int_0^{x_2} \boxed{\underline{f_X(x_1)} \cdots \underline{f_X(x_n)}}\,dx_1 \cdots dx_n.$$

Symmetric Function $g(x_1, \cdots, x_n) = g(x_{\sigma(1)}, \cdots, x_{\sigma(n)}) \qquad \sigma : P$

$$g(x_1, \cdots, x_n) = \prod_{k=1}^{n} f(x_k)$$



$$\frac{1}{\lambda} \qquad = E(S_{n+1}) - E(S_n \mid N(t)=n)$$

$$\lambda \qquad = \frac{n+1}{\lambda} - \frac{n}{n+1}\cdot t .$$

$$E(S_n \mid N(t)=n) = \int_0^t s\,\frac{n s^{n-1}}{t^n}\,ds. = \frac{1}{t^n}\int_0^t n s^n\,ds.$$

$$\underline{E(S_{N(t)+1} - S_{N(t)})} = \sum_{n=0}^{\infty}\left(\frac{n+1}{\lambda} - \frac{n}{n+1}t\right)\cdot\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$= \frac{1}{\lambda} + \frac{\lambda t}{\lambda}\sum_{n=0}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}\exp(-\lambda t) - \sum_{n=0}^{\infty}\left(1-\frac{1}{n+1}\right)t\,\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$t$$

$$\frac{1}{\lambda} + t\sum_{n=0}^{\infty}\frac{1}{n+1}\,\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$\frac{1}{\lambda} + \sum_{n=0}^{\infty}\frac{(\lambda t)^{n+1}}{(n+1)!}\exp(-\lambda t)$$

$$S_n(t_n \mid N(t)=n) = n\cdot\left(\frac{s}{t}\right)^{n-1}\cdot\frac{1}{t} = \underline{\frac{n s^{n-1}}{t^n}},\ \underline{0\le s\le t}$$

$$S_n \mid N(t)=n) = E(S_{n+1} - t \mid N(t)=n) + E(t - S_n \mid N(t)=n)$$

$$= E(S_{n+1} - t) + t - E(S_n \mid N(t)=n)$$

$$\frac{1}{\lambda} \quad = E(S_{n+1}) - E(S_n \mid N(t)=n)$$

$$\lambda \quad = \frac{n+1}{\lambda} - \frac{n}{n+1}\cdot t.$$

$$E(S_n \mid \underline{N(t)=n}) = \int_0^t s\,\frac{n s^{n-1}}{t^n}\,ds = \frac{1}{t^n}\int_0^t n s^n\,ds$$

$$\underline{E(S_{N(t)+1}-S_{N(t)})} = \sum_{n=0}^{\infty}\left(\frac{n+1}{\lambda}-\frac{n}{n+1}t\right)\cdot\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$= \frac{1}{\lambda} + \cancel{\frac{\lambda t}{\lambda}\sum_{n=0}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}\exp(-\lambda t)}\; t - \sum_{n=0}^{\infty}\left(1-\frac{1}{n+1}\right)t\,\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$= \frac{1}{\lambda} + t\sum_{n=0}^{\infty}\frac{1}{n+1}\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$x \le t$

$$= \frac{1}{\lambda} + \frac{1}{\lambda}\sum_{n=0}^{\infty}\frac{(\lambda t)^{n+1}}{(n+1)!}\exp(-\lambda t)$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{n!}\exp(-\lambda t)$$

$$B(t) = S_{N(t)+1} - t$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda}\left(1-\exp(-\lambda t)\right)$$

$$A(t) = t - S_{N(t)}$$

$$P(\boxed{A(t)>x}, B(t)>y) = \exp(-\lambda(x+y))$$