Image Processing and Analysis

Chapter 9 Morphological Image Processing

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Some basic concepts from set theory

- Let A be a set in Z^2. If $a = (a_1, a_2)$ is an element of A,then we write

  $$a \in A$$

  Similarly, if a is not an element of A, we write

  $$a \notin A$$

  The set with no elements is called the *null* or *empty set* and is denoted by the symbol $\varnothing$

Some basic concepts from set theory(cont.)

- If every element of a set A is also an element of another set B, then A is said to be a *subset* of B, denoted as

  $$A \subseteq B$$

  The *union* of two sets A and B, denoted by

  $$C = A \bigcup B$$

  is the set of all elements belonging to either A, B,or both. Similarly, the *intersection* of two sets A and B denoted by

  $$D = A \bigcap B$$

  is the set of all elements belonging to both A and B.

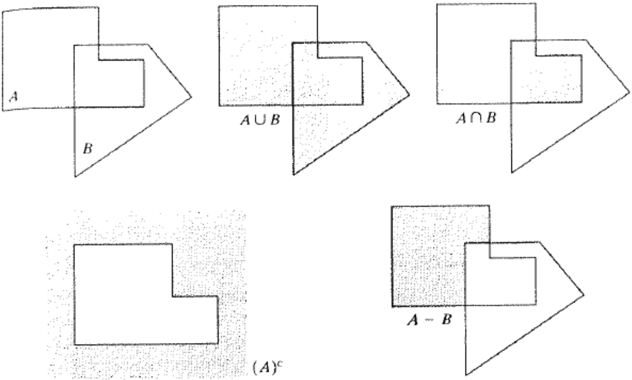
- Two sets A and B are said to be *disjoint or mutually exclusive*if they have no common elements. In this case,

  $$A \bigcap B = \varnothing$$

Some basic concepts from set theory(cont.)

- The *complement* of a set A is the set of elements not contained in A
  $$A^c = \{\omega | \omega \notin A\}$$
- The *difference* of two sets A and B, denoted $A - B$, is defined as
  $$A - B = \{\omega | \omega \in A, \omega \notin B\} = A \bigcap B^c$$

Some basic concepts from set theory(cont.)



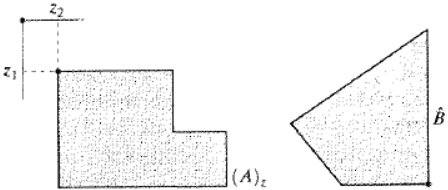
a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Some basic concepts from set theory(cont.)

- The *reflection* of set B denoted $\widehat{B}$, is defined as
  $$\widehat{B} = \{\omega | \omega = -b, b \in B\}$$
- The *translation* of set A by point $z = (z_1, z_2)$, denoted $(A)_z$, is defined as
  $$(A)_z = \{c | c = a + z, a \in A\}$$

Some basic concepts from set theory(cont.)



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Logic operations involving binary images

- The principal logic operations used in image processing are AND,OR and NOT(COMPLEMENT)

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Logic operations involving binary images(cont.)

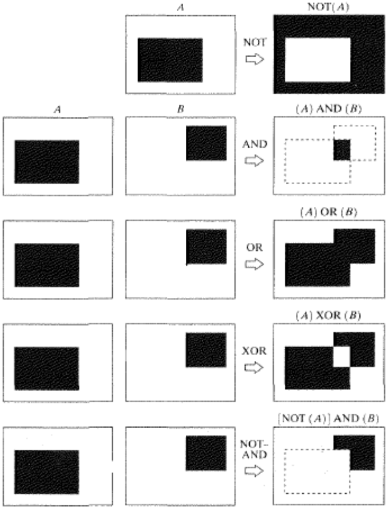


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

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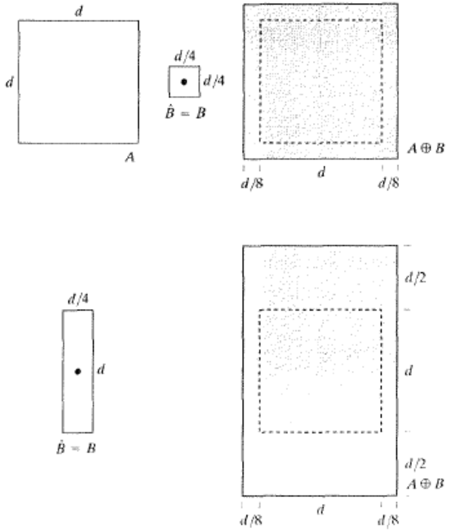
Dilation

- With A and B are sets in Z^2, the *dilation* of A by B, denoted $A \bigoplus B$,is defined as
$$A \bigoplus B = \{z | (\widehat{B})_z \bigcap A \neq \emptyset\}$$
- This equation is based on obtaining the reflection of B about its origin and shifting this reflection by z. The dilation of A by B then is the set of all *displacements*, z, such that $\widehat{B}$ and A overlap by at least one element.
$$A \bigoplus B = \{z | [(\widehat{B})_z \bigcap A] \subseteq A\}$$
Set B is commonly referred to as the *structuring element* in dilation,as well as in other morphological operations.

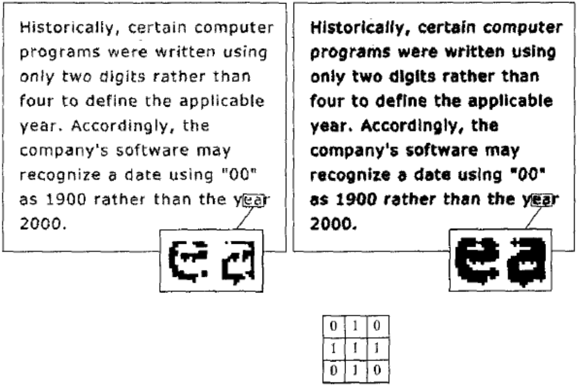
Dilation(cont.)

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Dilation(cont.)



a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion

- For sets A and B in Z^2 the erosion of A by B, denoted $A \ominus B$, is defined as

  $$A \ominus B = \{z | (B)_z \subseteq A\}$$

  the erosion of A by B is the set of all points z such that B, translated by z, is contained in A
- another definition of erosion

  $$(A \ominus B)^c = A^c \bigoplus \widehat{B}$$
  $$(A \ominus B)^c = \{z | (B)_z \subseteq A\}^c$$
  $$(A \ominus B)^c = \{z | (B)_z \bigcap A^c = \emptyset\}^c$$
  $$(A \ominus B)^c = \{z | (B)_z \bigcap A^c \neq \emptyset\}^c = A^c \bigoplus \widehat{B}$$

Erosion(cont.)

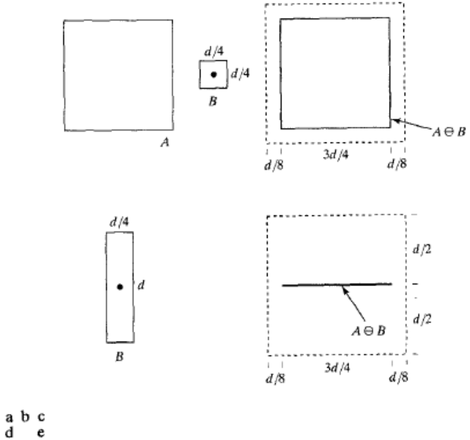
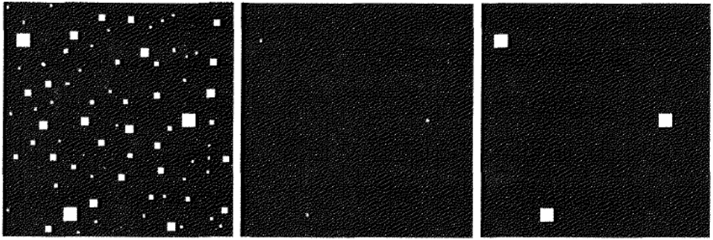


FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Erosion(cont.)



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Dilation and Erosion

- Dilation and erosion are duals of each other with respect to set complementation and reflection. That is

$$(A \ominus B)^c = A^c \bigoplus \widehat{B}$$

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1. Preliminaries
2. Dilation and Erosion
3. Opening and Closing
4. The Hit-or-Miss Transformation
5. Some Basic Morphological Algorithms

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Opening and Closing

- The opening of set A by structuring element B, denoted $A \circ B$, is defined as

  $$A \circ B = (A \ominus B) \oplus B$$

  thus, the opening A by B is the erosion of A by B, followed by a dilation of the result by B.

- The closing of set A by structuring element B, denoted $A \cdot B$, is defined as

  $$A \cdot B = (A \oplus B) \ominus B$$

  the closing A by B is simply the dilation of A by B, followed by the erosion of the result by B.

Geometric interpretation of Opening

- $A \circ B = \bigcup\{(B)_z | (B)_z \subseteq A\}$

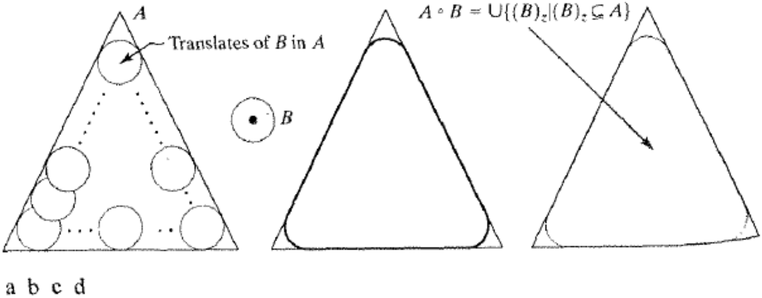
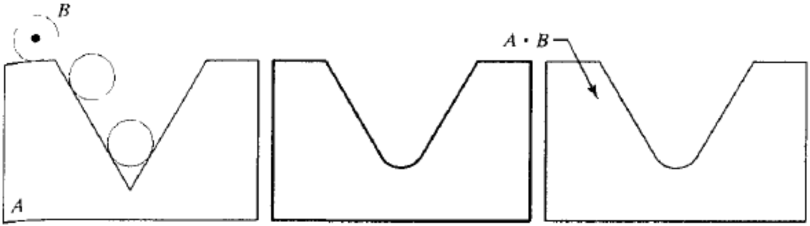


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Geometric interpretation of Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Opening and Closing(cont.)

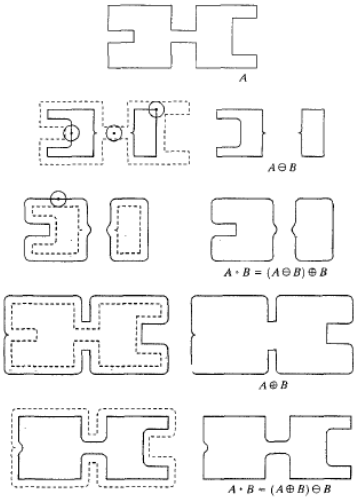


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Opening and Closing(cont.)

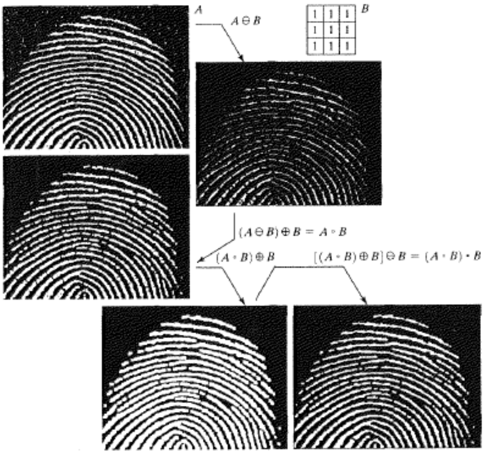
- Opening and closing are duals of each other with the respect to set complementation and reflection. That is,

$$(A \cdot B)^c = (A^c \circ \widehat{B})$$

Opening and Closing(cont.)

- The opening operation satisfies the following properties:
  - $A \circ B$ is a subset(subimage)of A.
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
  - $(A \circ B) \circ B = A \circ B$
- The closing operation satisfies the following properties:
  - A is a subset(subimage) of $A \cdot B$
  - If C is a subset of D, then $C \cdot B$ is a subset of $D \cdot B$.
  - $(A \cdot B) \cdot B = A \cdot B$
- In both cases that multiple opening or closing of a set have no effect after the operator has been applied once.

Opening and Closing(cont.)



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

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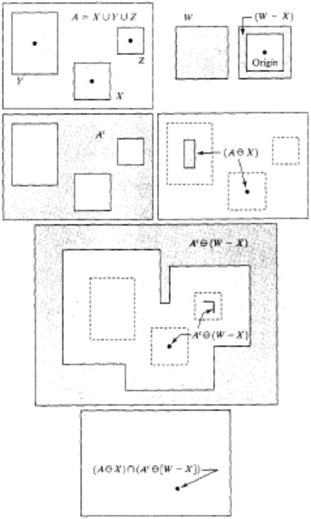
The Hit-or-Miss Transformation

- The morphological hit-or-miss transform is a basic tool for shape detection.

$$A \circledast B = (A \ominus X) \bigcap [A^c \bigcap (W - X)]$$
$$A \circledast B = (A \ominus B_1) \bigcap (A^c \ominus B_2)$$
$$A \circledast B = (A \ominus B_1) - (A \bigoplus \widehat{B}_2)$$

Any of the preceding three equations as the *morphological hit-or-miss transform*.

The Hit-or-Miss Transformation(cont.)



a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

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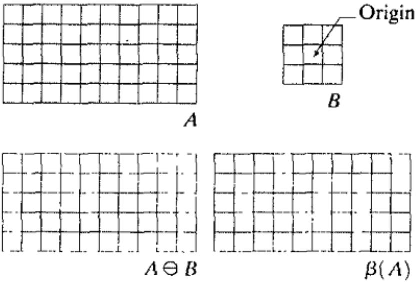
Boundary Extraction

- The boundary of a set A, denoted by $\beta(A)$, can be obtained by first eroding A by B and then performing the set difference between A and its erosion. That is

$$\beta(A) = A - (A - A \ominus B)$$

where B is a suitable structuring element.

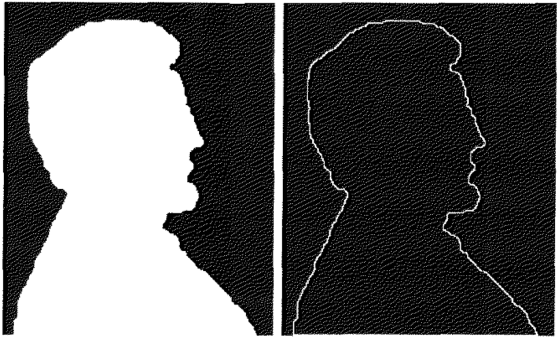
- The mechanics of boundary extraction:



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Boundary Extraction



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

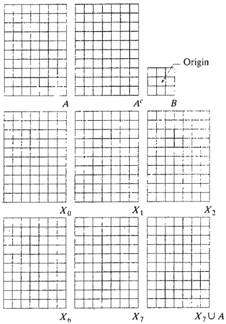
- Region filling algorithm can be implemented based on set dilations, complementation and intersections.

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, ...$$

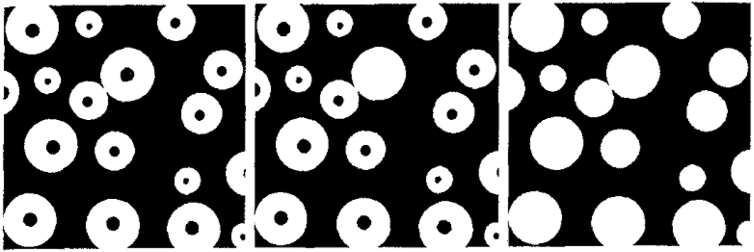
where $X_0 = p$*,*B *is the symmetric structuring element shown in Fig.9.15(c).*

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



Region Filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extraction of Connected Components

- The following iterative expression yields all the connected components of set A:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, ...$$

where $X_0 = p$, B is a suitable structuring element, as shown in Fig.9.17.

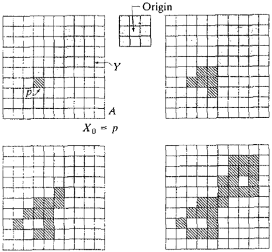
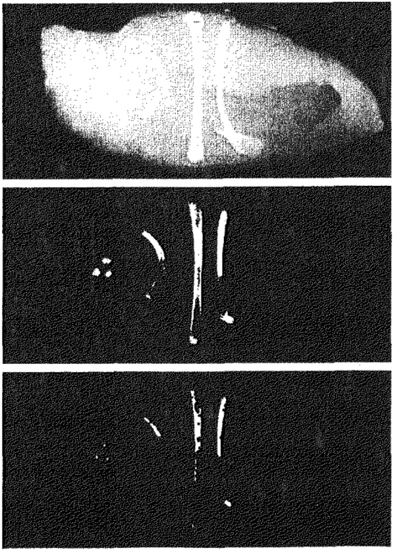


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Extraction of Connected Components

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- The convex hull C of an arbitrary set S is the smallest convex set containing S.
- Let $B^i, i = 1, 2, 3, 4$, represent the four structuring elements in Fig.9.19(a). The procedure consists of implementing the equation:
$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 k = 1, 2, 3, ...$$
where $X_0^i = A$.
- Now let $D^i = X_{conv}^i$, where the subscript "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. The the convex hull of A is
$$C(A) = \bigcup_{i=1}^{4} D^i$$

Convex Hull

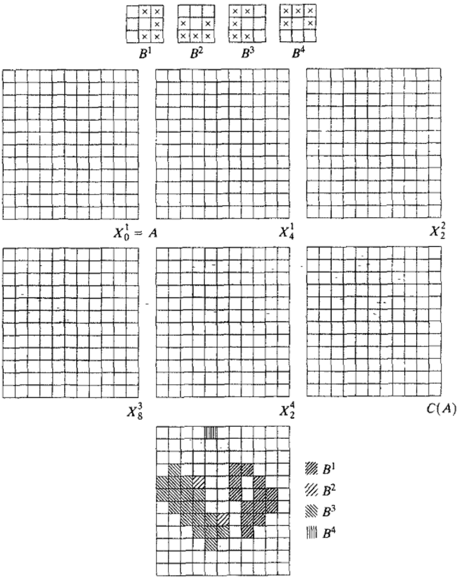


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Thinning

- The thinning procedure can be defined in terms of the hit-or-miss transform:

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

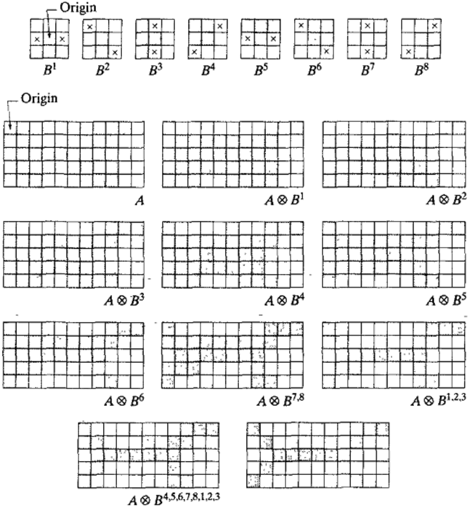
- A more usefull expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

- The process is to thin A by one pass with B^1, then thin the result with one pass of B^2, and so on, until A is thinned with one pass of B^n. The entire process is repeated until no further changes occur.

Thinning



a
b c d
e f g
h i j
k l

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

- Thickening is the morphological dual of thinning and is defined by the expression

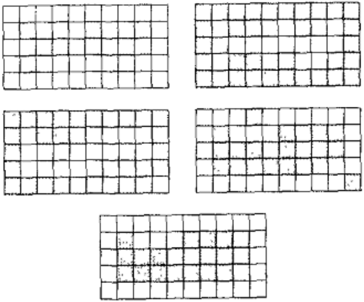
$$A \odot B = A \cup (A \circledast B)$$

- As in thinning, thickening can be defined as a sequential operation:

$$A \odot \{B\} = ((...((A \odot B^1) \odot B^2)...) \odot B^n)$$

- However, a separate algorithm for thickening is seldom used in practice. Instead, the usual procedure is to thin the background of the set in question and then complement the result. In other words, to thicken a set A, we form $C = A^c$, thin C, and then form C^c.

Thickening



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

- The skeleton of set A can be expressed in terms of erosions and openings. That is, it can be shown that

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and $(A \ominus kB)$ indicates k successive erosions of A:

$$(A \ominus kB) = (...((A \ominus B) \ominus B) \ominus ...) \ominus B$$

- K is the last iterative step before A erodes to an empty set. In other words,

$$K = \max\{k | (A \ominus kB) \neq \emptyset\}$$

Skeletons

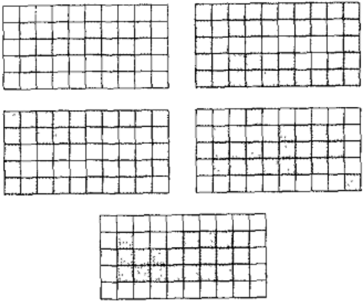
- A can be reconstructed from these skeleton subsets $S_k(A)$

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $(S_k(A) \oplus kB)$ denotes k successive dilations of $S_k(A)$; that is

$$(S_k(A) \oplus kB) = ((...(S_k(A) \oplus B) \oplus B) \oplus ...) \oplus B$$

Skeletons



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Pruning

- Step1:

$$X_1 = A \otimes \{B\}$$

- Step2:

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

- Step3:

$$X_3 = (X_2 \oplus H) \cap A$$

- Step4:

$$X_4 = X_1 \cup X_3$$

Pruning

a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.