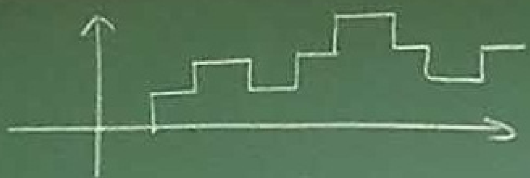


$$E(Y(t)) = \lambda \int_0^t E_A(h(t,\tau,A))\,d\tau.$$

$$E(h(t,\tau,A)) = 1 \cdot P(h(t,\tau,A)=1).$$

$$= P(A \geq t-\tau) = P(A \geq \max(t-\tau,0)) = \int_{\max(t-\tau,0)}^{\infty} f(x)dx$$

$$E(Y(t)) = \lambda \int_0^t \left(\int_{\max(t-\tau,0)}^{\infty} f(x)dx\right)d\tau \qquad f(x) = \mu\exp(-\mu x)$$

$$= \lambda\mu \int_0^t \left(\int_{\max(t-\tau,0)}^{\infty} \exp(-\mu x)dx\right)d\tau = \lambda\mu \int_0^t \left(\int_{t-\tau}^{\infty} \exp(-\mu x)dx\right)d\tau$$

Queueing Problem. Kleirock (UCLA) 1970's.

Income. , Outcome., Number of Resource.
$\overline{010010001}$

Poisson. Poisson (Exponential) M/G/K. Kendall

M/G/$\boxed{\infty}$ $\boxed{f(x)}$ Queue Length. 1.

$$Y(t) = \sum_{k=1}^{\boxed{N(t)}} h(t, \boxed{\tau_k}, \boxed{A_k}) \qquad h(t,\tau,\boxed{A}) = \begin{cases} 1 & \tau \leq t \leq \tau + A \\ 0 & \text{others.} \end{cases}$$

$A \geq 0$.

$$P(\bar{X}_k \mid \bar{X}_{k-1}, \dots, \bar{X}_1) = P(\bar{X}_k \mid \bar{X}_{k-1})$$ Markov Property.

Future — Now — Past

C — B — A

$$P(C \mid BA) = P(C \mid B).$$

$$\Updownarrow$$

$$P(CA \mid B) = P(C \mid B)P(A \mid B).$$

$$P(CA \mid B) = \underline{P(C \mid AB)}P(A \mid B) = P(C \mid B)P(A \mid B).$$

$$P(\bar{X}_3 = x_3 \mid \bar{X}_2 = x_2, \bar{X}_1 = x_1) = P(\bar{X}_3 = x_3 \mid \bar{X}_2 = x_2)$$

$$P(\bar{X}_3 \in A \mid \bar{X}_2 = x_2, \bar{X}_1 = x_1) \neq P(\bar{X}_3 \in A \mid \bar{X}_2 = x_2)$$

$$= \lambda \int_0^t \exp(-\mu(t-\tau))\,d\tau = \lambda \exp(-\mu t) \int_0^t \exp(\mu \tau)\,d\tau.$$

$$= \frac{\lambda}{\mu} \exp(-\mu t)(\exp(\mu t) - 1) = \boxed{\frac{\lambda}{\mu}}(1 - \underline{\exp(-\mu t)}).$$ Steady State

<u>Markov</u> Chains. 1905 $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$. $P(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)$

$$= P(\bar{X}_n \mid \bar{X}_{n-1}, \dots, \bar{X}_1)P(\bar{X}_{n-1}, \dots, \bar{X}_1) = P(\bar{X}_n \mid \bar{X}_{n-1}, \dots, \bar{X}_1)P(\bar{X}_{n-1} \mid \bar{X}_{n-2}, \dots, \bar{X}_1)P(\bar{X}_{n-2}, \dots, \bar{X}_1)$$

$$= \prod_{k=1}^{n} P(\bar{X}_k \mid \bar{X}_{k-1})$$ Constraint Nearest Neighborhood

$$P(\mathbb{X}_3 \in A \mid \mathbb{X}_2 = x_2, \mathbb{X}_1 = x_1) = \sum_{x_3 \in A} P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 = x_2, \mathbb{X}_1 = x_1).$$

$$= \sum_{x_3 \in A} P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 = x_2) = P(\mathbb{X}_3 \in A \mid \mathbb{X}_2 = x_2).$$

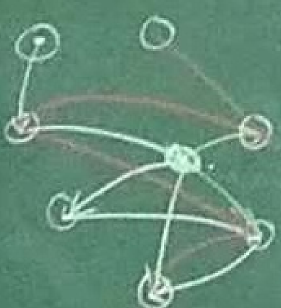
$$P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 = x_2, \mathbb{X}_1 \in A) = P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 = x_2)$$

$$\Longleftrightarrow P(\mathbb{X}_3 = x_3, \mathbb{X}_1 \in A \mid \mathbb{X}_2 = x_2) = P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 = x_2)\, P(\mathbb{X}_1 \in A \mid \mathbb{X}_2 = x_2)$$

$$P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 \in A, \mathbb{X}_1 = x_1) \neq P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 \in A)$$

$A = \Omega$. $P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 \in \Omega, \mathbb{X}_1 = x_1) = P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_1 = x_1)$

$$= P(\mathbb{X}_3 = x_3 \mid \mathbb{X}_2 \in \Omega) = P(\mathbb{X}_3 = x_3)$$



Jump!

State \ Time	Discrete	Continuous
Discrete	Markov Chains	Poisson.
Continuous	Time Series	Gaussian (Brown Motion)

$$x_n = f(x_{n-1}, w_n)$$

$$\cdots \sum_{x_1,\ldots} P(X_n=j, X_{n-1}=x_{n-1}, \ldots, X_1=x_1, X_0=i)$$

$$P(X_n=j \mid X_0=i) = \sum_k P(X_n=j, X_m=k \mid X_0=i)$$

$$= \sum_k \underline{P(X_n=j \mid X_m=k, X_0=i)}\ \underline{P(X_m=k \mid X_0=i)}$$

Chapman-Kolmogorov (C-K)

$$= \sum_k P(X_n=j \mid X_m=k)\, P(X_m=k \mid X_0=i)$$

Matrix Multiplication $\quad P_{ij}(n) = \sum_k P_{ik}(m)\, P_{kj}(n-m)$

$P(X_n = x_j \mid X_m = x_i)$. $n>m$. Transition Probability.

$$\underline{P(X_1=x_1, X_2=x_2, \ldots, X_n=x_n)} = \underline{\prod_{k=1}^{n} P(X_{k+1}=x_k \mid X_k=x_k)}$$

Local $\Rightarrow$ Global. $P(X_n=x_n \mid X_{n-1}=x_{n-1})$ One-Step T.P.

$P(X_n=x_j \mid X_m=x_i) = P_{ij}(n,m) = P_{ij}(n-m)$. Stationary Transition Probability

$$P(X_n=j \mid X_0=i) = P(X_n=j, X_0=i) / P(X_0=i).$$

$m<n$

$$P(n) = (P_{ij}(n))_{ij}. \Rightarrow P(n) = P(m)\cdot P(n-m).$$

$$P(n) = P(n-1)\cdot P(1) = P(n-2)(P(1))^2 = \cdots = (\boxed{P(1)})^n.$$

$$P(1) = P = \begin{pmatrix}\alpha & 1-\alpha\\ 1-\beta & \beta\end{pmatrix}_{0<\alpha,\beta<1} \quad P_{ij}\geq 0. \quad \sum_j P_{ij} = 1$$

Jordan Canonical Form. $P = B^{-1}\Lambda B \Rightarrow P^n = B^{-1}\Lambda^n\cdot B$ $\quad (\underline{P_{ii}>0})$

$$\det(\lambda I - P) = 0 \Rightarrow \lambda_1, \lambda_2$$

$(\lambda_1 I - P)x = 0.\ \lambda_1$ $\quad \det(\lambda I - P) = (\lambda-\lambda_1)^k G(\lambda).$

$(\lambda_1 I - P)^2 x = 0.$ $\quad \begin{matrix}\lambda_1 & 1\\ & \lambda_1\end{matrix}$

$(\lambda_1 I - P)^3 x = 0. \cdots,$ $\quad \begin{pmatrix}\lambda_1 & 1 & \\ & \lambda_1 & 1\\ & & \lambda_1\end{pmatrix}$

$$P(X_3=x_3 \mid X_2=x_2, X_1\in A) = P(X_3=x_3 \mid X_2=x_2)$$

$$\Leftrightarrow P(X_3=x_3, X_1\in A \mid X_2=x_2) = P(X_3=x_3 \mid X_2=x_2)\,P(X_1\in A \mid X_2=x_2)$$

$$P(X_3=x_3 \mid X_2\in A, X_1=x_1) \neq P(X_3=x_3 \mid X_2\in A)$$