MTH 261 — Computational Linear Algebra (S26) **Coding Homework 3**

Due: By **11:59pm** on **Monday, February 23, 2026**

Name: ____

Goal. The goal of this assignment is to become comfortable with **LU factorization** as an algorithm (i.e. not a black box), and to use it to solve a linear system by **forward- and back-substitution**.

What to turn in. Upload a single MATLAB file (`.m`) that contains:

- a function `[L,U] = LU(A)` that computes an LU factorization,
- a function `y = fwdsub(L,b)` that solves $Ly = b$ when L is lower triangular,
- a function `x = backsub(U,y)` that solves $Ux = y$ when U is upper triangular,
- and a short script section that uses these functions to solve the system in Part 2 and prints your final answer `x`.

(You may place the functions at the end of the same file as *local functions*.)

Restrictions. Inside your functions, do **not** use

`\  inv  lu  linsolve  rref`

or any built-in solver/factorization routine. You *may* use them **outside** the functions to check your work.

Assumptions (for this assignment).

- You may assume A is square.
- You may assume no row swaps are needed (i.e. the pivots you need are nonzero), so you can build U via Gaussian elimination *without pivoting*, and store the multipliers in L.
- Your L should have 1's on the diagonal.

Part 1 — LU factorization: `[L,U] = LU(A)`

Write a function `[L,U] = LU(A)` that accepts an $n \times n$ matrix A and outputs matrices L and U such that

$$A = LU,$$

where U is upper triangular (coming from Gaussian elimination) and L is lower triangular (holding the elimination multipliers), with diagonal entries equal to 1.

Verification. In your script, verify your factorization by checking (numerically) that $\|LU - A\|$ is close to 0. (Any matrix norm is fine; for example, `norm(L*U - A)` in MATLAB.)

Part 2 — Solve a linear system using LU

Use your `LU` factorization and substitution functions to solve

$$Ax = b, \qquad A = \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix}, \qquad b = \begin{bmatrix} 12 \\ 17 \\ 5 \end{bmatrix}.$$

Instructions.

1. Compute `[L,U] = LU(A)`.
2. Solve $Ly = b$ by forward substitution using `y = fwdsub(L,b)`.
3. Solve $Ux = y$ by back substitution using `x = backsub(U,y)`.
4. Print your final answer `x` (and optionally `L`, `U`, and `y`).

Check (recommended). Outside your functions, you may compare your answer to MATLAB's `x_check = A\b` to confirm correctness.

Style expectations

Avoid lines of code that are hard to interpret without explanation. Use clear variable names, short steps, and brief comments.