Research Proposal:

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April 13, 2022

Introduction

Mathematical modeling is a useful method for understanding of how infectious diseases spread through a population. The possible course of an outbreak and procedures to contain an epidemic can also be anticipated using this type of modeling. This host population model defines four major compartments as Susceptible, Exposed, Infectious and Recovered, hence is called SEIR model. The model introduces one more compartment for vaccination. The individual who are vaccinated are acquired life-long immunity. The vector population model defines three major compartments as Susceptible, Exposed and Infectious hence is called SEI model.

In this study, we look at an SEIR malaria model that combines treatment and vaccination strategies and covers both human and vector populations. The goal is to gain insight into the best interventions to reduce malaria disease transmission within the population and to study the effects of various intervention options, such as vaccination and treatment. The important dynamics are taken into account in this model. We will estimate the character of the vaccination compartment with time t using the first-order non-linear equation. Analytical epidemiology parameters relating to the vaccine compartment and estimate derive basic reproductive number.

We will determined the stability analysis of the disease-free and endemic equilibrium from the SEIR model relating to vaccination compartment description. The model analysis consists of the stability analysis of disease-free and endemic equilibrium. We will use numerical simulation to show the dynamical behavior of our vaccination strategies.

Mathematical Model

Model formulation

Consider SEIR epidemic model, where the compartmental model divides the total human host population, denoted by N_h into four compartments related to the epidemic, the susceptible (S_h), the exposed (E_h), the infectious (I_h) and the recovered (R_h). We formulate the problem based on the SEIR model with vital dynamics and study the effect of Vaccination and epidemiological factors related to it. The individuals in the (S_h) compartment are those who are vulnerable to become infected. An individual is in the (E_h) compartment are exposed to malaria parasites and the individuals in this compartment are not able to spread the disease (a compartment in which the disease is latent; infected but not infectious). Infectious individuals are in the (I_h) compartment and the immune individuals are in the (R_h) compartment. In malaria, there is no possibility of lifelong immunity after recovering from it, despite that the antibodies are developed and during active antibodies, it remains in the recovered compartment for a restricted period and then flows back to the susceptible compartment. We assume that vaccination will give life-long immunity either in one dose or with periodic boosters. The population which does not receive vaccination after recovering shall flow back in the compartment of susceptible S after completing the recovery period λ^{-1}. The reciprocals ϵ^{-1}, average disease incubation period, and μ^{-1} are average natural deaths. Λ and μ describe a model with vital dynamics (endemic model), which has an inflow of births into the class S_h at a rate Λ and outflow of deceased μS. This model is based on the assumptions proposed by Hethcote; the population size is constant and large enough so that we can consider the population of each compartment as a continuous model. The birth and death rates are equal and the population (fixed) is homogeneously mixed and uniform. The governing differential equations are:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h + \beta_h S_h I_v - (\mu_h + \alpha) S_h + \sigma E_h + \rho R_h \\
\frac{dE_h}{dt} &= \beta_h S_h I_v - (\mu_h + \sigma + \epsilon) E_h + \lambda V_h \\
\frac{dI_h}{dt} &= \epsilon E_h - (\delta + \mu_h + \gamma) I_h \\
\frac{dR_h}{dt} &= \gamma I_h - (\mu_h + \rho) R_h \\
\frac{dV_h}{dt} &= \alpha S_h - (\mu_h V_h + \lambda) V_h
\end{aligned}$$

Let S_h(t), V_h(t), E_h(t), I_h(t) and R_h(t) represent the number of individuals in the corresponding compartment at the time t, respectively. Thus, the total population at time t is denoted as N_h(t) satisfying:

$$N_h(t) = S_h(t) + V_h(t) + E_h(t) + I_h(t) + R_h(t)$$

The total vector population, denoted by N_v is divided into three compartment, the susceptible (S_v), the exposed (Ev) and the infectious (I_v). In (S_v) compartment are susceptible mosquitoes. The mosquitoes that are exposed in malaria parasite are in (E_v) compartment. The vector population model is given by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dS_v}{dt} &= \Lambda_v - \beta_v S_v I_h - \mu S_v \\
\frac{dE_h}{dt} &= \beta_v S_v I_h - (\mu + \omega) E_h \\
\frac{dI_h}{dt} &= \omega E - \mu I_v
\end{aligned}$$

Infected mosquitoes are in (I_v) compartment. The total vector population at time t is denoted as N_v(t) satisfying:

$$N_v(t) = S_v(t) + E_h(t) + I_h(t)$$

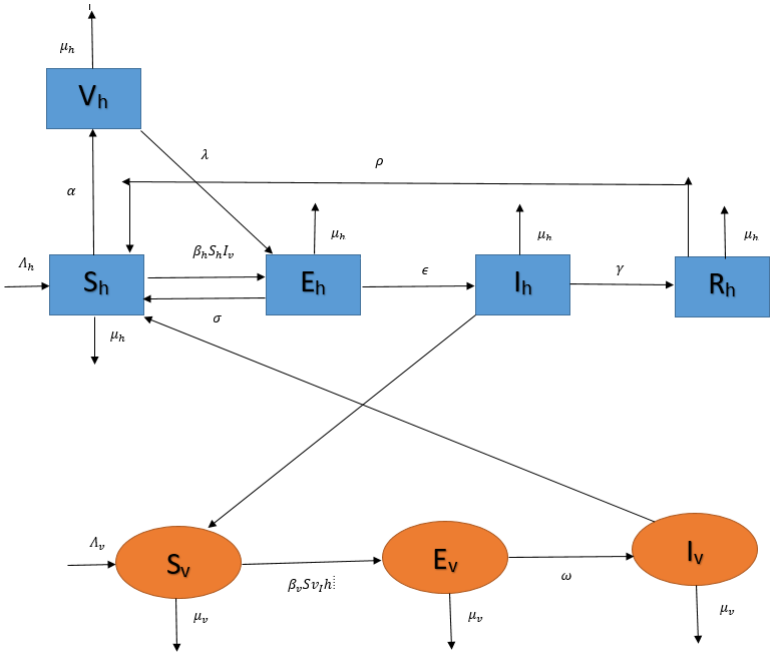

Figure 1: A schematic diagram for malaria transmission

Susceptible individuals are recruited at a rate Λ_h. Susceptible individuals acquire malaria through contact with infectious mosquitoes at a rate β_h. The rate of vaccination is α to the individuals present in Susceptible. Due to waning effect, some vaccinated individuals will move to the exposed class at a rate λ. Exposed individuals move to the infectious class at a rate ϵ. Individuals with malaria are treated under control, and are recovered spontaneously at rate γ. Loss of immunity for humans is ρ.

Susceptible mosquitoes are generated at a rate Λ_v and acquire malaria through contacts with infected humans at a rate β_v. Developing ω rate of exposed (mosquitoes) becoming infectious. Mosquitoes are assumed to suffer death due to natural causes and various control measures (insecticides, destruction of mosquitoes breeding sites, etc.) at a rate μ_v.

The vaccine will be administered to the susceptible individuals so that in addition to their natural immunity they can acquire vaccine-induced immunity too. This epidemic disease model predicts a peak of susceptible, exposed, infected, and recovered including vaccinated individuals per day as a function of time. The μ is defined as the rate of mortality, which includes both natural and due to malaria.

Table 1: Parameter for the model

Parameter	Description
Λ_h	Recruitment rate of humans
Λ_v	Recruitment rate of mosquitoes
μ_h	Natural death rate of humans
μ_v	Natural death rate of mosquitoes
β_h	Susceptible individuals acquire malaria through contact with infectious mosquitoes at a rate
β_v	Acquire malaria through contacts with infected humans at a rate
α	The rate of vaccination of individuals present in Susceptible
σ	The rate of inflow into susceptible from the exposed compartment
ρ	Loss of immunity for human
δ	Induce death rate of humans
ϵ	Developing rate of exposed (humans) becoming infectious
γ	Recover rate of humans.(removal rate)
λ	vaccinated individuals will move to the exposed class
ω	Developing rate of exposed (mosquitoes) becoming infectious

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