Stats 1 Formulae

Statistics 1 Formulae

Measures of Central Tendency

Mean

Continuous data

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- n total number of observations
- x_i each i^{th} observation

Continuous grouped data

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i m_i}{\sum f_i}$$

- m_i each group's middle value
- f_i frequency of each group

Discrete data

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{\sum f_i}$$

- x_i each discrete value
- f_i frequency of each discrete value

Adding a constant

$$\bar{y} = \bar{x} + c$$

- $\bar{x}$ is old mean
- $\bar{y}$ is new mean
- c is constant value added in each x_i or overall mean $\bar{x}$

Multiplying a constant

$$\bar{y} = \bar{x} \cdot c$$

- $\bar{x}$ is old mean
- $\bar{y}$ is new mean
- c is constant value multiplied with each x_i or overall mean $\bar{x}$

Median

> **Important**
>
> Data must be sorted

Odd frequency n

$$x_i = \frac{n+1}{2}$$

- n is frequency
- x_i is median value

Even frequency n

$$x_i = \frac{\frac{n}{2} + \frac{n+1}{2}}{2}$$

- n is frequency
- x_i is median value

Adding a constant

$$y_i = x_i + c$$

- $\bar{x}$ is old median
- $\bar{y}$ is new median

- c is constant value added in each x_i or overall median

Multiplying a constant

$$y_i = x_i \cdot c$$

- $\bar{x}$ is old median
- $\bar{y}$ is new mean
- c is constant value multiplied in each x_i or overall median $\bar{x}$

Mode

- Most occured value
- If data has 2 modes, then it's *bimodal*.
- If data has more than 2 modes, then it's *multimodal*.

Adding a constant

$$y_i = mode + c$$

- c is constant added to all terms
- y_i is new mode

Multiplying a constant

$$y_i = mode \cdot c$$

- c is constant value multiplied by all terms
- y_i is new mode

Measures of Dispersion

- It tells how much our data varies and spread.

Range

- Difference b/w maximum and minimum value

$$Range = max - min$$

Variance

- It means how far data is spread in a data set.
- The units will be squared, hence we can't measure the quantity.
- It is calculated by sum of squared deviation
- Deviation = $x_i - \mu$

Population variance

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

- n is total no. of observations
- μ is mean of all observations

Sample variance

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}$$

- n is total no. of observations
- μ is mean of all observations

Adding a constant

- Adding a constant does not effect the variance.
- As constant will be added to all terms as well as mean, so it cancelled out.

Multiplying a constant

$$\sigma^2 = var \cdot c^2$$

var is old variance

- c is the constant value being multiplied by each term.
- As we are squaring the deviations, hence we need to square the constant also.

Standard Deviation

- It also measures how much the data is dispersed.
- But, we can use it as it's calculated with same quantity.
- It is calculated by adding up all squared deviations and square root it.

> **Note**
>
> - Standard deviation is the square root of variance.
>
> $$\sigma = \sqrt{\sigma^2}$$

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$$

- x_i each observation
- n is total no. of observations
- μ is mean of all observations

Sample Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n - 1}}$$

- x_i each observation
- n is total no. of observations
- μ is mean of all observations

Adding a constant

- Same as variance, adding a constant does not change the standard deviation.
- As constant will be added to all terms and mean, hence it cancelled out.

Multiplying a constant

$$\sigma = \sqrt{std \cdot c^2}$$

- std is old standard deviation
- c is constant value being multiplied by each term.
- As we are squaring the deviations, and then taking total's root, same we are doing here.

Percentiles

- Percentiles tells us how much data is distributed on a k^{th} percentile.
- Data should be sorted in ascending order.

Even frequency

$$percentile = \frac{p}{100} \cdot \frac{n}{2} + \frac{(n+1)}{2}$$

- n is the total number of observations
- p is k^{th} percentile.

Odd frequency

$$percentile = \frac{p}{100} \cdot \frac{n}{2}$$

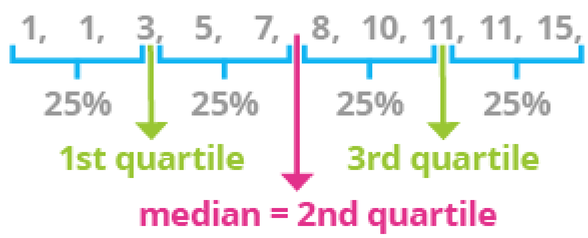
- $\frac{n}{2}$ should be rounded to next integer.

Qaurtile

1. Q1 - Lower (first) = 25% tile
2. Q2 - Middle (Median) = 50% tile
3. Q3 - Upper (Third) = 75% tile

Quartile range

1. Min - Q1
2. Q1 - Q2
3. Q2 - Q3
4. Q3 - Max



InterQuartile Range (IQR)

$$IQR = Q3 - Q1$$

Detecting outliers

$$Lower = Q1 - 1.5 \cdot IQR$$

$$Upper = Q3 + 1.5 \cdot IQR$$

Association b/w two categorical variables

Contingency table

	PC	Mac	Row Totals
Male	66	40	106 (0.475)
Female	30	87	117 (0.524)
Column Totals	96 (0.430)	127 (0.570)	223

Association b/w two numerical variables

Covariance

- It quantifies the strength of the linear association b/w two numerical variables.
- It shows how two variables are different.

> **Important**
>
> - Covariance is always measures b/w $-\infty$ and ∞
>
> $$-\infty \leq Cov(x, y) \leq \infty$$

Population covariance

$$Cov(x, y) = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{N}$$

- N is the total number of observations
- x is first variable / column
- y is second variable / column

Sample covariance

$$Cov(x, y) = \frac{\sum_{i=1}^{n-1} (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

- n is the total number of observations

- x is first variable / column
- y is second variable / column

Correlation

- It is used to interpret the *linear association* b/w two numerical variables.

- It shows how two variables are related.

- It derives from covariance.

- Correlation is always measures b/w -1 and 1

$$-1 \leq r \leq 1$$

Pearson correlation

- We are using pearson correlation to find a linear relationship b/w two variables.

Sample correlation

$$r = \frac{cov(x, y)}{S_x S_y}$$

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

- $cov(x, y)$ is covariance of x and y
- $\bar{x}$ is x variable mean.
- $\bar{y}$ is y variable mean.
- n is total number of observations.
- S_x is standard deviation of x variable/column.

- S_y is standard deviation of y variable/column.

Population correlation

$$\rho = \frac{cov(x,y)}{S_x S_y}$$

$$\rho = \frac{\sum_{i=1}^{N}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)}{\sqrt{\sum_{i=1}^{N}\left(x_i-\bar{x}\right)^2}\cdot\sqrt{\sum_{i=1}^{N}\left(y_i-\bar{y}\right)^2}}$$

- $cov(x,y)$ is covariance of x and y
- $\bar{x}$ is x variable mean.
- $\bar{y}$ is y variable mean.
- N is total number of observations.
- S_x is standard deviation of x variable/column.
- S_y is standard deviation of y variable/column.

Point Bi-serial Correlation

$$\rho = \frac{\bar{Y_0}-\bar{Y_1}}{S_s}\sqrt{P_0 P_1}$$

- We need to assign a value of 1 or 0 to each observation, depending on whether the observation is in the first group or the second group.
- Y_0 is the mean of the observations in the first group.
- Y_1 is the mean of the observations in the second group.
- S_s is the sample standard deviation of the entire sample.
- P_0 is the proportion of observations in the first group.
- P_1 is the proportion of observations in the second group.

Scatter plot

- In scatter plot we usually measure 4 things:

1. Direction (upwards, downwards)
2. Curve (linear, quadratic, cubic)

3. Clustered or spreaded
4. Outliers

Fitting a line

- A linear regression has a equation:

$$y = mx + c$$

- m is slope
- c is y-intercept

R^2

- It measures the goodness of fit of regression line.
- It's the error metric.
- Ranges from $0 \leq R^2 \leq 1$

Permuation and Combination

Permutation Formula

- The number of permutations of n objects when p if them are of one kind and rest distinct is given by:

$$^nP_r = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects where p_1 is of one kind, p_2 is of another kind and p_k is of k^{th} kind is given by:

$$\frac{n!}{p_1! * p_2! * ... p_k!}$$

Circular Permutation: Clockwise and Anti-clockwise

- We fix one element and then we have $n-1$ elements to arrange in $n-1$ ways.

$$(n-1)!$$

- If number of elements arranged in clockwise direction and anti-clockwise direction are same then:

$$\frac{(n-1)!}{2}$$

Combination Formula

- In general, each combination of r objects from n objects can give rise to $r!$ arrangements.
- The number of possible combinations of r objects from collection of n objects is denoted by:

$$^nC_r = \frac{n!}{r!(n-r)!}$$

- Also

$$\frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$$

- In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.

- $^nC_n = 1$ and $^nC_0 = 1$ for all values of n.

- $^nC_r = {}^{n+1}C_{r-1} + {}^{n+1}C_r \; ; 1 \leq r \leq n$

Probability

Conditional Probability

- The probability of an event A given that another event B has already occurred is called conditional probability.
- It is denoted by $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule

- The probability of two events A and B occurring together is given by:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

- The probability of three evennts A, B and C occuring together is given by:

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$$

$$P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}$$

Independent Events

- In case of independent events, the probability of two events occurring together is given by:

$$P(A|B) = P(A)$$

- In other words, the probability of A occurring given that B has occurred is equal to the probability of A occurring irrespective of whether B has occurred or not.

$$P(A \cap B) = P(A) \cdot P(B)$$

- Therefore the probability of A and B are independent of each other.

Independence of more than two events

- If A, B, C are independent events, then:

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

Law of Total Probability

- Let E and F be two events such that $E \cap F = \emptyset$.

$$P(E) = P(E \cap F) + P(E \cap F^c)$$
$$or$$
$$P(E) = P(E|F) \cdot P(F) + P(E|F^c) \cdot P(F^c)$$

Bayes' Theorem

$$P(B|A) = \frac{P(B \cap A)}{P(B)}$$
$$\text{or}$$
$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(B) \cdot P(A|B) + P(B^c) \cdot P(A|B)}$$

- Suppose that events F

Random Experiment

- A random experiment is an experiment whose outcome is not known in advance. It is an experiment whose outcome is determined by chance.
- The outcome of a random experiment is called a random variable.
- Sample Space: The set of all possible outcomes of a random experiment is called the sample space of the experiment.
  - Sample space is denoted by S.
  - Suppose a random experiment of throwing a die is performed. The sample space of the experiment is given by:

$$S = \{1, 2, 3, 4, 5, 6\}$$

  - The sample space of a random experiment of tossing two coins is given by:

$$S = \{(H,H),(H,T),(T,H),(T,T)\}$$

Discrete Random Variable

- A random variable that can take on at most a countable number of possible values is said to be a Discrete Random Variable.
  - Thus, any random variable that can tkae on only a finite number or countably infinite number of values is a discrete.

Continuous Random Variable

- There also exists a random variable that can take on an uncountably infinite number of values. Such a random variable is called a Continuous Random Variable.

Probability Mass Function

- The probablity mass function $p(x)$ is positive for at most a coutntable number values of x. That is, if Xmust assume one of the values $x_1, x_2, \ldots, x_n$ then:
  - $p(x_i) \geq 0$ for all i and $p(x_i) = 0$ for all $x \neq x_i$.
  - Since $p(x)$ is a probability, it must satisfy the following:

$$\sum_{i=1}^{\text{\textbackslash infin}} p(x_i) = 1$$

$$\text{Any } x_i \text{ should not be } \geq \text{ to } 0.$$

- The graph from the probablity mass function can take many shapes. The graph can be skewed positively or negatively. It can be symmetric or asymmetric. It can be unimodal or multimodal. It can be discrete or continuous. It can also be uniform or non-uniform.

Cumulative Distribution Function

- The cumulative distribution function $F(x)$ is defined as:

$$F(x) = P(X \leq x)$$

- The cumulative distribution function $F(x)$ is a non-decreasing function of x.

Expected Value of a Random Variable

- Let X be a Discreet Random variable taking values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively.
- The Expectation of X is denoted by $E(X)$ and is defined as:

$$E(X) = \sum_{i=1}^{n} x_i \cdot p_i$$

- Suppose we want to find out the expectation of a random variable X that takes on values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively.

> - We can find the expectation by multiplying each value of X by its probability and then adding the results.
> - The expectation of a random variable is the weighted average of the possible values of the random variable.
>
> $$E(g(X)) = \sum g(x_i) P(X = x_i)$$

- If the function $g(x)$ is multiplied by a constant c, then the expectation of the function is also multiplied by the same constant.

$$E(cg(X)) = cE(g(X))$$

- If we add a constant c to the random variable X, then the expectation of the random variable is also increased by the same constant.

$$E(X + c) = E(X) + c$$

- The expected value of the sum of random variables is equal to the sum of the individual expected values.

$$E(X + Y) = E(X) + E(Y)$$

Example

X	-1	0	1

X	-1	0	1
$P(X)$	0.2	0.5	0.3

- Let $Y = g(X) = X^2$. What is $E(Y)$
- $E(Y) = \sum_{i=1}^{n} y_i \cdot p_i$
- $E(Y) = [1 \cdot 0.2] + [0 \cdot 0.5] + [1 \cdot 0.3]$
- $E(Y) = 0.2 + 0.3 = 0.5$

X	-2	2	5
$P(X)$	0.25	0.3334	0.41667

- Calculate the expected value of $(2X + 1)^2$.
- $E(Y) = \sum_{i=1}^{n} y_i \cdot p_i$
- $E(Y) = [(2 \cdot -2 + 1) \cdot 0.25] + [(2 \cdot 2 + 1) \cdot 0.3334] + [(2 \cdot 5 + 1) \cdot 0.41667]$
- $E(Y) = 0.75 + 1.6667 + 4.5833

Variance of a Random Variable

- Let's denote expected value of a random variable X by the greek letter μ.
- Let X be a random variable with the expected value μ. Then the variance of X denoted by $Var(X)$ is defined as:

$$Var(X) = E(X - \mu)^2$$

$$\text{or}$$

$$Var(X) = E(X^2) - \mu^2$$

$$E(X) = \sqrt{Var(X) + \mu^2}$$

- In other words, the variance of random variable X measures the square of the difference of the random variable from its mean, μ, on the average.

Example

- Random Experiment: Roll a die once.

> - Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$
> - Random Variable: $X = \text{number of spots on the die}$ |X | 1 | 2 | 3 | 4 | 5 | 6 | | --- | --- | --- | --- | --- | --- | --- | |X^2 | 1 | 4 | 9 | 16 | 25 | 36 | |$P(X)$ | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | |$E(X)$ | 1/6 | 2/6 | 3/6 | 4/6 | 5/6 | 6/6 | |$E(X^2)$ | 1/6 | 4/6 | 9/6 | 16/6 | 25/6 | 36/6 |
> - $E(X) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3.5$
> - $E(X^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6} = 15.17$
> - $Var(X) = E(X^2) - \mu^2 = 15.17 - 3.5^2 = 15.17 - 12.25 = 2.92$

Bernoulli Random Variable

- A random variable that takes on either the value 1 or 0 is called a Bernoulli Random Variable.
- Let X be a Bernoulli random variable with p being the probability of success. Then the probability mass function of X is:

$$p(x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$$

$$Var(X) = p(1 - p)$$

Discrete Uniform random variable

- Let X be a random variable that is equally likely to occur on any of the n possible values $x_1, x_2, \ldots, x_n$. Then X is said to be a Discrete Uniform Random Variable.

$$Var(X) = \frac{(n^2 - 1)}{12}$$

Variance of a Random Variable when mulplied by a constant

$$Var(cX) = c^2 Var(X)$$

Variance of a Random Variable when added to a constant

$$Var(X + c) = Var(X)$$

Collorary

$$Var(aX + b) = a^2Var(X)$$

Variance of the sum of two random variables

- This is applicable only when the two random variables are independent.

$$E(X + Y) = E(X) + E(Y)$$

Variance of sum of many independent random variables

$$V(X_1 + X_2 \ldots\ X_k) = V(X_1) + V(X_2) + \ldots + V(X_k)$$

$$\text{or}$$

$$V(\sum_{i=1}^{k} X_i) = \sum_{i=1}^{k} V(X_i)$$

Standard Deviation of a Random Variable

$$SD(X) = \sqrt{Var(X)}$$

- Hence, the standard deviation is a positive square root of variance.
- Standard deviation is in the same units as the random variable.

Standard Deviation adding or being multiplied by a constant

Multplied by a constant

$$SD(cX) = \sqrt{c^2Var(X)} = c\sqrt{Var(X)}$$

$$\text{or}$$

$$SD(cX) = cSD(X)$$

Added to a constant

- The standard deviation of a random variable is not affected by adding a constant to the random variable.

$$SD(X + c) = SD(X)$$

Collorary

$$SD(aX+b) = aSD(X)$$

$$SD(aX+b) = \sqrt{a^2Var(X)} = a\sqrt{Var(X)}$$

Bernoulli Distribution

- X is a binomial random variable with praameters n and p that represents the number of successes in n independent Bernoulli Trails, when each trial is a succees witha probablity p. X takes the values $0, 1, 2, \ldots, n$ with the probablity.

$$P(X=i) = \ {}^{n}C_i \cdot p^i \cdot (1-p)^{n-i}$$

- We can predict the skewness of the distribution graph of X by the value of p.

> - If $p < 0.5$, then the distribution is skewed to the right.
> - If $p = 0.5$, then the distribution is symmetric.
> - If $p > 0.5$, then the distribution is skewed to the left.

- The Probablity Distribution of Bernoulli Random Variable is

X	0	1
$P(X)$	$1-p$	p

$$E(X) = 0 \cdot (1-p) + 1 \cdot p = p$$

Example : Tossing a coin

- $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
- Success: H and Failure: T
- X is a random variable which counts the number of heads in toe tosses of a coin. $n=3, p=0.5$ | X | 0 | 1 | 2 | 3 | | --- | --- | --- | --- | --- | | $P(X)$ | 0.125 | 0.375 | 0.375 | 0.125 |

$P(X = 3) = {}^3C_3 \cdot 0.5^3 \cdot (0.5)^{3-3} = 0.125$

Variance of a Bernoulli Random Variable

$$V(X) = p(1-p) = p - p^2$$

- The largest variance occurs when $p = 0.5$. In happens when the success and failure are equally likely. In other words the most uncertain outcome is when the probability of success and failure are equal.

Independent and Identically Distributed Random Variables

- Two random variables are said to be Independent and Identically Distributed if they are independent and have the same probability distribution.
- A collection of random variables is said to be Independent and Identically Distributed if each random variable in the collection is independent and has the same probability distribution.

Expectation of Binomial Random Variable

$$E(X) = np$$

Variance of Binomial Random Variable

$$V(X) = np(1-p)$$

Standard Deviation of Binomial Random Variable

$$SD(X) = \sqrt{np(1-p)}$$

Hypergeometric Distribution

- Let X be the number of items of type 1, then the probablity mass function of the discrete random variable, X, is called the hypergeometric distribution and is of the form:

$$P(X = i) = \frac{{}^mC_i \cdot {}^{N-m}C_{n-i}}{{}^NC_n}$$

Example: Choosing balls without replacement

- A bag consists of 7 balls of which 4 are white and 3 are black. A student randomly samples two balls without replacement. Let X be the number of black balls selected.
  - Here, $N = 7, m = 3, n = 2$
  - X has the values $0, 1, 2$
  - the probability mass function of X is:

$$P(X=0) = \frac{{}^3C_0 \cdot {}^{7-3}C_{2-0}}{{}^7C_2} = \frac{12}{42}$$

$$P(X=1) = \frac{{}^3C_1 \cdot {}^{7-3}C_{2-1}}{{}^7C_2} = \frac{24}{42}$$

$$P(X=2) = \frac{{}^3C_2 \cdot {}^{7-3}C_{2-2}}{{}^7C_2} = \frac{6}{42}$$

Expectation of Hypergeometric Random Variable

$$E(X) = \frac{n \cdot m}{N}$$

Variance of Hypergeometric Random Variable

$$V(X) = n \cdot \frac{m}{N} \cdot \frac{N-m}{N}$$

- The variance of the hypergeometric distribution is symmetric when both m and n are equal to $\frac{N}{2}$ or $\frac{m}{N} = \frac{1}{2}$.

Poisson Distribution

- The Poisson probablity distribution gives the probablity of a number of ecents occurring in a fixed interval of time or space.
- We assume that these events happen with a known average rate, λ, and independently of the time since the last event.
- Let X be the number of events in a given interval. Then the probability mass function of the discrete random variable, X, is called the Poisson distribution

and is of the form:

$$P(X = i) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

Things to note about the graphs of Poisson Distribution

- If the value of λ is very small, the the graph is skewed to the right.
- As the value of λ increases, the graph becomes more symmetric.

Poisson as Binomial Approximation

- Define "success" as exactly one event happening in a short interval of length δt .
- The n events happening in interval of length t can be viewed as n successes happening in n intervals of length δt, with each one of them being an Independent and Identical trails.
- Hence the problem can be viewed as

$$Bin(n, p = \frac{\lambda}{n}) = \ ^nC_1 \cdot (\frac{\lambda}{n})^x \cdot (\frac{n - \lambda}{n})^{n-x}$$

Expectation of Poisson Random Variable

- The expectation of a Poisson random variable is equal to the value of λ itself.

$$E(X) = \lambda$$

Variance of Poisson Random Variable

- The variance of a Poisson random variable is equal to the value of λ.
- $$V(X) = \lambda$$

Probablity Density Function

- Every Continuous Random Variable has a Probablity Density Function.

- That probablity distribution cruve of a continuous random variable is also called the Probablity Density Function. It is denoted by $f(x)$.

Area under the Probablity Density Function

- Consider any two points a and b where $a < b$.
- The probablility that X assumes a value that lies between a and b is given by the area under the probablity density function between a and b.

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Properties of the Probablity Density Function

- The area under the probablity distribution curve of a continuous random variable between any two points a and b is always between 0 and 1.
- The area under the whole probablity distribution curve is always equal to 1.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$1 \geq \int_a^b f(x)\,dx \geq 0$$

Expectation of a Continuous Random Variable

$$E(X) = \int \_{-\infty}^{\infty} x f(x)\,dx$$

Variance of a Continuous Random Variable

$$V(X) = \int_{-\infty}^{\infty} (x - E(X))^2 f(x)\,dx$$

Cumulative Distribution Function

- For a continuous random variable, the Cumulative Distribution Function is the integral of the Probablity Density Function.

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

Uniform Distribution

- A A random variable has the standard uniform distribution with minimum 0 and maximum 1 if its probability density function is given by

$$\begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- The standard uniform distribution plays an important role in random variate generation.

> - $f(x) \leq 0, \text{for} x \in \mathbb{R}$
> - $\int_{-\infty}^{\infty} f(x)\, dx = \int_0^1 f(x) dx = 1$

Expectation of Uniform Random Variable

$$E(X) = \frac{a+b}{2}$$

Variance of Uniform Random Variable

$$V(X) = \frac{(b-a)^2}{12}$$

Expoential Distribution

- A continuous random variable whose probablity density function is given, for some $\lambda > 0$, by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- These are called exponential random varaible or, called exponentially distributed random variables with parameter λ.

Expectation of Exponential Random Variable

$$E(X) = \frac{1}{\lambda}$$

Variance of Exponential Random Variable

$$V(X) = \frac{1}{\lambda^2}$$

Standart Deviation of Exponential Random Variable

$$\sigma = \sqrt{V(X)} = \frac{1}{\lambda}$$

Mean of Exponential Random Variable

$$\mu = E(X) = \frac{1}{\lambda}$$

Contributions:

Week 5-12 by *Kabir Maniar*

Week 1-4 Contributions by *Parampreet Singh*