IIT Madras

BSc Degree

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Maths 1

(Detailed notes)

— Akash Kumar

- Natural number : (no fraction or decimal)

  Sometime, $N = 1, 2, 3, 4 \ldots \infty$

  Generally, $N_0 = 0, 1, 2, 3, 4, \ldots \infty$

(no fraction or decimal)

- Integers : Integers, $Z = -\infty, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, \infty$

- Factors : It is combination of multiple which lead to end number.

  factor of 12 : (1, 12), (2, 6), (3, 4)

  (36) → (1, 36), (2, 18), (3, 12), (4, 9), (6)

- Prime number : It is the number which as only 2 factor. (1 and itself) (no decimal/fraction)

  1 is not prime only factor = 1

  2 is prime bcz factor = 1, 2

  3 is prime bcz factor = 1, 3

  5 is prime bcz factor = 1, 5

- Composite number : It is number has more than 2 factor.

  (no fraction or decimal)

  Eg → 4 has factor 1, 2, 4

  6 has factor 1, 2, 3

  1 is neither prime nor composite.

- Rational number : Thes are those number which which can be written as p/q.

  & $q \neq 0$.

Terminating decimal : 0.375 (Rational no)
Non-terminating recurring decimal : 0.66666...
Non-terminating & non-recurring decimal: $\sqrt{2} = 1.4142...$ (Irrational)

P, Q → integers
Rational number, Q = $\frac{2}{3}, \frac{5}{8}$

- **Irrational number** : These are numbers which are non-terminating or non-recuring decimal. It is square root of no which is not not perfect square.
eg :- $\sqrt{2} = 1.41421356 2...$
like that $\sqrt{3}$, ; $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ are irrational no.

- **Whole number** : Whole no are number from (0 to $+\infty$) (no fraction or decimal) It is if natural no starts from 1.

- **Real number** : Any number which can be plotted on number line is real number. It contain both rational & irrational number.

- **Imaginary number**: Imaginary number are those which can't be solved and we imagine no.
like, $\sqrt{-1} = i$
than $i^2 = i \times i = \sqrt{-1} \times \sqrt{-1} = -1$
here we imagine i is no whose $\sqrt{\ }$ is -1.
Eg → i, $3i$, bi, etc.

- **Complex number**: It is combination of real no and imaginary number.
Eg :- $3 + 7i$ = Complex no
Real + imaginary

(Standard infinite set)

- Notations : (All numbers)
  - N → Natural number
  - W → Whole number
  - R → Real number (R^+ = +ve real & R^- = -ve real)
  - Q → Rational number
  - I/Z → Intigers (I^+/Z^+ = +ve intigers & I^-/Z^- = -ve intigers)
  - $R - Q$ → Irrational numbers

○ GCD (Great common divisor):

GCD (18, 60):

$18 = 2 \cdot 3 \cdot 3$
$60 = 2 \cdot 2 \cdot 3 \cdot 5$

$= \text{Comon} = 2 \times 3 = \underline{6}$

Perfect square : 1, 4, 9, 16, 25...
Square roots : 1, 2, 3, 4, 5...

○ Set :

→ Set is a collection of item.
→ Set can be infinite.
→ Uniform type not required.
→ Sequence don't matter.
→ Duplicate don't matter. (It auto remove)
→ Not every collection of item is set. (∞ set)

◇ It has → Finite set, and → Infinite set

eg: factor of 24 : {1, 2, 3, 4, 6, 8, 12, 24}
Prime below 15 : {2, 3, 5, 7, 11, 13}
Players : {Kohli, Dhoni, Kholi}

□ Elements :

→ Item in set is called element

□ $x \in X$ means x is an element of X

Hence, we write :

$x \in X$

Element	Not element
$x \in X$	$x \notin X$

□ likewise, $5 \in Z$, $\sqrt{2} \notin Q$

Here,

Z = (intiger) means 5 is intiger.
X = (Rational no') means $\sqrt{2}$ is not rational number.

Subset :

- If X is a subset of Y.

Subset means every element of X is also an element of Y.

Hence, we write notation: $\boxed{X \subseteq Y}$

Subset	Not Subset
$X \subseteq Y$	$X \not\subseteq Y$
$X \subset Y$	

	Subsets	
$B = \{1\} \Rightarrow n(B) = 1$	$\{\}, \{1\}$	2 subset $= 2^1$
$C = \{1,2\} \Rightarrow n(C) = 2$	$\{\}, \underbrace{\{1\}, \{2\}}_{\text{Proper set}}, \{1,2\}$	4 Subset $= 2^2$
$D = \{1,2,3\} = n(D) = 3$	$\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$	8 subset $= 2^3$

Number of subset can be created by set is 2^n

Example : [n = cardinal no.] or [n = no. of element in set]

→ Kohli $\in$ {Dhoni, Kohli, Pujara}

→ {Kohli, Dhoni} $\subseteq$ {Dhoni, Kohli, Pujara}

→ Primes $\subseteq N$, $N \subseteq Z$, $Z \subseteq Q$, $Q \subseteq R$

- Every set is subset of itself.
- Empty set has no element. $\emptyset$, { }, (null set)
  empty element is subset of all sets.
  $\emptyset \in X$
- A set can contain other set.
- Powerset → It is set of subset of a set.
  $X = \{a, b\}$ (no. of element $= 2^{n(a)}$)
  Powerset $= \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- Singleton set = Set with only 1 element.

Subset: $a = \{1, 2\} = \{\}, \{1\}, \{2\}, \{1,2\}$ (Subset

Powerset: $a = \{1, 2\} = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$ (Set of subset)

& set become element

Symbols

$\in$ → Belongs to

$\forall$ → For all / For every

$\exists$ → There exist

$|$, $;$, $:$ → Such that, $,$ → and

N → Natural no.

Z → integers

Q → Rational no.

R → Real no

$\in$ = Element ($5 \in Z$)

$\subset$ = Subset ($\{Z\} \subset \{R\}$)

$\subseteq$ = Subset (or equal to) ($\{N\} \subseteq \{Z\}$)

$\cup$ = union

$\cap$ = Intersection

$A^c, \bar{A}, A', \sim A$ = A complement

$\setminus$ = Set difference

- **Equal set** = When element of set A match element of set B. is equal set.

  Eg = $A = \{1, 2\}$

  $B = \{x : x^2 - 3x + 2 = 0,\ x \in R\} \Rightarrow B = \{1, 2\}$

  Hence, $A = B$ (equal set)

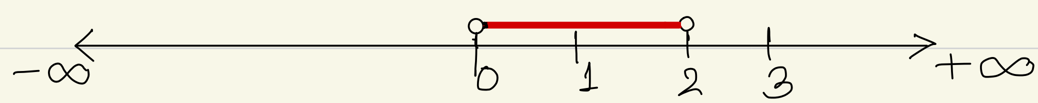
- **Equivalent set** = When cardinal no. are same in 2 set. (means no. of element in set C is equal to no. of element in set D)

  $C = \{1, 2\}$, $d = \{d, f\}$

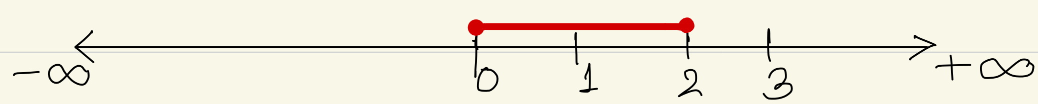
  $n(c) = n(d)$ (equivalant set)

- **Universal set** = The set that has all the element relevent to our question.

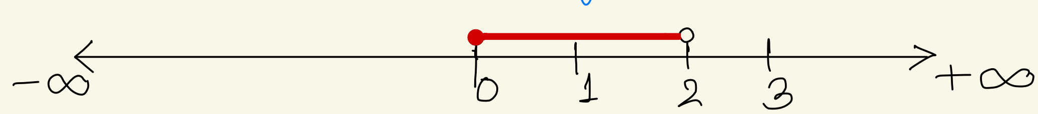
- **Intervals** = It is of 2 type :
  - **Open intervals** :- It is interval where end point are not included. Ex = $(0, 2)$ or $]0, 2[$



  - **Close interval** :- It is interval where end point are included. Eg = $[0, 2]$



  - **Semi open/close interval** :- In this type of interval one end point is included & 1 end is excluded. Eg = $[0, 2)$ or $[0, 2[$



after -6 to 6 $= -6 < z \leq 6$
from -6 to before $6 = -6 \leq z < 6$

Eg:-

Integers from -6 to 6
$\{z \mid z \in \mathbb{Z},\ -6 \leq z \leq 6\}$

- ☐ Set of n element has 2^n element.
- ☐ Subset of binary number:

→ $X = \{x_1, x_2, x_3, x_4, \dots x_n\}$
→ n bit binary no. It also has 2^n element:
- 3 bit : 000, 001, 010, 100, 011, 110, 111, 101

→ Digit i represent wether x_i is subset.
- $X = \{a, b, c, d\}$
- 0101 is $\{b, d\}$
- 0000 is $\emptyset$, 1111 is X.

→ 2^n is n bit number.

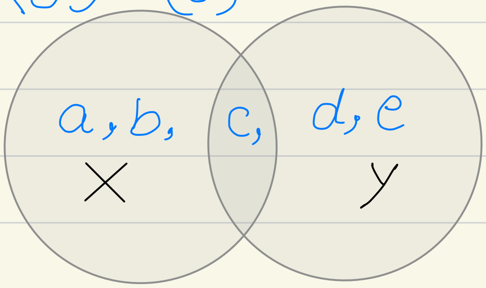
- ☐ Set comprehension:
  - The set of even integers.
    $\{x \mid x \in \mathbb{Z},\ x \bmod 2 = 0\}$
    → Begin with existing set $\mathbb{Z}$.
    (means divided by 2) → Apply condition in $x \in \mathbb{Z}$ so, $x \bmod 2 = 0$
    → Collect all element that match cond..
  - The set of perfect square.
    $\{m \mid m \in \mathbb{N},\ \sqrt{m} \in \mathbb{N}\}$
  - Set of rational in reduced form.
    $\{m/n \mid m, n \in \mathbb{Z},\ \gcd(m, n) = 1\}$

Cardinality means no. of element in a set.

Union → Combine X and Y, $X \cup Y$ ($\cup$)

Can be written $A \cup B$, A or B, A + B

$\{a,b,c\} \cup \{c,d,e\}$
$= \{a,b,c,d,e\}$

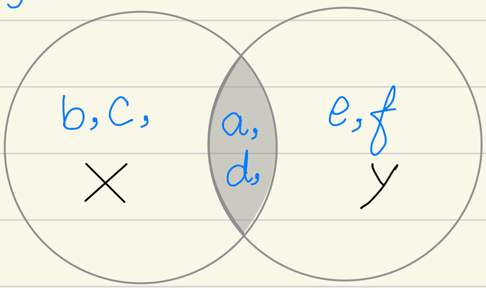


Intersection → Element common in X and Y. ($\cap$)

written as $X \cap Y$.

Can be written as $A \cap B$, $A \cdot B$, A and B

$\{a,b,c,d\} \cap \{a,d,e,f\}$
$= \{a,d\}$



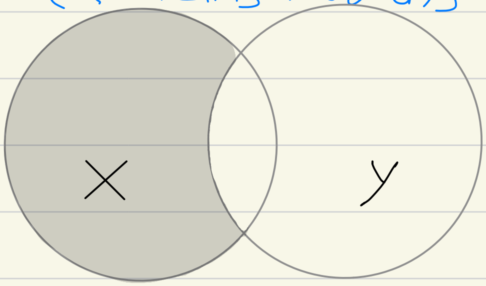
Set diffrence → element of X not in element Y.

Can writeas $X \setminus Y$ or $X - Y$ ($\setminus$ means not in)

$\{a,b,c,d\} \setminus \{a,d,e,f\}$
$= \{b,c\}$

In set $x - y \neq y - x$

Also written as : $A - (A \cap B)$, $A \cap B^c$

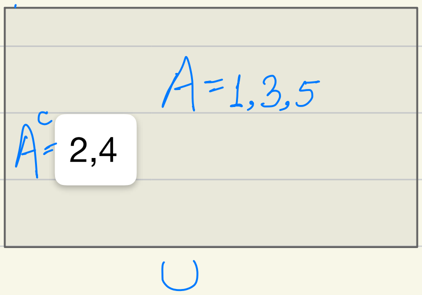


Complement → Element not in X, $\bar{X}$ or X^c.

Universal set → It is the set from your set come from.

$U = \{1,2,3,4,5\}$
$A = \{1,3,5\}$ $A^c = \{2,4\}$
$B = \{1,2,3,4,5\}$ $B^c = \{\}$
$C = \{\}$ $C^c = \{1,2,3,4,5\}$

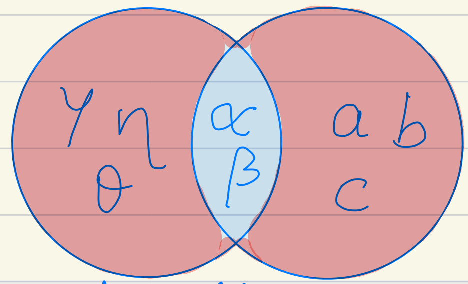
$(A^c)^c = A$, $n(A^c) + n(A) = n(U)$



Symmetric difference of 2 sets:

- Can be written as $A \Delta B$ or $A \oplus B$ or $(A \setminus B) \cup (B \setminus A)$

$A = \{\alpha, \beta, \gamma, \eta, \theta\}$
$B = \{\alpha, \beta, a, b, c\}$

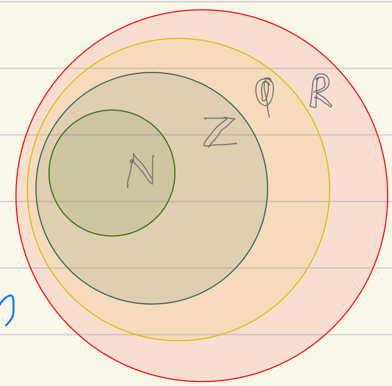


Can be written as:
$(A/B) \cup (B/A)$ or $(A-B) \cup (B-A)$ or $(A \cap B^c) \cup (B \cap A^c)$
$= (A \cup B) - (A \cap B)$

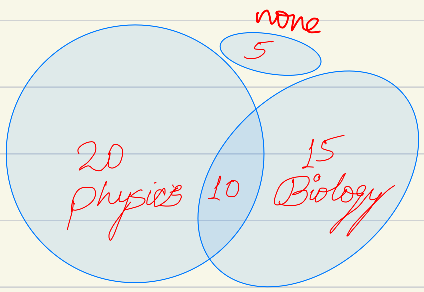
- Membership: $5 \in$ Prime, $5 \notin$ Prime.

Subset: Prime $\subset \mathbb{N}$, $\mathbb{N} \subset \mathbb{Z}$, $\mathbb{Z} \subset \mathbb{Q}$, $\mathbb{Q} \subset \mathbb{R}$

$\mathbb{Q}$ = Rational no.
$\mathbb{R}$ = Real no.



Euler-venn diagram



Total student = 50

$P \cup B = 45$ (Taken phy or bio or both)
$P \cap B = 10$ (Both phy & Bio taken)
$P \setminus B = 20$ (Only phy)
$B \setminus P = 15$ (Only Bio)
$\overline{P \cup B} = 5$ (taking neither) ↳ complement

- In a class of 60 student, 35 took physics, 30 took bio & 10 took neither. How many took both Ph & B.

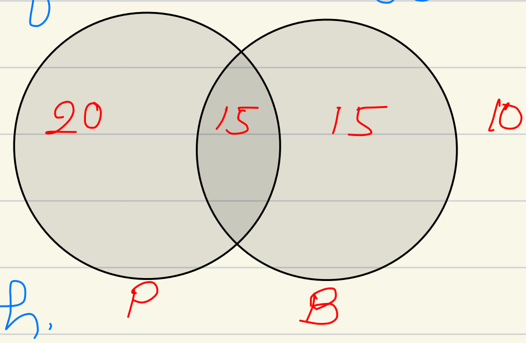
$\Rightarrow |Y| =$ Cardinality of Y (no. of element) $= 60$

$\Rightarrow |P| + |B| = 35 + 30 = 65$

If 10 ~~took~~ neither than:

$\Rightarrow |P \cup B| = 60 - 10 = 50$

So, $65 - 50 = 15$ taken both.



Cartesian product :-

- $A \times B = \{(a,b) \mid a \in A, b \in B\}$

  It is pair of element from A & B.

  - $A = \{0,1\}$, $B = \{2,3\}$

  $A \times B = \{(0,2), (0,3), (1,2), (1,3)\}$

$\rightarrow$ In pair order is Important :-

- $(0,1) \neq (1,0)$

- Combine cartesian product with set comprehension :-

  $\{(m,n) \mid (m,n) \in \mathbb{N} \times \mathbb{N}, n = m+1\}$

  $\{(0,1), (1,2), (2,3) \ldots (17,18), \ldots\}$

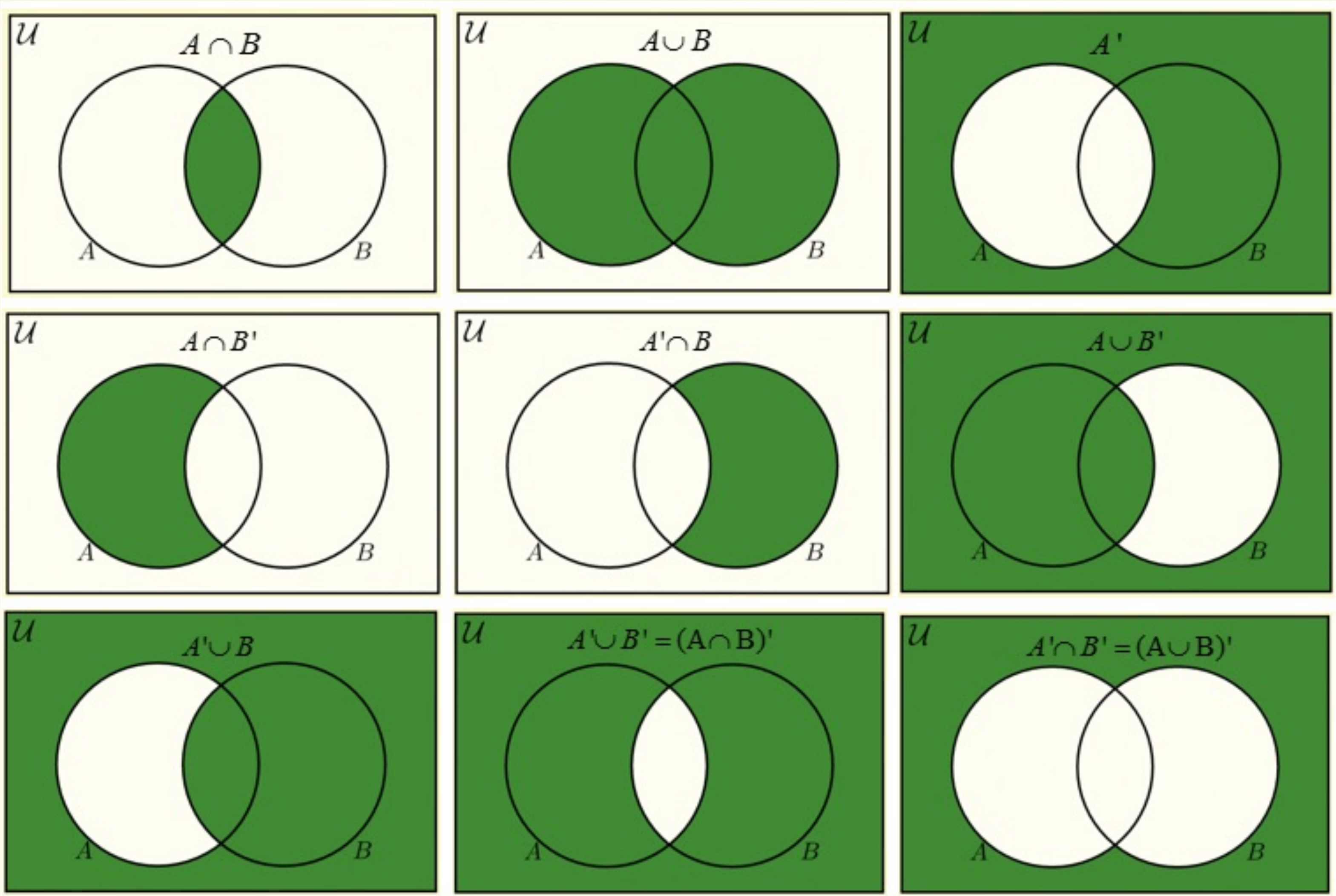
- Pairs (d,n) wher d is factor of n

  $- \{(d,n) \mid (d,n) \in \mathbb{N} \times \mathbb{N}, d \mid n\}$

  $\{(1,1), \ldots\ldots, (2,82), \ldots, (14,56), \ldots\}$

Binary relation $= R \subseteq A \times B$

notation, $(a,b) \in R$, $a R b$



$\mathcal{U}$
$A \cap B$
A
B
$A \cup B$
A'
$A \cap B'$
$A' \cap B$
$A \cup B'$
$A' \cup B$
$A' \cup B' = (A \cap B)'$
$A' \cap B' = (A \cup B)'$

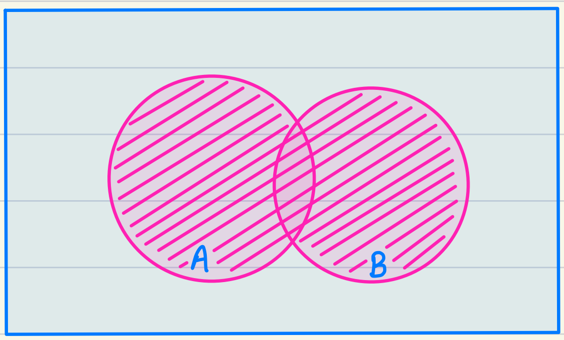
○ DE-MORGAN'S LAW :-

→ Law of complement
→ law of union & intersection

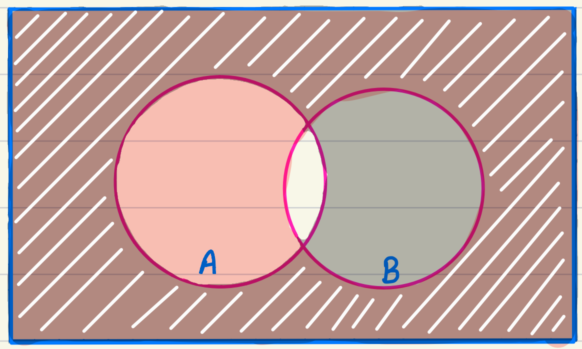
→ Law of complement :

① $(A \cup B)^c = A^c \cap B^c$
② $(A \cap B)^c = A^c \cup B^c$

① $(A \cup B)^c$ = $A^c \cup B^c$

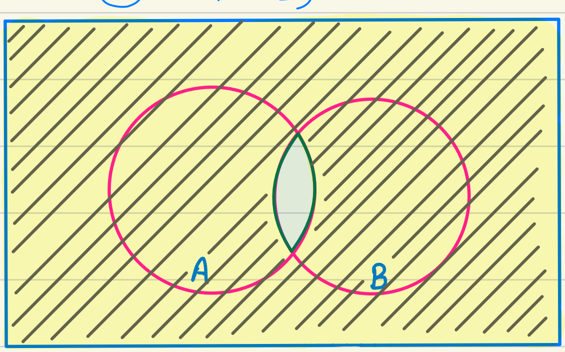


Blue area is $(A \cup B)^c$

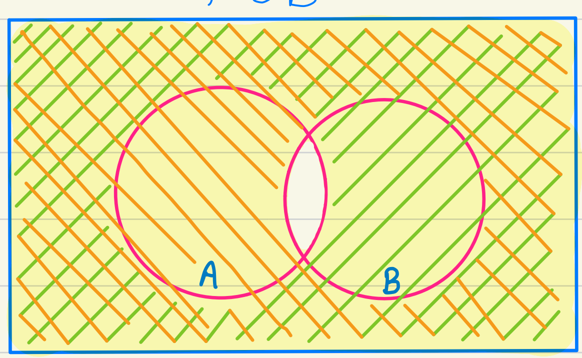


Area with both black & Red sade combine is $A^c \cup B^c$ (white line)

② $(A \cap B)^c$ = $A^c \cup B^c$



$A \cup B$ is green shaded part it's intersection is grey line area



A^c is green line area
and B^c is orange line area
and $A^c \cup B^c$ is both orange green line combine yellow.

→ law of union & intersection:

→ $\cup / \cap$ is distributive over $\cap / \cup$:-

① $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

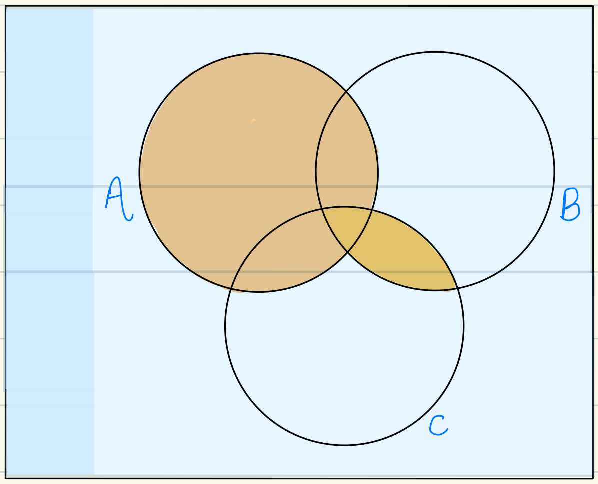
② $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

like removing().

$= a(b+c)$

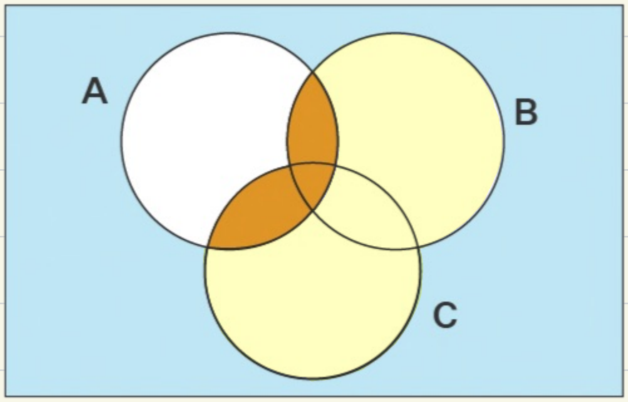
$\Rightarrow ab + ac$

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



$A \cup (B \cap C) =$

2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

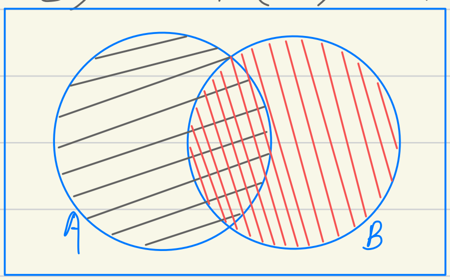


$A \cap (B \cup C)$ —

$(B \cup C)$ —

○ Addition theorem in set:-
(Inclusion & exclusion principle)

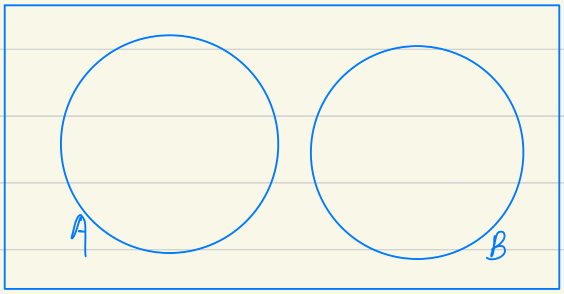
$\rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Bcz, to add {A} and {B} we have to remove red-black check line once bcz, if is added twice.

$\rightarrow n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Disjoint set →



$\rightarrow n(A \cup B) = n(A) + n(B)$

○ $x = 2, 4$ (If we write like this than order of 2,4 are not important
$x = (2,4)$ (In this case order is important like point in graph)
means : $2,4 = 4,2$ and $(2,4) \neq (4,2)$

○ Cartition product of sets:

$A = \{1, 2\}$ $B = \{A, B\}$

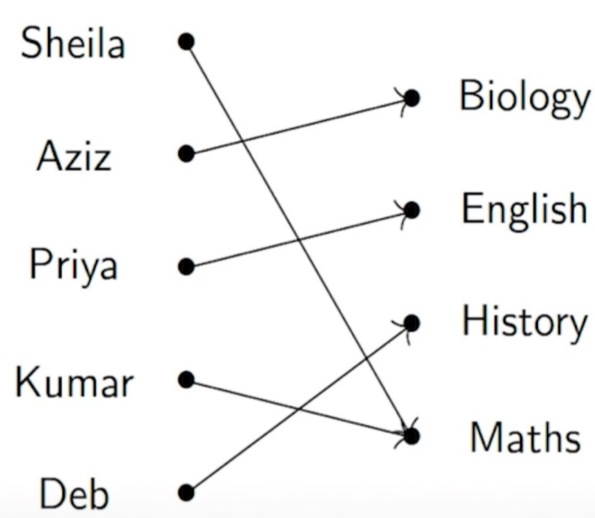
$A \times B = \{(1,A), (1,B), (2,A), (2,B)\}$ [Here $A \times B \neq B \times A$]

$B \times A = \{(A,1), (A,2), (B,1), (B,2)\}$

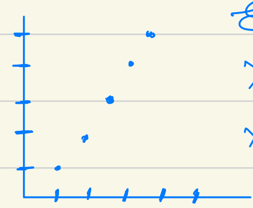
$n(A \times B) = n(A) \times n(B)$ & $n(A \times B) = n(B \times A)$

- Teachers and courses
  - T, set of teachers in a college
    C, set of courses being offered
  - $A \subseteq T \times C$ describes the allocation of teachers to courses
  - $A = \{(t,c) \mid (t,c) \in T \times C, t \text{ teaches } c\}$
- Mother and child
  - P, set of people in a country
  - $M \subseteq P \times P$ relates mothers to children
  - $M = \{(m,c) \mid (m,c) \in P \times P,$
    $m \text{ is the mother of } c\}$

A relation as a graph



○ **Identity relation:**
Same as reflexive but only identiy.
→ $I \subseteq A \times A$
○ $I = \{(a,b) \mid (a,b) \in A \times A, a = b\}$



Eg: $A = \{1,2\}$
✗ $R = \{(1,1)\}$
✗ $R = \{(1,1)(2,2)(3,1)\}$
✓ $R = \{(1,1)(2,2)\}$

○ **Reflexibe relation:**

→ $R \subseteq A \times A$, $I \subseteq R$
○ $\{(a,b) \mid (a,b) \in N \times N, a,b > 0, a \mid b\}$

○ **Symetric relation:** (a,b) can change place.
→ $(a,b) \in R$ if & only if $(b,a) \in R$
○ $\{(a,b) \mid (a,b) \in N \times N, \gcd(a,b) = 1\}$
Eg: $A = \{a,b,c\}$ ⇒ ✗ $R = \{(a,b)(b,a)(c,a)\}$ ✓ $R = \{(a,b)(b,a)\}$

○ **Transitive relation:** If c is not ther we asume true.
→ If $(a,b) \in R$ and $(b,c) \in R$ than $(a,c) \in R$
$= \{(a,b) \mid (a,b) \in \mathbb{R} \times \mathbb{R}, a < b\}$
Eg ⇒ ✓ $R_1 = \{(a,b)\}$ ✓ $R_2 = \{(1,1)\}$ ✓ $R = \{(1,2)(2,3)(1,3)\}$

$A = \{1, 2, 3\}$ ✗ $R = \{(1,2)(2,1)(1,1)(2,2)\}$

a b b a
but $b \neq a$

- **Antisymmetric relation:** Eg ✓ $R = \{(a,b)\}$, ✗ $R = \{(a,a)\}$
  - → If $(a,b) \in R$ and $a \neq b$, than $(b,a) \notin R$
  - ⇒ $M \subseteq P \times P$ relates mother to child then, $(P,C) \in M$ than $(C,P) \notin M$

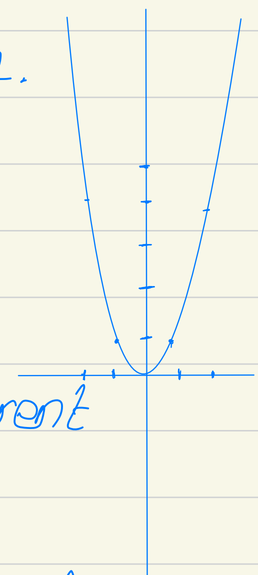
- **Reflexive, symmetric transitive:**
  - In equavalance -ion partition a set. (AM/PM)
  - Group of equavalant element is called equivalance class. Eg: $A = \{1, 2, 3\}$
  - ✓ $R = \{(1,1)(2,2)(3,3)(4,5)\}$ ✗ $R = \{(1,1)\}$ ✓ $R = \{(1,1)(2,2)(3,3)\}$

Functions:

It is rule to map input to output.

Conv. x to x^2

$x \longmapsto x^2$, $\boxed{f(x) = x^2}$



Function is injective: It means different input produces diff. output.

- $A \times B$ – Cartision product, all pair (a,b), $a \in A$ & $b \in B$
  - $A = \{1, 4, 8\}$ $B = \{2, 8, 9\}$
  - $A \times B = \{(1,2), (1,8), (1,9), (4,2), (4,8), (4,9), (8,2), (8,8), (8,9)\}$
  - $B \times A = \{(2,1), (8,1), (9,1), (2,4), (8,4), (9,4), (2,8), (8,8), (9,8)\}$

  Cartision product can ~~take~~ more than 2 sets.
  - $S \subseteq A \times B = \{(1,1), (4,16), (7,49)\}$
  - $S = \{(a,b) \mid (a,b) \in A \times B, b = a^2\}$

Divisiblity:

$D = \{(d,n) \mid (d,n) \in N \times N,\ d \mid n\}$

$D = \{(d,n) \mid (d,n) \in Z \times N,\ d \mid n\}$

+ve

+ve & -ve

Prime power:-

They are number which are multiple of itself many time.

eg: $\{(3,1), (5,625), (7,343)\} \ldots$
$\quad 3^0 \qquad 5^4 \qquad 7^3$

$P = \{P \mid P \in N,\ \text{factor}(P) = \{1, P\},\ P \neq 1\}$

$P_{power} = \{(P,n) \mid (P,n) \in P \times N,\ n = P^m \text{ for some } m \in N\}$

Function:

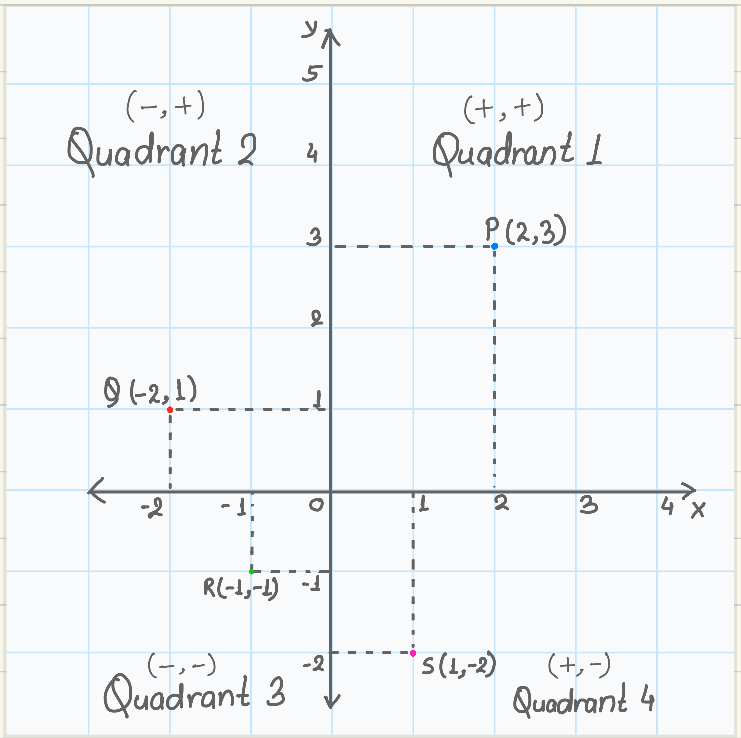
It is a rule to map input to output.

$x \to x^2$, $g(x) = x^2$

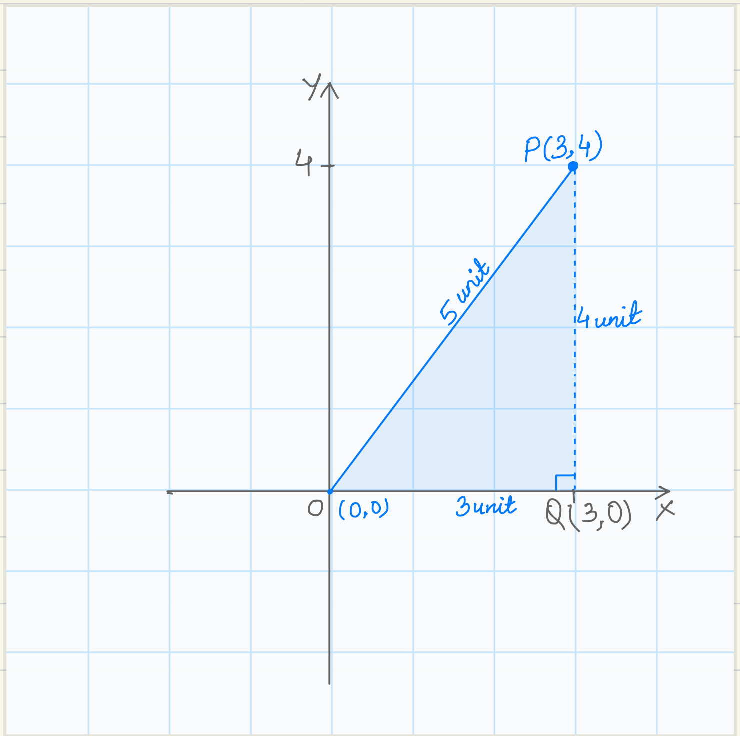
- If a num. divide a sum & it divide 1 part of sum than it must also divide other part of the sum.

Coordinate Geometry

- Rectangular coordinate system:



Coordinate Plane

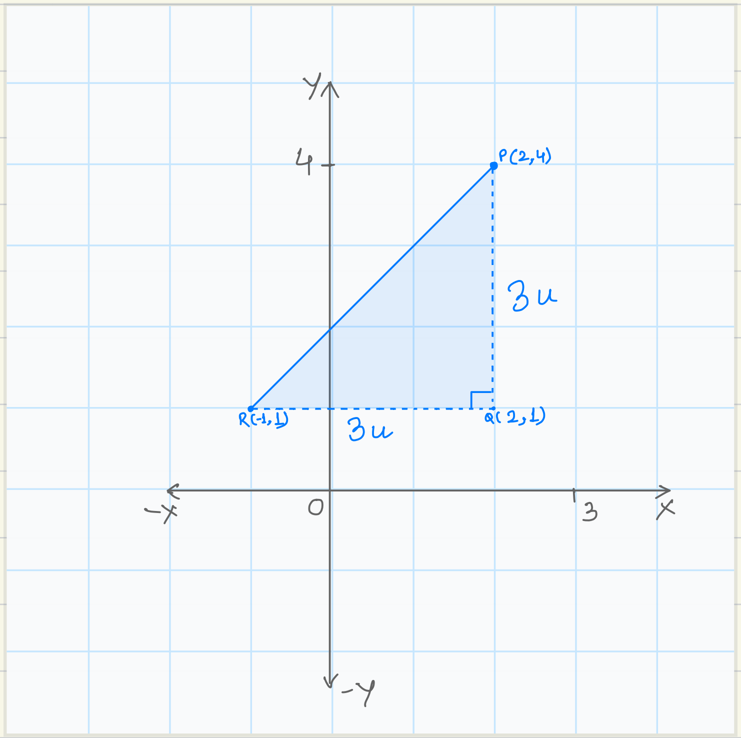


find distance (OP)

By pythagorean we know:

$$OP^2 = OQ^2 + QP^2$$

$$OP = \sqrt{3^2 + 4^2} = 5$$



By same formulae:

$$PR = \sqrt{3^2 + 3^2} = \sqrt{18}$$

$$= 3\sqrt{2}$$

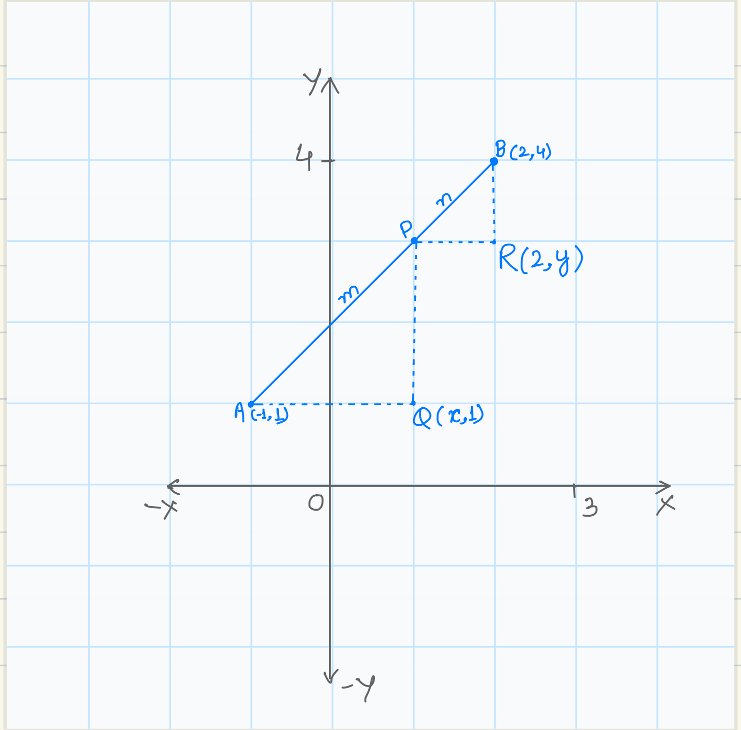
- Point
- line
- circle
- Parabola
- Elypse
- Hyperbola

All are conic section

o Section formula :

P cuts line AB in m : n ratio.
Find P coordinate.

→ Observe $\Delta AQP \sim \Delta PRB$.



$$\Rightarrow \frac{m}{n} = \frac{AP}{PB} = \frac{AQ}{PR} = \frac{PQ}{BR}$$

$$\Rightarrow \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y^1}{y_2 - y}$$

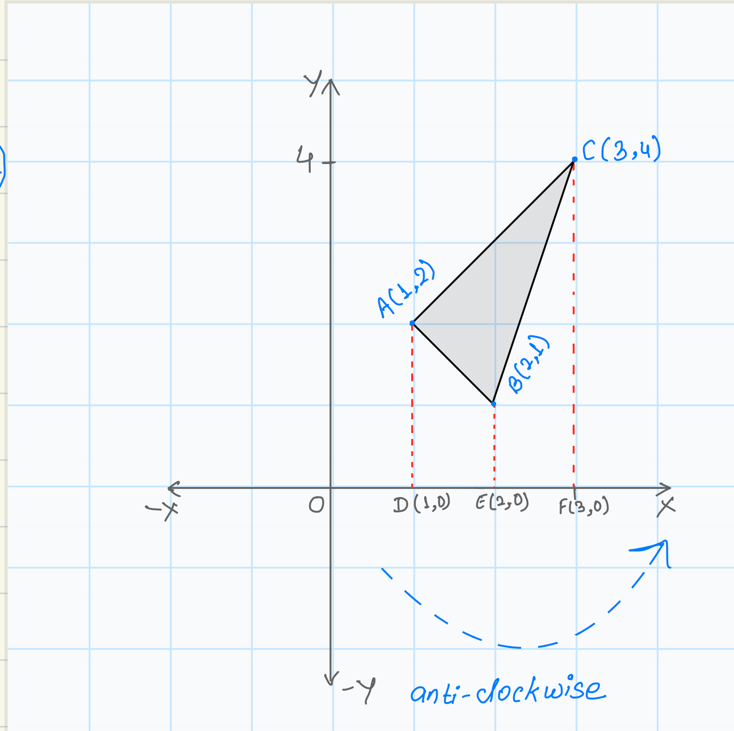
from this we will get:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

o Area of a triangle using co-ordinate :-

By constructing red ‖ line we created 3 trapisium. So,

$\triangleright A(\Delta ABC) = A(\square ADFC) - A(\square ADEB) - A(\square BEFC)$



Area of trapisium is $= \frac{1}{2}$(Sum of ‖ sides) × height

$$A(\square ADFC) = \frac{1}{2}(AD + FC) \times DF = \frac{1}{2}(y_1 + y_3)(x_3 - x_1)$$

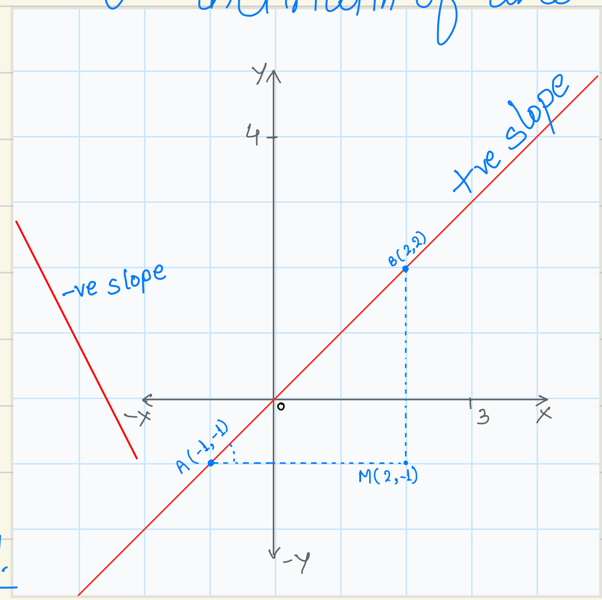
$$A(\square ADEB) = \frac{1}{2}(AD + EB) \times DE = \frac{1}{2}(y_1 + y_2)(x_2 - x_1)$$

$$A(\square BEFC) = \frac{1}{2}(BE + CF) \times EF = \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

After calculation :-

$$A(\Delta ABC) = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

θ = inclinatin of line



Slop of a line :

To find slope of line identify 2 pts. $A(x_1, y_1)$ and $B(X_2, Y_2)$

Construct $\triangle$ with pt. $M(x_2, y_1)$

$$\tan\theta = \frac{\Delta Y}{\Delta X}$$

$$m = \frac{MB}{AM} = \frac{y_1 - y_2}{x_1 - x_2} = \tan\theta$$

→ Vertical line slope = not defined '⊥'

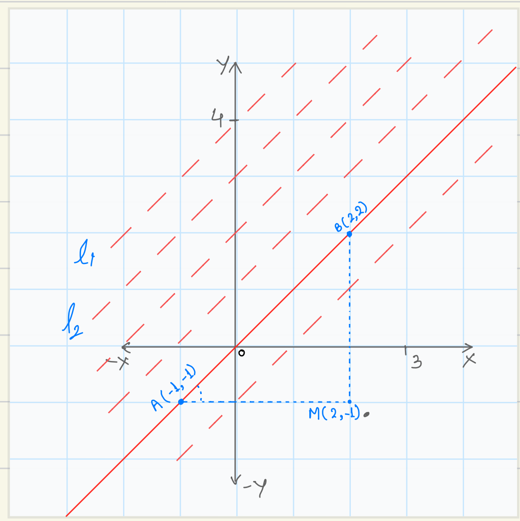
→ Horizontal " = 0. '∥'

$$\text{Slope of a line} = \frac{\Delta \text{ in } Y}{\Delta \text{ in } x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3}{3} = 1$$

(not define) $\tan\theta = \frac{\pi}{2} = \perp$ x-ax

There can be many line with same slope.

Here we can see many dashed line with same inclanation.

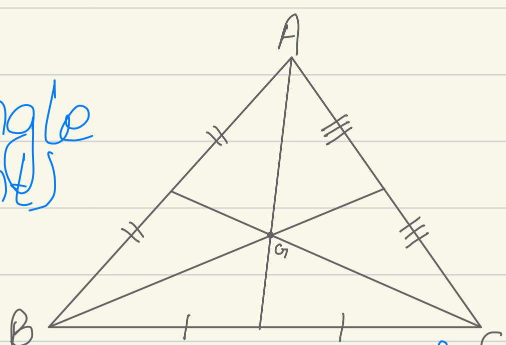


- If two line are parallel in graph than slope of both line are equal and vice-versa

Centroid : 3 median of a trangle is always concurrent. (met at a point)

This point is called centroid (G).

'G' divides median in 2:1 (ratio).



$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$, If all 3 coordinate of Δ is given.

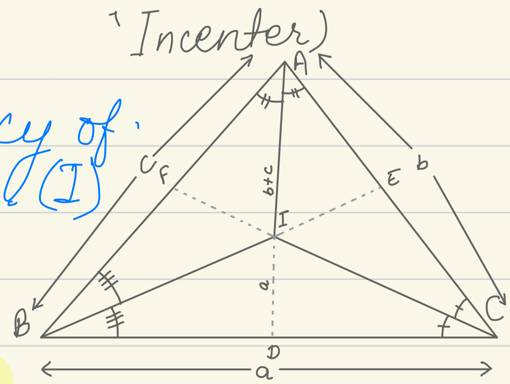
○ Incenter : Point of concurrency of internal ∠ bisector of Δ is incenter (I)

→ $BD : CD = c : b$

→ $AI : ID = b+c : a$

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

(Incenter)



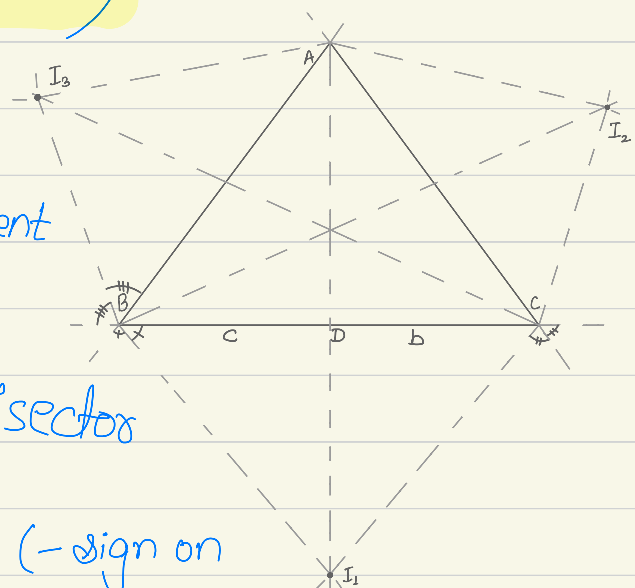
○ Excenter: I_1, I_2, I_3 are excenter of ΔABC. It is point of concurrent of 2 exterior angle bisector and 1 interior angle bisector.

→ I_1 divides the internal angle bisector AD in $b+c : a$. (means, $A : I_1 : AD$)

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

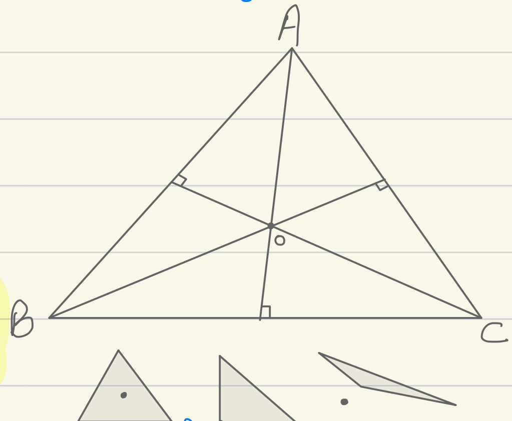
(− sign on begining of 'a' bcz I_1 is front of A)

I_2 & I_3 vice-versa of I_1.



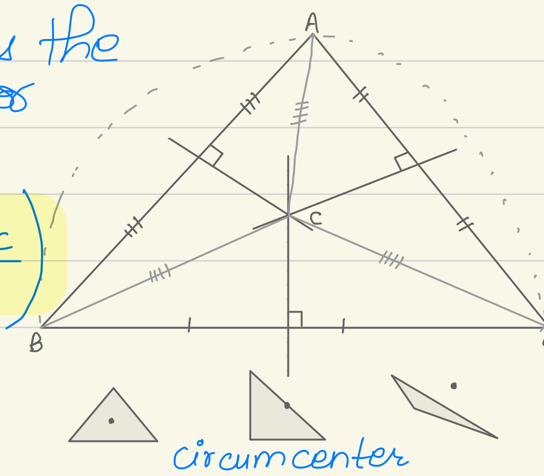
○ Orthocenter:- Point of concurrency of 3 altitude is orthocenter. (can be outside Δ)

$$O = \left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C} \right)$$

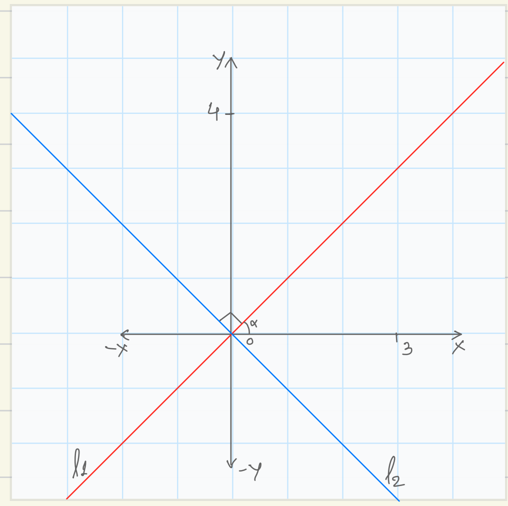


○ Circumcenter:- Circumcenter is the point of concurrency of three ⊥ bisector of sides of Δ.

$$C = \left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C} \right)$$



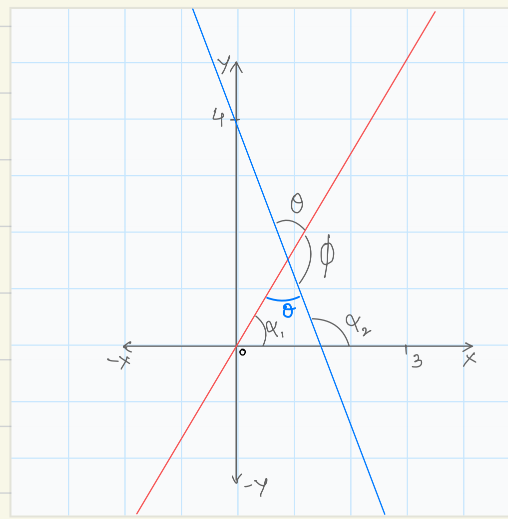
- 3 points are collinear if:
  - → $AB + BC = AC$
  - → area $(\Delta ABC) = 0$
  - → If 'B' divides AC in m : n



If $l_1 \perp l_2$, than product of slop of $l_1 \cdot l_2 = -1$ and vice. versa.

Here suppose:
- $\alpha_1 \& \alpha_2 \neq 90°$

& so, $\theta = \alpha_2 - \alpha_1$



θ or $\angle$ between 2 line :-

$$\tan\theta = \left|\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \cdot \tan\theta_1}\right| = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|$$

- Inclination: It is the angle made by straight line with the direction of x-axis in anticlock wise direction.
  Eg = α_1, α_2 range = $\pi > \alpha \geq 0$

- In case of verticle line all the cordinate of x-axis are same in respect to y-axis.
  if $x = b$ than coordinates of that line are (b, y)
- In case of horizontal line all the cordinate of y-axis are same in respect to x-axis.
  if $y = a$ than coordinates of that line are (x, a)

Equation of line : In point slope form:-

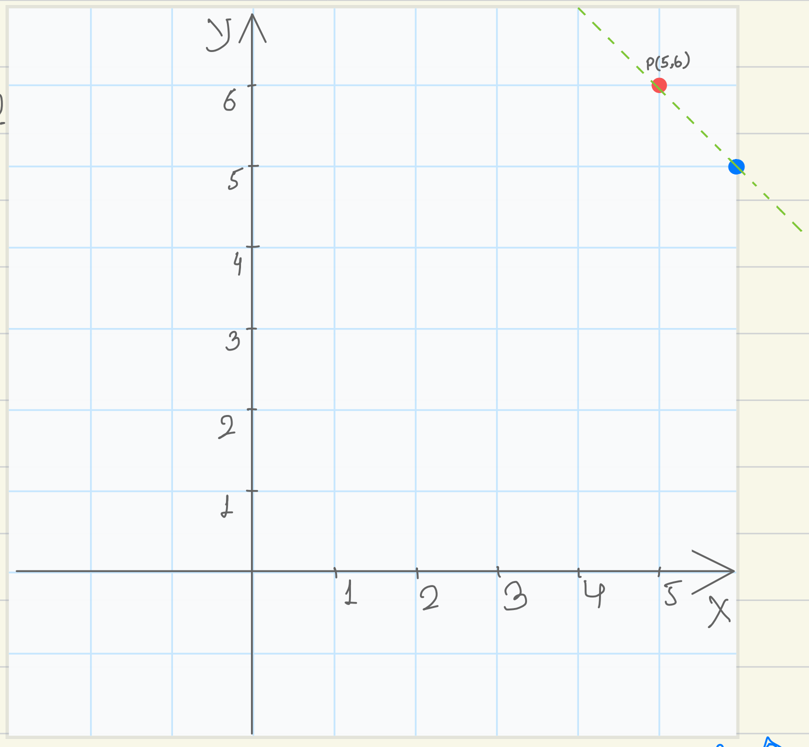
For a non-vertical line l, with slope m and a fix point $P(x_0, y_0)$ on a line, to find equation of line through this point:-

1) 1st we have to take a point $Q(x, y)$ collinier or on the same line and find slope m.

by, $m = \frac{\Delta y}{\Delta x} \Rightarrow m = \frac{y - y_0}{x - x_0}$

$\Rightarrow y - y_0 = m(x - x_0)$ is in point slope form.

Q Find equation of line through point P(5,6) and slope -2.



→ let's take arbitary pt. on this line $Q(x, y)$

→ Slope of line is -2 so,

$\Rightarrow -2 = \frac{(y-6)}{(x-5)}$

$\Rightarrow -2(x-5) = y-6$

$\Rightarrow y = 16 - 2x$

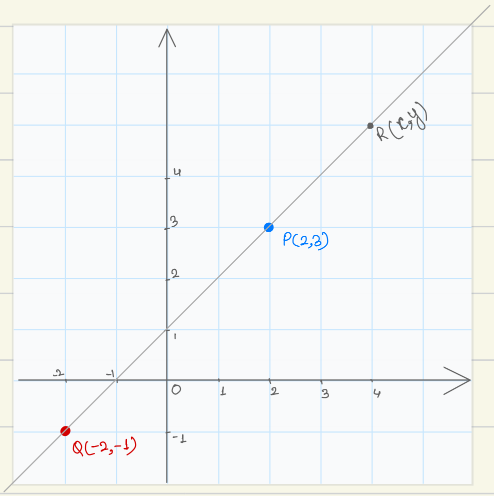
let say, if $x = 5$, than $y = 6$, so line look somthing like

- Equation of a line (two point form):

A line l pass from $P(x_1, y_1)$ & $Q(x_2, y_2)$
Assume $R(x, y)$ is arbitary pts on l line.
Then point P, Q, R are collinear.
Hence, slope of PR = slope of PQ.
Therefore, $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

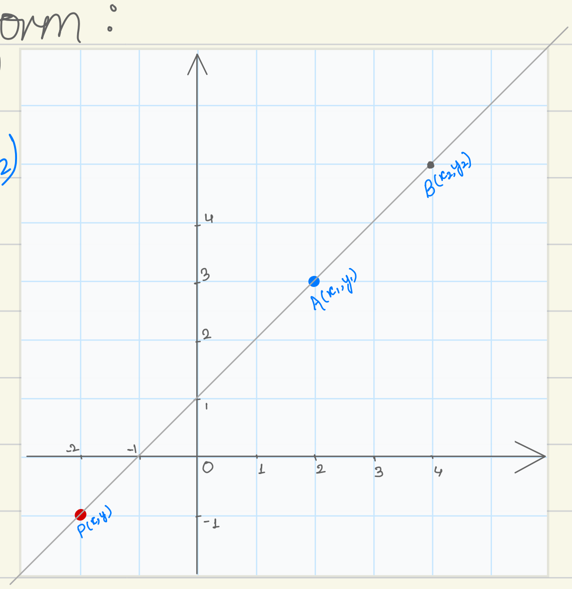
Q. Find equation for P&Q pts.



→ putting these point in 2 pt. form equation.

$$(y-3) = \frac{-1-3}{-2-2}(x-2)$$

- Equation of line in determinant form:



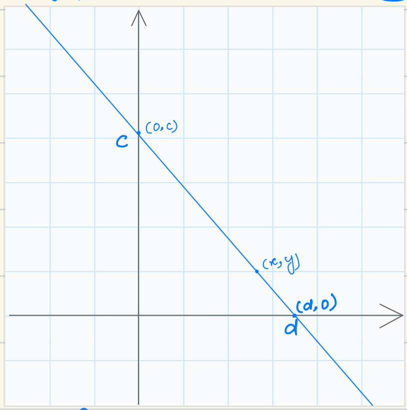
→ Eq. of line passing through (x_1, y_1) & (x_2, y_2)
→ If P is on that line than it must be collinear.
→ So, area ΔPAB should be 0.

so, $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Equation of straight line in determinant form.

○ Equation of line (Slope-intercept form)

When a line l with slope m cuts y-axis at C.
Than c is called y-intercept of line l.



$$m = \frac{y-c}{x-0} \Rightarrow y - c = mx$$

$$\Rightarrow \boxed{y = mx + c}$$

And, when line l with slope m cuts x-axis at d.
than x-axis is called x-intercept of line l

So, $m = \dfrac{y-0}{x-d} \Rightarrow \boxed{y = m(x-d)}$

Q find equation of line with slope ½ & y-intercept = $-3/2$

$\Rightarrow y = mx + c \qquad y = \frac{1}{2}x + \left(-\frac{3}{2}\right) \Rightarrow y = \frac{1}{2}x - \frac{3}{2}$

Q find eq. of line with slope ½ and x-intercept 4.

$\Rightarrow \quad y = \frac{1}{2}(x-4) \quad$ or $\quad 2y - x + 4 = 0$

□ Equation of line : Intercept form

When a line makes x-intercept at a and y-intercept at b than two point of line are (a,0) and (0,b)

Using 2-point form:-

$\rightarrow (y-0) = \dfrac{b-0}{0-a}(x-a)$ or $\dfrac{x}{a} + \dfrac{y}{b} = 1$

Q Find equation of line with x-intercept at -3 and y- intercept at 3.

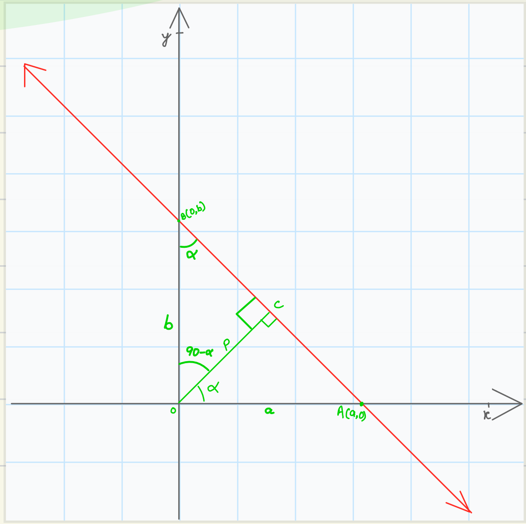
$\Rightarrow \dfrac{x}{-3} + \dfrac{y}{3} = 1$

General equation of a line:

Diff. form of equation of line	Representation	General form $Ax+By+C=0$
Slope-point form	$(y-y_0) = m(x-x_0)$	$m = -\dfrac{A}{B}$, $y_0 - mx_0 = -\dfrac{C}{B}$
Slope-intercept form	$y = mx+c$ or $y = m(x-d)$	$m = -\dfrac{A}{B}$, $c = -\dfrac{C}{B}$ or $d = -\dfrac{C}{A}$
Two point form	$(y-y_1) = \dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$	$\dfrac{y_2-y_1}{x_2-x_1} = -\dfrac{A}{B}$, $y_1 + \dfrac{A}{B}x_1 = -\dfrac{C}{B}$
Intercept form	$\dfrac{x}{a} + \dfrac{y}{b} = 1$	$a = \dfrac{-C}{A}$, $b = -\dfrac{C}{B}$

- Normal/$\perp$ form of line:-

Eq. of line at distance 'p' units from origin and $\perp$ line from origin is making $\angle$ 'α' with x-axis.



$\cos\alpha = \dfrac{\text{base}}{\text{hypotenus}} = \dfrac{p}{a}$, $\sin\alpha = \dfrac{\Delta BCO, \perp}{\text{hypotenus}} = \dfrac{p}{b}$

$a = p\sec\alpha$ (x-inter) $b = p\,\text{cosec}\,\alpha$ (y-inter)

Hence, eq. of line:

$\dfrac{x}{p\sec\alpha} + \dfrac{y}{p\,\text{cosec}\,\alpha} = 1$

$$\boxed{x.\cos\alpha + y.\sin\alpha = p}$$

General form = $Ax + By + C = 0$

Any equation in form $Ax + By + C = 0$, where $A, b \neq 0$ simultinously, is called general linear equation equation of line. (This line will not be vertical.

Q. Equation of line is $3x - 4y + 12 = 0$. find slope, x-intercept & y-intersept on the line.

⇒ Here, $A = 3$, $B = -4$ and $C = 12$

Using intersept foom, $a = -C/A = -4$
& , $b = -C/B = 3$ } Intersepts

Slope intercept form = $y = mx + c$

Changing $3x - 4y + 12 = 0$ in that foom.

$3x + 12 = 4y$

$y = \frac{3x}{4} + 3$

So, $m = \frac{3}{4}$

Slope $= 3/4$.

If line are parallel:
- $a_1 b_2 = a_2 b_1$
- $a_1 a_2 + b_1 b_2 = 0$

→ In ∥ line their slope are equal.
→ In ⊥ line product of their slope is -1.

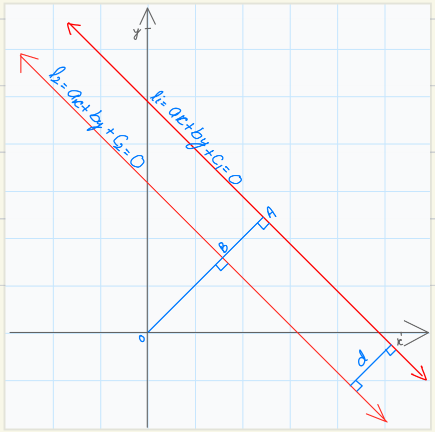
☐ Distance b/2 2 ∥ line

Distance is same every whor.

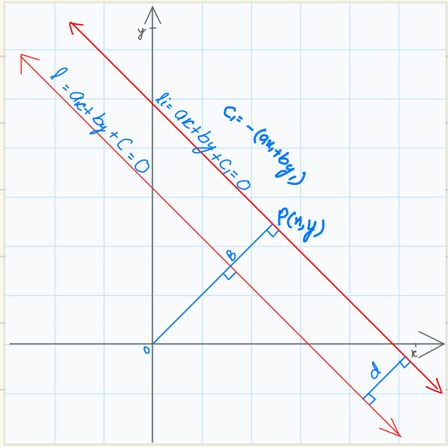
d o, $OA = \left| \frac{C_1}{\sqrt{a^2 + b^2}} \right|$ $OB = \left| \frac{C_2}{\sqrt{a_2 b_2}} \right|$ & , $AB = OA - OB$

$AB = \left| \frac{C_1 - C_2}{\sqrt{a^2 + b^2}} \right|$

- Coff. must be same.
- C_1 & C_2 on same side.



☐ Distance b/w point (x_1, y_1) and a line $ax + by + c = 0$



$\Rightarrow$ Construct a l_1 // line from l on point P.

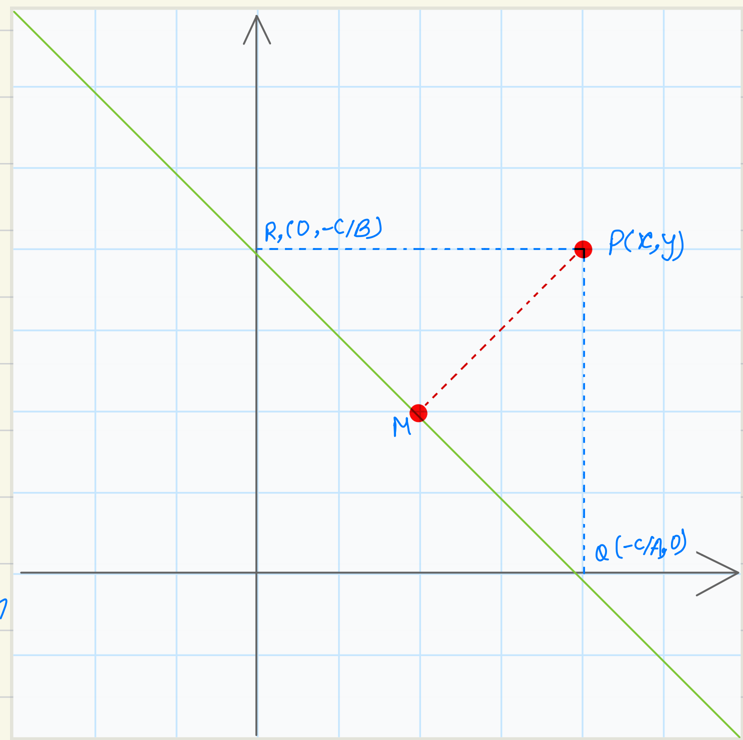
Putting x_1, y_1 in line formula to find C_1.

$\Rightarrow ax_1 + by_1 + C_1 = 0$

$\Rightarrow C_1 = -(ax_1 + by_1)$

$\Rightarrow$ Puttin in distance of // line formula $\Rightarrow d = \left| \dfrac{ax_1 + by_1 + C}{\sqrt{a^2 + b^2}} \right|$

◯ Distance of a point $P(x_1, y_1)$ from line l having equation $Ax + By + C = 0$



$\Rightarrow$ For $A, B \neq 0$

By intercept form:-

$\Rightarrow \dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow Ax + By + C = 0$

$\Rightarrow Ax + By = -C$

$\Rightarrow \dfrac{Ax}{-C} + \dfrac{By}{-C} = \dfrac{-C}{-C}$ | that form

$\Rightarrow x = \dfrac{-C}{A} \quad y = \dfrac{-C}{B}$

$A(\Delta PQR) = \frac{1}{2} QR \times PM$. Hence, $PM = 2A(\Delta PQR)/QR$

By, area of Δ by quardine formula:

$A(\Delta ABC) = \dfrac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$

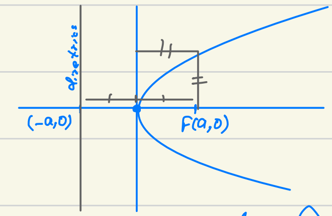
$\Delta PQR = \dfrac{1}{2} \left| x_1 \left(\dfrac{-C}{B}\right) - \dfrac{C}{A}\left(y_1 + \dfrac{C}{B}\right) \right| = \dfrac{1}{2} \dfrac{|c|}{|AB|} |Ax_1 + By_1 + C|$

Quadratic Function

A quadratic function is described as equation in the form of: (Where square is present)

- $f(x) = \underline{ax^2} + \underline{bx} + \underline{c}$, (Quadratic fn) where $a \neq 0$

Quadratic term   Linear term   Constant



- $f(x) = mx + c$, (linear fn)

Parabola is a locus of point (x,y) which move such that its dist. from line is same as dist. from point. The graph of any quadratic fn is parabola.

Making suppose b & $c = 0$

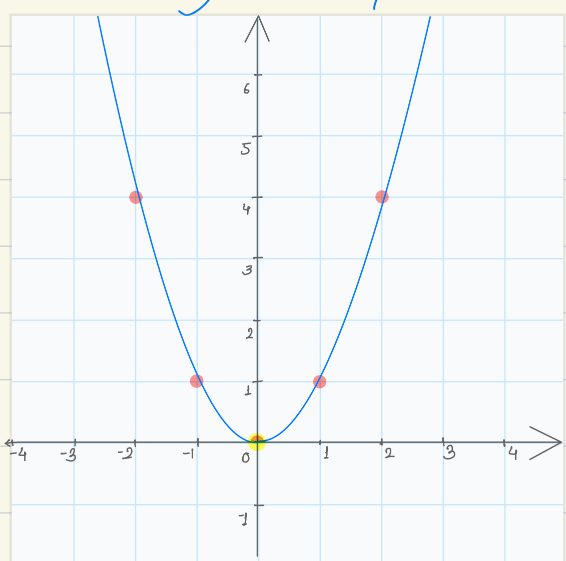
$f(x) = ax^2 + bx + c$

if $x = 1$ $y = 1$

$x = 2$ $y = 4$

$x = -1$ $y = 1$

$x = -2$ $y = 2$

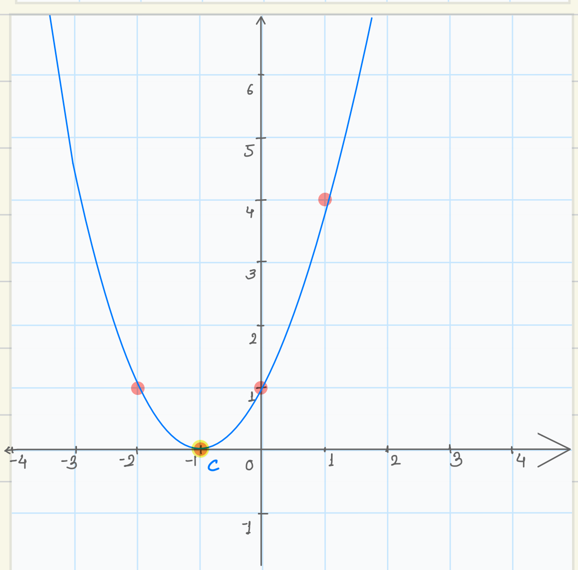


- Plot graph for $f(n) = x^2 + 2x + 1$

We give value of x and get y.

x	y
-2	1
-1	0
0	1
1	4

→ vertex (Minimum/maximum)



→ All parabola has axis of symmetry. (both side match)
→ Y intercept of quadratic function is c.
like:

- putting 0 in x ⇒ $a(0)^2 + b(0) + c = c$
- Axis of symmetry equation: $x = -b/(2a)$
- X-coordinate of vertex: $-b/(2a)$
- If $a > 0$ graph will be upward & vice-versa.

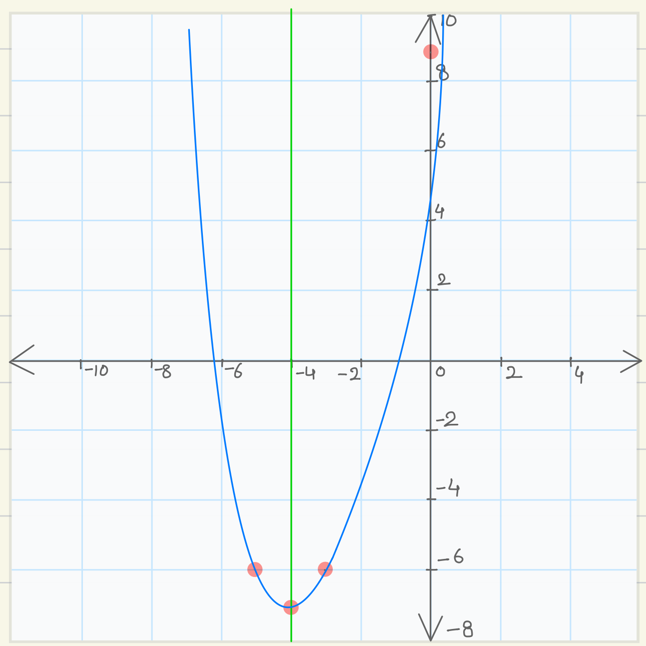
Eg: Graph a function: $f(x) = x^2 + 8x + 9$ $a > 0$

Y intercept by $(x=0) = 9$

Axis of symmetry $= \frac{-8}{2(1)} = -4$

The vertex: $(-4, -7)$

x	y
0	9
-3	-6
-4	-7
-5	-6



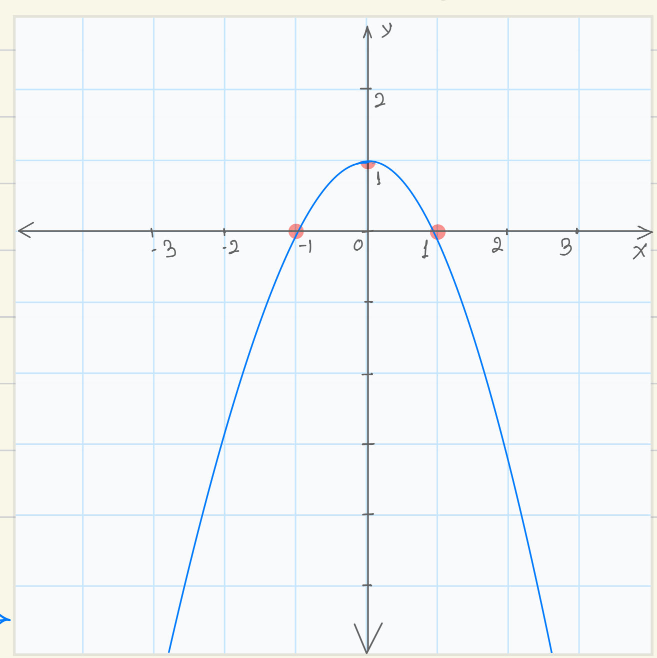
Graph eq. $f(x) - x^2 + 1$.

Y intercept $= 1$

Axis of symmetry $= 0$

vertex $= (0, 1)$

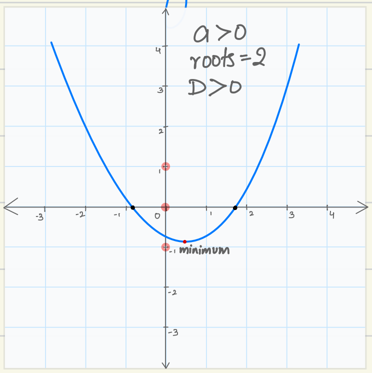
x	y
-1	0
0	1
1	0

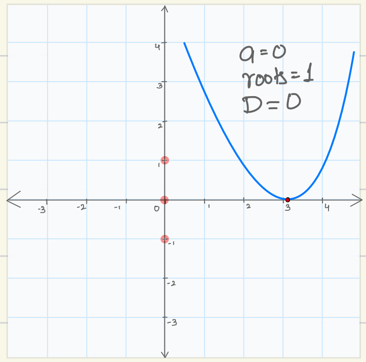


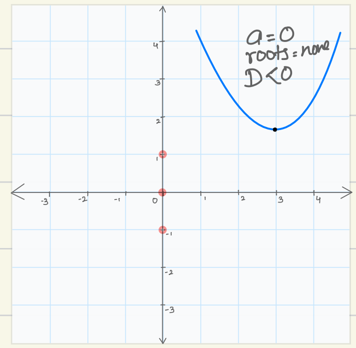
$a < 0$ →

- When $a > 0$ it make upward graph & vice versa.

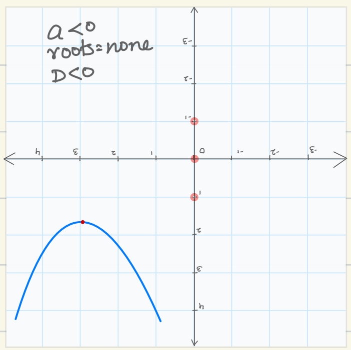
→ No. of roots determine how many time it cuts.

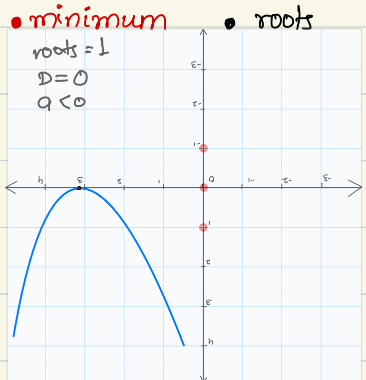


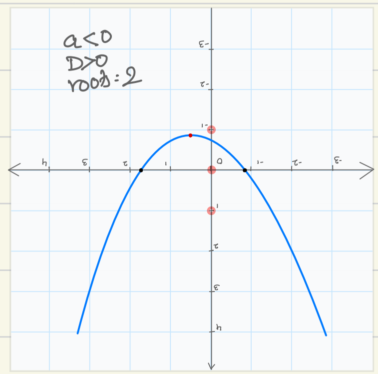




• minimum • roots

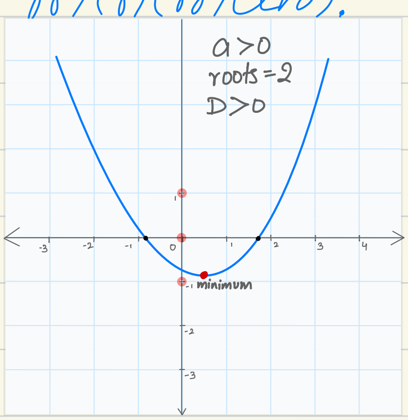




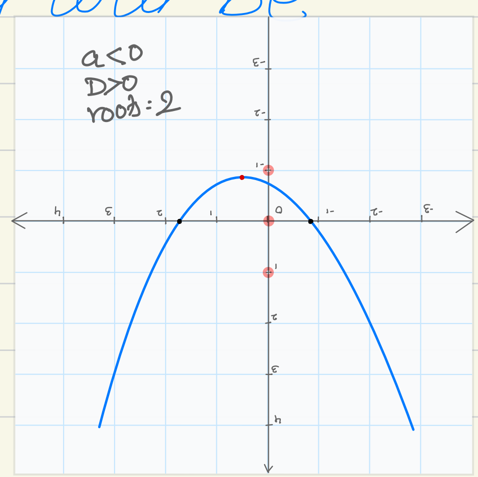


- putting 0 in x ⇒ $a(0)^2 + b(0) + c = c$
- Axis of symmetry equation: $x = -b/(2a)$
- X-coordinate of vertex: $-b/(2a)$
- If $a > 0$ graph will be upward & vice-versa.

- When x will be $\frac{-b}{2a}$ than y will be minimum.



for: $ax^2 + bx + c$



minimum $x = -b/2a$ max $= \infty$

minimum $y = -D/4a$

Range: $\left[\frac{-D}{4a}, \infty\right)$

min $= -\infty$

maximum $x = -b/2a$

maximum $y = -D/4a$

Range: $\left(-\infty, \frac{-D}{4a}\right]$

If vertex is given & a is given than quadratic eq. will be:

$y - k = a(x-h)^2$ | Eg, vertex $= (5, 4)$ & $a = 1$ $\Rightarrow y - 4 = 1(x-5)^2 \Rightarrow y = (x-5)^2 + 4$

Eg. Let $f(x) = x^2 - 6x + 9$

- Determine whether f has min or max value. If so, than what is value?
- State domain and range of f.

→ Domain of rang is entire line.

$f(x) = x^2 - 6x + 9$

here, $a = 1$, $b = -6$ & $c = 9$

since $a > 0$ function open up. so, It has min-value

- min-value is vertex of parabola.

$= -b/2a = 6/(2 \times 1) = 3$

so, $f(3) = 9 - 18 + 9 = 0$

Q A tour bus in chennai has 500 customer/day. Charge = 40₹. Owner estamate it lose 10 passanger/day for each ₹4 hike. How much should fare to maximise income?

→ Let x = no. of ₹4/- fare hike. than price/pasanger $= 40 + 4x$ & no. of passanger is $(500 - 10x)$. So, income is:

$I(x) =$

$\Rightarrow (500 - 10x)(40 + 4x) \Rightarrow 40x^2 + 1600x + 20000$

Maximum is possible bcz A is -ve ∩

vertex $= x = -b/2a$

$= \frac{+1600}{+80} = 20$ so, $I(20) = 36000$

This mean company could make 20 time hike: $40 + 4 \times 20 = 120$ rs.

- Slop of quadratic equation :

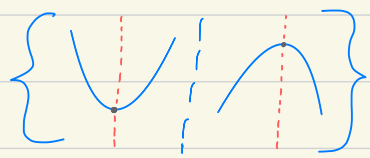
Given quadratic fn , $f(x) = ax^2 + bx + c$, where $a \neq 0$ how to determine slope of function.

Slope of parabola or curve line is :

$$\boxed{m = 2ax + b}$$

If $f(x) = ax^2 + bx + c \quad a \neq b$
then, $g(x) = 2ax + b$

If somehow slope of quadratic fn = 0, then
$g(x) \Rightarrow 2ax + b = 0$
where, $x = \dfrac{-b}{2a}$ vertex =



Means, the slope of a quadratic f(x) at vertex will be zero.

$$m = 2ax + b$$

- Quadratic function : Intercept form

- Common roots :

→ Both roots are common = $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

→ One common root = If: $(a_1c_2 - c_1a_2)^2 = (a_1b_2 - b_1a_2)(b_1c_2 - c_1b_2)$
If this condition match.

→ When no roots are common → Above cond. not satisfy.

Quadratic formula:

$\Rightarrow ax^2 + bx + c = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\alpha = \frac{-b + \sqrt{d}}{2a}, \quad \beta = \frac{-b - \sqrt{d}}{2a}$

Discriminant :- $d = b^2 - 4ac$

If, $b^2 - 4ac = 0$ (1 root)
$b^2 - 4ac < 0$ (no real root)
$b^2 - 4ac > 0$ (2 real root)

Roots of Quadratic equation property: $= \alpha, \beta$

$\alpha + \beta = \frac{-b}{a}, \quad \alpha \times \beta = \frac{c}{a}$

Diff. of roots, = $|\alpha - \beta| = \sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{\sqrt{D}}{|a|}$

$\Rightarrow$ If $\Rightarrow D = 0$, than roots will be equal and real.
$\Rightarrow$ If $\Rightarrow D > 0$, than roots will be real & distinct.
$\Rightarrow$ If $\Rightarrow D < 0$, than roots will be imaginary & distinct. it always lie in a pair.

POLYNOMIALS

- **Layman's prespective :**

  A polynomial is a kind of methomatical expression which is a sum of several methematical terms. Mathematical expression can be number, variable or product of saveral variables.

- **Mathematician prespective :**

  It is a algebric expression in which only arthematic in addition, substraction, multiplication and 'natural' (non-ve) exponents of variables.

- **Degree of a polynomial :**

  → The exponent on the variable in term is called degree of variable in that term.

  Eg → $4x^2y^2$ $deg(x) = 2$, $deg(y) = 2$

  → Degree of a term is sum of degree of variable in that term.

  Eg = $4x^2y^2 = deg(x) + deg(y) = 2+2 = 4$

  → Degree of polynomyal is largest degree of any one of the term with non-zero cofficient.

  Eg → $3x^2 + 4x^2y^2 + 10y + 1 = degree(p) = 4$.

Power should be whole no.

Degree	Name	Example
0	Cons. poly	$c, 1, 5$
1	Linear poly	$2x+4, ax+b$
2	Quadratic poly	$3x^2+2, 4xy+2x$
3	Cubic poly	$3x^2+2, 4x^2y + 2y+1$
4	Quarticpoly	$10x^4+y^4, x^4+10x+1$

→ Addition of Polynomial :

Degree

$P(x) < Q(x) = Q(x)$

$P(x) = Q(x) = \text{any}$

$P(x) > Q(x) = P(x)$

1. $P(x) = x^2 + 4x + 4$, $q(x) = 10$

$\Rightarrow$
$$P(x) = x^2 + 4x + 4$$
$$q(x) = 0x^2 + 0x + 10$$
$$P(x) + q(x) = x^2 + 4x + 14$$

2. $P(x) = x^4 + 4x$, $q(x) = x^3 + 1$

$\Rightarrow x^4 + x^3 + 4x + 1$

3. $P(x) = x^3 + 2x^2 + x$, $q(x) = x^2 + 2x + 2$

$\Rightarrow x^3 + 3x^2 + 3x + 2$

→ Substraction of Polynomial :

1. $P(x) = x^2 + 4x + 4$, $q(x) = 10$

$\Rightarrow$
$$P(x) = x^2 + 4x + 4$$
$$q(x) = (0x^2 \pm 0x \pm 10) \times -1$$
$$P(x) - q(x) = x^2 + 4x - 6$$

2. $P(x) = x^4 + 4x$, $q(x) = x^3 + 1$

$\Rightarrow x^4 - x^3 + 4x - 1$

3. $P(x) = x^3 + 2x^2 + x$, $q(x) = x^2 + 2x + 2$

$\Rightarrow x^3 + x^2 - x - 2$

Let $P(x) = \sum_{k=0}^{n} a_k x^k$, and $Q(x) = \sum_{j=0}^{m} b_j x^j$. then

$$P(x) + q(x) = \sum_{k=0}^{m \vee n} (a_k + b_k) x^k$$

→ Multiplication of polynomial:

1. $P(x) = x^2 + x + 1$ and $Q(x) = 2x^3$

$$\Rightarrow P(x)\,Q(x) = (x^2+x+1)(2x^3)$$
$$= 2x^{3+2} + 2x^{3+1} + 2x^3$$
$$= 2x^5 + 2x^4 + 2x^3$$

2. $P(x) = x^2 + x + 1$ and $q(x) = 2x+1$

$$\Rightarrow P(x)\,Q(x) = (x^2+x+1)(2x+1)$$
$$= (x^2+x+1)(2x) + (x^2+x+1)$$
$$= 2x^{2+1} + 2x^{1+1} + 2x + x^2 + x + 1$$
$$= 2x^3 + 2x^2 + 3x + x^2 + 1$$
$$= 2x^3 + 3x^2 + 3x + 1$$

To generalise:

$P(x) = a_2x^2 + a_1x + a_0$ and $Q(x) = b_1x + b_0$

$$\Rightarrow P(x)\,Q(x) = (a_2x^2 + a_1x + a_0)(b_1x + b_0)$$
$$= (a_2x^2 + a_1x + a_0)(b_1x) + (a_2x^2 + a_1x + a_0)(b_0)$$
$$= (a_2b_1x^{2+1} + a_1b_1x^{1+1} + a_0b_1x) + (a_2b_0x^2 + a_1b_0x + a_0b_0)$$
$$= a_2b_1x^3 + a_1b_1x^2 + a_0b_1x + a_2b_0x^2 + a_1b_0x + a_0b_0$$
$$= a_2b_1x^3 + a_1b_1x^2 + a_2b_0x^2 + a_1b_0x + a_0b_1x + a_0b_0$$
$$= a_2b_1x^3 + (a_1b_1 + a_2b_0)x^2 + (a_1b_0 + a_0b_1)x + a_0b_0$$

Let $P(x) = \sum_{k=0}^{n} a_k x^k$, and $Q(x) = \sum_{j=0}^{m} b_j x^j$. then

$$P(x)\,Q(x) = \sum_{k=0}^{m+n} \sum_{j=0}^{k} (a_j b_{k-j}) x^k$$

Eg: $P(x) = x^2 + x + 1$ and $q(x) = x^2 + 2x + 1$

By above formulae:

k	a_k	b_k
0	1	1
1	1	2
2	1	1

Cofficient

Putting in formula:-

Cofficient	Calculation.
$a_0 b_0$	1
$a_1 b_0 + a_0 b_1$	$1+2=3$
$a_0 b_2 + a_1 b_1 + a_2 b_0$	$1+2+1=4$
$a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0$	$0+1+2+0=3$
$a_0 b_4 a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0$	$0+0+1+0+0=1$

So, result $= P(x) Q(x) = x^4 + 3x^3 + 4x^2 + 3x + 1$

→ Division of polynomial:

Q $\dfrac{3x^2 + 4x + 3}{x} = 3x + 4 + \dfrac{3}{x}$

- Division of polynomial by another polynomial:

$$= \frac{3x^2 + 4x + 1}{x+1} = \frac{3x^2 + 3x + x + 1}{x+1} = \frac{3x(x+1) + 1(x+1)}{x+1}$$

$$= \frac{(3x+1)\cancel{(x+1)}}{\cancel{(x+1)}} = 3x + 1$$

If it is not factor of $x+1$ so,

$$= \frac{3x^2+4x+4}{x+1} = \frac{3x^2+3x+x+4}{x+1}$$

$$= \frac{3x\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{x+4}{x+1} = 3x + \frac{x+1+3}{x+1} = 3x + \frac{\cancel{(x+1)}^{1}}{\cancel{(x+1)}} + \frac{3}{x+1}$$

$$= 3x+1+\frac{3}{x+1}$$

Algorithem →

$P(x) = x^4+2x^2+3x+2$ by $q(x) = x^2+x+1$

$$= \frac{x^4+2x^2+3x+2}{x^2+x+1}$$

$$= \frac{x^4+x^3+x^2-x^3+x^2+3x+2}{x^2+x+1}$$

$$= \frac{x^2\cancel{(x^2+x+1)}}{\cancel{(x^2+x+1)}} + \frac{-x^3+x^2+3x+2}{x^2+x+1}$$

$$= x^2 + \frac{-x^3-x^2-x+2x^2+4x+2}{x^2+x+1}$$

$$= x^2 - x\frac{\cancel{(+x^2+x+1)}}{\cancel{(x^2+x+1)}} + \frac{2\cancel{(x^2+x+1)}}{\cancel{(x^2+x+1)}} \frac{+2x}{(x^2+x+1)}$$

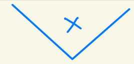
$$= x^2-x+2+\frac{2x}{x^2+x+1}$$

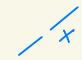
Dividend

So, $\dfrac{P(x)}{Q(x)} = \underbrace{x^2-x+2}_{\text{Quotient}} + \dfrac{2x}{q(x)}$ → Remainder

Divisor

Graph of polynomials:

→ Graph don't have sharp corners. 

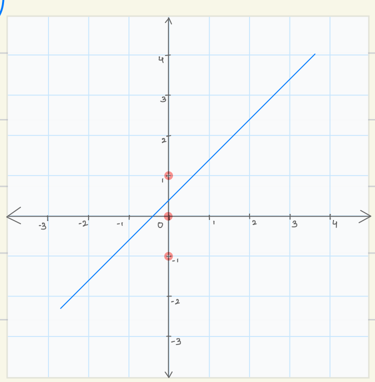
→ Polynomial fn(n) don't have any break. 

Zeros of polynomial f(n):-

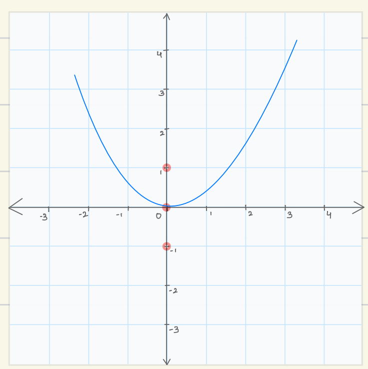
The value of x for which $f(x) = 0$ are zeros of f.

If zeros are even multiple than graph touch x axis and bounce back or bounceback after touching x-axis.

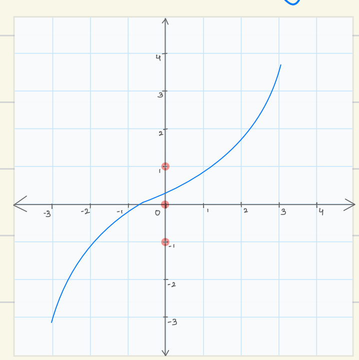
If zeros are odd multiple it cuts directly.



$(x+1)$



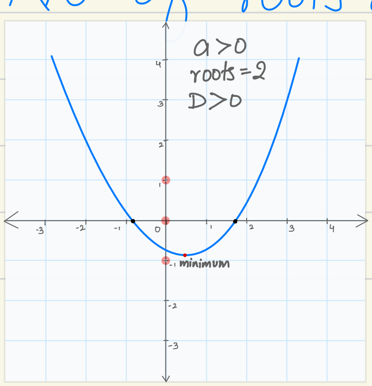
$(x+1)^2$

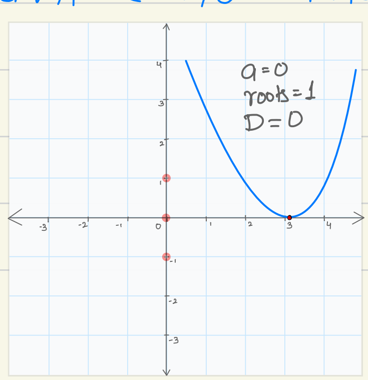


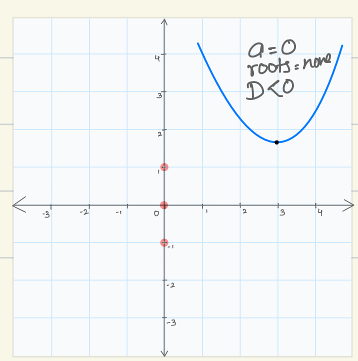
$(x+1)^3$

When $a > 0$ it make upward graph & vice versa.

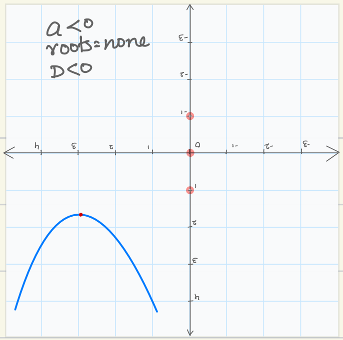
→ No. of roots determine how many time it cuts.

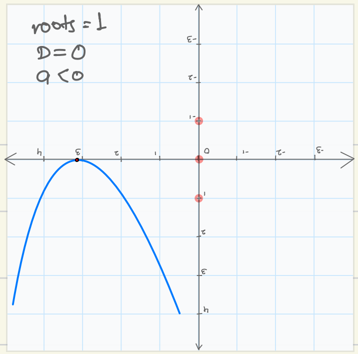


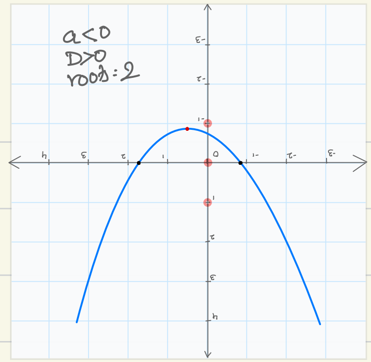




• minimum • roots







☐ End behavior of polynomials:

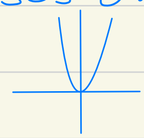
To determine how graph will plot after crossing all its zeros.

So, for:

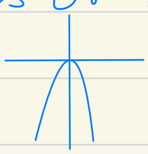
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

Here $a_n x^n$ dominate end behavior of polynomial.

→ If $a_n > 0$ and x^n has even power exponent as x increases or decreases f(x) always go to +ve infinity.



→ If $a_n < 0$ and x^n has even power exponent as x increases or decreases f(x) always go to −ve infinity.



→ If an > 0 and x^n is of odd power as x increases fx also increases and x decreases fx also decreases. & both go to ∞ on in +∞ & ↓ in −∞.

○ Intermediate value theorem: Let f be polyn. fn, It states that if f(a) and f(b) have opp. sign, then there exists atleast one value c b/w a and b for which f(c) = 0.

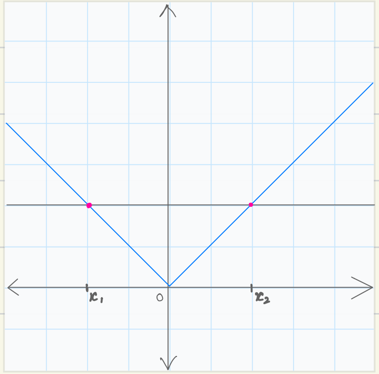
Domain A	Domain B	
1 x	more than 1 $f(x)$	Not a fn.
More than 1 $f(x)$	One $f(x)$	Non-revesable fn.
1 x	one $f(x$	It is fn & reversable.

- One to one function :

A function $f: A \rightarrow B$ is called one-to-one function. If, for any $x_1 \neq x_2 \in A$,

then, $f(x_1) \neq f(x_2)$

- Horizontal line test :- In vertical line test we check if $f(x)$ passes for more than 1 point it is not one-to-one function.



Here $f(x_1) = f(x_2)$

→ If f is an increasing or decreasing fn then f is one to one fn.

○ **Relations :-** Generally denoted by 'R'

→ Relation will be defined between two sets:

i.e. $\boxed{R : A \longrightarrow B}$ $\boxed{W : B \longrightarrow A}$

Read like R is a relation such that it is defined from set (A) to set (B).

$A = \{1, 2, 3, 4, 5\} \xrightarrow{R} B = \{1, 2, 4, 9, 10\}$ → Visual Representation

$R = \{(a, b) : a \in A, b \in B, a^2 = b\}$ → Set-builder form

$R = \{(1, 1), (2, 4), (3, 9)\}$ → Rosted form

○ For above set-builder in below set:

Pre-Image: 1, -1, 0, 2, -2

Image: 1, 0, 4

Domain of Relation : $D_{R1} = \{1, -1, 0, 2, -2\}$

It is set of all the pre-images.

Range of relation : $R_{R1} = \{1, 0, 4\}$
It is set of all the images.

Co-domain of relation : $Co\text{-}D_{R_1} = \{1, 2, 0, 3, 4\}$

It is set of all the element of Set B.
(Complete set B)

$$\boxed{\text{Range} \subseteq \text{Co-Domain}}$$

- Relation & Cartesion product:

Set A	Set B
1	1
2	4
3	

$R = \{(a, b) : a \in A, b \in B, a^2 = b\}$
$R = \{(1,1), (2,4)\}$
Relation

$A \times B = \{(1,1), (1,4), (2,1), (2,4), (3,1), (3,4)\}$
Cartesion product

Hence any Relation of A & B will be subset of Cartesion product of A & B.

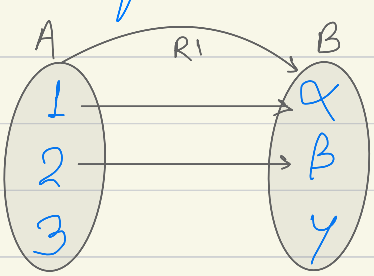
$(R : A \rightarrow B) \subseteq (A \times B)$

If $n(A) = p$, $n(B) = q$ and $n(A \times B) = p \times q$

Total no of Relation that can be defined in Set (A) and set (B) $= 2^{p \cdot q}$

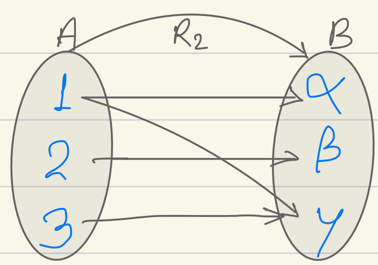
- **Function :-** Functions are basically special type of relation.
  - → Functions can be defined b/w two sets.
  - → $f : A \to B$
    - Element of set A → Input (x)
    - Element of set B → Output (y)
  - → Functions are the mapping which maps each and every element of set A to give unique element of set B.

① ✗



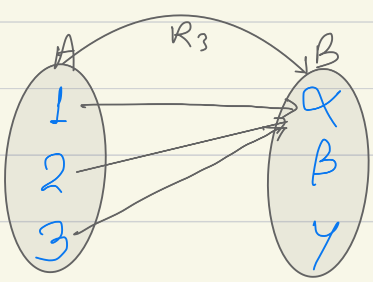
R_1 is not fn

② ✗



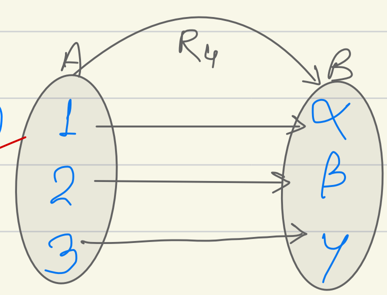
R_2 is not fn

③ ✓



R_3 is fn

④ ✓



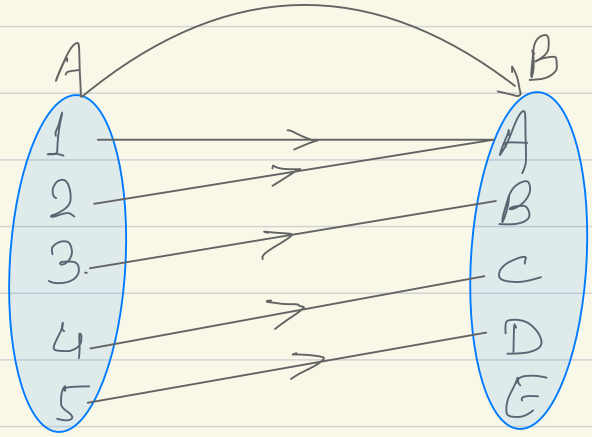
R_4 is a fn

All relation are not function.
All function are relation.

Like relation set also has pre-images:

Pre-Image:

$\{1, 2, 3, 4, 5\}$



Images:

$\{A, B, C, D\}$

Domain of f(n) = $\{1, 2, 3, 4, 5\}$ (Pre-image)

Range of f(n) = $\{A, B, C, D\}$ (Image)

Co-Domain of f(n) = $\{A, B, C, D, E\}$ (All set B)

$R_f \subseteq$ Co-D_f

$$y = x^2$$

$x \rightarrow$ input (Independent variable)

$y \rightarrow$ Output (Dependent variable)

For $y = x^2$,

All possible input = D_f $(-\infty, \infty)$

All possible output = $R_f = [0, \infty)$

○ Type of function :

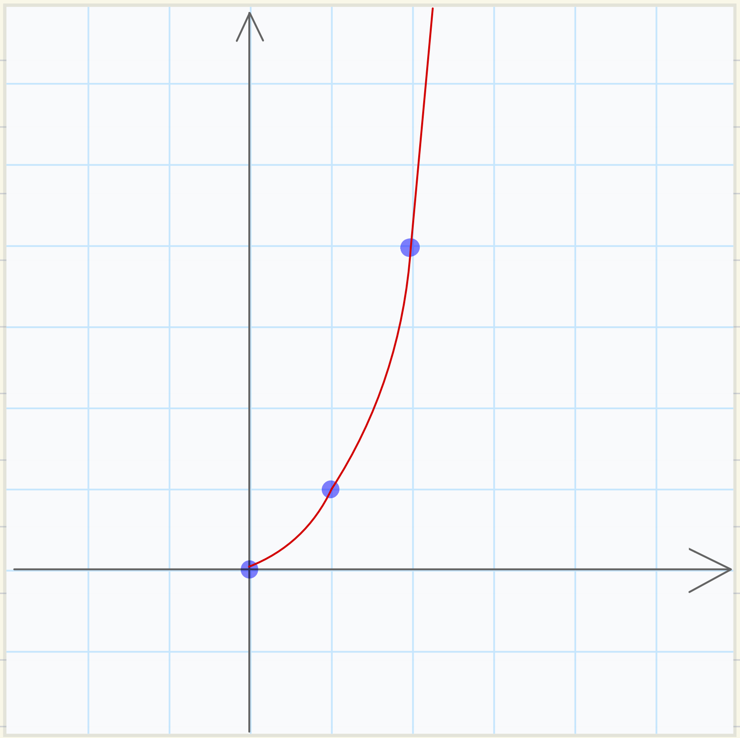
1. Square root function :-

$f(x) = \sqrt{x}$ or $y = \sqrt{x}$

$D_f = [0, \infty) \longrightarrow$ Domain
$R_f = [0, \infty) \longrightarrow$ Range

Graph of $f(x) \longrightarrow$

x	y
0	0
1	1
4	2
9	3



2. Exponential function :-

$f(x) = a^x$

a = base (constant) (Here ($a>0, a\neq 1$))
x = exponent, index, power (variable)
Here, $(-\infty - x - +\infty)$

Exponential property :-

$2^3 = 8$
$2^0 = 1$
$2^{-2} = \frac{1}{4}$, $2^{-1} = \frac{1}{2}$
$0^2 = 0$
0^{-2} = undefined
0^0 = undefined
$1^5 = 1$

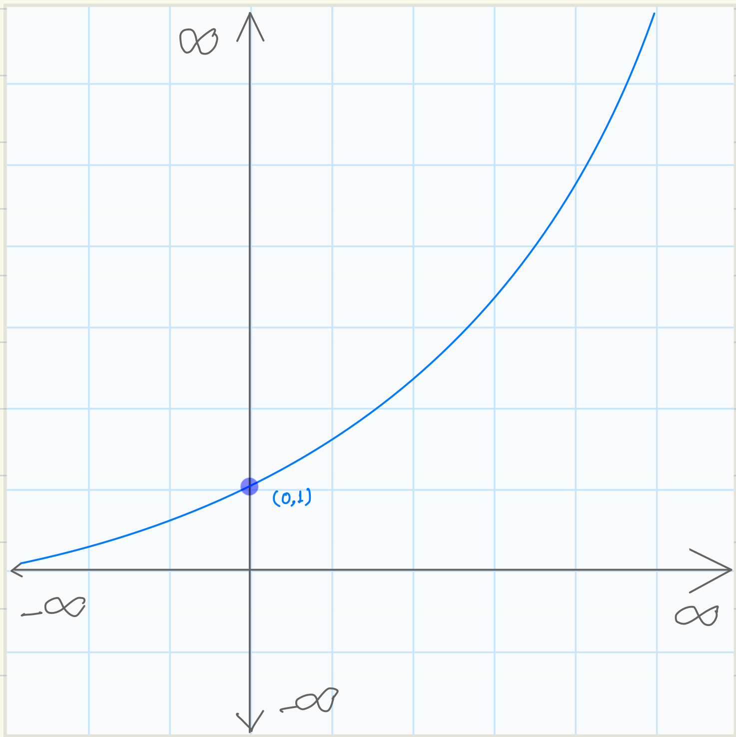
not -ve
not zero

$$D_f = (-\infty, +\infty)$$
$$R_f = (0, +\infty)$$

(This is used in zero means numbers more than zero are included. If you have to include a number you use [ ].

for $\Rightarrow f(x) = 2^x$

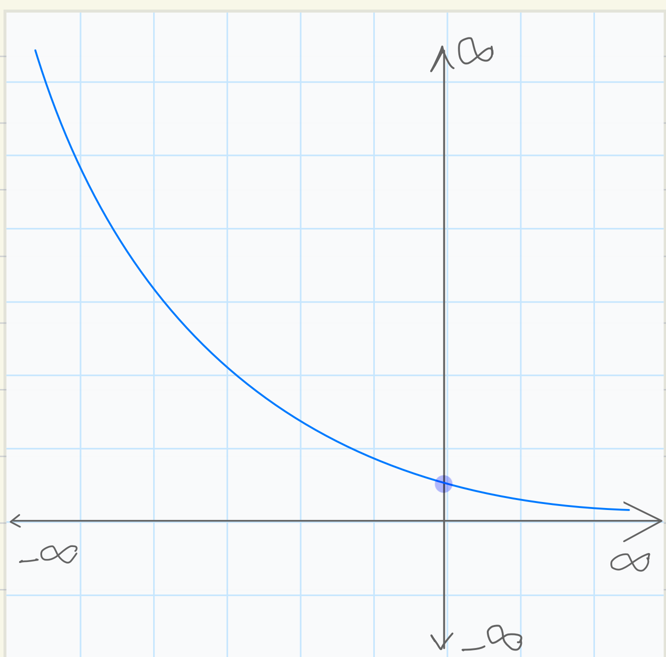
$y = a^x$
when, $a > 1$



$2^{-\infty} = 0^+$
$\vdots$
$2^{-2} = 1/4$
$2^{-1} = 1/2$
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$\vdots$
$2^{\infty} = \infty$

& for $f(x) = \frac{1}{2}^x$

$y = a^x$
when, $a < 1$



$\left(\frac{1}{2}\right)^{\infty} = \frac{1}{2^{\infty}} = \frac{1}{\infty} = 0$

$\left(\frac{1}{2}\right)^{-\infty} = \frac{1}{2^{-\infty}} = 2^{\infty} = \infty$

$\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$

$\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^{3} = \frac{1}{8}$

Law of exponents:

1.) $a^x \cdot a^y = a^{x+y}$

2.) $(a^s)^t = a^{st}$

3.) $(a \cdot b)^s = a^s \times b^s$

4.) $\dfrac{a^n}{a^m} = a^{n-m}$

Natural Exponential function:

In limits theory : $\left(1+\frac{1}{n}\right)^n \longrightarrow e$ as $n \longrightarrow \infty$

By substituting n we get value:

n	$\left(1+\frac{1}{n}\right)^n$
1	2
10	2.5937
1000	2.7169
100000	2.7182

- e is an irrational no.
- $e \approx 2.71828\ldots$

1% intrest

Bank intrest $\longrightarrow$ Quarterly $= \left(1+\frac{0.01}{4}\right)^4$

Monthly $= \left(1+\frac{0.01}{12}\right)^{12}$

any time $= \left(1+\frac{0.01}{n}\right)^n$

$= 1 \times e^{0.01t}$

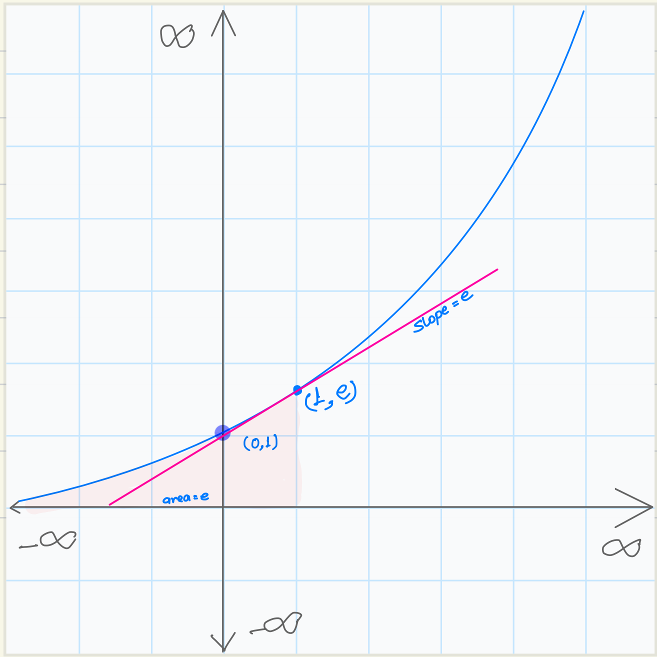
$\Rightarrow \left(1+\frac{x}{n}\right)^{nt} = e^{xt}$

Natural exponential :-

$f(x) = e^x$

$D_{f(x)} = \mathbb{R}$ $\quad R_{f(x)} = (0,\infty)$

& , $e > 1$



Composite Function:

⇒ Suppose a store is giving discount of 15% and after that giving additional 3000 as disc.

So, Steps for function for this is:

$f(x) = .85x$

$g(x) = x - 3000$

After combining both we get:

$h(x) = f(x) - 3000$

$h(x) = (g \circ f)(x) = g(f(x))$

Let $x = 14000$

$(g \circ f)(x) = g(f(x))$ $\qquad g(f(14000))$

$= f(x) - 3000$

$= .85x - 3000$

$= 8900$

The composition of function f & g is denoted as $(f \circ g)$ & is defined by:

$$(f \circ g)x = f(g(x))$$

→ Domain of composite fn:-

- X is in domain of g.
- $g(x)$ is in domain of f.

dom(g) → $x \xrightarrow{g}$ dom(f) → $g(x) \xrightarrow{f} f(g(x))$ ← Range(f)

Eg $\Rightarrow$ $f(x) = 3x-4$ $\quad g(x) = x^2$

find $(g\circ f)(x)$ & $(f\circ g)(x)$

$(g\circ f)(x) = g(f(x)) = (3x-4)^2$

$f(g(x)) = (f\circ g)(x) = 3x^2-4$

- Determine domain of composite function:

$(f\circ g)(x) = f(g(x))$

- Following value should be excluded from input x:

$\rightarrow$ If x is not in domain(g)

means, • $x \notin Dom(g) = x \notin Dom(f\circ g)$

$\rightarrow$ Set of all x which $g(x)$ don't belong to domain

• $\{x \mid g(x) \notin Dom(f)\}$ must not be included

- Inverse Function:- The inverse of a function f, f^{-1} is a function such that:

$\rightarrow f^{-1}\circ f = f^{-1}(f(x)) = x \quad \because x \in Dom(f) = Range(f)$

& $f\circ f^{-1} = f(f^{-1}(x)) = x \quad \because x \in Dom(f^{-1}) = Range(f^{-1})$

Here, $f^{-1} = f^{-1}(x) \neq \frac{1}{f}$ | But, $(f(x))^{-1} = \frac{1}{x}$

Eg: $g(x) = x^3$ & $g^{-1}(x) = 3\sqrt{x} = x^{1/3}$

verify, $g^{-1}(g(x))$ & $g(g^{-1}(x))$

1) $g^{-1}(x^3) = (x^{1/3})^3 \quad x^{3/3}$

2) $g(x^{1/3}) = (x^3)^{1/3}$

map with only 1 output Dont have any images left.

- If a f^n is one to one (Injective) & onto (Surjective) than it is called bijective function. / Invertable fn. / by-uniform fn / Non-Singular fn.

→ If 'f' & 'g' are inverse of each other than:

$$f: A \to B$$
$$f^{-1}: B \to A$$

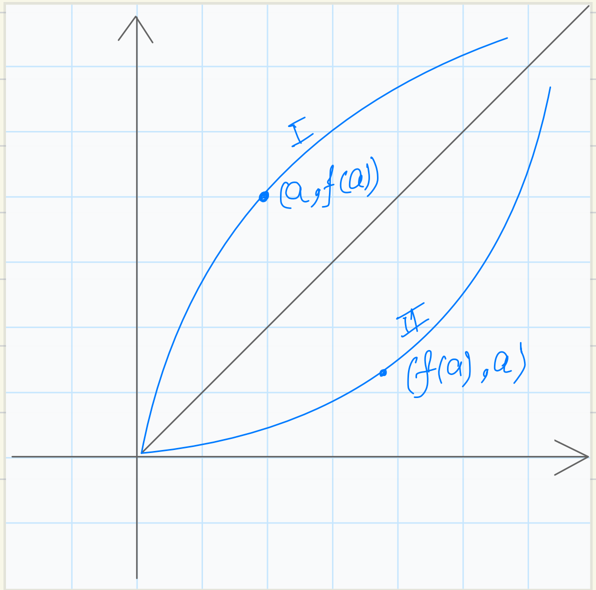
- Inverse can be defined only for the function which are one-one and onto function.

- Graph of a inverse:
  - If $(a, f(a))$ is on graph of f
  - then $(f(a), a)$ is on graph of f^{-1}

$$x = a \xrightarrow{f} f(a) = y$$
$$f^{-1}(f(a)) \xleftarrow{f^{-1}} f(a)$$

- Input of 'f' is output of 'f^{-1}' & vice-versa.

The inverse of $f(x)$ will be symmetric.



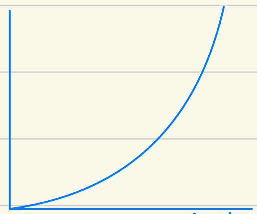
- Log is inverse of exponential So, Range of Log is domain of exponent & vice-versa.

Eg:

If $f : [0, \infty) \rightarrow [0, \infty)$

$f(x) = x^2$. Find inverse if it exists.

$\Rightarrow$



(one-one) (onto)

$R_f = [0, \infty)$

$Co\text{-}D_f = [0, \infty)$

$R_f = Co\text{-}D_f$.

Steps to find inverse:-

Step 1 $\rightarrow$ let $f(x) = y$

$y = x^2$

$m = \log_a x$, $a^m = x$

$\because$ We are asking that we are given base 'a' so what power should I raise to get 'x'.

Step 2 $\rightarrow$ Express 'x' in terms of 'y'

$y = x^2$

$x = \pm\sqrt{y}$

Step 3 $\rightarrow$ Replace $x \rightarrow f^{-1}(x)$

Replace $y \rightarrow x$

$\circ\ f^{-1}(x) = \pm\sqrt{x}$

Step 4 $\rightarrow$ Reject extra answer:

$\circ\ f^{-1}(x) = \sqrt{x}$ ✓ & $\circ\ f^{-1}(x) = -\sqrt{x}$ ✗

Because range can't be -ve acc. to question.

$a^{\log_a x} = x = \log_a(a^x) = x$

○ Laws of Logarithem :

Let $r \in \mathbb{R}$, $0<a<1$ or $a>1$; $M, N > 0$,

① $\log_a (MN) = \log_a M + \log_a N$

② $\log_a (M/N) = \log_a M - \log_a N$

③ $\log_a (1/N) = -\log_a N$

④ $\log_a M^n = n \log_a M$ $\quad n$ = real no.

⑤ $a^{\log_b c} = c^{\log_b a}$ $\quad$ ⑥ $a^{\log_a x} = x$

⑦ $\log_a 1 = 0$ $\quad$ ⑧ $\log_a a = 1$

⑨ $\log_a m = \dfrac{\log_c m}{\log_c a}$ or $\dfrac{\ln m}{\ln a}$

⑩ $\log_a m = \dfrac{1}{\log_m a}$

⑪ $\log_{r^a} m = \dfrac{1}{a} \log_r m$

⑫ $\log_{a^r} x = \dfrac{1}{r} \log_a x$

Warning:

$\ln\left(\frac{x}{a}\right) \neq \frac{\ln(x)}{\ln(a)}$

$\log_a(M+N) \neq \log_a M + \log_a N = \log_a(MN)$

$\log_a(M-N) \neq \log_a M - \log_a N = \log_a(M/N)$

⑤ Change in base rule:

If, $\underbrace{0<a<1 \text{ or } a>1}_{\text{Old base}}$ & $\underbrace{0<b<1 \text{ or } b>1}_{\text{New base}}$

then, for $x>0$,

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$$

Eg → $\log_5 89 = \frac{\ln 89}{\ln 5} \approx 2.78$.

We can change any log fn. to our familiar common log ($\log_{10} x$) or natural log ($\log_e x$)

3. Logarithmic functions :

$f(x) = \log_a x$ (It is inverse f(x) of exponentiation)

$\underset{\text{Base}}{2}^{3 \to \text{Exponent}} = 8 \to$ Number (output)

$\log_2 8 = 3$

The input of exponential is output of log & vice-versa.

input	2	3	4
	↓	↓	↓
		$y = 2^x$	
	↓	↓	↓
Output	4	8	16

input	4	8	16
	↓	↓	↓
		$y = \log_2 x$	
	↓	↓	↓
output	2	3	4

if, $x = a^y$

$f(x) = \log_a x \longrightarrow$ Number (Input)

$a \to$ Base of log

Here, $x > 0$,

$a > 0$; $a \neq 1$

- Output of log can be +ve, -ve & zero.
- Input of log can only be +ve
- Base of log must be +ve except 1.

$Df = (0, \infty)$ (>0)

$Rf = (-\infty, \infty)$

$\log_2 1 = 0$

$\log_2 2 = 1$

$\log_2 4 = 2$

$\log_2 8 = 3$

$\log_2 16 = 4$

means,

$2^x = 16$

$x = 4$

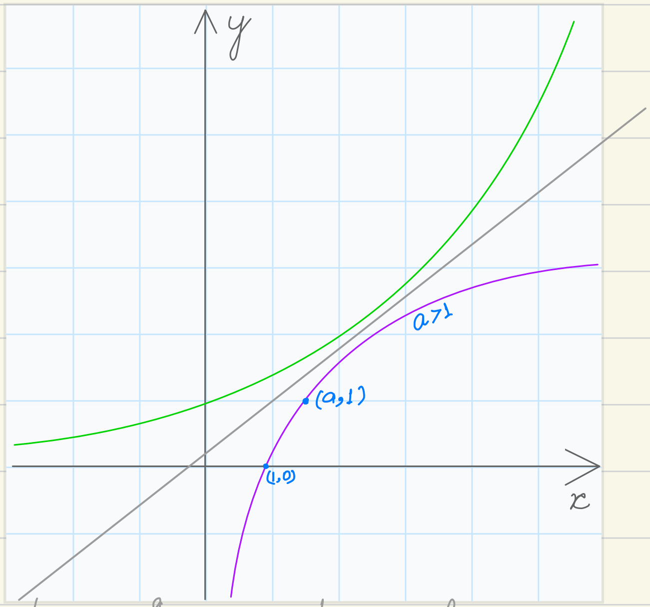
- $\log_e x$ or $\ln x$ $\longrightarrow$ Natural log $e = 2.718...$
- $\log_{10} x$ $\longrightarrow$ Common log.

Graph of log:

$y = \log_a x$

$y = 2^x$

$y = \log_2 x$



mirror image bcz of inverse fn.

If, $\log_a x = y$

then, $a^y = x$

$y = \left(\frac{1}{2}\right)^x$

$y = \log_{\frac{1}{2}} x$

4. Step function:

- Floor function:

$f(x) = \lfloor x \rfloor$ $\qquad \lfloor x \rfloor \leq x$

$\lfloor\ \rfloor$ here denotes G.I.F

It is also known as: (Step wise function)

Read as greatest integer 'x'.

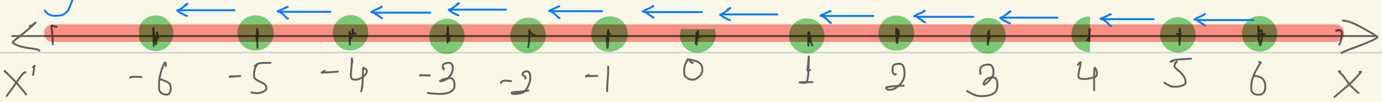
or, Bracketed 'x'

It gives same output for intiger and just next smallest integer for non-intiger value.

Eg → $[0]=0$, $[2]=2$, $[-2]=-2$, $[3.1]=3$, $[0.9]=0$, $[4.9]=4$, $[-6.2]=-7$, $[-4.9]=-5$, $[\sqrt{2}]=1$, $[\sqrt{3}+1]=2$, $[\pi]=3$

$D_f = (-\infty, \infty) = R$

$R_f = Z$



It is known as greatest integer fn bcz it gives greatest integer from all smallest integer from non-integer number.

$\lfloor 2.1 \rfloor = 2$

$\lfloor 0.9 \rfloor = 0$

$\lfloor 2.1 \rfloor = 2$

$\lfloor 3.8 \rfloor + 5 = 8$

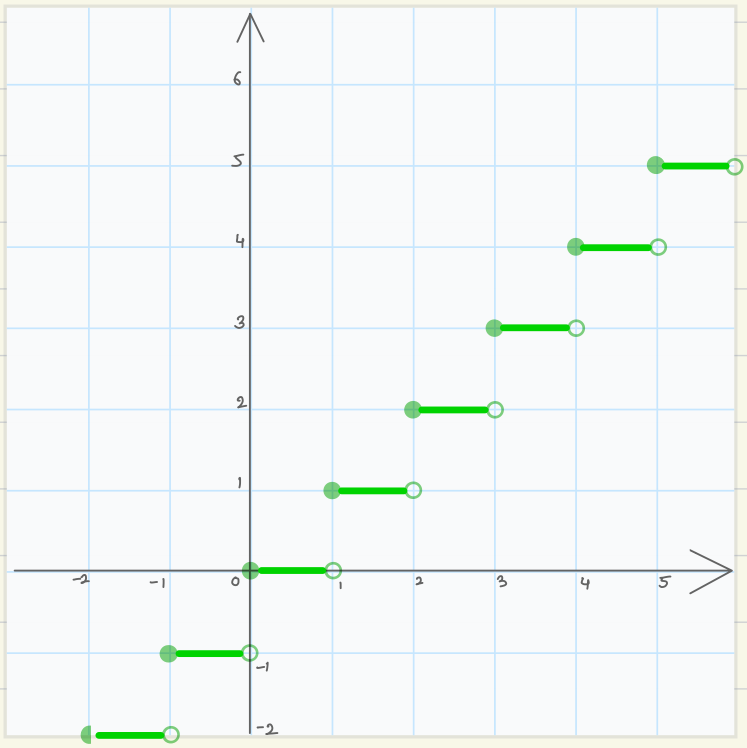
$\lfloor 3.8 \rfloor + 5 = 8$

You can only take int. out:

$3.8 + \lfloor 5 \rfloor = 3.8 + 5$

$x + \lfloor -x \rfloor$ — For non-integer → -1; For integer → 0

$\lfloor x \rfloor \leq x$



○ Ceiling function:

It is denoted by: $f(x) = \lceil x \rceil$

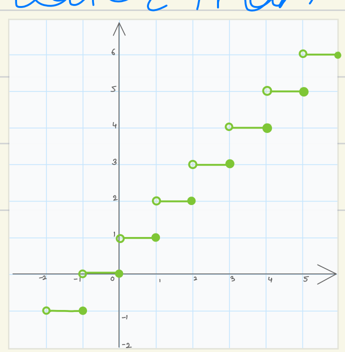
It is opp. of floor function is is smallest integer greater than given fn.

Eg:- $\lceil 3.8 \rceil = 4$

$\lceil 4.1 \rceil = 5$

$\lceil -5.2 \rceil = -5$

$\lceil 5 \rceil = 5$



| | representation		Denote
Real number	$= \|-2\|$	Modulus
Complex number	$= \|2+5i\|$	Magnitude of C.N
Vector	$= \|\vec{A}\|$	Magnitude of vector
Matrix	$= \|A\|$	Determinant
Set	$= \|A\|$	Cardinal number

5.) Modulus function : (Absolute value function)

$f(x) = |x|$; $x \in R$

$|2| = 2$

$|0| = 0$

$|-3| = 3$

$|\sqrt{2}-1| = \sqrt{2}-1$

$|\sqrt{2}-2| = -(\sqrt{2}-2)$
$= 2-\sqrt{2}$

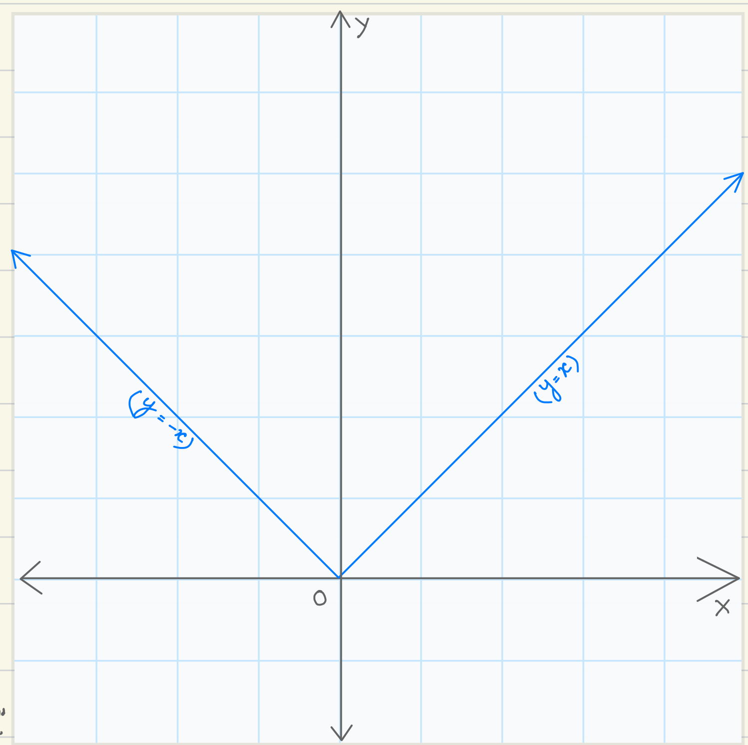
→ Input can be any real number.
→ Output must be non-ve (0, +ve).

$f(x) = |x|$ — outputs, → x ; if $x \geq 0$
→ $-(x)$; if $x < 0$

$D_f = (-\infty, \infty) = R$

$R_f = [0, \infty)$ = Whole no.

Graph of $f(x) = |x|$



$|x| = 0$ (critical point)

Eg :-
$|x-2|$ → $x=2$ is critical point.
$|x^2-4|$ → $x = -2, 2$ chang it to 0.

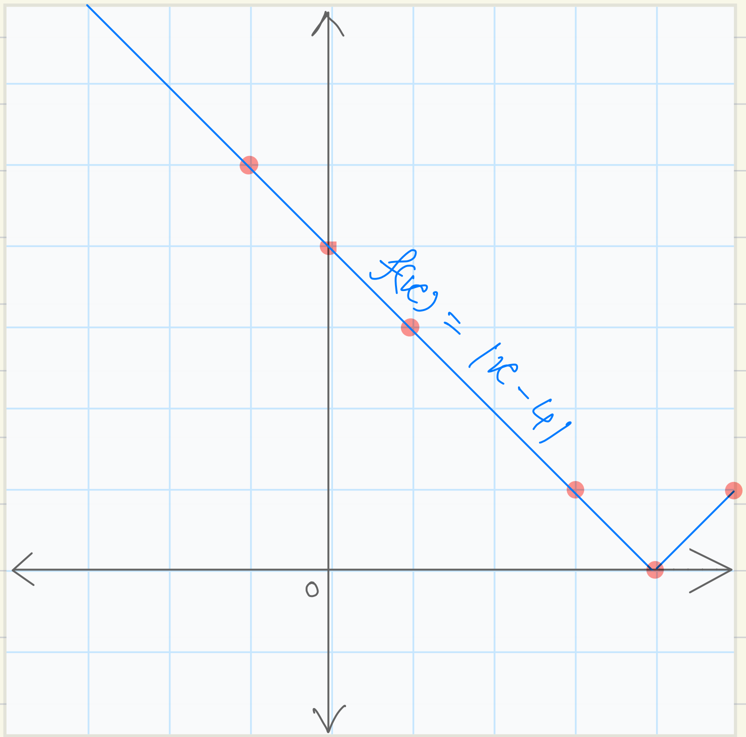
find value of x satisfying:

① $|x-2| = 4$
⇒ $x = 6, -2$

② $||x|-2| = 5$
$x = 7$

$x = -3$
not possible

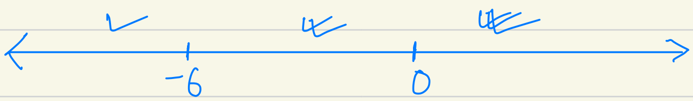
Q Draw graph of $f(x) = |x-4|$



$x = 0 \quad y = 4$
$x = 1 \quad y = 3$
$x = -1 \quad y = 5$
$x = 3 \quad y = 1$
$x = 4 \quad y = 0$ (critical point)
$x = 5 \quad y = 1$

Q find value of x satisfying
$|x+4| - |x| = 4$
$\Rightarrow x = 0 \qquad$ Ans $= [0, \infty)$
$x =$ any whole number $x \geq 0$.

Q find $|x+6| + |x| = 8$



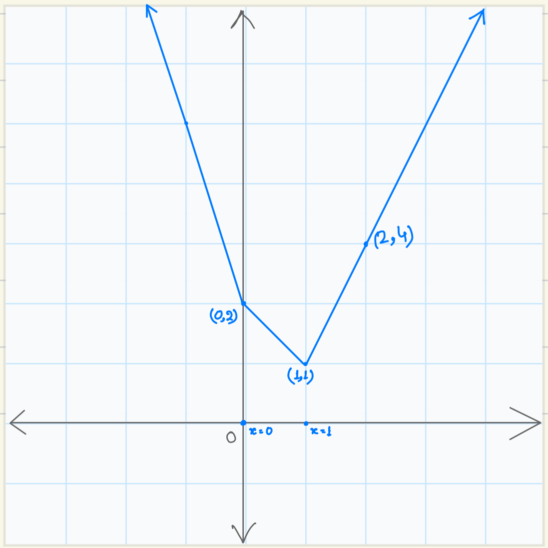
$\Rightarrow$

Case 1	Case 2	Case 3
if $x \leq -6$	if $0 > x > -6$	if $x > 0$
$-(x+6) - (x) = 8$	$(\cancel{x}+6) - \cancel{x} = 8$	$x + 6 + x = 8$
$-x - 6 - x = 8$	$8 = 6$	$2x = 2$
$-2x = 14$	false	$x = 1$
$x = -7$		

Q Make graph for $f(x) = |x| + 2|x-1|$

Solution:
Critical pt. of both modulus:

$x = 0$ & $x = 1 \qquad x = 2 \qquad x = -1$
than, $y = 2 \qquad y = 1 \qquad y = 4 \qquad y = 5$



6.) Signum function :

$f(x) = \text{Signum}(x)$

or

$\text{sgn}(x)$

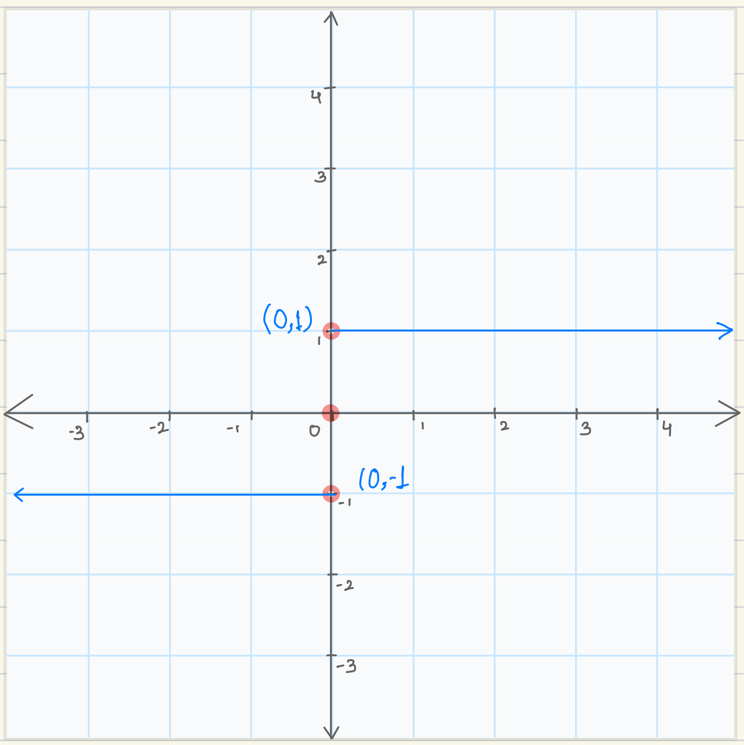
Eg :-

Input	Output
+ve	1
−ve	−1
0	0
Sign(6)	1
Sign(−9)	−1
Sign(0)	0
Sign(π)	1
Sign(1−√2)	−1

$D_f = (-\infty, \infty)$

$R_f = (-1, 0, 1)$

Graph of $f(x) = \text{Sign}(x)$



$$\text{Sign}(x) \begin{cases} 1 & \text{if } x > 1 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 1 \end{cases}$$

Can be written as:-

$$\text{Sign}(x \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

7.) Identity function :

$f(x) = x$

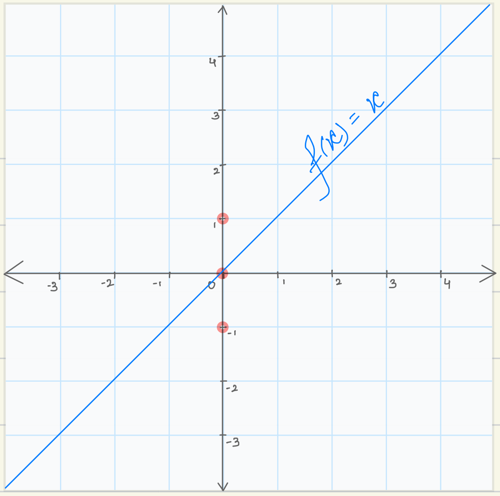
$f(0) = 0$

$f(-2) = -2$

$f(9) = 9$

$D_f = (-\infty, \infty) = R$
$R_f = (-\infty, \infty) = R$

Graph of identity fn)

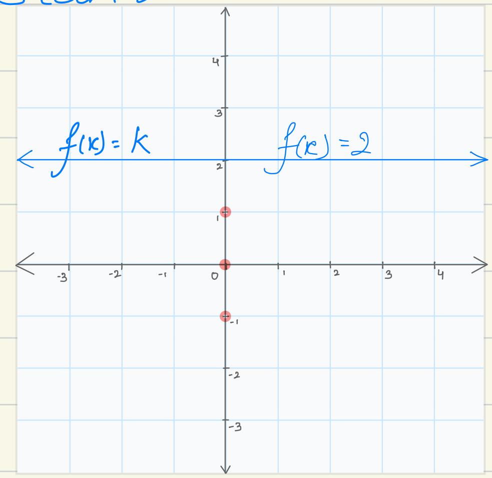


8.) Constant function:

$f(x) = K$, K is constant

$D_f = (-\infty, \infty) = R$
$R_f = K$

Graph of fn:



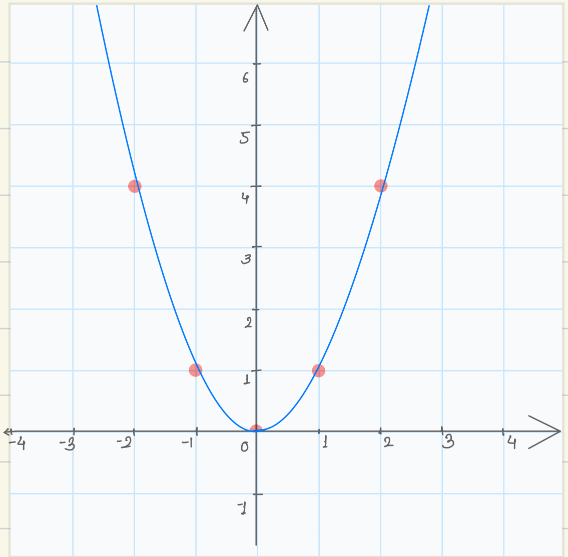
Eg →
For $f(x) = 2$
$f(0) = 2$
$f(99) = 2$
$f(x) = 0$
$f(9) = 0$
$f(6) = 0$

9.) Simple parabolic function:

$f(x) = x^2$

$D_f = (-\infty, \infty) = R$
$R_f = (0, \infty)$

$g(x) = x^2 - 2$
$h(x) = x^2 + 4 + 1$
All these graph will be parabola.



Eg →
$f(1) = 1$
$f(2) = 4$
$f(-2) = 4$
$f(0) = 0$
$f(4) = 16$
$f(-3) = 9$

10.) Polynomial functions :

$f(x) = 5$ (Constant polynomial) (except $f(x)=0$) — 0 degree

$f(x) = x+1$ (linear polynomial) — 1 degree

$f_2(x) = x^2+x+1$ (Quadratic polynomial) — 2 degree

$f_3(x) = x^3-x^2+4x-1$ (Cubic polynomial) — 3 degree

$f_4(x) = x^4+x^3+1$ (By-quadratic polynomial) — 4 degree

$f_5(x) = x^5-1$ (5^{th} degree polynomial) — 5 degree

$f(x) = 0$ (Zero polynomial)

General polynomial :-

$$f(x) = a_0x^n + a_1x^{(n-1)} + a_2x^{(n-2)} + \dots + a_n$$

& , $n \in W$

$a_1, a_2, \dots, a_n \in R$ (cofficient)

Degree of polynomial $= n$ (largest power) & $a \neq 0$

a_0 = leading cofficent

$D_f = (-\infty, \infty) = R$

R_f = Depends on polynomial to polynomial)

Algebra of function :

① $f(x) \pm g(x) = (f \pm g)x$

$D_{f \pm g} = D_f \cap D_g$

② $f(x) \times g(x) = (f \times g)x$

$D_{f \times g} = D_f \cap D_g$

③ $\frac{f(x)}{g(x)} = \frac{f}{g}x$ $\quad g(x) \neq 0$

$D_{\frac{f}{g}} = D_f \cap D_g$ $\quad g(x) \neq 0$

④ Multiple of $f(x)$ with scaler 'K'
$= K \cdot f(x)$

$D_{f(x) \times K} = D_{f(x)}$

We can't comment on range depends on $f(x)$

Q. If $f(x) = 2x+1$
$g(x) = \sqrt{x}$
than find $f+g$, $f-g$, $f\times g$, $\frac{f}{g}$.

① $(f+g)(x) = 2x+1+\sqrt{x}$
$(f\cdot g)(x) = (2x+1)\cdot\sqrt{x}$
$\frac{f}{g}(x) = \frac{2x+1}{\sqrt{x}}$

○ Functional equation :

→ $g(x) + f(-x) = 2x+1$

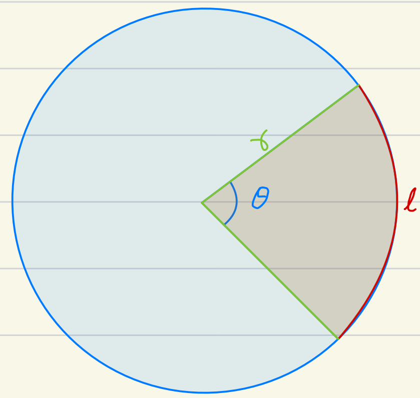
→ $f(x)\cdot f\left(\frac{1}{x}\right) = 1$

→ $f(x+1) + f(x+2) + f(x+3) = f(x+4) + f(x+4)$

Q. If $f(2x+1) = 3x$

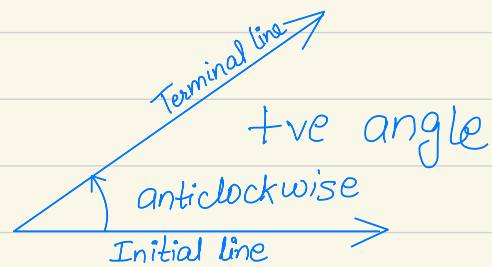
$f(x) = ?$

Trignometric function :



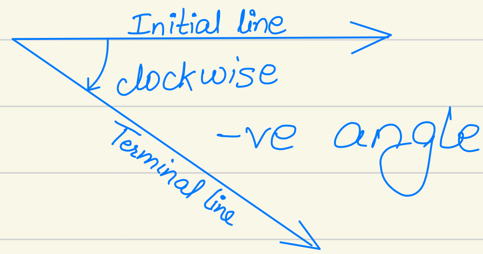
$$\theta = \frac{l}{r} = \frac{(arc)}{radius}$$

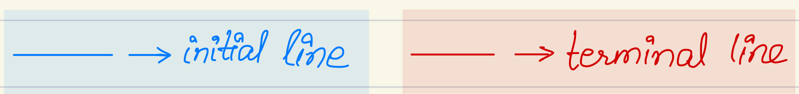
Here S.I unit comes in 'radian' not in degree.

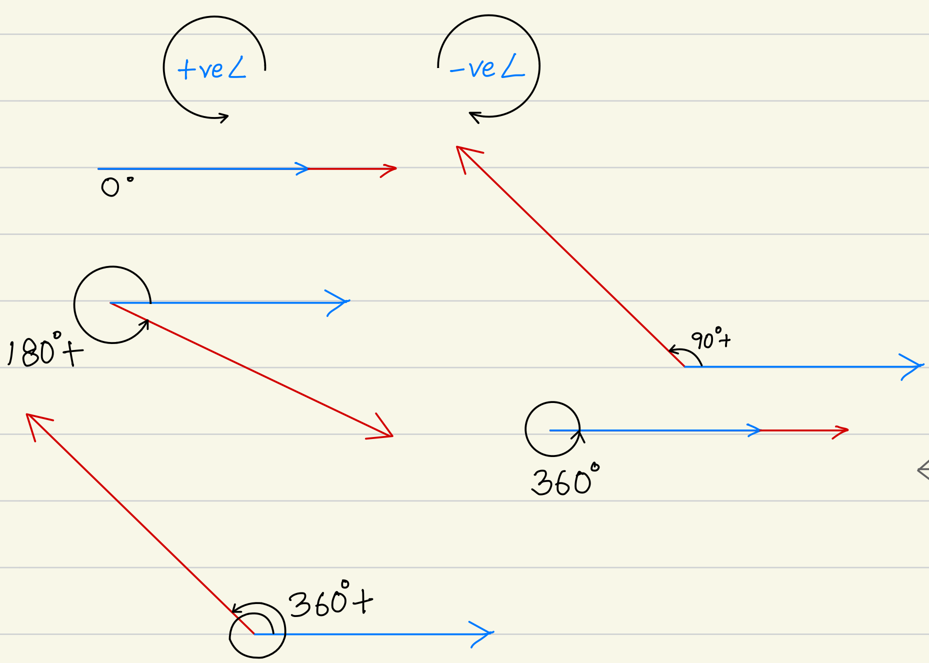


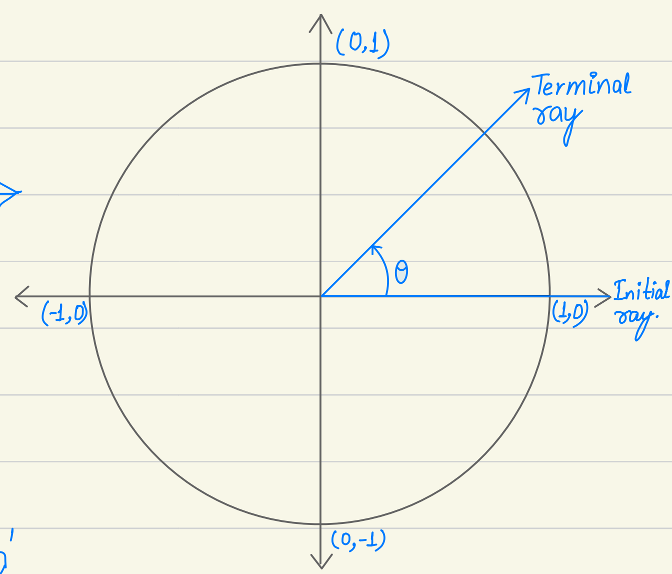
Eg ⇒ arc = 2m , r = 4m

$$\theta = \frac{2}{4} = \frac{1}{2} \text{ radian}$$







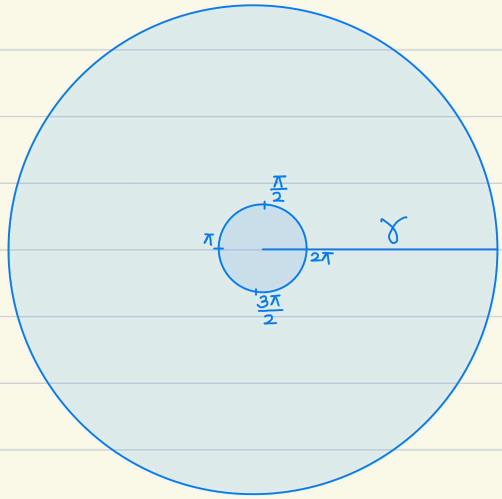


Like this angle can approch '∞' and '-∞'.

Unit Circle

RADIAN:

The radian can be measured by finding how many times r (radius) travel in circumference to get to that specific angle.



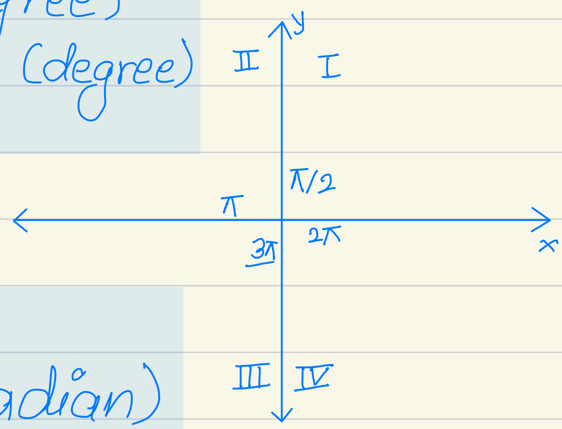
Circumfrence $= 2\pi r$

Then the whole circle takes $\frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi^c$

Here we get:

$2\pi^c$ (radian) $= 360°$ (degree)

1^c (radian) $= \frac{360}{2\pi} = \frac{180°}{\pi}$ (degree)



Vice-versa:

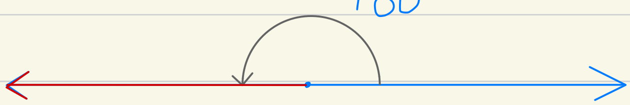
$360°$ (degree) $= 2\pi^c$ (radian)

$1°$ (degree) $= \frac{2\pi}{360} = \frac{\pi}{180}^c$ (radian)

Now we can calculate:

- $30° = \frac{\cancel{2}\pi}{\cancel{360}} \times \cancel{30}^{\cancel{12}\ 6} = \frac{\pi}{6}^c$
- $\frac{\pi}{3}^c = \frac{\cancel{360}^{\cancel{120}\ 60}}{\cancel{2\pi}} \times \frac{\cancel{\pi}}{\cancel{3}_1} = 60°$

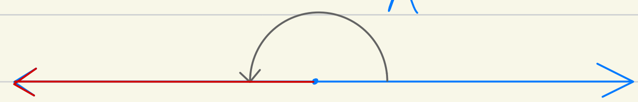
Degree measurement



Straight angle = 180°

$180^\circ \rightarrow$ Degree

Radian measurement



Straight angle = π^c

π or $\pi^c \rightarrow$ Radian

From this we can conclude that:

$180^\circ = \pi$

$\pi = 180^\circ,$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

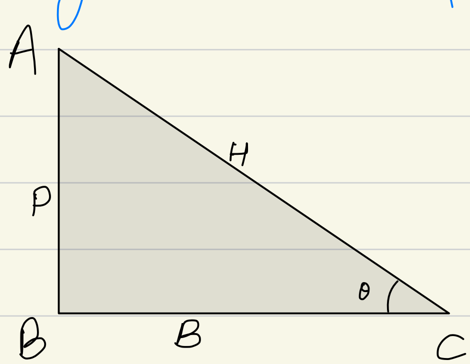
$1^\circ = 60'$ (minute)

$1' = 60''$ (second)

$1^\circ = 3600''$

Trignometric Ratio :

$H^2 = P^2 + B^2$



$\sin\theta = \frac{P}{H}$ $\quad$ $\operatorname{cosec}\theta = \frac{H}{P}$

$\cos\theta = \frac{B}{H}$ $\quad$ $\sec\theta = \frac{H}{B}$

$\tan\theta = \frac{P}{B}$ $\quad$ $\cot\theta = \frac{B}{P}$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trignometric identities :

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$H^2 = P^2 + B^2$$

$$\Rightarrow \frac{H^2}{H^2} = \frac{P^2}{H^2} + \frac{B^2}{H^2}$$

$$\Rightarrow 1 = \sin^2\theta + \cos^2\theta$$

&, vice-versa.

Advance identities :

$$\sec^2\theta \cdot \operatorname{cosec}^2\theta = \sec^2\theta + \operatorname{cosec}^2\theta$$

$$\tan^2\theta \cdot \sin^2\theta = \tan^2\theta + \sin^2\theta$$

$$\cot^2\theta \cdot \cos^2\theta = \cot^2\theta - \cos^2\theta$$

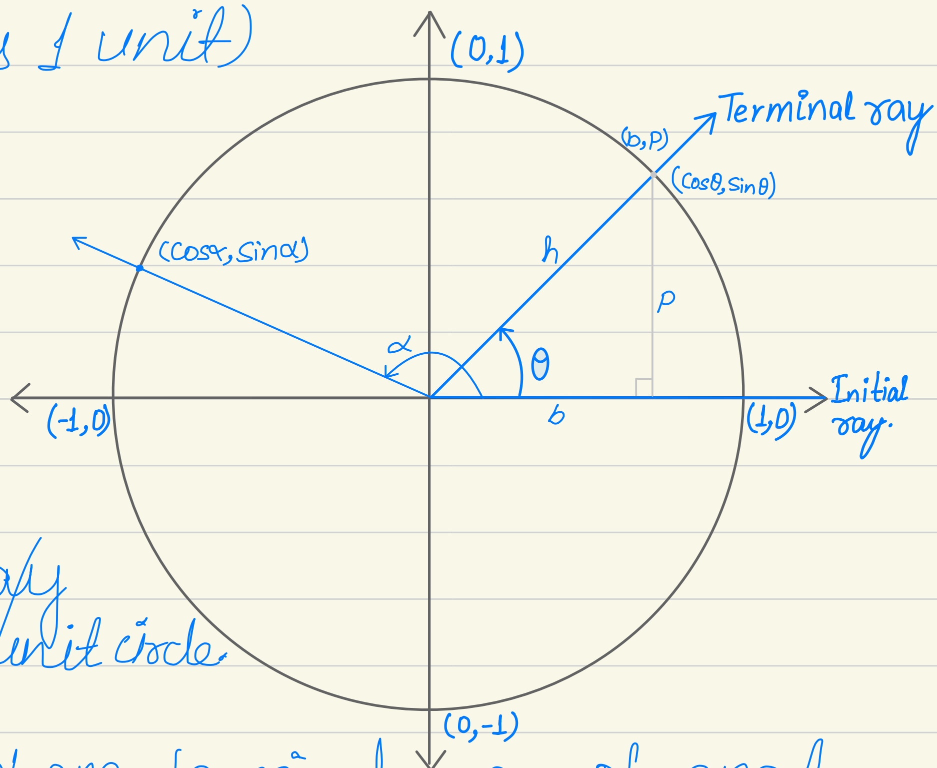
$$L.H.S = \sec^2\theta \cdot \operatorname{cosec}^2\theta = \frac{1}{\cos^2\theta \cdot \sin^2\theta}$$

$$= \frac{\cancel{\sin^2\theta}}{\cos^2\theta \cdot \cancel{\sin^2\theta}} + \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta} \cdot \sin^2\theta} = \sec^2\theta + \operatorname{cosec}^2\theta$$

○ <u>Unit Circle</u>: (Radius 1 unit)

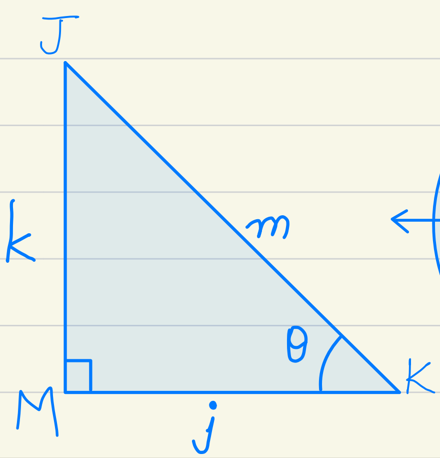
$$\sin\theta = \frac{P}{H} = \frac{P}{1} = P$$

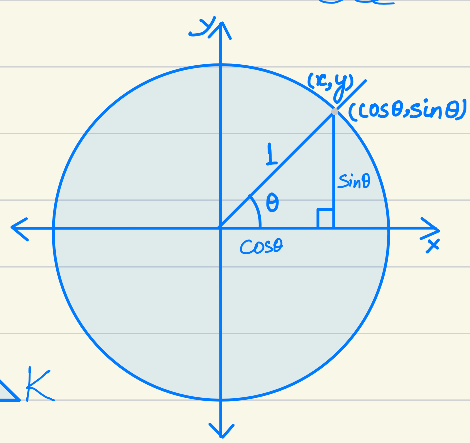
$$\cos\theta = \frac{B}{H} = \frac{b}{1} = b$$



$\cos\theta$ = x co-ordinate where terminal ray of angle intersect unit circle.

$\sin\theta$ = y co-ordinate where terminal ray of angle intersect unit circle.

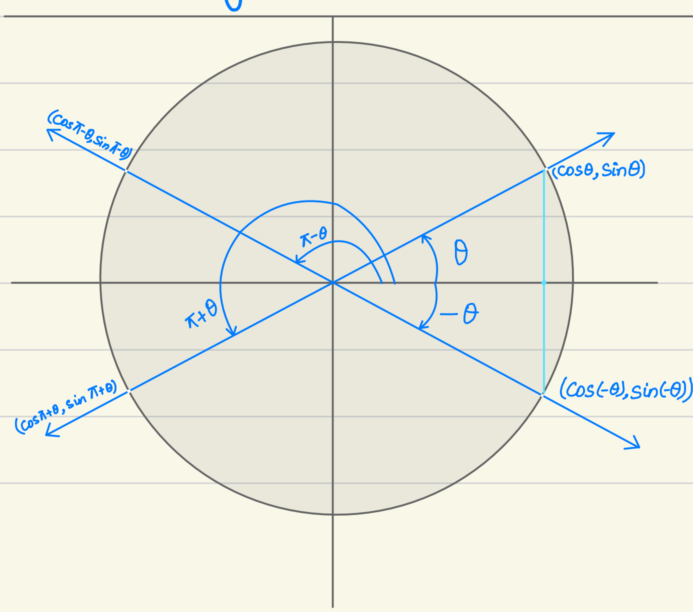




$\sin\angle MKJ = \sin\theta = \dfrac{y}{1} = \dfrac{k}{m}$

$\cos\angle MKJ = \cos\theta = \dfrac{x}{1} = \dfrac{j}{m}$

$\tan\angle MKJ = \tan\theta = \dfrac{y}{x} = \dfrac{j}{k}$



From this we get:

$\cos\theta = \cos(-\theta)$

$\sin(-\theta) = -\sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\sin(\pi+\theta) = -\sin\theta$

Similary from this:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan(\pi-\theta) = \frac{\sin(\pi-\theta)}{\cos(\pi-\theta)} = \frac{-\cos\theta}{\sin\theta} = -\tan\theta$$

$$\tan(\pi+\theta) = \frac{\sin(\pi+\theta)}{\cos(\pi+\theta)} = \frac{+\sin\theta}{+\cos\theta} = \tan\theta$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

○ $f(x) = \sin(x)$

$\sin x = \frac{P}{H^+}$

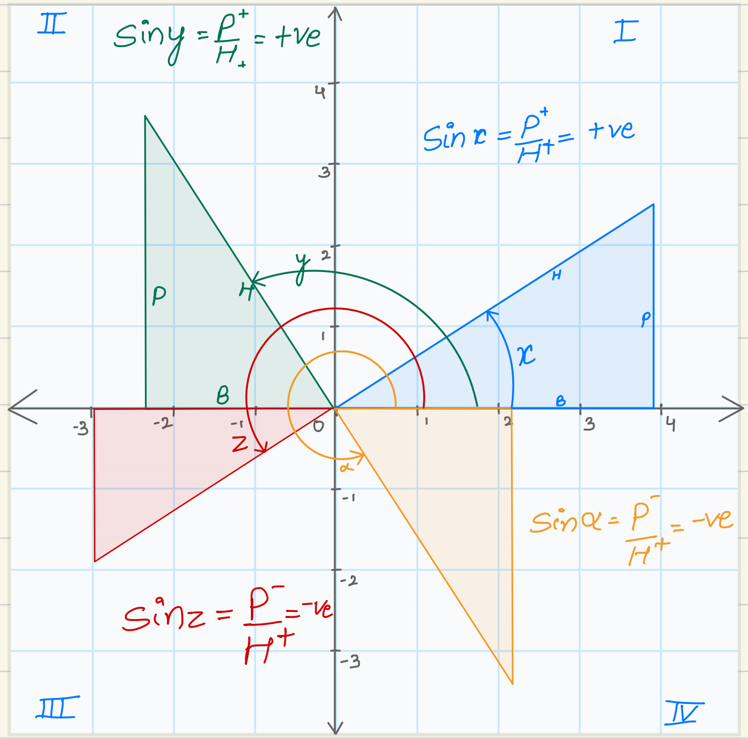
h will always +ve

bcz, $h^2 = p^2 + b^2$

Domain of $f(x) = (-\infty, \infty)$

$P \leq H$, $\frac{P}{H} \leq 1$ so, $\sin x \leq 1$

Range of $f(x) = [-1, 1]$



○ $f(x) = \cos(x)$

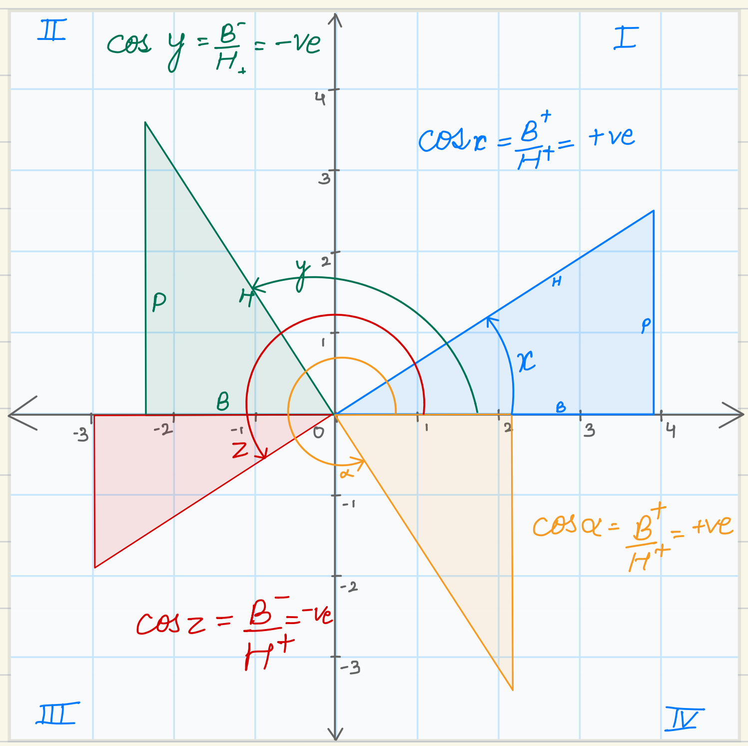
$\cos x = \frac{B}{H^+}$ (always +ve H)

Domain of $f(x) = (-\infty, \infty)$

Range of $f(x) = [-1, 1]$

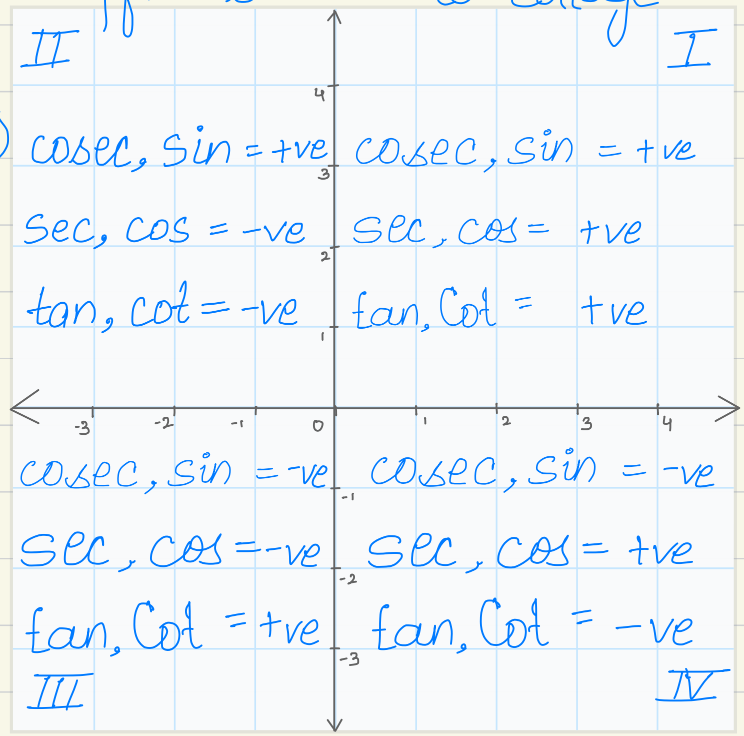
eg → $\cos 140^\circ = -ve$,

$\sin 140^\circ = +ve$,



$f(x) = \cos(x)$ & $f(x) =$

Add suger to coffee.
After school to collage



○ $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

here $\cos x$ can't be 0 (denom∴)
& $\cos x$ 0 are on $\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

Domain of $\tan x \rightarrow$

$D_f = R - \left\{ \frac{(2n+1)\pi}{2},\ n \in I \right\}$

All real no' except (odd x of $\frac{\pi}{2}$)

Range of $\tan x \rightarrow R = (-\infty, \infty)$

○ $f(x) = \cot x = \dfrac{\cos x}{\sin x}$

here also $\sin x$ can't be 0 bcz it is denominator.
$\sin x$ are 0 on $\pi, 2\pi, 3\pi, -2\pi, -3\pi$, etc.

Domain of function:
$D_f = R - \{ n\pi,\ n \in Z \}$

Range of $\cot x \rightarrow R = (-\infty, \infty)$

○ $f(x) = \sec x = \dfrac{1}{\cos x}$ (Here also $\cos x$ in denominator so, it can't be 0.)

Domain of $\sec x \rightarrow$

$D_f = R - \left\{ \frac{(2n+1)\pi}{2},\ n \in I \right\}$

Range of $\sec x \rightarrow R = (-\infty, -1] \cup [1, \infty)$

○ $f(x) = \text{cosec}\,x = \frac{1}{\sin x}$ (Here also sin x in denominator so, it can't be 0.)

Domain of function:

$D_f = R - \{n\pi, n \in Z\}$

Range of cosec x → $R = (-\infty, -1] \cup [1, \infty)$

○ Trignometric table:-

Radian	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Degree	0	30°	45°	60°	90°
Sin θ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	—
Cot θ	—	$\sqrt{3}$	1	$1/\sqrt{3}$	0
Sec θ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	—
Cosec θ	—	2	$\sqrt{2}$	$2/\sqrt{3}$	1

$90° = \frac{\pi}{2}$

$180° = \pi$

$270° = \frac{3\pi}{2}$

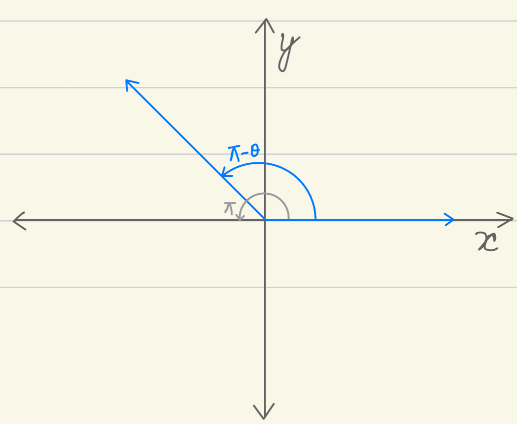
$360° = 2\pi$

○ Trignometric ratio of an angle:

$\sin(90° \text{ or } \pi/2 - \theta)$	$= \cos\theta$
$\cos(90° \text{ or } \pi/2 - \theta)$	$= \sin\theta$
$\tan(90° \text{ or } \pi/2 - \theta)$	$= \cot\theta$
$\cot(90° \text{ or } \pi/2 - \theta)$	$= \tan\theta$
$\sec(90° \text{ or } \pi/2 - \theta)$	$= \text{cosec}\,\theta$
$\text{cosec}(90° \text{ or } \pi/2 - \theta)$	$= \sec\theta$

In degree = 90°

In radian = $\pi/2$



○ $(\pi-\theta)$ Relation :-

$\sin(\pi-\theta) = +\sin\theta$
$\cos(\pi-\theta) = -\cos\theta$
$\tan(\pi-\theta) = -\tan\theta$
$\cot(\pi-\theta) = -\cot\theta$
$\sec(\pi-\theta) = \sec\theta$
$\operatorname{cosec}(\pi-\theta) = +\operatorname{cosec}\theta$

Note :

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\cot(-\theta) = -\cot\theta$
$\sec(-\theta) = \sec\theta$
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Cos and sec only be +ve in IV.

○ Expansion formulae :

- $\sin(A+B) = \sin A \cos B + \cos A . \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A . \sin B$

- $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
- $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

- $\cot(A+B) = \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$
- $\cot(A-B) = \dfrac{\cot A \cdot \cot B - 1}{\cot B - \cot A}$

Eg. find value of $\sin 75°$ and $\cos 15°$.

$\rightarrow$ $\sin 75° = \sin(45°+30°)$

$= \sin 45° \cdot \cos 30° + \cos 45° \cdot \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\rightarrow$ $\cos 15°$:-

Hence, $\sin(90° - \theta) = \cos\theta$

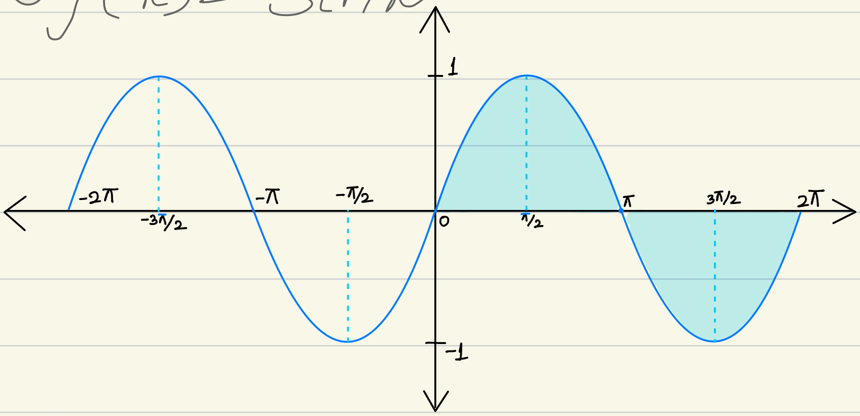
So, $\sin(90° - 15) = \cos 15°$

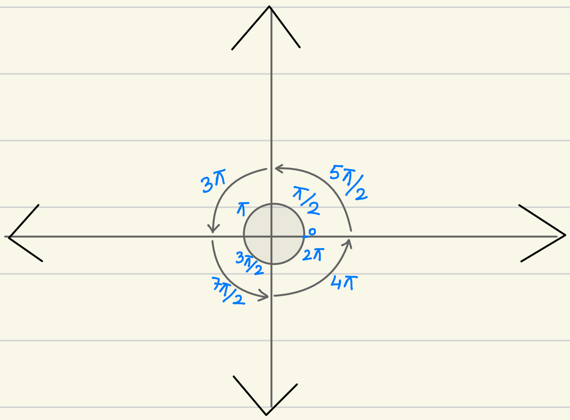
Same as, $\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\cos A = \sqrt{1 - \sin^2 A}$

$\sin A = \sqrt{1 - \cos^2 A}$

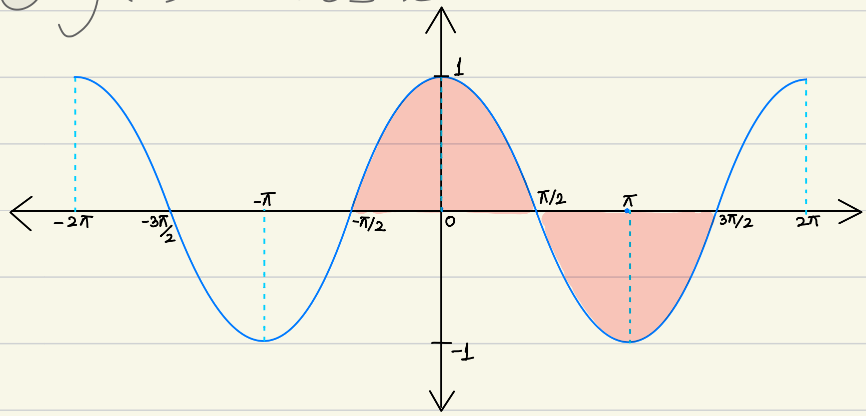
- $f(x) = \sin x$



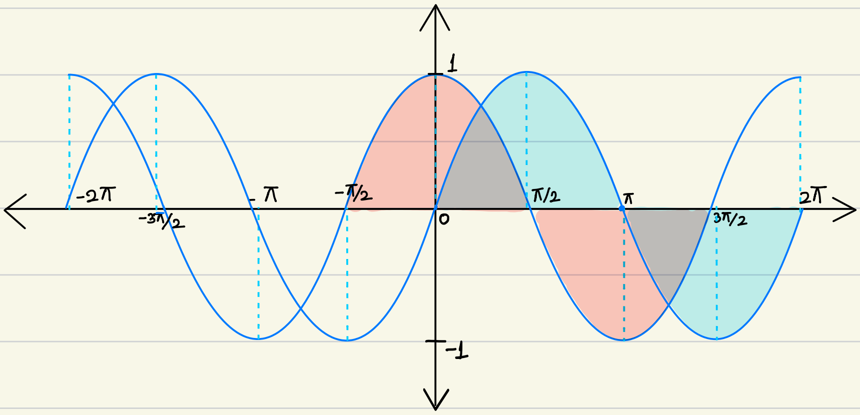


Period of sin x is 2π
can be, $(-\pi$ to $\pi)$
or, $(0$ to $2\pi)$

- $f(x) = \cos x$



- When sin x & cos x graphed together.



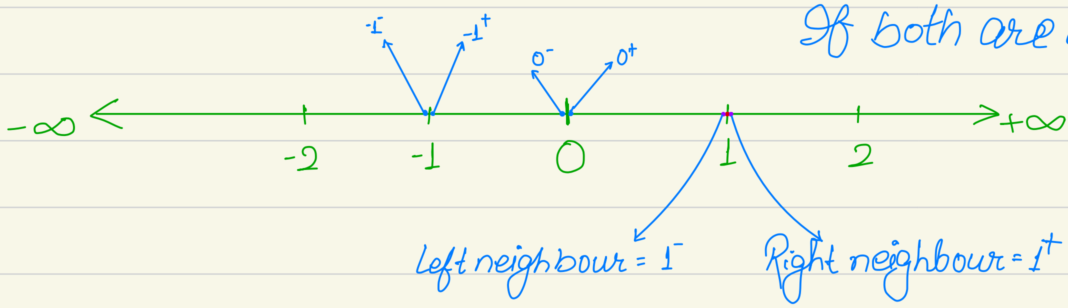
Limits : (PW)

$$\lim_{x \to a} f(x)$$

$$\lim_{x \to a^-} f(x) = a$$

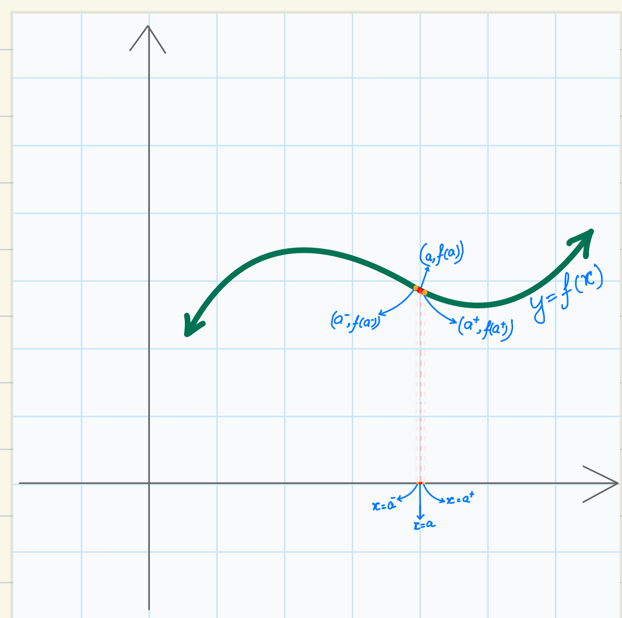
$$\lim_{x \to a^+} f(x) = a$$

If both are correct lim exist.



$x = a \Rightarrow$ left neighbour $\Rightarrow x = a^-$, right neighbour $\Rightarrow x = a^+$

In this graph $\longrightarrow$



i) Value of $f(x)$ at $x = a \Rightarrow f(a)$

ii) Neighbour of $f(x)$ in neighbourhood of $x = a$

$\Rightarrow f(a^-), f(a^+)$

$\lim_{x \to a} f(x)$ means value of $f(x)$ in neighbourhood of $x = a$.

Roughly, limit is study of neighbourhood.

7 undetermined quantity :

$0/0$, ∞/∞ , $\infty-\infty$, $0\times\infty$, ∞^0 , 0^0 , 1^∞

$\downarrow$

Undetermined/ Indeterminate Quantities

Note:

→ Zero written above are not exact zero. They are limiting zero/approximate zero.

→ One written above are not exact One. They are limiting One /approximate One.

1.) $\frac{\infty}{\infty}$ form :

We can't directly tell the value of ∞/∞ we can only tell it by seeing source.

$$\lim_{x\to\infty} \frac{x}{x} \left(\frac{\infty}{\infty}\right) = 1$$

$$\lim_{x\to\infty} \frac{2x}{x} \left(\frac{\infty}{\infty}\right) = 2$$

$$\lim_{x\to\infty} \frac{3x}{x} \left(\frac{\infty}{\infty}\right) = 3$$

$$\lim_{x\to 0^+} \frac{x}{x} \left(\frac{0^+}{0^+}\right) = 1$$

$$\lim_{x\to 0^+} \frac{2x}{x} \left(\frac{0^+}{0^+}\right) = 2$$

$$\lim_{x\to 0^+} \frac{3x}{x} \left(\frac{0^+}{0^+}\right) = 3$$

$\Rightarrow x \to 2 \neq x = 2$

$\Rightarrow x \to$ means, $x = 2^+$ or $x = 2^-$

Evaluate :-

- $\lim_{x\to 2} \frac{x^2-4}{x-2} = \left(\frac{0}{0}\right) \Rightarrow \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = \lim_{x\to 2} (x+2)$

putting 2 inplace of x, we get 4.

- $\lim_{x\to -1} \frac{x^3+1}{x^2-1} = \left(\frac{0}{0}\right) = \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x-1)} = \frac{3}{-2} = -\frac{3}{2}$

by putting ($x = -1$) putting $x = -1$

~ Existance of limit :-

- $\lim_{x\to 0} [x]$, where [ ] is G.I.F

→ It's neighbours are 0^- & 0^+

G.I.F of $0^- = -1$

&, G.I.F of $0^+ = 0$

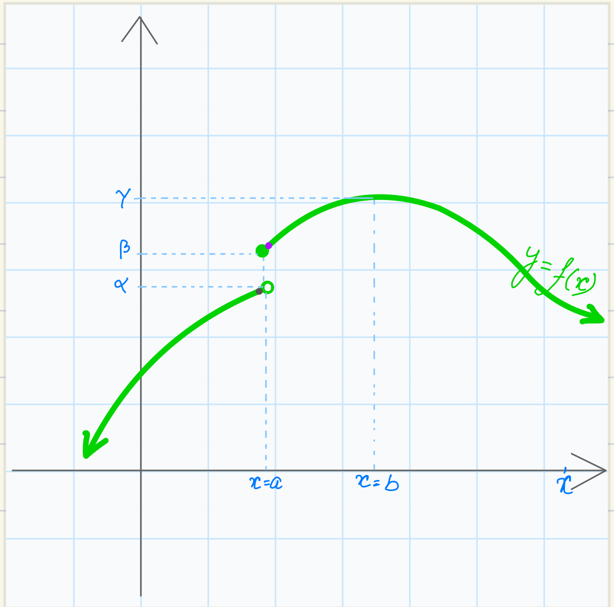
Hence, LHS ≠ RHS (limits don't exist)

Find, $\lim_{x\to a^-} f(x) = \alpha$

$\lim_{x\to a^+} f(x) = \beta$

$\lim_{x\to a} f(x) =$ limit don't exist

$\lim_{x\to b} f(x) = \gamma$



$\lim_{x \to a} f(x)$ will exist if $\underset{\substack{x=a \\ f(a^-)}}{LHS} = \underset{\substack{x=a \\ f(a^+)}}{RHS}$ = finite Quantity

① $\lim_{x \to 0} \{x\}$ where $\{\ \}$ is fractional part fn $= \underset{0^- = 1^-}{LHS} \neq \underset{0^+}{RHS}$

(By fractional part fn)

② $\lim_{x \to 0} |x|$ Here, $LHS = RHS$

$|0^-| \quad |0^+|$

$0^+ \quad 0^+$

$0 \quad 0$

Limit exists & $|x| = 0$

(limit means which number it is approaching)

Standard limits:-

- Trignometric limit
- Exponential limit
- Logarithemic limit
- Algebric limit.

– Trignometric limit:

$$\lim_{x \to 0} \frac{\sin x}{x} = 0.99\ldots.99 \approx 1 \text{ or } 1^-$$

Exponential limit

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a \quad (a>0)$$

Logarithmic limit

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

Algebric limit

$$\lim_{x\to a} \frac{x^n - a^n}{x-a} = n \times a^{n-1}$$, when n is rational no. ($\mathbb{Q}$).

Ques:- $$\lim_{x\to 1} \frac{x^{100}-1}{x-1} = \lim_{x\to 1} \frac{x^{100}-1^{100}}{x-1} = 100 \times 1^{100-1} = 100$$ Ans

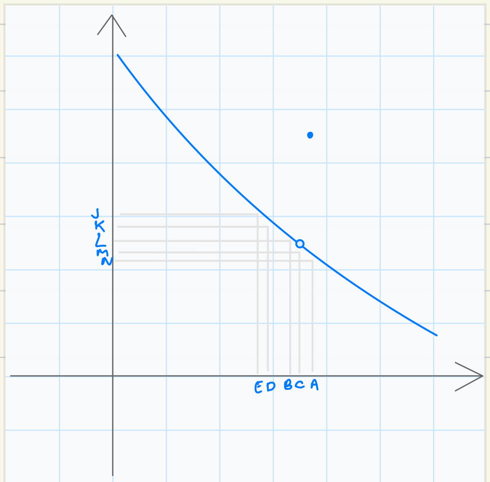
$$\lim_{x\to 1} \frac{x-1}{x^{1/3}-1} = \lim_{x\to 1} \frac{1}{\frac{x^{1/3}-1^{1/3}}{x-1}} = \frac{1}{\frac{1}{3}\times 1^{\frac{1}{3}}} = \frac{1}{\frac{1}{3}} = 3$$

- [ ] Limits : (KA)

limits helps us to know where the function is approching.

Here we don't worry about wether P is there or not we just care about function before p & after approching it or not.

$$\lim_{x\to p} f(x) = P$$



$\Rightarrow$ If, $\lim\limits_{x \to c}$ where ever we see x we can use $\lim\limits_{x \to c}(\ )$

like, $\lim\limits_{x \to c} (f(x))^{g(x)} = \lim\limits_{x \to c} f(x)^{\lim\limits_{x \to c} g(x)}$

Eg, $\lim\limits_{x \to c} K f(x) = K \lim\limits_{x \to c} f(x)$

$\lim\limits_{x \to c} (f(x))^m = \left(\lim\limits_{x \to c} f(x)\right)^m$

$\lim\limits_{x \to c} (K)^{f(x)} = K^{\lim\limits_{x \to c} f(x)}$

$\lim\limits_{x \to c} h(f(x)) = h\left(\lim\limits_{x \to c} f(x)\right)$

- $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

Calculating $\lim\limits_{x \to a} f(x)$

A. Direct substitution

Try to evaluate the function directly.

$$f(a)$$

$$f(a) = \frac{b}{0}$$

where ***b*** is not zero

B. Asymptote (probably)

example:

$$\lim_{x \to 1} \frac{1}{x-1}$$

Inspect with a graph or table to learn more about the function at ***x=a***.

$$f(a) = b$$

where ***b*** is a real number

C. Limit found (probably)

example:

$$\lim_{x \to 3} x^2 = (3)^2 = 9$$

$$f(a) = \frac{0}{0}$$

D. Indeterminate form

example:

$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$

Try rewriting the limit in an equivalent form.

E. Factoring

example:

$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$

can be reduced to

$$\lim_{x \to -1} \frac{x-2}{x-3}$$

by factoring and cancelling.

F. Conjugates

example:

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

can be rewritten as

$$\lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$

using conjugates and cancelling.

G. Trig identities

example:

$$\lim_{x \to 0} \frac{\sin(x)}{\sin(2x)}$$

can be rewritten as

$$\lim_{x \to 0} \frac{1}{2\cos(x)}$$

using a trig identity.

Try evaluating the limit in its new form.

H. Approximation

When all else fails, graphs and tables can help approximate limits.

Limits: (IITM)

Rules:

1) If $\lim_{x \to a} f(x) = F$, $\lim_{x \to a} g(x) = G$ then, $\lim_{x \to a} (f+g)(x) = F+G$

2) If $\lim_{x \to a} f(x) = F$, $\lim_{x \to a} g(x) = G$, then $\lim_{x \to a} (f-g)(x) = F-G$

3) If $\lim_{x \to a} f(x) = F$ and $c \in R$, then $\lim_{x \to a} (cf)(x) = cF$

4) If $\lim_{x \to a} f(x) = F$, $\lim_{x \to a} g(x) = G$, then $\lim_{x \to a} (fg)(x) = FG$

5) If $\lim_{x \to a} f(x) = F$, $\lim_{x \to a} g(x) = G \neq 0$, than

$$\text{function}\left(\frac{f}{g}\right) = \lim_{x \to a} \frac{f}{g}(x) = \frac{F}{G}, \qquad (G \neq 0)$$

6) The sandwich principle: If $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = L$

and, $h(x)$ is function as $f(x) \leq h(x) \leq g(x)$,

then, $\lim_{x \to a} h(x) = L$.

Rules for limit :

If $a_n \longrightarrow a$ and $b_n \longrightarrow b$, then

① $a_n + b_n \longrightarrow a + b$

② $c a_n \longrightarrow c a$

③ $a_n - b_n \longrightarrow a - b$

④ $a_n b_n \longrightarrow ab$

⑤ $f(a_n) \longrightarrow f(a)$, where f is any polynomial fn.

⑥ $\dfrac{a_n}{b_n} \longrightarrow \dfrac{a}{b}$, if $b \neq 0$

⑦ $c^{a_n} \longrightarrow c^{a}$, for any real no. c, where c^{a_n} is a real no. for each $n \in \mathbb{N}$.

⑧ $\log_c(a_n) \longrightarrow \log_c(a)$, if $a_n > 0$ for all $n \in \mathbb{N}$, & $a, c > 0$.

⑨ $c_n \longrightarrow a$, if $a_n \leq c_n \leq b_n$, for all $n \in \mathbb{N}$.

Sequence

Limit

- STANDARD LIMITS :-

1) $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

2) $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

3) $$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

If $\lim_{n \to \infty} a_n = L$ (converses)

$\lim_{n \to \infty} a_n = \infty / -\infty$ / Don't have. (Diversant)

eg. $\lim_{n \to \infty} a_n = \frac{1}{n}$ an leads to 0. (convesing)

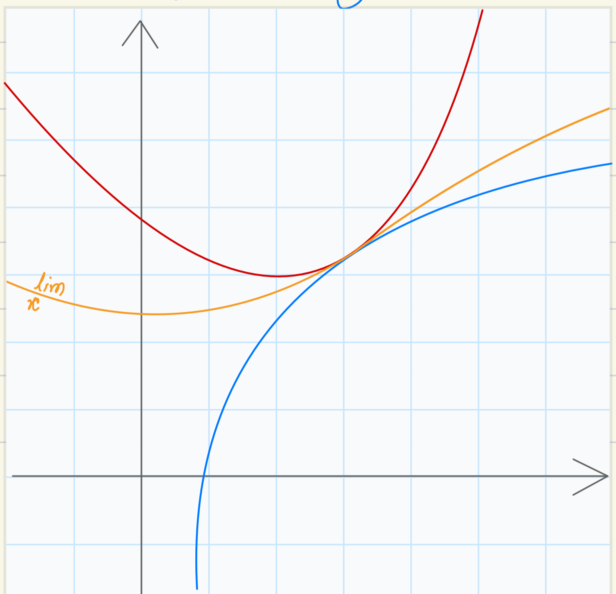
To find limit n domain should be a natural number.

To find limit of sequence we have to consider:

$\lim_{n \to \infty} a_n$

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ \frac{-1}{6n-5} = & \frac{-+1}{+5} = & \frac{-1}{1} = & \frac{-1}{7} & \frac{-1}{13} \\ & -.2 & -1 & -.14 & -.08 \end{array}$$

Sandwitch principle means, if $f(x) \leq g(x) \leq h(x)$ for all x and



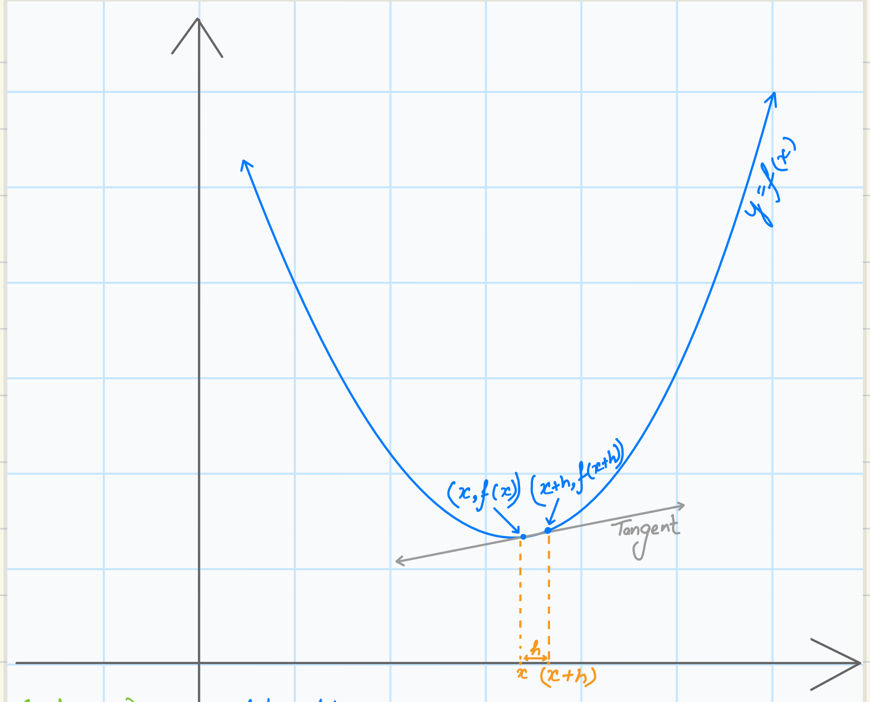
$\lim_{x \to 4} f(x) = \lim_{x \to 4} h(x) = Z$

then, $\lim_{x \to 4} = Z$

$$\frac{y_2 - y_1}{x_2 - x_1} = \text{slope} = f'(x)$$

Derivative: $f'(x)$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x}$$



Diff. b/w both point is 'h'.
If we want 'h' to become 0.
then, diff. b/w both pts.
in parabola also come to 0.
and line become tangent.

That will be:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x} \leftarrow \text{(Slope)} \rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(h)}{(x+h) - x} = \frac{f(x+h) - fh}{h} = f'(x)$$

when **h** is tending to 0. (Tangent)

Δ method / ab-initio mtd.

Note: $f'(x)$ or $\frac{dy}{dx}$ or y' or y_1 all are same.

Tangent can be called limits of secent.

Linear approximation: $l(x)$
- slope $= f'(x)$
- point $= (x, f(x))$

$$m = \frac{y - y_0}{x - x_0}$$

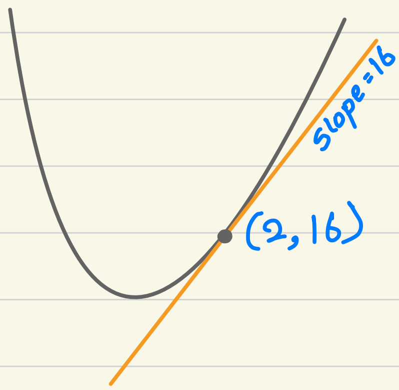
$\Rightarrow y - y_0 = m(x - x_0)$
is in point slope form.

Let, $x = a$,
So, slope $= f'(a)$
$\Rightarrow$ point $= (a, f(a))$
Point slope form, $y - y_0 = m(x - x_0)$
So, $y = m(x - x_0) + y_0$

So, by putting value, $l(x) = f'(x)(x-a) + f(a)$

Secant (2point) in curve which became tangent with limit is $= \frac{f(x+h) f(x)}{x+h-x} (y-x)$

Q/ Find eq. of tangent of parabola $f(x) = 4x^2$ at $(\underline{2}, 16)$?



$\Rightarrow$ Slope of $f(x) = 4x^2 = 8x = f'(x)$

So, $f'(2) = 8 \times 2 = 16$

putting in formula : $l(x) \Rightarrow y = 16(x-2) + 16$

$y = 16x - 32 + 16$

$y = 16x - 16$

$y = 16(x-1)$

Q/ Let $f(x)$ is diffrenciable at $x = 1$ & x be represented by the curve C. If a line passes through point $(5,8)$ is tangent to curve C at pt$(1,0)$. find $f'(1)$.

$\Rightarrow$ Slope $= (8-0)/(5-1) = 2$

$f'(1) = 2$

Slope formulae

$l(x) = f'(x)(x-a) + f(a)$

$8 = f'(1)(5-1) + 0$

$f'(1) = \frac{8}{4} = 2$

Linear approximation

Q) $f(x+y) = f(x) f(y) \; \forall \; x, y \in R$, $f(9) = 6$, $f'(0) = 4$, So, $f'(9) = ?$

$\Rightarrow$ It means:

$$f'(9) = \lim_{x \to 0} \frac{f(9+h) - f(9)}{h}$$

$$= \lim_{h \to 0} \frac{f(9+h) - f(9+0)}{h}$$

$$= \lim_{h \to 0} \frac{f(9) f(h) - f(9) f(0)}{h}$$

$$= f(9) \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= f(9) f'(0)$$

given $f(9) = 6$ so, $4 \times 6 = 24$

Power rule: It says:

$$f(x) = x^n, \quad n \neq 0$$

$$f'(x) = n x^{n-1}$$

Eg $\Rightarrow$ $f(x) = x^3$, $f(x) = 3x^2$

$f(x) = x^3 + x^2$

Constant rule: $\frac{d}{dx} k = 0$

Constant multiple rule: $\frac{d}{dx} [k . f(x)] = k . \frac{d}{dx} . f(x)$

Sum rule : $\frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$

Difference rule: $\frac{d}{dx} [f(x) - g(x)] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$

Basic differentiation formulae:

1. $D(\sin x) = \cos x$
2. $D(\cos x) = -\sin x$
3. $D(\tan x) = \sec^2 x$
4. $D(\sec x) = \sec x \cdot \tan x$
5. $D(\cot x) = -\operatorname{cosec}^2 x$
6. $D(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
7. $D(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$
8. $D(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$
9. $D(\tan^{-1} x) = \dfrac{1}{1+x^2}$
10. $D(\cot^{-1} x) = \dfrac{-1}{1+x^2}$
11. $D(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$
12. $D(\operatorname{cosec}^{-1} x) = \dfrac{-1}{|x|\sqrt{x^2-1}}$
13. $D(e^x) = e^x$
14. $D(a^x) = a^x \cdot \ln a \quad (a>0)$
15. $D(\ln x) = \dfrac{1}{x}$
16. $D(\log_a x) = \dfrac{1}{x \ln a}$
17. $D(x^n) = n \cdot x^{n-1} \, (n \in \mathbb{Q})$
18. $D\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$
19. $D(\sqrt{x}) = \dfrac{1}{2\sqrt{x}}$
20. $D(\text{constant}) = 0$

Eg, find derivative of $\dfrac{2^x}{3^x}$ Formulae: $D(a^x) = a^x \cdot \ln a$

$\Rightarrow$ So, $y = \dfrac{2^x}{3^x} \Rightarrow y = \left(\dfrac{2}{3}\right)^x \Rightarrow \boxed{y = \left(\dfrac{2}{3}\right)^x \cdot \ln \dfrac{2}{3}}$

Theorem of Diffrentiation :

(Addition rule)
(Substraction rule)

① $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

(Product rule)

② $\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

ie, $(f \cdot g \cdot h)' = f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

(Quotient rule)

③ $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

(Chain rule)

④ $\left[f(g(x))\right]' = f'(g(x)) \times g'(x)$

ie $\left[f(g(h(x)))\right]' = f'(g(h(x))) \times g'(h(x)) \times h'(x)$

Eg : If $f(x) = \sin(e^x)$, find $f'(x)$ (Chain rule)

$\Rightarrow f(x) = \sin(e^x) \Rightarrow f'(x) = \cos(e^x) \cdot e^x$

Q: If $f(x) = (\sin x)/x$. find $f'(x)$ (Quotient rule)

$\Rightarrow f(x) = \dfrac{\sin x}{x} \Rightarrow f'(x) = \dfrac{(\cos x) \cdot x - \sin x \cdot 1}{x^2}$

Q. If $f(x) = x \cdot e^x \cdot \sin x$, find $f'(x)$ (Product rule)

$\Rightarrow f x = x \cdot e^x \cdot \sin x$

$\Rightarrow f'x = e^x \cdot \sin x + x \cdot e^x \cdot \sin x + x \cdot e^x \cdot \cos x$

Q. If $f(x) = \dfrac{x^4 + x^2 + 1}{x^2 + x + 1}$. Find $f'(500)$

$f'(500)$ means $f'(x)$ x in 500

$$\Rightarrow f(x) = \frac{x^4 + x^2 + 1}{x^2 + x + 1}$$

$$\Rightarrow f(x) = \frac{\cancel{(x^2+x+1)}(x^2 - x + 1)}{\cancel{(x^2+x+1)}} = x^2 - x + 1$$

$f(x) = x^2 - x + 1$

$f'(x) = 2x - 1$

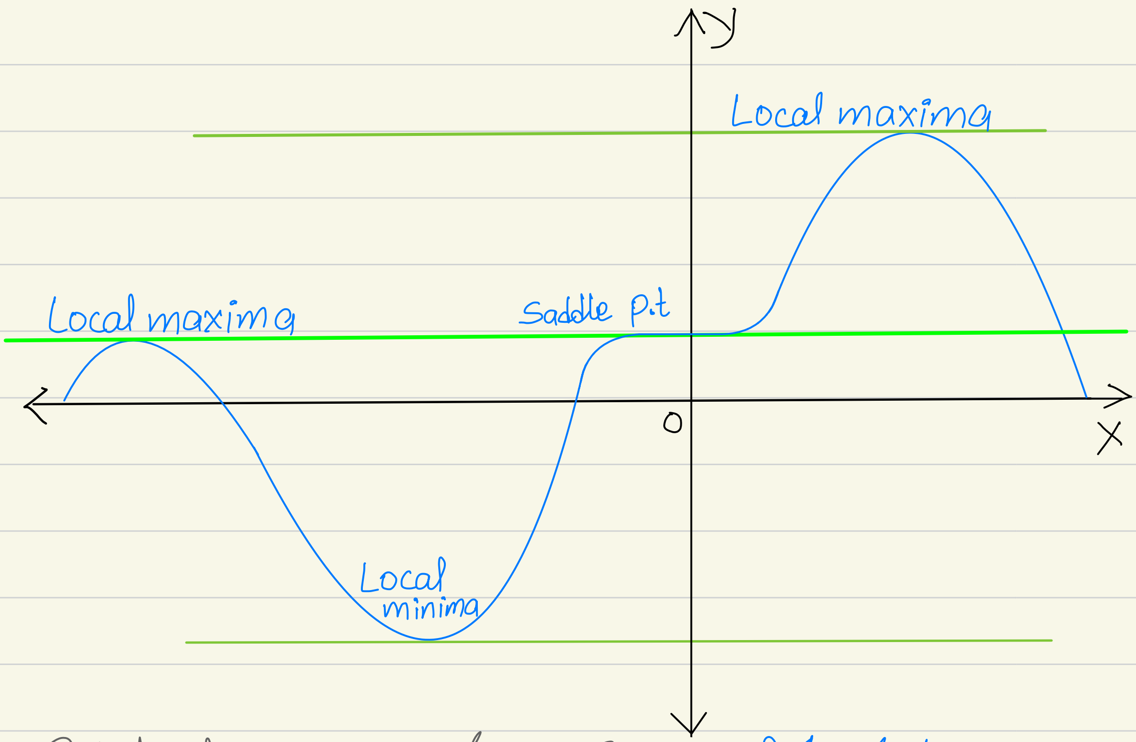
$f'(500) = 2 \times 500 - 1 = 999$

- Indeterminate form:

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \infty^0, 0^\infty, 1^\infty$

- L'Hôpital's rule:

$$\lim_{x \to 0} \frac{\log_e(1+x)}{x}$$



Global minima/maxima → Max/Min pt. of the whole graph.

Tangent which are horizontal to x are at,

$$y = f'(a)(x-a) + f(a)$$

- **Critical point:** It is point where $f(x)$ is either not differenciable or $f'(a) = 0$. (Horizontal)

- **Saddle point:-** It is critical point which exclude local maximum & minimum.

If a is critical pt. $f''(a) > 0$ so, (a = local minimum) +ve

If a is critical pt. $f''(a) < 0$ so, (a = local maximum) -ve

If a is critical pt. $f''(a) = 0$ so, (a = inconclusive) 0

Diffrenciation:-

Shortcut is (times and takes)

$$y = 5x^3 + 4x^2 - 8x + 3$$
$$= 5 \times 3x^{3-1} + 4 \times 2 \cdot x^{2-1} - 8 \times 1 \cdot x^{1-1} + 3$$ (vanish bcz don't have x)
$$\Rightarrow \frac{dy}{dx} = 15x^2 + 8x - 8$$

Eg:

$$f(x) = x^3 - 12x$$

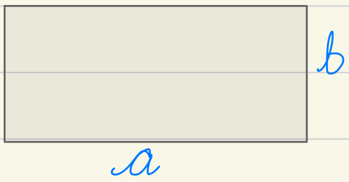
$f'(x) = 3x^2 - 12$, Setting it to 0.
$\Rightarrow 3x^2 - 12 = 0$
$\Rightarrow x = \pm 2$ Critical pt $= \pm 2$

$f''(x) = 6x$, so, $f''(2) = 12 > 0$
$f''(-2) = -12 < 0$
So, 2 is local minimum & -12 is local maximum

Eg: $f(x) = \cos(x)$
$f'(x) = -\sin(x)$, setting to 0.
$-\sin(x) = k\pi$ (k = int)
$f''(x) = -\cos(x)$ hence, $f(k\pi) = -\cos(k\pi)$ $\begin{cases} \text{even} \to -1 \\ \text{odd} \to 1 \end{cases}$

Eg: $f(x) = x^3 + x^2 - x + 5$, $f'(x) = 3x^2 + 2x - 1$ setting to 0.
$\Rightarrow 3x^2 + 3x - x - 1 = 3x(x-1) + 1(x+1) \Rightarrow x = -1, +1/3$
$f''(x) = 6x + 2 \Rightarrow$ putting zeros, $-6+2 = -4$, $6 \times \frac{1}{3} + 2 = 4$
4 (local maxima), -4 (local minima)

Area of rectangle:

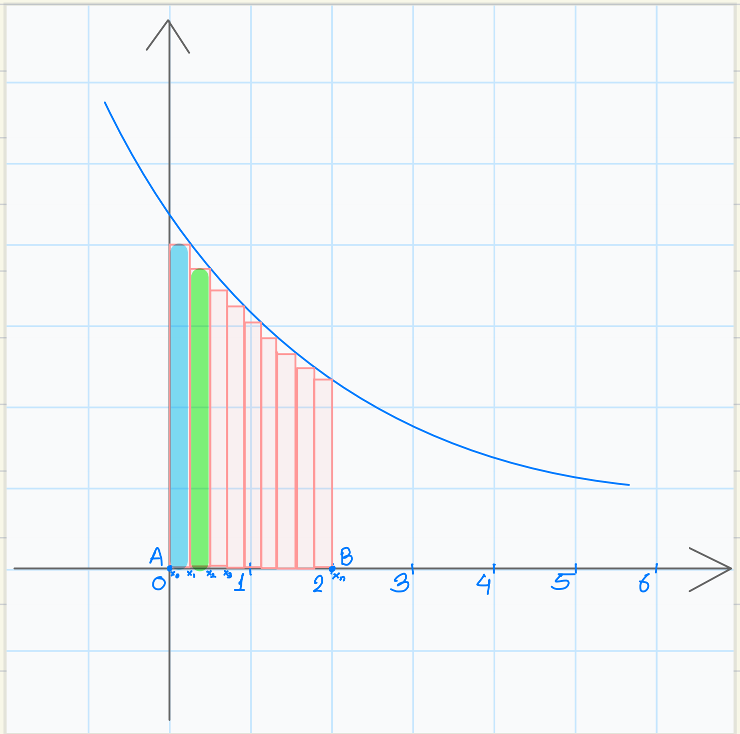


$\text{Area} = f(a,b)$

if, $a = a\times 2 = f(2a,b) = 2f(a,b)$

In methematical term:

To find area under curve from interval $[A,B]$ in domain D.



- A partition from $[A,B]$, i.e. a choice of intermediate point $A = x_0 < x_1 < x_2 < {}_3 < \dots x_n = B$
- A choice of $x_i^* \in [x_{i-1}, x_i]$

Define $\Delta x_i = x_i - x_{i-1}$, and
$\|P\| = \max\{\Delta x_i\}$

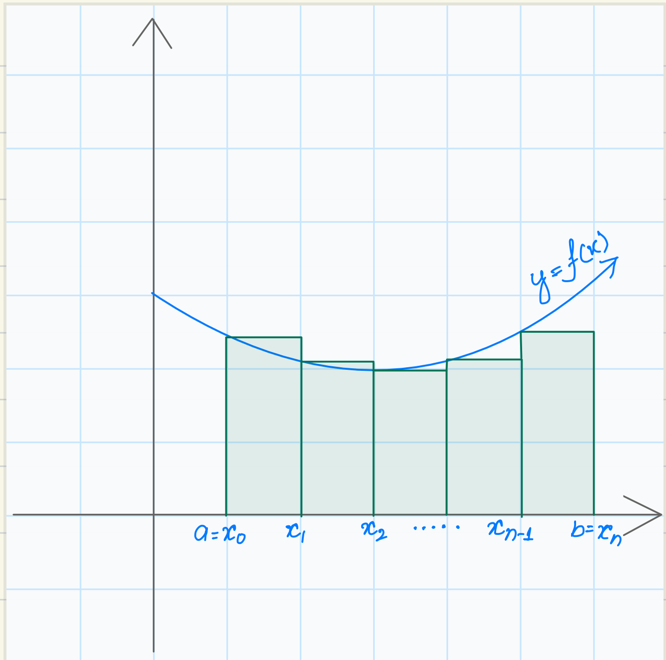
$x_i^* = x_{i-1}$

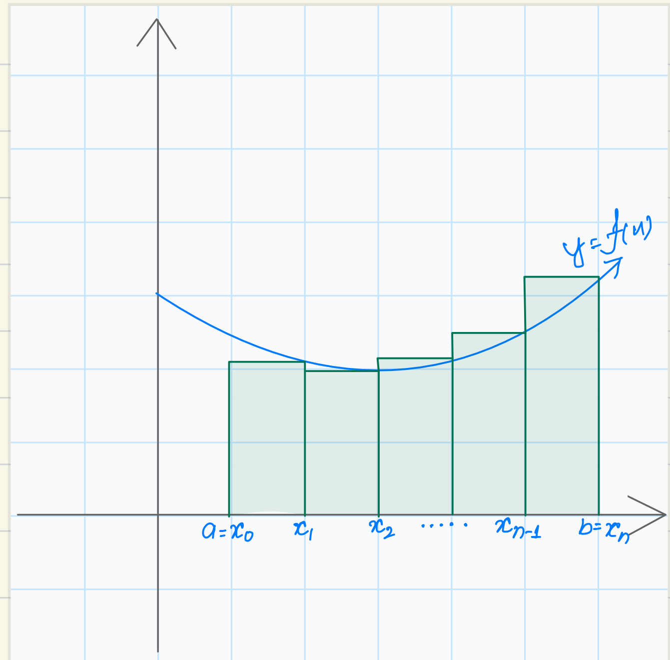
$$f(x_0)(x_1 - x_0) + f(x_1)(x_2 - x_1) + \dots + f(x_{n-1})(x_n - x_{n-1})$$

So, Riemann sum is:-

$$S(P) = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

Here, $\Delta X = \frac{b-a}{n}$



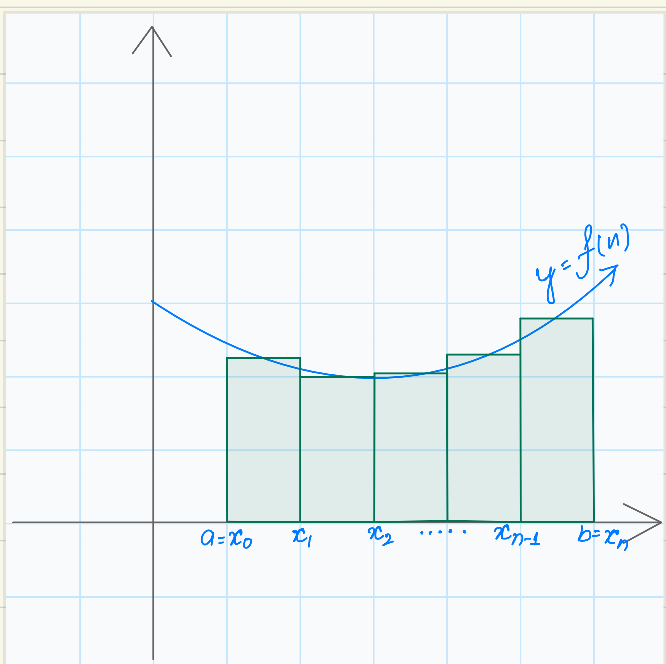


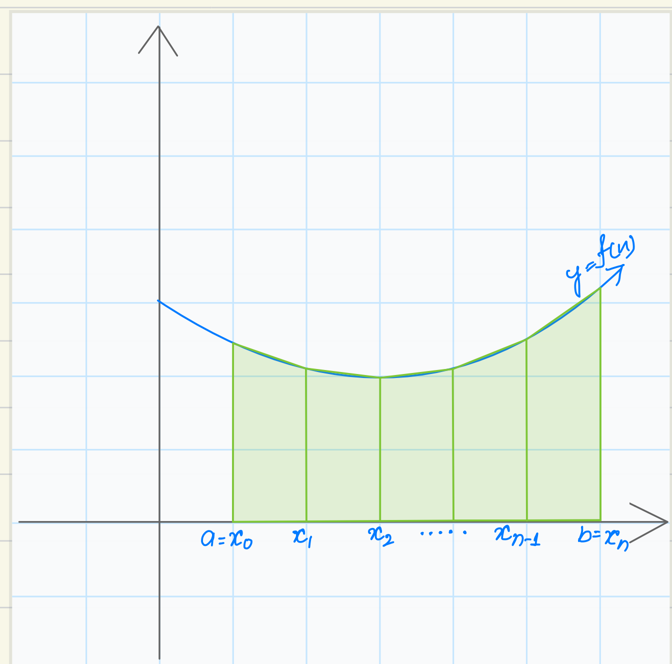
In left remansum area will be:

$$\sum_{i=1}^{n} f(x_{i-1})\Delta X$$

In right remansum area will be:

$$\sum_{i=1}^{n} f(x_i)\Delta X$$





In mid-point remansum area will be:

$$\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta X$$

In trapazoid area will be.

$$\sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2}\Delta x$$

- Integral of fn:

let f be fn from D to R ~~for~~ some domain $D \subseteq C$.
Suppose interval $[a,b]$ is in domain D.

The (definite) integral of f from a to b is:

$$\lim_{\|P\| \to 0} S(P) = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i$$

it is denoted by: $\int_a^b f(x)\,dx$.

If $f \geq 0$, than area under graph of $\Gamma(f)$ above interval $[a,b]$ measured by

$$\int_a^b f(x)\,dx \quad (\text{resp.} -\int_a^b f(x)\,dx)$$

Infinitely small Δx

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i-1}) \Delta x \text{, where } \Delta x = \frac{b-a}{n} = \int_a^b f(x)\,dx$$

Like BODMAS here we solve in ILATE

I → Inverse trignometric ($\sin^{-1}x$, $\cos^{-1}x$)
L → Logerithemic ($\log$, $-\log$)
A → Algebric (x, x^2, $\sqrt{x}$)
T → Trignometric ($\sin x$, $\cos x$)
E → Exponential (e^x, 3^x)

□ Power Rule:

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
- $\int K dx = Kx + C$

Constant rule:

Take constant outside and

- $\int_a^b kx^2 dx = k \int_a^b x^2 dx$

Eg:- $I = \int_4^6 3x^2 dx = 3 \int_4^6 x^2 dx = \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_4^6$

$= [x^3]_4^6 = 6^3 - 4^3 = 216 - 64 = \underline{152}$

Here we have to subtract upper limit - lower limit place those inplace of x.

Eg2:- $I = \int_2^4 6x^2 - 3x + 11\, dx = \left[\cancel{6}^{2} \frac{x^3}{\cancel{3}_1} - 3\frac{x^2}{2} + 11x\right]_2^4$

$\left[2\times4^3 - \frac{3\times4^2}{2} + 11\times4\right] - \left[2\times2^3 - \frac{3\times\cancel{2^2}}{\cancel{2}} + 11\times2\right]$

$= [128 - 24 + 44] - [16 - 6 + 22] = 148 - 32 = 116$

The property of integrals:

Indefinite integral :-

$\rightarrow \int c f(x)\,dx = c\int f(x)\,dx$

$\rightarrow \int (f+g)(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\rightarrow \int (f-g)(x)\,dx = \int f(x)\,dx - \int g(x)\,dx$

$\rightarrow$ Integration by part: ($f(u)$ & $g(u)$ are 2 function)

$\int (fg')(x)\,dx = (fg)(x) - \int (f'g)(x)\,dx$

$\Rightarrow \int f(u)\cdot g(u)\,du = f(u)\cdot \int g(u)\,du - \int f'(u)\left(\int g(u)\cdot du\right)\cdot du$

Definite integral :-

$\rightarrow \int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

$\rightarrow \int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

$\rightarrow \int_a^b (f-g)(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$\rightarrow \int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

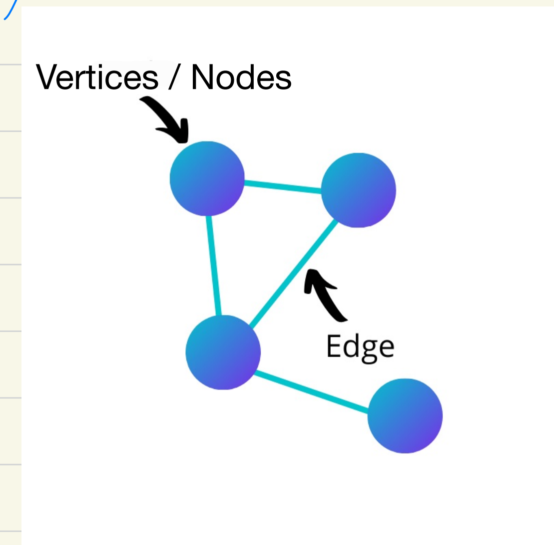
$\rightarrow$ For any $c \in \mathbb{R}$, $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$\rightarrow$ If $f(x) \geq g(x)$ for all but finitely many p.t in interval $[a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

GRAPHS

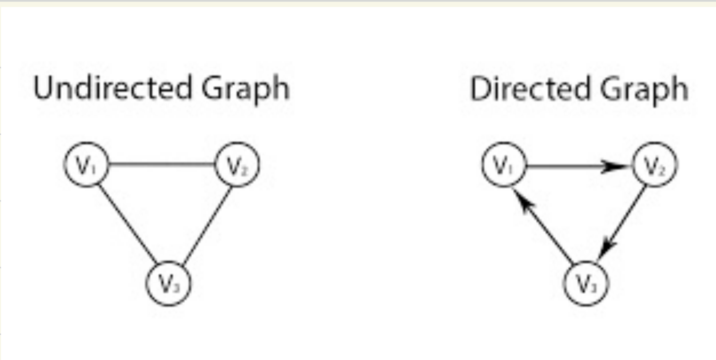
A graphs represent relation b/w entities.

→ In graph entries are vertices/nodes. (V)
→ Relationships are edge. (E)



Graph can be of 2 type:

→ Directed (Family tree)
→ Undirected. (Friend circle)



Path is sequence of connected edges.

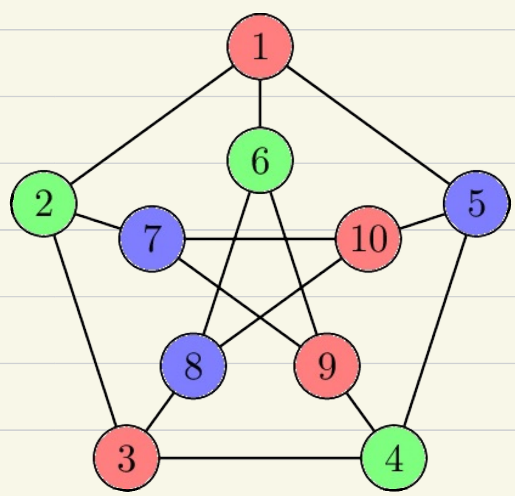
☐ Graph colouring:

We have graph, $G=(V,E)$, and set of color C.

→ Colouring is a function assigned to every v from set C.
- Pair of v connecte should diff. c.

such that, $C: V \rightarrow c$

$$(u,v) \in E \Rightarrow c(u) \neq c(v)$$



Generally 4 colour is sufficient for maps.

Situations :-

→ Can be used to make scheduling and effeciently allocating classroom by coloring vertex of same which run in same time.

→ Vertex cover :

- Where to allot ambulance so it reaches most area in min time.
- Camera in corridor.

Independent set :

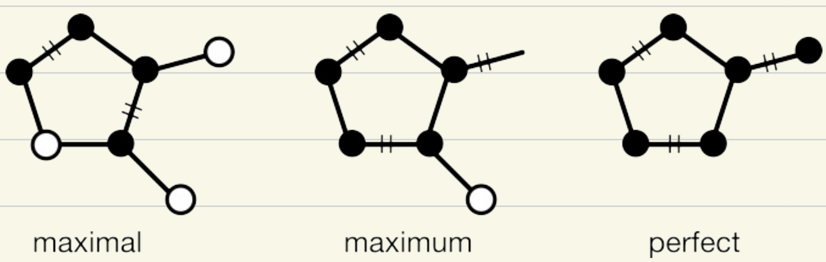
- Subset of vertices so no two are connected by edge.

Matching :

- $G = (V, E)$ as undirected graph.
- A matching is subset $M \subseteq E$ of mutually disjoint edge.

Maximal matching

Perfect matching :- It connects every vertex in graph.



Graph $G = (V, E)$

$V \rightarrow$ Set of vertices.

$E \rightarrow$ Set of edges.

- A path is sequence of $v_1, v_2 \ldots v_k$ connected by edge :

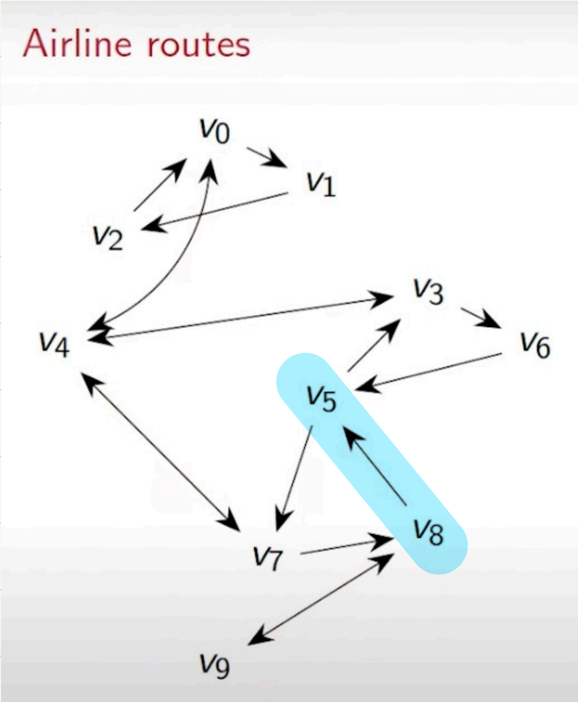
$$\text{for } 1 \leq i < k, (v_i, v_{i+1}) \in E$$

- Vertex v is rechable from vertex u if there is path/edge from u to v.

- Let $|V| = n$

Assume, $V = \{0, 1, 2, \ldots, n-1\}$

- Assume newpair (i, j), where $0 \leq i, j < n$. (usually asume $i \neq j$) loop.



Directed Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	0	0	0
4	1	0	0	1	0	0	0	1	0	0
5	0	0	0	1	0	0	0	1	0	0
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	1	0	0	0	1
9	0	0	0	0	0	0	0	0	1	0

- Adjacent metrix:
  - Here row and column are numbered $\{0, 1, \ldots, n-1\}$
  - $A[i, j] = 1$ if $(i, j) \in E$

- In directed graph:
  Row - outgoing, column - incoming

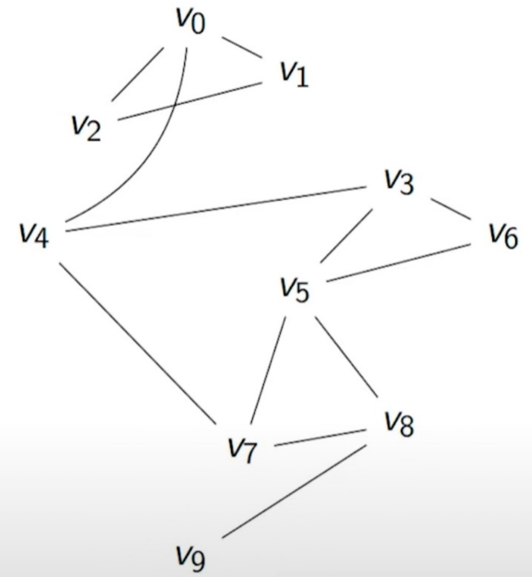
- Degree of a vertex i, no-connect $= 0$, all connect $= n-1$.

- Size of adjacency metrix has n^2 edges.

- Undirected graph
  - $A[i,j] = 1$ iff $A[j,i] = 1$
  - Symmetric across main diagonal

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	0	1	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	1	0	0	0
4	1	0	0	1	0	0	0	1	0	0
5	0	0	0	1	0	0	1	1	1	0
6	0	0	0	1	0	1	0	0	0	0
7	0	0	0	0	1	1	0	0	1	0
8	0	0	0	0	0	1	0	1	0	1
9	0	0	0	0	0	0	0	0	1	0

Airline routes, all routes bidirectional



Undirected graph

○ Adjacency list:
It only represent neighbour of each vertex.

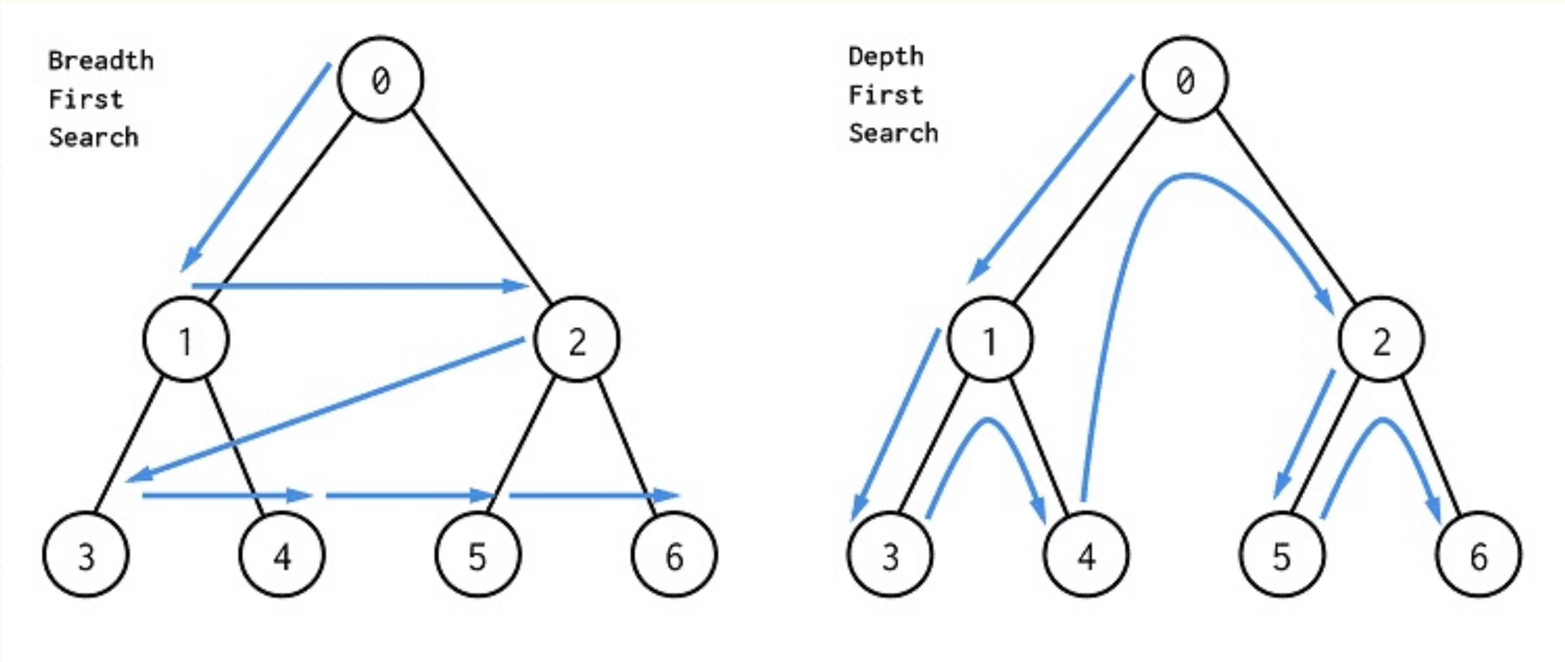
0	{1,4}
1	{2}
2	{0}
3	{4,6}
4	{0,3,7}

Directed graph

5	{3,7}
6	{5}
7	{8}
8	{5,9}
9	{8}

- Tree → Minimal connected acyclic graph.
- Forest → collection of trees.

▷ Breath first search: In this we search root level by level.
- It telles what all the vertices are rechable from the starting node.
- First in first out.
- If diff. b/w level of vertices is more than 1 than there can't be edge connection of if we have level diff. 1 or less there can be edge.



▷ Dept first search(DFS) → First in last out.
→ It is used to find reachability.
→ Used to find cycles.
→ In this we can only have edge in same branch.

Forward edge → It is edge which go forward.
not in DFS but in original tree. A → B → C

Backward edge → It is edge which go backward and create cycle.

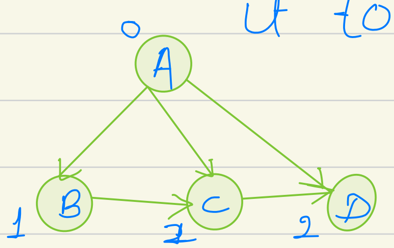
☐ Facts of Graphs, vertices and degrees:

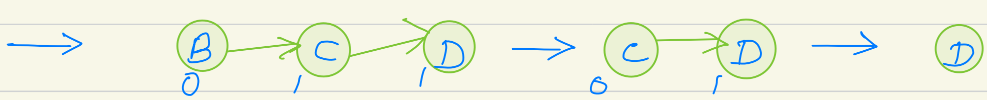
- Any vertex in graph having 'n' vertices can have maximum degree of (n-1).
- Sum of degree of all the vertices will be even.
- If 2 vertices in a graph have degree (n-1), it means there will be no vertex with degree 1.

- Planar graph:
  - All planar graph can be colored with 4 color.
  - No edge can cross another edge.

- DAG: (Directed acyclic graph) $A \longleftrightarrow B$ ×
  - → There atleast 1 vertex with indegree 0.
  - → There atleast 1 vertex with outdegree 0.
  - → If we remove a vertex with indegree 0 than graph will be DAG.

- Topological sequence :- In this we keep removing vertex with 'n' degree 0 and adding it to a sequance list.





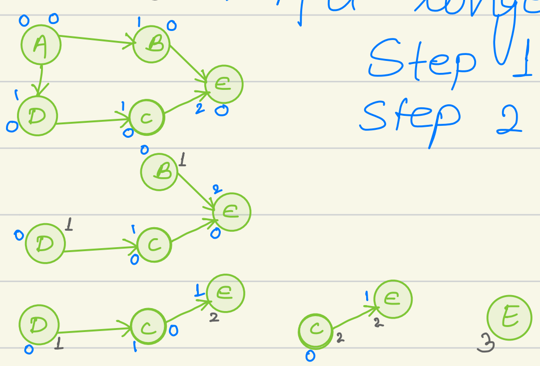
TS → A, B, C, D

Steps:
1) Compute all the indegree of vertices in graph.

2) Pick the vertex 'i' which has indegree 0.
3) Remore the vertex 'i' & all the edge which has incident with vertex 'i'.
4) Update the indegree of all remaining vertex.
5) Repeat from step (2) to (4).

Longest path in DAG:

→ All longest path will be 0.



Step 1 → $LPT(i) = 0 \quad i \in \{A, B, C, D, E\}$

Step 2 → $LPT(B) = \max\{LPT(B), 1 + LPT(A)\}$

$= \max\{0, 1+0\} = 1$

$LPT(D) = \max\{LPT(B), 1 + LPT(A)\} = \max\{0, 1+0\} = 1$

$LPT(E) = \max\{LPT(E), 1 + LPT(B)\}$

$= \max\{0, 1+1\} = 2$

$LPT(C) = \{0, 1+1\} = 2$, $LPT(E) = \max\{LPT(E), 1 + LPT(C)\} = \max\{2, 1+2\} = 3$

Steps:

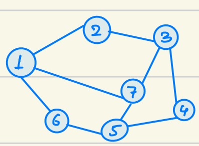
1) Compute indegree for every vertex 'i' in the DAG.
2) Initialize longest path (i) = 0 for all $i \in V$.
3) Find a vertex 'u' with indegree 0 and remove it from DAG.
4) Update indegree for all vertices that are adjacent to vertex 'u'.
5) Update longest path with:

longest path to (i) = Max { longest path to (i), 1 + longest path to 'u' }

Maximum independent set:

Here we find largest set (A) of vertices such that no two vertices in A should have direct edge.

Eg →



7 vertices, 9 edge.

$A = \{2, 4, 6, 7\}$

Algorithm to use:

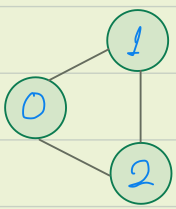
- **Dijkstra's algorithm:**
  It is used to find shortast path from a source vertex to every other vertex. (No -ve edge)

- **Bellman-ford algorithm:**
  It is used to find shortast path from a source vertex to every other vertex. (With or without -ve edge but no -ve cycle.

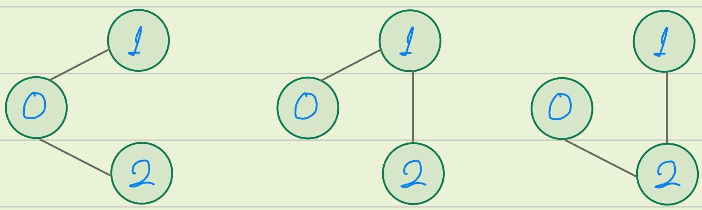
- **Floyd-Warshall algorithm:**
  All pair shortest path. (With or without -ve edge but no -ve cycle.)

- **Prim's algorithm and Prim's algorithm:**
  These algorithms are used to find MCST. (Minimum cost spanning tree).

- Spanning tree → It is undirected graph (subgraph) which include all vertex G.



Graph G

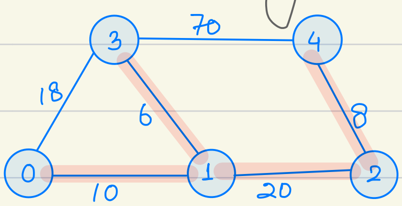


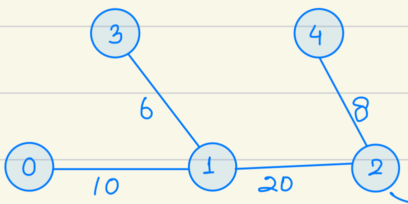
Possible spanning tree

→ Minimum spaning tree:-

- It should be minimum weited/distance edge graph to all vertices.
- It should be acyclic graph.

○ Prim's algorithm : (Undirected graph only.)

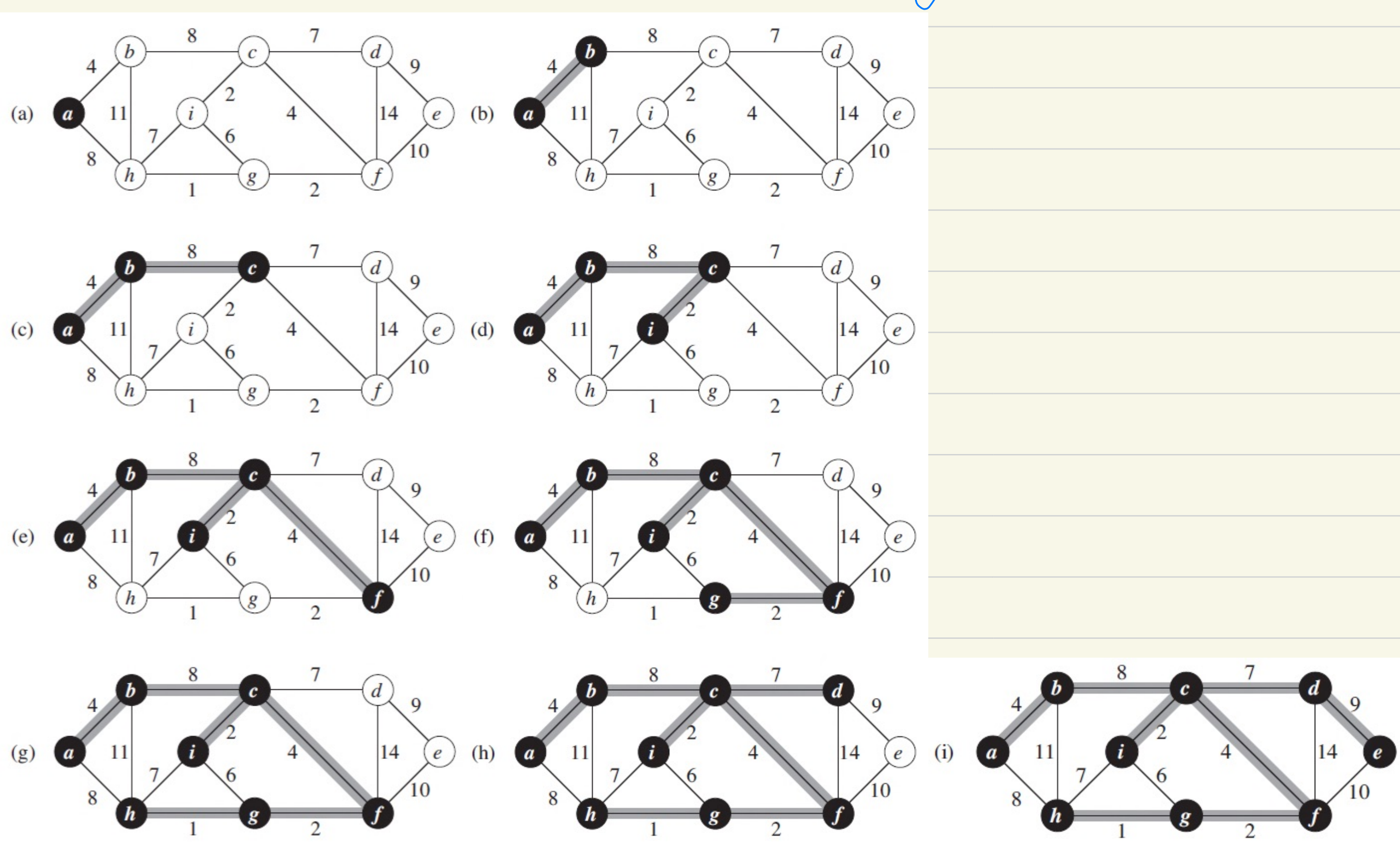




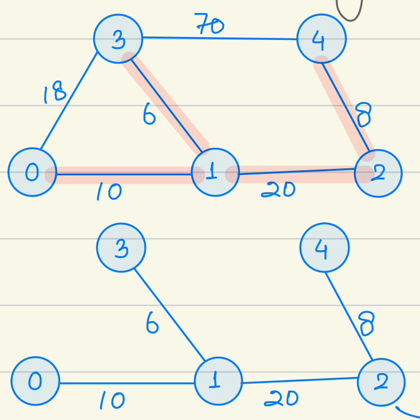
Steps:-

1.) Select smallest edge 'i'. (Here $3 \xrightarrow{6} 1$)
2.) Select smallest neighbour of 'i' (Here $0 \xrightarrow{10} 1$) and add in previously selected edge.
3) Keep repeating step2 until we get all vertices.

→ Minimum spanning tree



○ Kruskal's algorithm:



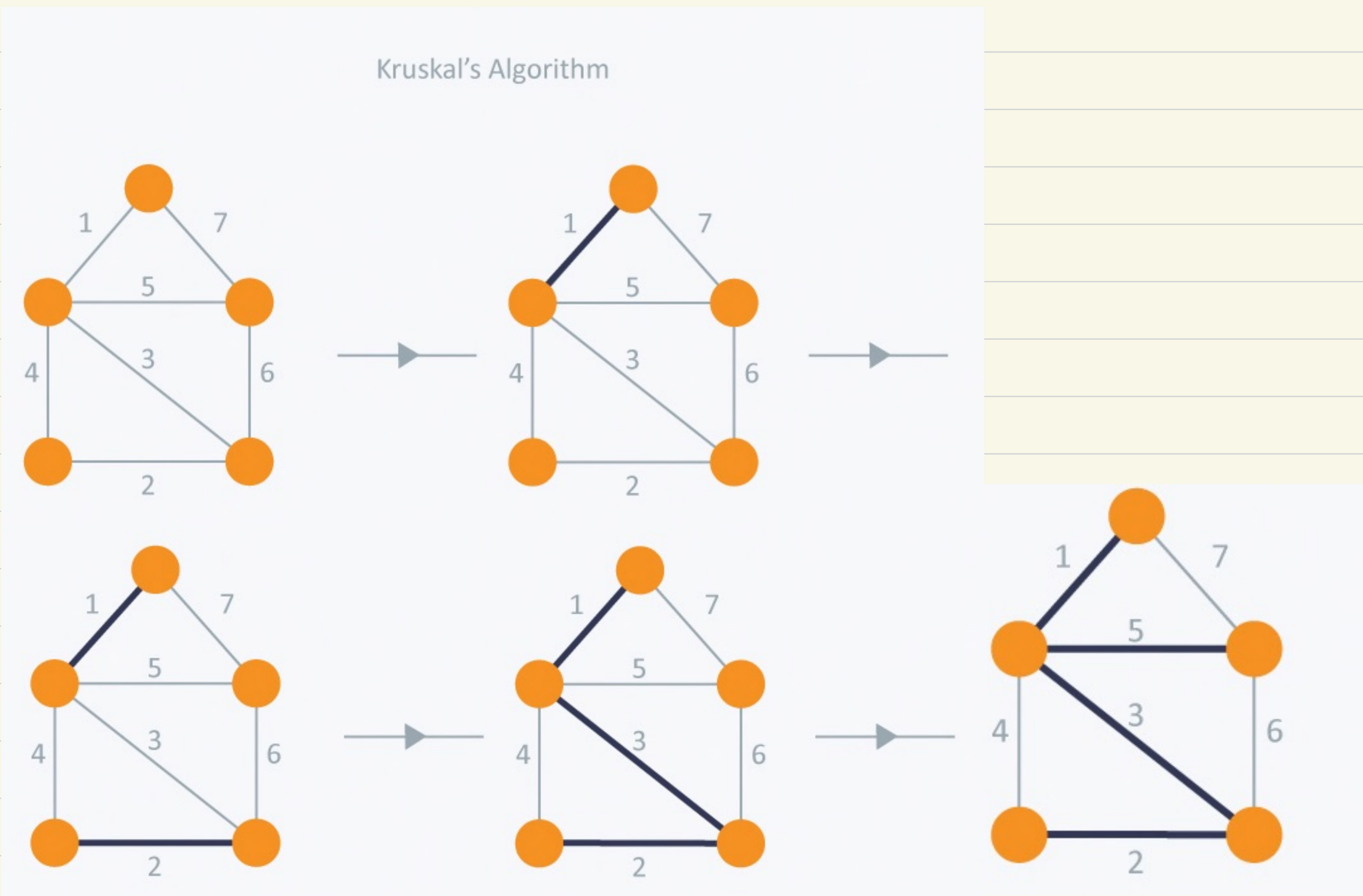
Steps:

1.) Select smallest edge 'i'. (Here $3 \xrightarrow{6} 1$)

2.) Here insted of neighbour we see smallest in remaining edges. (Here $2 \xrightarrow{8} 4$)

3) Keep repeating step2 until we get all vertices.

→ Minimum spanning tree

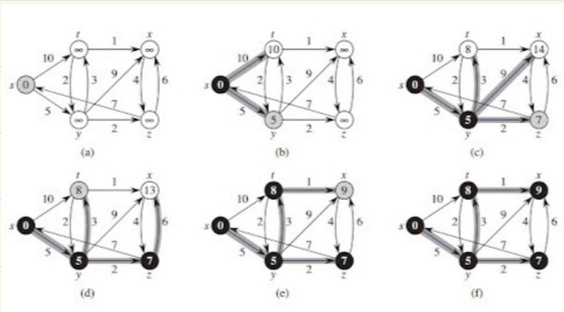


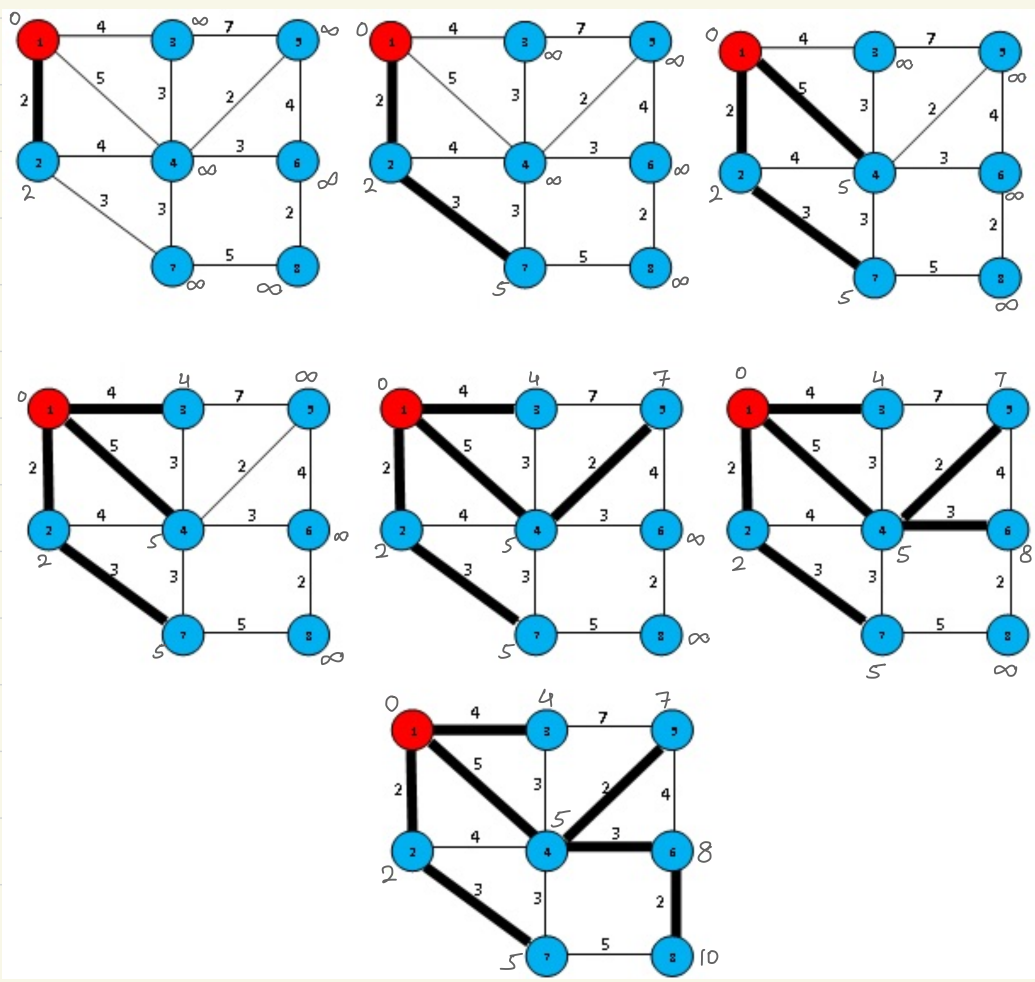
- Dijkstra's algorithm:

It is used to find shortest path from a source vertex to every other vertex. (No -ve edge)

Steps:-

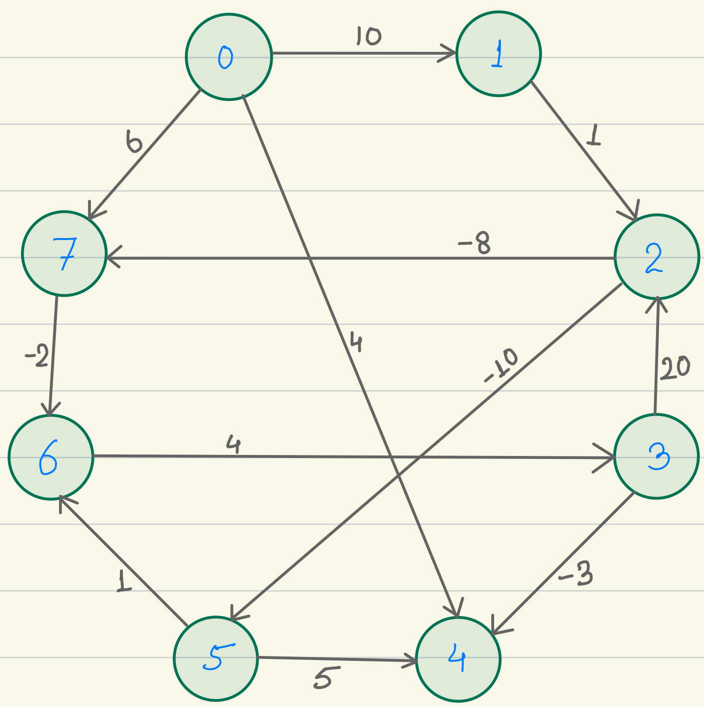
1) It starts with source vertex i.
2) Initialise distance value of vertex "i" to 0 and ∞ to all other vertex.
3) Update distance of all neighbour vertex from i.
4) Each time visit the minimum distanced vertex and update update minimum distance value to it if it have larger value.





- Belman ford algorithem:
  It is used to find sortest part from one vertex to each other vertex.

V								D(V)
0	0	0	0	0	0	0	0	0
1	∞	10	10	10	10	10	10	10
2	∞	∞	11	11	11	11	11	11
3	∞	∞	∞	8	8	5	5	5
4	∞	4	4	4	4	4	2	2
5	∞	∞	∞	1	1	1	1	1
6	∞	∞	4	4	1	1	1	1
7	∞	6	6	3	3	3	3	3



Steps:

1.) Here we has to find origin or vertices with inward edge 0 and assign distance 0 and every other vertex as '∞'.
2.) Write distance of adjacent edge from 0 in next column.
3.) Keep doing that and if we clash with new edge reaching previously reached vertex from other edge we keep smaller one.
4.) Keep repeating step 2 and 3 until we get to vertex with outward edge 0.

- Floyd-Warshall algorithm: (For directed weighted graph)

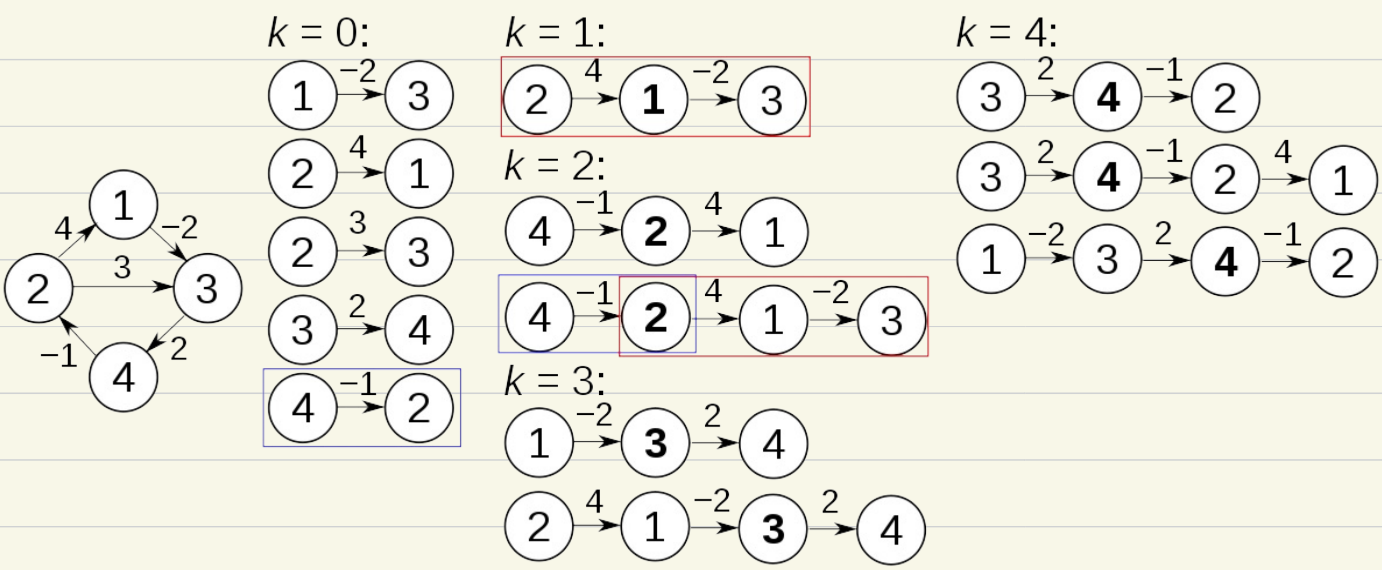
It is used to find all pair shortest path.

(For both +ve & -ve edge weight but no -ve cycle.)

- $Sp^k[i,j]$ is length of shortest path from vertex 'i' to vertex 'j' using vertices in $\{1,2,3,\ldots,k-1\}$

Eg:- If we want to compute Sp^3 it means we need to find shortest distance from any 'i' to vertex 'j' via vertices $\{0,1,2\}$

- If no path b/w vertices than distance is ∞.



k = 0	j = 1	2	3	4
i = 1	0	∞	−2	∞
2	4	0	3	∞
3	∞	∞	0	2
4	∞	−1	∞	0

k = 1	j = 1	2	3	4
i = 1	0	∞	−2	∞
2	4	0	**2**	∞
3	∞	∞	0	2
4	∞	−1	∞	0

k = 2	j = 1	2	3	4
i = 1	0	∞	−2	∞
2	4	0	2	∞
3	∞	∞	0	2
4	**3**	−1	**1**	0

k = 3	j = 1	2	3	4
i = 1	0	∞	−2	**0**
2	4	0	2	**4**
3	∞	∞	0	2
4	3	−1	1	0

k = 4	j = 1	2	3	4
i = 1	0	**−1**	−2	0
2	4	0	2	4
3	**5**	**1**	0	2
4	3	−1	1	0