The most probable questions from UNIT 01

1. What are matter waves? Derive the equation for the wavelength of matter waves

According to de Broglie every moving particle is associated with a wave and called as matter wave. The above is treated as the de Broglie hypothesis.

From the wave particle dualisms, we know that,

From Einstein's mass energy relation, we have $E = mc^2$

From Planck's radiation law we have $E = h\upsilon = \frac{hc}{\lambda}$

From the above equations we can write $E = mc^2 = \frac{hc}{\lambda}$ (or) $\lambda = \frac{h}{mc}$

The de Broglie extended the wave particle dualism of light for the matter also. According to him, every moving particle is associated with a wave whose wavelength $\lambda = \frac{h}{mv}$

Where m is the mass of the particle and v is the wavelength of the particle.

$$\therefore \, de\ Broglie\ wavelength\ \lambda = \frac{h}{mv}$$

Here m is relativistic mass and given by $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

The wave nature of the particle is noticeable only when the associated wavelength is comparable with the dimensions of the particle.

If E is the energy of particle, $E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{P^2}{2m}$ $(or)\ p = \sqrt{2mE}$

$$(or)\lambda = \frac{h}{\sqrt{2mE}}$$

If the considered particle is electron with mass m and accelerated with a potential V, then E=eV

$$\text{Hence wavelength associated with the electron}, \lambda = \frac{h}{\sqrt{2meV}}$$

Ignoring the relativistic condition, if we substitute all the constants,

$$(or)\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}}$$

$$(or)\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}}\ m$$

Characteristics of Matter waves:

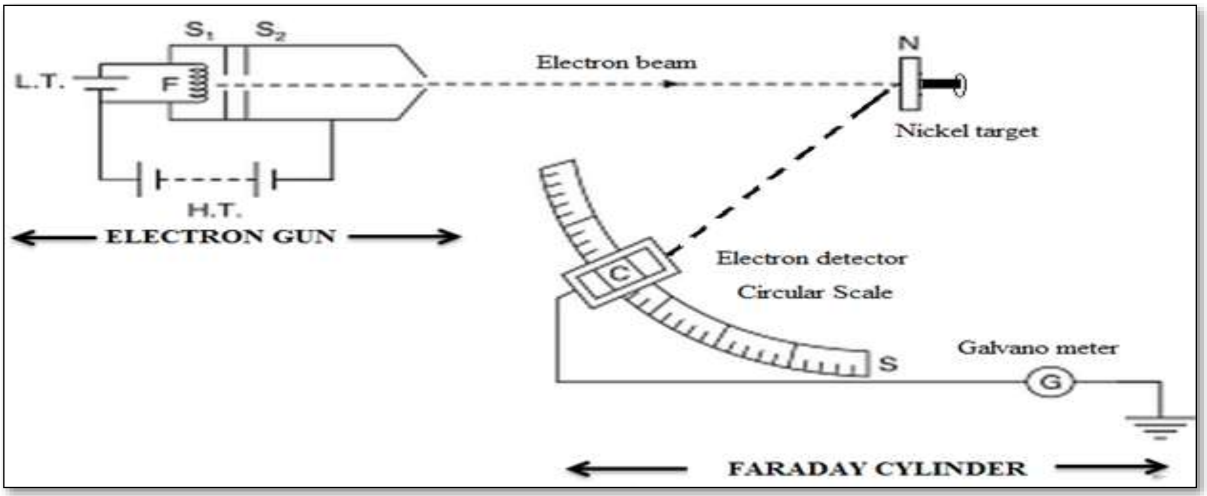
1. Lighter the particle, greater is the wavelength associated with it.
2. Lesser the velocity of the particle, longer the wavelength associated with it.
3. For $v = 0$, $\lambda = \infty$. Hence the matter waves are associated with only moving particles.
4. Whether the particle is charged or not, matter waves are associated with it.
5. No single phenomena exhibit both particle nature and wave nature simultaneously.

2. ***What are matter waves? With neat diagram, explain how the Davisson and Germer's Experiment proved the existence of matter waves***

Every moving particle is associated with a wave and called as matter wave. This was stated by the de Broglie and called as de Broglis hypothesis. However, the experimental evidence for the matter waves was given by an experiment designed to by Davisson and Germer

Experimental Setup:

- **Electron Source:** The electrons were produced by heating the filament F with a low-tension battery
- **Electron Beam:** These electrons were accelerated with the help of high-tension battery and collimated into fine pencil of beam with the help of slits S1 and S2. The energy of the electrons was adjustable by varying the voltage.
- **Crystal:** This beam of electron is made to fall on a single large nickel crystal. The crystal structure act like a diffraction grating for the electrons.
- **Detector:** It is an arrangement to find the direction of diffraction maxima of electrons. This electron detector can move on a circular scale between 20° to 90° and connected to a galvanometer.



Experimental Procedure

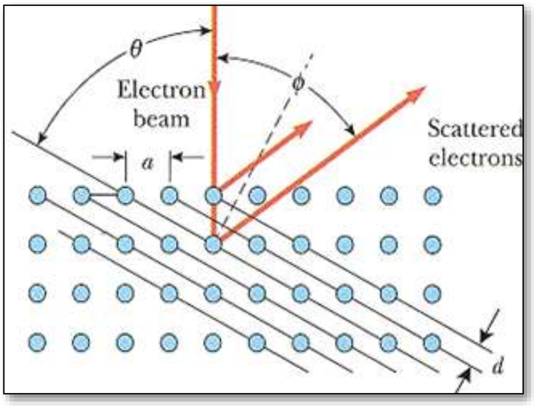
The electrons were produced by the electron gun and accelerated towards the target by controlling the voltage. The Target will act as the Diffraction grating for the electrons and diffract them in various directions. The direction in which the electrons were diffracted maximum will be measured by moving Faraday's cylinder. The change in Galvano meter recorded will be recorded for regular intervals of angles and the angle of diffraction maxima will be measured.

Once the diffraction maxima were found, the Faraday cylinder will be fixed at that position and the voltage for which the Galvano meter reading is maxima is measured by charging the voltage in small intervals

Experimental Observations:

The pronounced scattering direction was found to be 50° and for a voltage of 54 volts.

Analysis



Interatomic distance of Ni $a = 2.14\times10^{-10}$ m

Inter planar spacing $d = a \sin\varphi = 2.14\times10^{-10} \sin 25^\circ$

$= 0.909\times10^{-10}$ m

From Braggs law we have $2d\sin\theta = n\lambda$

For first order diffraction we can write

$2\times0.909\times10^{-10}\sin(90-25) = 1\times\lambda$

(or) $\lambda = 1.65\times10^{-10}$ m

Similarly from de Broglie principle, we have

$$\lambda = \frac{12.27\times10^{-10}}{\sqrt{V}}m = \frac{12.27\times10^{-10}}{\sqrt{54}}m$$

(or) $\lambda = 1.67\times10^{-10}$ m

This value is in agreement with the experimental value. Hence the experiment proves the wave nature of matter.

3. Derive the Schrödinger's Time Independent Wave Equation and time dependent wave equation. Discuss the physical significance of wavefunction.

Consider a system of stationary waves associated with a particle. Let *x,y,z* be the coordinates of particle and Ψ be the wave displacement for de Broglie at any instant of time 't'. This wave function is represented by $\Psi(x,y,z,t)$.

According to classical theory, the differential equation of wave motion is given by

$$\frac{\partial^2\psi}{\partial t^2} = \mathrm{v}^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) \qquad 1$$

$$(or)\,\frac{\partial^2\psi}{\partial t^2} = \mathrm{v}^2\nabla^2\psi$$

$$\frac{\partial^2\psi}{\partial t^2} - \mathrm{v}^2\nabla^2\psi = 0 \qquad (2)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \rightarrow Laplacian\ operator$$

Equation (2) is a second order differential equation. Let the trial solution is in the form of

$$\psi = \psi_0 \sin \omega t$$

$$\frac{\partial\psi}{\partial t} = \omega\psi_0 \cos \omega t$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\,\psi_0 \sin \omega t = -\omega^2\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -(2\pi\upsilon)^2\psi = -\frac{4\pi^2\mathrm{v}^2}{\lambda^2}\psi \quad //\, \upsilon = \frac{\mathrm{v}}{\lambda}$$

From de Broglie's equation we have $\lambda = \frac{h}{mv}$

$$\frac{\partial^2\psi}{\partial t^2} = -(2\pi\upsilon)^2\psi = -\frac{4\pi^2\mathrm{v}^2}{h^2}(mv)^2\psi - - - - (3)$$

Substituting (3) in (2)

$$\nabla^2\psi + \frac{4\pi^2}{h^2}(m\mathrm{v})^2\psi = 0 \qquad (4)$$

We know the total energy of the system E= Kinetic Energy + Potential Energy

$$E = \frac{1}{2}m\mathrm{v}^2 + V \quad (or)\ (m\mathrm{v})^2 = 2m(E - V) \qquad (5)$$

Substituting (5) in (4)

$$\nabla^2\psi + \frac{8m\pi^2}{h^2}(E - V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad //\ \hbar = \frac{h}{2\pi}$$

The above two equations are called as Schrödinger time independent wave equations

Schrodinger time dependent wave equation:

The Schrodinger time dependent wave equation can be obtained by eliminating "*E*" from Schrödinger time independent wave equation.

Consider ψ to be complex function of space and time coordinates.

$$Let\ \psi = \psi_0 e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\omega e^{-i\omega t} = -i\omega\psi$$

$$But\ \omega = 2\pi v = \frac{2\pi E}{h} = \frac{E}{\hbar}$$

$$\therefore\ \frac{\partial \psi}{\partial t} = \frac{-iE\psi}{\hbar} = \frac{E\psi}{i\hbar}$$

$$(or) E\psi = i\hbar\,\frac{\partial \psi}{\partial t}$$

Substituting in Schrodinger time independent wave equation, we can write

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\,\frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$i\hbar\,\frac{\partial \psi}{\partial t} - V\psi = -\frac{\hbar^2}{2m}\nabla^2\psi$$

$$i\hbar\,\frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi\ (or) \hat{E}\psi = \hat{H}\psi$$

$$\hat{E} = i\hbar\,\frac{\partial}{\partial t} \rightarrow Energy\ operator$$

$$\hat{H} = \frac{\hbar^2}{2m}\nabla^2 + V \rightarrow Hamiltonian\ operator$$

Physical Significance of wave function

According to Max Born, the product of the wave function (ψ) with its complex conjugative (ψ*) gives the probability density i.e. the probability of finding particle. he probability of finding particle in a volume dxdydz

$$\iiint |\psi|^2 dxdydz = 1$$

The above condition is known as normalization condition. Ψ satisfying the above equation is called as normalized wave function

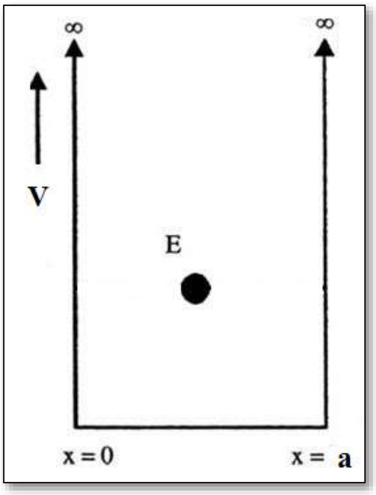
Limitations of Wave function

1. It must be finite for all values of x,y,z.
2. It must be single valued i.e. for each set of x,y,z, ψ must have only one value.
3. It must be continuous in all regions except where the potential energy is infinite.
4. Ψ is analytic i.e. it possess continuous first order derivatives.
5. Ψ vanishes at boundaries.

4. Discuss the energies of a particle in one dimensional potential well. Derive the equation for wavefunction

Consider a particle of mass *m* is bouncing back and forth between the walls of a one dimensional box of length *a*. Let the motion of particle is restricted only along the *x* axis i.e. in between *x=0* and *x=a*. Let the potential energy of the walls of the box is infinity and potential energy inside the box is uniform. For simplicity, this uniform potential energy can be considered as zero.

Hence we can write the potential function as



$V(x) = 0\ for\ 0 < x < a$, inside the box

$V(x) = \propto\ \ for\ x < 0 \&\ x > a$, outside the potential well.

Hence the Schrodinger wave equation inside the potential well can be written us

$$\frac{d^2\psi}{dx^2} + \left(\frac{2m}{\hbar^2}\right) E\psi = 0$$

$$(or)\ \frac{d^2\psi}{dx^2} + k^2\psi = 0$$

$$where\ k^2 = \frac{2mE}{\hbar^2} \qquad 1$$

The above equation is a second order differential equation. Let the trail solution is

$$\psi(x) = A \sin kx + B \cos kx$$

where *A* and *B* are arbitrary constants. The values of A & B can be obtained by applying the boundary conditions.

I Boundary condition:

As the particle cannot penetrate the potential well, *ψ(x) = 0 at x =0*

Hence, we can write 0= $A \sin k(0) + B \cos k(0)$ (or) *B=0*

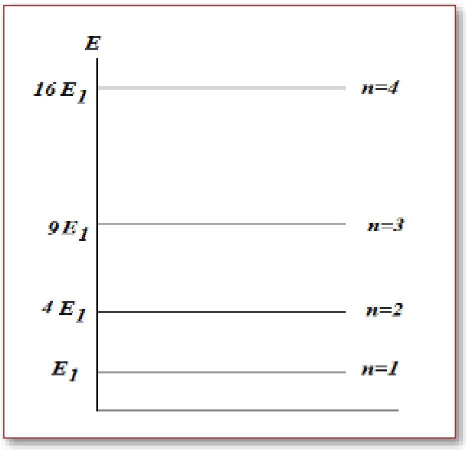
Hence the trial solution reduces to $\psi(x) = A \sin kx$

II Boundary Condition:

Similarly we can write *ψ(x) = 0 at x =a*

$$(or)\ 0 = \sin ka \qquad (or) k = \pm\frac{n\pi}{a} \qquad (2)$$

$$\therefore \psi(x) = A \sin\left(\frac{n\pi}{a}\right) x$$



From equations (1) and (2) , we can write

$$k = \left[\frac{2mE}{\hbar^2}\right]^{\frac{1}{2}} = \pm\frac{n\pi}{a}$$

$$(or) E = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2h^2}{8ma^2}$$

$$for\ n = 1, \qquad E_1 = \frac{\pi^2\hbar^2}{2ma^2} - Zero\ point\ energy$$

$$for\ n = 2, \qquad E_2 = 4 \times \frac{\pi^2\hbar^2}{2ma^2} = 4E_1$$

Hence in general form we can write the equation as $E_n = n^2 E_1$

This shows that the energy of the particle in one dimensional box is quantized.

Here the integers corresponding to *n=1,2,3* are called quantum numbers for corresponding E_n.

The wave function corresponding to is called as eigen function of the particles.

To calculate the value of A

To calculate the value of A, let us apply the normalization condition.

The normalization condition in one dimension can be written as $\int_0^a |\psi|^2\, dx = 1$

$$= \int_0^a A^2 \sin^2\left(\frac{n\pi}{a}\right) x\, dx = 1$$

$$= \frac{A^2}{2}\int_0^a \left[1 - \cos\left(\frac{2n\pi}{a}\right)x\right] dx = 1 \ \ //\cos 2\theta = 1 - 2\ \sin^2\theta$$

$$= \frac{A^2}{2}\left[x - \frac{a}{2\pi n}\ \sin\left(\frac{2\pi n}{a}\right)x\right]_0^a = 1 = \frac{A^2 a}{2}$$

$$(\text{or}) A = \sqrt{\frac{2}{a}}$$

Therefore the wave equation in one dimensional potential box will become

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}\right) x$$