Week 4 Summary: Preferences and utility. Expected utility maximization.

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Contents

1 Terminology

Term	Definition/Explanation	Notes
Terminal node	Decision tree, the leafs where payoffs are attached	
Payoff function	Function that maintains transitivity of pre-existing preference relations	

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Term	Definition/Explanation	Notes
Degenerate lottery	A deterministic situation (probably pointlessly) modelled as a lottery	Probabilities of states arising based off chosen actions would be 1 or 0
Intensity	Essentially, magnitude	E.g. intensity of preference is not invariant to positive monotonic transforms (it's cardinal, not ordinal)
Risk neutral	Indifferent between a lottery and sure situation given that the expected payout is the same	
Risk averse	Strictly prefers a sure situation to a lottery, given that the expected payout is the same	
Risk loving	Strictly prefers a lottery to a sure situation, given that the expected payout is the same	Not as ridiculous at it seems *prima facie*. In situations where you're wealthy/have nothing substantial to lose, risk is fun.
Risk neutral	Indifferent between a lottery and sure situation given that the expected payout is the same	
Degenerate lottery	A deterministic situation (probably pointlessly) modelled as a lottery	Probabilities of states arising based off chosen actions would be 1 or 0

2 Tadelis Ch1

2.1 Completeness Axiom in Preference Relations

Complete iff any two outcomes can be ranked by preference relation

2.2 Rational Choice Paradigm

2.2.1 Assumptions

Full understanding of the following:

1. All possible acts
2. All possible outcomes
3. Mappings for every state/outcome pair
4. His rational preferences (payoffs) over outcomes

2.2.2 Meaning

Rational if payoff-maximizing.

Manner of maximization is unclear in the text (is it expectation maximization?) but various methods were covered in `Week 2`.

2.2.3 `Homo Economicus`

Means "economic man". Essentially, the perfect rational agent.

2.2.3.1 Fun Facts

- Wikipedia describes this as a "caricature" and "mythical species", although in a way that seems to be a tool, rather than pejoratively
- Oppositionally, see homo reciprocans ("reciprocating man").
  - "cooperative actors who are motivated by improving their environment."
- Sometimes used by non-economists to critique economic approaches (distilled down to the "people aren't rational" argument)

3 Tadelis Ch2.1-2.3 + Peterson CH4.1 - 4.3, 4.6

3.1 Simple and Compound Lotteries

3.1.1 Meaning of Simple Lottery

- Stochastic state distribution
- Probability of states are contingent upon performed acts, which suggest that acts are not necessarily seen as independent of states. This is different from what we learnt in class (assumption of causal independence)

3.1.2 Compound Lotteries

- A simple lottery after a simple lottery (multi-level decision tree)
- Can be transformed into a simple lottery (take each path as a single decision, then consider terminal nodes only)

3.2 Backward Induction

It's easiest to work backwards then dealing with multi-stage decisions trees.

3.2.1 Relation to Dynamic Programming

It's one strategy for it, but certainly not the only one (e.g. you could just as easily work forwards in shortest path problems)

3.3 Maximizing Expectation

3.3.1 Units

Unit	Explanation/Notes
Monetary value	$$$
Value	Monetary value, adjusted for diminishing marginal 'value' of money to the agent
Utility	Yes. Not well defined, but POV is important. Allows for non-monetary evaluation

Utility, value, monetary value, payoff

3.3.2 Keyne's Objection

"In the long run we are all dead".

Essentially, a criticism of relying on LLN.

3.4 Axiomatic Approach

Arrives at the principle of maximizing utility without using LLN (thereby sidestepping Keynes' problem).

3.4.1 Axioms

1. If all outcomes of an act have utility u, then the utility of the act is u
2. For acts, utility relations follow dominance relations (e.g. strict dominance implies higher utility)
3. Every decision probable can be transformed into a rival formalization with equiprobable (possibly repeated) states, which maintain the utilities of all acts
4. Setup: 1) two equiprobable outcomes 2) The better outcome is made slightly worse Claim) Overall utility of act can be preserved by adding some amount of utility to the other outcome

These 4 axioms lead to the following conclusion: the utility of an act = its expected utility

3.5 Misc

3.5.1 `de minimis` Principle as a Solution to the St Petersburg Paradox

Roughly put, sufficiently improbable outcomes should be ignored

3.5.2 Two-Envelope Paradox (infinite swapping reasoning)

- No established solution exists.
- This is **not** the same as the Monty Hall problem

3.5.2.1 Set Up

- Two seemingly identical envelopes (A and B)
- 1 envelope contains twice as much as the other
- You don't know which is which, can only choose 1

3.5.2.2 Choice

- Make a decision
- Given chance to swap decision

3.5.2.3 The paradox

- $\mathbb{E}[swap] = \frac{1}{2} \cdot 2x + \frac{1}{2} \cdot \frac{x}{2} = \frac{5}{4}x$, so logically, swap
- Given choice again just before picking
- $\mathbb{E}[swap] = \frac{1}{2} \cdot 2y + \frac{1}{2} \cdot \frac{y}{2} = \frac{5}{4}y$, so logically, swap
- Given choice again, just before picking
- Repeat ad infinitum

3.5.2.4 Comments The problem is not to find an alternate formulation of the given situation. The problem is to find the issue with this current formulation, which gives nonsensical results.