Week 1 Summary: Introduction. Decision Trees.

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Contents

1 Effective Decision Rules

1.1 Maximin and Maximax

They're the same, just at different extreme ends.

1.1.1 Maximin

Informal Principle of choosing the act with the largest minimal outcome obtainable

Formal $a_i \succeq a_j \iff \min(a_i) \geq \min(a_j)$

1.1.2 Maximax

Informal: Principle of choosing the act with the largest maximal outcome obtainable

Formal: $a_i \succeq a_j \iff \max(a_i) \geq \max(a_j)$

1.2 Further Constraints: Leximin and Leximin Modifications

Below is for leximin. Leximax is just the opposite

1.2.1 Explanation

Description: Essentially a way to filter the dominance space from `maximin` (also an `effective decision rule`)

Procedure: Iteratively compare the next minimal outcomes under the states until you find a difference. Remove the act(s) with a lower outcome

Equivalence: The only remaining equivalent acts are acts which are equivalent in every state

1.2.2 Formal Definition

$a_i \succ a_j \iff$ there exists some positive integer n such that $\min^n(a_i) > \min^n(a_j)$ and $\min^m(a_i) = min^m(a_j)$ for all $m < n$

1.3 Combination: Optimism-Pessimism Rule (a.k.a alpha-index rule)

Informal: Weighted combination of maximin and maximax. Weight parameter α reflects optimism

Formal: $a_i \succeq a_j \iff \alpha \cdot \max(a_i) + (1-\alpha) \cdot min(a_i) \geq \alpha \cdot \max(a_j) + (1-\alpha) \cdot \min(a_j)$

1.3.1 Random Thought

You can probably combine with leximin/leximax to some degree too, to get an even better framework

1.4 Problem: Relevance of non-extreme values [all]

Particularly easy to see when the mins are close. E.g. below.

Applying the maximin principle selects option a_2, but intuitively a_1 seems far better.

Note: Example formulated for `maximin/leximin`. Just flip the logic for `maximax`

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
a_1	1	0.99	99999	99999	99999	99999	99999
a_2	1	1	1	1	1	1	1

Pedantic Note: you don't know the probability distribution of the state space. Maybe a_2 is better after all…

1.5 Problem: Unintuitive equivalence [Minimax, Maximax]

Note: Example formulated for `maximin`. Just flip the logic for `maximax`

	s_1	s_2
a_1	1	99999
a_2	1	1

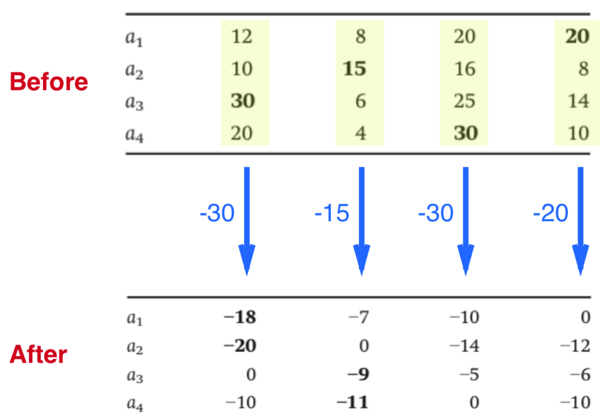
Under vanilla `maximin`, both acts are equally reasonable. Obviously, this is weird

1.6 Minimax Regret

1.6.1 Explanation

- Essentially an attempt to formalize the concept of `regret`

1.6.2 Procedure (won't formally describe)



1.6.3 Note

It's not globally accepted that this concept is relevant to rational decision making. But a substantial number of theorists think it is.

1.6.4 Problem: Argument from irrelevant alternatives

- Ranking can be altered by adding a non-optimal alternative
- Breaks intuition about how a "normatively plausible decision rule must not be sensitive tothe addition of irrelevant alternatives"

1.6.4.1 Example: Addition of a_5

Table 3.14

a_1	12	8	20	20
a_2	10	**15**	16	8
a_3	**30**	6	25	14
a_4	20	4	**30**	10
a_5	−10	10	10	**39**

Table 3.15

a_1	−18	−7	−10	**−19**
a_2	−20	0	−14	**−31**
a_2	0	−9	−5	**−25**
a_3	−10	−11	0	**−29**
a_5	**−40**	−5	−20	0

1.6.4.2 Counter

- Prima facie intuition is wrong. It's "rational to compare alternatives with the entire set of alternatives".

2 Transformative Decision Rules

2.1 Principle of Insufficient Reason

Pro: Decision under ignorance → Decision under risk

2.1.1 Problem: Which states should be considered? Modeling problem

- More states means lower probability for each state (direct influence on choice strategy)
- Choosing relevant states is often not easy
- Traditional argument for `ir` is from symmetric states (e.g. dice sides). Many problems have no such symmetry

2.1.2 Problem: Uniform probability assumption seems arbitrary

Under ignorance, any probability distribution *seems to be* equally justifiable as any other. Assumption of equality seems arbitrary

2.1.2.1 Counter: Symmetry

- Assume every probability distribution is equally justifiable
- Use lens of toy 2-state case $S = \{s_1, s_2\}$
- Every probability distribution has a symmetric partner (e.g. $\{p_{s_1} = 0.6, p_{s_2} = 0.4\}$ has $\{p_{s_1} = 0.4, p_{s_2} = 0.6\}$

  - Exception: Uniform distribution. Suggests uniform distribution is a collapsed state of (in this case) 2 identical probability distributions. Making it multiplicatively more reasonable as any other case (in this case, 2x more reasonable)

2.1.2.1.1 Problem

- Beautiful argument, but the assumption of the uniform distribution being an additively collapsed one is a bit dubious imo

2.1.3 Problem: Practically, it's often not complete ignorance

You generally know some things or at least have a sense of ordinal ranking for the probabilities of some of the states. Why not use it?

2.2 Randomized Acts

2.2.1 Procedure

- Create a new act with expected values as outcomes
- If your decision making strategy selects random act, then randomly choose one of those initial acts
- Note: Choosing the random act is not a choice on its own, but a procedure to arrive at an actual choice

2.2.1.1 Example Introduce random act a_3

Table 3.18

a_1	1	0
a_2	0	1

Table 3.19

a_1	1	0
a_2	0	1
a_3	1/2	1/2

2.2.2 Potential Problem

I'm assuming the random choice doesn't have to be uniformly distributed. But this opens up a whole can of worms by allowing you to tweak the probability distribution of the random function to bias it towards whatever choice you irrationally want.

3 Axiomatic Analysis of the Decision Rules

Taken directly and shamelessly from the textbook

3.0.1 Descriptions of Axioms

1. **Ordering**: $\succeq$ is transitive and complete. (See Chapter 5.)
2. **Symmetry**: The ordering imposed by $\succeq$ is independent of the labelling of acts and states, so any two rows or columns in the decision matrix could be swapped.
3. **Strict Dominance**: If the outcome of one act is strictly better than the outcome of another under every state, then the former act is ranked above the latter.
4. **Continuity**: If one act weakly dominates another in a sequence of decision problems under ignorance, then this holds true also in the limit decision problem under ignorance.
5. **Interval scale**: The ordering imposed by $\succeq$ remains unaffected by a positive linear transformation of the values assigned to outcomes.
6. **Irrelevant alternatives**: The ordering between old alternatives does not change if new alternatives are added to the decision problem.
7. **Column linearity**: The ordering imposed by $\succeq$ does not change if a constant is added to a column.
8. **Column duplication**: The ordering imposed by $\succeq$ does not change if an identical state (column) is added.
9. **Randomisation**: If two acts are equally valuable, then every randomisation between the two acts is also equally valuable.
10. **Special row adjunction**: Adding a weakly dominated act does not change the ordering of old acts.

3.0.2 Axiomatic Analysis

	Maximin	Optimism–pessimism	Minimax regret	Insufficient reason
1. Ordering	⊗	⊗	⊗	⊗
2. Symmetry	⊗	⊗	⊗	⊗
3. Strict dominance	⊗	⊗	⊗	⊗
4. Continuity	⊗	⊗	⊗	⊗
5. Interval scale	×	⊗	×	×
6. Irrelevant alternatives	⊗	⊗	–	⊗
7. Column linearity	–	–	⊗	⊗
8. Column duplication	⊗	⊗	⊗	–
9. Randomisation	⊗	–	⊗	×
10. Special row adjunction	×	×	⊗	×

Symbol	Meaning
-	Incompatible with decision rule
×	Compatible with decision rule
⊗	Necessary and sufficient for decision rule