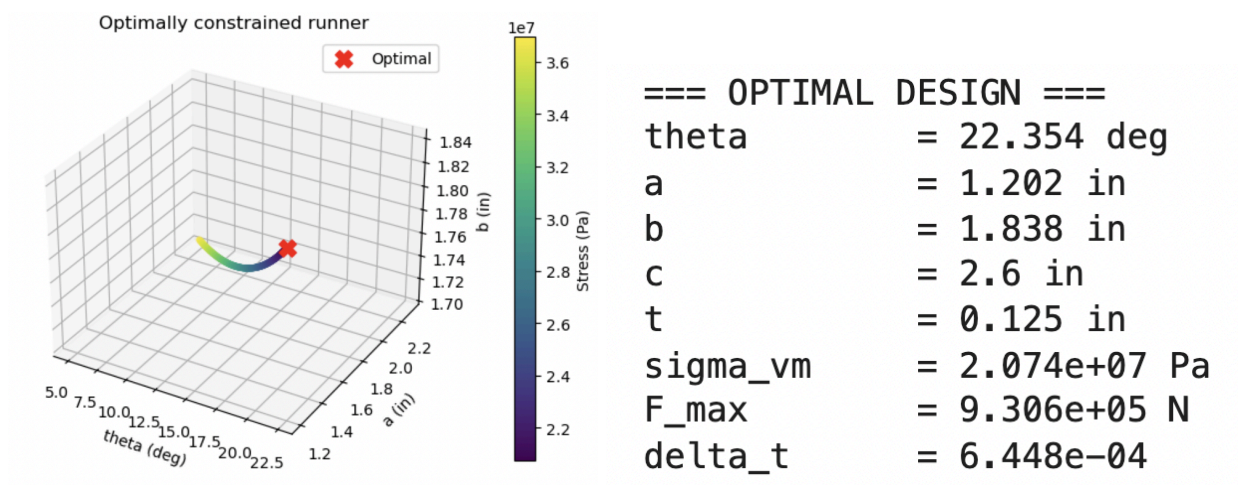
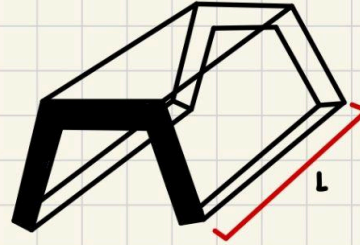
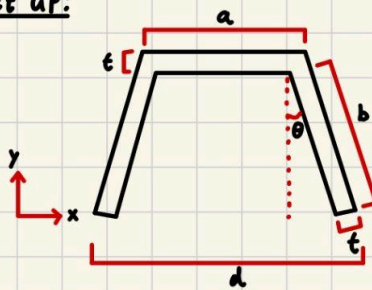


Runner Design

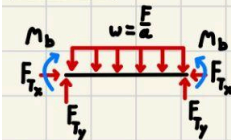


Optimization Code

Set up:



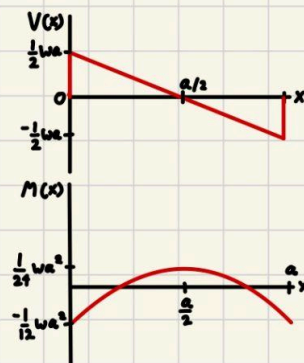
Top Piece:



Equilibrium:

$$\begin{aligned}\sum F_y &= 2F_y - wa = 0 \\ \rightarrow F_y &= \frac{1}{2}wa \\ \frac{F_{tx}}{F_y} &= \tan\theta \\ \rightarrow F_{tx} &= \frac{wa}{2}\tan\theta\end{aligned}$$

Diagrams:



Bending stress:

$$\begin{aligned}\sigma_b &= -\frac{M_{max} y}{I} \\ M_{max} &= \frac{1}{12}wa^2 \\ y &= \frac{t}{2} \\ I &= \frac{1}{12}Lt^3 \\ \sigma_b &= \frac{-\frac{1}{12}wa^2 \cdot \frac{t}{2}}{\frac{1}{12}Lt^3} \\ &= -\frac{wa^2}{4Lt^2}\end{aligned}$$

Normal stress:

$$\begin{aligned}\sigma_N &= \frac{F_{tx}}{tL} \\ &= -\frac{wa\tan\theta}{2tL}\end{aligned}$$

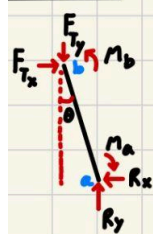
Stresses:

$$\begin{aligned}\sigma_x &= -\frac{wa^2}{4Lt^2} - \frac{wa\tan\theta}{2tL} \\ \sigma_y &\approx 0 \\ \tau_{xy} &\approx 0\end{aligned}$$

Von Mises stress:

$$\begin{aligned}\sigma_{vm} &= \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \\ &= |\sigma_x| \\ &= \frac{wa^2}{4Lt^2} + \frac{wa\tan\theta}{2tL} \\ &= F\left(\frac{a}{4Lt^2} + \frac{\tan\theta}{2tL}\right)\end{aligned}$$

Side Piece:



Equilibrium:

$$\begin{aligned}\sum F_x &= F_{tx} - R_x = 0 \\ \rightarrow F_{tx} &= R_x = \frac{wa}{2}\tan\theta \\ \sum F_y &= R_y - F_{ty} = 0 \\ \rightarrow F_{ty} &= R_y = \frac{1}{2}wa\end{aligned}$$

Normal stress:

$$\begin{aligned}F_N &= \sqrt{F_{tx}^2 + F_{ty}^2} \\ &= \frac{wa}{2}\sec\theta \\ \sigma_N &= \frac{-F_N}{tL} \\ &= -\frac{wa\sec\theta}{2tL} \\ &= -\frac{F\sec\theta}{2tL}\end{aligned}$$

Buckling:

$$\begin{aligned}P_{crit} &= \frac{n^2 EI}{kl^2} \\ I &= \frac{1}{12}Lt^3 \\ P_{crit} &= \frac{n^2 E \cdot \frac{1}{12}Lt^3}{\frac{1}{4}b^2} \\ &= \frac{3n^2 ELt^3}{b^2}\end{aligned}$$

Hand Calcs