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Day 4 of DSA.

* Tasks:-

Check
Box

Recursion.



Understanding Recursion :- Basic Recursive Function (Factorial, Fibonacci)



Recursive Backtracking : simple problems like generating subsets, permutation



Implement recursive algorithms : Factorial, Fibonacci



Solve problems involve recursive backtracking.

* Recursion:-

It is a technique in which a function calls itself repeatedly until a given condition is satisfied.

Syntax

```
return-type recursive-func {
```

```
    // Base Condition
```

```
    // Recursive Case
```

* Recursive function:-

A function that calls itself is called a recursive function.

- Base Condition:- For terminating the recursion

- Recursive case:- It contains multiple recursive calls

* eg.

```
int sum(int n)
{
    if (n == 0) {
        return 0;
    }
    int res = n + sum(n-1);
    return res;
}
```

Dry run

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Sum(3)

$$res = 3 + (3-1) \quad \leftarrow 3+3 = 6$$

Sum(2)

$$res = 2 + (2-1) \quad \leftarrow 2+1 = 3$$

Sum(1)

$$res = 1 + (1-1) \quad \leftarrow 1+0 = 1$$

Sum(0)

n == 0

return 0

————— X ————— X —————

Types of Recursion

Direct
Recursion

- Head Recursion.
- Tail Recursion.
- Tree Recursion

Indirect recursion

* Basic Example :-

(1) Fibonacci Series:-

```
int fib(int n){
```

```
if( n==0) return 0;
```

```
if( n==1 || n==2) return 1
```

```
else return (fib(n-1) + fib(n-2));
```

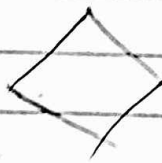
Fib. Series = 0 1 1 2 3 5 8 13
0 1 2 3

Fib(3)

Fib(3-1) + Fib(3-2)

Fib(2) + Fib(1)

return 1



fib

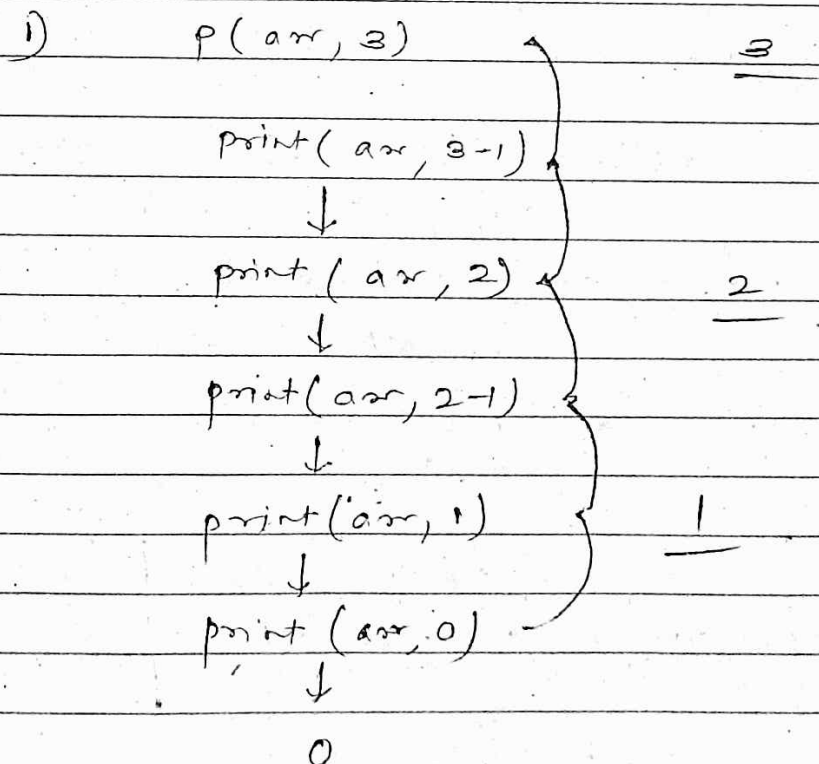
return 1 + 1 = 2

② Print array in reverse order using Recursion:-

```
void print (int * arr, int n)
{
    if (n == 0) return;

    print (arr, n-1);
    cout << arr[n-1] << " ";
}
```

arr[3] = {1, 2, 3}



① Program of printing N to 1 Numbers in reverse order

Approach:-

Base condition = if ($n \leq 0$) return;
else cout << n;

Recursive call = printReverse($n-1$);

Time complexity: $O(N)$

Space complexity: $O(N)$

② Print 1 to N Numbers.

Approach:-

Base condition: \rightarrow if ($n > 0$)

d

Recursive call \rightarrow print ($n-1$);

cout << n << " ";

y

Time complexity: $O(N)$

Space complexity: $O(N)$

* Tail Recursive Functⁿ :-

It is defined as a recursive function in which the recursive call is the last statement that is executed by the function.

- Nothing is left to execute after the recursion call

Eg.

```
void print (int n) {
    if (n <= 0) return;
    cout << " " << n;
    print (n-1);
}
```

Time Complexity : $O(N)$ Space Complexity : $O(N)$

Time Complexity : $O(\log n)$

Space Complexity : $O(\log n)$

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* Check if a number is palindrome.

1) We make 3 Functions

2) `int OneDigit(int num)`

{
 return (num >= 0 && num < 10);
}

Checks if number is present betⁿ 0 to 10

3) 'Recursive Funct'

`bool isPalUtil(int num, int *dupNum)`

{
 if (OneDigit(num))
 return (num == (*dupNum) % 10);
 // Comparing 1st & last digit.

if (!isPalUtil(num/10, dupNum))
 return false;

every time we decrease num/10 &
check 1st & last element of
num & dupNum respectively

* `dupNum/10 = 10;`

return (num % 10 == (*dupNum) % 10);

3) `isPal(int num)`

{
 if (num < 0) { num = -1; }
 int *dupNum = new int(num);
 return isPalUtil(num, dupNum);
}

Time Complexity = $O(\log N)$
Space complexity = $O(\log N)$

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*

Sum of the digits of a given number :-

Input = 666

Output = 18

Approach :-

Base Case : if (no == 0) return 0;

Recursive call = return (no/10) + sumdigits(no/10);

Dry Run.

666

① no = 666

$(666/10) + \text{sum}(666/10)$

6 + (66)

↓

6 + 12 = 18

18

② no = 66

$(66/10) + \text{sum}(66/10)$

6 + 6

↓

6 + 6 = 12

12

③ no = 6

$(6/10) + \text{sum}(6/10)$

6 + 0.6

6 + 0 = 6

6

④ no = 0

no == 0 return 0

* Printing all subsets from a set.

Approach:-

Subset (string str, int index = -1, string curr = "")

Base case = if (index == n) return;

— cout << curr << "\n";

Recursive call - For (int i = index + 1; i < n; i++)

{

curr += str[i];

subset (str, i, curr);

curr.erase (curr.size() - 1);

}

return;

Time Complexity : $O(2^n)$

Space Complexity : $O(n)$

* Tower of Hanoi :-

Approach :-

```
toh( int n, char from_rod, char to_rod,
      char aux_rod)
```

```
{
```

Base case - if ($n = 0$) return;

}

D

Recursive calls - toh($n-1$, from_rod, aux_rod, to_rod);

H

print statements -

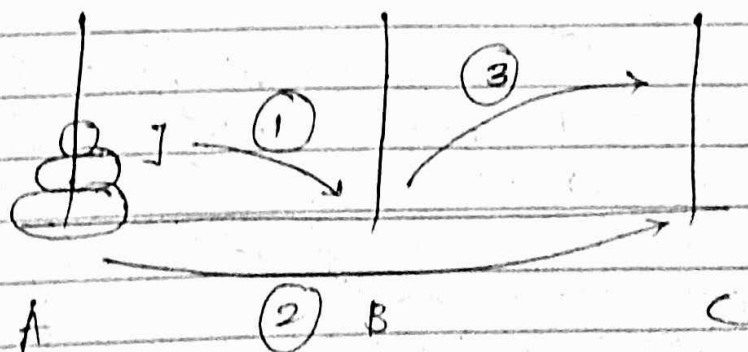
cout << "Move disk " << n << " from rod " <<
from_rod << " to rod " << to_rod
<< endl;

toh($n-1$, aux_rod, to_rod, from_rod);

```
}
```

Time Complexity : $O(2^n)$

Space Complexity : $O(N)$



* Josephus problem:-

Base case:- if $(n=1)$ return 1;

Recursive call: return

$$j(n-1, k) + k - 1) \% n + 1;$$

$$n = 5, k = 2$$

$$(1) \quad (j(4, 2) + 1) \% 5 \quad (4+1) \% 5 = 5$$

$$\downarrow n=4, k=2$$

$$j(3, 2) + 1) \% 4; \quad (3+1) \% 4 = 4$$

$$\downarrow n=3, k=2$$

$$j(2, 2) + 1) \% 3; \quad (2+1) \% 3 = 3$$

$$\downarrow n=2, k=2$$

$$j(1, 2) + 1) \% 2; \quad (1+1) \% 2 = 2$$

$$\downarrow n=1$$

1

Time Complexity: $O(N)$

Space Complexity: $O(N)$

1 2 3 4 5