Boson Sampling
for Harry

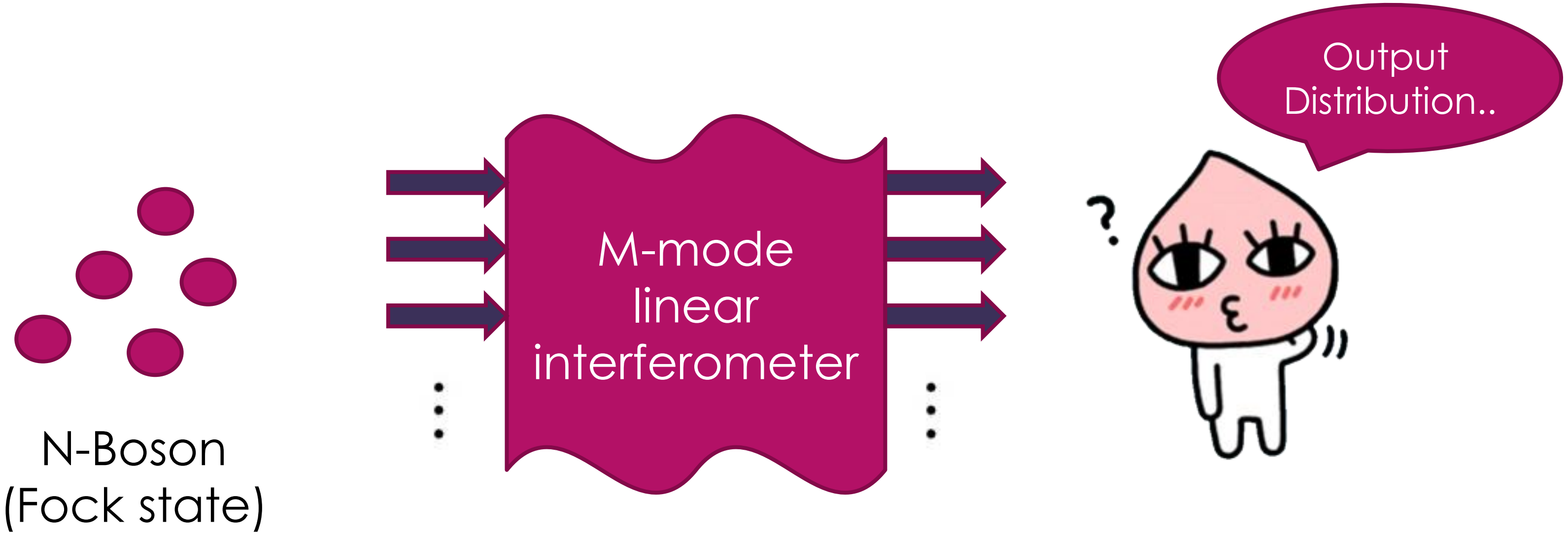
2019. 1. 27

YOONJEONG SHIN

INTERNSHIP AT CENTER FOR QUANTUM INFORMATION, KIST

MOTIVATED BY PROF.HUH'S TALK

Boson Sampling Problem



- Suggested by Aaronson & Arkhipov (2013)
- Exponentially hard to solve classically
- Qunatum computer

Notorious Permanent

- Initial Fock state : $|G\rangle = |g_1 g_2 \ldots g_m\rangle = a_1^{\dagger g_1} a_2^{\dagger g_2} \ldots a_m^{\dagger g_m} |\odot\rangle$
- Probability amplitude of |G>→|H> : $\langle H|U_F|G\rangle = \dfrac{\mathrm{per}(U_{G,H})}{\sqrt{g_1!..g_m!h_1!..h_m!}}$

A. Crespi *et al.*, Nature Photonics, 2013

Notorious Permanent

- The same as determinant except all positive terms

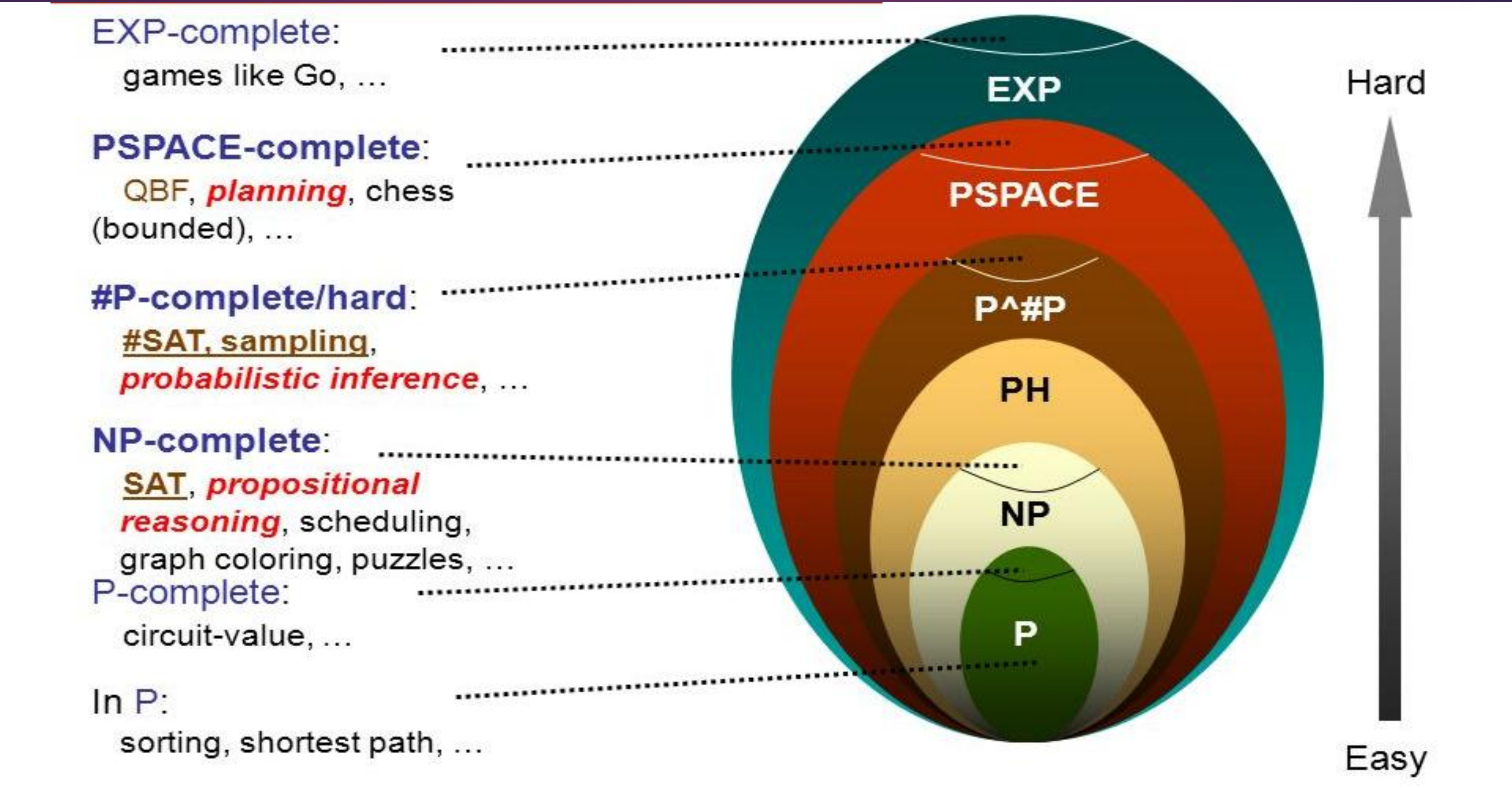
$$\operatorname{perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}. \quad \operatorname{perm}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad + bc,$$

- Express the wave function of identical bosons (symmetric under exchange)

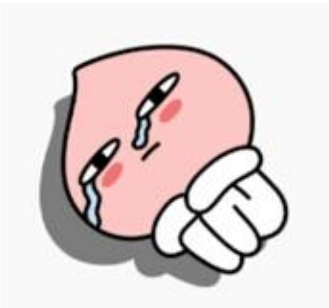
  cf. Slater determinant for identical fermions

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix} \equiv \langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N | \chi_1, \chi_2, \cdots, \chi_N \rangle,$$

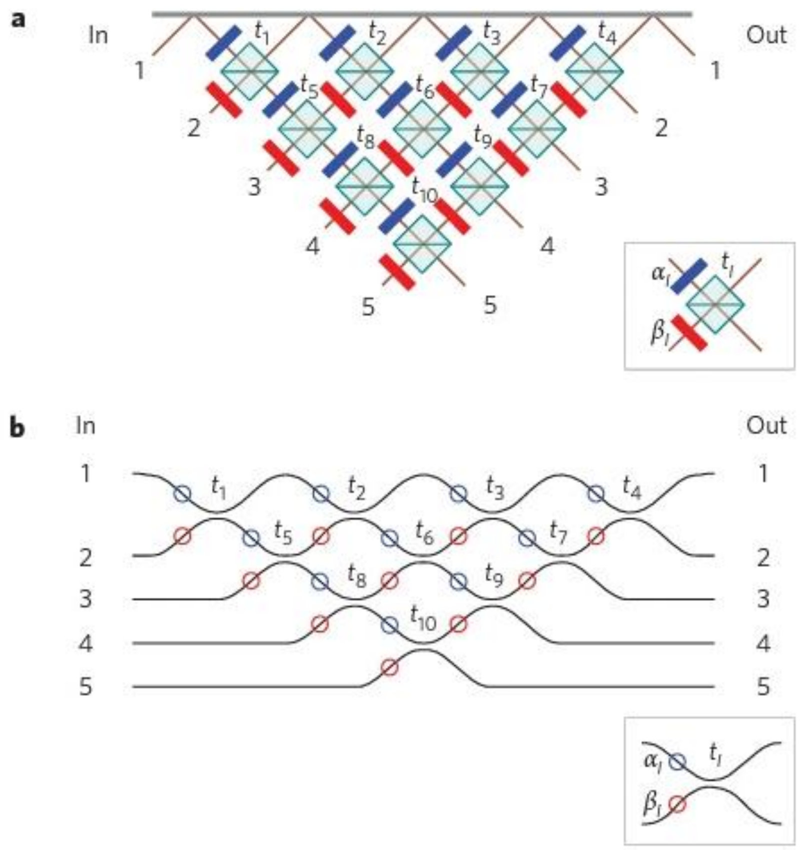
Notorious Permanent



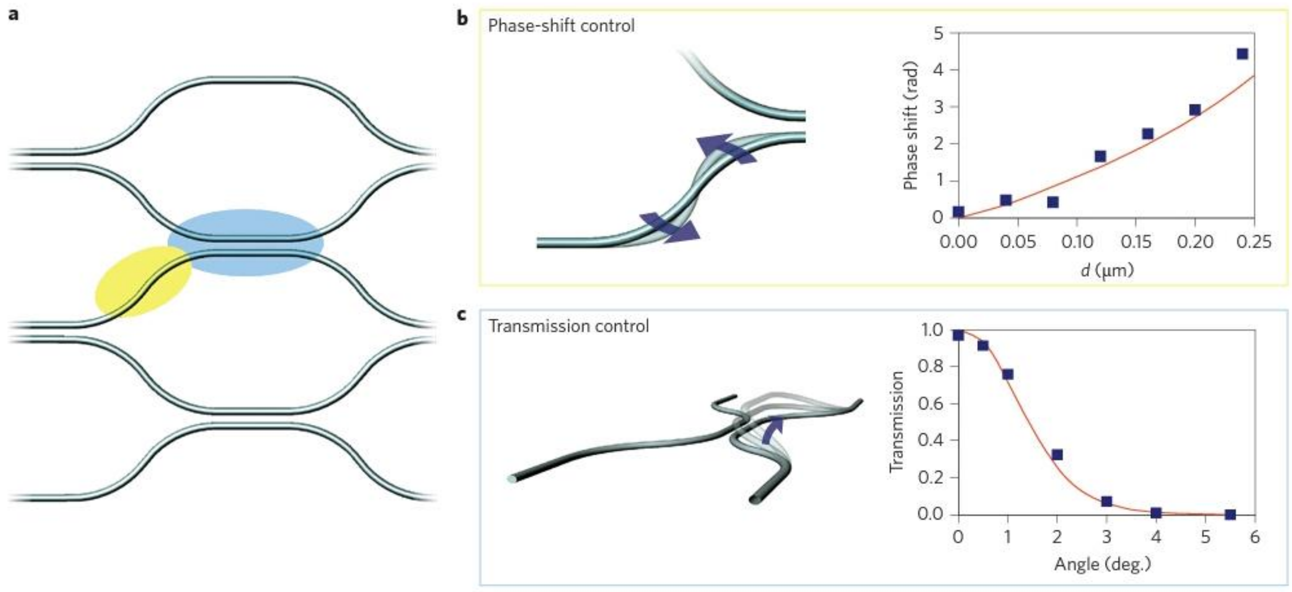
- Complexity class : #P
- exponentially with the size of marix



On Photonic Chips

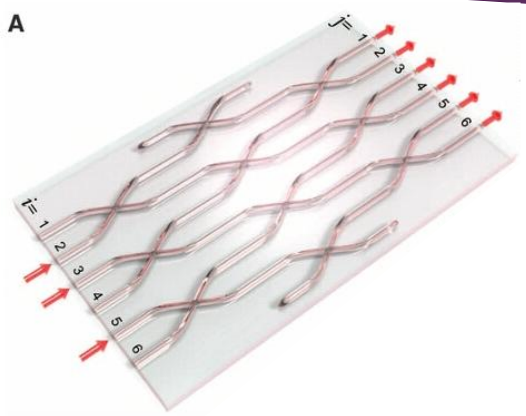


- Fock states are well developed
- Scalable
- controllable

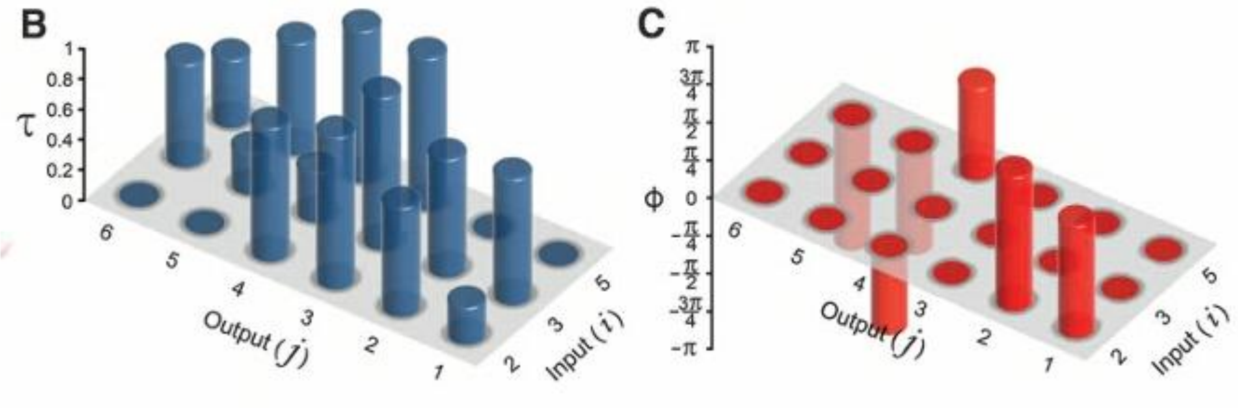


A. Crespi *et al.*, Nature Photonics, 2013

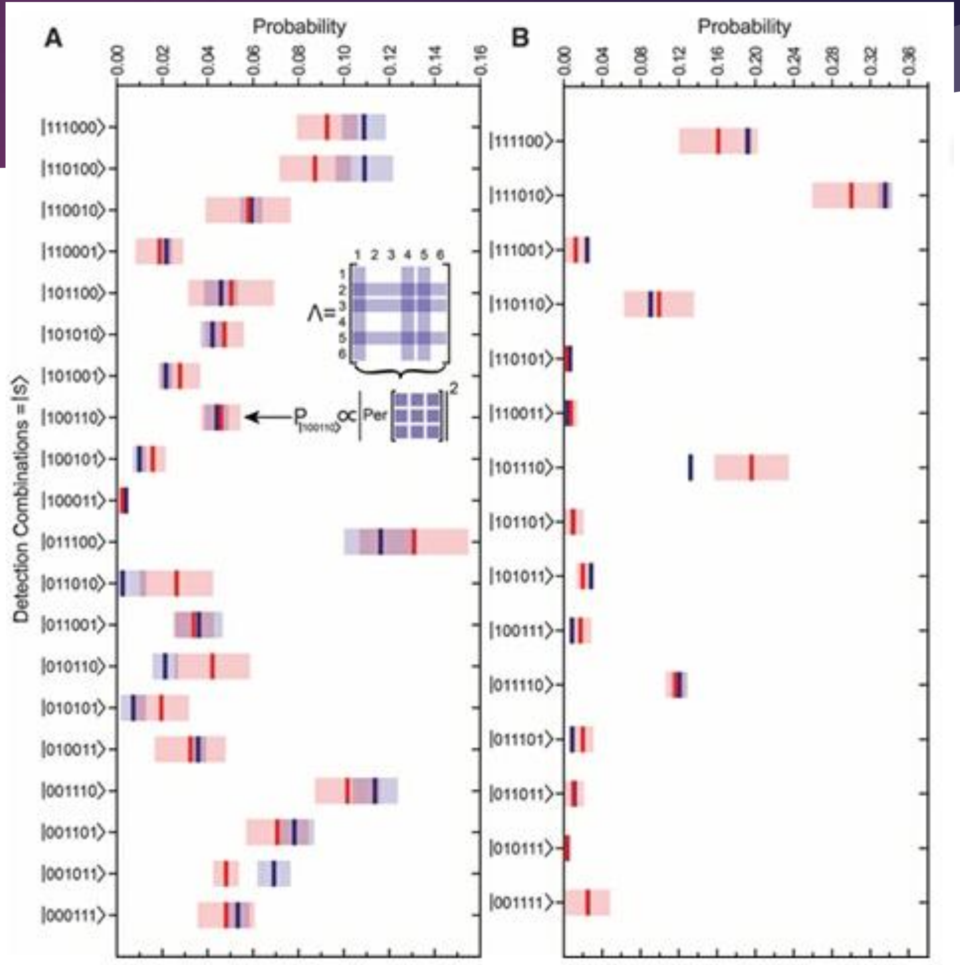
On Photonic Chips



photonic circuit
with 6-mode



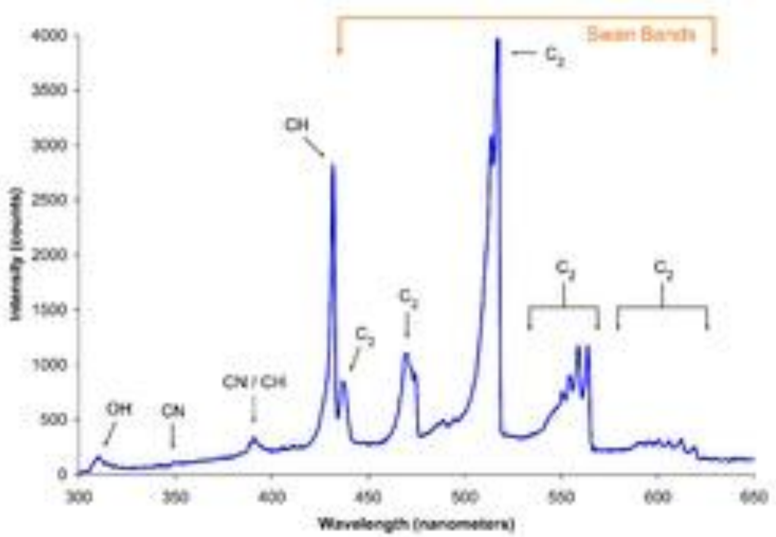
Element of the linear transformantion

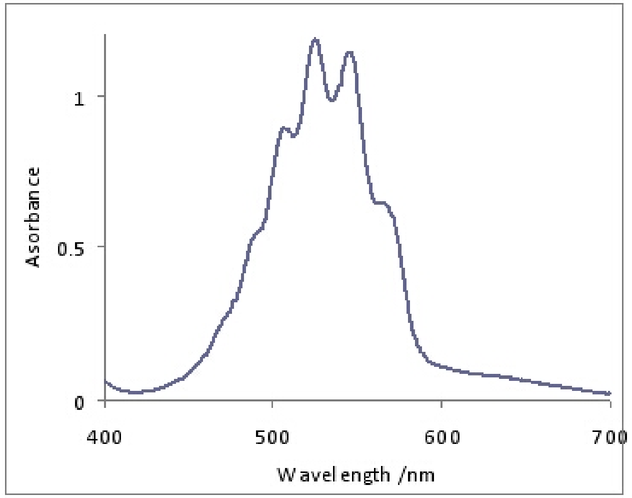


Boson sampling results
(a) N=3, (b) N=4

Boson Sampling for Molecular Vibronic Spectra

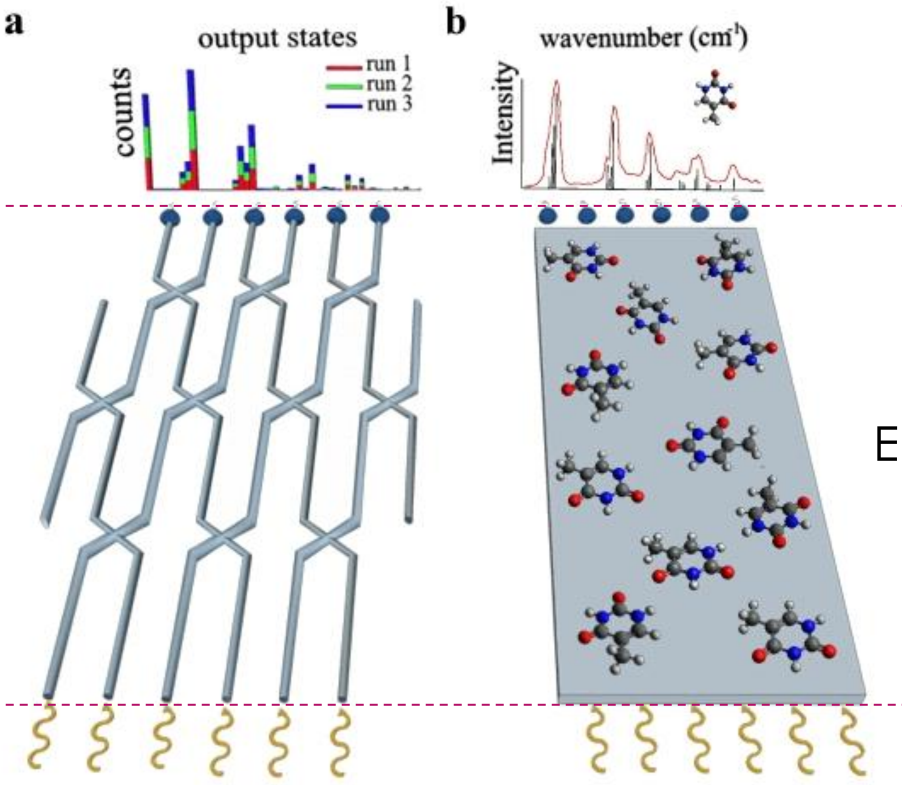
- Vibronic spectroscopy
  - For figuring out molecular properties
  - Absoption, emission, photoelectron, Raman resonace





- Obtain vibronic spectrum using boson sampling simulation (instead of real molecule)

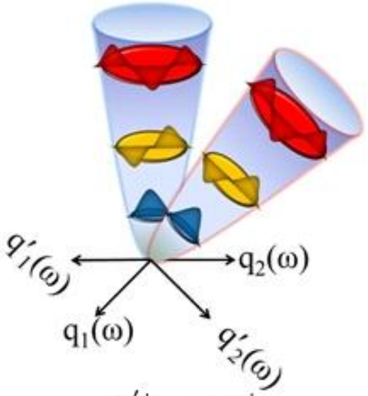
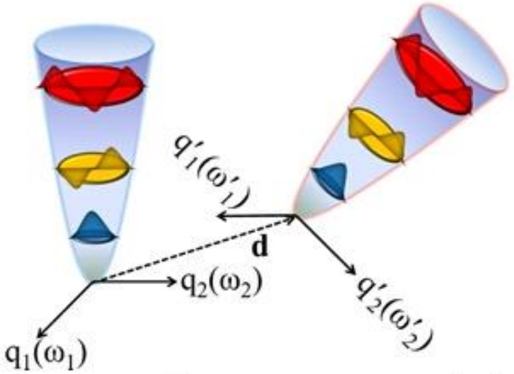
Boson Sampling for Molecular Vibronic Spectra



Output distribution	Vibtational spectrum
Linear interferometer	Ensemble of identical molecules
Photons	Cohrerent lights

J. Huh *et al.*, Nature Photonics, 2015

Boson Sampling for Molecular Vibronic Spectra

	Boson Sampling	Vibronic Transitions
Harmonic oscillators		
Linear transform	$\hat{\mathbf{a}}'^{\dagger} = \mathbf{U}\hat{\mathbf{a}}^{\dagger}$	$\hat{\mathbf{a}}'^{\dagger} = \frac{1}{2}\left(\mathbf{J} - (\mathbf{J}^{t})^{-1}\right)\hat{\mathbf{a}} + \frac{1}{2}\left(\mathbf{J} + (\mathbf{J}^{t})^{-1}\right)\hat{\mathbf{a}}^{\dagger} + \frac{1}{\sqrt{2}}\boldsymbol{\delta}$
Unitary operators	Rotation	Displacement, Squeezing and Rotation
Particle to simulate	Photon	Phonon
Particle in simulator	Photon	Photon
Outcome of simulation	$\lvert\text{Permanent}\rvert^2$	Franck-Condon profile (spectrum)

N-photon
M-mode

=

N-phonon
M-vibrational mode

J. Huh *et al.*, Nature Photonics, 2015

Boson Sampling for Molecular Vibronic Spectra

- Duschinsky relation :
  for computing vibronic profiles $\mathbf{q}' = \mathbf{U}\mathbf{q} + \mathbf{d}\,,$
- In terms of ladder operator : $\hat{\mathbf{a}}'^{\dagger} = \frac{1}{2}\left(\mathbf{J} - (\mathbf{J}^{t})^{-1}\right)\hat{\mathbf{a}} + \frac{1}{2}\left(\mathbf{J} + (\mathbf{J}^{t})^{-1}\right)\hat{\mathbf{a}}^{\dagger} + \frac{1}{\sqrt{2}}\boldsymbol{\delta}\,,$

$$\Rightarrow \quad \hat{\mathbf{a}}'^{\dagger} = \hat{U}^{\dagger}_{\mathrm{Dok}}\hat{\mathbf{a}}^{\dagger}\hat{U}_{\mathrm{Dok}}, \quad \hat{U}_{\mathrm{Dok}} = \hat{D}_{\boldsymbol{\delta}/\sqrt{2}}\hat{S}^{\dagger}_{\boldsymbol{\Omega}'}\hat{R}_{\mathbf{U}}\hat{S}_{\boldsymbol{\Omega}}\,.$$

Part of states preparation

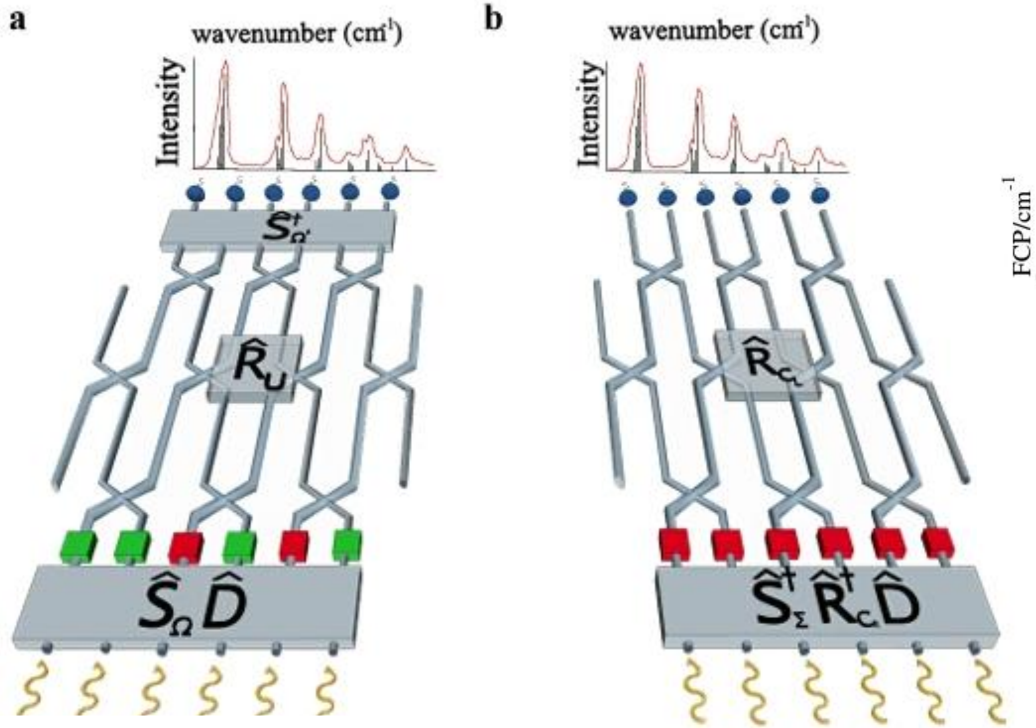
$$\Rightarrow \quad \hat{U}_{\mathrm{Dok}} = \hat{S}^{\dagger}_{\boldsymbol{\Omega}'}\hat{R}_{\mathbf{U}}\hat{S}_{\boldsymbol{\Omega}}\hat{D}_{\mathbf{J}^{-1}\boldsymbol{\delta}/\sqrt{2}}\,. \quad \Rightarrow \quad \hat{U}_{\mathrm{Dok}} = \hat{R}_{\mathbf{C}_{\mathrm{L}}}\hat{S}^{\dagger}_{\boldsymbol{\Sigma}}\hat{R}^{\dagger}_{\mathbf{C}_{\mathrm{R}}}\hat{D}_{\frac{1}{\sqrt{2}}\mathbf{J}^{-1}\boldsymbol{\delta}}\,.$$

Hard to realize

Requires Inonlinear interation between limited photons

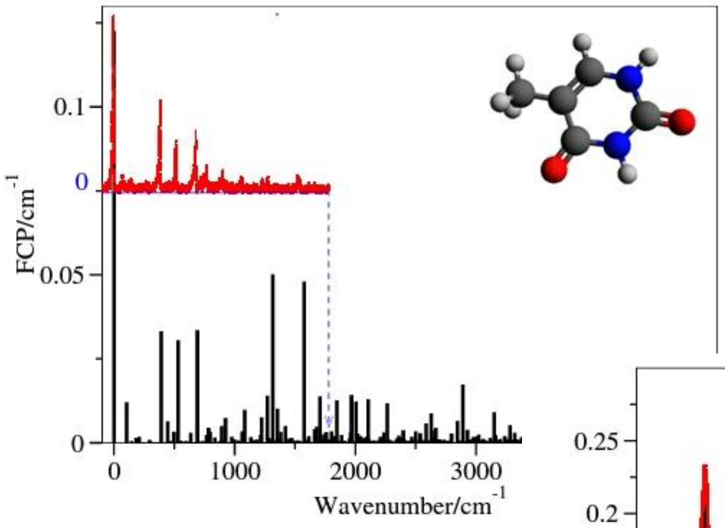
J. Huh *et al.*, Nature Photonics, 2015

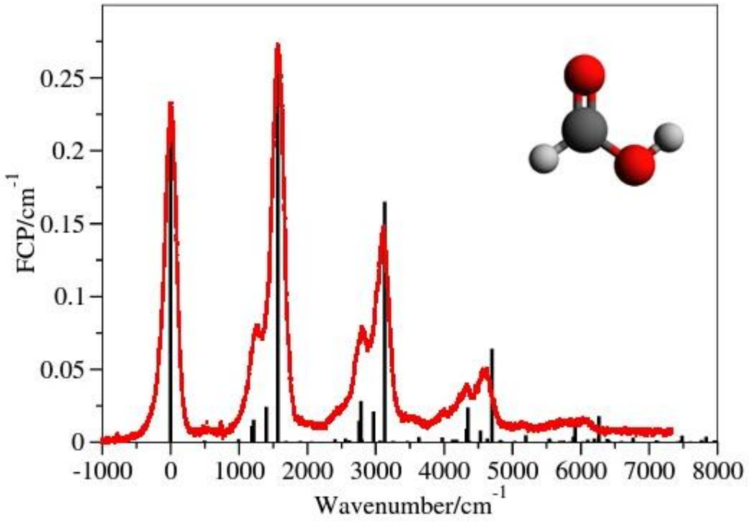
Boson Sampling for Molecular Vibronic Spectra



$$\hat{U}_{\mathrm{Dok}} = \hat{S}^{\dagger}_{\mathbf{\Omega}'} \hat{R}_{\mathbf{U}} \hat{S}_{\mathbf{\Omega}} \hat{D}_{\mathbf{J}^{-1}\boldsymbol{\delta}/\sqrt{2}} \, .$$

$$\hat{U}_{\mathrm{Dok}} = \hat{R}_{\mathbf{C}_{\mathrm{L}}} \hat{S}^{\dagger}_{\mathbf{\Sigma}} \hat{R}^{\dagger}_{\mathbf{C}_{\mathrm{R}}} \hat{D}_{\frac{1}{\sqrt{2}}\mathbf{J}^{-1}\boldsymbol{\delta}} \, .$$





J. Huh *et al.*, Nature Photonics, 2015

Non-universial quantum computer

- Universial :
- Non-universial :

Universial :	Qubit Specific state	Quantum correlation Across the system	measurement
Non-universial :	Bosons Indistingushable	Linear state evolution Low decoherence	measurement

- Problems with verification and application
- " a new window into the hidden computational power of quantum mechanical devices "

Bryan T *et al.*, From Atomic to Mesoscale, chap.8, 2014