Tristepper is NP-Complete

CSCI 361 Theory of Computation

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Abstract

This academic paper examines the computational complexity of the online game *Tristepper* through the lens of Grid Graph Hamiltonian Path framework. We provide a hallway-lobby gadget implementation to show that *Tristepper* can be reduced to a Grid Graph Hamiltonian Path problem, hence proving that it is NP-Complete.

I. Introduction

Tristepper is an online single-player game that is available on coolmathgames.com. It involves a three-eyed monster that can move left, right, up, or down. Every three moves, it opens one eye and shoots a ball of goo forward, in the direction of the last move. Each level has a threshold value (less than or equal to the maximum possible value) of goo balls needed to be thrown in order to win the level. The monster dies when there is no more space to shoot the goo on its third step.

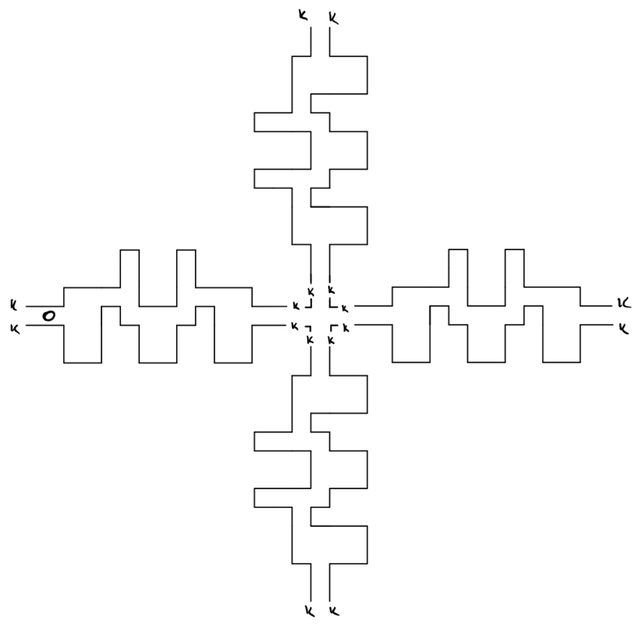
II. Source Problem

This section describes the undirected Grid Graph Hamiltonian Path problem (UHAMPATH) used as the source problem for our reduction. A Hamiltonian Path in an undirected graph G is a path that goes through each node exactly once. What makes it a grid graph is that every vertex lies on an integer space in a coordinate plane. The Hamiltonian Path problem is NP-complete[1]. In addition, it also has the property of *polynomial verifiability* - we can determine whether any path is a valid Hamiltonian Path in polynomial time[2].

III. Reduction and Proof

a. Reduction

To be able to use UHAMPATH as the source problem, our reduction converts a grid graph G into a viable Tristepper puzzle. For each node x in G, a single-width hallway of length k is joined at the node in all four directions. If node x has a neighboring node, a lobby gadget will be placed in between the neighboring nodes, joining the two hallways together. However, if there is a direction with no neighboring node to x, a simple cap will be produced to seal off the end of the hallway. The hallways and the lobby gadgets as a collective will be referred to as the gadget.



This gadget shown above simulates a four degree vertex in G[3]. It is an undirected gadget that allows for only one entrance and one exit. If the monster enters from the west, it could exit from the North, East, or South.

The gadget as a whole only allows one entry and one exit. This is because in a hallway of width one, in order to not block the tristepper's exit path, it must shoot at least k goo in the opposite direction which clogs up any possible entrance from the path opposite to its exit. For example, if coming from the West, the process of exiting North, blocks off entry from the South while clogging the northbound helper gadget, therefore prohibiting any possibility of visiting the gadget twice since three of four directions are blocked off. Another case is if the tristepper enters from the West leaves East. Still, the tristepper creates a trail of at least 2k goos behind it, successfully blocking off the center tile and prohibiting a second entrance between the North and South directions.[4]

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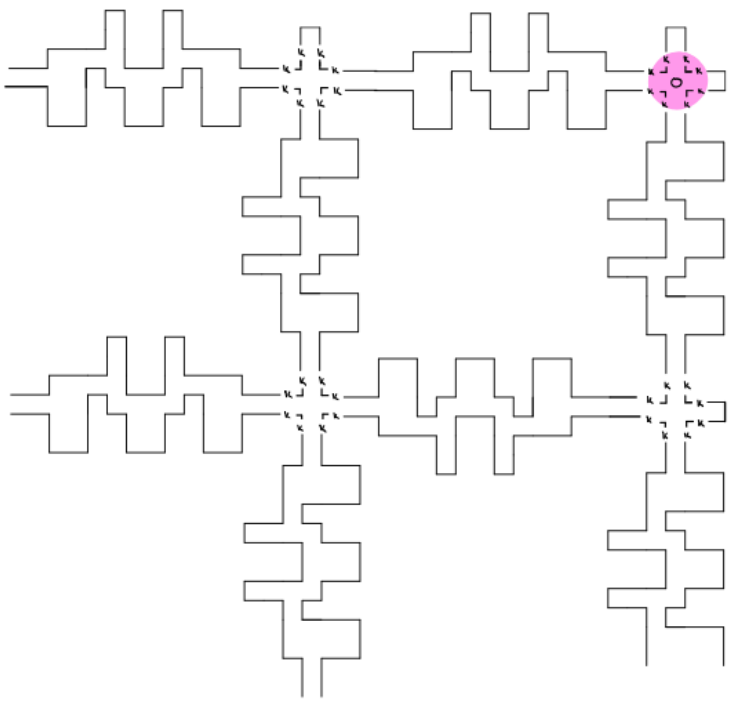
[1] Sipser, M. (2013) "7.5," in *Introduction to the Theory of Computation*. Australia: Course Technology Cengage Learning, pp. 314–319.

[2] Sipser, M. (2013) "7.5," in *Introduction to the Theory of Computation*. Australia: Course Technology Cengage Learning, pp. 292–293.

[3] Refer to Appendix A (Fig. 1a and 1b) for degree 3 and degree 2 vertices

[4] Refer to Appendix B: Gadget Breakdown



As seen above, each hallway (the single-cell-wide paths in the center) in the gadget is of length K. Each lobby (the transition sections in the outer part of the gadget) contains 34 cells. $K > 34n$, where n is the number of nodes in G. To complete the level, one must shoot at least $4Kn$ balls of goo which acts as the threshold value.

An example starting cell of the game is highlighted in pink. It must be located at the intersection of the hallway so it still has the opportunity to goo 4k spots. In order for the monster to maximize the number of spaces filled with goo while progressing forward, it must shoot the goo in the opposite direction of where it is headed. This is done most efficiently by taking two steps forward and one step back. Effectively, this would place the goo going forward one step at a time. To adjust spacing the tristepper can also take 3 steps backwards.

To prove that Tristepper inhabits the same complexity class as UHAMPATH, we must prove that for a given UHAMPATH problem, there exists a solution if and only if there exists a solution for the corresponding Tristepper level. To do this, we first prove that Yes instances of UHAMPATH produce solvable Tristepper levels.

b. Yes to Yes Proof

If there is a Hamiltonian path in G, there is a path that goes through each node exactly once. The corresponding Tristepper level only has a solution if there is a valid path that fills $4kn$ cells, where n is the number of edges in G. The Tristepper level for G has a degree 2, 3, or 4 gadget corresponding to a node in the original grid graph. As shown above, each of these gadgets allow only one entry and one exit, just like a node along a Hamiltonian path. To solve the level, the monster must enter each of the n gadgets in the Tristepper level and shoot $4K$ balls of goo, filling up all 4 hallways in each gadget. Since a Hamiltonian path exists in the original grid graph G, it is possible for the monster to enter all n gadgets. As shown above, it is possible for the monster to shoot $4K$ balls of goo once it is within a gadget using the two steps forward, one-step back approach. Therefore, a Yes instance of UHAMPATH for a given grid graph G always produces solvable levels of Tristepper using our reduction.

c. No to No Proof

To complete this proof, we must show that Yes instances of Tristepper map to Yes instances of UHAMPATH. To do this, we can prove the contrapositive: No instances of UHAMPATH map to No instances of Tristepper.

If there is no Hamilton path in G, there is no path that goes through each node exactly once. To solve the corresponding Tristepper level, the monster must enter each of the n gadgets in the Tristepper level and shoot $4K$ balls of goo, filling up all 4 hallways in each gadget. For each gadget that it enters, the monster is able to shoot $4K$ balls of goo. However, since there is no Hamiltonian path in G, the monster is only able to enter x gadgets, where $x < n$. Therefore, the monster can only shoot $4Kx + y$ balls of goo, where y is the total number of balls of goo shot in the lobbies. However, $y \leq 34n < K$, so $4Kx + y < 4Kn$. Since the maximum number of balls of goo that the monster can shoot is less than $4Kn$, the Tristepper level is not solvable. Therefore, a No instance of UHAMPATH always produces a No instance of Tristepper.

Appendix A.

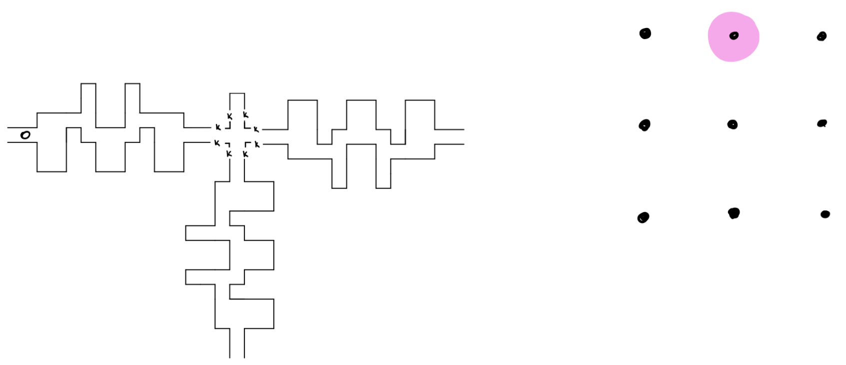


Figure 1a. Degree 3 Vertex

Figure 1b. Degree 2 Vertex

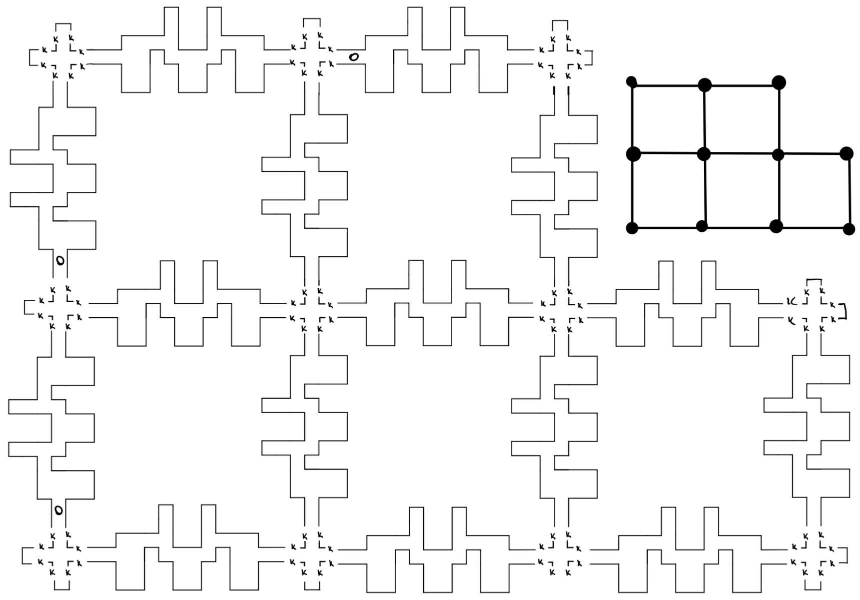


Figure 2. Example Map

Appendix B.

Gadget Breakdown

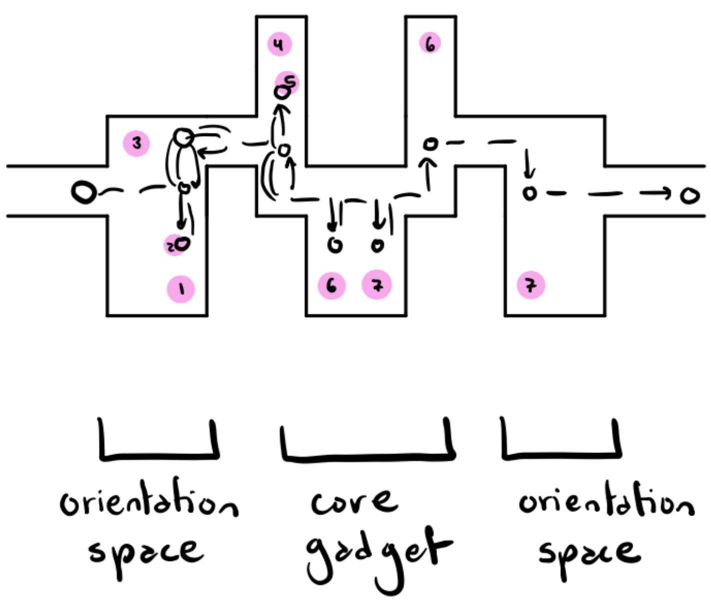


Figure 4. Lobby Section

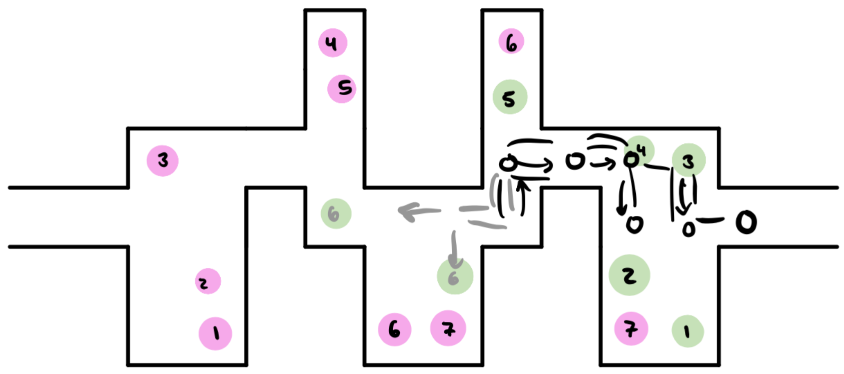


Figure 5. Blocked Re-entry (Example)

The attempt for re-entry in Figure 4, with the new goos shown in green, demonstrates how the structure of the core gadget prevents the monster from going past the lobby stage. Because of the location of 6 and 7 goos from initial exit (shown in pink), it is not possible to reuse this space.

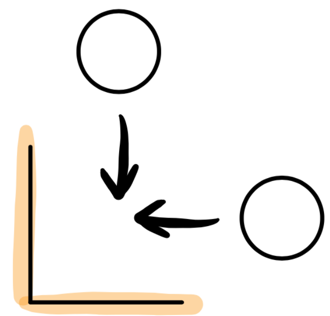


Figure 5. A Tristepper's 3rd step can never be in a corner

Because this core section of the lobby has L-shaped corners (highlighted in orange), it is impossible for the monster to ever sit in spot A without gooing itself. This is because no matter how the tristepper approaches the corner, its forward goo will block itself as highlighted in figure 5. If it was possible to sit in A, the movement in figure 6 would be possible, and the gadget would be ineffective and allow for two passes.

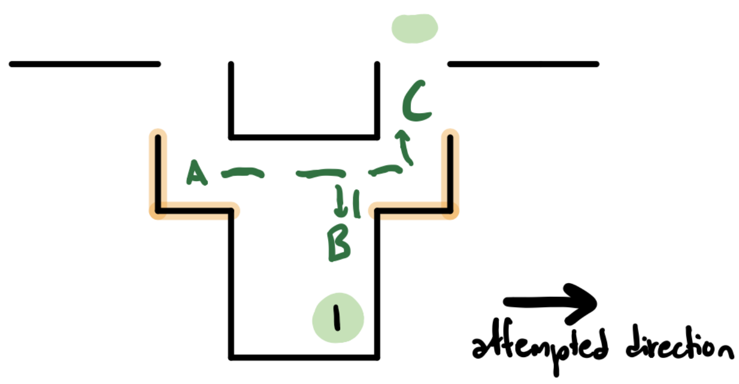


Figure 6. Impossible mapping of gadget