Signal Processing and Linear Systems
Lecture 1. Introduction

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The University of Aizu, 2019

Contents

- Course Information
- Introduction
  - Basic Concepts
  - Signals and Classification
  - Basic Continuous-Time Signals
  - Basic Discrete-Time Signals
  - Systems and Classification
  - Application Examples

Outline

- Systems
  - Continuous-time and discrete-time, linear and nonlinear, time-variant and time-invariant
- Signals
  - Speech, audio, image, video, radar, vital signs
- Signal processing
  - Transformation, detection, decomposition, reconstruction, compression, representation
  - Filter structures and designs, digital signal filtering
- A prerequisite course for further study on other fields
  - Speech processing, image processing, audio and video data compressing, pattern recognition, biomedical signal processing, and so forth.

Objectives

- To provide students with the foundations and tools of signal processing, particularly linear time-invariant systems in both continuous and discrete domains.
- To enhance topics such as signal representation in time domain, Fourier transform, sampling theorem, linear time-invariant system, discrete convolution, z-transform, discrete Fourier transform, and digital filter design.
- To understand how to analyze a given signal or system using various transforms; how to process signals to make them more useful and significant; how to design and implement a digital signal filter for a given real-world problem.

Schedule

No.	Lectures	Quiz + Ex	Date
1	Introduction to Signals and Systems	1	4/9
2	Linear Time-Invariant System (continuous-time)	2	4/12
3	Linear Time-Invariant System (discrete-time)	3	4/16
4	Continuous Fourier Series and Fourier Transform	4	4/19
5	Discrete Fourier Series, Fourier Transform, FFT	5	4/23
6	Fourier Transform Analysis of Signals and Systems	6	4/26
7	Midterm exam		5/7
8	Laplace Transform	8	5/10
9	Z-Transform	9	5/17
10	Structures for Digital Filters I: FIR Filter	10	5/21
11	Digital Filter Design I: FIR Filter	11	5/24
12	Structures for Digital Filters II: IIR Filter	12	5/28
13	Digital Filter Design II: IIR Filter	13	5/31
14	Applications of Signal Processing		6/4

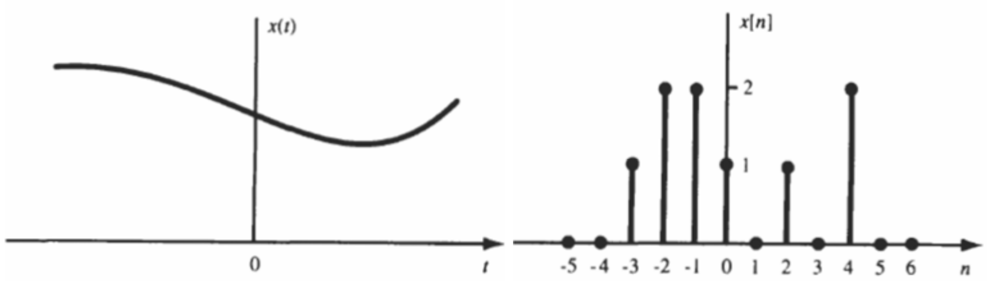
Teachers/TAs and Grading

- Faculty
  - Japanese: 陳 文西、朱 欣
  - English: C-T. Truong
- TA
  - Japanese: 李天恵、徐建波、小名達也
  - English: Hoang Anh Nguyen
- Grading method
  - Mid-term exam: 20%
  - Final exam: 30%
  - Exercises: 40%
  - Quiz: 10%

Webs, Textbooks and References

- Homepages
  - UoA http://web-int.u-aizu.ac.jp/course/spls/
  - MIT OpenCourseWare https://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/index.htm
- Textbooks:
  - Schaum's Outline of Signals and Systems, 3rd Edition (Schaum's Outlines) 2013, Hwei P Hsu ca. 2500 Yen
  - Schaums Outline of Digital Signal Processing, 2nd Edition (Schaum's Outlines) 2011, Monson H. Hayes ca. 3000 Yen
- Reference books:
  - Digital Signal Processing, 2011 ca.6,000 Yen, Sanjit K. Mitra
  - ディジタル信号処理(第2版)、萩原将文、森北出版、約2300円
  - MATLAB対応ディジタル信号処理、樋口龍雄、川又政征、森北出版、約3500円

Continuous-time & Discrete-time



(a) (b)

Analog and Digital Signals

- If a continuous-time signal $x(t)$ can take on any value in the continuous interval (a, b), where a may be $-\infty$ and b may be $+\infty$, then the continuous-time signal $x(t)$ is called an analog signal.
- If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a digital signal.

Real and Complex Signals

- A signal $x(t)$ is a real signal if its value is a real number.
- A signal $x(t)$ is a complex signal if its value is a complex number.
- A general complex signal $x(t)$ is a function of the form $x(t) = x_1(t) + jx_2(t)$

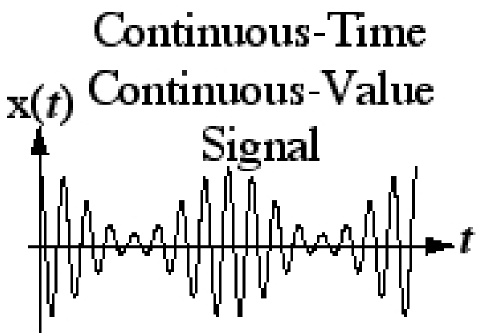
where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$.

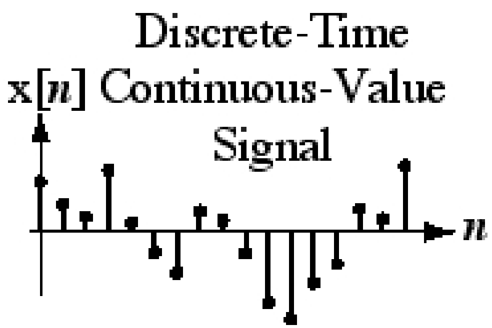
t represents either a continuous or a discrete variable.

Deterministic and Random Signals

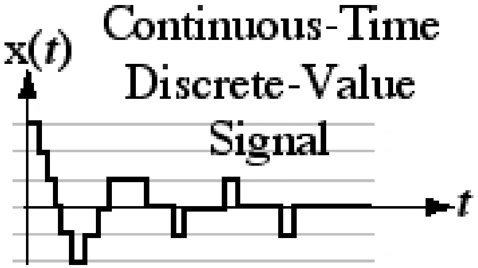
- Deterministic signals are those signals whose values are completely specified for any given time. It can be modeled by a known function of time t.
- Random signals are those signals that take random values at any given time and must be characterized statistically.

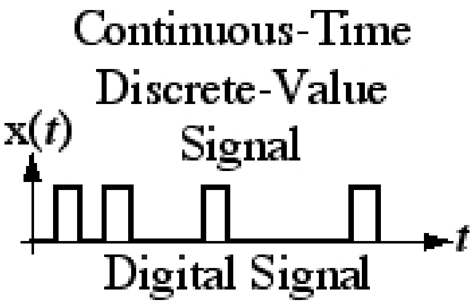
Various Signals



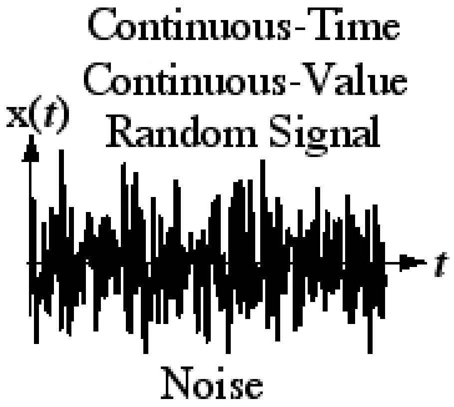


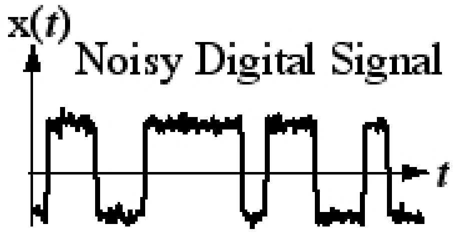
Various Signals





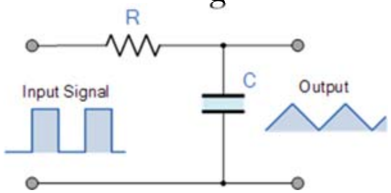
Various Signals





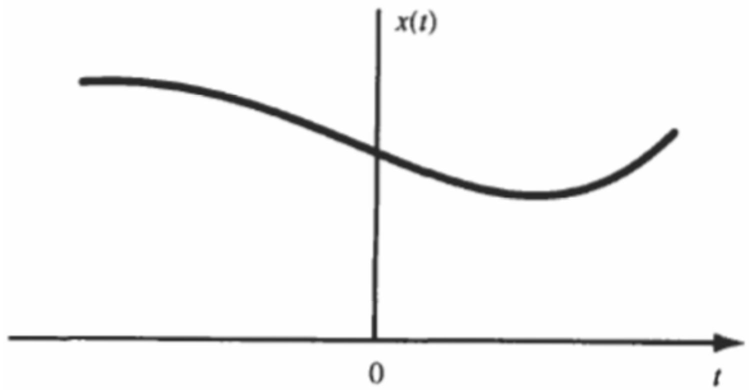
Definition of a Signal

- A *signal* is a function representing a physical quantity or variable, and typically contains information about the behavior or nature of a phenomenon.
- A *signal* is represented as a function of an independent variable t. Usually t represents time.
- A *signal* is denoted by $x(t)$.
- In a RC circuit, a *signal* may represent the voltage across the capacitor or the current flowing in the resistor.



Definition of a Continuous-Time Signal

- A signal $x(t)$ is a continuous-time signal if t is a continuous variable.

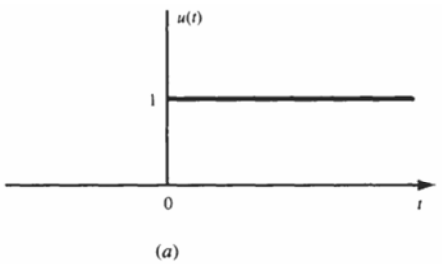


Basic Continuous-Time Signals

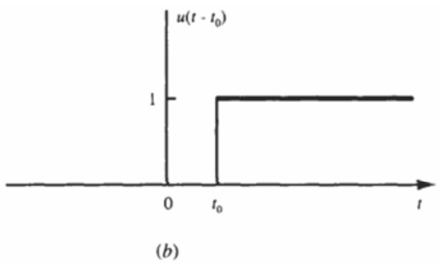
A. Unit Step Function:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$



Unit step function

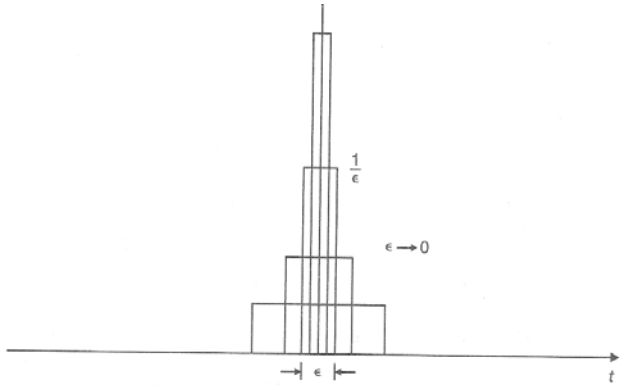


Shifted unit step function

Basic Continuous-Time Signals

B. Unit Impulse Function $\delta(t)$, Dirac Delta Function

defined as the limit of a suitably chosen conventional function having unity area over an infinitesimal time interval

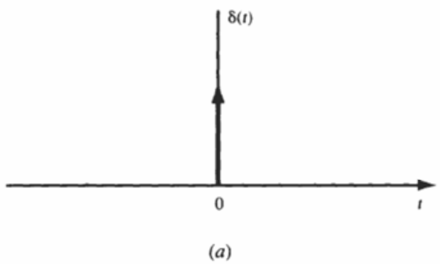


Basic Continuous-Time Signals

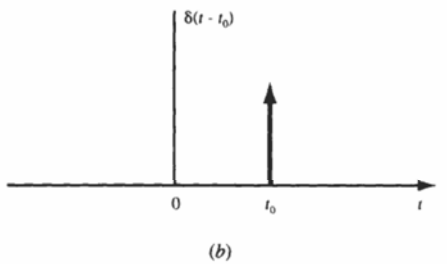
B. Unit Impulse Function:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \qquad \delta(t - t_0)$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(t)\, dt = 1 \qquad \int_{-\infty}^{\infty} \phi(t)\delta(t)\, dt = \phi(0) \qquad \int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)\, dt = \phi(t_0)$$



Unit impulse function



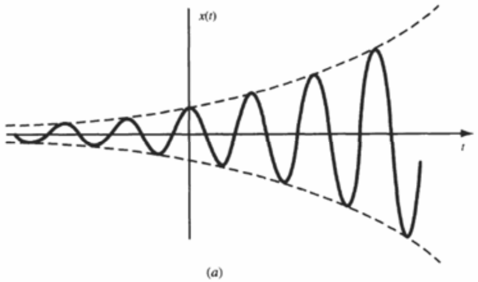
Shifted unit impulse function

Basic Continuous-Time Signals

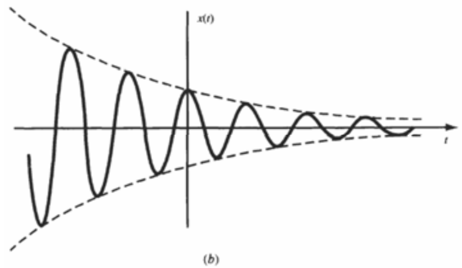
C. Complex Exponential Signals: $x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

Let $s = \sigma + j\omega$ be a complex number.

$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(\cos \omega t + j \sin \omega t) \qquad T_0 = \frac{2\pi}{\omega_0}$$



Exponentially increasing sinusoidal signal（σ>0）



Exponentially decreasing sinusoidal signal（σ<0）

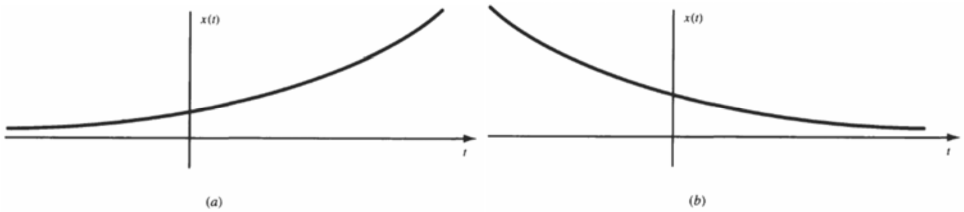
Basic Continuous-Time Signals

Real Exponential Signals:

If $s = \sigma$ is a real number, we have a real exponential signal

$$x(t) = e^{\sigma t}$$

a growing exponential | a decaying exponential



Continuous-time real exponential signal (σ>0)

Continuous-time real exponential signal (σ<0)

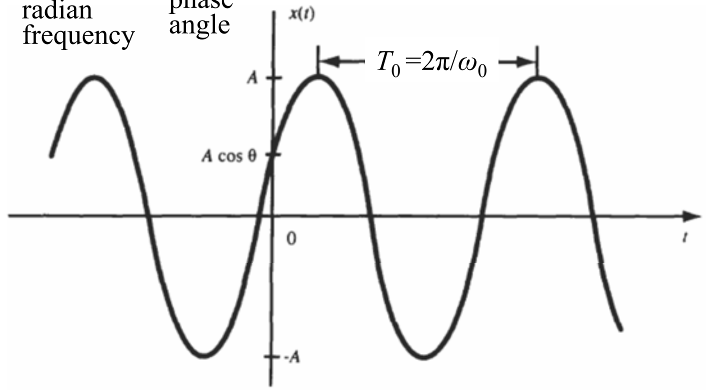
Basic Continuous-Time Signals

D. Sinusoidal Signals:

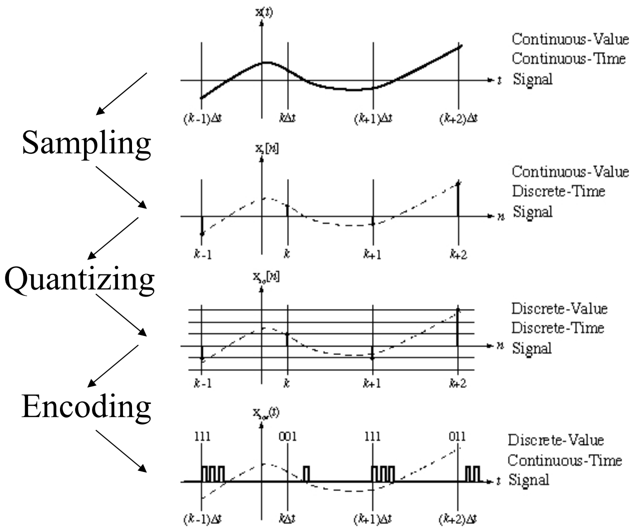
$$x(t) = A\cos(\omega_0 t + \theta)$$

amplitude (A), radian frequency (ω_0), phase angle (θ)

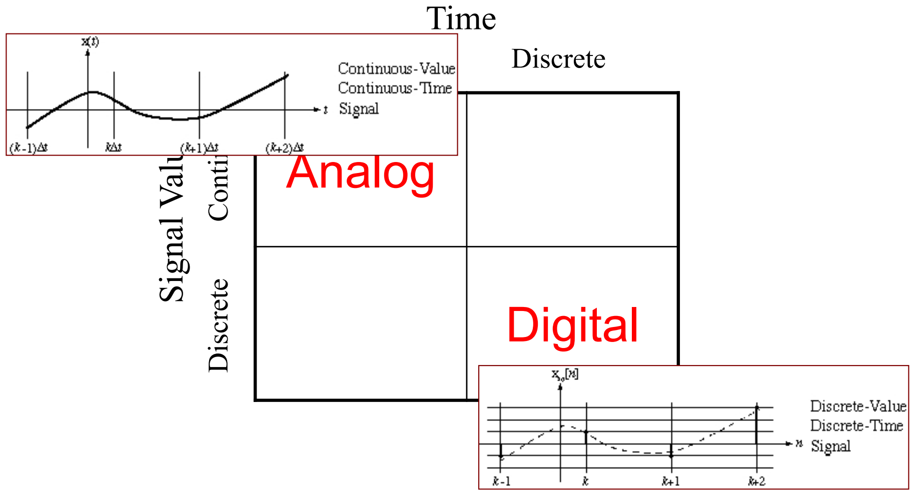
Sinusoidal signal $x(t)$ is periodic with
fundamental period $T_0 = 2\pi/\omega_0$ (sec)
fundamental frequency $f_0 = 1/T_0$ (Hz)
fundamental angular frequency $\omega_0 = 2\pi f_0$



Conversion of a Continuous Signal

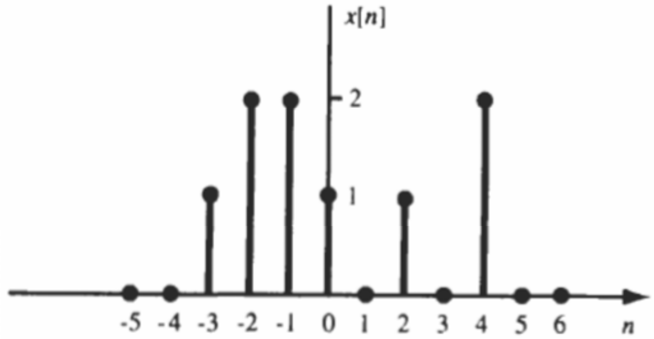


Analog vs. Digital



Definition of a Discrete-Time Signal

- If t is a discrete variable, and $x(t)$ is defined at discrete times, then $x(t)$ is a discrete-time signal.
- A discrete-time signal is identified as a sequence of numbers, denoted by $\{x_n\}$ or $x[n]$, where n = integer.

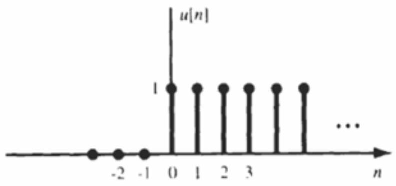


Basic Discrete-Time Signals

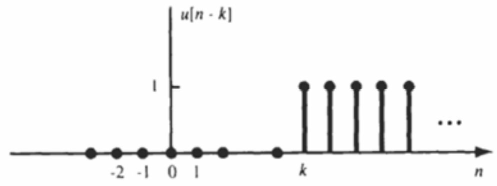
A. Unit Step Sequence:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases}$$



(a) Unit step sequence



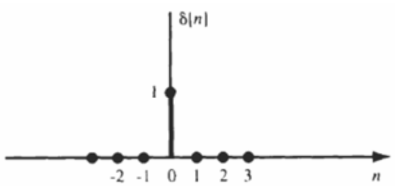
(b) Shifted unit step sequence

Basic Discrete-Time Signals

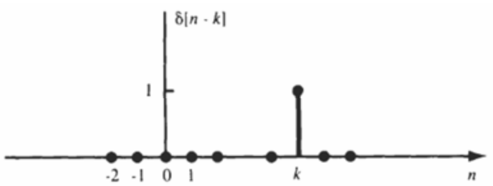
B. Unit Impulse Sequence:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$



(a) Unit impulse sequence



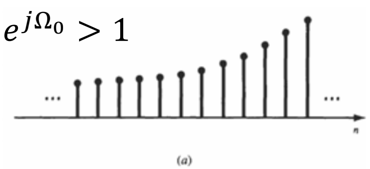
(b) Shifted unit impulse sequence

Basic Discrete-Time Signals

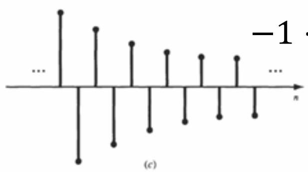
C. Complex Exponential Sequence

$$x[n] = e^{j\Omega_0 n} = \underbrace{\cos\Omega_0 n}_{\text{Real part}} + \underbrace{j\sin\Omega_0 n}_{\text{Imaginary part}}$$ Euler's formula

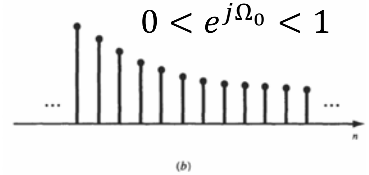
$e^{j\Omega_0} > 1$



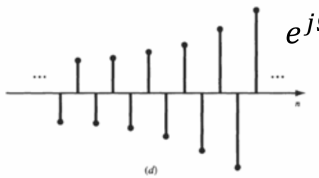
$-1 < e^{j\Omega_0} < 0$



$0 < e^{j\Omega_0} < 1$

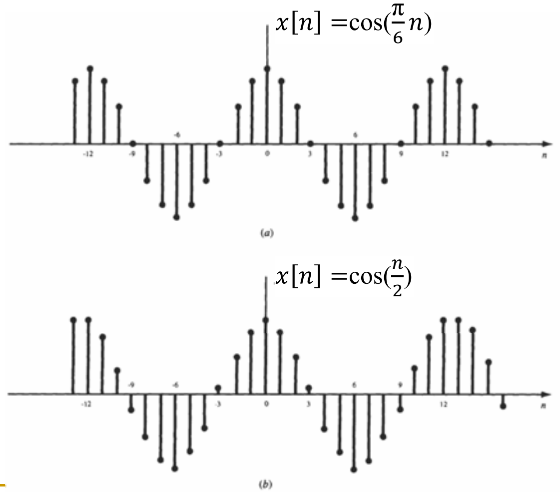


$e^{j\Omega_0} < -1$



Basic Discrete-Time Signals

D. Sinusoidal Sequence $x[n] = A\cos(\Omega_0 n+\theta) = A\text{Re}\{e^{j(\Omega_0 n+\theta)}\}$



periodic
fundamental period=12

$$N_0 = m\left(\frac{2\pi}{\Omega_0}\right)$$

not periodic

System Representation

A. Single/Multiple Input and Output Systems

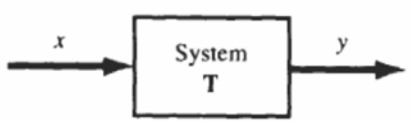
A *system* is a mathematical model of a real-world process that relates the *input* (or *excitation*) signal to the *output* (or *response*) signal.

Let *x* and *y* be the input and output signals, respectively, of a system. Then the system is viewed as a *transformation* (or *mapping*) of *x* into *y*.

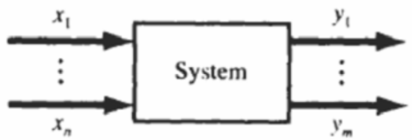
This transformation is represented by the mathematical notation

$$y = \mathbf{T}x$$

where **T** is the *operator* representing some well-defined rule by which *x* is transformed into *y*.



Single input and output system

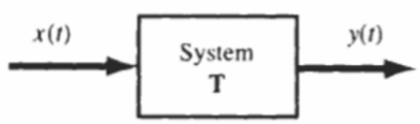


Multiple input and output system

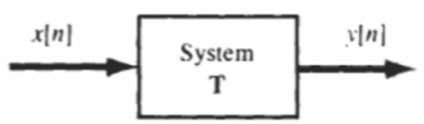
System Classification

B. Continuous-Time and Discrete-Time Systems

- *continuous-time system* = the input and output signals *x* and *y* are continuous-time signals.
- *discrete-time system* = the input and output signals *x* and *y* are discrete-time signals or sequences.



continuous-time system



discrete-time system

System Classification

C. Systems without Memory and with Memory

- *memoryless system* = the output at any time depends on only the input at that same time.
  - An example is a resistor R with the input $x(t)$ taken as the current and the voltage taken as the output $y(t)$.
  - The input-output relationship (Ohm's law) of a resistor is $y(t) = Rx(t)$
- *memory system* = the output at any time depends on not only the input at that same time but also the inputs at past time.
  - An example is a capacitor *C* with the current as the input $x(t)$ and the voltage as the output $y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$
  - Another example is a discrete-time system whose input and output sequences are related by $y[n] = \sum_{k=-\infty}^{n} x[k]$

System Classification

D. Causal and Noncausal Systems

- *causal system* = its output $y(t)$ at an arbitrary time $t = t_0$ depends on only the input $x(t)$ for $t \le t_0$.
  - its output at the present time depends on only the present and/or past values of the input, not on its future values.
  - it is not possible to obtain an output before an input is applied to the system.
- *noncausal system* = its output $y(t)$ at an arbitrary time $t = t_0$ depends on not only the input $x(t)$ for $t \le t_0$ but also the input $x(t)$ for $t \ge t_0$.
  - Examples of noncausal systems are

$$y(t) = x(t+1)$$
$$y[n] = x[-n]$$

  - All memoryless systems are causal, but not vice versa.

System Classification

E. Linear and Nonlinear Systems

- *linear system* = a system **T** satisfies the following two conditions
  1. **Additivity**:

  Given that $\mathbf{T}x_1 = y_1$ and $\mathbf{T}x_2 = y_2$, then $\mathbf{T}\{x_1+x_2\} = y_1+y_2$ for any signals x_1, x_2.

  2. **Homogeneity** (or **Scaling**):

  $\mathbf{T}\{\alpha x\} = \alpha y$ for any signals x and any scalar α.

  3. **Superposition**: above two conditions can be combined into one

  $\mathbf{T}\{\alpha_1 x_1+\alpha_2 x_2\}=\alpha_1 y_1+\alpha_2 y_2$

  - Examples of linear systems are $y(t) = Rx(t)$, $y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$
- *nonlinear system* = a system that does not satisfy the above conditions
  - Examples of nonlinear systems are $y=x^2$, $y=\cos x$

System Classification

F. Time-Invariant and Time-Varying Systems

- *time-invariant system* = a time shift (delay or advance) in the input signal causes the same time shift in the output signal.
  - A continuous-time-invariant system = $\mathbf{T}\{x(t-\tau)\} = y(t-\tau)$ for any real value τ.
  - A discrete-time-invariant (or shift-invariant) system = $\mathbf{T}\{x(n-k)\} = y(n-k)$ for any integer k.
- *time-varying system* = a system which does not satisfy the above two equations.
- To check a system for time-invariance, we can compare the shifted output with the output produced by the shifted input.

System Classification

G. Linear Time-Invariant Systems

- *linear time-invariant* (*LTI*) *system* = a system which is linear and also time-invariant.

System Classification

H. Stable Systems

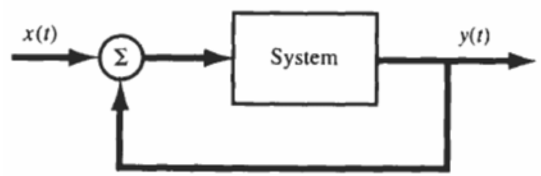
- *bounded-input/bounded-output* (BIBO) *stable system*
  - for any bounded input x defined by $|x| \le k_1$
  - the corresponding output y is also bounded defined by $|y| \le k_2$

    where k_1 and k_2 are finite real constants.
  - there are many other definitions of stability.

System Classification

I. Feedback Systems

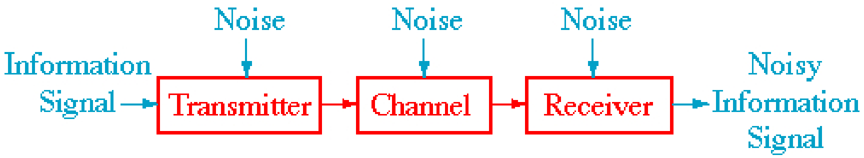
- *feedback system* = the output signal is fed back and added to the input to the system



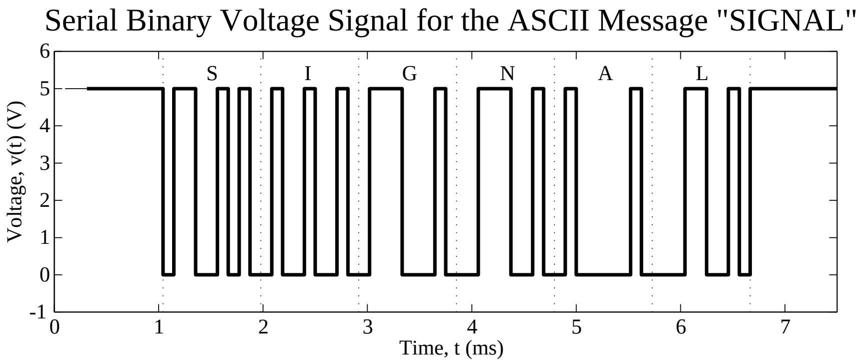
Application Examples

A Communication System

- informative signal but contaminated by noise
- an interconnection of smaller systems

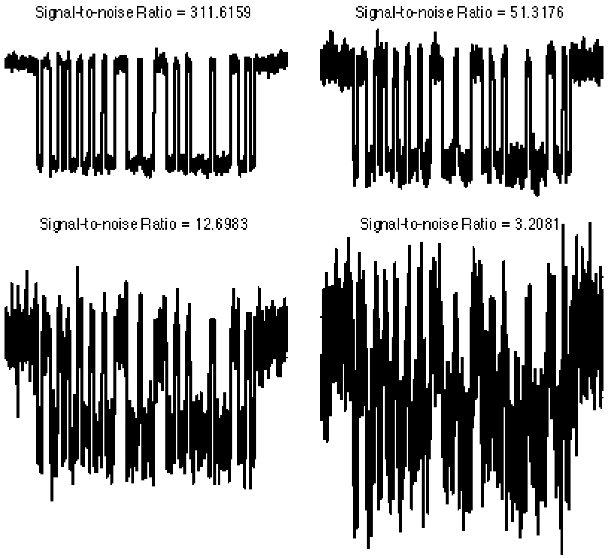


Message Encoded in ASCII

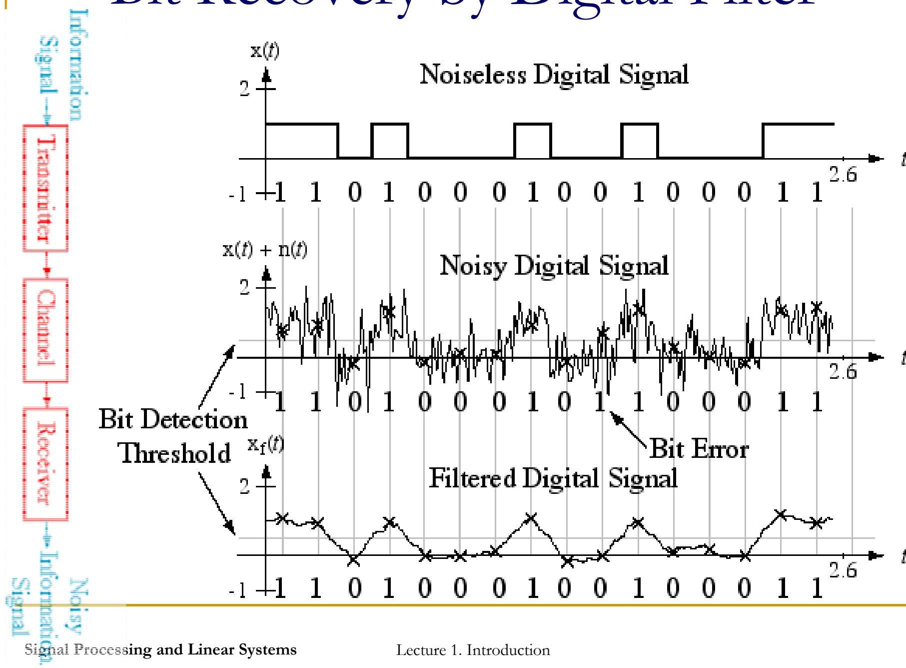


Noisy Message Encoded in ASCII

Progressively noisier signals



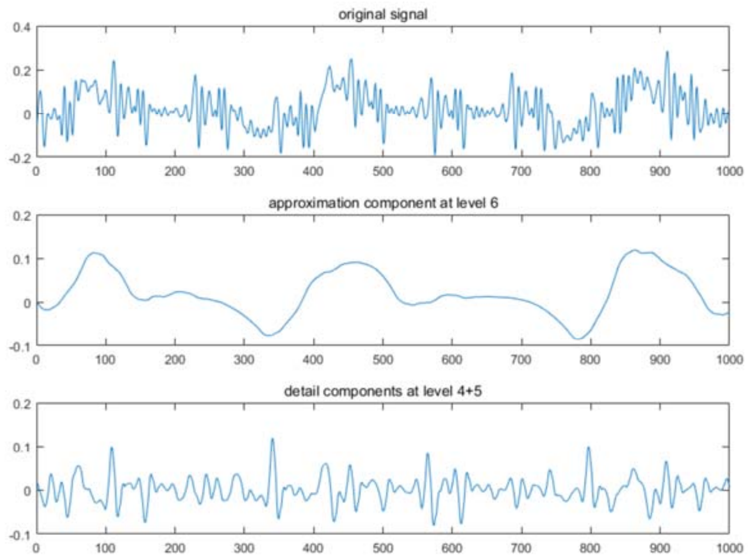
Bit Recovery by Digital Filter



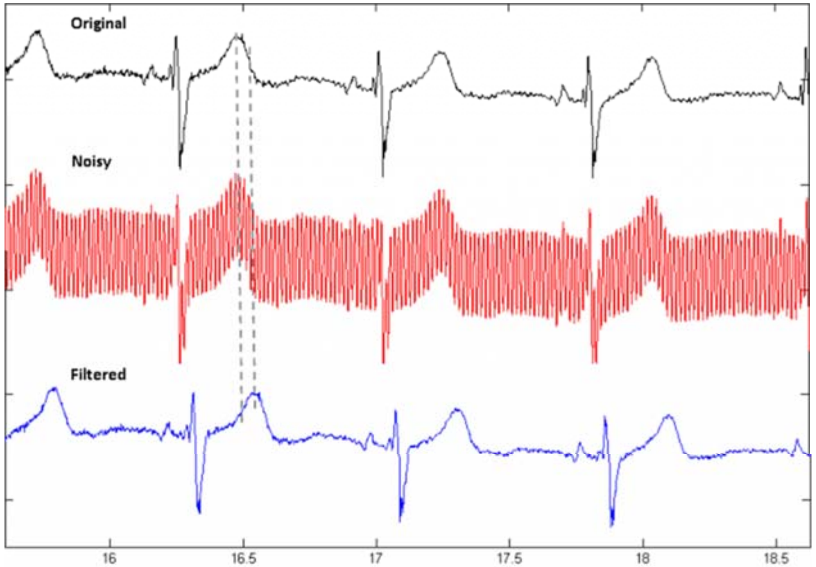
Some Signal Processing Topics

- Signal transform
- Noise suppression
- Signature detection

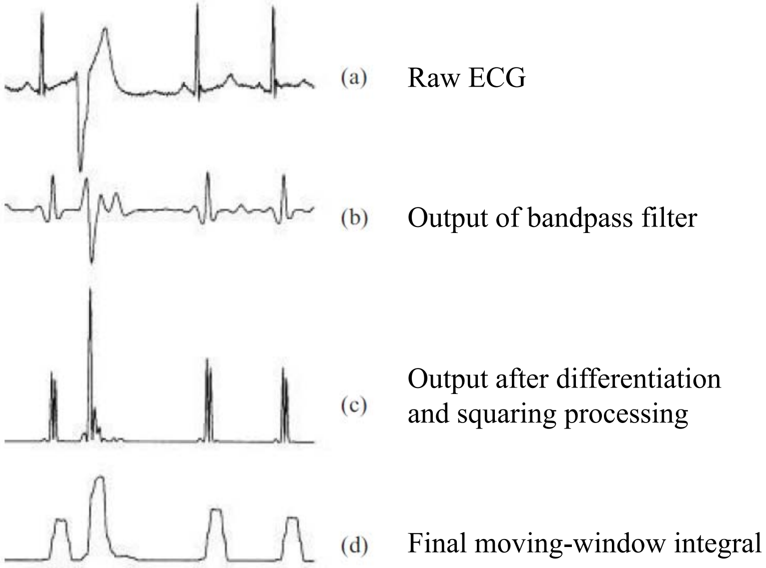
Signal Transform



Noise Suppression



Signature Detection



Future Topics

- Signals and Systems in mathematic forms
  - Linear Time-Invariant System
- Fourier series / Fourier transform
- Analysis of Signals and Systems using Fourier transform
- Other transforms / representations
  - Z-transform, Laplace transform, State space
- Applications