Information Sharing in the Supply Chains of Products With Seasonal Demand

Yeu-Shiang Huang, Chia-Hsien Ho, and Chih-Chiang Fang

***Abstract*—This study considers a two-echelon supply chain with one supplier and one retailer for products with seasonal demand. The effects that information sharing has on coordination and benefits of the supply chain are investigated. In order to better forecast the seasonal demand, the supplier initiates the information sharing process by offering incentives to the retailer which are proportional to the degree of information sharing. Since the variance in the supplier's inventory would marginally decrease as the degree of information sharing increases, the benefits gained by the supplier due to information sharing are thus a convex function. This can be used to obtain the optimal degree of information sharing with the aim of maximizing profits, which is a tradeoff between the benefits gained and costs incurred by information sharing. In this study, the seasonal demand is described by a SARMA time series model. Bayesian analysis is employed to investigate the value of information and determine the optimal degree of information sharing. Constructive properties are derived to provide managerial insight for effective decision making. The results of sensitivity analyses show that the correlations of demand for successive periods and estimation errors would both have great effects on the benefits gained by information sharing.**

***Index Terms*—Information sharing, seasonal demand, supply chain coordination.**

Managerial Relevance Statement

This study considers a two-echelon supply chain with seasonal consumer demand, in which the impacts of the degree of information sharing on the supplier's profits are investigated. In considering both the benefit and cost of sharing information, information sharing can be profitable for both the supplier and retailer in the supply chain. However, continuously increasing the degree of information sharing would eventually result in a decrease in the supplier's net profit. In fact, an excessive amount of information sharing can result in high costs for the supplier, and thus determining an optimal degree of information sharing to benefit both the supplier and retailer is essential for effective supply chain management, and proposing a way to achieve this is the main contribution of this study. The derived results can be applied in practice to a decentralized supply chain in any industry with seasonal demand.

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I. Introduction

In a supply chain, a supplier needs to forecast demand from a retailer for its own material handling, inventory control, and production planning. The demand forecasting includes some uncertain factors, which can be described as demand variability. However, the demand variability and the lead time of delivery will cause bullwhip effect in different degree, no matter how carefully the supplier predicts the demand. In recent years, it has become a major issue for supply chain management. In order to mitigate such bullwhip effects, information sharing among members of a supply chain would be a useful solution that can achieve a win–win situation for a supplier and a retailer if both agree the information sharing policy. Information sharing is a collaborative mechanism in which the supplier can obtain and utilize the demand and inventory status of the retailer.

Firms often deal with products with uncertain or seasonal demand, such as airplane tickets and air conditioners. The demand of these products often has great variation that results in inaccurately forecasting of demand. The phenomenon of seasonal demand will also amplify the bullwhip effect in a supply chain that can potentially cause overstocking or stock-out in supply chains and increase the costs of inventory and stock-outs. As a result, the effectiveness of the overall supply chain is damaged. However, if members in a supply chain are willing to share their information appropriately, the bullwhip effect can be significantly reduced.

Information sharing can not only enable members in a supply chain to mutually communicate, but also enhance performance of the entire supply chain. However, not every member can benefit from information sharing. For example, when a retailer shares information with a supplier, the demand uncertainty is reduced, and the supplier can lower its inventory cost. The retailer, on the other hand, though can trim down its own inventory level based on the demand information as well, has to bear the costs of collecting information. In such a case, the supplier should share the increased benefits due to information sharing to coordinate with the retailer. This can thus increase the retailer's willingness to share information. For the supplier, information sharing is cost effective, and both parties of the supply chain benefit from coordination. If information sharing is too costly or the benefit due to information sharing is not substantial, the supplier would not be willing to share its revenues and initiate the information sharing process. Therefore, the supplier has to decide the optimal degree of information sharing to obtain the maximized profits.

In sum, the seasonal demand can be forecasted more accurately by the use of the shared information to further develop an appropriate inventory strategy. Therefore, the objective of this study is that, by considering the tradeoff between the benefit due to information sharing and the cost of collecting information, the optimal degree of information sharing can be obtained with the aim of maximizing the benefits. Accordingly, this study considers a two-echelon supply chain with seasonal consumer demand, in which the impacts that the degree of information sharing has on the supplier's profits are investigated. The consumer's seasonal demand is described by an seasonal autoregressive moving average (SARMA) time series model. Bayesian analysis is also introduced into the proposed model for examining the value of information sharing and thus determining the optimal degree of information sharing. This study contributes to the literature on 1) proposing a model to determine the optimal degree of information sharing under the case of seasonal demand in supply chain, and 2) employing Bayesian analysis to investigate the value of information sharing.

The remainders study is organized as follows: Section II reviews the relevant literature regarding the issue of the research topic. Section III states the research problem and presents the research framework. Section IV describes the construction of the proposed model. Section V provides numerical examples to demonstrate the effectiveness of the proposed approach. Finally, Section VI gives the concluding remarks and suggests some directions for future research.

II. Literature Review

Most studies on information sharing focus on the differences that arise with and without information sharing, and some consider different types of information sharing, such as complete, partial, or even no information sharing [29], [30], [32]. Since these studies use discrete instead of continuous approaches in investigating the effects that information sharing have on supply chain performance, the profits of the entire supply chain may not be optimally maximized. In addition, the influence of information sharing on retailers is rarely investigated. Information sharing can benefit the entire supply chain, but if only the retailer needs to bear the costs of collecting and sharing information, then this can discourage it from collaborating with suppliers [22], [36], [39]. Therefore, suppliers must understand the retailer's information costs and share the related benefits, so as to encourage the latter to collect and share information [7], [13], [37]. Moreover, if the degree of information sharing is higher, suppliers also need to pay a higher cost of information sharing to retailers [9]. Therefore, this study considers the benefits and costs of information sharing in a supply chain for products from the perspective of the supplier, with an aim of obtaining the optimal degree of information sharing which maximizes profits.

Information sharing plays an essential role in achieving supply chain coordination. If firms engage in information sharing, then this can reduce uncertainties and increase benefits for the entire supply chain. Zhang *et al.* [38] considered the benefits of sharing shipment information in a one-stage supply chain, and stated that the arrived shipment quantities may be less than the ordered ones due to the supplier's imperfect service, which is known as shipment quantity uncertainty. Yao *et al.* [33] considered a supply chain with one supplier and two value-added retailers, in which the retailers have their own value-added cost structures which are unknown to the supplier, and constructed a three-stage game theoretic model. They suggested that the upstream supplier can benefit from the accurate information which is provided by the retailers, and information sharing benefits both parties in order to achieve a win–win situation. Mishra *et al.* [24] examined the incentives of a supplier and a retailer to share their demand forecasts. If the savings from inventory holding and shortage costs are sufficiently high due to information sharing, then a side payment contract that induces Pareto-optimal information sharing is feasible.

Information sharing benefits the entire supply chain by enabling greater coordination. Cachon and Fisher [7] stated that orders are the only information that firms exchange for inventory management in traditional supply chains, but noted that emerging information technology applications enable firms to share demand and inventory information with all the members in a supply chain, and that can then be utilized to decrease the lead time and purchase lot size, thus reducing total costs and benefiting the entire supply chain. Zhou and Benton [37] investigated the effects of information sharing, and concluded that it can significantly enhance the effectiveness of a supply chain as a whole. Lee *et al.* [23] proposed a two-level supply chain to investigate the benefits of information sharing, and concluded that the manufacturer would benefit more from information sharing when demand and lead time are highly correlated. Fiala [18] utilized a system dynamic model to analyze the impacts of information sharing from a multilevel network structure of supply chain. Such simulation method can handle more complicate structure of supply chain to avoid traditional mathematical analysis. Bhattacharya *et al.* [4] evaluated green supply chain performance by using collaborative decision-making approach. Bhattacharya *et al.* [3] proposed dynamic decision-making tools. The proposed tools could be employed for capturing the trade-offs among multiple and conflicting-in-nature criteria of supply chain. Nudurupati *et al.* [26] assessed potential strategic suppliers based on the mediating collaborative partnership among the suppliers by using case-based action research. Ouyang [27] investigated the importance of the bullwhip effect, and found that sharing customer sales and inventory information would significantly reduce this. Zhang and Chen [40] concluded that sharing of information among the members of any supply chain is essential because it is an effective means of mitigating bullwhip effects. It can be described as a situation where the variations of demand in supply chains are amplified as they move up the supply chain.

Generally, products with seasonal demand are often sold to consumers using seasonal marketing strategies, such as end-season promotions and discounts, and suppliers of such products cannot merely use historical data to accurately forecast market demand, since the promotion activities may be very different for each season [1], [10], [19]. Firms thus have to keep more inventories to cope with the fluctuating demand, which incurs higher holding costs and lowers the performance of the entire supply chain. Sreenivasan and Sumathi [31] utilized a generalized parameters technique to formulate a seasonal ARMA model.

Their model and mathematical ways give the basis for seasonal demand forecast because it is simple and can be applied to any time series model. Cho and Lee [11] proposed a two-echelon supply chain which seasonal demand was considered, and stated that the value of information sharing seems to be sensitive to the seasonal phenomenon. Therefore, focusing on seasonal effects may result in a growth in overall supply chain profits. Nagaraja *et al.* [25] found that the relationship between lead-time and seasonal lag to be the key to studying the bullwhip effect magnitude for SARMA models. However, with effective supply chain management, all the members of a supply chain can benefit from a good coordination mechanism, although this depends on information sharing among members during the coordination process to reduce the bullwhip effect due to changing demand. If this can be achieved, then the profitability of the entire supply chain can be improved [7], [16], [21]. Sharing information about seasonal demand is thus essential in supply chains, and can enhance both profits and the accuracy of demand forecasting [6], [17], [28].

While suppliers benefit from information sharing due to a decrease in uncertainty, retailers have to pay the cost of collecting this information. However, since information sharing is beneficial to the entire supply chain, suppliers should actively share the related benefits to compensate for the cost of collecting information paid by retailers. Moreover, Chu and Lee [8] noted that retailers can determine whether to collect better information about market demand and share it with their suppliers or not, often depending on the cost of information sharing and the nature of the market demand signals that are obtained. They thus suggested that the upstream members of the supply chain should share the costs of information sharing to enhance the downstream members' willingness to share information, thus enhancing the competitiveness of the entire supply chain. Yue and Liu [34] investigated the benefits of sharing information for both traditional and direct retail channels, and stated that information sharing can greatly benefit both parties in the supply chains of each channel. Wu and Cheng [32] stated that when both parties in a supply chain share accurate demand information, the supplier should then share the benefits of this by offering a lower wholesale price to the retailer, as this can eliminate information distortion, and both parties can benefit from sharing information by using a revenue sharing contract. Cachon and Lariviere [5] considered a supply chain in which the manufacturer encounters stochastic demand and offers demand information along with a capacity allocation contract to its supplier, and stated that when the manufacture has high forecast demand, the information which is shared to the supplier would not be very credible, and that this can then harm the performance of the supply chain.

Based on the aforementioned discussion, although information sharing is beneficial, it is subject to cost or revenue sharing, and the supplier has to decide the optimal degree of information sharing to obtain the maximum profits. Moreover, the demand for a seasonal product, which varies over time and results in an increase of the inventory cost, can be forecast more accurately with the use of the shared information to further determine an appropriate inventory strategy [41], [42]. Therefore, the tradeoff between the benefits due to information sharing and the cost of collecting information would both be considered to determine the optimal degree of information sharing for the supplier of seasonal products, with an aim to maximize the benefits. The subsequent section will present the problem, the necessary assumptions, and the notations in detail.

III. Information Sharing in a Supply Chain for Seasonal Products

As the above mentioned, this study considers a two-echelon supply chain with a supplier and a retailer, in which the supplier offers a product, and the retailer deals with the consumer's seasonal demand for the product, which can be described by a SARMA time series model [12]. The retailer would observe the consumer demand at *t*, i.e., D_t, and then determine the inventory level and place an order with the supplier. Upon receiving the retailer's order, the supplier would place an order with the upstream manufacturer based on the received order. The retailer may conduct market research to collect information regarding consumer demand, which can then be used to enhance the consumer demand forecast for the next period, D_{t+1}, and then determine the new inventory level, $S_t^{r'}$. With regard to information sharing, the retailer can share both the new inventory level and the collected demand information with the supplier, so that the latter can update its anticipated order quantity for the next period, Y_{t+1}. Note that the shared demand information from the retailer is assumed to be always accurate.

The variance of order quantity can be reduced based on the shared information, which can be treated as a benefit of information sharing, π. The supplier may enhance the retailer's willingness to collect and share information by sharing the related revenues, and these can thus be treated as the supplier's expenses with regard to gaining the shared information, C. By trading off between the benefits and expenses, the optimal degree of information sharing can thus be determined for the supplier to maximize its profits. On the other hand, the retailer can not only obtain the shared revenues from the supplier due to information sharing, but also utilize the collected information to gain benefits by reducing the variance in its own inventory.

Without information sharing, the supplier can consider the retailer's order quantity at period *t*, Y_t, as a basis along with the estimation of retailer's demand during the lead time to predict the retailer's order quantity at period $t+1$, Y_{t+1}, which is assumed to be normally distributed as $Y_{t+1} \sim N(\mu_t^s, v_t^s)$. In contrast, with information sharing, the supplier can obtain the consumer's demand information, D_t, and the updated inventory level, $S_t^{r'}$, from the retailer, which can be utilized to update the forecast order quantity from Y_{t+1} to Y'_{t+1}. Here, Y'_{t+1} would still be normally distributed, but with a revised variance, $v_t^{s'}$ according to the Bayesian updating process. Note that the variance reduces as the degree of information sharing increases, and $v_t^{s'} < v_t^s$. Also note that the lead times for replenishment are assumed to be one period for both the retailer and supplier to simplify the analytic process, which can also be extended for longer lead times.

This study assumes that both the supplier and retailer use the order-up-to policy for replenishment (the assumption is supported by the references [1], [2], [12], [14], [15], [20], [25],

[35]), and that no member in the supply chain can dominate the determination of the degree of information sharing, which would depend only on the gains and losses due to this sharing. Without information sharing, the supplier would have difficulty in estimating consumer demand due to seasonal effects, which may result in overestimating or underestimating it. If the retailer can share the demand information with the supplier according to the coordinated degree of information sharing, the supplier would be able to reduce the variance in its inventory and thus lower the expected inventory cost. In sum, the retailer can collect consumer information to update its forecast of consumer demand, which can not only reduce its own inventory variance but also gain some shared revenue from the supplier, if such, the information is shared with the supplier. Information sharing can thus benefit both the supplier and retailer in the two-tier supply chain.

The assumptions of this study are as follows:

1) The two-echelon supply chain has a supplier who offers a seasonal product to a retailer.
2) Consumer demand can be described by an SARMA model.
3) The retailer is willing to join the coordination of information sharing as long as the shared revenue can cover the cost of collecting market information and the requested premium.
4) The demand information is fully and honestly shared by the retailer.
5) The supplier and retailer both adopt the order-up-to policy for replenishment.

The notations used in this study are as follows:

D_t The seasonal demand confronted by the retailer at period *t*, which can be described by $SARMA(1,0) \times (0,1)_{sl}$.
C The revenue that the supplier shares with the retailer, which is the cost of sharing information for the supplier.
Y_t The retailer's order quantity, forecast by the supplier without information sharing.
S_t^r The retailer's inventory level.
S_t^s The supplier's inventory level.
v_t^s The variance of the inventory level forecast by the supplier without information sharing.
v_t^{s*} The variance of the inventory level forecast by the supplier with information sharing.
p^r The retailer's shortage cost.
h^r The retailer's inventory holding cost.
p^s The supplier's shortage cost.
h^s The supplier's inventory holding cost.
ϕ The correlation coefficient of the demand of successive periods.
θ The correlation coefficient of the demand of successive seasonal cycles.
α_t The random error of demand at *t*, where $\alpha_t \overset{iid}{\sim} N(0, \sigma^2)$.
π The benefit gained by the supplier due to information sharing.
B The supplier's net profit after information sharing.
N The total amount of available information.
n The degree of information sharing.
k^s The service level of the supplier's inventory.

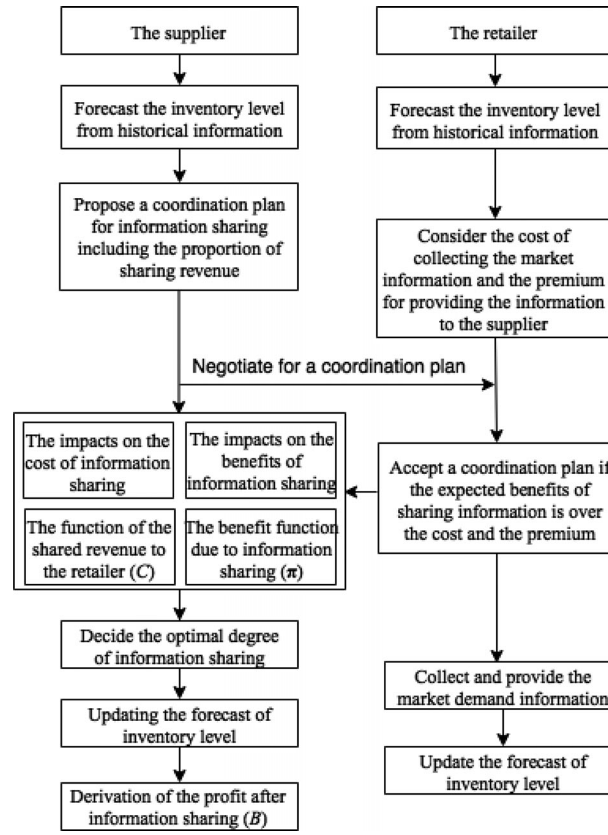

Fig. 1. Research steps of this study.

k^r The service level of the retailer's inventory.
X_t The market demand information which is collected by the retailer at period *t*.

Fig. 1 shows the research steps used in this study. First, the supplier has to use historical information to predict the retailer's order quantity and determine its own inventory level. However, in considering more relevant market information from the retailer for refining its prediction, the supplier would propose a coordination plan of information sharing to the retailer. The decision that the retailer has to make to join the information sharing coordination depends on whether the shared revenue from the supplier can cover both the cost of collecting the market information and the premium for providing the information to supplier. Once the information sharing coordination is accepted for the both sides, the supplier can determine the optimal degree of information sharing with consideration of the trade-off between the cost and benefit due to information sharing. In sum, with information sharing, the supplier sponsors the retailer to collect consumer demand information through coordination, and then updates the previously forecast order quantity based on the new information, which can reduce the variance in inventory and thus enhance the supplier's revenue (π). However, the supplier has to share some of this as an incentive for the retailer to share information, C, and the optimal degree of information sharing can thus be determined by maximizing the net profit ($\pi - C$) of the supplier.

IV. Determination of the Optimal Degree of Information Sharing

Suppose that the seasonal consumer demand can be described by a time series model $\mathrm{SARMA}(1,0) \times (0,1)_{sl}$ according to Cho and Lee [11], which is a combination of $\mathrm{AR}(1,0)$ and $\mathrm{MA}(0,1)_{sl}$, where sl denotes the number of periods in a seasonal cycle. For example, when $sl = 12$, this indicates that the seasonal effect occurs every 12 months (one year). Current demand is related to both the demand of the previous period and the demand error of the previous season, and is given as

$$D_t = d + \phi D_{t-1} + \alpha_t - \theta\alpha_{t-sl} \tag{1}$$

where D_t denotes the demand at period t, d is the estimated average demand, and α_t denotes the random error of the demand at period t, which is normally distributed, $\alpha_t \overset{iid}{\sim} \mathrm{N}(0, \sigma^2)$. ϕ and θ are constants between -1 and 1. ϕ is the correlation coefficient of the demand for successive periods, and θ is the correlation coefficient of the demand for successive seasonal cycles. Note that the random error of demand at *t*, α_t, is assumed to be far less than the average demand to ensure that the probability of negative demand is close to zero ($d > 0$).

A. Retailer's and Supplier's Inventory Strategies

The retailer observes the consumer demand and determines the order quantity at the end of period t to achieve the inventory up to level, and then places an order with the supplier which would be fulfilled and delivered in the next period. The retailer's order quantity, Y_t, is the sum of consumer demand at period t and the increase in inventory, which is given by

$$Y_t = D_t + S_t^r - S_{t-1}^r \tag{2}$$

where D_t is the consumer demand at t, and S_t^r denotes the retailer's optimal inventory up to level at period t. Since the order would be delivered in the next period, i.e., the lead time is one period, the retailer's inventory at period t, S_t^r, has to satisfy the consumer demand at period $t+1$, D_{t+1}. Suppose that the insufficient inventory has a unit shortage, p^r, and overstock would cause a unit holding cost, h^r, the total inventory cost is thus given by

$$\mathrm{TC}(S_t^r) = h_r \max(0, S_t^r - D_{t+1}) + p_r \max(0, D_{t+1} - S_t^r). \tag{3}$$

Since the retailer faces seasonal demand at period $t+1$, $D_{t+1} = d + \phi D_t + \alpha_{t+1} - \theta\alpha_{t-s+1}$, where D_{t+1} is normally distributed as $N(\mu_t^r, v_t^r)$, and μ_t and v_t are the mean and variance conditional on D_t, which are given by

$$\mu_t^r = E[D_{t+1}|D_t] = d + \phi D_t \tag{4}$$

and

$$v_t^r = \mathrm{Var}[D_{t+1}|D_t] = (1+\theta^2)\sigma^2 \tag{5}$$

respectively. This is a standard newsvendor problem, and the retailer's optimal inventory level, S_t^r, can be determined as

$$S_t^r = \mu_t^r + k^r\sqrt{v_t^r} \tag{6}$$

where $k^r = \Phi^{-1}(\frac{p^r}{p^r+h^r})$ and Φ is the cumulative distribution function of the standard normal distribution.

On the other hand, the supplier faces the retailer's demand and has to forecast the retailer's order quantity, Y_t, to determine its own inventory level. The retailer's order quantity at period *t* can be rewritten according to the derivations of D_t and S_t^r, which are given by

$$\begin{aligned} Y_t &= D_t + S_t^r - S_{t-1}^r \\ &= D_t + (d + \phi D_t + k^r\sqrt{(1+\theta^2)\sigma^2}) \\ &\quad - (d + \phi D_{t-1} + k^r\sqrt{(1+\theta^2)\sigma^2}) \\ &= D_t + \phi(D_t - D_{t-1}). \end{aligned} \tag{7}$$

The retailer's order quantity after the lead time, i.e., for period $t+1$, can be rewritten as

$$\begin{aligned} Y_{t+1} &= D_{t+1} + S_{t+1}^r - S_t^r \\ &= d + \phi Y_t - \phi\alpha_t + (1+\phi)\alpha_{t+1} \\ &\quad - (1+\phi)\theta\alpha_{t-s+1} + \phi\theta\alpha_{t-s}. \end{aligned} \tag{8}$$

Without information sharing, the supplier can only use the retailer's order quantity at *t*, Y_t, to forecast the expected value and variance of the retailer's order quantity during the lead time. That is, Y_{t+1} has a normal distribution, $N(\mu_t^s, v_t^s)$, where μ_t^s and v_t^s are the mean and variance under the condition in which the retailer's order quantity at period *t* is known as Y_t, and which are given by

$$\mu_t^s = E[Y_{t+1}|Y_t] = d + \phi Y_t \tag{9a}$$

and

$$v_t^s = \mathrm{var}[Y_{t+1}|Y_t] = (1+\theta^2)\sigma^2\left[(1+\phi)^2 + \phi^2\right]. \tag{9b}$$

Likewise, the supplier's optimal inventory level is thus given by

$$S_t^s = \mu_t^s + k^s\sqrt{v_t^s} \tag{10}$$

where $k^s = \Phi^{-1}(\frac{p^s}{p^s+h^s})$.

B. Bayesian Information Sharing Model

With regard to information sharing, the retailer collects consumer demand information through market surveys during the lead time period of and uses this to update its demand forecast as D'_{t+1}, which would have a smaller variance, and can be used to derive the optimal inventory level, $S_t^{r'}$. In the meantime, the retailer can also share the optimal inventory level, $S_t^{r'}$, and the collected demand information with the supplier, which can be both used to revise the supplier's forecast of the retailer's order quantity as Y'_t. As a result, the revised inventory level, $S_t^{s'}$, can be derived with a smaller inventory variance for the supplier. In general, complete information sharing can benefit the supplier the most. However, as the degree of information sharing increases, the supplier has to share more revenue with the retailer, so the benefits and costs of information sharing should be carefully investigated to determine the optimal degree to carry out with the aim of maximizing profits.

Bayesian analysis is often applied to decision making under conditions of risk. First, previous experiences or subjective judgments are adopted as a basis to predict the probability of an uncertain event, i.e., the prior. Then, a likelihood function can be obtained by collecting actual information, such as doing experiments. With the obtained likelihood, the prior can be revised to obtain the posterior, which would then be closer to the actual probability. However, since Bayesian analysis is often complex to perform, the natural conjugate family is adopted to simplify the updating process, in which the prior and posterior have the same kernel function. One of the common natural conjugate families is that, when the prior has a normal distribution and the likelihood function is from a normally sampling scheme, then the posterior also has a normal distribution, but with different parameters, which are revised by considering both the prior and likelihood.

This study assumes that the natural conjugate family of the normal distribution can be used for the Bayesian updating process. Suppose that the retailer uses the consumer demand at t, D_t, to predict D_{t+1} and form the prior distribution, i.e., $D_{t+1} \sim N(\mu_t^r, v_t^r)$. The market demand information which is collected by the retailer can be modeled as the likelihood function, and the retailer can thus update the consumer demand forecast and learn the new posterior consumer demand, D'_{t+1}, i.e., $D'_{t+1} \sim N(\mu_t^{r'}, v_t^{r'})$. The retailer can use the new D'_{t+1} to recalculate its inventory level, $S_t^{r'}$, and in the meantime, share such information with the supplier. The supplier can then reforecast the retailer's order quantity, Y'_t, and determine the new inventory level, $S_t^{s'}$.

Suppose that the likelihood function, which is collected by the retailer from a normal sample of market demand, is normally distributed as $X_t \sim N(\bar{x}, \frac{s^2}{n})$, where $\bar{x}$ and $\frac{s^2}{n}$ denote the sample mean and variance of market demand, and n denotes the sample size. The more data the retailer collects, the more informative the collected data would contain. According to the Bayesian updating of the normal natural conjugated family, the updated (posterior) consumer demand would be normally distributed as $D'_{t+1} \sim N(\mu_t^{r'}, v_t^{r'})$, with the mean and variance being given by

$$\mu_t^{r'} = \frac{s^2}{n\sigma^2(1+\theta^2)+s^2}\mu_t^r + \frac{n\sigma^2(1+\theta^2)}{n\sigma^2(1+\theta^2)+s^2}\overline{x} = \frac{n\sigma^2(1+\theta^2)\bar{x}+s^2(d+\phi D_t)}{s^2+n\sigma^2(1+\theta^2)} \tag{11a}$$

and

$$v_t^{r'} = \frac{\sigma^2(1+\theta^2)s^2}{n\sigma^2(1+\theta^2)+s^2} \tag{11b}$$

respectively, and the inventory level is thus given by

$$S_t^{r'} = \mu_t^{r'} + k^r\sqrt{v_t^{r'}}. \tag{11c}$$

The posterior distribution of demand obtained by the Bayesian updating process indicates that as more data are collected by the retailer, i.e., as n rises, the variance of the updated demand forecast would be reduced more. The supplier can use the shared posterior information to predict the order quantities Y'_t at t and Y'_{t+1} during the lead time period t+1, which can then be used to recalculate the new inventory level, $S_t^{s'}$ during the lead time period. This updating process would reduce the variance of the supplier's inventory, and thus increase the profits of the supplier chain.

When the retailer updates the demand forecast and inventory level during the lead time and shares such information with the supplier, the supplier can then reforecast the retailer's order quantity, Y'_t,which is given by

$$Y'_t = D_t + S_t^{r'} - S_{t-1}^r = D_t + \frac{n\sigma^2(1+\theta^2)(d-\overline{x}+\phi D_t)}{s^2+n(1+\theta^2)\sigma^2} + k^r\left(\sqrt{\frac{s^2(1+\theta^2)\sigma^2}{s^2+n(1+\theta^2)\sigma^2}} - \sqrt{(1+\theta^2)\sigma^2}\right). \tag{12}$$

The supplier considers the retailer's new order quantity, Y'_{t+1}, during the lead time period to determine its own inventory level, $S_t^{s'}$. The new order quantity forecast during the lead time period t+1 can be rewritten as

$$Y'_{t+1} = D'_{t+1} + S_{t+1}^{r'} - S_t^{r'} = D'_{t+1} + \frac{s^2\phi(D_{t+1}-D_t)}{s^2+n(1+\theta^2)\sigma^2} = D'_{t+1} + \frac{s^2\phi(d+(\phi-1)D_t+\alpha_{t+1}-\theta\alpha_{t-s+1})}{s^2+n(1+\theta^2)\sigma^2} \tag{13}$$

with mean and variance given by

$$\mu_t^{s'} = E[Y'_{t+1}|X_t, D'_{t+1}, S_{t+1}^{r'}] = \frac{n\sigma^2(1+\theta^2)\bar{x}+s^2(d+\phi D_t)}{s^2+n\sigma^2(1+\theta^2)} + \frac{s^2\phi d}{s^2+n(1+\theta^2)\sigma^2} + \frac{s^2\phi(\phi-1)}{s^2+n(1+\theta^2)\sigma^2}D_t \tag{14a}$$

and

$$v_t^{s'} = Var[Y'_{t+1}|X_t, D'_{t+1}, S_{t+1}^{r'}] = \frac{\sigma^2(1+\theta^2)s^2}{n\sigma^2(1+\theta^2)+s^2} + \left(\frac{s^4\phi^2}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)(1+\theta^2) = \frac{s^2(1+\theta^2)\left[n\sigma^4(1+\theta^2)+s^2(\sigma^2+\phi^2)\right]}{n\sigma^2(1+\theta^2)+s^2}. \tag{14b}$$

Therefore, upon using the shared information, the supplier's optimal inventory level can be given as

$$S_t^{s'} = \mu_t^{s'} + k^s\sqrt{v_t^{s'}}. \tag{14c}$$

With information sharing, both the retailer and supplier can reduce the variances of their inventory levels, and more importantly, the variances can be reduced more as the sample size of the collected data, n, increases. However, the retailer has to bear the cost of collecting information, such as market research, staff salaries, and other related expenses, which would increase along with the size of sample. If the supplier can share more profits gained by utilizing the shared information with the retailer to compensate for the retailer's efforts and expenses with regard to information collection, which is thus the cost of information sharing for the supplier, the retailer would be more willing to collect and share more data. The more information that can be collected, the more the variance in the inventory level can be reduced, resulting in more profits along with a higher cost of information sharing, and thus the determination of the optimal degree of information sharing is carried out based on the trade-off between benefits and costs, with the goal of maximizing the supplier's net profit. This issue is discussed in more detail below.

C. Optimal Degree of Information Sharing

The supplier shares the increased revenue with the retailer as an incentive to enhance the latter's willingness to collect information, and the way this increased revenue is shared, has a significant impact on the supplier's cost model. We consider two cases in which the shared revenue linearly or exponentially increases along with the sample size (*n*) to derive the optimal degree of information sharing, and investigate their impacts on the profits of the supply chain.

We first investigate the case in which the shared revenue linearly increases with *n*. Suppose the supplier would share τ proportion of its revenue with the retailer, and the total amount of available information, N, which is determined during the information collection process, is a positive integer greater than *n*. Therefore, the shared revenue is given by $C = \pi\tau\frac{n}{N}$, where π denotes the benefit gained by the supplier due to information sharing, and *N* denotes the total amount of available information. The supplier's benefit due to information sharing often corresponds to the reduction of variance in the inventory level. Therefore, the supplier's benefit can be derived by considering the holding and shortage costs. According to Lee *et al.* [23], the total expected inventory cost is given by

$$E\left(h^s\int_0^{St}(S_t - x)dF_t(x) + p^s\int_{S_t}^{\infty}(x - S_t)dF_t(x)\right) = \sqrt{v_t}[(h^s + p^s)L(k^s) + h^s k^s].$$

Therefore, the benefit gained due to information sharing, π, can be obtained by comparing the difference between the prior and the posterior variances of the inventory levels, and this is given by

$$\pi = \left(\sqrt{v_t^s} - \sqrt{v_t^{s'}}\right)[(h^s + p^s)L(k^s) + h^s k^s] \tag{15}$$

where $k^s = \Phi^{-1}(\frac{p^s}{(p^s+h^s)})$ and $L(k^s) = \int_{k^s}^{\infty}(z - k^s)d\Phi(z)$ is the right-hand linear loss function of the standard normal distribution. Therefore, the supplier's net profit after information sharing is obtained as $B = \pi - C = \pi(1 - \frac{n}{N}\tau)$.

As the degree of information sharing increases, the supplier's shared revenue would increase, which eventually decreases its marginal benefits. This indicates that the supplier's maximum net profit can be obtained by an optimal degree of information sharing, n^*. Fig. 2 shows the relationship between the cost of sharing information and the supplier's benefits.

Proposition 1: For the linear case, the supplier's net profit is a convex function, and there exists an optimal degree of information sharing, n^*, which can maximize the net profit.

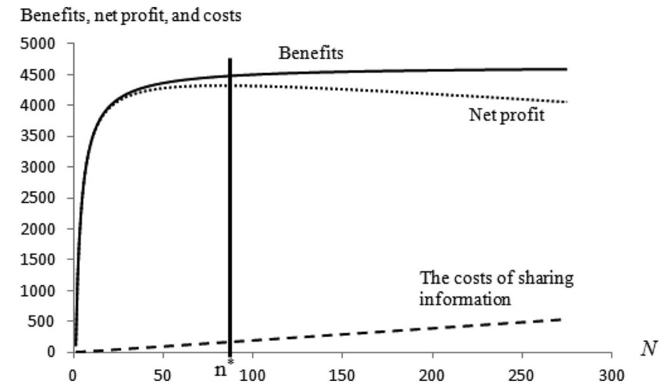

Fig. 2. Relationship between the benefits of information sharing and the sample size for the linear case.

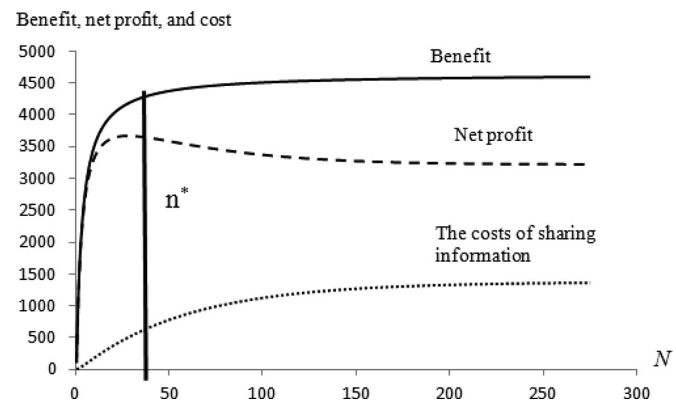

Fig. 3. Relationship between the benefits of information sharing and the sample size for the exponential.

Proof: Please see Appendix.

The supplier's benefit due to information sharing, π, is relatively higher at the beginning, but gradually decreases as the degree of information sharing rises. For the case that the shared revenue exponentially increases with *n*, this indicates that when the supplier gains more benefits due to information sharing, then it would share more revenue with the retailer. he supplier's benefit due to information sharing, π, is relatively higher at the beginning, but gradually decreases as the degree of information sharing rises. For the case that the shared revenue exponentially increases with *n*, this indicates that when the supplier gains more benefits due to information sharing, then it would share more revenue with the retailer. The shared revenue is thus given by $C = \tau\pi(1 - (e^{-\lambda n}))$, where $\lambda > 0$ is a parameter which adjusts the rate of sharing revenue. A small λ makes the shared revenue slowly increase with *n*, while a large λ makes it increase rapidly. Again, τ denotes the proportion of the revenue gained from information sharing that the supplier is willing to share.

Likewise, the benefit gained by the supplier due to information sharing, π, is given by $\pi = (\sqrt{v_t^s} - \sqrt{v_t^{s'}})[(h^s + p^s)L(k^s) + h^s k^s]$, and the net profit, B, is thus given by $B = \pi - C = \pi(1 - (1 - e^{-\lambda n})\tau)$. Fig. 3 shows the relationship between the cost of sharing information and the supplier's benefit for the exponential case.

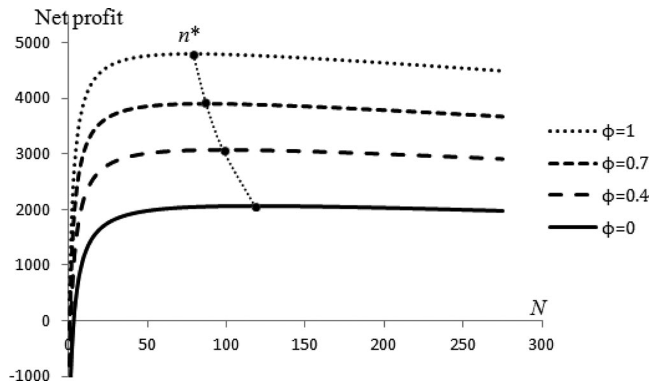

Fig. 4. Relationship between the supplier's net profit and ϕ.

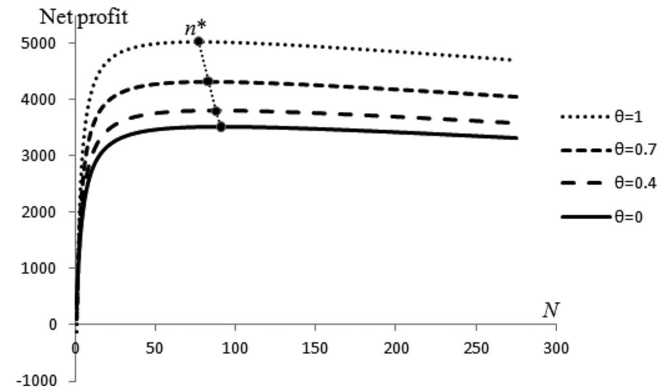

Fig. 5. Relationship between the supplier's net profit and θ.

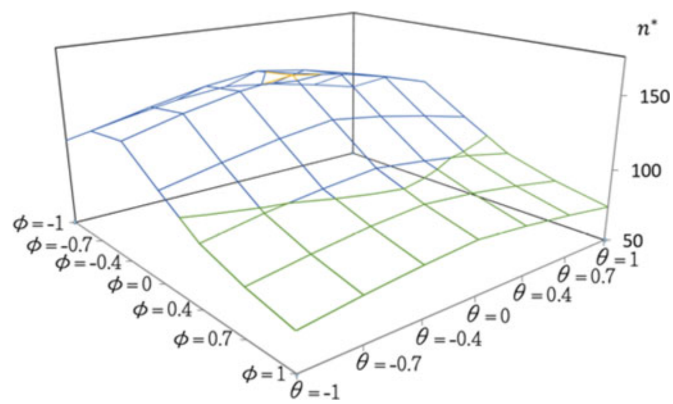

Fig. 6. Relationship among the optimal degree of information sharing ϕ and θ.

Proposition 2: For the exponential case, the supplier's net profit is a convex function, and there exists an optimal degree of information sharing, n^*, which can maximize the net profit.

Proof: Please see Appendix.

From the aforementioned discussion, it can be seen that exponentially increasing revenue sharing decreases the supplier's net profit faster than the linear case. However, no matter whether in the linear or exponential cases, the supplier can determine the optimal degree of information sharing, n^*, and the retailer can reduce its inventory variance by collecting the optimal amount of information and then obtaining the shared revenue from the supplier. Information sharing thus benefits both the supplier and retailer, and creates a win–win situation.

Proposition 3: The inventory holding cost, h^s, and the shortage cost, p^s, would not affect the optimal degree of information sharing, n^*, but their changes would affect the net profit.

Proof: Please see Appendix.

Proposition 4: When the correlation coefficient of demand for successive periods is within the range of $-\frac{2s^4+ns^2(-4-4\theta^2)\sigma^2-n^2(1.4142+1.4142\theta^2)^2\sigma^4}{s^4+ns^2(4+4\theta^2)\sigma^2+n^2(1.4142+1.4142\theta^2)^2\sigma^4} < \phi < 0$, information sharing would not benefit the supplier.

Proof: Please see Appendix.

Based on the aforementioned discussion, information sharing is profitable for both the supplier and retailer. That is, information sharing benefits the entire supply chain, and thus enhances its competitiveness.

D. *Sensitivity Analyses*

Since the optimal degree of information sharing may change with different parameter settings, this section investigates the relationship between this optimal degree of information sharing and the related parameters by performing sensitivity analyzes to have a better understanding of their mutual interactions. We use the case of linearly increasing cost in this investigation. Fig. 4 first shows the relationship between the supplier's net profit due to information sharing and the correlation coefficient of successive demand, ϕ.

As can be seen in Fig. 4, the supplier's net profit due to information sharing increases as the successive demand becomes more positively correlated. However, the optimal degree of information sharing decreases as ϕ increases since a more positive correlation results in less marginal profit. This can be explained by the fact that more positively correlated demand would make the future demand more predictable by looking at earlier demand, and thus the additional data needed for Bayesian updating would be less necessary. Likewise, the correlation coefficient of the successive seasonal cycle, θ, is also investigated, with the results shown in Fig. 5.

As can be seen in Fig. 5, similar impacts are found here as in the analysis of ϕ. Moreover, when both ϕ and θ change simultaneously, the optimal degree of information sharing would change dramatically, with the results shown in Fig. 6.

As can be seen in Fig. 6, when ϕ is close to -0.4 and θ is close to 0, the optimal degree of information sharing is the highest, and then decreases as ϕ increases in this case. The changes in θ do not significantly impact the optimal degree of information sharing, although those in ϕ do have a significant effect on this. Note that the curves in the middle of grid stand for the contour lines that connect the discrete points of the optimal information sharing degree together to form the surface. It is observed that the left side of the surface seems to have relatively high optimal degrees of information sharing. Accordingly, the decision makers should notice the tendency and react to possible situations.

V. Numerical Illustration

In order to illustrate the usefulness of the proposed model, a hypothetical dataset is utilized to investigate the impacts of

TABLE I
PARAMETER SETTINGS

Parameter	Value
h^s	850
p^s	964
ϕ	0.84
θ	0.7
σ	2
s	3.9
λ	3
D_0	18 649
N	700

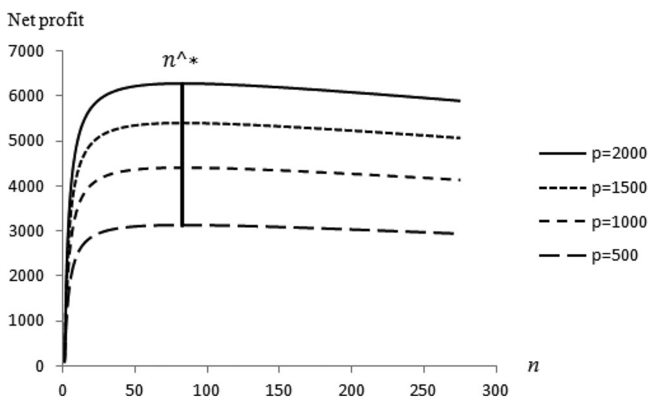

Fig. 7. Impact of the inventory cost.

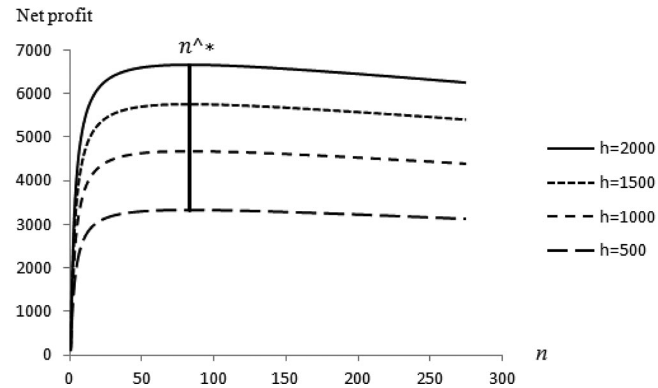

Fig. 8. Impact of the shortage cost.

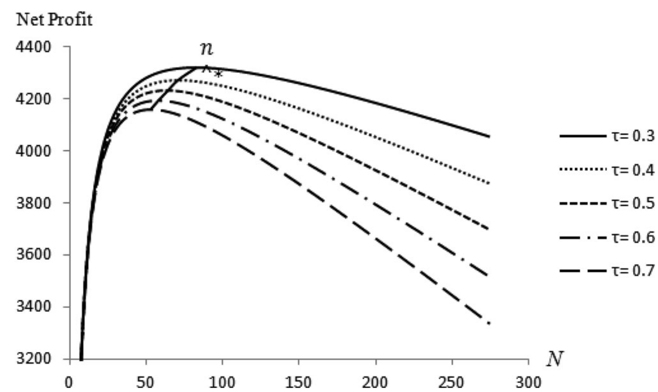

Fig. 9. Impact of the proportion of shared revenue.

information sharing. Suppose a two-echelon supply chain in which the supplier and retailer share information according to the proposed revenue sharing mechanism. Table I shows the parameter settings of the numerical example.

Based on these settings, we can obtain the service level of the inventory level, $k^s = \Phi^{-1}(\frac{p^s}{p^s+h^s}) = 0.5314$, and the right-hand linear loss function of the standard normal distribution, $L(k^s) = k^s \int_{k^s}^{\infty} (z - k^s) d\Phi(z) = 0.2976$. Accordingly, the optimal degree of information sharing, n^*, can be obtained for the cases of linearly and exponentially increasing revenue sharing, which are 96 and 39, respectively.

We further investigate the impacts that other related parameters have on the supplier's profits and the optimal degree of information sharing, such as the inventory cost, shortage cost, proportion of the shared revenue, exponentially increasing factor, and standard deviation of the demand error. Note that the sensitivity analysis is performed based on the case of linearly increasing revenue sharing. Figs. 7 and 8 show the impacts that the inventory and shortage costs have on the supplier's net profit with different sample sizes, respectively.

Figs. 7 and 8 verify Proposition 3, that changes in the inventory and shortage costs would not impact the supplier's optimal degree of information sharing, but would affect the supplier's net profit.

The proportion of shared revenue is the percentage of the benefit gains due to information sharing that the supplier is willing to share with the retailer. The higher the proportion is, the higher the cost of sharing information the supplier has to bear. Fig. 9 shows the relationships among the proportion of shared revenue, net profit, and optimal degree of information sharing with different sample sizes.

As can be seen in Fig. 9, when the proportion of shared revenue rises, the shared revenue increases dramatically, which forms a steeper curve. Therefore, under the condition in which the revenue increases slowly, the supplier's net profit would marginally decrease much earlier, and as a result the optimal degree of information sharing would decrease as the proportion of shared revenue increases.

For the case of exponentially increasing revenue sharing, we investigate the impacts that the adjusting factor, λ, has on the net profits with different sample sizes. Fig. 10 shows the results.

As can be seen in Fig. 10, a larger adjusting factor would result in a dramatic increase in the cost of sharing information, which decreases the supplier's net profit. As a result, the optimal degree of information sharing would decrease as λ increases, since collecting information becomes much more expensive.

Finally, the impact that the standard deviation of the random error of demand has on the net profit with different sample sizes is investigated, with the results shown in Fig. 11.

As can be seen in Fig. 11, when the standard deviation of demand decreases, i.e., the demand is less uncertain, less benefit would be gained by information sharing. A smaller n would result in a less significant increase in the supplier's benefit. In contrast, when the standard deviation of demand increases,

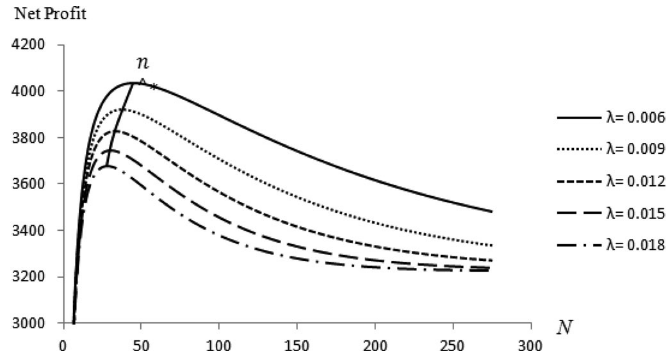

Fig. 10. Impact of the exponential adjusting factor.

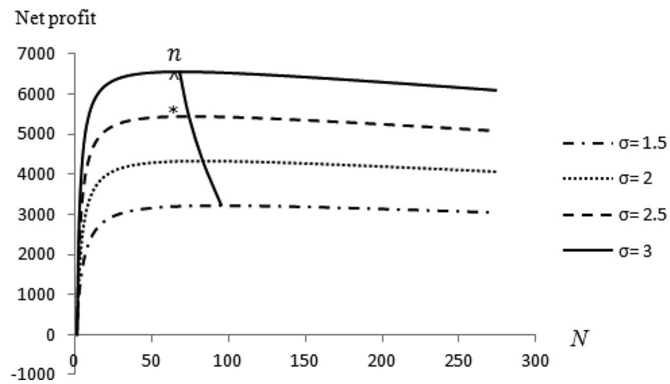

Fig. 11. Impact of the standard deviation of demand.

the benefit gained by information sharing also rises. Since the supplier has to bear more of the cost of information sharing when it obtains more benefits, this would thus result in a decrease in the optimal degree of information sharing.

VI. Conclusion

Information sharing plays an important role in the coordination of a supply chain, and previous studies have stated that it can benefit all members. However, the degree of information sharing is not always proportional to the benefits that are gained. In fact, the retailer often does not directly benefit from information sharing in practice. Accordingly, the retailer has no incentive to collect and share information to the supplier because of collecting data is costly. The supplier should thus share some proportion of the benefits it gains to encourage the retailer to collect and share information. As a result, the supplier has to consider the tradeoff between the benefit gained and the shared revenue to determine the optimal degree of information sharing, with the aim of maximizing profits. This study considers a two-echelon supply chain with seasonal consumer demand, in which the impacts of the degree of information sharing on the supplier's profits are investigated. The supplier obtains information from the retailer, which includes the consumer demand and inventory level and utilizes this to update its forecast of the retailer's order quantity, and thus reduce both inventory variance and inventory cost. Different supply chains may have different concerns in determining their optimal degrees of information sharing. This study considers two cases in which shared revenue is linear or exponentially increasing, and investigates their effects on the optimal degree of information sharing. The results show that both cases can benefit the supplier and the optimal degrees of information sharing can be obtained. The retailer can collect consumer information to update its forecast of consumer demand, which can not only reduce its own inventory variance but also gain some shared revenues from the supplier. Information sharing can thus benefit both the supplier and retailer in the supply chain. Since an excessive amount of information sharing can result in high costs for the supplier, and thus determining an optimal degree of information sharing to benefit both the supplier and retailer is essential for effective supply chain management, and proposing a way to achieve this is thus the main contribution of this study. In addition, the size of the variance of demand would influence the benefit of information sharing. In other words, if the demand is more uncertain, the information sharing will be more important to both the supplier and retailer. However, the optimal degree of information sharing decreases as the variance of demand increases. With regard to the shared revenue, if the proportion of shared revenue rises, the optimal degree of information sharing would decrease. Therefore, the decision maker should appropriately adjust the degree of information sharing under different conditions to ensure anticipated benefits are attainable.

This study can be extended by considering more players in a supply chain, as this would better reflect reality, although a game theoretic model may be needed to determine the equilibrium optimal degree of information sharing in this case. Since in practice inventories operate under various constraints, future studies may consider the cases of limited and deteriorating inventories. Finally, future research could also consider information sharing with stochastic lead times.

Appendix

Proof of Proposition 1

The second-order derivative of the net profit function with respect to n is given by unnumbered equation as shown at the bottom of the next page.

Since ϕ and θ are all quadratic, their signs would be irrelevant. In addition, since other parameters are all positive, the only thing that needs to be discussed is the following expression: $\frac{0.25s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+2\phi^2))^2}{\left(\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)^{1.5}} - \frac{(s^2+n(1+\theta^2)\sigma^2)^2(n(1+\theta^2)\sigma^4+s^2(\sigma^2+3\phi^2))}{\left(\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)^{0.5}}$. Since the denominator of the antecedent is greater than that of the consequent, and the parameters are all significantly less than the degree of information sharing, n, this means that the numerator of the antecedent is less than that of the consequent, and the difference between them would thus be negative. That is, there exists an optimal degree of information sharing, n^*, which can maximize the net profit, and this is given by

$$n^* = \arg\max_n \begin{pmatrix} [(h+p)L(k_s)+hk_s]\,(1-\frac{n}{N}\tau) \\ \times\left(\sqrt{(1+\theta^2)\sigma^2((1+\phi)^2+\phi^2)}\right. \\ \left.-\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}}\right) \end{pmatrix}.$$

Proof of Proposition 2

The second-order derivative of the new profit function with respect to *n* is given as unnumbered equation as shown at the bottom of the page, where, $t=(1+\theta^2)\sigma^2$.

Since ϕ and θ are all quadratic, whether they are negative or not is irrelevant. First, the denominators are positive in the first term within the square brackets, and $(\frac{1}{(nt+s^2)^2}-\frac{2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^3})$ in the numerator is obviously less than 0, so the first term is negative. The second term is affected by $(t(\phi^2+(1+\phi)^2))^{0.5}-(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2})$. The antecedent consists of different correlation coefficients and is less than the consequent due to the square root. When summing up the third term within the parentheses, the denominator of the antecedent, $\frac{0.25s^2(nt(t-2(1+\theta^2)\sigma^2)+s^2(t-2(1+\theta^2)(\sigma^2+\phi^2)))^2}{(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2})^{1.5}}$, has a greater power than that of the consequent. Therefore, the denominator of the antecedent is significantly less than that of the consequent, $\frac{(s^2+nt)^2(nt(-2t+3(1+\theta^2)\sigma^2)+s^2(-2t+3(1+\theta^2)(\sigma^2+\phi^2)))}{(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2})^{0.5}}$. The numerator of the consequent is greater than that of the antecedent due to the greater impacts of the degree of information sharing, n. Therefore, the second-order derivative of the net profit with respect to *n* is negative, which indicates that there exists an optimal degree of information sharing, n^*, which can maximize the net profit, and is given by

$$n^* = \arg\max_n \begin{pmatrix} [(h+p)L(k_s)+hk_s]\left(1-(1-\mathrm{e}^{(-\lambda n)})\tau\right) \\ \times\left(\sqrt{(1+\theta^2)\sigma^2((1+\phi)^2+\phi^2)}\right. \\ \left.-\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}}\right) \end{pmatrix}.$$

Proof of Proposition 3

By letting the first-order derivation of the net profit with respect to *n* be zero, the optimal degree of information sharing can be obtained, which is unrelated to the holding cost and the shortage cost, and this can be supported by the following equation: as shown at the top of the next page.

$$\frac{\partial^2}{\partial n}\left[\left(\sqrt{(1+\theta^2)\sigma^2((1+\phi)^2+\phi^2)}-\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}}\right)[(h+p)L(k_s)+hk_s]\left(1-\frac{n}{N}\tau\right)\right]$$
$$=\frac{1}{(s^2+n(1+\theta^2)\sigma^2)^6}ks^2(1+\theta^2)^2\sigma^2\begin{pmatrix} -\frac{(s^2+n(1+\theta^2)\sigma^2)^3\tau(n(1+\theta^2)\sigma^4+s^2(\sigma^2+2\phi^2))}{N\left(\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)^{0.5}} \\ +(1.+\theta^2)\sigma^2\left(1-\frac{n\tau}{N}\right)\begin{pmatrix} \frac{0.25s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+2\phi^2))^2}{\left(\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)^{1.5}} \\ -\frac{(s^2+n(1+\theta^2)\sigma^2)^2(n(1+\theta^2)\sigma^4+s^2(\sigma^2+3\phi^2))}{\left(\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}\right)^{0.5}} \end{pmatrix} \end{pmatrix}.$$

$$\frac{\partial^2}{\partial n}\left[\left(\sqrt{(1+\theta^2)\sigma^2((1+\phi)^2+\phi^2)}-\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}}\right)[(h+p)L(k_s)+hk_s]\left(1-(1-\mathrm{e}^{(-\lambda n)})\tau\right)\right]$$
$$=[(h+p)L(k_s)+hk_s]\begin{bmatrix} \frac{\mathrm{e}^{-n\lambda}s^2t^2\lambda\tau\left(\frac{1}{(nt+s^2)^2}-\frac{2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^3}\right)}{\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)^{0.5}}+\mathrm{e}^{-n\lambda}\lambda^2\tau\begin{pmatrix}(t(\phi^2+(1+\phi)^2))^{0.5} \\ -\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)\end{pmatrix}^{0.5} \\ +\frac{1}{(s^2+nt)^6}s^2t^2(1+(-1+\mathrm{e}^{-n\lambda})\tau)\begin{pmatrix}\frac{0.25s^2(nt(t-2(1+\theta^2)\sigma^2)+s^2(t-2(1+\theta^2)(\sigma^2+\phi^2)))^2}{\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)^{1.5}} \\ -\frac{(s^2+nt)^2(nt(-2t+3(1+\theta^2)\sigma^2)+s^2(-2t+3(1+\theta^2)(\sigma^2+\phi^2)))}{\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)^{0.5}}\end{pmatrix}\end{bmatrix}<0$$

$$\frac{\partial}{\partial n}\left[\left(\sqrt{(1+\theta^2)\sigma^2\left((1+\phi)^2+\phi^2\right)}-\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{\left(s^2+n(1+\theta^2)\sigma^2\right)^2}}\right)\left[(h+p)L(k_s)+hk_s\right]\left(1-\frac{n}{N}\tau\right)\right]$$

$$=\left[(h+p)L(k_s)+hk_s\right]\left(\frac{\frac{0.5s^2(1+\theta^2)t(N-n\tau)(nt\sigma^2+s^2(\sigma^2+2\phi^2))}{(s^2+nt)^3\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)^{0.5}}-\tau\left(\left(t(\phi^2+(1+\phi)^2)\right)^{0.5}-\left(\frac{s^2(1+\theta^2)(nt\sigma^2+s^2(\sigma^2+\phi^2))}{(s^2+nt)^2}\right)^{0.5}\right)}{N}\right)=0.$$

Proof of Proposition 4

The variance of the inventory levels without and with information sharing are $v_t^s=\sqrt{(1+\theta^2)\sigma^2\left((1+\phi)^2+\phi^2\right)}$, and $v_t^{s'}=\sqrt{\frac{s^2(1+\theta^2)(n(1+\theta^2)\sigma^4+s^2(\sigma^2+\phi^2))}{(s^2+n(1+\theta^2)\sigma^2)^2}}$, respectively. If the supplier can gain benefit due to information sharing, the former variance divided by the latter should be greater than 1. It is solved that when $-\frac{2s^4+ns^2(-4-4\theta^2)\sigma^2-n^2(1.4142+1.4142\theta^2)^2\sigma^4}{s^4+ns^2(4+4\theta^2)\sigma^2+n^2(1.4142+1.4142\theta^2)^2\sigma^4}<\phi<0$ the supplier benefits from information sharing. Note that since the correlation coefficient is between 1 to –1, if $-\frac{2s^4+ns^2(-4-4\theta^2)\sigma^2-n^2(1.4142+1.4142\theta^2)^2\sigma^4}{s^4+ns^2(4+4\theta^2)\sigma^2+n^2(1.4142+1.4142\theta^2)^2\sigma^4}$ is less than –1, the boundary is limited to –1<ϕ<0, i.e., only when ϕ is positive, the supplier benefits from information sharing.

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