The Bullwhip Effect in an Online Retail Supply Chain: A Perspective of Price-Sensitive Demand Based on the Price Discount in E-Commerce

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***Abstract*—This paper investigates the difference in bullwhip effects in online and offline retail supply chains, offering insights into how frequent price discounts in e-commerce influence the bullwhip effect in the online retail supply chain. We consider a two-level online retail supply chain with a manufacturer and an online retailer in which the demand faced by the retailer is price sensitive and based on the price discount. Assuming that the online retailer employs an optimal order-up-to inventory policy with an optimal minimum mean-squared error forecasting technique, we derive the expression of the bullwhip effect in the online retail supply chain and make analysis and comparison. Finally, we develop a dual-channel supply chain model to directly observe the impact of price discounts in e-commerce on the bullwhip effect. The results suggest that price discounts in the online retail market generally amplify the bullwhip effect in the online retail supply chain, but in certain conditions, the bullwhip in the online supply chain may be smaller than that in the offline supply chain. We also find that the relationship between the lead time and the bullwhip effect in the online supply chain presents a distinctive feature contrary to the conclusions of previous studies. Based on the analysis, we develop important managerial insights regarding online retail supply chains.**

***Index Terms*—Bullwhip effect, e-commerce, online retail market, online retail supply chain, price discount.**

Nomenclature

a	Market demand scale.
b	Price-sensitivity coefficient.
r	Price discount sensitivity coefficient.
s	Expectation price difference sensitivity coefficient.
δ	Standard deviation of the market price.
σ	Standard deviation of the market demand.
L	Lead time.
z	Safety factor.
μ	Constant in price model.
ρ	Price autoregression coefficient.
κ	Ratio of the online retail market demand to the total market demand.
d_t	Market demand in period t.
η_t	Price random disturbance term in period t.
ε_t	Demand random disturbance term in period t.
p_t	Market price in period t.
$p_{\min}$	Minimum expectation price.
r_t	Reference price in period t.
I_t	Inventory level in period t.
q_t	Order quantity in period t.
y_t	Order-up-to point in period t.
$\hat{\sigma}_t^L$	Estimate of the standard deviation of the L periods forecasting error in period t.
$\hat{d}_t$	Demand prediction in period t.
D_t^L	Lead-time demand in period t.
$\hat{D}_t^L$	Estimate of the lead-time demand in period t.
BWE_{online}	Bullwhip effect in the online retail supply chain.
BWE_{offline}	Bullwhip effect in the offline retail supply chain.

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I. Introduction

With the rapid growth of the global digital economy, a famous event in e-commerce is the "Double 11" promotion event in China, during which Tmall has achieved amazing sales records. Total transactions have rocketed from ¥19.1 billion in 2012 to ¥91.2 billion in the latest grand shopping festival in 2015. These promotion activities have strong appeal to online consumers largely due to the substantial price discounts. Frequent price discounts and dynamic pricing are marked features of e-commerce, because advanced information technologies have made it much easier and faster to adjust prices in online retails than in traditional retails. Although a well-planned promotion campaign usually generates generous profits, in the past, many firms were reluctant to consider sales campaigns, in part because event planning and implementation typically require large amounts of money, time, and effort [1]. However, the rise of the internet as a retailing channel has now provided firms with an ideal environment for testing the effects of price promotion activities and dynamic pricing on online sales. In particular, prices posted on a website can be changed instantly at a little cost for sellers [2]. In the traditional offline market, quantifying the effects of price promotion and dynamic pricing on consumer demands have been widely studied by researchers [3], [4]. However, due to the complexity of the shopping environment in e-commerce, especially with the emergence of powerful price comparison sites, recommendation systems, spike activities, and other auxiliary tactics in recent years, quantitative research of demand in the online retail market has become more complicated and difficult. Most studies do not report the impact on online retail market demands from the perspective of price discounts in e-commerce.

How do frequent price discounts in e-commerce affect the demand of online consumers? Here is an example. On October 21th, 2015, Mindy, who lives in China, was searching for e-books on Tmall and ultimately chose the Hanvon E920 e-book, which was priced at ¥1759, after comparison. Then, she opened a price comparison website called Huihui shopping assistant, in which she found the price history chart of that e-book and the complete past price information since October 21, 2014. Mindy observed the line chart and paid special attention to three points: the minimum price of ¥1600 during "Double 11" in 2014, which was the same as the presale price of "Double 11" in 2015, the recent price of ¥1699 last week, and the current price of ¥1759. In the end, Mindy decided to wait until November 11th to purchase the e-book, at the price of ¥1600, because her demand for the e-book was not that urgent and she anticipated that the price would drop to the minimum of ¥1600 again during the "Double 11" event. From this, it can be preliminarily inferred that the observed prices, which were lower than the current price, had a negative impact on Mindy's current demand for that e-book. As such, the observed price information as a reference can directly change the purchasing decisions of individual online consumers and subsequently affect the overall online demands.

As for the relationship between price discounts in e-commerce and the bullwhip effect in the online retail supply chain, we gain inspiration from the case of the grand promotion activity "Double-11" in China. To prepare for the event and make large profits, online retailers usually stock up on products in advance according to the prediction of future demands. It is a game of inventory and sales for online retailers. If online retailers make the right forecast and win the game, they will not only make a fortune but also gain new customers from the event. In contrast, if online retailers overestimate the sales in "Double-11," they will backlog huge stock and need a few months to deal with the inventory or even longer if the daily sales volume is low. Actually, after the "Double-11," many e-commerce companies transfer into the inventory-clearing period. High inventory means costs, the occupancy of capital and disrupted operation schedules, resulting in damage to the interests of companies, which is so called the "bullwhip effect." It follows that price discounts in e-commerce will impact the bullwhip effect in the online retail supply chain.

In this paper, we address the following question: How do frequent price discounts in e-commerce influence the online consumers' demand and the bullwhip effect in an online retail supply chain? We are specifically interested in the features of the bullwhip effect in the online supply chain and its difference with that in the traditional offline supply chain. With distinctive features, online retail supply chains differ from the traditional supply chains in many aspects. However, no research has been conducted on the bullwhip effect in the online retail supply chain. To explore this issue, based on reference price theory, we propose a demand model with price discounts in the online retail market. Considering a two-level online supply chain with a manufacturer and an online retailer, we further derive the analytical expression of the bullwhip effect in the online supply chain with an optimal order-up-to inventory policy and optimal minimum mean-squared error (MMSE) forecasting technique. After analyzing the properties of the effect, we compare it with the bullwhip effect in the offline supply chain. Finally, to directly observe the impact of price discounts on the bullwhip effect, we develop a dual-channel supply chain model.

The main contribution of this study is fourfold: First, we develop a demand model with price discounts in the e-commerce market. Second, we quantify and analyze the bullwhip effect in an online retail supply chain. Third, we make a comparison between the bullwhip effects in online and offline supply chains. Finally, we reveal the impact of price discounts in e-commerce on the bullwhip effect in the online retail supply chain.

The remainder of this paper is organized as follows. Section II reviews the literature. Section III presents the demand model and the price process in a two-level online retail supply chain. Section IV introduces the order-up-to inventory policy model and the MMSE forecasting technique. Section V derives the analytical expression of the bullwhip effect in the online retail supply chain. Section VI provides analysis and comparison. Section VII concludes this paper and suggests follow-up research directions.

II. Literature Review

Our research builds on two lines of literature: The literature on the bullwhip effect and the literature on the impact of the reference price on online consumer purchasing behaviors.

A. *Bullwhip Effect*

The bullwhip effect is a phenomenon of information distortion as ordering information moves upstream. It occurs when a downstream demand fluctuation leads to larger fluctuations in the variance of upstream ordering. The discovery of the effect can be traced back to Forrester [5], [6] who investigated its causes and possible remedies in the context of industrial dynamics. Forrester's work has drawn a great attention to the existence of this phenomenon in supply chains. To date, bullwhip effects in traditional offline supply chains have been studied extensively by researchers.

According to Wang and Disney [7], the five main elements in bullwhip modeling are demand, the lead time (a.k.a. time delay), the forecasting policy, the ordering policy, and information-sharing mechanisms.

First is the impact of demand. In bullwhip modeling, commonly used demand models include auto-regressive moving average (ARMA) models, price-sensitive models, and mixed models, respectively. The ARMA models are most broadly studied by researchers, especially the special form autoregressive AR(1) model [8]14]. Lee *et al.* [8], [9] were the first to apply the AR(1) demand model to the analysis of the bullwhip effect. They also provided a formal definition of the bullwhip effect and systematically analyzed four main causes. Chen *et al.* [11], [12] made a great contribution by recognizing the role of demand forecasting as a filter for the bullwhip effect with the AR(1) demand process. In addition to the AR(1) model, the general types of ARMA models have been applied [15]–[17]. The bullwhip effect has also been examined in price-sensitive demand processes [18]–[21]. Ma *et al.* [18] analyzed the bullwhip effects on product orders and on inventory using a price-sensitive demand model. Wang *et al.* [20] considered linear and isoelastic demand functions that both depend on the prices in multiple periods. Ma *et al.* [21] analyzed the bullwhip effect in two parallel supply chains with interacting price-sensitive demands. A relatively small amount of studies have considered the mixed demand model [22], [23]. Zhang and Burke [23] proposed a mixed demand model and discussed the impacts of the demand autoregression coefficient and price-sensitive coefficient. The choice of demand model depends on the complexity of the supply chain structure, the market environment, the research objectives, etc. This paper intends to study the bullwhip effect in a two-level online retail supply chain and analyze the impact of the online retail channel with price discounts on the supply chain. Because the price-dependent model is beneficial for the analysis of price dynamics, we develop a price-sensitive model to survey the bullwhip. In addition, in light of Ma *et al.* [18], the price-sensitive model involves in more practical demand and price characteristic parameters (i.e., price sensitivity, demand elasticity, and market scale), providing more managerial insights.

Second is the impact of lead time. Forrester [6] found that the lead time is a driving factor of demand amplification. Lee *et al.* [9] and Chen and Simchi-Levi [12] revealed that bullwhip effects increase with the lead time, which is supported by Steckel *et al.* [24] and Agrawal *et al.* [14]. Over the years, others have discovered that this relationship does not always hold [25], [26]. Luong [25] found that when the second-order autoregressive coefficient is negative, the bullwhip effect will not always increase as the lead time increases and instead presents a complex changing trend. Ma and Ma [26] showed that a smaller lead time does not always lead to a lower bullwhip effect, but a much greater lead time does result in a higher bullwhip effect. In this paper, we also investigate the relationship between the lead time and the bullwhip effect in the online supply chain. It is remarkable that our finding is different from all the above research.

Finally, as for forecasting policy and ordering policy, we suppose that online retailers adopt the optimal ordering policy (i.e., order-up-to policy [10]) and the optimal forecasting technique (i.e., MMSE forecasting technique [27]).

B. Impact of Reference Price on Online Consumer Purchasing Behaviors

Consumers often set their price expectation prior to paying for a product or service in online and offline markets. According to Lewis and Shoemaker [28], the price expectation, or the amount customers expect to pay for a product or service, is used as a reference point to compare prices and guide purchase decisions [29]. The reference price is usually based on a variety of price information, such as regular prices, presale prices, historical prices, and discounted prices.

Most consumers will inevitably be affected by the reference price in making their purchasing decisions. Moon *et al.* [30] found that the majority of consumers, approximately 91%, follow reference price mechanisms in their buying behaviors. Consumers, especially online shoppers, continually monitor the price environment and change their purchasing behavior accordingly. Moreover, they use the information they own, which is based on their memory of the last time they purchased products or the most recent brand promotion. Erdem *et al.* [31] revealed that the reference price is based on economically rational behavior and used as a signal of quality or a predictor of future prices. Hence, past prices are used as reference prices to predict future prices and unobserved or imperfectly observed quality.

Online price comparison sites, or shopbots, provide consumers with easy access to price comparison for desired products. Because shopbots greatly reduce the cost of searching for information about products and facilitate better and more efficient purchase decisions, they have become extremely popular among consumers who shop on the internet. Since 2011, shopbots, such as NexTag.com, PriceScan.com, and Skinflint.co.uk, have introduced line charts displaying a product's full price history, in addition to providing different sellers' offers for any product in the form of price comparison tables. NexTag.com calls this chart the "price history," whereas on pricescan.com, it is called the "price trend graph." These price history charts are easily available for online consumers around the world. As shown by Klein and Oglethorpe [32], historical price is a type of reference price that represents consumers' personal purchasing experiences. Similarly, Kopalle and Lindsey-Mullikin [33] suggested that information about a product's price history is a source of reference prices and should, therefore, stimulate consumer purchasing behavior. Danziger and Segev [34] argued that price expectations and purchase timing are conceptually related. Thus, price history charts provided by shopbots should support consumers in forming expectations about future prices and enforce strategic buying behavior with respect to buying now or later. In addition, in a shopping context, reference prices lower than the offer price represent negative stimuli. Buyers will attach more weight to lower price information [35]. Furthermore, Briesch *et al.* [36] revealed that memory or observed information of recent prices has a greater reference effect than earlier prices. Taken together, these studies imply a possible correlation between online demand and historical price information as reference prices.

To the best of our knowledge, we are the first to quantify and analyze the bullwhip effect in the online retail supply chain and investigate the difference in bullwhip effects between online and offline supply chains. In the context of the e-commerce market, we develop a demand model dependent on the price and price discounts based on the reference price theory. On that basis, we further measure the impact of price discounts on the bullwhip effect in the online retail supply chain.

III. Demand Model

Consider a simple two-level online retail supply chain with a manufacturer and an online retailer. The external demand for a single product occurs at the retailer and is price sensitive. In this context, we build a demand model with price discounts in e-commerce marketplace.

A. Demand Model With Price Discounts in E-Commerce

With the wide diffusion of e-commerce, online consumers can easily find the full price histories of any products through price history charts provided by shopbots or price comparison sites. Historical prices as a type of reference prices have guiding impacts on online consumers' demands. The relationship between reference prices and consumers' purchasing behavior is supported by many researchers [30], [32], [33], [35], [37], [38]. Prior studies provide significant afflatus for building a demand model in the e-commerce marketplace, where the price history and other price information can be seen by online shoppers through price comparison sites, and, thus, used as reference price points in their purchasing decisions. According to Birnbaum and Stegner [35], reference prices, lower than the currently offered price, represent negative stimuli on current demands. Furthermore, Briesch *et al.* [36] revealed that memory or observed information of recent prices has a greater reference effect than earlier prices, and the latest price observed by the consumer is the strongest determinant of the consumer reference price. Mayhew and Winer [39] and Krishnamurthi [40] used the price on the last occasion p_{t-1} as the reference price and correspondingly $p_{t-1} - p_t$ to describe the gain and loss of utility for consumers. In addition, Rajendran and Tellis [38] and Birnbaum and Stegner [35] showed that buyers will attach more weight to lower price information. The lowest price is recognized as an important reference price point [37] and was incorporated into the reference price model by Rajendran and Tellis [38]. Especially in e-commerce, the minimum discount price (i.e., the presale price in "Double 11," the minimum price in the historical price chart) is particularly important because it is usually posted in a conspicuous spot on the website to draw the eye and is usually deliberately guided by shopbots.

As such, we choose the last period price p_{t-1} and the minimum discount price $p_{\min}$ as the reference price points influencing online consumers' purchasing behaviors and build a demand model in the e-commerce market

$$d_t = D_t(p_t) + R_t(r_t, p_t) + \varepsilon_t \tag{1}$$

where

$$D_t(p_t) = a - bp_t \tag{2}$$

$$R_t(r_t, p_t) = b(r + s)(r_t - p_t) = b(r + s)\left(\left(\frac{r}{r+s}p_{t-1} + \frac{s}{r+s}p_{\min}\right) - p_t\right). \tag{3}$$

According to Popescu and Wu [41] and Huh *et al.* [42], for each period t, the demand d_t can be separated into two components. The first component is demand in the absence of the reference price effect, which is denoted by $D_t(p_t)$. The second component is an adjustment owing to the reference price effect, denoted by $R_t(r_t, p_t)$. Similarly, we develop a demand model with the reference price effect in (1)–(3).

By simplification, we obtain the following demand model:

$$d_t = a - bp_t - rb(p_t - p_{t-1}) - sb(p_t - p_{\min}) + \varepsilon_t \tag{4}$$

where $a > 0$, $b > 0$, $0 \le r < 1$, and $0 \le s < 1$.

In this formula, b denotes the price-sensitivity coefficient, and bp_t stands for the demand part influenced by the current period price. Demand decreases as the current period price increases. r denotes the price discount-sensitivity coefficient, and $rb(p_t - p_{t-1})$ represents the demand part affected by the current price discount range by comparing the current period price with the previous period price. If $p_t < p_{t-1}$, a current price discount will occur; thus, the demand will increase. When the demand elasticity of products is small, which is equivalent to low-price sensitivity, consumers' demands will be relatively rigid and urgent. Correspondingly, consumers will have lower price discount sensitivity. This suggests that the price discount sensitivity is positively correlated with the price sensitivity; hence, the expression of the price discount sensitivity is rb. $p_{\min}$ is the consumers' expected price formed from observing the minimum discount price information. It is defined as the minimum expectation price and equals the minimum value between the minimum promotion price in the past and an observed discount price in the near future (i.e., the presale price). s denotes the expectation price difference sensitivity coefficient, and $sb(p_t - p_{\min})$ is the demand part influenced by the difference between the current price and the minimum expectation price. When this difference range increases, the demand will decrease. Likewise, there is a positive correlation between the expectation price difference sensitivity and the price sensitivity, and we express the expectation price difference sensitivity as sb.

In this paper, we define the comparison between the current price and the reference prices (i.e., the historical price, the observed price, etc.) as the real-time price discount, which is denoted by $r_t - p_t$. $p_t < r_t$ means a positive price discount, and $p_t > r_t$ means a negative price discount. Thus, the real-time price discount information includes $p_t - p_{t-1}$ and $p_t - p_{\min}$.

When $r = 0$ and $s = 0$, the demand model becomes (5). It has been used in previous studies [18], [19] to describe the demand in offline markets, where consumers usually have no complete information about the full price history of a product and their purchasing decisions are based solely on the current period price, and they cannot capture the real-time price discounts by comparing the current price with the previous prices, as online consumers can. Therefore, we see that whether the real-time price discounts can be perceived by consumers is an important mark of distinction between the online retail market and the offline market.

$$d_t = a - bp_t + \varepsilon_t \tag{5}$$

where ε_t is an independent identically distributed (i.i.d.) normally distributed demand shock with mean zero and variance σ^2. We further assume that ε_t has no relation with the market price. Therefore, the covariance between the error term and the market price is zero: $\mathrm{Cov}(p_t, \varepsilon_{t'}) = 0,\ (\forall t, t')$.

We consider an online retail market setting in which the retailer sells on a perfectly competitive market and exerts no control over the market clearing price, and the market price evolution is determined by the overall market demand and supply. The market price p_t in (4) is an AR (1) pricing dynamic process

$$p_t = \mu + \rho p_{t-1} + \eta_t, 0 < \rho < 1 \tag{6}$$

where η_t is an i.i.d. normally distributed effect of overall market shocks on the price across time with mean zero and variance δ^2. The condition of $0 < \rho < 1$ ensures that the AR (1) pricing process is stationary. Similar to Zhang and Burke [23], we assume that the error term η_t and the market price have a covariance structure: $\mathrm{Cov}(p_t, \eta_{t'}) = 0, (t < t')$.

Thus, from (4) and (6), we obtain the expects and variances of price and demand, respectively: $\mu_p = E(p_t) = \mu/(1-\rho)$, $\sigma_p^2 = \mathrm{Var}(p_t) = \delta^2/(1-\rho^2)$, $\mu_d = E(d_t) = a + sbp_{\min} - (1+s)b\mu_p$, $\sigma_d^2 = \mathrm{Var}(d_t) = ((1+r+s)^2 + r^2 - 2(1+r+s)r\rho)b^2\sigma_p^2 + \sigma^2$.

We further deduce the covariance of η_t and ε_t: $\mathrm{Cov}(p_t, \varepsilon_{t'}) = \mathrm{Cov}(\eta_t, \varepsilon_{t'}) = 0, (\forall t, t')$ for any t or t'. In other words, the error terms are independent across time and are not correlated contemporaneously.

The following can be derived from (4) and (6):

$$d_t = w + \lambda d_{t-1} + h_t \tag{7}$$

where $\lambda = \rho$, $h_t = \varepsilon_t - (1+r+s)b\eta_t + rb\eta_{t-1} - \rho\varepsilon_{t-1}$, and $w = a + sbp_{\min} - (a\rho + \mu b(1+s) + sb\rho p_{\min})$. This equation reveals that the demand in (4) and the price dynamics in (6) can be simplified to an autoregressive demand process. Using an actual consumption demand data, Erkip and Nahmias [43] observed that the autoregression coefficient of the demand in adjacent periods hovers at approximately 0.7. The empirical work of Lee *et al.* [10] showed that the autoregression correlation coefficients of the demand of most products are greater than zero and fluctuate between 0.26 and 0.89 (see Lee *et al.* [9], [10] for more details). As shown by (7), the autoregression coefficients of the demand and the price are equal, so this paper sets the autoregression coefficient $0 < \rho < 1$. It should be noted that h_t is a function of the two error terms η_t and ε_t. Thus, the simplified demand model is an autoregressive demand process but not an AR(1) process.

IV. Ordering Process

We adopt the following sequence of events during the replenishment period. The online retailer observes consumer demand d_{t-1} at the end of period $t-1$ and places an order of quantity q_t to the manufacturer at the beginning of period t according to its current inventory level. After lead time L, the retailer receives the product from the manufacturer at the beginning of period $t+L$. In this section, we introduce the order-up-to inventory policy and the MMSE forecasting technique to calculate the order-up-to level and the ordering quantity.

A. *Order-up-to Inventory Policy*

The order-up-to policy is one of the most widely studied policies of supply chain models. When demands are stationary, resupply is infinite, product purchase cost is stationary, and there is no fixed order cost, the order-up-to policy is considered as the locally optimal inventory policy, which can minimize the total discounted holding and shortage costs [9], [10]. Assuming that the online retailer adopts the order-up-to inventory policy, the ordering decision is as follows:

$$q_t = y_t - (y_{t-1} - d_{t-1}). \tag{8}$$

The order-up-to level consists of an anticipation stock that is retained to meet the expected lead-time demand and a safety stock for hedging against unexpected demand. Therefore, the order-up-to level is updated in every period according to the following:

$$y_t = \hat{D}_t^L + z\hat{\sigma}_t^L \tag{9}$$

where D_t^L is the lead-time demand equal to the sum of demands of L periods in the interval $[t, t+L)$, $\hat{D}_t^L$ is an estimate of the mean lead-time demand, z is a constant that has been set to meet a desired service level and is often referred to as the safety factor [11], [12], and the estimate of the standard deviation of the L period forecasting error is $\hat{\sigma}_t^L = \sqrt{Var(D_t^L - \hat{D}_t^L)}$.

Gaalman and Disney [44] showed that the order-up-to policy is optimal when the demand is normally distributed. We have shown that our demand model in (4) and price dynamics model in (6) can be simplified to an autoregressive demand process as shown in (7). Because the errors η_t and ε_t are both i.i.d. normally distributed and are not correlated contemporaneously, the price p_t and the demand d_t are also both normally distributed. Therefore, in this research, the online retailer uses the optimal order-up-to inventory policy.

Substituting (9) into (8), the order quantity q_t can be rewritten as follows:

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + d_{t-1} + z\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right). \tag{10}$$

B. *MMSE Forecasting Technique*

To calculate the order-up-to level y_t, the online retailer should use certain forecasting techniques to estimate the mean lead-time demand $\hat{D}_t^L$. The three most commonly used forecasting techniques are MMSE, moving average, and exponential smoothing. Among them, MMSE has the smallest error. MMSE is provided by the conditional expectation given to previous observations and is considered as an optimal forecasting procedure that minimizes the mean-squared forecasting error [27]. Thus, this paper uses the MMSE forecasting technique to analyze the bullwhip effect in the online retail supply chain.

We suppose that the retailer in the online retail supply chain has adopted the optimal inventory policy and the optimal forecasting technique.

V. Bullwhip Effect in the Online Retail Supply Chain

In this section, we derive analytical expression of the bullwhip effect in the online retail supply chain. First, we analyze the retailer's order quantity, which is a forward step of computing the bullwhip effect.

A. *Online Retailer's Ordering Decision*

It has been shown that the MMSE forecast of period $t+i, (i=0,1,2,3,...)$ is the conditional expectation of d_{t+i} given previous observations $d_{t-1}, d_{t-2}, ...$ [27]. Let $\hat{d}_{t+i}$ be the demand forecast of period $t+i, (i=0,1,2,3,...)$ made at the end of period $t-1$. In the case of the AR (1) demand process, it has been shown that the MMSE forecast of $\hat{d}_{t+i}$ is given by $\hat{d}_{t+i} = E(d_{t+i}|d_{t-1})$ [10]. This paper considers a price-sensitive demand function in which the price follows an AR (1) process. Let $\hat{p}_{t+i}$ be the market price forecast of period $t+i$ made at the end of period $t-1$. According to Ma *et al.* [18], [19], for the AR (1) pricing process, it can be done in a similar way that $\hat{p}_{t+i}$ can be given as the future price conditioned on the actual price observed up to period $t-1$, i.e., $\hat{p}_{t+i} = E(p_{t+i}|p_{t-1})$. Thus, we can derive the demand forecast of period $t+i$ as follows:

$$\hat{d}_{t+i} = a - (1+r+s)\, b\hat{p}_{t+i} + rb\hat{p}_{t+i-1} + sbp_{\min}. \tag{11}$$

By recursively applying (6), we can obtain the following formulas:

$$\begin{aligned} p_{t+i} &= \mu + \rho p_{t+i-1} + \eta_{t+i} \\ &= (1+\rho)\,\mu + \rho^2 p_{t+i-2} + (\rho\eta_{t+i-1} + \eta_{t+i}) \\ &= ... = \frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1} p_{t-1} + \sum_{j=0}^{i} \rho^{i-j}\eta_{t+j} \end{aligned} \tag{12}$$

and

$$p_{t+i-1} = \frac{1-\rho^i}{1-\rho}\mu + \rho^i p_{t-1} + \sum_{j=0}^{i-1} \rho^{i-1-j}\eta_{t+j}. \tag{13}$$

Thus, the market price forecasts of period $t+i$ and period $t+i-1$ are as follows:

$$\hat{p}_{t+i} = E\left(p_{t+i}\,|p_{t-1}\right) = \frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1} p_{t-1} \tag{14}$$

and

$$\hat{p}_{t+i-1} = E\left(p_{t+i-1}\,|p_{t-1}\right) = \frac{1-\rho^{i}}{1-\rho}\mu + \rho^{i} p_{t-1}. \tag{15}$$

Substituting (14) and (15) into (11), we can compute the demand forecast of period $t+i$

$$\begin{aligned} \hat{d}_{t+i} = a - (1+r+s)\, b\left(\frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1}p_{t-1}\right) \\ + rb\left(\frac{1-\rho^{i}}{1-\rho}\mu + \rho^{i}p_{t-1}\right) + sbp_{\min}. \end{aligned} \tag{16}$$

Thus, the estimate of the mean lead-time demand is the following:

$$\begin{aligned} \hat{D}_t^L &= \sum_{i=0}^{L-1} \hat{d}_{t+i} \\ &= La + Lsbp_{\min} - \frac{(1+r+s)\,\mu b\,(L-\rho\Lambda_L)}{1-\rho} \\ &\quad + \frac{r\mu b\,(L-\Lambda_L)}{1-\rho} + (r\Lambda_L - (1+r+s)\,\rho\Lambda_L)\, bp_{t-1} \end{aligned} \tag{17}$$

where $\Lambda_L = \frac{1-\rho^L}{1-\rho}$.

Substituting (17) into (10), we obtain the following:

$$\begin{aligned} q_t = b\,(r\Lambda_L - (1+r+s)\,\rho\Lambda_L)\,(p_{t-1} - p_{t-2}) \\ + d_{t-1} + z\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right). \end{aligned} \tag{18}$$

Lemma 1: When the MMSE technique is used to forecast the lead-time demand, the variance of forecasting error for the lead-time demand in the online retail supply chain remains constant over time and is given by the following:

$$\left(\hat{\sigma}_t^L\right)^2 = L\sigma^2 + (1+r+s)^2 b^2\delta^2 + \Phi b^2\delta^2/(1-\rho)^2 \tag{19}$$

where

$$\begin{aligned} \Phi = {} & (1+r+s)^2\left(L - 1 - \frac{2\left(\rho^2 - \rho^{1+L}\right)}{1-\rho} + \frac{\rho^4 - \rho^{2+2L}}{1-\rho^2}\right) \\ & + r^2\left(L - 1 - \frac{2\left(\rho - \rho^L\right)}{1-\rho} + \frac{\rho^2 - \rho^{2L}}{1-\rho^2}\right) \\ & - 2r\,(1+r+s) \\ & \left(L - 1 - \frac{\rho - \rho^L}{1-\rho} + \frac{\rho^3 - \rho^{2L+1}}{1-\rho^2} - \frac{\rho^2 - \rho^{1+L}}{1-\rho}\right). \end{aligned} \tag{20}$$

Proof: See Appendix A.

From Lemma 1, it is obvious that the variance of the lead-time demand forecasting error in an online retail supply chain is independent of time with the MMSE technique, which means $\hat{\sigma}_t^L = \hat{\sigma}_{t'}^L, (\forall t, t')$. Substituting $\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L = 0$ into (18), we obtain the retailer's order quantity

$$q_t = b\,(r\Lambda_L - (1+r+s)\,\rho\Lambda_L)\,(p_{t-1} - p_{t-2}) + d_{t-1}. \tag{21}$$

B. *Bullwhip Effect in the Online Retail Supply Chain*

The bullwhip effect is computed as the ratio of the order variance of the retailer to the demand variance of the customer

[9], [10]. When the ratio is larger than 1, it means that a bullwhip effect exists. This information distortion represents potential costs for the supply chain, including misguided capacity plans, missed production schedules, and inactive transportation [8]. Therefore, it is desirable to reduce the bullwhip effect. Using (21), the expression of the bullwhip effect in an online retail supply chain is shown by Theorem 1.

Theorem 1: Assuming that an online retailer uses the order-up-to inventory policy and the MMSE forecasting technique, the bullwhip effect in a two-level online retail supply chain can be expressed as follows:

$$BWE_{\text{online}} = \frac{\mathrm{Var}(q_t)}{\mathrm{Var}(d_t)} = 1 + \frac{\frac{2b^2\delta^2}{1+\rho}\begin{pmatrix}(r\Lambda_L - (1+r+s)\,\rho\Lambda_L)^2 \\ -(r\Lambda_L - (1+r+s)\,\rho\Lambda_L)\,(1+2r+s)\end{pmatrix}}{\sigma_d^2} \tag{22}$$

where $\sigma_d^2 = ((1+r+s)^2 + r^2 - 2(1+r+s)r\rho)b^2\sigma_p^2 + \sigma^2$, $\Lambda_L = \frac{1-\rho^L}{1-\rho}$ and $\sigma_p^2 = \frac{\delta^2}{1-\rho^2}$.

Proof: See Appendix B.

From the expression of the bullwhip effect in Theorem 1, we know that the bullwhip effect in the online retail supply chain depends on the following seven parameters: the price-sensitivity coefficient b, the price autoregression coefficient ρ, the price discount-sensitivity coefficient r, the expectation price difference sensitivity coefficient s, the lead time L, and the variances of error terms σ^2 and δ^2. In contrast, the market demand scale a and the minimum expectation price $p_{\min}$ have no effect on the bullwhip effect in the online retail supply chain.

VI. Analysis and Comparison

In this section, we analyze the properties of the bullwhip effect in an online retail supply chain, compare the bullwhip effects in online and offline supply chains and investigate the impact of price discounts in e-commerce on the bullwhip effect.

A. Condition of Eliminating the Bullwhip Effect in the Online Retail Supply Chain

To simplify the analysis, we set $\sigma^2 = \delta^2$ and obtain the simplified expression of BWE_{online} as follows:

$$BWE_{\text{online}}^{\text{simplified}} = 1 + \frac{2b^2\,(r-(1+r+s)\,\rho)\,(1-\rho^L)\begin{pmatrix}(r-(1+r+s)\,\rho)\,(1-\rho^L)\\ -(1-\rho)\,(1+2r+s)\end{pmatrix}}{(1-\rho)\left(1-\rho^2+b^2\left(r^2+(1+r+s)^2-2r\,(1+r+s)\,\rho\right)\right)}. \tag{23}$$

Proposition 1: When $\rho \le \frac{r}{1+r+s} \le \rho + \frac{1-\rho^2}{\rho-\rho^L}$ or $b = 0$ holds, the bullwhip effect in the online retail supply chain will be eliminated.

Proof: see Appendix C.

To illustrate the condition in Proposition 1, the following numerical example is studied.

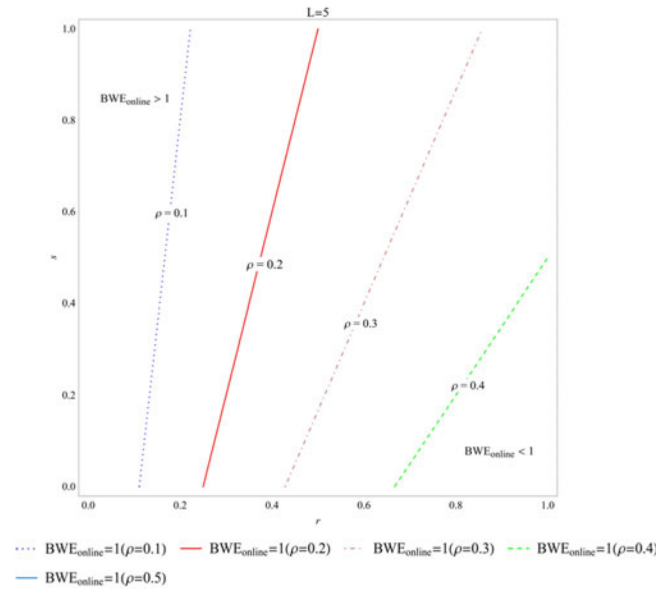

Fig. 1. Condition of eliminating the bullwhip effect in the online retail supply chain.

The "areas" (different combinations of the price discount-sensitivity coefficient r and the expectation price difference sensitivity coefficient s) for the bullwhip effect in the online retail supply chain are shown in Fig. 1 for $\rho = 0.1, 0.2, 0.3, 0.4, 0.5$, respectively. Every line in the figure stands for the borderline of $BWE_{\text{online}} = 1$. The left area of each borderline represents the case when $BWE_{\text{online}} > 1$, whereas the right area of each borderline means the case when $BWE_{\text{online}} < 1$.

Fig. 1 shows that the area of $BWE_{\text{online}} < 1$ reduces as the price autoregression coefficient ρ increases. In particular, when $\rho \ge 0.5$, the area of $BWE_{\text{online}} < 1$ disappears. As we have proven in Appendix C, the maximum value of ρ that satisfies the inequation of eliminating the bullwhip effect is less than 0.5.

Property 1: When the price autoregression coefficient ρ and the expectation price difference sensitivity coefficient s are smaller and the price discount-sensitivity coefficient r is larger, the bullwhip effect in an online retail supply chain will be smaller and even eliminated.

B. Impacts of Price-Sensitivity Coefficient and Lead Time on the Bullwhip Effect in the Online Retail Supply Chain

The impacts of the price autoregression coefficient, the price discount-sensitivity coefficient, and the expectation price difference sensitivity coefficient on the bullwhip effect in an online retail supply chain have been discussed in the above section. In this section, we analyze the impacts of the price-sensitivity coefficient and the lead time on the bullwhip effect.

1) Impact of Price-Sensitivity Coefficient: Proposition 2: The impact of the price-sensitivity coefficient b on the bullwhip effect in the online retail supply chain satisfies the following properties.

1) When $\rho \ge \frac{r}{1+r+s}$, in the interval $(0, +\infty)$, $\frac{\partial BWE_{\text{online}}}{\partial b} \ge 0$. Hence, BWE_{online} increases with the price-sensitivity coefficient b.

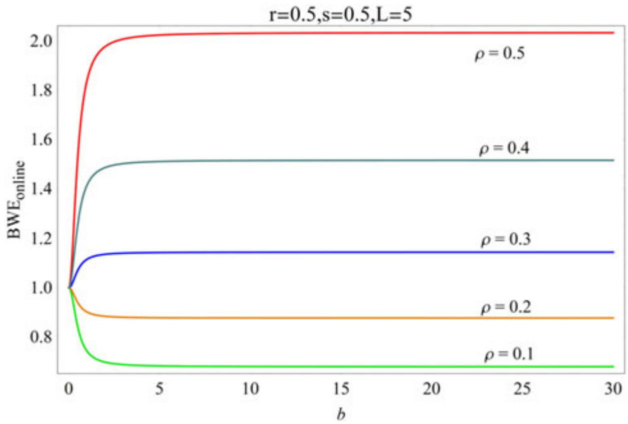

Fig. 2. Impact of b on the bullwhip effect in an online retail supply chain at different values of ρ.

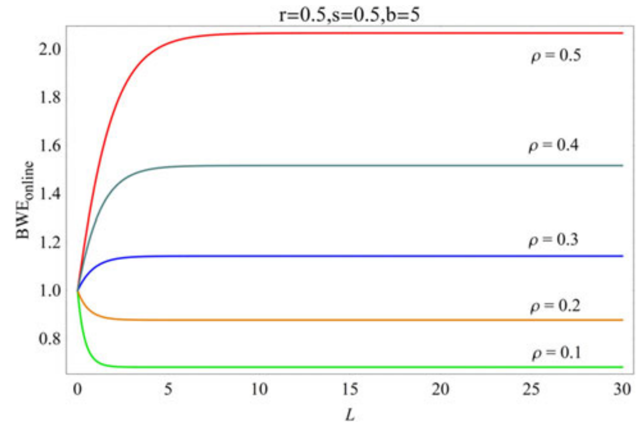

Fig. 3. Impact of L on the bullwhip effect in an online retail supply chain at different values of ρ.

2) When $\rho < \frac{r}{1+r+s}$, in the interval $(0, +\infty)$, $\frac{\partial BWE_{\text{online}}}{\partial b} < 0$. Hence, BWE_{online} decreases with the price-sensitivity coefficient b.

Proof: see Appendix D.

The two characteristics in Proposition 2 are illustrated in Fig. 2. Fig. 2 presents the different impacts of b on the bullwhip effect in an online retail supply chain at different values of ρ. The bullwhip effect decreases with b if ρ is less than 0.25, and the effect increases with b if ρ is larger than 0.25. In addition, Fig. 2 validates the property in Proposition 1 that if ρ is less than 0.25, the bullwhip effect in the online retail supply chain will be less than one; otherwise, the effect will be larger than one.

Property 2: When the values of other parameters are given, if the price autoregression coefficient ρ is relatively small, the bullwhip effect in the online retail supply chain decreases with the price-sensitivity coefficient b; otherwise, the bullwhip effect increases with the price-sensitivity coefficient b.

2) Impact of Lead Time: Proposition 3: The impact of the lead time L on the bullwhip effect in the online retail supply chain satisfies the following properties.

1) When $\rho \geq \frac{r}{1+r+s}$, in the interval $(0, +\infty)$, $\frac{\partial BWE_{\text{online}}}{\partial L} \geq 0$. Hence, BWE_{online} increases with lead time L.
2) When $\rho < \frac{r}{1+r+s}$, in the interval $(0, +\infty)$, $\frac{\partial BWE_{\text{online}}}{\partial L} < 0$. Hence, BWE_{online} decreases with lead time L.

Proof: see Appendix E.

The two characteristics in Proposition 3 are illustrated in Fig. 3. Fig. 3 reveals the different impacts of L on the bullwhip effect in an online retail supply chain at different values of ρ. The bullwhip effect decreases with L if ρ is less than 0.25, and the effect increases with L if ρ is larger than 0.25. This property indicates that in online retail supply chains, a larger lead time will cause a smaller bullwhip effect when the price autoregression coefficient is relatively small, which is contrary to the conclusions of previous research [9], [25], [26]. Lee *et al.* [9] and many researchers revealed that bullwhip effects increase with the lead time. Luong [25] found that when the second-order autoregressive coefficient is negative, the bullwhip effect presents a complex changing trend in the lead time. Ma and Ma [26] showed that a smaller lead time does not always lead to a lower bullwhip effect, but a much greater lead time does result in a higher bullwhip effect. It is remarkable that our finding is different from all the above research. In addition, Fig. 3 verifies the property in Proposition 1 that if ρ is less than 0.25, then the bullwhip effect in the online retail supply chain will be less than one; otherwise, the effect will be larger than one.

Property 3 : When the values of other parameters are given, if the price autoregression coefficient ρ is relatively small, the bullwhip effect in the online retail supply chain decreases with the lead time L; otherwise, the bullwhip effect increases with the lead time L.

C. Comparison of Bullwhip Effects in Online and Offline Supply Chain

1) Bullwhip Effect in the Offline Retail Supply Chain: The bullwhip effect in an online retail supply chain can be computed by (22) in the above section. For the bullwhip effect in a traditional offline supply chain, we directly quote the conclusion of Ma *et al.* [18]. The demand faced by the offline retailer is shown by (5).

Supposing that the retailer uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect in a two-level offline supply chain is as follows:

$$BWE_{\text{offline}} = 1 + 2b^2\rho \frac{\left(1-\rho^L\right)\left(1-\rho^{L+1}\right)\delta^2}{(1-\rho)\left(\left(1-\rho^2\right)\sigma^2 + b^2\delta^2\right)}. \quad (24)$$

2) Comparison of Bullwhip Effects in Online and Offline Supply Chains: By letting $BWE_{\text{online}} < BWE_{\text{offline}}$, where BWE_{online} is given by (22) and BWE_{offline} is given by (24), (25) as shown at the top of next page. we can obtain the condition when the bullwhip effect in an online retail supply chain is smaller than that in an offline supply chain, as shown by Proposition 4. Again, to simplify the analysis, we set $\sigma^2 = \delta^2$ and equalize the mutual parameters in two equations.

Proposition 4: The bullwhip effect in the online retail supply chain is smaller than that in the offline retail supply chain, when the following condition holds.

$$\frac{\begin{pmatrix}(r-(1+r+s)\,\rho)\left(b^2+1-\rho^2\right)\\ \times\left((r-(1+r+s)\,\rho)\left(1-\rho^L\right)-(1-\rho)\,(1+2r+s)\right)\end{pmatrix}}{\rho\left(1-\rho^{L+1}\right)\left(1-\rho^2+b^2\left(r^2+(1+r+s)^2-2r\,(1+r+s)\,\rho\right)\right)}<1. \tag{25}$$

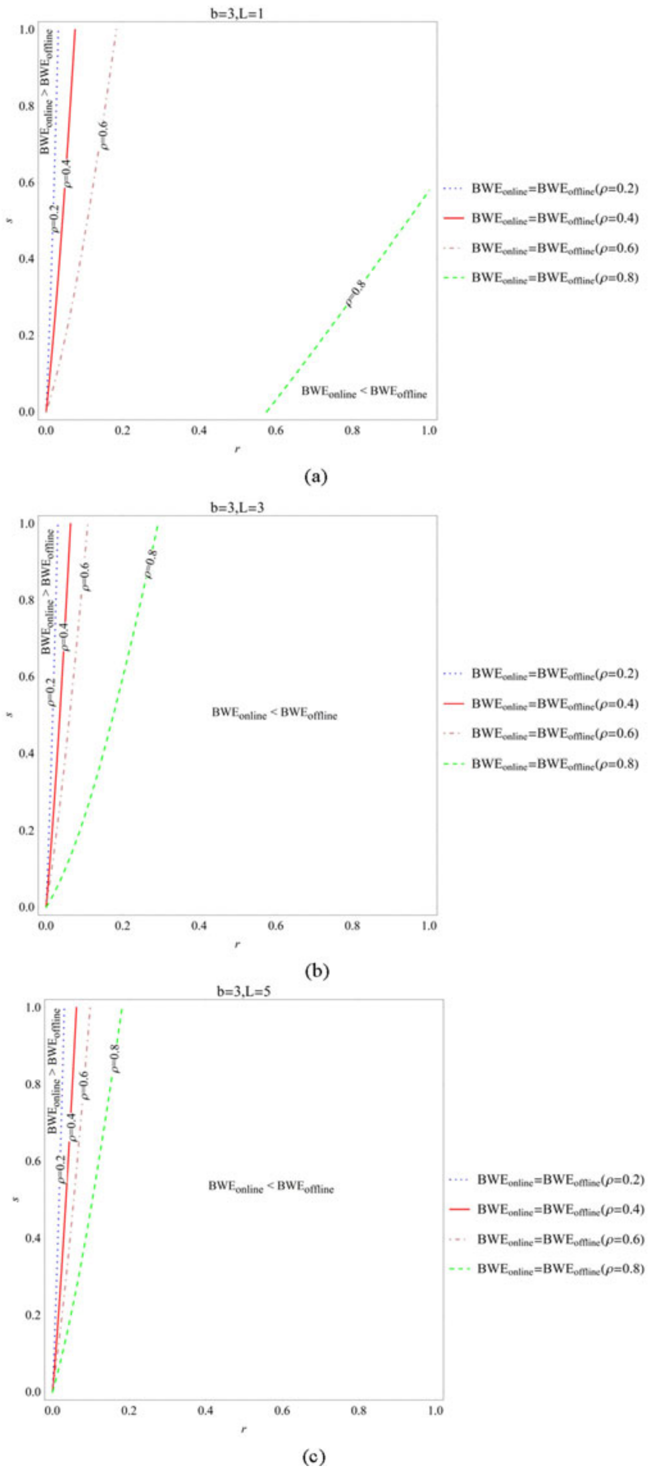

Fig. 4. (a) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 1$ and $b = 3$. (b) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 3$ and $b = 3$. (c) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 5$ and $b = 3$.

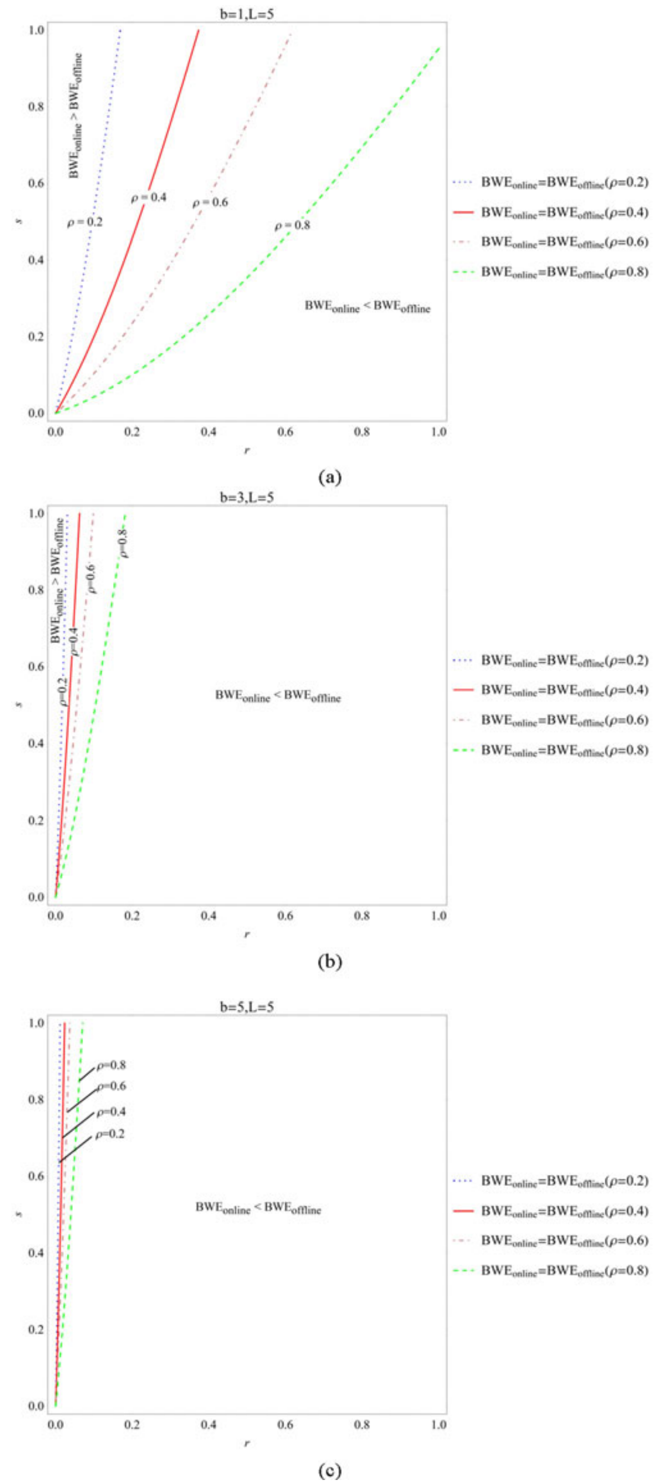

Fig. 5. (a) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 5$ and $b = 1$. (b) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 5$ and $b = 3$. (c) Impact of ρ on the bullwhip effect in an online retail supply chain when $L = 5$ and $b = 5$.

To illustrate the condition in Proposition 4, numerical examples are studied in Figs. 4 and 5.

The "areas" (different combinations of the price discount-sensitivity coefficient r and the expectation price difference sensitivity coefficient s) for comparing bullwhip effects in online and offline retail supply chains are shown in Fig. 4 for $L = 1, 3, 5$ in the cases of $\rho = 0.2, 0.4, 0.6, 0.8$ and in Fig. 5 for $b = 1, 3, 5$ in the cases of $\rho = 0.2, 0.4, 0.6, 0.8$, respectively. The results are not influenced by the parity of parameters. Every line in each picture stands for the borderline of $BWE_{\text{online}} = BWE_{\text{offline}}$ for the given values of parameters. The left area of each borderline represents the case when $BWE_{\text{online}} > BWE_{\text{offline}}$, whereas the right area of each borderline stands for the case when $BWE_{\text{online}} < BWE_{\text{offline}}$. Fig. 4 displays the comparable results of bullwhip effects in online and offline supply chains for $L = 1, 3, 5$. Fig. 5 shows the comparable results of bullwhip effects in online and offline supply chains for $b = 1, 3, 5$.

According to Figs. 4 and 5, we obtain the following.

1) Fig. 4(a) displays how the borderline of $BWE_{\text{online}} = BWE_{\text{offline}}$ changes as ρ changes in the case of $b = 3$ and $L = 1$. When the values of other parameters are given, the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ increases with ρ, and the area of $BWE_{\text{online}} < BWE_{\text{offline}}$ decreases with ρ. When ρ decreases from 0.8 to 0.6, the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ dramatically reduces. If ρ continues to decrease, the trend of reduction becomes flattened. The same conclusion can be obtained from Figs. 4(b), and (c) and 5.
2) Fig. 4 overall depicts how the borderline of $BWE_{\text{online}} = BWE_{\text{offline}}$ changes as L changes at different values of ρ. All in all, when the values of other parameters are given, the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ decreases with L, and the area of $BWE_{\text{online}} < BWE_{\text{offline}}$ increases with L. When ρ is large (i.e., $\rho = 0.8$), the reduction of the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ is sharp as L increases. In contrast, if ρ is not that large (i.e., $\rho = 0.2, 0.4, 0.6$), the reduction is flat.
3) Fig. 5 reveals globally how the borderline of $BWE_{\text{online}} = BWE_{\text{offline}}$ changes as b changes at different values of ρ. First, when the values of other parameters are given, the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ decreases with b, and the area of $BWE_{\text{online}} < BWE_{\text{offline}}$ increases with b. When b increases from 1 to 3, the area of $BWE_{\text{online}} > BWE_{\text{offline}}$ drastically reduces. When b increases from 3 to 5, the trend of reduction becomes flat.
4) Figs. 4 and 5 illustrate the condition of $BWE_{\text{online}} < BWE_{\text{offline}}$. If ρ is small (i.e., $\rho = 0.2$), $BWE_{\text{online}} < BWE_{\text{offline}}$ almost always holds. If not, when b and L are both large (i.e., $b \geq 5$ and $L \geq 5$), $BWE_{\text{online}} \leq BWE_{\text{offline}}$ is also tenable. If all of the above are invalid, in the case that s is small and r is large, $BWE_{\text{online}} < BWE_{\text{offline}}$ will still exist.

Property 4: When there is a small price autoregression coefficient ρ, or a large price-sensitivity coefficient b and lead time L, or a large price discount-sensitivity coefficient r and small expectation price difference sensitivity coefficient s in an online supply chain, the bullwhip effect in the online retail supply chain will be smaller than that in the offline retail supply chain.

D. Bullwhip Effect in the Dual-Channel Supply Chain

This section develops a dual-channel supply chain model. We analyze the impact of the ratio of the online retail market demand to the total market demand κ on the bullwhip effect to study the impact of the online retail channel on the supply chain. Based on the analysis, the impact of price discounts on the bullwhip effect can be directly observed.

Consider a simple two-level dual-channel supply chain with a manufacturer and a dual-channel retailer who sells products in the online market and offline market simultaneously. To simplify the supply chain models, we make strong assumptions that the prices and the price-sensitivity coefficients of two markets are the same and that consumers in the offline market are loyal to the offline retail channel and have no access to the historical prices and real-time price discount information in the online retail market. The total demand function faced by the dual-channel retailer is then the following:

$$\begin{aligned} d_t &= \kappa d_t^{\text{online}} + (1-\kappa)\, d_t^{\text{offline}} + \varepsilon_t \\ &= \kappa\left(a - bp_t - rb\left(p_t - p_{t-1}\right) - sb\left(p_t - p_{\min}\right)\right) \\ &\quad + (1-\kappa)\left(a - bp_t\right) + \varepsilon_t \\ &= a - bp_t - \kappa rb\left(p_t - p_{t-1}\right) - \kappa sb\left(p_t - p_{\min}\right) + \varepsilon_t \end{aligned} \tag{26}$$

where $0 \leq \kappa \leq 1$. a is the total market demand scale (i.e., potential demand if free of charge). κ is the percentage share of the demand going to the online retail channel, and $1-\kappa$ goes to the offline retail channel. κ reflects the consumers' preference for the online retail channel. The assumptions and models for $p_t, \varepsilon_t, p_{\min}$, a, s, r, and b are the same as those mentioned above.

The simplified expression of the bullwhip effect in a dual-channel supply chain (i.e., $(\sigma^2 = \sigma^2)$) is as follows:

$$\begin{aligned} &BWE_{\text{dual-channel}}^{\text{simplified}} \\ &= 1 + \frac{2b^2\left(r\kappa - (1 + r\kappa + s\kappa)\rho\right)\left(1-\rho^L\right) \times \begin{pmatrix} \left(r\kappa - (1 + r\kappa + s\kappa)\rho\right)\left(1-\rho^L\right) \\ -(1-\rho)(1 + 2r\kappa + s\kappa) \end{pmatrix}}{(1-\rho)\left(1 - \rho^2 + b^2\begin{pmatrix} r^2\kappa^2 + (1 + r\kappa + s\kappa)^2 \\ -2r\kappa(1 + r\kappa + s\kappa)\rho \end{pmatrix}\right)}. \end{aligned} \tag{27}$$

Proposition 5: The impact of the ratio of the online retail market demand to the total market demand κ on the bullwhip effect in a dual-channel supply chain satisfies the following property.

In the interval $[0, 1]$, $\frac{\partial BWE_{\text{dual-channel}}^{\text{simplified}}}{\partial \kappa} \geq 0$. Hence, $BWE_{\text{dual-channel}}^{\text{simplified}}$ increases with κ.

Proof: see Appendix F.

Proposition 5 indicates that the online retail channel increases the bullwhip effect in the dual-channel supply chain. This result is not surprising because the online market demand has a smaller mean and larger variance than the offline market. From (4) and (5), we have $\mu_s^{\text{online}} = a - b\mu_p + sb(p_{\min} - \mu_p)$ $\mu_d^{offline} = a - b\mu_p, \sigma_{d,online}^2 = ((1 + s)^2 + 2r(1 + r + s)(1 - \rho))b^2\sigma_p^2 + \sigma^2$ and $\sigma_{d,\text{offline}}^2 = b^2\sigma_p^2 + \sigma^2$.

Property 5: The bullwhip effect in the dual-channel supply chain increases with the ratio of the online retail market demand.

Property 5 reveals that generally the online retail channel has a larger bullwhip effect than the offline retail channel due to its price transparency and the availability of real-time price discount information. It is worth reemphasizing that whether the real-time price discounts can be perceived by consumers is an important mark of the distinction between the online retail market and the offline market, and the availability of real-time price discount information of the online retail channel leads to a larger demand volatility in the online retail supply chain. Therefore, we conclude that the price discounts in the online retail market amplify the bullwhip effect in the online supply chain.

VII. Conclusion

This paper studies the bullwhip effect in an online retail supply chain for the first time. Based on the features of frequent price discounts and price information transparency in the online retail market, we establish a demand model dependent on price discounts with reference price theory and derive the expression of the bullwhip effect in the online retail supply chain. In addition, we deduce the conditions of eliminating the bullwhip effect in the online retail supply chain, analyze the properties, and compare the bullwhip effects in online and offline supply chains. Finally, to directly observe the impact of price discounts in e-commerce on the bullwhip effect in the online retail supply chain, we develop a dual-channel supply chain model. Our analytical findings help us to generate several managerial insights.

1) All in all, price discounts in the online retail market amplify the bullwhip effect in the online retail supply chain. Managers who neglect the price transparency feature of the online retail channel will underestimate the bullwhip effect. This is because the availability of real-time price discount information of the online retail market leads to larger demand volatility. Although frequent price promotion event in e-commerce may cause the bullwhip effect and increase inventory costs, it is still a useful channel for dealing with the perennial backlog of inventory, especially for garment and textile industries.
2) Despite the above conclusion, the bullwhip effect in the online retail supply chain can be smaller than that in the offline supply chain, as long as the price autoregression coefficient is small, or the price sensitivity coefficient and lead time are both large, or the price discount sensitivity coefficient is large and the expectation price difference sensitivity coefficient is small. When one of these conditions is satisfied, traditional companies should consider developing new e-commerce initiatives or strengthening their current e-commerce presence to reduce the bullwhip effect and costs in supply chains.
3) If the price autoregression coefficient is relatively small, the bullwhip effect in the online retail supply chain will decrease with the lead time; if not, the bullwhip effect will increase with the lead time. This finding indicates that in online retail supply chains, a larger lead time will cause a smaller bullwhip effect in certain conditions. This distinctive relationship between the lead time and the bullwhip effect in the online supply chain is different from the conclusions of all previous research [9], [25], [26]. From the perspective of supply chain managers, they should determine the appropriate lead time with their cooperators for a lower bullwhip effect according to the actual online supply chain conditions, rather than blindly pursue a shorter lead time.
4) When the price autoregression coefficient is relatively small, the bullwhip effect in the online retail supply chain decreases with the price-sensitivity coefficient; otherwise, the bullwhip effect increases with the price-sensitivity coefficient. This suggests that in the case of a small price autoregression coefficient, commodities with larger demand elasticity in the online market will lead to a smaller bullwhip effect in the online retail supply chain.
5) When the price autoregression coefficient and the expectation price difference sensitivity coefficient are smaller and the price discount-sensitivity coefficient is larger, the bullwhip effect in the online retail supply chain will be smaller and even eliminated.

Our research offers a method to measure the bullwhip effect in an online retail supply chain from a new perspective. The results reveal the amplification effect of frequent price discounts in e-commerce on the bullwhip effect and provide novel insights about the relationships between the bullwhip effect and its influencing parameters. Our findings could help managers better understand the bullwhip effect in the online retail supply chain and its difference with that in the offline supply chain, which will help them improve the online supply chain performance.

This research suggests several future directions to enhance our understanding of the bullwhip effect in the online retail supply chain. First, our model considers only the order-up-to inventory policy. Whether and how our findings will change under other inventory policies require further study. In addition, extending the two-level supply chain to the multilevel chain is an interesting topic that deserves investigation. Finally, comparing more forecasting techniques and choosing the best one to minimize the bullwhip effect in the online retail supply chain have a great deal of significance and should be a priority for future research in this area.

Appendix

Appendix A. Proof of Lemma 1

The variance of the forecasting error can be computed as follows:

$$\left(\hat{\sigma}_t^L\right)^2 = \operatorname{Var}\left(D_t^L - \hat{D}_t^L\right)$$
$$= \operatorname{Var}\sum_{i=0}^{L-1}\left(d_{t+i} - \hat{d}_{t+i}\right)$$

$$= \mathrm{Var} \sum_{i=0}^{L-1} \begin{pmatrix} -b(1+r+s)(p_{t+i} - \hat{p}_{t+i}) \\ +rb(p_{t+i-1} - \hat{p}_{t+i-1}) + \varepsilon_{t+i} \end{pmatrix}$$

$$= \mathrm{Var} \sum_{i=0}^{L-1} \begin{pmatrix} -b(1+r+s) \sum_{j=0}^{i} \rho^{i-j} \eta_{t+j} \\ +rb \sum_{j=0}^{i-1} \rho^{i-1-j} \eta_{t+j} + \varepsilon_{t+i} \end{pmatrix}$$

$$= \mathrm{Var} \begin{pmatrix} rb \sum_{i=1}^{L-1} \sum_{j=0}^{i-1} \rho^{i-1-j} \eta_{t+j} + \sum_{i=0}^{L-1} \varepsilon_{t+i} \\ -b(1+r+s) \sum_{i=0}^{L-1} \sum_{j=0}^{i} \rho^{i-j} \eta_{t+j} \end{pmatrix}$$

$$= \mathrm{Var} \begin{pmatrix} rb \sum_{i=1}^{L-1} \eta_{t+i-1} \sum_{j=0}^{L-1-i} \rho^{j} + \sum_{i=0}^{L-1} \varepsilon_{t+i} \\ -b(1+r+s) \sum_{i=0}^{L-1} \eta_{t+i} \sum_{j=0}^{L-1-i} \rho^{j} \end{pmatrix}$$

$$= \mathrm{Var} \begin{pmatrix} rb \sum_{i=1}^{L-1} \left(\frac{1-\rho^{L-i}}{1-\rho}\right) \eta_{t+i-1} + \sum_{i=0}^{L-1} \varepsilon_{t+i} \\ -b(1+r+s) \sum_{i=0}^{L-1} \left(\frac{1-\rho^{L-i}}{1-\rho}\right) \eta_{t+i} \end{pmatrix}$$

$$= \mathrm{Var} \begin{pmatrix} \sum_{i=0}^{L-2} \eta_{t+i} \left(\frac{-b(1+r+s)(1-\rho^{L-i})}{1-\rho} + \frac{rb(1-\rho^{L-1-i})}{1-\rho} \right) \\ -b(1+r+s)\eta_{t+L-1} + \sum_{i=0}^{L-1} \varepsilon_{t+i} \end{pmatrix}$$

$$= \mathrm{Var} \left(\sum_{i=0}^{L-2} \eta_{t+i} \left(\frac{-b(1+r+s)(1-\rho^{L-i})}{1-\rho} + \frac{rb(1-\rho^{L-1-i})}{1-\rho} \right) \right) + L\sigma^2 + (1+r+s)^2 b^2 \delta^2, \quad (28)$$

where

$$\mathrm{Var} \sum_{i=0}^{L-2} \eta_{t+i} \left(\frac{-b(1+r+s)(1-\rho^{L-i})}{1-\rho} + \frac{rb(1-\rho^{L-1-i})}{1-\rho} \right)$$
$$= \frac{\delta^2 b^2}{(1-\rho)^2} \sum_{i=0}^{L-2} \left(-(1+r+s)(1-\rho^{L-i}) + r(1-\rho^{L-1-i}) \right)^2$$
$$;= \frac{(1+r+s)^2 b^2 \delta^2 \left(L - 1 - \frac{2(\rho^2 - \rho^{L+1})}{1-\rho} + \frac{\rho^4 - \rho^{2L+2}}{1-\rho^2} \right)}{(1-\rho)^2}$$
$$+ \frac{r^2 b^2 \delta^2 \left(L - 1 - \frac{2(\rho - \rho^L)}{1-\rho} + \frac{\rho^2 - \rho^{2L}}{1-\rho^2} \right)}{(1-\rho)^2}$$
$$- \frac{2r(1+r+s) b^2 \delta^2 \left(L - 1 - \frac{\rho - \rho^L}{1-\rho} + \frac{\rho^3 - \rho^{2L+1}}{1-\rho^2} - \frac{\rho^2 - \rho^{L+1}}{1-\rho} \right)}{(1-\rho)^2}. \quad (29)$$

Substituting (29) into (28), we see that $(\hat{\sigma}_t^L)^2$ is a constant. This completes the proof.

Appendix B. Proof of Theorem 1

The variance of the order quantity can be derived from (21) as follows:

$$\begin{aligned} &\mathrm{Var}(q_t) = \\ &\mathrm{Var}(b(r\Lambda_L - (1+r+s)\rho\Lambda_L)(p_{t-1} - p_{t-2}) + d_{t-1}) \\ &= b^2(r\Lambda_L - (1+r+s)\rho\Lambda_L)^2 \mathrm{Var}(p_{t-1} - p_{t-2}) \\ &+ 2b(r\Lambda_L - (1+r+s)\rho\Lambda_L)\mathrm{Cov}(d_{t-1}, p_{t-1} - p_{t-2}) \\ &+ \mathrm{Var}(d_{t-1}) \end{aligned} \quad (30)$$

where $\mathrm{Var}(d_{t-1}) = \sigma_d^2$, and

$$\begin{aligned} \mathrm{Var}(p_{t-1} - p_{t-2}) &= 2\mathrm{Var}(p_t) - 2\mathrm{Cov}(p_{t-1}, p_{t-2}) \\ &= 2\sigma_p^2 - 2\rho\sigma_p^2 \\ &= \frac{2\delta^2}{1+\rho} \end{aligned} \quad (31)$$

and

$$\begin{aligned} &\mathrm{Cov}(d_{t-1}, p_{t-1} - p_{t-2}) \\ &= \mathrm{Cov}(-b(1+r+s)p_{t-1} + rbp_{t-2} + \varepsilon_{t-1}, p_{t-1} - p_{t-2}) \\ &= -b(1+r+s)\mathrm{Cov}(p_{t-1}, p_{t-1}) + b(1+r+s) \\ &\mathrm{Cov}(p_{t-1}, p_{t-2}) - rb\mathrm{Cov}(p_{t-2}, p_{t-2}) \\ &+ rb\mathrm{Cov}(p_{t-2}, p_{t-1}) \\ &= -(1-\rho)(1+2r+s)b\sigma_p^2 \\ &= \frac{-(1+2r+s)b\delta^2}{1+\rho}. \end{aligned} \quad (32)$$

Substituting (31) and (32) into (30) and dividing both sides of (30) by σ_d^2, we derive the expression of the bullwhip effect in the online retail supply chain. This completes the proof.

Appendix C Proof of Proposition 1

According to (23), to eliminate the bullwhip effect in an online retail supply chain, the values of the parameters should satisfy the following:

$$\frac{\begin{pmatrix} b^2(r - (1+r+s)\rho)(1-\rho^L) \\ \times((r - (1+r+s)\rho)(1-\rho^L) - (1-\rho)(1+2r+s)) \end{pmatrix}}{(1-\rho)\left(1 - \rho^2 + b^2\left(r^2 + (1+r+s)^2 - 2r(1+r+s)\rho\right)\right)} \leq 0. \quad (33)$$

Because the denominator of (33) satisfies

$$\begin{aligned} &(1-\rho)\left(1-\rho^2+b^2\left(r^2+(1+r+s)^2-2r(1+r+s)\rho\right)\right) \\ &= (1-\rho)\left(1-\rho^2+b^2\left((1+s)^2+2r(1+r+s)(1-\rho)\right)\right) \\ &> 0 \end{aligned} \quad (34)$$

$$\frac{\partial BWE^{\text{simplified}}_{\text{dual-channel}}}{\partial \kappa} = \frac{\begin{array}{c}-4b^4 r(r\kappa-(1+r\kappa+s\kappa)\rho)^2\left(1-\rho^L\right)\\ \times\left((r\kappa-(1+r\kappa+s\kappa)\rho)\left(1-\rho^L\right)-(1-\rho)(1+2r\kappa+s\kappa)\right)\end{array}}{(1-\rho)\left(1-\rho^2+\left(\kappa^2 r^2+(1+r\kappa+s\kappa)^2-2r\kappa(1+r\kappa+s\kappa)\rho\right)b^2\right)^2} \tag{43}$$

and $b^2(1-\rho^L)\geq 0$, we have the following:

$$(r-(1+r+s)\rho)\begin{pmatrix}(r-(1+r+s)\rho)\left(1-\rho^L\right)\\ -(1-\rho)(1+2r+s)\end{pmatrix}\leq 0. \tag{35}$$

Let $(r-(1+r+s)\rho)(1-\rho^L)-(1-\rho)(1+2r+s)\leq 0$ and $r-(1+r+s)\rho\geq 0$. Reducing the two inequations, we obtain

$$\rho\leq\frac{r}{1+r+s}\leq\rho+\frac{1-\rho^2}{\rho-\rho^L}. \tag{36}$$

If $(r-(1+r+s)\rho)(1-\rho^L)-(1-\rho)(1+2r+s)\geq 0$ and $r-(1+r+s)\rho\leq 0$, there is no solution.

Therefore, we conclude that when $\rho\leq\frac{r}{1+r+s}\leq\rho+\frac{1-\rho^2}{\rho-\rho^L}$ or $b=0$ holds, the bullwhip effect in the online retail supply chain will be eliminated. Because $0\leq r<1$ and $0\leq s<1$, we can easily obtain $0\leq\frac{r}{1+r+s}<0.5$. Then, the maximum value of ρ that satisfies (36) is less than 0.5. This completes the proof.

Appendix D Proof of Proposition 2

Taking the first derivative of (23) with respect to b, we obtain the following:

$$\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial b} = \frac{\begin{pmatrix}4b(1+\rho)(r-(1+r+s)\rho)\left(1-\rho^L\right)\\ \times\left((r-(1+r+s)\rho)\left(1-\rho^L\right)-(1-\rho)(1+2r+s)\right)\end{pmatrix}}{\left(1-\rho^2+\left(r^2+(1+r+s)^2-2r(1+r+s)\rho\right)b^2\right)^2} \tag{37}$$

where

$$\frac{b(1+\rho)\left(1-\rho^L\right)}{\left(1-\rho^2+\left(r^2+(1+r+s)^2-2r(1+r+s)\rho\right)b^2\right)^2}\geq 0 \tag{38}$$

and

$$\begin{aligned}&(r-(1+r+s)\rho)\left(1-\rho^L\right)-(1-\rho)(1+2r+s)\\ &=-\left(r\left(1+\rho^L\right)(1-\rho)+(1+s)\left(1-\rho^{L+1}\right)\right)\leq 0.\end{aligned} \tag{39}$$

Therefore, if $r-(1+r+s)\rho\leq 0$, then $\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial b}\geq 0$; if not, $\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial b}<0$.

Solving the equation $\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial b}=0$, we have $b^*=0$.

Then, we can obtain the proposition. This completes the proof.

Appendix E Proof of Proposition 3

Taking the first derivative of (23) with respect to L, we obtain the following:

$$\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial L} = \frac{\begin{pmatrix}2\ln(\rho)(r-(1+r+s)\rho)b^2\rho^L\\ \times\left((1-\rho)(1+2r+s)-2(r-(1+r+s)\rho)\left(1-\rho^L\right)\right)\end{pmatrix}}{(1-\rho)\left(1-\rho^2+b^2\left(r^2+(1+r+s)^2-2r(1+r+s)\rho\right)\right)} \tag{40}$$

where $\ln(\rho)\leq 0$, $\rho^L\geq 0$, and

$$\begin{aligned}&(1-\rho)(1+2r+s)-2(r-(1+r+s)\rho)\left(1-\rho^L\right)\\ &\geq(1-\rho)(1+2r+s)-2(r-(1+r+s)\rho)\\ &=1+\rho+s+s\rho>0\end{aligned} \tag{41}$$

and

$$\begin{aligned}&(1-\rho)\left(1-\rho^2+b^2\left(r^2+(1+r+s)^2-2r(1+r+s)\rho\right)\right)\\ &=(1-\rho)\left(1-\rho^2+b^2\left((1+s)^2+2r(1+r+s)(1-\rho)\right)\right)\\ &\geq 0.\end{aligned} \tag{42}$$

Therefore, if $r-(1+r+s)\rho\leq 0$, then $\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial L}\geq 0$; if not, $\frac{\partial BWE^{\text{simplified}}_{\text{online}}}{\partial L}<0$.

Then, we can obtain the proposition. This completes the proof.

Appendix F Proof of Proposition 5

Taking the first derivative of (27) with respect to κ, we obtain the following: (43) as shown at the top of this page. where

$$\frac{-4b^4 r(r\kappa-(1+r\kappa+s\kappa)\rho)^2\left(1-\rho^L\right)}{(1-\rho)\left(1-\rho^2+\begin{pmatrix}\kappa^2 r^2+(1+r\kappa+s\kappa)^2\\ -2r\kappa(1+r\kappa+s\kappa)\rho\end{pmatrix}b^2\right)^2}\leq 0 \tag{44}$$

and

$$\begin{aligned}&(r\kappa-(1+r\kappa+s\kappa)\rho)\left(1-\rho^L\right)-(1-\rho)(1+2r\kappa+s\kappa)\\ &=-\left(r\kappa\left(1+\rho^L\right)(1-\rho)+(1+s\kappa)\left(1-\rho^{L+1}\right)\right)\leq 0.\end{aligned} \tag{45}$$

Therefore, we obtain $\frac{\partial BWE^{\text{simplified}}_{\text{dual-channel}}}{\partial \kappa}\geq 0$.

Then, we can obtain the proposition. This completes the proof.

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