An In-Depth Analysis of Contingent Sourcing Strategy for Handling Supply Disruptions

Yong He, Shanshan Li, Henry Xu, and Chunming Shi

***Abstract*—In this paper, we consider a make-to-stock production-inventory system where a manufacturer's production may be entirely interrupted due to a supply disruption. Customers react dynamically to the subsequent inventory shortage, depending on factors including market condition, customer characteristic, and behavioral interaction. The manufacturer can adopt contingent sourcing to manage the disruption. Consequently, the postdisruption demand and inventory exhibit complicated dynamics in terms of customer behavior, demand recovery, and the adoption of contingent sources. We first model and forecast the postdisruption customer behavior. Customers are classified into two types based on brand loyalty and the interaction is captured as "demand learning" within each type. Using differential models, we analytically characterize customers' postdisruption behavior in five possible scenarios, depending on customers' constitution, transient reaction, brand loyalty, and competition intensity. Next, we propose dynamic contingent sourcing strategies to mitigate the supply disruption, and the optimal sourcing time is derived. Finally, by conducting numerical analysis, we obtain further managerial insights on how to adapt dynamic contingent sourcing strategies according to various contributing factors.**

***Index Terms*—Customer behavior, dynamic contingent sourcing, risk management, supply disruption.**

I. Introduction

As SUPPLY chains expand globally, supply disruption risks increase [41]. As a result, supply disruption risk management has attracted much attention from both academics and practitioners [46]. Numerous strategies and approaches have been proposed to cope with supply disruptions [44]. However, different strategies can lead to contrasting consequences. The Philips's semiconductor plant in New Mexico is a case in point.

In 2000, a fire hit the plant which impacted Nokia and Ericsson, back then the two largest mobile phone companies. While Nokia adopted contingent sourcing strategy immediately, Ericsson reacted to the incident passively. As a result, Nokia survived and extended its market share by three percent one year later, whereas Ericsson ultimately withdrew from the mobile phone market [29]. This famous and much-discussed case revealed that the response time appears to be a crucial factor when dealing with the disruption, as the postdisruption demand usually highly depends on the time of impact [5]. In addition, factors related to market condition and customer characteristics (e.g., the availability of competing products and the brand loyalty of customers) also play considerably important roles. If Ericsson's customers were highly loyal or no substitutable product existed in the market, they would not have reacted so sensitively to the disruption. Therefore, it is critical to understand and predict the dynamics of the postdisruption demand when designing an effective supply risk mitigation strategy.

Demand forecasting has gained an increased importance among practitioners in recent years [38]. Nonetheless, little research work has been done in forecasting postdisruption demand [32]. In the literature, the postdisruption behavior of customers is mainly characterized as a fixed rate of backorders [8], [22], [23], or captured as customer sensitivity (patience) [43]. As a result, most existing studies on supply chain disruptions assume that the postdisruption demand decreases over time linearly or exponentially, or follow a given stochastic distribution [9]. However, in the presence of a disruption, managers could face fast-changing demand. A model with only several parameters to describe the demand dynamics cannot capture such changes effectively [34]. Thus, in this paper, we develop differential models to predict postdisruption customer behavior (reaction) in view of market condition, customer characteristics, and customer interaction.

Additionally, customer purchasing preference may be severely influenced by stock-outs. In consequence, it could lead to long-term loss of market share [49]. With a reduced market share, the demand cannot automatically return to the original level after the disruption is over. Certain marketing incentives, which need financial investment, may be required to gradually recover the demand. Thus, in alignment with Hohenstein *et al.* [26], we consider both responsiveness and recovery time in a postdisruption phase. The responsiveness refers to the reaction time to the disruption until the start of the recovery phase, where our investigation concerns on customer reaction. The recovery phase is the time to return to normal performance (i.e., the original demand level in this research work) after turbulence. We assume demand is recovered linearly with an additional demand recovery cost and time.

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In view of customer reaction and demand recovery, we analyse the dynamics of the postdisruption demand and inventory incorporating these two phases. Based on the analysis of the dynamics, we then explore the optimal design of contingent sourcing strategy to alleviate the negative impact caused by the disruption. In practice, contingent sourcing strategy appears to be preferred over alternatives such as the multiple sourcing strategy [13], the optimal allocation procurement strategy [18], and the recovery strategy [30].

Facing a supply disruption, a manufacturer can get supply replenished from a secondary source (spot market or a backup supplier), probably at a higher price. As commonly accepted in the literature (see, e.g., [36]), we assume that the order for contingent sources is processed instantaneously and the transit lead-time is zero. The postdisruption demand may hold steady without lost sales if the manufacturer reroutes to a secondary source immediately after a stock-out until the primary supplier is back to normal. Nevertheless, contingent sourcing can be expensive. Thus, the manufacturer may wait for some time to see if the additional cost of contingent sourcing could be avoided. For instance, if the primary supplier can quickly recover, the manufacturer can still get replenishments at a normal price. On the contrary, if the disruption lasts long, the manufacturer could suffer short-term losses of the demand, and substantially long-term losses due to negative impact through direct and indirect influences, including market share reduction triggered by reputation effects [21]. The longer the response, the higher the negative impact of the disruption could be. This means that the timing for contingent sourcing is critical for the manufacturer. Therefore, there are two important research questions: What is the optimal time to reroute? When may the contingent sourcing strategy fail to hedge against disruptions? To address these two questions, we propose dynamic contingent sourcing strategies by considering customer behavior, demand recovery, contingent sourcing cost, and disruption duration. The concept "dynamic" in this paper refers to the dynamic nature in terms of the timing for rerouting to minimize the disruption impact in light of the dynamics of the postdisruption demand and inventory. Particularly, the dynamic contingent sourcing strategies consist of three alternatives: rerouting to a secondary source instantaneously when inventory shortage occurs (IS), implementing contingent sourcing after waiting for some time (WS), and not seeking any replenishment during the disruption period (NS).

This study provides both academics and practitioners with two unique contributions. First, we develop a new method to predict the dynamics of the postdisruption demand incorporating customer behavior and demand recovery. Customer behavior is analysed in view of market condition, customer characteristic, and customer interaction. Our analysis confirms that the postdisruption dynamics cannot be adequately described as a single distribution, as assumed in most of the existing studies. Moreover, we analytically reveal that there are five possible scenarios for the postdisruption demand in the absence of mitigation countermeasures, depending on factors like customer characteristic and market condition. Second, we propose dynamic contingent sourcing strategies based on demand forecasting. Those strategies can help answer two critical questions: what is the optimal time for contingent sourcing? When may the contingent sourcing strategy become ineffective for mitigating disruptions? By employing numerical analysis, we obtain further managerial insights on how the dynamic contingent sourcing strategies are affected by various factors.

The rest of this paper is organized as follows. In Section II, the related work is briefly reviewed. Section III describes the problem. In Section IV, we focus on predicting postdisruption customer behavior. Based on the analysis of the dynamics of demand and inventory, dynamic contingent sourcing strategies are proposed in Section V. Section VI further examines the dynamic contingent sourcing strategies via numerical analysis. Finally, conclusions are drawn and future research work is suggested in Section VII.

II. Related Literature

There has been extensive research work on supply disruption risk management, including multisourcing or dual-sourcing [24], [39], [45], [52], contingent (emergency) sourcing, backup supplier [3], [51], supplier (portfolio) selection [1], [7], [10], [12], [33], inventory policy [37], [40], and resilient supply network design [2], [4], [31]. However, two streams of research work are more closely related to this study, namely, contingent sourcing strategy and demand uncertainty, which will be briefly reviewed below.

A. *Contingent Sourcing Strategy*

Tomlin [47] suggested that contingent rerouting is possible if a reliable supplier has volume flexibility. He further found that contingent rerouting is often a component of a firm's optimal disruption-management strategy and can significantly reduce the firm's costs. Later, Tomlin [48] evaluated 12 possible disruption-management strategies and indicated that contingent sourcing is preferred over supplier diversification as supply risk increases. In contrast, diversification is preferred as the demand risk increases. He also found that risk aversion makes contingent sourcing preferable over a broad set of supply- and demand-risk combinations.

More recently, Qi [36] considered the possibility that a retailer may want to wait before rerouting to its backup supplier. The paper then examined the optimal sourcing and replenishment decisions for the retailer. He *et al.* [18] investigated two competing manufacturers using emergency procurement strategy and optimal allocation procurement strategy. They found that the emergency procurement strategy can be more profitable than the optimal allocation procurement strategy when all suppliers are unreliable. This study was further extended to incorporate price competition in He *et al.* [19]. Yang and Xu [50] investigated the optimal contingency tactics of a grain supply chain when grain processors face shortages due to natural disasters. Gupta *et al.* [16] studied the contingent sourcing strategy in a supply chain where two competing manufacturers sell substitutable products. They found that supply disruption and procurement time are important factors. Seok *et al.* [42] demonstrated the advantages of the intelligent contingent sourcing protocol, which guides collaboration with both internal and external suppliers.

As can be seen from the brief review above, extant research work on contingent sourcing has mainly focused on comparing different contingent sourcing strategies. Little research work has been done to examine the optimal timing for contingent sourcing, or whether a contingent sourcing strategy is effective to mitigate disruptions under various circumstances. We will address this research work gap in this paper.

B. Demand Uncertainty

In the context of supply chain disruption management, demand uncertainty has been largely discussed with regards to stochastic distributions and forecasting. Federgruen and Yang [14] analyzed how to procure supplies from multiple sources using a planning model where uncertain demand can be modeled with a cumulative distribution function. More recently, de Treville *et al.* [9] analyzed the impact of three types of demand distribution on the optimal production and sourcing choices: demand with constant volatility, with stochastic volatility, or with heavy-tail. Schmitt *et al.* [41] considered both deterministic and stochastic demand in designing supply disruption management systems. They found that when demand is deterministic and supply may be disrupted, a decentralized inventory system is optimal for a risk-averse firm.

With respect to demand forecasting, a number of methodologies have been developed such as regression models, moving average, weighted moving average, exponential smoothing, grey prediction method, and machine learning algorithm [34]. Particularly, Beutel and Minner [6] adopted regression models to precisely forecast demand in the process of planning safety stock and illustrated how estimation errors can be utilized to set the required level of safety stock. To predict market demand after transportation disruption, Liu *et al.* [32] designed an improved model of grey neural networks and tested its feasibility. Paul *et al.* [35] developed a fuzzy inference system to predict future demand changes and proposed a predictive mitigation planning approach for managing those changes.

To the best of our knowledge, there has been no study on incorporating the postdisruption behavior of customers into demand forecasting or developing a dynamic contingent sourcing plan based on demand forecasting. In this paper, we will address this important research work gap. Specifically, we will examine how demand evolves after a supply disruption while considering dynamic customer behavior, and explore the optimal design of contingent sourcing strategy for managing the disruption.

III. Problem Description

In this paper, we consider a manufacturer's make-to-stock production system in the presence of market competition. In the absence of disruptions, the demand rate at time t is a constant $A,$ and the inventory system is depicted in Fig. 1, following a typical EPQ system [25]. The production cycle time T consists of the production uptime T_m and downtime $(T - T_m)$. Obviously, $T_m = I/(P - A)$, and $T = I/(P - A) + I/A$, where P is the production rate ($P > A$) and I is the inventory capacity. Suppose that a supply disruption occurs at time t_0 in the 1st original production cycle, i.e., $t_0 \in (0, T]$. In the presence of a disruption, the production stops immediately, and the inventory is consumed at the rate of demand A if $t_0 \in (0, T_m]$. As the disruption continues, a stock-out occurs when the inventory is

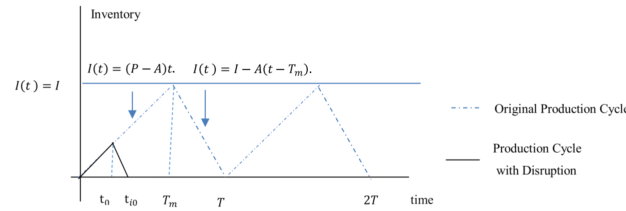

Fig. 1. Inventory with and without disruption when no mitigation action is taken.

completely depleted at time t_{i0}, as shown in Fig. 1. Conversely, if $t_0 \in (T_m, T]$, both demand and inventory remain stable in the 1st original cycle in view that the production is fulfilled in $(0, T_m]$. The dynamics in the 2nd original cycle, i.e.,$(T, 2T]$, can be analysed in a similar way. Therefore, we focus on $\in t_0 (0, T_m)$ when disruptions occur during the production uptime.

Facing stock-out, customers react in two ways: to leave (switching to a substitutable product of a competitor) or to stay (waiting for delayed delivery). As pointed out in [20], customer reactions are heavily impacted by brand loyalty and the availability of substitutable products in the market. Therefore, in view of brand loyalty, we classify customers into two types: type-1 (with the quantity of Aa_1), and type-2 (with the quantity of Aa_2), where $a_1 + a_2 = 1$. Type-1 customers display no brand loyalty to the manufacturer's product. Type-2 customers have a certain level of brand loyalty to the product. Their purchasing decisions are partly based on their satisfaction with the brand in the past.

In this study, we model customer behavior via customer utility, a well-developed concept in marketing research (see, e.g., [17]). Without loss of generality, we normalize customer utility to be 1 before supply disruption. With no brand loyalty, type-1 customers obtain a utility of 1 for switching to a substitutable product after disruption and a utility of 0 for waiting. As for type-2 customers, with their brand loyalty, their utility is $u(t)$ for staying with the manufacturer, and λ for leaving. $u(t)$ depends upon the brand loyalty of type-2 customers. The parameter $0 \leq \lambda \leq 1$ reflects the competition intensity among substitutable products in the market. In particular, $\lambda = 0$ represents no competition and $\lambda = 1$ represents the situation where the manufacturer's product is completely substitutable.

In addition to brand loyalty and competition intensity, the interactions among customers over time also influence their reactions to disruptions. For example, Zinn and Liu [54] confirmed that the patronage is influenced by positive or negative word-of-mouth from other customers. In recent years, due to the quick development of the Internet and e-commerce, consumer information can be disseminated easily via websites and social media [53]. As a result, allelomimetic behavior or demand learning might occur among customers. However, customers tend to process the same information differently [27]. Therefore, we model the dynamics of customer interactions by analyzing "demand learning" among each type of customers. Specifically, we model customer postdisruption behavior in two steps (to be detailed in Section IV). First, we use the contributing factors of customer behavior, i.e., market condition and customer characteristic. We describe market condition based on competition intensity λ and customer segmentation (A, a_2) (the number of type-1 and type-2 customers). Customer characteristic is modeled with brand loyalty $(1-\theta)$ and transient reaction. Second, to characterize the interactions among customers, we consider demand learning through evolutionary dynamics in the differential models, as done in Kwon *et al.* [28].

Unless otherwise noted, in this study, customer behavior refers to how the customers dynamically react to disruption when the manufacturer takes no mitigation measure. It is quantitatively modeled with the postdisruption demand without mitigation countermeasures (to be detailed in Section IV). The word "original" in this paper is used to distinguish the variables before and after a disruption. For instance, the original demand A is the demand before a disruption. Parameter ε is a positive number close to zero, representing the error of demand estimation. In other words, the manufacturer has no customer demand if the demand reaches ε. This is reasonable as the demand estimation error is inevitable. For conciseness, in addition to the acronyms of "WS", "NS," and "IS," "WSNS" means the pattern of repeating "WS" and "NS" periodically, and "WSIS" means the pattern of repeating "WS" and "IS" periodically.

IV. Forecasting of the Postdisruption Customer Behavior

In this section, we first formulate differential models for demand learning to predict how the numbers of type-1 and type-2 customers change after a stock-out. Then, we investigate the dynamics of the postdisruption demand in the absence of mitigation strategy.

A. Dynamics of Type-1 Customers

The market demand remains stable until inventory drops to zero at time t_{i0}. Facing the stock-out at time t_{i0}, x_0 of type-1 customers choose to leave for competing products, and $(1-x_0)$ of them are willing to wait for backorder. As described in Section III, without brand preference, type-1 customers have the utility of 1 for leaving and 0 for waiting. However, because of "demand learning" among customers, the decision of each of type-1 customers at time t depends on not only the utility of each option ("leaving" or "staying"), but also their observations of other customers' behavior. Thus, type-1 customers will evaluate a "reference utility" when they decide to leave or stay. Here, "reference utility" means the utility of all type-1 customers, $\overline{u_1} = x(t)\cdot 1 + [1-x(t)]\cdot 0 = x(t)$ [28]. Suppose $x(t)$ of type-1 customers switch to substitutable products at time t. Following the game-theoretic dynamics proposed by Fudenberg and Levine [15], we formulate the differential model for predicting the evolution of type-1 customers as follows:

$$\begin{cases} \frac{dx(t)}{dt} = x(t)\left(u_{1l} - \overline{u_1}\right); \\ x(t)|_{t=t_{i0}} = x_0 \end{cases}$$

where u_{1l} is the utility that type-1 customers obtain for leaving. The difference $(u_{1l} - \overline{u_1})$ represents the excess between the utility of leaving and the reference utility. Clearly, given the same value of $(u_{1l} - \overline{u_1})$, a greater number of customers leaving earlier results in a higher rate of leaving in the present period. Therefore, we also consider the switching rate $x(t)$ as the utility sensitivity of type-1 customers [11]. It indicates how quickly customers learn to react to the utility change. In short, these two factors, the utility sensitivity and the excess of customers' utility $(u_{1l} - \overline{u_1})$, characterize how fast the switching rate $x(t)$ changes. Substituting u_{1l} and $\overline{u_1}$ into the differential equation above and solving the model, we have

$$x(t) = 1 - \frac{1}{1+ce^{(t-t_{i0})}};\ c = \frac{x_0}{1-x_0}. \tag{1}$$

Given the switching rate $x(t)$ in (1), the number of type-1 customers remaining at time t can be calculated, which is $Aa_1[1-x(t)]$. From (1) we can also see $Aa_1(1-x(t)) = \varepsilon$ when $t = t_{i0} + \ln[(Aa_1-\varepsilon)/(c\varepsilon)]$. Denoting the time $\{t_{i0} + \ln[(Aa_1-\varepsilon)/(c\varepsilon)]\}$ by $t1_\varepsilon$, the dynamics of type-1 customers can be described as follows. Before time t_{i0}, the number of type-1 customers is always Aa_1 at time t. When a stock-out occurs at t_{i0}, the number of type-1customers instantaneously decreases to $Aa_1(1-x_0)$. As the disruption continues without any countermeasures taken by the manufacturer, the number of type-1 customers drops continuously to zero at $t = t_{1\varepsilon}$.

B. Dynamics of Type-2 Customers

As stated in Section III, type-2 customers have brand loyalty and exhibit some degree of tolerance for disruption. y_0 of them choose to instantaneously switch to alternative products at time t_{i0}. Different from type-1 customers, their utility derived from staying is $u(t)$, and from leaving is λ, where $u(t)$ depends on the brand loyalty and λ represents the competition intensity among alternative products in the market. Because the utility derived from brand loyalty decreases during the waiting period, we model $u(t)$ as $u(t) = 1-\theta(t-t_{i0})$, where $(t-t_{i0})$ is the waiting time, and $(1-\theta)$ represents the brand loyalty of type-2 customers ($0 \le \theta \le 1$). Clearly, when $\theta = 0$, type-2 customers possess the maximal brand loyalty. As a result, they derive utility 1 for staying, no matter how long the disruption lasts. On the contrary, when $\theta = 1$, type-2 customers have the minimal brand loyalty, and they obtain utility 0 for waiting. Denoting the time when $u(t) = 0$ by $t_{2\theta}$, we have $t_{2\theta} = t_{i0} + 1/\theta$. In other words, at time $t_{2\theta}$, type-2 customers' brand loyalty will be lost completely.

Let $y(t)$ be the percentage of type-2 customers who choose to leave at time t. We can model the dynamics of type-2customers as follows:

$$\begin{cases} \frac{dy(t)}{dt} = y(t)\left(u_{2l} - \overline{u_2}\right); \\ y(t)|_{t=t_{i0}} = y_0 \end{cases} \tag{2}$$

where u_{2l} denotes type-2 customers' utility for leaving, $\overline{u_2}$ is the mean utility (or reference utility) of type-2 customers and $\overline{u_2} = y(t)\lambda + [1-y(t)]ut$. According to $u(t)$, we explore the model in the next two stages.

1) Stage One. When $u(t) > 0$: As discussed before, $u(t) > 0$ corresponds to the period of $t < t_{2\theta}$. Given $u(t) = 1-\theta(t-t_{i0}) > 0$ and $u_{2l} = \lambda$, $\overline{u_2}$ is calculated as $\overline{u_2} = y(t)\lambda + [1-y(t)]u \quad t = y(t)\lambda + [1-y(t)](1-\theta(t-t_{i0}))$.

Substituting u_{2l} and $\overline{u_2}$ into (2) and solving the model, we have

$$y(t) = 1 - \frac{1}{1 + be^{v(t)}};\ b = \frac{y_0}{1 - y_0};$$
$$v(t) = \frac{1}{2}\,\theta(t - t_{i0})^2 + (\lambda - 1)\,(t - t_{i0})\,. \quad (3)$$

Based on (3), the number of type-2 customers at time t is determined, i.e., $Aa_2(1 - y(t))$. By checking the first derivative of $y(t)$, we find that $y(t)$ decreases before time $t_{2b} = t_{i0} + (1 - \lambda)/\theta$, and increases after t_{2b}. In other words, due to brand loyalty, the number of type-2 customers choosing to leave decreases before time t_{2b} as disruption continues. However, due to competition from other substitutable products, a greater number of customers will leave after time t_{2b}, even though they reserve a certain degree of brand loyalty to the manufacturer's product. As the disruption continues, all type-2 customers could leave before their brand loyalty is completely lost. That is, the value of $Aa_2(1 - y(t))$ at time $t_{2\theta}$ is less than ε, i.e., $Aa_2[1 - y(t)]|_{t_{2\theta}} < \varepsilon$. Given $t_{2\theta} = t_{i0} + 1/\theta$ and $y(t)$ in (3), the inequality can be simplified into $(\lambda - 1/2)/\theta > \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$. In other words, type-2 customers are fully lost before time $t_{2\theta}$ (in Stage one) if $(\lambda - 1/2)/\theta > \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$. Denote the critical time when the number of type-2 customers approaches zero as $t_{2\varepsilon 1}$. We can obtain $t_{2\varepsilon 1} = t_{i0} + [1 - \lambda + \sqrt{(1 - \lambda)^2 + 2\theta \ln \frac{Aa_2 - \varepsilon}{b\varepsilon}}]/\theta$ by solving $Aa_2(1 - y(t)) = \varepsilon$.

Conversely, a fraction of type-2 customers still stay at time $t_{2\theta}$ when the disruption continues, if $(\lambda - 1/2)/\theta \le \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$. We denote the switching proportion of type-2 customers at time $t_{2\theta}$ as y_{01}. Based on (3), it can be verified that $y_{01} = y(t)|_{t = t_{2\theta}} = 1 - 1/[1 + be^{(\lambda - 1/2)/\theta}]$. The number of all remaining type-2 customers at time $t_{2\theta}$ is then identified as $Aa_2[1 - y_{01}]$. To further analyze the dynamics of type-2 customers after time $t_{2\theta}$, we proceed to Stage two.

2) Stage Two. When $u(t) = 0$: Given $u_{2l} = \lambda$ and $u(t) = 0$, we see $\overline{u_2} = y(t)\lambda + [1 - y(t)]u\ t = y(t)\lambda$. Following (2), the dynamics of type-2 customers after time $t_{2\theta}$ is modeled as follows:

$$\begin{cases} \frac{dy(t)}{dt} = y(t)\,[\lambda - \lambda y(t)]\,; \\ y(t)|_{t = t_{i0} + 1/\theta} = y_{01}. \end{cases}$$

Solving the model, we have

$$y(t) = 1 - \frac{1}{1 + b_1 e^{\lambda\left(t - t_{i0} - \frac{1}{\theta}\right)}},\ b_1 = \frac{y_{01}}{1 - y_{01}}. \quad (4)$$

According to (4), the switching rate $y(t)$ keeps increasing over time. Clearly, with this trend, all the type-2 customers will be lost if no mitigation strategy is implemented. Denote the critical time when the number of type-2 customers reaches zero as $t_{2\varepsilon 2}$. We can derive it as $t_{2\varepsilon 2} = t_{i0} + 1/\theta + (1/\lambda)\ln[(Aa_2 - \varepsilon)/(b_1\varepsilon)]$ by solving $Aa_2(1 - y(t)) = \varepsilon$.

We summarize the dynamics of type-2 customers in Proposition 1, which is also illustrated in Fig. 2.

Proposition 1

1) If $(\lambda - 1/2)/\theta > \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$, the number of type-2 customers evolves as follows. Before stock-out, it remains at Aa_2 at time t in the market. When a stock out takes place at time t_{i0}, the number drops to $Aa_2(1 - y_0)$ instantaneously. If no mitigation countermeasure is implemented, due to their brand loyalty, the number of type-2 customers gradually recovers to a certain level at time t_{2b}, following the function of $Aa_2/[1 + be^{v(t)}]$. However, due to market competition, starting at time t_{2b}, the number decreases until it reaches zero at time $t_{2\varepsilon 1}$ [shown in Fig. 2(a)].
2) If $(\lambda - 1/2)/\theta \le \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$, the number of type-2 customers evolves as follows. It exhibits the same pattern as Proposition 1 (i) until time t_{2b}, when the number starts to decrease. However, the decrease follows with the function $Aa_2/[1 + be^{v(t)}]$ within the time interval $(t_{2b},\, t_{2\theta}]$, and with $Aa_2/(1 + b_1 e^{\lambda(t - 1/\theta)})$ afterward. Finally, the number of type-2 customers becomes zero at time $t_{2\varepsilon 2}$[shown in Fig. 2(b)], where $t_{2b} = t_{i0} + (1 - \lambda)/\theta$, $t_{2\varepsilon 1} = t_{i0} + \{1 - \lambda + \sqrt{(1 - \lambda)^2 + 2\theta \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]}\}/\theta, t_{2\theta} = t_{i0} + 1/\theta$, and $t_{2\varepsilon 2} = t_{i0} + 1/\theta + (1/\lambda)ln[(Aa_2 - \varepsilon)/(b_1\varepsilon)]$.

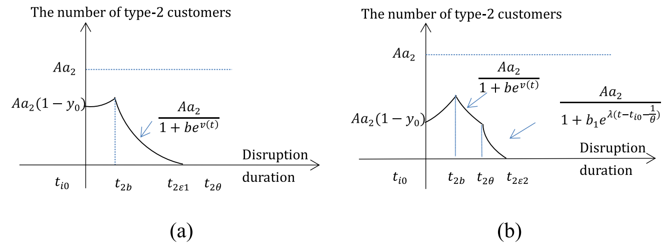

Fig. 2. The dynamics of type-2 customers: (a) if $(\lambda - 1/2)/\theta > \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$; and (b) if $(\lambda - 1/2)/\theta \le \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$.

Proposition 1 depicts an interesting trend of the dynamics of type-2 customers. During the first period $(t_{i0}, t_{2b}]$ after stock-out, the number of type-2 customers automatically recovers to a certain level after the initial decrease. In other words, some of type-2 customers choose to return after switching to other brands. This is consistent with the findings in the literature (see, e.g., [55]). However, the recovery will not sustain long and ends at time t_{2b}, as the negative impact of supply disruption increases over time.

C. Postdisruption Demand Without Mitigation

According to the above-mentioned analysis, we can examine how the numbers of type-1 and type-2 customers will change after a disruption. By incorporating the dynamics of these two classes of customers, the evolution process of the total market demand can be divided into five scenarios. Tables I and II show the prerequisite conditions and the specific demand functions during the disruption periods in various scenarios.

As indicated in Table I, without implementation of mitigation policies, the dynamics of the total market demand falls into five scenarios, depending on the complicated relationship among the following parameters: $\theta, b, b_1, \lambda, A,$ and a_2. In other words, the pattern of the demand dynamics is critically determined

TABLE I
CONDITIONS FOR ALL THE DEMAND EVOLUTION SCENARIOS

Scenarios of the demand evolution	The conditions for each scenario	
Scenario 1	$\frac{\lambda-1/2}{\theta} > \ln\frac{Aa_2-\varepsilon}{b\varepsilon}$	$\ln\frac{Aa_1-\varepsilon}{c\varepsilon} <= \frac{1-\lambda+\sqrt{(1-\lambda)^2+2\theta\ln\frac{Aa_2-\varepsilon}{b\varepsilon}}}{\theta}$
Scenario 2		$\frac{1-\lambda+\sqrt{(1-\lambda)^2+2\theta\ln\frac{Aa_2-\varepsilon}{b\varepsilon}}}{\theta} < \ln\frac{Aa_1-\varepsilon}{c\varepsilon}$
Scenario 3	$\frac{\lambda-1/2}{\theta} <= \ln\frac{Aa_2-\varepsilon}{b\varepsilon}$	$\ln\frac{Aa_1-\varepsilon}{c\varepsilon} \leq \frac{1}{\theta}$
Scenario 4		$\frac{1}{\theta} < \ln\frac{Aa_1-\varepsilon}{c\varepsilon} < \frac{1}{\theta} + \frac{1}{\lambda}\ln\frac{Aa_2-\varepsilon}{b_1\varepsilon}$
Scenario 5		$\frac{1}{\theta} + \frac{1}{\lambda}\ln\frac{Aa_2-\varepsilon}{b_1\varepsilon} < \ln\frac{Aa_1-\varepsilon}{c\varepsilon}$

TABLE II
DEMAND FUNCTION $A(t)$ DURING THE DISRUPTION PERIODS

Scenarios	The demand function $A(t)$.					
Scenario 1	Disruption duration	$[0, t_{i0}]$	$(t_{i0}, t_{1\varepsilon}]$	$(t_{1\varepsilon}, t_{2\varepsilon1}]$	$(t_{2\varepsilon1}, \infty)$	
	$A(t)$	A	$A_1 + A_{21}$	A_{21}	0	
Scenario 2	Disruption duration	$[0, t_{i0}]$	$(t_{i0}, t_{2\varepsilon1}]$	$(t_{2\varepsilon1}, t_{1\varepsilon}]$	$(t_{1\varepsilon}, \infty)$	
	$A(t)$	A	$A_1 + A_{21}$	A_1	0	
Scenario 3	Disruption duration	$[0, t_{i0}]$	$(t_{i0}, t_{1\varepsilon}]$	$(t_{1\varepsilon}, t_{2\theta}]$	$(t_{2\theta}, t_{2\varepsilon2}]$	$(t_{2\varepsilon2}, \infty)$
	$A(t)$	A	$A_1 + A_{21}$	A_{21}	A_{22}	0
Scenario 4	Disruption duration	$[0, t_{i0}]$	$(t_{i0}, t_{2\theta}]$	$(t_{2\theta}, t_{1\varepsilon}]$	$(t_{1\varepsilon}, t_{2\varepsilon2}]$	$(t_{2\varepsilon2}, \infty)$
	$A(t)$	A	$A_1 + A_{21}$	$A_1 + A_{22}$	A_{22}	0
Scenario 5	Disruption duration	$[0, t_{i0}]$	$(t_{i0}, t_{2\theta}]$	$(t_{2\theta}, t_{2\varepsilon2}]$	$(t_{2\varepsilon2}, t_{1\varepsilon}]$	$(t_{1\varepsilon}, \infty)$
	$A(t)$	A	$A_1 + A_{21}$	$A_1 + A_{22}$	A_1	0

by the transient behavior of these two types of customers, the competition intensity, the customers' constitution, and the brand loyalty of type-2 customers. The results in Table I reveal two interesting findings. First, given that demand estimation error ε is close to zero, we have $\ln\frac{Aa_2-\varepsilon}{b\varepsilon} > 0$. Hence, we can conclude that Scenarios 1 and 2 merely happen with small $\{\theta, Aa_2\}$ and large λ. In other words, the demand dynamics can be described by Scenario 1 or 2 only when market competition is intense (large λ) and the number of type-2 customers is small (small Aa_2) but high in brand loyalty level (small θ). Otherwise, the demand dynamics will be depicted by one of the remaining scenarios (i.e., Scenarios 3–5). Second, a particular observation is found based on the threshold $\frac{1}{\theta}$. Under a market with a low level of competition, the demand dynamics falls in Scenario 3 if type-2 customers exhibit a sufficiently high level of brand loyalty.

Table II gives the demand function $A(t)$ in Scenarios 1–5 with closed forms, where

$$A_1 = \frac{Aa_1}{1+ce^{(t-t_{i0})}}\ ,\ A_{21} = \frac{Aa_2}{1+be^{v(t)}}$$

$$A_{22} = \frac{Aa_2}{1+b_1e^{\lambda(t-t_{i0}-1/\theta)}}.$$

$t_{2\theta}$ represents the critical time when the brand loyalty of type-2 customers is totally lost. $t_{1\varepsilon}$ is the time when all type-1 customers choose to leave. $t_{2\varepsilon1}$ denotes the time when all type-2 customers leave under the condition of $(\lambda - 1/2)/\theta \geq \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$, and $t_{2\varepsilon2}$ for $(\lambda - 1/2)/\theta < \ln[(Aa_2 - \varepsilon)/(b\varepsilon)]$.

According to Table II, the demand dynamics during disruption periods is comprehensively described by the five types of segmental demand functions. During the periods before the time

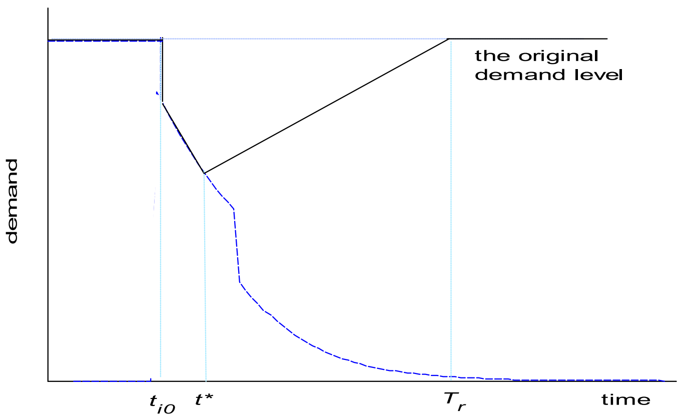

Fig. 3. Dynamic demand with contingent sourcing implemented at t^*.

$\min\{t_{1\varepsilon}, t_{2\varepsilon 1}, t_{2\varepsilon 2}\}$, some of both type-1 and type-2 customers remain in the market. As the disruption continues, these two types of customers disappear in turn. The number of type-2 customers reaches zero before type-1 customers in Scenarios 2 and 5. By using Tables I and II, we can analytically identify how the total demand dynamically reacts to disruptions according to market condition and customer characteristic. This can be done in two steps. First, we identify the applicable scenario for the demand. Second, the corresponding demand function can be found in Table II.

V. Model for Dynamic Contingent Sourcing Strategies

In this section, we propose dynamic contingent sourcing strategies by considering customer behavior and demand recovery. To this end, we examine how the demand and inventory will change if contingent sourcing is adopted to mitigate supply disruption. We then construct a model to determine the optimal contingent sourcing time to minimize the disruption impact. Finally, we develop dynamic contingent sourcing strategies.

A. Dynamics of Demand and Inventory With Contingent Sourcing

1) Dynamics of Demand: Suppose the manufacturer decides to reroute to a secondary source at time t^* to alleviate the disruption risk. With contingent sourcing, the production resumes and customers face stock-out no more. As a result, the remaining demand is satisfied, and the lost demand could be gradually recovered by implementing some marketing incentives. Denote the time when demand is completely recovered to A as $T_r(t^*)$, and the recovery speed as $1/t_r$. Intuitively, $T_r(t^*)$ depends on time t^*, the quantity $(A - A(t^*))$ required to be recovered, and the recovery speed. We describe it as $T_r(t^*) = t^* + t_r(A - A(t^*)$. The dynamic demand after time t^* is plotted in Fig. 3.

In Fig. 3, the blue dashed curve represents the demand without any mitigation countermeasures. The black solid line depicts the demand with contingent sourcing implemented at time t^*, and this demand can be expressed as follows:

$$D(t, t^*) = \begin{cases} A, & t \in (0, t_{i0}]; \\ A(t), & t \in (t_{i0}, t^*]; \\ A(t^*) + \frac{A - A(t^*)}{T_r(t^*) - t^*}(t - t^*), & t \in (t^*, T_r(t^*)]; \\ A, & t \in (T_r(t^*), +\infty) \end{cases} \tag{5}$$

where $A(t)$ is summarized in Table II. As indicated in (5), the dynamics of the postdisruption demand goes through four stages: being stable before the stock-out, dynamic evolving during the stock-out, recovering after supply is resumed from a secondary source, and back to the original level after recovery.

2) Dynamics of Inventory: According to $D(t, t^*)$, we know that demand changes dynamically over time t^*, and remains stable after time $T_r(t^*)$. Correspondingly, inventory is accumulated at a $t^* - \mathrm{dependent}$ rate during the production uptime in $[t^*, T_r(t^*)]$, and consumed dynamically during the downtime. We denote the sth production cycle after t^* as $(T^n_{s-1}(t^*), T^m_s(t^*)] \cup (T^m_s(t^*), T^n_s(t^*)]$, where $s \geq 1$, and $T^n_0(t^*) = t^*$. The production uptime is $(T^n_{s-1}(t^*), T^m_s(t^*)]$, downtime is $(T^m_s(t^*), T^n_s(t^*)]$. The inventory level at time $t \geq t^*$ is captured as $I(t, t^*)$, denoted as $I_{1s}(t, t^*)$ during the production uptime, and as $I_{2s}(t, t^*)$ during the downtime.

Assume that $T^n_{i-1}(t^*) < T_r(t^*) \leq T^n_i(t^*)$, that is, the demand is fully recovered in the ith production cycle , $i \geq 1$. Accordingly, the inventory dynamics, i.e., $I(t, t^*)$, has three stages: before, during, and after the ith cycle. Considering the possibility that demand could be entirely recovered in the production uptime and downtime, i.e., $T^n_{i-1}(t^*) < T_r(t^*) \leq T^m_i(t^*)$ and $T^m_i(t^*) < T_r(t^*) \leq T^n_i(t^*)$, the inventory dynamics during the ith cycle (Stage 2) could exhibit two patterns as depicted in Fig. 4. For notational simplicity, we omit the argument t^* and t in the following figures and tables.

In Fig. 4, the dashed triangle lines represent the inventory pattern in each original production cycle without disruption, and the solid lines correspond to the inventory with disruption and the adoption of contingent sourcing at t^*. The inventory at Stages 1–3 for $T^n_{i-1}(t^*) < T_r(t^*) \leq T^m_i(t^*)$ is shown in Fig. 4(a), and Stage 2 for $T^m_i(t^*) < T_r(t^*) \leq T^n_i(t^*)$ is given in Fig. 4(b). Next, we analytically identify $I(t, t^*)$ in Fig. 4.

a) Stage 1. The inventory dynamics before the ith cycle: After a disruption occurs at time t_0, the production stops immediately, and the inventory is consumed at the rate of demand A. As the disruption continues, a stock-out arises at time t_{i0}. During $(t_{i0}, t^*]$, demand is subject to $A(t)$ [see (5)], and the inventory level remains zero due to production stoppage. Since time t^* [see Fig. 4(a)], by using contingent sourcing, the manufacturer restores the 1st cycle after the disruption. During the production uptime $(T^n_0(t^*), T^m_1(t^*)]$ of the 1st cycle, products are manufactured at the rate of P, and consumed at the rate of $D(t, t^*)$. The inventory level $I_{11}(t, t^*)$ keeps increasing at the rate of $[P - D(t, t^*)]$ until it reaches the maximal capacity I at time $T^m_1(t^*)$. Then, during the production downtime $(T^m_1(t^*), T^n_1(t^*)]$, the production pauses. The inventory level

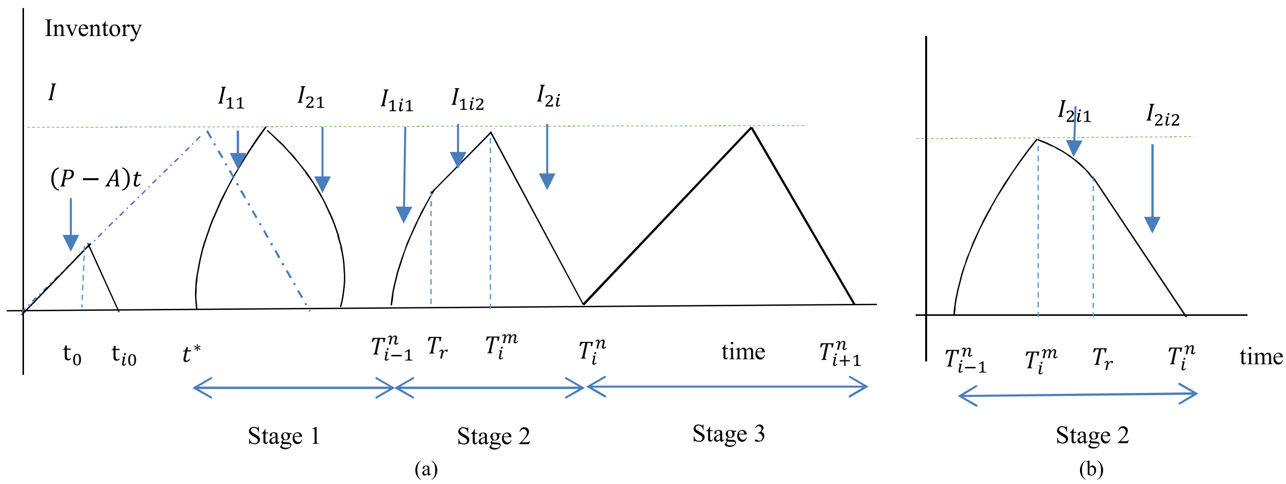

Fig. 4. Inventory: (a) when $T_{i-1}^n(t^*) < T_r(t^*) \leq T_i^m(t^*)$, and in the ith cycle (b) when $T_i^m(t^*) < T_r(t^*) \leq T_i^n(t^*)$.

$I_{21}(t, t^*)$ declines at the rate of $D(t, t^*)$ until it reaches zero at time $T_1^n(t^*)$. Clearly, the production time $[T_1^m(t^*) - T_0^n(t^*)]$ deviates from T_m, and the length $[T_1^n(t^*) - T_0^n(t^*)]$ of the 1st cycle after the disruption also deviates from T. Both the production time and length depend on the inventory dynamics, while the inventory is also related to the demand as given in (5). To build a model for determining the inventory dynamics in the 1st cycle, we take the following steps.

1) Obtain the expression of $I_{11}(t, t^*)$ based on $D(t, t^*)$ and $T_0^n(t^*) = t^*$.
2) Determine $T_1^m(t^*)$ by solving $I_{11}(t, t^*) = I$.
3) Specify the expression of $I_{21}(t, t^*)$ based on $D(t, t^*)$ and $T_1^m(t^*)$.
4) Determine $T_1^n(t^*)$ by solving $I_{21}(t, t^*) = 0$.

This pattern of inventory is repeated in the following production runs. Therefore, I_{1s} and I_{2s} $(s = 1, 2, \ldots, i)$ can be determined by sequentially extending the above-mentioned formulation in the 1st cycle to the following 2nd, ..., ith cycles, as illustrated in Fig. 5.

Based on Fig. 5, we can derive I_{1s} and I_{2s} $(s = 1, 2, \ldots, i)$ as follows:

$$I_{1s} = \int_{T_{s-1}^n(t^*)}^{t} [P - D(\tau, t^*)] d\tau \tag{6}$$

$$I_{2s} = I - \int_{T_s^m(t^*)}^{t} D(\tau, t^*) \, d\tau. \tag{7}$$

By using the demand $D(t, t^*)$ in (5), we calculate I_{1s} and I_{2s} for Stage 1 $(s \leq i-1)$ as follows:

$$I_{1s} = \int_{T_{s-1}^n(t^*)}^{t} \left[P - A(t^*) - \frac{A - A(t^*)}{T_r(t^*) - t^*}(\tau - t^*)\right] d\tau \tag{8}$$

$$I_{2s} = I - \int_{T_s^m(t^*)}^{t} \left[A(t^*) + \frac{A - A(t^*)}{T_r(t^*) - t^*}(\tau - t^*)\right] d\tau. \tag{9}$$

The inventory dynamics $I(t, t^*)$ at Stage 1 is identified by the above-mentioned I_{1s} and I_{2s}.

b) Stage 2. The inventory dynamics during the ith cycle: As indicated in Fig. 4, $I(t, t^*)$ in the ith cycle could display two patterns considering $T_{i-1}^n(t^*) < T_r(t^*) \leq T_i^m(t^*)$ and $T_i^m(t^*) < T_r(t^*) \leq T_i^n(t^*)$.

1) $T_{i-1}^n(t^*) < T_r(t^*) \leq T_i^m(t^*)$.
In this scenario, the demand is completely recovered to the original level A in the production uptime of the ith cycle. As shown in Fig. 4(a), the inventory I_{1i} during the production uptime is expressed in two distinctive segments. Let I_{1i1} and I_{1i2} denote the inventory before and after $T_r(t^*)$, respectively. The model for determining I_{1i1}, I_{1i2}, and I_{2i} is given in Fig. 5

$$I_{2i} = I - A(t - T_i^m(t^*)). \tag{11}$$

Equations (10) shown at the bottom of this page, and (11) are obtained by substituting $D(t, t^*)$ into (6) and (7).

2) $T_i^m(t^*) < T_r(t^*) \leq T_i^n(t^*)$.
The demand is recovered during the production downtime $(T_i^m(t^*), T_i^n(t^*)]$. As depicted in Fig. 4(b), the inventory I_{2i} during production downtime is composed of I_{2i1} and I_{2i2}, and Eq. (12) shown at the bottom of the next page, where I_{2i1} represents the inventory before time $T_r(t^*)$, and I_{2i2} corresponds to the inventory after time $T_r(t^*)$. In (12), I_{1i} and I_{2i1} are obtained by substituting (5) into (6) and (7), and I_{2i2} is derived from I_{2i1}.

c) Stage 3. The inventory dynamics after the ith *cycle:* The demand is completely recovered to A at time $T_r(t^*)$ in the ith cycle, and the production remains stable after time t^*.

$$I_{1i} = \begin{cases} I_{1i1} = \int_{T_{i-1}^n(t^*)}^{t} \left[P - A(t^*) - \frac{A - A(t^*)}{T_r(t^*) - t^*}(\tau - t^*)\right] d\tau, & \text{if } t \leq T_r(t^*), \\ I_{1i2} = (P - A)(t - T_r(t^*)) + I_{1i1}(T_r(t^*)), & \text{if } T_r(t^*) < t \leq T_i^m(t^*) \end{cases} \tag{10}$$

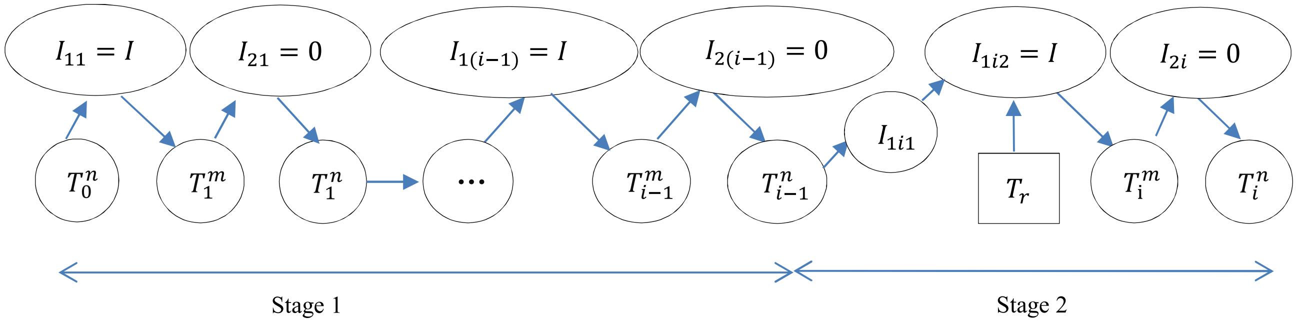

Fig. 5. Model for identifying inventory dynamics at Stages 1–2 if $T_{i-1}^n(t^*) < T_r(t^*) \le T_i^m(t^*)$.

Therefore, the inventory since the $(i+1)$th cycle exhibits the same pattern as that before the disruption. Similar to Stage 1, we can obtain the inventory I_{1k} in $(T_{k-1}^n(t^*), T_k^m(t^*)]$ and I_{2k} in $(T_k^m(t^*), T_k^n(t^*)]$ $(k = i+1, i+2, \ldots)$ as follows:

$$I_{1k} = \int_{T_{k-1}^n(t^*)}^{t} [P - D(\tau, t^*)] d\tau = (P - A)\left(t - T_{k-1}^n(t^*)\right) \tag{13}$$

$$I_{2k} = I - \int_{T_k^m(t^*)}^{t} D(\tau, t^*)\, d\tau = I - A(t - T_k^m(t^*)). \tag{14}$$

Due to the constant demand A and the constant production rate P, both the length and production time of the *k*th cycle also remain constant

$$T_k^n(t^*) = T_i^n(t^*) + (k-i)T, \text{ and } T_k^m(t^*) = T_i^n(t^*) + (k-i-1)T + T_m. \tag{15}$$

By summing up Stages 1–3, we can present the inventory $I(t, t^*)$ after getting replenishment from contingent sources at time t^* in Table III.

Table III generates the dynamics of inventory $I(t, t^*)$ with closed forms, where $I_{1s}, I_{2s}, \ldots, I_{1i1}, I_{1i2}, I_{2i}, I_{2i1}, I_{2i2}, I_{k1}$, and I_{k2} are defined in (8)–(14). Three stages are involved in $I(t, t^*)$, which falls into two patterns at Stage 2 based on $T_r(t^*)$. At Stages 1 and 2, as a result of dynamic demand, the inventory is accumulated and depleted at a varying time-dependent rate. In consequence, the lengths of the production cycle and the uptime change dynamically, and cannot be described analytically in closed forms. However, by identifying the production cycle times with the sequential model in Fig. 5, the inventory dynamics can be analytically determined by Table III. At Stage 3 where the demand is fully recovered to the original level, the inventory exhibits the same pattern as that before the disruption.

B. Disruption Impact

To identify the optimal contingent sourcing strategy for mitigating disruption, we need to evaluate the disruption impact. Since time t^*, with contingent sourcing, the production is restored to satisfy the demand. In the meantime, the demand level is gradually recovered at a constant rate (as in Fig. 3). Therefore, in addition to the production cost and inventory holding cost found in a common make-to-stock production-inventory system, the manufacturer needs to consider three more types of cost: the lost sales cost incurred from unsatisfied demand, the contingent sourcing cost referred as the markup price of contingent sources, and the demand recovery cost. Among them, the production cost incorporates both the capital cost and the variable production cost, such as equipment, materials, labor, etc. The demand recovery cost can be the cost of financial investment to restore the demand to its original level. For example, some marketing incentives may be needed to restore the reputation of the product or brand loyalty of type-2 customers.

The evaluation of disruption impact essentially depends on the demand and inventory dynamics and disruption duration. In Section V-A, we find that demand and inventory after time t^* exhibit two different patterns, depending on $T_r(t^*)$. Suppose the supply resumes at the jth cycle after time t^*, that is, $T_{j-1}^n(t^*) < T_d + t_0 \le T_j^n(t^*)$. Considering that the supply disruption could be restored during the production uptime or downtime, the disruption impact falls into four possible scenarios, as summarized in Table IV.

In Scenario 1, the demand begins to recover at time t^* and back to the level of A during the production uptime of the ith cycle, while the supply disruption is restored in the production uptime of the jth cycle. For the sake of convenience, we use (i-uptime, j-uptime) to represent Scenario 1. Accordingly, Scenarios 2–4 correspond to (i-uptime, j-downtime), (i- downtime, j-uptime), and (i- downtime, j- downtime), respectively. Furthermore, considering the supply restoration could occur after or before the cycle in which the demand is entirely recovered, i.e., $i < j$ and $i \ge j$, each scenario involves two subscenarios.

The disruption impacts in these scenarios are denoted by $\Delta C_S^1, \Delta C_S^2, \ldots, \Delta C_S^8$. Next, we explore each ΔC_S^p from

$$I_{1i} = I_{1i1}\ , \text{ and } I_{2i} = \begin{cases} I_{2i1} = I - \int_{T_i^m(t^*)}^{t} \left[A(t^*) + \frac{A - A(t^*)}{T_r(t^*) - t^*}(\tau - t^*)\right] d\tau, & \text{if } T_i^m(t^*) < t \le T_r(t^*); \\ I_{2i2} = I_{2i1}(T_r(t^*)) - A(t - T_r(t^*)), & \text{if } T_r(t^*) < t \le T_i^n(t^*) \end{cases} \tag{12}$$

TABLE III
INVENTORY LEVEL $I(t, t^*)$ AFTER TIME t^*

Stages	*Stage1*		*Stage2:* the i^{th} cycle				*Stage3*	
$I(t,t^*)$	$(T^n_{s-1}, T^m_s]$	$(T^m_s, T^n_s]$	$T^n_{i-1} < T_r \le T^m_i$	$(T^n_{i-1}, T_r]$	$(T_r, T^m_i]$	$(T^m_i, T^n_i]$	$(T^n_{k-1}, T^m_k]$	$(T^m_k, T^n_k]$
	I_{1s}	I_{2s}		I_{1i1}	I_{1i2}	I_{2i}	I_{1k}	I_{2k}
			$T^m_i < T_r \le T^n_i$	$(T^n_{i-1}, T^m_i]$	$(T^m_i, T_r]$	$(T_r, T^n_i]$		
				I_{1i1}	I_{2i1}	I_{2i2}		

TABLE IV
IMPACTS CAUSED BY DISRUPTIONS FOR ALL $T_r(t^*)$ AND T_d

Scenarios for $T_r(t^*)$ and T_d				Disruption impact ΔC_S
Scenario 1	$T^n_{i-1}(t^*) < T_r(t^*) \le T^m_i(t^*)$	$T^n_{j-1}(t^*) < T_d + t_0 \le T^m_j(t^*)$	$i \ge j$	ΔC_S^1
			$i < j$	ΔC_S^2
Scenario 2		$T^m_j(t^*) < T_d + t_0 \le T^n_j(t^*)$	$i \ge j$	ΔC_S^3
			$i < j$	ΔC_S^4
Scenario 3	$T^m_i(t^*) < T_r(t^*) \le T^n_i(t^*)$	$T^n_{j-1}(t^*) < T_d + t_0 \le T^m_j(t^*)$	$i \ge j$	ΔC_S^5
			$i < j$	ΔC_S^6
Scenario 4		$T^m_j(t^*) < T_d + t_0 \le T^n_j(t^*)$	$i \ge j$	ΔC_S^7
			$i < j$	ΔC_S^8

the five types of costs mentioned earlier: production cost $C^p_{\text{production}}$, inventory holding cost C^p_{holding}, lost sales cost C^p_{lost}, demand recovery cost C^p_{recovery}, and contingent sourcing cost $C^p_{\text{contingent}}$, where $p = 1, 2, \ldots, 8$. In this section, we use the following notations: unit production cost c_p; unit inventory holding cost per unit of time c_h; unit lost sales cost c_l; unit demand recovery cost per unit of demand rate c_r; and unite contingent sourcing cost c_s.

Scenario 1: (i-uptime, j-uptime).

1) *When $i > j$:* The supply disruption is restored in the production uptime before $T_r(t^*)$ (see Fig. 6). The whole recovery period $(t^*, T_r(t^*)]$ consists of two intervals: $(t^*, T_d + t_0]$ and $(T_d + t_0, T_r(t^*)]$. In the first interval, meeting the demand fully relies on the contingent replenishments. However, in the second period $(t^*, T_d + t_0]$, no contingent sourcing is needed as the supply resumes. Therefore, the cost of contingent rerouting is only incurred in $(t^*, T_d + t_0]$. Based on the analysis of the demand and inventory in Section V-A, next, we can specify the total disruption cost.

As depicted in Fig. 6, the period impacted by the disruption lasts till time $T^n_i(t^*)$. During $(0, t_{i0}]$, the total cost consists of production and inventory holding costs, but not lost sales cost. During $(t_{i0}, t^*]$, all remaining demand is lost in the absence of any mitigation actions, resulting in lost sales cost. During $(t^*, T_d + t_0]$, the manufacturer resumes production by contingent sourcing. As a result, both the remaining and recovered demands are satisfied. Therefore, the total cost consists of costs related to production, inventory holding, lost sales, demand recovery, and contingent sourcing. During $(T_d + t_0, T_r(t^*)]$, no contingent sourcing cost is incurred. During $(T_r(t^*), T^n_i(t^*)]$, the disruption has phased out and the demand is completely recovered. Therefore, only production and inventory holding costs need to be considered. We can generate the total cost in the impacted period $(0, T^n_i(t^*)]$ as follows.

1) The inventory holding cost

$$C^1_{\text{holding}} = c_h \left\{ \frac{1}{2}(P - A) t_0 t_{i0} + \sum_{s=1}^{i-1} \left(\int_{T^n_{s-1}(t^*)}^{T^m_s(t^*)} I_{1s} dt + \int_{T^m_s(t^*)}^{T^n_s(t^*)} I_{2s} dt \right) + \left(\int_{T^n_{i-1}(t^*)}^{T_r(t^*)} I_{1i1} dt + \int_{T_r(t^*)}^{T^m_i(t^*)} I_{1i2} dt \right) + \frac{1}{2} I \left[T^n_i(t^*) - T^m_i(t^*) \right] \right\}. \tag{16}$$

2) The lost sales cost

$$C^1_{\text{lost}} = c_l \left[A(t^* - t_{i0}) + \int_{t^*}^{T_r(t^*)} (A - D(\tau, t^*)) d\tau \right]. \tag{17}$$

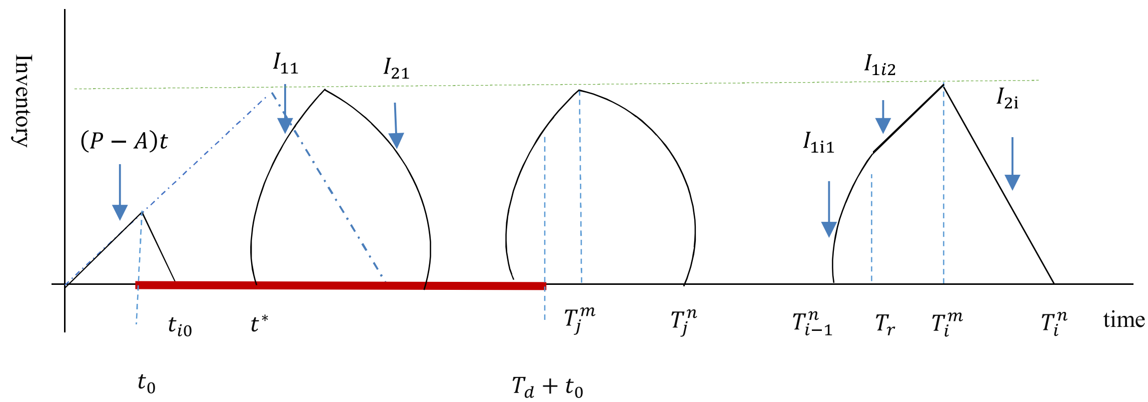

Fig. 6. Inventory pattern in Scenario 1 while $i > j$.

3) The demand recovery cost

$$C^1_{\text{recovery}} = c_r\left(A - A(t^*)\right). \tag{18}$$

4) The production cost

$$C^1_{\text{production}} = c_p\,P\left[t_0 + \sum_{s=1}^{i}\left(T_s^m(t^*) - T_{s-1}^n(t^*)\right)\right]. \tag{19}$$

5) The contingent sourcing cost

$$C^1_{\text{contingent}} = c_s P\left[\left(T_d(t^*) - T_{j-1}^n(t^*)\right) + \sum_{s=1}^{j-1}\left(T_s^m(t^*) - T_{s-1}^n(t^*)\right)\right]. \tag{20}$$

where $A(t^*)$ and $D(\tau, t^*)$ are given in (5). I_{1s}, I_{2s}, I_{1i1}, I_{1i2}, and $T_1^m(t^*), T_1^n(t^*), \ldots, T_i^m(t^*), T_i^n(t^*)$ are determined in (8)–(11).

In (16), the first item is the inventory holding cost in $(0, t_{i0}]$, the second is for $(t^*, T_{i-1}^n(t^*)]$, the third is for $(T_{i-1}^n(t^*), T_i^m(t^*)]$, and the last is for $(T_i^m(t^*),\ T_i^n(t^*)]$. Equation (17) describes the lost sales cost derived from $(t_{i0}, t^*] \cup (t^*, T_r(t^*)]$. Equation (18) gives the cost for recovering demand $A(t^*)$ to the original level A. The production cost in $(0, t_{i0}]$ and the cycles $(t^*, T_i^n(t^*)]$ are captured in (19). In (20), $P[(T_d(t^*) - T_{j-1}^n(t^*)) + \sum_{s=1}^{j-1}(T_s^m(t^*) - T_{s-1}^n(t^*))]$ is the production quantity manufactured in $(t^*, T_d(t^*)]$ from contingent sources. Therefore, the contingent sourcing cost is derived as the unit contingent sourcing cost c_s multiplied by the production quantity.

2) *When $i = j$:* $T_d(t^*)$ could lie before or beyond $T_r(t^*)$. However, we can verify that the total cost is captured with the same function as $i > j$.

On the other hand, the production cost is $c_p T_m P$ in each original production cycle, and the inventory holding cost is $\frac{1}{2}c_h IT$. Therefore, with no disruption, the total cost in $(0, T_i^n(t^*))$ can be expressed as $(\frac{1}{2}c_h IT + c_p T_m P)\frac{T_i^n(t^*)}{T}$. Given the total cost in (16)–(20), the disruption impact, with contingent sourcing adopted at time t^*, can be presented as follows:

$$\Delta C^1_S = C^1_{\text{holding}} + C^1_{\text{lost}} + C^1_{\text{recovery}} + C^1_{\text{production}} + C^1_{\text{contingent}} - \left(\frac{1}{2}c_h IT + c_p T_m P\right)\frac{T_i^n(t^*)}{T} \tag{21}$$

where T is the length of each original production cycle, and T_m is the production uptime.

3) *When $i < j$:* The disruption lasts beyond time $T_r(t^*)$ and ends during the production uptime in the jth cycle (as shown in Fig. 7). The period impacted by the disruption is $(0, T_j^n(t^*)]$.

Following the discussions of Scenario 1 (1), we find that the disruption impact $\Delta C2_S$ in $(0, T_j^n(t^*)]$ is formulated as the same expression as ΔC^1_S. That is

$$\Delta C^2_S = \Delta C^1_S \tag{22}$$

which is explained in details in Appendix. Different from ΔC^1_S, j is larger than i in ΔC^2_S. Hence, in addition to $\{t^*, T_1^m(t^*), \ldots, T_i^m(t^*), T_i^n(t^*)\}$ required in (21), $\{T_{i+1}^m(t^*), T_{i+1}^n(t^*), \ldots, T_j^n(t^*)\}$ as in (15) should be taken into account for the evaluation of ΔC^2_S.

Scenario 2: (i-uptime, j-downtime).

Scenario 2 differs from Scenario 1 only in that the disruption restoration here takes place in the production downtime of the jth cycle. In other words, the total production uptime in $(t^*, T_d]$ is given by $\sum_{s=1}^{j}(T_s^m(t^*) - T_{s-1}^n(t^*))$. The contingent sourcing cost is then derived as follows:

$$C^3_{\text{contingent}} = c_s\,P\sum_{s=1}^{j}\left(T_s^m(t^*) - T_{s-1}^n(t^*)\right).$$

Based on Fig. 5, we can describe the disruption impact in Scenario 2 as follows:

$$\Delta C^3_S = C^1_{\text{holding}} + C^1_{\text{lost}} + C^1_{\text{recovery}} + C^1_{\text{production}} + C^3_{\text{contingent}} - \left(\frac{1}{2}c_h IT + c_p T_m P\right)\frac{T_i^n(t^*)}{T}. \tag{23}$$

Scenarios 3–4: (i- downtime, j-uptime), and (i- downtime, j- downtime).

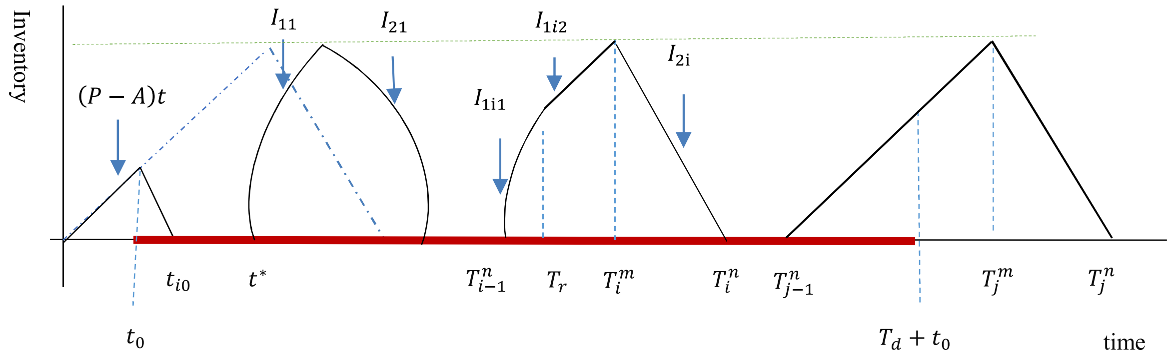

Fig. 7. Inventory pattern in Scenario 1 while $i < j$.

The demand is completely recovered during the production downtime $(T_i^m(t^*), T_i^n(t^*)]$ of the ith cycle. Based on the discussions of Scenarios 1 and 2 and the comparison of inventory patterns between Scenarios 1–2 and Scenarios 3–4 (see Figs. 4 and 6), we can obtain the disruption impact as follows:

$$\Delta C_S^5 = C_{\text{holding}}^5 + C_{\text{lost}}^1 + C_{\text{recovery}}^1 + C_{\text{production}}^1 + C_{\text{contingent}}^1 - \left(\frac{1}{2}c_h IT + c_p T_m P\right)\frac{T_i^n(t^*)}{T} \tag{24}$$

$$\Delta C_S^7 = C_{\text{holding}}^5 + C_{\text{lost}}^1 + C_{\text{recovery}}^1 + C_{\text{production}}^1 + C_{\text{contingent}}^3 - \left(\frac{1}{2}c_h IT + c_p T_m P\right)\frac{T_i^n(t^*)}{T} \tag{25}$$

$$\Delta C_S^6 = \Delta C_S^5 \text{ and } \Delta\, C_S^8 = \Delta C_S^7 \tag{26}$$

where, Eq. (27) shown at the bottom of this page.

The first item of C_{holding}^5 is the inventory holding cost in $(0, t_{i0})$, the second is for $(t^*, T_{i-1}^n(t^*)]$, the third represents for $(T_{i-1}^n(t^*), T_i^m(t^*)]$, and the last is for $(T_i^m(t^*),\ T_i^n(t^*)]$.

In summary, (21)–(26) evaluate the impact caused by the disruption with contingent sourcing adopted at time t^*. The level of disruption impact can be defined by four distinctive functions. Furthermore, the demand recovery time critically influences the total inventory holding cost, while the disruption end-time determines the contingent sourcing cost. Therefore, we need to consider not only the disruption length, the main contributing time factor, but also factors such as the demand recovery time and the disruption end-time.

C. Dynamic Contingent Sourcing Strategies

Based on the evaluation of the disruption impact above, the model for determining the optimal sourcing time t^* for contingent sourcing can be formulated as follows:

$$t^* \in arg\min \Delta\, C_S. \tag{28}$$

$$\text{Subject to } t_{i0} \le t^* \le \min\{T_d + t_0, t_2\} \tag{29}$$

where ΔC_S is given in Table IV. Equation (29) ensures that contingent replenishment is sought after time t_{i0} when the reserved stock is completely depleted, and before t_2 when the total demand (from type-1 and type-2 customers) is entirely lost or before $(T_d + t_0)$ when the supply is restored. The specific expression of $t_2 = \min\{t_{1\varepsilon}, t_{2\varepsilon 1}, t_{2\varepsilon 2}\}$ is given in Table I. From (28), (29) and Tables I and IV, we can see that the optimal sourcing time t^* depends on various factors.

As shown in Table V, one set of contributing factors that predominantly control customer reaction to disruptions (discussed in Section III) include the competition intensity (λ), brand loyalty $(1-\theta)$ of type-2 customers, transient reactions ($x_0,\ y_0$) of each type of customers, and customers' constitution (A, a_2). Another two sets of contributing factors that reflect the difficulty of sourcing and demand recovering include the contingent sourcing cost c_s, market recovery cost c_r, and time t_r. Last but not least, the factor that characterizes disruption itself is the disruption duration T_d.

With the optimal sourcing time t^* obtained from the above model, we can answer the research questions raised earlier: whether and when to temporarily implement contingent sourcing to mitigate a supply disruption? With the answers in hand, dynamic contingent sourcing strategies for managing disruptions can be established, which provide guidance on how to set the contingent sourcing time. In particular, $t^* = t_{i0}$ suggests that the manufacturer immediately reroutes to secondary sources at the time of stock-out. Conversely, $t^* = T_d + t_0$ or $t^* = t_2$ indicates that no action is needed during the whole disruption period. In other words, contingent sourcing strategy is not superior to passive acceptance (doing nothing) in these two cases. The difference is, $t^* = T_d + t_0 < t_2$ suggests that the manufacturer waits for resupply from the primary supplier when the duration of disruption is short. When $t^* = t_2$, it reveals that the contingent sourcing strategy is ineffective for mitigation on this

$$C_{\text{holding}}^5 = c_h \left\{ \begin{array}{l} \frac{1}{2}(P-A)\, t_0 t_{i0} + \sum_{s=1}^{i-1}\left[\int_{T_{s-1}^n(t^*)}^{T_s^m(t^*)} I_{1s}(t,t^*)\, dt + \int_{T_s^m(t^*)}^{T_s^n(t^*)} I_{2s}(t,t^*)\, dt\right] + \int_{T_{i-1}^n(t^*)}^{T_i^m(t^*)} I_{1i}(t,t^*)\, dt \\ + \left[\int_{T_i^m(t^*)}^{T_r(t^*)} I_{2i1}(t,t^*) dt + \frac{1}{2} I_{2i1}(T_r(t^*), t^*)\,(T_i^n(t^*) - T_r(t^*))\right] \end{array} \right\} \tag{27}$$

TABLE V
CONTRIBUTING FACTORS OF THE OPTIMAL SOURCING TIME t^*

t^* depends on the following factors						
Customer behavior				Sourcing cost	demand recovery	Disruption duration
λ	θ	(x_0, y_0)	(A, a_2)	c_s	(c_r, t_r)	T_d

22

occasion. Instead, the manufacturer should seek other potential strategies to manage supply disruptions, such as order splitting and supplier portfolio selection.

The whole process of proposing dynamic contingent sourcing strategies based on demand prediction is illustrated in Fig. 8.

Due to the complex scenarios of ΔC_S (in Table IV) and demand $D(t, t^*)$ [as given in Table II and (5)] and inventory $I(t, t^*)$ (as given in Table III), models (28) and (29) cannot be analytically examined. Therefore, we develop the following algorithm to investigate the optimal contingent sourcing time t^*.

Step 1: Calculate $t_{1\varepsilon}, t_{2\theta}, t_{2\varepsilon 1}, t_{2\varepsilon 2}, t_2$, and determine the demand function $A(t)$ by combining Table I and Table II.

Step 2: Start with $k_1 = 1$, and initialize $T_d(k_1) = t_{i0} - t_0$.

Step 3: Start with $k_2 = 1$, and initialize $t^*(k_2) = t_{i0}$, and compute $T_r(t^*(k_2))$.

Step 4: Start with $i(k_2) = 1$, and calculate $I_{1i(k_2)}, I_{2i(k_2)}$ using (8) and (9), and solve $T^m_{i(k_2)}(t^*(k_2))$ from $I_{1i(k_2)} = I$, and $T^n_{i(k_2)}(t^*(k_2))$ from $I_{2i(k_2)} = 0$. Update $i(k_2) = i(k_2) + 1$ until $T^n_{i(k_2)}(t^*(k_2)) > T_r(t^*(k_2))$ or $T^m_{i(k_2)}(t^*(k_2)) > T_r(t^*(k_2))$. The value of $i(k_2)$ is determined in this step.

Step 5: Replace $T^m_{i(k_2)}(t^*(k_2))$ by solving $I_{1i1} = I$, and $T^n_{i(k_2)}(t^*(k_2))$ by $I_{1i2} = 0$ if $T^m_{i(k_2)}(t^*(k_2)) > T_r(t^*(k_2))$. Replace $T^n_{i(k_2)}(t^*(k_2))$ by solving $I_{2i2} = 0$ if $T^n_{i(k_2)}(t^*(k_2)) > T_r(t^*(k_2))$. We obtain $\{T^m_1(t^*(k_2)), T^n_1(t^*(k_2)), \ldots, T^m_{i(k_2)}(t^*(k_2)), T^n_{i(k_2)}(t^*(k_2))\}$ before (including) the $i(k_2)$th cycle in this step.

Step 6: If $T_d(k_1) \le T^n_{i(k_2)}(t^*(k_2))$: start with $j(k_1) = 1$, search $j(k_1)$ subject to $T^m_{j(k_1)}(t^*(k_2)) < T_d(k_1) \le T^n_{j(k_1)}(t^*(k_2))$ or $T^n_{j(k_1)-1}(t^*(k_2)) < T_d(k_1) \le T^m_{j(k_1)}(t^*(k_2))$.

Step 7: If $T_d(k_1) \ge T^n_{i(k_2)}(t^*(k_2))$, $j(k_1)$ is defined by $j(k_1) = i(k_2) + \lceil T_d(k_1)/T \rceil$. Then, calculate $\{T^m_{i(k_2)+1}(t^*(k_2)), T^n_{i(k_2)+1}(t^*(k_2)), \ldots, T^m_{j(k_1)}(t^*(k_2)), T^n_{j(k_1)}(t^*(k_2))\}$ from (15). The $j(k_1)$th cycle in which the supply is restored, and the related time $\{T^m_1(t^*(k_2)), \ldots, T^m_{j(k_1)}(t^*(k_2)), \quad T^n_{j(k_1)}(t^*(k_2))\}$ are determined in Steps 6 and 7.

Step 8: Calculate $\Delta C_S(k_1, k_2)$ for $\{T_d(k_1), t^*(k_2)\}$ from Table IV. Set $k_2 = k_2 + 1$ and go back to Step 4 until $t^*(k_2) \ge T_d(k_1) + t_0$ or $t^*(k_2) \ge t_2$.

Step 9: Given $T_d(k_1)$, search the minimum $\Delta C_S(k_1, k_2)$ for $t^*(k_2) \in [0, \min\{T_d(k_1) + t_0, t_2\}]$. Set $k_1 = k_1 + 1$ and go back to Step 3. The optimal sourcing time t^* for disruption with the length of T_d is determined.

Employing Steps 1–9, the optimal contingent sourcing time t^* for (28) and (29) is obtained. The dynamic contingent sourcing strategies incorporating customer behavior and demand recovery are then proposed.

VI. Dynamic Contingent Sourcing Strategies Under Different Markets and Customer Behavior

In this section, we conduct numerical analysis to visually examine dynamic contingent sourcing strategies. We further generate insights into the important roles of customer behavior, demand recovery, disruption duration, and contingent sourcing cost. As illustrated in Table V, the factors influencing customer behavior (reaction) to a disruption include $\theta, x_0, y_0, \lambda,$ and a_2.

A. Dynamic Cotingent Sourcing Strategies Under Different Values of $\boldsymbol{\theta}, \boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{\lambda}$, and $\boldsymbol{a}_2$

Before investigating dynamic contingent sourcing strategies under diverse customer behavior, we start by visually showing the strategy under a certain pattern of customer behavior. To this end, we establish a basic set of parameter values as follows: $c_l = 9, t_0 = 2, c_p = 3, c_s = 7, c_h = 1.5, t_r = 0.5, c_r = 13, \lambda = 0.6, \theta = 0.4, P = 15, A = 10, I = 20, a_2 = 0.2, x_0 = 0.6, y_0 = 0.2$. Through the algorithm proposed in Section V-C, the optimal sourcing time t^* for a disruption with a variable length T_d can be obtained, and the dynamic contingent sourcing strategy is depicted in Fig. 9(a).

In Fig. 9(a), the *x*-axis is the time when the supply is restored, reflecting the duration of a disruption. The *y*-axis represents the optimal sourcing time t^*. t_2 indicates the time when the total demand is completely lost if no mitigation strategy is implemented during the disruption. The black line represents the dynamic contingent sourcing strategy under the given customer behavior, denoted as "NS-WS-WSNS" in the study. It reveals that the strategy has three stages. At the first stage when the disruption ends before 4, the manufacturer waits for the main supplier to resume supply rather than reroute to the secondary source, i.e., NS. At the second stage when the disruption ends during (4, 24), the optimal decision is to wait for some time before rerouting to secure contingent replenishments, i.e., WS. Note that the waiting time also varies according to the disruption duration. At the third stage when disruption lasts longer than $T_r(t_2) = t_2 + t_r A \approx 20 + 5$, the optimal sourcing time exhibits a periodic pattern.

To shed more light on the periodic pattern (WSNS) for managing long supply disruptions, we focus on the zoomed

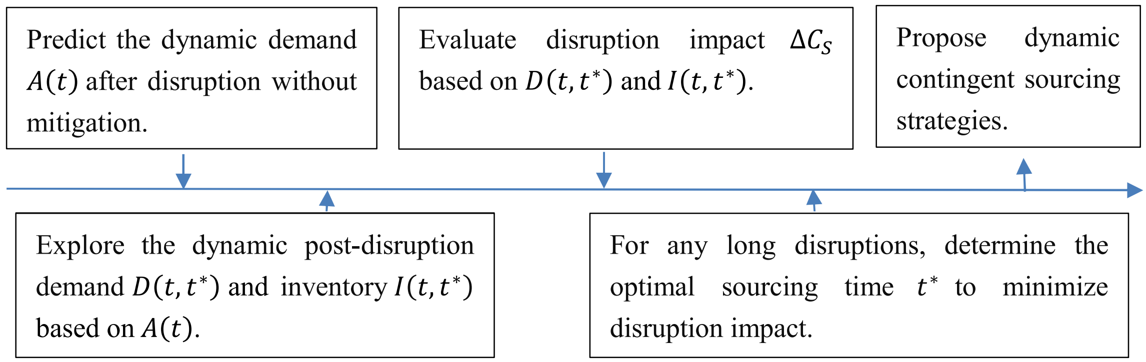

Fig. 8. Process of proposing dynamic contingent sourcing strategies.

line in (30, 36) [in Fig. 9(a)]. It shows that the contingent sourcing strategy should not be considered as a countermeasure for hedging against disruptions, the duration of which satisfies $(T_d + t_0) \in (33.5, 36)$. However, for disruptions with $(T_d + t_0) \in (30, 33.5)$, we can still mitigate them by using the optimal sourcing time t^*. Based on the results in Section V-B, we also find that the production uptime in each cycle after time $T_r(t_2)$ is $T = I/(P - A) = 4$, and the production downtime is $T_m = 2$. Obviously, the repeated periodic pattern (WSNS) here corresponds to each production cycle. "WS" is for managing long disruptions ending in the production up time, and "NS" is for downtime. In other words, the length of the disruption duration is not the only factor we should consider when designing optimal mitigation strategies. The time when the supply is restored is also important.

From Sections IV and V, we observe that customer behavior is mainly influenced by $\theta, x_0, y_0, \lambda$, and a_2. To generate further insights into the role of customer behavior, we then focus on how dynamic contingent sourcing strategies are influenced by thee parameters, as illustrated in Fig. 9.

Fig. 9(b) illustrates how contingent sourcing strategies change with brand loyalty $(1 - \theta)$ of type-2 customers. It shows that these strategies exhibit the same pattern of "NS-WS-WSNS." t^* decreases in θ for long disruptions. However, when disruptions are short, θ has little impact on t^*. Hence, lower brand loyalty means that for managing long disruptions, the manufacturer needs to reroute to secondary replenishments earlier. However, for short disruptions, the impact of brand loyalty on the optimal sourcing time is negligible. It is also worth mentioning that for long disruptions, when the brand loyalty is quite high (> 0.7), t^* can be very sensitive to brand loyalty. This is because, through interactions among customers, the information generated by highly loyal customers influences other customers more effectively. Therefore, a small drop in brand loyalty could greatly change market demand and consequently lead to a noticeable variation of time t^*.

Fig. 9(c) and (d) provides the insights regarding the roles of type-1 and type-2 customers' transient behavior [represented as (x_0, y_0)]. Our results reveal that the transient behavior of these two types of customers play different roles in determining the optimal sourcing time t^*. Specifically, y_0 substantially affects the periodic pattern "WSIS" for long disruptions. Meanwhile, x_0 plays a significant role in determining t^* for short disruptions. As x_0 grows, the components of the contingent sourcing strategies evolve from "IS-WS" to "NS-WS" and then back to "IS-WS." When x_0 is large (> 0.5), larger x_0 requires employing contingent sources earlier to cope with short disruptions. With respect to y_0, higher y_0 results in giving up the strategy for shorter disruptions and rerouting earlier.

Fig. 9(e) depicts the complicated effect of a_2 on the contingent sourcing strategies. For short disruptions (< 12), a_2 influences the optimal sourcing time without a clear pattern. It is a result of the complex demand, as some type-1 customers remain in the market for a short period. For longer disruptions, the pattern becomes more distinctive and the optimal contingent sourcing time increases to a_2. In other words, it suggests rerouting later and abandoning contingent sourcing strategy for longer disruptions if there are more type-2 customers.

Fig. 9(f) illustrates that the dynamic contingent sourcing strategies relate to competition intensity λ, which exhibit the same pattern "NS-WS-WSNS." We observe that the pattern of the strategies mainly concerns the periodic pattern for long disruptions, i.e., "WSNS." As for relatively short disruptions, the optimal sourcing time barely changes with λ. In the periodic pattern "WSNS," a higher λ suggests relinquishing contingent sourcing for shorter disruptions and implementing contingent replenishments earlier when the disruption ends during the production uptime. Another interesting observation from Fig. 9(f) is that the optimal sourcing time dramatically changes with λ if λ is small, but not so much if λ is high. This result is reasonable because the demand after disruption decreases quickly in a highly competitive market.

B. Dynamic Contingent Sourcing Strategies Under Different Values of $\boldsymbol{c_s}, \boldsymbol{t_r}$, and $\boldsymbol{c_r}$

As mentioned earlier, another set of factors influencing the optimal sourcing time consists of the contingent sourcing cost c_s, recovering cost c_r, and recovering time t_r. We further examine the roles of these factors in designing dynamic contingent sourcing strategies in this section and present how these strategies evolve along with these factors in Fig. 10.

Fig. 10(a) shows how the dynamic contingent sourcing strategies change with c_s. We find that the pattern of the strategies

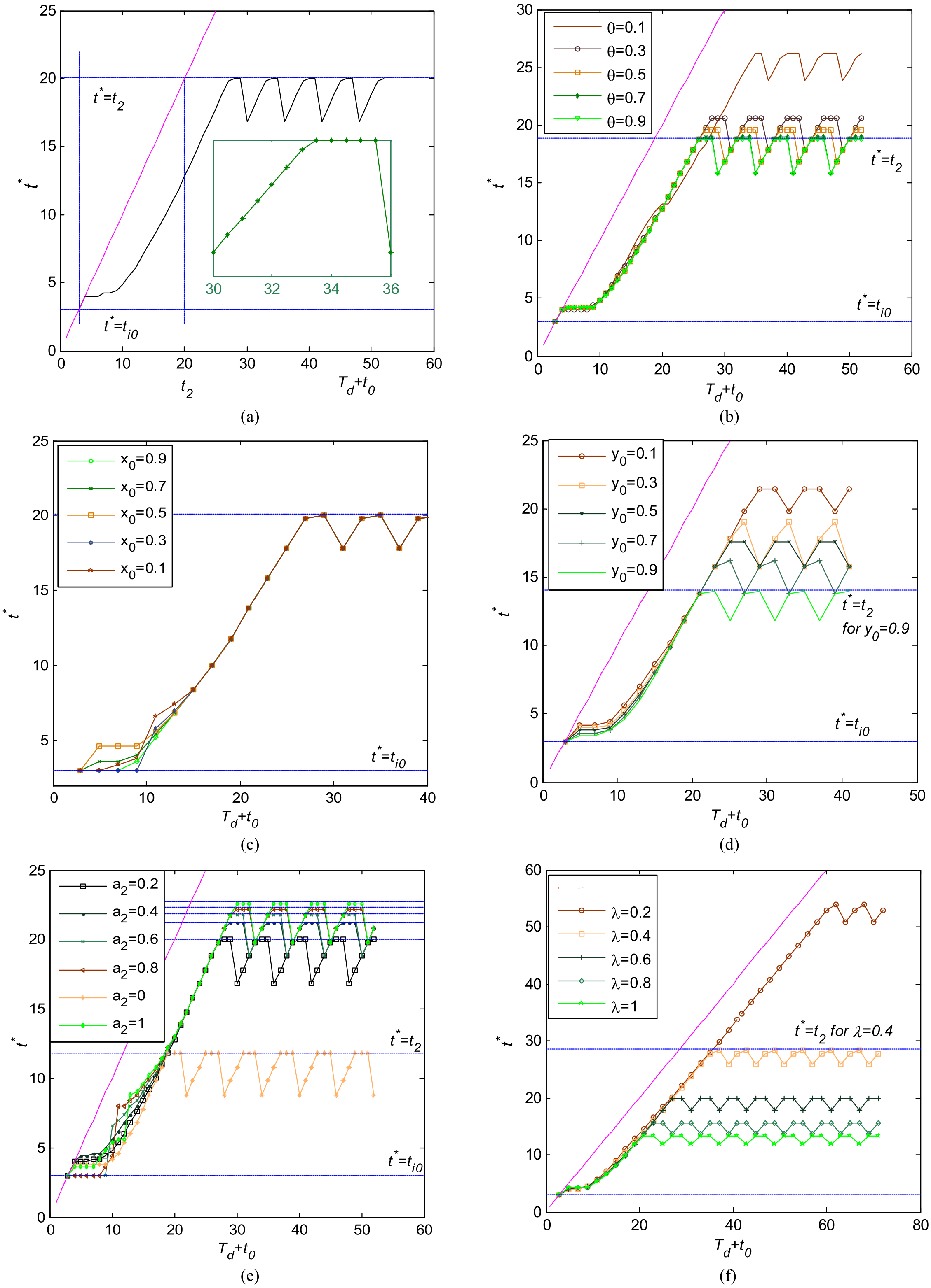

Fig. 9. Dynamic contingent sourcing strategies for the base values (a), and for different values of (b) θ, (c) x_0, (d) y_0, (e) a_2, and (f) λ.

evolves as c_s increases. When c_s is small (i.e., $c_s = 1$), the strategies start to exhibit periodic pattern "WSIS" for relatively short disruptions. It suggests the use of contingent sourcing immediately or shortly after inventory shortage. When c_s is moderate ($c_s = 7, 13, 19$), the periodic pattern only occurs for long disruptions, with the form of "WSNS." When c_s is large (>25), no periodic pattern occurs, and the contingent sourcing strategy is ineffective for managing long disruptions. Another interesting observation is that for short disruptions, larger c_s necessitates contingent replenishments later. And contingent

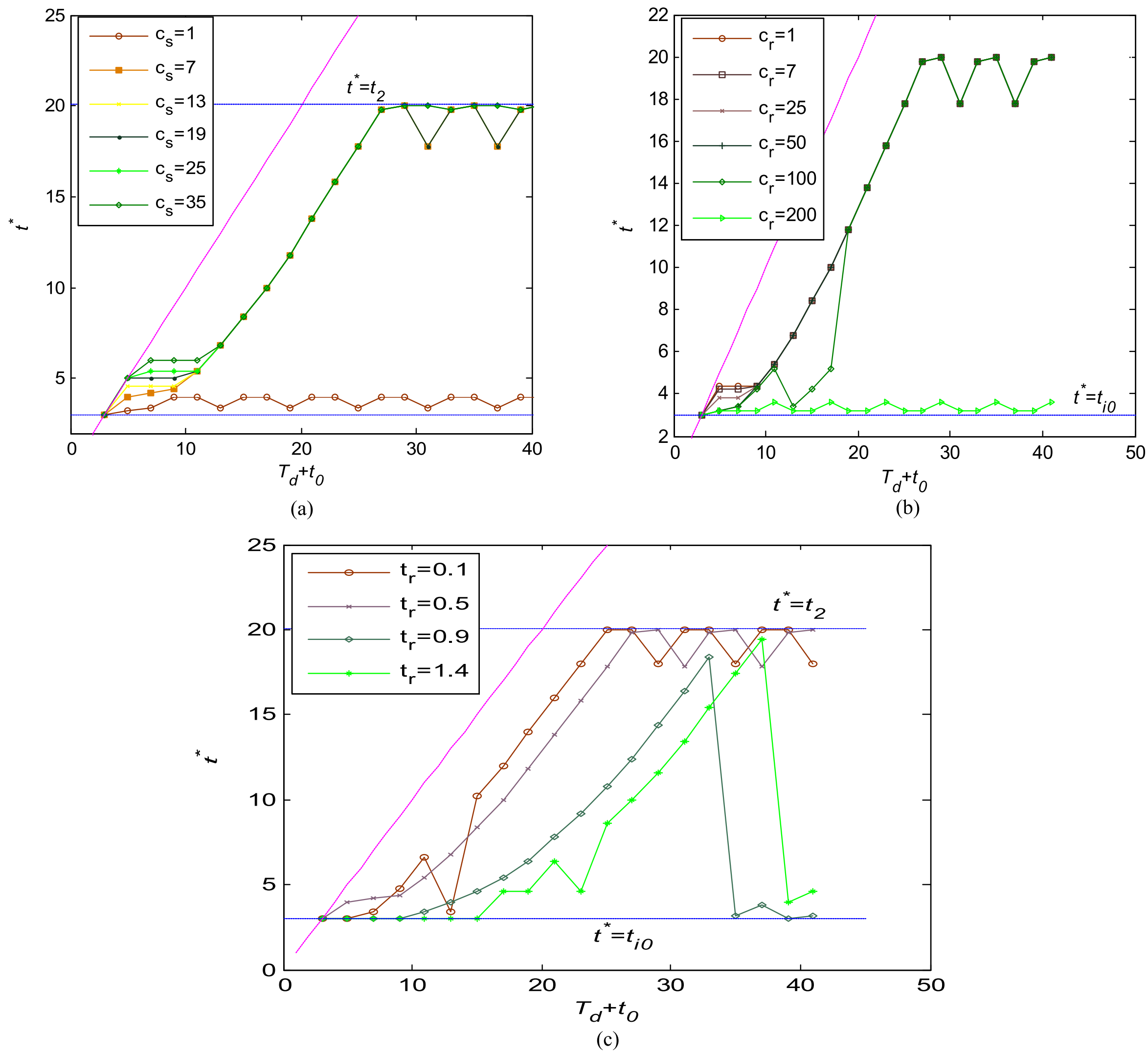

Fig. 10. Dynamic contingent sourcing strategies for different values of (a) c_s, (b) c_r, and (c) t_r.

sourcing strategies tend to be "NS" during the whole disruption period if the contingent sourcing cost is high. Nonetheless, "NS" represents distinctive strategies for long and short disruptions. It indicates that the manufacturer waits for the primary supplier to resume its supply if the disruption is short. However, it suggests that other countermeasures rather than contingent sourcing should be taken if the disruption lasts long.

Fig. 10(b) and (c) indicates that both t_r and c_r could significantly influence the patterns of contingent sourcing strategies. The periodic components for long disruptions are modified from "WSNS" to "WSIS," while t_r or c_r is increased to a certain degree. Nevertheless, t_r appears to have greater influence than c_r on the optimal sourcing time. As shown in Fig. 10(b), when c_r is small, only the optimal contingent sourcing time for short disruptions exhibits a downward trend. Meanwhile, according to Fig. 10(c), larger t_r suggests rerouting earlier for long disruptions. Therefore, we can conclude that contingent sourcing strategies should be implemented instantaneously at the appearance of inventory shortage if it requires large cost or long time for demand recovery.

VII. Conclusion and Future Research

In this paper, we consider a maker-to-stock system where a manufacturer may halt production due to a supply disruption. There are competing products in the market, and customers will be gradually lost if the disruption-induced stock-outs continues long enough. By considering customer behavior and demand recovery, we first forecast postdisruption demand and inventory dynamics, and then develop dynamic contingent sourcing strategies to mitigate disruption impacts.

We classify customers into two types in terms of their brand loyalty levels and consider the learning effect within each customer group. By employing differential models, we analytically predict that the postdisruption demand in the absence of mitigation countermeasures may exhibit one of the five possible scenarios, depending on market condition and customer characteristic, i.e., competition intensity, customers' transient reactions, brand loyalty, and customer constitution.

As the disruption continues, the manufacturer may order from a second supply source. Hence, we study how the demand and inventory change with the adoption of contingent replenishments. Our investigation confirms that in addition to customer behavior and demand recovery, the dynamics of the demand and inventory is also critically linked to the time when contingent sources are employed. By developing a sequential model incorporating various related factors, we obtain the dynamic patterns of the demand and inventory with closed forms. A model for developing dynamic contingent sourcing strategies is then formulated to identify the optimal sourcing time to minimize the impact caused by disruptions. To visually examine the dynamic contingent sourcing strategies, and generate further managerial insights, we conduct numerical analyses based on our proposed algorithm. These numerical analyses reveal that the dynamic contingent sourcing strategies have diverse patterns, with components of "IS," "WS," "NS," "WSIS," and "WSNS." We also discuss some key managerial insights for practitioners.

Our results first show that the dynamic contingent sourcing strategies may be ineffective to cope with long disruptions in some circumstances, especially when disruptions are restored during the production downtime. For long disruptions, we suggest that the manufacturer acquire replenishments from a secondary supplier instantaneously or soon after inventory shortage, if one of the following occasions occurs: it is very difficult for the demand to recover to its original level; the contingent sources can be used at a price comparable to that of the primary supplier. On the other hand, we observe that the dynamic contingent sourcing strategies tend to be ineffective for managing relatively shorter disruptions when one of the following occasions arises: the demand recovery time is short; the intensity of competition is high; type-2 customers exhibit lower brand loyalty; a greater number of type-2 customers choose to leave immediately after stock-out; or a larger proportion of type-1 customers in the market. When the dynamic contingent sourcing strategies are effective for disruption risk mitigation, we should acquire contingent sources earlier when one of these conditions is met: demand recovery requires higher cost; there are a smaller number of type-2 customers in the market; or the contingent sourcing cost is small.

Moreover, we find that the dynamic contingent sourcing strategies are not only impacted by the length of the disruption duration, but also by the time when the disruption ends. Specifically, the periodic patterns "WSIS" "WSNS" normally arise for long disruptions. The "WS" of "WSIS" "WSNS" arises when the disruption ends in the production uptime, and "IS" "NS" occurs when the disruption ends in the downtime. As for the contingent sourcing cost and the demand recovery time and cost, they markedly affect the decisions of switching among "IS," "WS," and "NS" for short disruptions, and between "WSIS" and "WSNS" for long disruptions. However, among these three factors, the market recovery time appears to have the greatest influence. On the contrary, customer behavior does not influence the periodic pattern, but affects the length of waiting time in "WSNS." Interestingly, among the determining factors of customer behavior, the transient behavior of type-1 and type-2 customers exhibits remarkably different effects on the sourcing time of short and long disruptions, respectively. The influence of competition intensity on the strategies becomes weakened when competition is more intense. And the optimal sourcing time is sensitive to the brand loyalty of type-2 customers.

This study raises several directions for future research work. For example, in this paper, we consider a single disruption in the production system, and demand could be completely recovered to the original level with enough time and cost. A future study could extend by considering multiple disruptions or the possibility of not able to fully recover demand. Another area worthy of exploration is to incorporate inventory decisions into the development of contingent sourcing strategies.

APPENDIX

The calculation for *Scenario 1(3)* in Section V-B.

Scenario 1(3): (i-uptime, j-uptime) when $i < j$.

In this subscenario, the period impacted by the disruption is $(0, T_j^n(t^*)]$. We investigate ΔC_S^2 in two intervals $(0, T_i^n(t^*)]$ and $(T_i^n(t^*), T_j^n(t^*)]$.

1) *During* $(0, T_i^n(t^*)]$*:* Comparing Figs. 6 with 7, we can have

$$C^2_{\text{holding}} = C^1_{\text{holding}};\ C^2_{\text{lost}} = C^1_{\text{lost}};\ C^2_{\text{recovery}} = C^1_{\text{recovery}};$$
$$C^2_{\text{production}} = C^1_{\text{production}}$$
$$C^2_{\text{contingent}} = c_s\ P \sum_{s=1}^{i} \left(T_s^m(t^*) - T_{s-1}^n(t^*)\right).$$

Given that the total cost in $(0,\, T_i^n(t^*)]$ without disruption is $(\frac{1}{2} c_h IT + c_p T_m P) \frac{T_i^n(t^*)}{T}$ (detailed in Scenario 1), the disruption impact during $(0,\, T_i^n(t^*)]$, denoted by ΔC_{S1}^2, can be derived as follows:

$$\begin{aligned}
\Delta C^2_{S1} &= C^2_{\text{holding}} + C^2_{\text{lost}} + C^2_{\text{recovery}} + C^2_{\text{production}} \\
&\quad + C^2_{\text{contingent}} - \left(\frac{1}{2} c_h IT + c_p T_m P\right) \frac{T_i^n(t^*)}{T} \\
&= C^1_{\text{holding}} + C^1_{\text{lost}} + C^1_{\text{recovery}} + C^1_{\text{production}} \\
&\quad + c_s P \sum_{s=1}^{i} \left(T_s^m(t^*) - T_{s-1}^n(t^*)\right) \\
&\quad - \left(\frac{1}{2} c_h IT + c_p T_m P\right) \frac{T_i^n(t^*)}{T}.
\end{aligned}$$

2) *During* $(T_i^n(t^*), T_j^n(t^*)]$*:* As shown in Table III and (5), starting from the $(i+1)$th production cycle, demand and inventory exhibit the same pattern as that before the disruption. Thus, the contingent sourcing cost is the only impact caused by the disruption in $(T_i^n(t^*), T_j^n(t^*)]$, which is

$$\begin{aligned}
\Delta C^2_{S2} = c_s\ P \Bigg[&\left(T_d(t^*) - T_{j-1}^n(t^*)\right) \\
&+ \sum_{s=i+1}^{j-1} \left(T_s^m(t^*) - T_{s-1}^n(t^*)\right) \Bigg].
\end{aligned}$$

The disruption impact $\Delta C2_S$ in $(0, T_j^n(t^*)]$ is formulated as follows:

$$\begin{aligned}\Delta C_S^2 &= \Delta C_{S1}^2 + \Delta C_{S2}^2\\ &= C_{\text{holding}}^1 + C_{\text{lost}}^1 + C_{\text{recovery}}^1 + C_{\text{production}}^1 + c_s P\\ &\quad \times \left[\left(T_d(t^*) - T_{j-1}^n(t^*)\right) + \sum_{s=1}^{j-1}\left(T_s^m(t^*) - T_{s-1}^n(t^*)\right)\right]\\ &\quad - \left(\frac{1}{2}c_h IT + c_p T_m P\right)\frac{T_i^n(t^*)}{T} = \Delta C_S^1.\end{aligned}$$

The disruption impact ΔC_S^2 has the same expression of ΔC_S^1.

References

[1] P. Amorim, E. Curcio, B. Almada-Lobo, A. Barbosa-Povoa, and I. E. Grossmann, "Supplier selection in the processed food industry under uncertainty," *Eur. J. Oper. Res.*, vol. 252, no. 3, pp. 801–814, 2016.

[2] A. Azaron, K. N. Brown, S. A. Tarim, and M. Modarres, "A multi-objective stochastic programming approach for supply chain design considering risk," *In. J. Prod. Econ.*, vol. 116, no. 1, pp. 129–138, 2008.

[3] V. Babich, A. N. Burnetas, and P. H. Ritchken, "Competition and diversification effects in supply chains with supplier default risk," *Manuf. Serv. Oper. Manage.*, vol. 9, no. 2, pp. 123–146, 2007.

[4] A. Baghalian, S. Rezapour, and R. Z. Farahani, "Robust supply chain network design with service level against disruptions and demand uncertainties: A real-life case," *Eur. J. Oper. Res.*, vol. 227, no. 1, pp. 199–215, 2013.

[5] A. Ben-Tal, B. Do Chung, S. R. Mandala, and T. Yao, "Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains," *Transp. Res. B, Methodol.* vol. 45, no. 8, pp. 1177–1189, 2011.

[6] A. L. Beutel and S. Minner, "Safety stock planning under causal demand forecasting," *Int. J. Prod. Econ.*, vol. 140, no. 2, pp. 637–645, 2012.

[7] J. Y. Chai and E. W. T. Ngai, "Multi-perspective strategic supplier selection in uncertain environments," *Int. J. Prod. Econ.*, vol. 166, pp. 215–225, 2015.

[8] P. Chu, K. L. Yang, S. K. Liang, and T. Niu, "Note on inventory model with a mixture of back orders and lost sales," *Eur. J. Oper. Res.*, vol. 159, no. 2, pp. 470–475, 2004.

[9] S. de Treville, N. Schurhoff, L. Trigeorgis, and B. Avanzi, "Optimal sourcing and lead-time reduction under evolutionary demand risk," *Prod. Oper. Manage.*, vol. 23, no. 12, pp. 2103–2117, 2014.

[10] S. J. Deng and W. Elmaghraby, "Supplier selection via tournaments," *Prod. Oper. Manage.*, vol. 14, no. 2, pp. 252–267, 2005.

[11] L. Fan, T. L. Friesz, T. Yao, and X. Chen, "Strategic pricing and production planning using a Stackelberg differential game with unknown demand parameters," *IEEE Trans. Eng. Manage.*, vol. 60, no. 3, pp. 581–591, 2013.

[12] C. Fang, X. X. Liao, and M. Xie, "A hybrid risks-informed approach for the selection of supplier portfolio," *Int. J. Prod. Res.*, vol. 54, no. 7, pp. 2019–2034, 2016.

[13] J. R. Fang, L. Zhao, J. C. Fransoo, and T. Van Woensel, "Sourcing strategies in supply risk management: An approximate dynamic programming approach," *Comput. Oper. Res.*, vol. 40, no. 5, pp. 1371–1382, 2013.

[14] A. Federgruen and N. Yang, "Selecting a portfolio of suppliers under demand and supply risks," *Oper. Res.*, vol. 56, no. 4, pp. 916–936, 2008.

[15] D. Fudenberg, and D. K. Levine. *The Theory of Learning in Games*, vol. 2. Cambridge, MA, USA: MIT Press, 1998.

[16] V. Gupta, B. He, and S. P. Sethi, "Contingent sourcing under supply disruption and competition," *Int. J. Prod. Res.*, vol. 53, no. 10, pp. 3006–3027, 2015.

[17] B. R. Gordon, "A dynamic model of consumer replacement cycles in the PC processor industry," *Marketing Sci.*, vol. 28, no. 5, pp. 846–867, 2009.

[18] B. He, H. Huang, and K. F. Yuan, "The comparison of two procurement strategies in the presence of supply disruption," *Comput. Ind. Eng.* vol. 85, pp. 296–305, 2015.

[19] B. He, H. Huang, and K. F. Yuan, "Managing supply disruption through procurement strategy and price competition," *Int. J. Prod. Res.*, vol. 54, no. 7, pp. 1980–1999, 2016.

[20] R. Helm, T. Hegenbart, and H. Endres, "Explaining costumer reactions to real stockouts," *Rev. Managerial Sci.*, vol. 7, no. 3, pp. 223–246, 2013.

[21] K. B. Hendricks and V. R. Singhal, "An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm," *Prod. Oper. Manage.*, vol. 14, no. 1, pp. 35–52, 2005.

[22] H. Hishamuddin, R. A. Sarker, and D. Essam, "A disruption recovery model for a single stage production-inventory system," *Eur. J. Oper. Res.*, vol. 222, no. 3, pp. 464–473, 2012.

[23] H. Hishamuddin, R. A. Sarker, and D. Essam, "A recovery mechanism for a two echelon supply chain system under supply disruption," *Econ. Model.* vol. 38, pp. 555–563, 2014.

[24] H. Huang and H. Y. Xu, "Dual sourcing and backup production: Coexistence versus exclusivity," *Omega – Int. J. Manage. Sci.*, vol. 57, pp. 22–33, 2015.

[25] H. Huang, Y. He, and D. Li, "EPQ for an unreliable production system with endogenous reliability and product deterioration," *Int. Trans. Oper. Res.*, vol. 24, no. 4, pp. 839–866, 2017.

[26] N. O. Hohenstein, E. Feisel, E. Hartmann, and L. Giunipero, "Research on the phenomenon of supply chain resilience: A systematic review and paths for further investigation," *Int. J. Phys. Distrib. Logist. Manage.*, vol. 45, no. 1/2 pp. 90–117, 2015.

[27] J. Jin, Y. Liu, P. Ji, and H. G. Liu, "Understanding big consumer opinion data for market-driven product design," *Int. J. Prod. Res.*, vol. 54, no. 10, pp. 3019–3041, 2016.

[28] C. Kwon, T. L. Friesz, R. Mookherjee, T. Yao and B. Feng, "Non-cooperative competition among revenue maximizing service providers with demand learning," *Eur. J. Oper. Res.*, vol. 197, no. 3, pp. 981–996, 2009.

[29] A. Latour, "Trial by fire: A blaze in Albuquerque sets off major crisis for cell-phone giants," *Wall Street J.* vol. 1, no. 29, 2001.

[30] S. Li, Y. He and L. Chen, "Dynamic strategies for supply disruptions in production-inventory systems," *Int. J. Prod. Econ.*, vol. 194, no. 88–101, 2017.

[31] C. C. Lin and T. H. Wang, "Build-to-order supply chain network design under supply and demand uncertainties," *Trans. Res. B, Methodol.* , vol. 45, no. 8, pp. 1162–1176, 2011.

[32] C. X. Liu, T. Shu, S. Chen, S. Y. Wang, K. K. Lai, and L. Gan, "An improved grey neural network model for predicting transportation disruptions," *Expert Syst. With Appl.*, vol. 45, pp. 331–340, 2016.

[33] Y. Merzifonluoglu, "Risk averse supply portfolio selection with supply, demand and spot market volatility," *Omega – Int. J. Manage. Sci.*, vol. 57, pp. 40–53, 2015.

[34] S. O'Neil, X. Y. Zhao, D. Sun, and J. C. Wei, "Newsvendor problems with demand shocks and unknown demand distributions," *Decis. Sci.*, vol. 47, no. 1, pp. 125–156, 2016.

[35] S. K. Paul, R. Sarker, and D. Essam, "A quantitative model for disruption mitigation in a supply chain," *Eur. J. Oper. Res.*, vol. 257, no. 3, pp. 881–895, 2017.

[36] L. Qi, "A continuous-review inventory model with random disruptions at the primary supplier," *Eur. J. Oper. Res.*, vol. 225, no. 1, pp. 59–74, 2013.

[37] L. Qi, Z. J. M. Shen, and L. V. Snyder, "A continuous-review inventory model with disruptions at both supplier and retailer," *Prod. Oper. Manage.*, vol. 18, no. 5, pp. 516–532, 2009.

[38] A. Samvedi and V. Jain, "A grey approach for forecasting in a supply chain during intermittent disruptions," *Eng. Appl. Artif. Intell.*, vol. 26, no. 3, pp. 1044–1051, 2013.

[39] T. Sawik, "Joint supplier selection and scheduling of customer orders under disruption risks: Single vs. dual sourcing. *Omega – Int. J. Manage. Sci.*, vol. 43, pp. 83–95, 2014.

[40] A. J. Schmitt and L. V. Snyder, "Infinite-horizon models for inventory control under yield uncertainty and disruptions," *Comput. Oper. Res.*, vol. 39, no. 4, pp. 850–862, 2012.

[41] A. J. Schmitt, S. A. Sun, L. V. Snyder, and Z. J. M. Shen, "Centralization versus decentralization: Risk pooling, risk diversification, and supply chain disruptions," *Omega – Int. J. Manage. Sci.*, vol. 52, pp. 201–212, 2015.

[42] H. Seok, K. Kim, and S. Y. Nof, "Intelligent contingent multi-sourcing model for resilient supply networks," *Expert Syst. Appl.* vol. 51, pp. 107–119, 2016.

[43] X. F. Shao and M. Dong, "Supply disruption and reactive strategies in an assemble-to-order supply chain with time-sensitive demand," *IEEE Trans. Eng. Manage.*, vol. 59, no. 2, pp. 201–212, May 2012.

[44] L. V. Snyder, Z. Atan, P. Peng, Y. Rong, A. J. Schmitt, and B. Sinsoysal, "OR/MS models for supply chain disruptions: A review," *IIE Trans.*, vol. 48, no. 2, pp. 89–109, 2016.

[45] F. J. Sting and A. Huchzermeier, “Dual sourcing: Responsive hedging against correlated supply and demand uncertainty,” *Naval Res. Logist.*, vol. 59, no. 1, pp. 69–89, 2012.
[46] O. Tang and S. N. Musa, “Identifying risk issues and research advancements in supply chain risk management,” *Int. J. Prod. Econ.*, vol. 133, no. 1, pp. 25–34, 2011.
[47] B. Tomlin, “On the value of mitigation and contingency strategies for managing supply chain disruption risks,” *Manage. Sci.*, vol. 52, no. 5, pp. 639–657, 2006.
[48] B. Tomlin, “Disruption-management strategies for short life-cycle products,” *Naval Res. Logist.*, vol. 56, no. 4, pp. 318–347, 2009.
[49] T. Wu, S. Huang, J. Blackhurst, X. Zhang, and S. Wang, “Supply chain risk management: An agent-based simulation to study the impact of retail stockouts,” *IEEE Trans. Eng. Manage.*, vol. 60, no. 4, pp. 676–686, Nov.2013.
[50] Y. F. Yang and X. R. Xu, “Post-disaster grain supply chain resilience with government aid,” *Transp. Res. E, Logist. Transp. Rev.*, vol. 76, pp. 139–159, 2015.
[51] Z. B. Yang, G. Aydin, V. Babich, and D. R. Beil, “Supply disruptions, asymmetric information, and a backup production option,” *Manage. Sci.*, vol. 55, no. 2, pp. 192–209, 2009.
[52] H. S. Yu, A. Z. Zeng, and L. D. Zhao, “Single or dual sourcing: Decision-making in the presence of supply chain disruption risks,” *Omega – Int. J. Manage. Sci.*, vol. 37, no. 4, pp. 788–800, 2009.
[53] M. Yu, L. Debo, and R. Kapuscinski, “Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods,” *Manage. Sci.*, vol. 62, no. 2, pp. 410–435, 2015.
[54] W. Zinn and P. C. Liu, “Consumer response to retail stockouts,” *J. Bus. Logist.* vol. 22, no. 1, pp. 49–71, 2001.
[55] W. Zinn and P. C. Liu, “A comparison of actual and intended consumer behavior in response to retail stockouts,” *J. Bus. Logist.* , vol. 29, no. 2, pp. 141–159, 2008.

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