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A Distributed Model Predictive Control Strategy for Bullwhip Reducing Inventory Management Policy

Dongfei Fu, Hai-Tao Zhang, *Senior Member, IEEE,* Ying Yu, Clara Ionescu, *Member, IEEE,* El-Houssaine Aghezzaf, and Robin De Keyser

***Abstract*—Given the input/output constraints and cross-couplings of supply chain (SC) nodes, model predictive control (MPC) is efficient to seek the optimal solutions to the problems posed by interacting nodes to satisfy customer demands. In supply chain applications, due to the growing spatial distribution and interactions between the supply network elements, the information flow management becomes a challenging yet significant task. To reduce numerical complexity while maintain implementability, a distributed MPC strategy is proposed. The scheme aims at finding the Nash equilibrium where the controller of each subsystem communicates with other ones in the presence of non-cooperative interaction and strong coupled inputs due to the ordering decisions. Extensive numerical simulations verify that the strategy outperforms conventional policies in terms of substantially reduced SC operating cost.**

***Index Terms*—Predictive control, supply chain management, inventory control, large-scale systems, distributed industrial control.**

I. Introduction

SUPPLY chain (SC) is a large-scale distribution network where multiple highly interconnected organizations contribute to move products or services from sources (suppliers) to destinations (customers) [1]. The ever increasing complexity of SC systems due to growing spatial distribution and interactions among network nodes poses challenges to both operations and management. In this context, it requires effective methods to coordinate the multiple operational decisions along the chain, leading to cooperation between various SC players [2]. The coordination is realized via information exchange between agents/controllers of SC organizations, which results in various control structures [3]–[5].

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D. Fu and H.-T. Zhang are with School of Automation and State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: fudongfei@hust.edu.cn, zht@mail.hust.edu.cn).

Y. Yu is with the Energy Conservation Business Unit, Dongfang Electronics Co., Ltd, Yantai 264000, China (e-mail: dongfangy.yu@outlook.com).

C. Ionescu and R. De Keyser are with the Department of Electrical Energy, Metals, Mechanical Constructions and Systems, Ghent University, Gent 9000, Belgium (e-mail: ClaraMihaela.Ionescu@UGent.be, Robain.DeKeyser@UGent.be).

E.-H. Aghezzaf is with the Department of Industrial Systems Engineering and Product Design, Ghent University, Gent 9000, Belgium (e-mail: ElHoussaine.Aghezzaf@UGent.be).

Supply chain management (SCM) handles *a priori* the movement and processing of materials, finance and information in a manner that it can address customer demand uncertainties without creating too much inventories [6]. It involves various decisions at three hierarchical levels with respect to the time scales (i.e. length of the planning horizon) and their impacts (i.e. the importance of the decisions), that is, long-term strategic planning, mid-term decisions and short-term execution, of which the last two are grouped into the operational level [7], [8]. Most of operational decisions have been traditionally made by managers at each site, leading to a decentralized decision-making structure. Disregarding the strong interactions among nodes may lead to an unreliable decentralized control [9]. Recognition of this fact gives rise to a significant change in the way of managing these interacting SC partners, which is characterized by a distributed decision-making process. In recent years, distributed control strategies has attracted more and more attentions from SC scholars [5], [10], [11] due to its capability of maintaining the topology and flexibility of decentralized decision-making structure, especially reducing the bullwhip effect.

In this paper, the SC operations are treated as a tracking control problem in which the inventory position is kept as close as possible to the target. A proper inventory control scheme takes an important role in mitigating both the inventory disruption and the bullwhip effect [12], [13]. However, the SCs are complex systems requiring multi-objective decision-making, where various operational goals need to be considered simultaneously. Furthermore, some physical restrictions are commonly encountered in practical SC operations and must be taken into account. Significantly, along the broad range of control engineering techniques, model predictive control (MPC) is one of the most suitable frameworks for both operations and management of SCs [14] and more general networked system control [15]–[17] due to its capability of handling both input/output constraints and cross-couplings of interacting nodes. One recent trend towards SCM is to apply distributed MPC (DMPC) schemes to the decision-making processes, where local agents of SC nodes exchange predication information to achieve global optimized performance. The DMPC is featured by two aspects i.e., i) interactions between subsystems are considered explicitly in local prediction models, and ii) information is transmitted among controllers of interacting subsystems. The existing DMPC strategies can be classified with respect to two criteria [18]:

- The information exchange can be made according to either of the following protocols: *non-iterative algorithms*

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where information is transmitted only once by local agents within each sampling period; and *iterative algorithms* where information is transmitted multiple times until all the agents have reached consensus.
- Each local agent can optimize either a local cost function in *independent algorithms* or a global cost function in *cooperative algorithms*.

So far, there are only a few efforts devoted to DMPC in SC engineering. Dunbar & Desa [19] applied a distributed nonlinear MPC method to a three-node supply network, where each node was regulated and optimized locally in parallel with the other nodes. During the optimization process, each local subsystem was required to communicate the most recent control profile to those coupling nodes. Maestre *et al*. [10] proposed a DMPC scheme based on cooperative gaming, which is applied to a two-node SC system. Subramanian *et al*. [11] presented cooperative and parallel algorithms based on a DMPC structure, which were afterwards implemented to a two-node SC as an illustrative example. Till now, the dynamic relationship between SC behaviors and MPC is still unclear, thus only a few works are conducted for DMPC applications to SCM. To fill the gap, an independent and iterative DMPC approach is proposed in this paper to provide an optimal replenishment protocol for a benchmark SC. More precisely, this approach seeks assistance from the Nash equilibrium. Upon attaining such a Nash equilibrium, no local agent can improve its performance by changing its decision. The possibility of iteratively exchanging information with the rest of agents in the local optimization guarantees the availability of globally optimized decisions, and hence the whole SC reaches coordination. Finally, the condition that the whole SC system can reach the equilibrium is provided to guarantee the convergence of the proposed DMPC algorithm.

The remainder of the paper is organized as below. Section II presents the detailed modeling and control problem. Distributed replenishment strategy is proposed in Section III. Numerical simulations are conducted in Section IV. Finally, the conclusions are drawn in Section V.

The following notations will be used throughout the paper. $\mathbb{C}$, $\mathbb{Z}$ and $\mathbb{R}^+$ denote the complex, positive integer and non-negative real number sets, respectively. "$*(t+k|t)$" ($t,k \in \mathbb{Z}$) is the prediction of a variable "$*$" at time $t+k$ based on the information available at time t. "Δ" is an operator taking $\Delta*(t) = *(t) - *(t-1)$. The notation $\|\cdot\|_Q$ stands for the matrix Q-weighted Euclidean norm, i.e. $\|\zeta\|_Q = \sqrt{\zeta^{\mathrm{T}} Q \zeta}$. Matrix $\mathbf{I}$ denotes the unit matrix with a compatible dimension. With slight abuse of notations, "z^{-1}" represents one-step backward shift operator.

II. Problem Formulation

The SC in red box of Fig. 1 is a cascade production and distribution system, which shares a similar structure to the models in [20]–[22]. For notational convenience, the SC nodes and the node i's neighbors are collected in the sets $\mathbb{N} := \{1, 2, \cdots, n\}$ and $\mathcal{N}_i := \{j \in \mathbb{N} \mid |i-j| = 1\}$, respectively.

The SC is characterized by counter-current flows of ordering information and product. The orders propagate upstream from end-customers to factory n, and products are shipped along the opposite direction. A periodic review strategy is adopted where the operational decisions are made at evenly separated time instants $t \cdot \xi$, where ξ is the review period and $t \in \mathbb{T} := \{1, 2 \cdots\}$. First, take any arbitrary node $i \in \mathbb{N}$ to analyze micro-dynamics of the SC network. There are four events performed within each review period $[t\cdot\xi, (t+1)\cdot\xi)$ with the following sequence. Node i first receives goods from supplier, then observes demand from customer and satisfies the standing demand. Finally, an order is placed following a certain replenishment policy. Accordingly, let $s_i \in \mathbb{R}^+$, $g_i \in \mathbb{R}^+$, $d_i \in \mathbb{R}^+$ and $u_i \in \mathbb{R}^+$ be the shipment of goods from supplier $i+1$, the delivery to customer $i-1$ of the current node i, the demand and the replenishment orders, respectively.

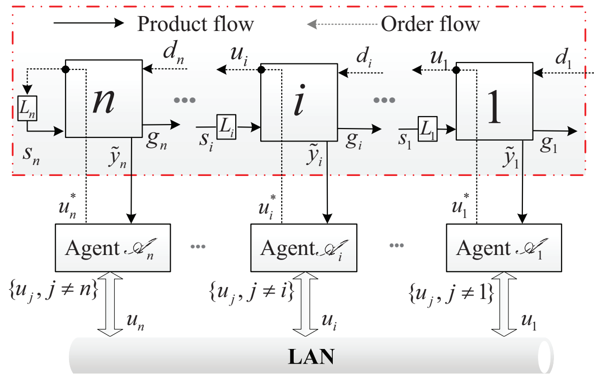

Fig. 1. The distributed control structure of the benchmark supply chain.

The end-customer demand d_1 enters the SC system at node 1 as random disturbance and propagates along the direction of order flow. It should be noted that d_1 is not assumed to follow any specified statistical property but accounts for any standard distribution typically analyzed in the practical problem. Denote by $x_i(t) \in \mathbb{R}^+$ the stock level and $y_i(t) \in \mathbb{R}^+$ the inventory position (IP) at time t. Note that the ordering information is transmitted instantaneously, but an order placed at t can only be processed at time instant $t+1$ due to the administrative delay. Therefore, the standing orders $v_i(t) \in \mathbb{R}^+$ is defined as the amount of orders to be handled at $t+1$. Without loss of generality, zero initial conditions on all the variables are assumed. From the law of conservation, node i's inventory, inventory position, and standing orders are given as:

$$x_i(t) = x_i(t-1) + s_i(t-L_i) - g_i(t), \tag{1}$$

$$y_i(t) = y_i(t-1) + s_i(t) - g_i(t), \tag{2}$$

$$v_i(t) = v_i(t-1) + d_i(t) - g_i(t), \tag{3}$$

where the replenishment lead time $L_i = m_i \cdot \xi$ and $m_i \in \mathbb{Z}$ is assumed to be invariable over time horizon of interest. Note that the following DMPC strategy is also applicable under stochastic lead-time scenario due to it is inherently robust to system uncertainty and modeling errors.

The decisions of goods dispatch $g_i(t)$ plays a significant role in accurately representing SC dynamic responses. It depends on the sufficiency of $x_i(t-1)$ to fulfill $v_i(t-1)$ [23]. Thus, $g_i(t) = \min\{x_i(t-1), v_i(t-1)\}$. No unfilled demand will be lost, instead it will be backlogged until deliveries can be

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made. Since each node's stock conditions are time-varying, the SC is naturally a switched system [24] with $\{\eta_i(t) = x_i(t-1) - v_i(t-1), i \in \mathbb{N}\}$ serving as the switching variables. To capture the SC dynamics with a switched system approach, a proposition is introduced.

Proposition 1: The SC node i is a switched system where the general dynamic IP model (2) falls into either of the following cases:

C1. If $\eta_{i+1}(t) \geq 0$ and $\eta_i(t) \geq 0$, the SC node i operates under *Infinite Supply High Stock* (*ISHS*) mode at time instant t, and the actual IP $y_i(t)$ is

$$y_i(t) = \frac{z^{-1}}{1-z^{-1}}\big[u_i(t) - d_i(t)\big]; \tag{4}$$

C2. If $\eta_{i+1}(t) \geq 0$ and $\eta_i(t) < 0$, the SC node i operates under *Infinite Supply Low Stock* (*ISLS*) mode at time instant t and

$$y_i(t) = \big(z^{-1} + z^{-2} + \cdots + z^{-(L_i+1)}\big)u_i(t); \tag{5}$$

C3. If $\eta_{i+1}(t) < 0$, the SC node i operates in *Limited Supply* (*LS*) mode at time instant t and

$$y_i(t) = \begin{cases} \dfrac{z^{-1}}{1-z^{-1}}\big[x_{i+1}(t) - d_i(t)\big], & x_i(t) \geq z^{-1}d_i(t); \\ \dfrac{z^{-1}(1-z^{-(L_i+1)})}{1-z^{-1}}x_{i+1}(t), & \text{otherwise.} \end{cases} \tag{6}$$

∎

Proof: Refer to Appendix A. ∎

The complete description of SC dynamics is realized by above switched system in *Proposition 1* when it is complemented with an ordering policy. The most frequently encountered ordering policy is "*Order-Up-To*" (OUT), where the inventory position is reviewed periodically and an "*Order*" $u_i(t) = w_i(t) - y_i(t)$ is placed to bring the IP "*Up-To*" a predefined level $w_i(t)$. The target is updated by a forecast $w_i(t) = (L_i+2)\hat{d}_i(t)$, where the estimate can be approached by exponential smoothing $\hat{d}_i(t) = \frac{1}{1+\alpha_i}[d_i(t) + \alpha_i \hat{d}_i(t-1)]$ with an average age α_i of the demand data, or moving average $\hat{d}_i(t) = \frac{\sum_{k=0}^{T_w-1} d_i(t-k)}{T_w}$ with T_w being the number of review periods within the forecast window. Although it is optimal to minimize the variance of x_i, it is not the best choice when considering the bullwhip effect [21], [25], [26]. To quantify the bullwhip effect, the following index is given in *Definition 1*.

Definition 1: In an SC, when both two sets of matching "in" and "out" time series data $\{d(t)\}_{t=1}^{|\mathbb{T}|}$, $\{u(t)\}_{t=1}^{|\mathbb{T}|}$ are available for demand and upstream orders, respectively, then the magnitude of the bullwhip effect (Ω) is defined as:

$$\Omega := \frac{\sigma_u^2/\mu_u}{\sigma_d^2/\mu_d}, \tag{7}$$

where $|\mathbb{T}|$ denotes the cardinality of $\mathbb{T}$, and σ^2, μ are variance and mean values, respectively. ∎

Remark 1: Although $\Omega := \sigma_u^2/\sigma_d^2$ is a widely used index since [27], a variation of Ω suggested by [28] in Eq. (7) quantifies more appropriately for practical utilization [29]. The metric is applicable to the following two cases: i) a single decision-making unit, and ii) a system with a more macro-type transformation mechanism. $\Omega > 1, \Omega = 1$ and $\Omega < 1$ imply demand signal amplification, i.e. the bullwhip effect, neutrality and damping, respectively. As an aggregate indicator for efficiency and effectiveness of SC operations, Ω is normally expected to be reduced through a more proper design of ordering policies. ∎

However, SC performance improvement does not lie solely in reducing the bullwhip effect. Other challenges of practical SCM goals, e.g. dynamic inventory management policy, multi-objective decision-making, and resilience to uncertain demand also need to be addressed. The following MPC-based replenishment is a natural solution to meet these requirements due to its inherent control mechanism.

The DMPC Replenishment Problem: The DMPC replenishment scheme is illustrated in Fig. 1. For the SC composed of n subsystems $\{\mathscr{S}_i, i \in \mathbb{N}\}$, each associated agent $\mathscr{A}_i$ designs a local MPC law with feedback information from its neighbors $\{\mathscr{A}_j, j \in \mathcal{N}_i\}$ transmitted via a local area network (LAN), such that the optimal $u_i^*(t)$ is obtained to match $y_i(t)$ to $w_i(t)$ while mitigating Ω_i and overall Ω, $i \in \mathbb{N}$, $t \in \mathbb{T}$.

III. DMPC Replenishment Strategy

A. The Nash Optimality for SC System

Fig.1 shows the DMPC approach to derive replenishment rules. In practice, the SC members are generally not quite willing to completely share their collected data [3], [29]. However, the information exchange underpins the proposed DMPC strategy. To overcome this dilemma, we consider iterative exchange of the presumed, predictive information among agents, rather than the exchange of real data among SC members. Therein, each $\mathscr{A}_i$ uses both partial information of $\mathscr{S}_i$ and the presumed, iteratively optimized ordering signals from $\{\mathscr{A}_j, j \in \mathcal{N}_i\}$ for decision-making of u_i by optimizing a local cost function J_i. The local goals of different $\mathscr{S}_i$ are reconciled by seeking the Nash equilibrium [30], where each agent minimizes J_i only with respect to its local control actions ΔU_i assuming that neighbors' optimal solutions $\{\Delta U_j^*, j \in \mathcal{N}_i\}$ have been obtained:

$$\left.\frac{\partial J_i}{\partial \Delta U_i}\right|_{\Delta U_j^*(j \in \mathcal{N}_i)} = 0, \quad i \in \mathbb{N}. \tag{8}$$

When the iteration stops, the resulted $\Delta U^* = [\Delta U_1^*, \cdots, \Delta U_n^*]$ is called Nash optimal solution if for all $\{\Delta U_i^*, i \in \mathbb{N}\}$, the following condition stands:

$$\begin{aligned} J_i(\Delta U_1^*, \cdots, \Delta U_i^*, \cdots, \Delta U_n^*) \leq \\ J_i(\Delta U_1^*, \cdots, \Delta U_i, \cdots, \Delta U_n^*). \end{aligned} \tag{9}$$

The overall SC system reaches the Nash equilibrium if every agent has achieved the Nash optimal solution. Before giving the DMPC algorithm, we need the following assumptions.

Assumption 1: Each local agent $\{\mathscr{A}_i, i \in \mathbb{N}\}$ implements the MPC algorithm and uses identical control and prediction horizons with neighbors, i.e. $N_u^i = N_u^j = N_u$, $N_2^i = N_2^j = N_2$, $j \in \mathcal{N}_i$, respectively. ∎

Assumption 2: At each time instant t, $\{\mathscr{A}_i, i \in \mathbb{N}\}$ exchanges the optimized control action $\Delta U_i^{(l_i(t))}(t)$ of the $[l_i(t)]^{\text{th}}$ iteration with its neighbors via LAN iteratively, s.t.

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$T_i(t) = \sum_{l_i(t)=1}^{\min\{\bar{l}_i(t),\, l_i^*(t)\}} \tau_i(l_i(t)) \leq \xi$, where $\tau_i(l_i(t))$ is the optimization time at the $[l_i(t)]^{\text{th}}$ iterative step, the design limit $\bar{l}_i(t)$ is the maximum permissible iteration within $[t,\, t+1)$, and $l_i^*(t)$ satisfies $\|\Delta U_i^{(l_i^*(t))} - \Delta U_i^{(l_i(t))}\| \leq \varepsilon_i$ with an error threshold $\varepsilon_i > 0$. ∎

Remark 2: This assumption is an immediate result of SCM operations. It ensures the triviality of LAN communication network-induced imperfections [31] in order to accomplish the global objective, i.e. $\Delta U_i^{(l_i(t))}(t)$ can be transmitted multiple times from $\mathscr{A}_i$ to $\{\mathscr{A}_j, j \in \mathcal{N}_i\}$ within $[t,\, t+1)$. ∎

Assumption 3: $\{\mathscr{A}_i,\, \forall i \in \mathbb{N}\}$ are synchronous in the sense that $\{\Delta U_i^*(t),\, \forall i \in \mathbb{N}\}$ are obtained at the same time t. ∎

Remark 3: Generally, ξ and $\tau_i(l_i(t))$ are not of the same time scale. Since $T_i(t) \ll \xi$, Assumption 3 is mild. ∎

B. The MPC Formulation for a Local Agent

We now use the receding horizon control-based [33] extended predictive self-adaptive control (EPSAC) strategy [32] to formulate the local agent. To acquire the internal model, assume node i satisfies *ISHS* [34]–[36]. Then, the IP model (2) can be rewritten as similar to (4) as below:

$$y_i(t) = \frac{z^{-1}}{1 - z^{-1}}[u_i(t) - u_{i-1}(t)]. \tag{10}$$

From Eq. (10), $\mathscr{S}_i$ and $\{\mathscr{S}_j,\, j \in \mathcal{N}_i\}$ are interacted via coupled inputs. In EPSAC, the local model is implemented as

$$\widetilde{y}_i(t) = y_i(t) + n_i(t) \quad i \in \mathbb{N}, \tag{11}$$

where $y_i(t)$ is the model output (IP calculated from model (10)), and $\widetilde{y}_i(t)$ is SC output (measured IP) of $\mathscr{S}_i$. Agent $\mathscr{A}_i$ has access to the model and the measured IP $\widetilde{y}_i$, and receives the optimized input trajectory from $\{\mathscr{A}_j,\, j \in \mathcal{N}_i,\}$ as well. The random disturbance $n_i(t)$ is a colored noise process as

$$n_i(t) = \frac{C_i(z^{-1})}{D_i(z^{-1})} e_i(t), \tag{12}$$

which is excited by white noise $e_i(t)$ through a design filter $C_i(z^{-1})/D_i(z^{-1})$.

The future N_2-step output of the i^{th} subsystem is

$$\widetilde{y}_i(t+k|t) = y_i(t+k|t) + n_i(t+k|t), \tag{13}$$

where $k = 1, \cdots, N_2, i \in \mathbb{N}$, the predictions of $y_i(t+k|t)$ and $n_i(t+k|t)$ are calculated by recursion of (10) and (12), respectively. Eq. (13) is rewritten in a compact form as

$$\begin{aligned} Y_i &= Y_{i\_\text{base}} + G_{ii} U_i + \sum_{j \in \mathcal{N}_i} G_{ij} U_j^* \\ &= Y_{i\_\text{base}} + Y_{i\_\text{opt}}, \end{aligned} \tag{14}$$

with $Y_i = [\widetilde{y}_i(t+N_1|t), \cdots, \widetilde{y}_i(t+N_2|t)]^\intercal$, $U_i = [\delta u_i(t|t), \cdots, \delta u_i(t+N_u-1|t)]^\intercal$. Here, we have $\delta u_i(t+k|t) = u_i(t+k|t) - u_{i\_\text{base}}(t+k|t)$ where $u_{i\_\text{base}}(t+k|t)$ is a basic future control scenario defined *a priori* and $u_i(t+k|t)$ the optimal ordering decision. The future evolution of $\widetilde{y}_i$ is calculated based on the most recent N_u steps of input signals. So, the future N_2 steps of output $Y_{i\_\text{base}} = [y_{i\_\text{base}}(t+N_1|t), \cdots, y_{i\_\text{base}}(t+N_2|t)]^\intercal$ represents cumulative effect of the past ordering decisions u_i and IP $\widetilde{y}_i$, the pre-specified future base control sequence $U_{i\_\text{base}} = [u_{i\_\text{base}}(t|t), \cdots, u_{i\_\text{base}}(t+N_u-1|t)]^\intercal$ and the predicted disturbance. Meanwhile, Eq. (14) indicates that the optimizing response Y_{i_opt} depends not only on U_i of $\mathscr{S}_i$ but also on the optimized control signals U_j^*, $j \in \mathcal{N}_i$. In (14), G_{ij} is a Toeplitz matrix

$$G_{ij} = \begin{bmatrix} h_{N_1}^{ij} & h_{N_1-1}^{ij} & \cdots & h_{N_1-N_u+2}^{ij} & g_{N_1-N_u+1}^{ij} \\ h_{N_1+1}^{ij} & h_{N_1}^{ij} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N_2}^{ij} & h_{N_2-1}^{ij} & \cdots & h_{N_2-N_u+2}^{ij} & g_{N_2-N_u+1}^{ij} \end{bmatrix},$$

where h^{ij} and g^{ij} are the coefficients of the unit impulse and step responses respectively of the i^{th} subsystem to u_j^*. Let $\Delta U_j = [\Delta u_j(t|t), \cdots, \Delta u_j(t+N_u-1|t)]^\intercal$, one has $\Delta U_j = A_j U_j + b_j$ with $A_j = \mathbf{I}_{N_u} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{I}_{N_u-1} & \mathbf{0} \end{bmatrix}$ and $b_j = \big[(u_{j\_\text{base}}(t|t) - u_j(t-1)),\, (u_{j\_\text{base}}(t+1|t) - u_{j\_\text{base}}(t|t)),\, \cdots,\, (u_{j\_\text{base}}(t+N_u-1|t) - u_{j\_\text{base}}(t+N_u-2|t))\big]^\intercal$.

Next, recall that the objective of the DMPC replenishment is to minimize both the variance of x_i and Ω_i. To this end, the agent obtains control input U_i for $\mathscr{S}_i$ by minimizing a local cost function over the prediction horizon N_2 as below,

$$\begin{aligned} J_i(t,\, \Delta U_i) = \sum_{k=N_1}^{N_2} \Big\{ & q_i(k)[r_i(t+k|t) - \widetilde{y}_i(t+k|t)]^2 \\ & + p_i(k)[\Delta u_i(t+k-N_1|t)]^2 \Big\}, \end{aligned} \tag{15}$$

s. t. $\Delta u_i(t+\bar{k}|t) = 0, \bar{k} \in [N_u, N_2 - N_1]$, where N_1, N_2, N_u, q_i, p_i are the minimum, maximum prediction horizon, control horizon, tracking error weights and control effort weights, respectively. The control goal is to steer the IP from the current state $\widetilde{y}_i(t)$ to the target trajectory $\{r_i(t+k|t), k \in [N_1, N_2]\}$. For conciseness, Eq. (15) is rewritten in a compact form of $J_i(t,\, \Delta U_i) = \|(R_i - Y_i)\|_{Q_i}^2 + \|\Delta U_i\|_{P_i}^2$ with reference trajectory $R_i = [r_i(t+N_1|t), \cdots, r_i(t+N_2|t)]^\intercal$ and weighting matrices $Q_i = \text{diag}\big(q_i(N_1), \cdots, q_i(N_2 - N_1 + 1)\big)$, $P_i = \text{diag}\big(p_i(N_1), \cdots, p_i(N_u)\big)$. Note that in the right-handed side of Eq. (15), the first term penalizes the tracking errors, and the second term aims to mitigate the bullwhip effect by alleviating the variations of ordering process [37]. Define $W_{ii} = R_i - Y_{i\_\text{base}} - \varsigma_i$, $H_{ii} = G_{ii} A_i^{-1}$, and $H_{ij} = G_{ij} A_j^{-1}$, where $\varsigma_i = -\sum_{j=1}^{n} H_{ij} b_j$. It can be transformed into the standard quadratic cost index in ΔU_i as

$$J_i(t,\, \Delta U_i) = \Delta U_i^\intercal \widetilde{H}_i \Delta U_i + 2 \widetilde{f}_i^\intercal \Delta U_i + \widetilde{c}_i, \tag{16}$$

with $\widetilde{H}_i = H_{ii}^\intercal Q_i H_{ii} + P_i$, $\widetilde{f}_i = -H_{ii}^\intercal Q_i^\intercal \big[W_{ii} - \sum_{j \in \mathcal{N}_i} H_{ij} \Delta U_j^*\big]$, $\widetilde{c}_i = \big[\sum_{j \in \mathcal{N}_i} H_{ij} \Delta U_j^*\big]^\intercal Q_i \big[\sum_{j \in \mathcal{N}_i} H_{ij} \Delta U_j^*\big] + W_{ii}^\intercal Q_i W_{ii}$ $-2 W_{ii}^\intercal Q_i \sum_{j \in \mathcal{N}_i} H_{ij} \Delta U_j^*$.

Before presenting the non-cooperative DMPC architecture, it is necessary to give the local MPC problem as below.

Definition 2: Given a SC network consisting of n subsystems $\mathscr{S}_i, i = 1, \cdots, n$, within each review period $[t \cdot \xi, (t+1) \cdot \xi)$, each agent $\mathscr{A}_i$ firstly uses the IP model (11) to predict the future evolution of the local subsystem. Then, it derives the control signal $u_i(t)$ by optimizing (16) subject to the system dynamics (14) and analogous input/output constraints formulated in [4], [12] of the following linear inequalities:

$$\widetilde{A}_i \Delta U_i \leq \widetilde{b}_i \tag{17}$$

where $\widetilde{A}_i = \begin{bmatrix} \mathbf{I}_{N_u} \\ -\mathbf{I}_{N_u} \\ A_i^{-1} \\ -A_i^{-1} \\ H_{ii} \\ -H_{ii} \end{bmatrix}$, $\widetilde{b}_i = \begin{bmatrix} \overline{\Delta U_i} \\ -\underline{\Delta U_i} \\ \overline{U_i} + A_i^{-1} b_i \\ -\underline{U_i} - A_i^{-1} b_i \\ \overline{Y_i} - Y_{i\_\text{base}} - \sum_{j \neq i} H_{ij} \Delta U_j^* - \varsigma_i \\ Y_{i\_\text{base}} - \underline{Y_i} + \sum_{j \neq i} H_{ij} \Delta U_j^* + \varsigma_i \end{bmatrix}$ with $\overline{*}$ and $\underline{*}$ representing the upper bounds and lower bounds of variables $*$, respectively. ∎

The local optimization in *Definition 2* is then converted to a quadratic programming (QP) problem, which can be solved by the MATLAB function *quadprog*.

C. Non-cooperative DMPC

In order to calculate ΔU_i^*, Eq. (14) indicates that it is necessary for $\mathscr{A}_i$ to receive the optimal decisions ΔU_j^*, $j \in \mathcal{N}_i$ of its neighbors. Now, we are ready to give the non-cooperative DMPC Algorithm 1 as below.

Algorithm 1 The Iterative DMPC replenishment rule

Step 1 Each $\mathscr{A}_i$ receives $\widetilde{y}_i(t)$, initializes the optimal local control actions with an estimate $\Delta \hat{U}_i$, and exchanges it with $\mathscr{S}_j$, $j \in \mathcal{N}_i$.
Set $l_i(t) = 0$, $\Delta U_i^{(l_i(t))} = \Delta \hat{U}_i$.
Step 2 Each $\mathscr{A}_i$ solves the local MPC problem in Definition 2 to update $\Delta U_i^{(l_i(t)+1)}$.
Step 3 If the termination conditions $\|\Delta U_i^{(l_i(t)+1)} - \Delta U_i^{(l_i(t))}\| \leq \varepsilon_i \vee l_i(t) + 1 \geq \bar{l}_i(t)$ are satisfied, then set $\Delta U_i^* = \Delta U_i^{(l_i(t)+1)}$, $l_i^*(t) = l_i(t) + 1$, end the iteration, and proceed to Step 4;
Otherwise, set $l_i(t) = l_i(t) + 1$, and $\mathscr{A}_i$ transmits $\Delta U_i^{(l_i(t))}$ to neighbors, and return to Step 2;
Step 4 Calculate the optimal control action $\Delta u_i^*(t|t) = [1, 0, \cdots, 0]_{1 \times N_u} \cdot \Delta U_i^*$ and implement the decision to $\mathscr{S}_i$ as $u_i^*(t|t) = u_i(t-1) + \Delta u_i^*(t|t)$.
Step 5 The initial estimation for $\mathscr{A}_i$ at $t+1$ will be reassigned as $\Delta \hat{U}_i \leftarrow \Delta U_i^*$.
Step 6 Set $t = t + 1$, go back to Step 1; ∎

Remark 4: Note that only the first element of ΔU_i^* is implemented as the ordering decision $u_i^*(t|t) = u_i(t-1) + \Delta u_i^*(t|t)$ at t and the whole procedure is repeated during $[(t+1)\cdot\xi, (t+2)\cdot\xi)$. This is the receding horizon mechanism of MPC. The DMPC in Algorithm 1 is capable to handle explicitly the constraints arising from physical restrictions in practical SC operations. Moreover, it can address multi-objective decision-making for SCs. Some control methods [21], [37] can also be implemented in a distributed way. However, they fail to achieve above goals due to their inherent control mechanisms. ∎

Remark 5: The implementation of DMPC for SC optimization is not a trivial problem due to apparent disciplinary boundaries. While the DMPC strategy is straightforward to control engineers, the SCM are executed by human beings with decision-making based on psychological risk perceptions and preferences. Hence, a more practical way is to encapsulate the daunting mathematical formulations into a software simulator with a friendly interface. It can facilitate the decision-making process by eliminating the bias caused by human planners. Although developing such a tool is not difficult under current technology, it requires in-depth collaboration between the SC managers and control specialists to fill the gaps. ∎

Before giving the main technical result concerning the convergence of Algorithm 1, we need to present a lemma.

Lemma 1: [38], [39] Let $A \in \mathbb{C}^{m \times m}$ be a nonnegative matrix with spectral radius $\rho(A)$ and $\Upsilon_i(A)(\Theta_i(A))(i = 1, 2, \cdots, m)$ be the rows (columns) sum of A. Then the limit $\Upsilon(A) = \lim_{k \to \infty} \max_i (\frac{\Upsilon_i(A^n B^k)}{\Upsilon_i(B^k)})^{1/n}$ exists for any $k \in \mathbb{Z}$, $n \in \mathbb{Z}^+$, and $\rho(A) \leq \Upsilon(A)$, where $B = (A + \mathbf{I}_{m \times m})^{m-1}$. Analogously, above results hold for the column sums. ∎

We are ready to give the main technical result in the following *Theorem 1*.

Theorem 1: For the SC network composed of subsystems $\{\mathscr{S}_i,\ i \in \mathbb{N}\}$ with the set of independent agents $\{\mathscr{A}_i,\ i \in \mathbb{N}\}$, given the control law calculated by Algorithm 1, the ordering decisions $\{u_i,\ i \in \mathbb{N}\}$ at time instant t converge to the optimal values $\{u_i^*,\ i \in \mathbb{N}\}$, i.e. $\{\lim_{t \to T_i(t)} u_i(t) = u_i^*(t|t),\ \forall i \in \mathbb{N}\}$ if $\min\{\Upsilon(T_0), \Theta(T_0)\} < 1$ with

$$T_0 = \begin{bmatrix} 0 & -T_{11}H_{12} & \cdots & -T_{11}H_{1n} \\ -T_{22}H_{21} & 0 & \cdots & -T_{22}H_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -T_{nn}H_{n1} & -T_{nn}H_{n2} & \cdots & 0 \end{bmatrix}_{nN_u \times nN_u} \tag{18}$$

and $\{T_{ii} = \left[H_{ii}^{\intercal} Q_i H_{ii} + P_i\right]^{-1} H_{ii}^{\intercal} Q_i, i \in \mathbb{N}\}$. ∎

Proof: For the MPC problem in *Definition 2*, the exclusion of constraints (17) will not affect the convergence analysis. At each interval $[t, t+1)$, the output prediction Y_i for $\mathscr{S}_i$ in the $[l_i(t)]^{\text{th}}$ iteration can be obtained from (14) as:

$$Y_i^{(l_i(t))} = Y_{i\_\text{base}} + H_{ii}\Delta U_i + \sum_{j=1, j \neq i}^{n} H_{ij} \Delta U_j^{*(l_i(t))} + \varsigma_i. \tag{19}$$

where $\Delta U_j^{*(l_i(t))}$ is the optimized control for $\mathscr{S}_j$ at the $[l_i(t)]^{\text{th}}$ iteration. The explicit solution to Nash equilibrium is derived by locally taking $\frac{\partial J_i}{\partial \Delta U_i}\Big|_{\Delta U_j^{*(l_i(t))}(j \neq i)}$, resulting in $\Delta U_i^{(l_i(t)+1)} = T_{ii}\big[W_{ii} - \sum_{j=1, j \neq i}^{n} H_{ij} \Delta U_j^{*(l_i(t))}\big]$. If $\{\mathscr{S}_i,\ i = 1, \cdots, n\}$ arrive at the Nash equilibrium, then $\{\mathscr{A}_i,\ \forall i \in \mathbb{N}\}$ have achieved the optimal solutions and the integral control action for the whole system is:

$$\Delta U^{(l(t)+1)} = T_1 W + T_0 \Delta U^{(l(t))}, \tag{20}$$

where T_0 is specified as in Eq. (18), and $l(t)$ is the iteration index for ΔU, $\Delta U := \left[(U_1)^{\intercal}, \cdots, (U_n)^{\intercal}\right]^{\intercal}$, $T_1 := \text{blkdiag}\left(T_{11}, \cdots, T_{nn}\right)$, $W := \left[(W_{11})^{\intercal}, \cdots, (W_{nn})^{\intercal}\right]^{\intercal}$. The constant term $T_1 W$ contributes no effect to the iteration, and hence the convergence in (20) is equivalent to that in

$$\Delta U^{(l(t)+1)} = T_0 \Delta U^{(l(t))}. \tag{21}$$

The generated power sequence by (21) converges if and only if the spectral radius of T_0 is less than 1 [38]. From

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Lemma 1, the upper bound of $\rho(-T_0)$ can be approached by $\min\{\Upsilon(-T_0), \Theta(-T_0)\}$. Thereby, the iterative Algorithm 1 converges if $\min\{\Upsilon(-T_0), \Theta(-T_0)\} < 1$. ■

Remark 6: The parameters are generally picked as below. N_2 and N_u are picked with $N_2 \geq N_u$. In order to guarantee the convergence, the weighting factors are picked with $p_i > q_i \geq 0$. Once N_2 and N_u are selected, then H_{ii}, H_{ij} are fixed and the convergence is solely determined by Q_i, P_i. ▮

IV. Numerical Case Study

Consider a typical SC consisting of a retailer, a wholesaler, a distributor and a factory [12], where $|\mathbb{N}| = 4$, $\xi = 1$ week and the operations are evaluated over $|\mathbb{T}| = 100$ weeks. In the numerical study, 15 types of real-world demand patterns from Procter & Gamble's home-care products [40] are examined as the end-customer's demand d_1. Extensive experiments have been conducted to validate the performance of DMPC by applying the demands to the SC under three different replenishment rules, i.e. PID-based ordering policy (PID), decentralized EPSAC replenishment (EPSAC) [4] and the proposed DMPC scheme. To ensure a fair comparison, the SC is configured identically under these three scenarios except for the replenishment policy. The superiority of DMPC replenishment strategy to the other two lies in its independence of the specific demand pattern. Thus, only one case is presented as an illustrative example, in which d_1 is generated by an ARMA model (22)

$$\begin{cases} d_1(0) = e(0) + \mu, \\ d_1(t) = \varphi(d_1(t-1) - \mu) + e(t) - \theta e(t-1) + \mu. \end{cases} \tag{22}$$

where $\varphi = 0.711$, $\theta = -0.133$ and the mean μ is assumed to be 10 units. Each $\mathscr{S}_i$ is subject to 2 weeks of lead time L_i, with 10 units of products in initial inventory and initial Work-In-Progress (WIP), respectively. The constraints are constant over N_2 as $\Delta\overline{u}_1(\cdot) = 3$, $\Delta\overline{u}_2(\cdot) = 4$, $\Delta\overline{u}_3(\cdot) = 5$, $\Delta\overline{u}_4(\cdot) = 6$; $\{\underline{y}_i(\cdot) = 0, \forall i \in \mathbb{N}\}$, $\overline{y}_1(\cdot) = \overline{y}_2(\cdot) = 40$, $\overline{y}_3(\cdot) = 45$, and $\overline{y}_4(\cdot) = 50$. The experiment results in Fig. 2 and Fig. 3 are obtained for a specific sample function of the ARMA stochastic process (22). In addition, a summary of numerical results is presented in Tables I and II for the 10 representative demand patterns among 15 types.

As presented in [4], the parameters for PID are tuned to take account of economic cost reduction, constraints and bullwhip mitigation. In EPSAC, each controller assigns the tuning parameters as $N_1 = 1$, $N_2 = 5$, $N_u = 1$ and $q_i = 1$, $p_i = 10$. However, in the current DMPC replenishment, we set $N_1 = 1$ and $N_2 = 5$ for all $\mathscr{S}_i$. The prediction horizon N_2 is sufficiently long for $\mathscr{A}_i$ to encompass the IP responses. The ordering $u_i(t)$ influences x_i after 3 weeks, i.e. L_i weeks plus 1 week nominal ordering delay, whereas $u_{i-1}(t)$ affects $\mathscr{S}_i$'s IP after 1 week nominal ordering delay. Meanwhile, the control horizon is picked according to $N_u \in [1, N_2]$. The IP tracking abilities of the 4 subsystems are assumed to be equally important, i.e. $Q_i = \mathbf{I}_{N_2 - N_1 + 1}$ and analogously, $P_i = 10 * \mathbf{I}_{N_u}$, $\forall i \in \mathbb{N}$. Moreover, the picked parameters P_i, Q_i guarantee the convergence of the iterative algorithm. The simulation result of DMPC is presented in Fig. 2. In

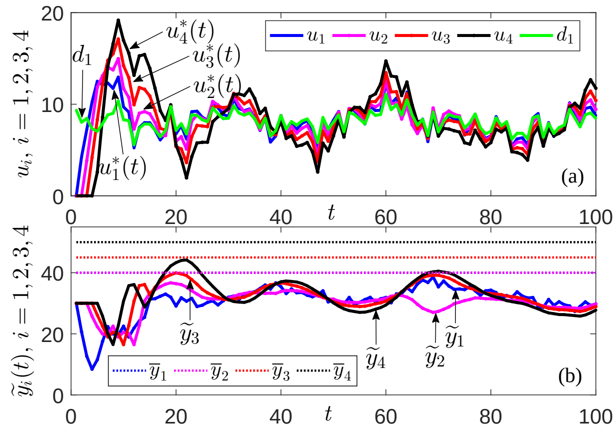

Fig. 2. The evolution of (a) orders and (b) inventory positions under the DMPC control scheme.

comparison with PID and EPSAC replenishment rules, it generates a closer tracking of IP target with much smoother IP variation.

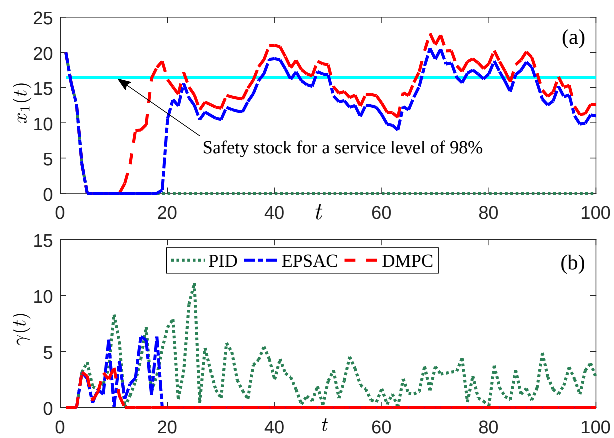

Fig. 3. (a) The evolution of retailer's inventory and (b) the end-customer's satisfaction level under three different control schemes (PID, EPSAC and DMPC). Here, $\gamma(t) := |g_1(t) - d_1(t)|$.

The objective of controlling SC are two-fold, i.e. i) to increase customer satisfaction, and ii) to minimize operation costs. More precisely, goal i) is assessed by measuring $\gamma(t) = |g_1(t) - d_1(t)|$ over $t \in \mathbb{T}$ and the average measure of customer satisfaction (AMCS). Here, $AMCS := \frac{1}{|\mathbb{T}|}\sum_{t=1}^{|\mathbb{T}|}\gamma(t)$ and the AMCS for PID, EPSAC and DMPC are 2.17, 0.37 and 0.14, respectively. Smaller γ and AMCS mean a higher level of end-customer satisfaction. The lower figure of Fig. 3 and above AMCS measures indicate that the two MPC-based strategies cause higher customer satisfaction level than PID. Meanwhile, they can maintain x_1 around the safety stock with 98% service level. Goal ii) is evaluated by the whole SC operation costs determined by the average excess inventory (*AEI*) and the average backorder (*ABO*). Here, $AEI := \frac{1}{|\mathbb{T}|}\sum_{t=1}^{|\mathbb{T}|}\sum_{i=1}^{n} x_i(t)$, $ABO := \frac{1}{|\mathbb{T}|}\sum_{t=1}^{|\mathbb{T}|} BO(t)$ with the backorder for the overall SC, $BO(t) := \sum_{i=1}^{n}[d_i(t) - x_i(t-1) - s_i(t - L_i)]$. To assess SC's economic performances, the total cost is defined as $TC := I_c + BO_c$, following the cost structure in [41], where $I_c := \sum_{t=1}^{|\mathbb{T}|}\sum_{i=1}^{n} c' \cdot x_i(t)$ and $BO_c := \sum_{t=1}^{|\mathbb{T}|} c'' \cdot BO(t)$. A

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TABLE I
THE ECONOMIC PERFORMANCE INDICES FOR 10 DEMAND PATTERNS UNDER 3 REPLENISHMENT RULES

θ	φ	PID replenishment			EPSAC replenishment				DMPC replenishment			
		AEI	*ABO*	*TC*	*AEI*	*ABO*	*TC*	*TC Reduction*	*AEI*	*ABO*	*TC*	*TC Reduction*
0.128	0.629	17.83	26.05	13060.36	47.04	7.39	12446.19	4.7%	61.04	2.59	12393.30	5.1%
0.342	0.673	16.04	17.41	12975.53	46.46	4.51	11929.89	8.1%	62.13	1.53	11184.43	13.8%
-0.597	0.611	19.38	29.89	13388.76	44.29	6.72	11680.80	12.76%	56.13	1.41	11466.81	14.35 %
$*$-0.133	0.711	20.00	35.75	17322.42	57.96	1.94	14777.06	14.69%	58.22	0.92	14490.13	16.35%
0.074	0.371	19.39	26.97	13357.17	48.00	5.98	12883.01	3.6%	62.64	1.47	12251.40	8.3%
-0.296	0.607	20.25	38.82	17120.56	47.38	9.64	13352.84	22.0%	60.24	3.22	13328.38	22.1%
0.459	0.641	17.24	22.63	13131.07	47.79	5.25	12210.20	7.0%	62.48	2.07	12387.06	5.7%
-0.072	0.694	23.33	26.56	13313.50	50.52	3.74	11763.40	7.5%	59.29	1.76	12321.01	5.2%
-0.454	-0.351	16.05	18.59	12615.70	45.93	3.77	11572.40	8.3%	61.21	0.99	11307.90	10.3%
-0.295	-0.018	17.78	23.89	12917.72	46.87	4.47	12650.17	2.1%	61.82	1.22	11984.20	7.2%

TABLE II
COMPARISON OF BULLWHIP FOR 10 DEMAND PATTERNS UNDER 3 REPLENISHMENT RULES

θ	φ	Ω_i & Ω under PID Replenishment					Ω_i & Ω under EPSAC Replenishment					Ω_i & Ω under DMPC Replenishment				
		Re.	*Wh.*	*Di.*	*Fa.*	*SC*	*Re.*	*Wh.*	*Di.*	*Fa.*	*SC*	*Re.*	*Wh.*	*Di.*	*Fa.*	*SC*
0.128	0.629	1.87	2.81	2.19	2.08	24.05	3.05	2.19	1.90	1.80	22.98	3.54	1.58	1.63	1.73	15.99
0.342	0.673	2.51	2.89	2.08	1.95	24.57	2.23	2.07	1.74	1.64	13.28	2.53	1.45	1.40	1.55	8.06
-0.597	0.611	1.17	1.85	1.81	1.77	7.05	1.79	1.72	1.62	1.53	7.68	1.97	1.19	1.35	1.42	4.55
$*$-0.133	0.711	1.17	2.42	1.80	1.65	8.52	1.54	1.61	1.53	1.42	5.45	1.61	1.23	1.32	1.36	3.59
0.074	0.371	1.57	2.70	2.14	2.29	21.02	2.53	2.21	1.89	1.88	20.06	2.91	1.58	1.62	1.83	13.75
-0.296	0.607	1.61	2.37	2.15	2.18	18.09	2.32	1.99	1.88	1.76	15.46	2.60	1.39	1.66	1.73	10.55
0.459	0.641	1.60	3.22	2.22	2.29	26.45	3.00	2.13	1.82	1.73	20.30	3.53	1.53	1.46	1.59	12.67
-0.072	0.694	1.57	2.03	1.94	1.75	10.93	2.13	1.83	1.71	1.54	10.34	2.36	1.34	1.56	1.55	7.77
-0.454	-0.351	1.14	2.76	2.03	2.06	13.25	2.26	2.07	1.79	1.70	14.39	2.59	1.40	1.41	1.64	8.47
-0.295	-0.018	1.43	2.83	2.03	2.14	17.74	2.51	2.02	1.81	1.77	16.40	2.79	1.43	1.56	1.78	11.20

specific case in Table I is $c' = \$1$, $c'' = \$2$ per unit per period for inventory holding cost and backorder penalizing cost, respectively. From Table I, DMPC control scheme substantially reduced TC compared to the PID approach.

From *Remark 1*, the bullwhip of each node and the whole SC are quantified by $\Omega_i = \frac{\sigma^2_{u_i}/\mu_{u_i}}{\sigma^2_{d_i}/\mu_{d_i}}, \forall i \in \mathbb{N}$ and $\Omega = \frac{\sigma^2_{u_4}/\mu_{u_4}}{\sigma^2_{d_1}/\mu_{d_1}}$, respectively. The Ω and Ω_i in Table II show that information sharing mechanism in DMPC is beneficial to mitigating both the order variation and SC instability. In addition, minimizing the control cost (16) helps limit the excessive movement of $\{u_i,\, i \in \mathbb{N}\}$ and hence reduce the ordering variability.

V. Conclusion

In this paper, we propose a DMPC scheme for the inventory management problem of abundant SC networks. With such a scheme, the network achieves superior performance compared with existing approaches like PID in terms of economic cost indices for SC operations. By iteratively exchanging information of current and future decisions with neighbors, each agent determines the optimal ordering policy to reach the Nash equilibrium of the whole network. By this means, the DMPC scheme yields better control performances while maintaining the flexibility and topology of the network. The effectiveness and superiority of the proposed scheme are verified by extensive numerical simulations.

As prospects of future research, we are considering extension of the current series SC to multi-echelon and multi-product framework, accordingly of the DMPC replenishment problem for the complex network. It is an interesting topic to investigate the effect of distinct extents of information sharing on the SC performance. Meanwhile, to further decrease communication cost, a non-iterative, cooperative algorithm is a promising remedy.

Appendix A
Proof of Proposition 1

C1.: According to the definition of v_i, the delivery of $\mathscr{S}_i$ at review period t is determined by $v_i(t-1)$:

$$g_i(t) = v_i(t-1). \tag{A.1}$$

Substitute (A.1) to (3) yields $v_i(t) = d_i(t)$, which implies $g_i(t+1) = d_i(t)$. Thus the delivery of $\mathscr{S}_i$ is determined by its customer's order u_{i-1} with one review period delay, i.e.,

$$g_i(t) = d_i(t-1) = u_{i-1}(t-1), \tag{A.2}$$

which immediately leads to $s_i(t) = g_{i+1}(t) = u_i(t-1)$. Substituting this and (A.2) to Eq. (2) yields model (4).

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C2.: The condition $\eta_i(t) < 0$ of *ISLS* operation indicates that $\mathscr{S}_i$ keeps a low stock and thus the delivery is limited by its inventory:

$$g_i(t) = z^{-1}x_i(t). \tag{A.3}$$

Substituting (1) into (A.3) yields

$$\begin{aligned} g_i(t) &= \frac{z^{-(L_i+1)}}{1-z^{-1}}s_i(t) - \frac{z^{-1}}{1-z^{-1}}g_i(t) \\ &= z^{-(L_i+1)}s_i(t). \end{aligned} \tag{A.4}$$

The condition $\eta_{i+1}(t) \geq 0$ implies that the supplier $\mathscr{S}_{i+1}$ has sufficient stock, so that its delivery to $\mathscr{S}_i$ is determined according to

$$s_i(t) = g_{i+1}(t) = v_{i+1}(t-1) = z^{-1}u_i(t). \tag{A.5}$$

Substituting (A.5) into (A.4) yields $g_i(t) = z^{-(L_i+2)}u_i(t)$. Using this relation and (A.5), the IP becomes

$$y_i(t) = \frac{z^{L_i+1}-1}{z^{L_i+1}(z-1)}u_i(t). \tag{A.6}$$

Factoring the numerator of Eq. (A.6) directly leads to Eq. (5).

C3.: The supplier $\mathscr{S}_{i+1}$ of current node $\mathscr{S}_i$ has insufficient inventory, then $g_{i+1}(t)$ is limited by its stock, i.e.,

$$s_i(t) = g_{i+1}(t) = z^{-1}x_{i+1}(t). \tag{A.7}$$

Then Eq. (2) becomes $y_i(t) = \frac{1}{1-z^{-1}}\left(z^{-1}x_{i+1}(t) - g_i(t)\right)$. When $\mathscr{S}_i$ keeps a high stock, noting Eq. (A.5), one has

$$y_i(t) = \frac{z^{-1}}{1-z^{-1}}\left(x_{i+1}(t) - d_i(t)\right). \tag{A.8}$$

When $\mathscr{S}_i$ has a low stock, then IP model becomes

$$\begin{aligned} y_i(t) &= \frac{z^{-1}}{1-z^{-1}}\left(x_{i+1}(t) - x_i(t)\right) \\ &= \frac{z^{-1}}{1-z^{-1}}\left(x_{i+1}(t) - y_i(t) - \frac{z^{-L_i}-1}{1-z^{-1}}s_i(t)\right). \end{aligned} \tag{A.9}$$

Substituting Eq. (A.7) into Eq. (A.9) yields

$$\left(1+\frac{z^{-1}}{1-z^{-1}}\right)y_i(t) = \frac{z^{L_i+1}-1}{(z-1)^2 z^{L_i}}x_{i+1}(t). \tag{A.10}$$

Further simplification of Eq. (A.10) gives

$$y_i(t) = \frac{z^{-1}(1-z^{-(L_i+1)})}{1-z^{-1}}x_{i+1}(t). \tag{A.11}$$

Therefore, Eq. (A.8) and Eq. (A.11) equals Eq. (6).

The proof is thus completed.

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Dongfei Fu obtained the B.Sc. degree in Automation and the M.Sc. degree in Control Theory and Control Engineering from Ocean University of China, Qingdao, China, in 2007 and 2010, respectively. He received the Ph.D. degree in electro-mechanical engineering from Ghent University, Gent, Belgium, in September 2016.

He is currently a postdoctoral researcher at the School of Automation, and State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, China. His main research interests are model predictive control, multi-agent system control and supply chain management.

Hai-Tao Zhang (M'07–SM'13) received the B.E. and Ph.D. degrees in control science and engineering from the University of Science and Technology of China, Hefei, China, in 2000 and 2005, respectively.

From January to December 2007, he was a Postdoctoral Researcher at the University of Cambridge, Cambridge, U.K. Since 2005, he has been with Huazhong University of Science and Technology, Wuhan, China, where he was an Associate Professor from 2005 to 2010 and has been a Full Professor since 2010. He serves as an Associate Editor of the IEEE Transactions On Circuits And Systems II, Scientific Reports, and Asian Journal of Control, a Conference Editorial Board member of the IEEE Control Systems Society (IEEE CSS) and the Chair of IEEE CSS Wuhan Chapter. He is currently a "Cheung Kong" Young Scholar. His research interests include model predictive control and multiagent systems control.

Ying Yu received the B.Sc. degree in Electrical Engineering and Automation from Qingdao Agricultural University and the M.Sc. degree in Control Theory and Control Engineering from Ocean University of China, both in Qingdao, China, in 2007 and 2010, respectively.

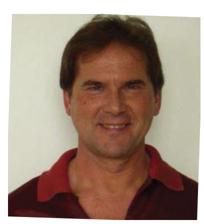
From 2010 to 2016, she was an engineer with the Third Division of China Aerospace Science and Industry Co., Beijing, China. Now she is with Energy Conservation Business Unit, Dongfang Electronics Co., Ltd as a researcher.

Clara Mihaela Ionescu (M'07) received the M.Sc. degree in industrial informatics and automation from Dunarea de Jos University, Galati, Romania, in 2003, and the Ph.D. degree in biomedical engineering from Ghent University, Gent, Belgium, in 2009.

She is currently a tenured professor with Ghent University, involved in several international projects with both industrial and biomedical applications in identification and control. She was the holder of a prestigious postdoctoral grant from the Flanders Research Center during 2011–2016.

El-Houssaine Aghezzaf is a professor of industrial systems engineering and operations research at the Ghent University. He holds an M.Sc. and Ph.D. in applied mathematics and operation research from CORE!Center for Operations Research and Econometrics at UCL-Universit catholique de Louvain.

He is an associate editor of "International Journal of Production Research" and member of editorial boards of three other journals. He is member of IFAC - Manufacturing and Logistics Systems. His main research interests are integrated systems optimization and simulation, stochastic and data analysis, supply and utility networks engineering.



Robin De Keyser received the M.Sc. degree in electro-mechanical engineering and the Ph.D. degree in control engineering from Ghent University, Ghent, Belgium, in 1974 and 1980, respectively. He is a Full Professor of control engineering with the Faculty of Engineering, Ghent University.

He is the author or co-author of over 300 publications in journals, books, and conference proceedings. He has acted as an external review expert in several European Commission Research Programs and is one of the pioneers who produced the original concepts of predictive control during the 1980s. His research interests include model predictive control, auto-tuning and adaptive control, modeling and simulation, and system identification.