Unit II (Transient Analysis) (1)

When the circuit is switched from one condition to another (by change in circuit elements or by switching operation), there is a translational period during which the currents and voltages in the branches change from their initial values to final values. The way in which they change from initial values to final values represent the transient response of the circuit.

Even in the transitional conditions, the circuit should satisfy Kirchoff's Laws. The network equations for a circuit containing energy storage elements obtained by applying Kirchoff's laws are Linear Integro differential equations with constant co-efficients. The solution of the above differential equation represents the response of the circuit.

The solution for the differential equation contains two parts. The first part is the solution of the differential equation with energy sources (or) excitation set eqn to zero (homogeneous part of Differential equation). This is called <u>complementary function</u> (or) transient response (or) natural response (or) Source free response of the netw

- This response solely depends on the circuit elements and their response of the intercon

This part of the response is independent of th sources acting in the circuit. This response goes to zero in relatively short time and hence it is called the TRANSIENT part of the solution.

The second part of the solution is called <u>Particular Integral</u> or the steady s Solution. This depends on the energy source and the configuration of the circuit.

The solution of Differential equation contains arbitary constants and these are to be evaluated using initial conditions. Normally in mathematics, the initial conditions are given, but in engineering systems, they ar to be evaluated. The number of constants will be equal to <u>order</u> of the differentia equation.

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Initial Conditions :-

Initial conditions are those conditions that exist in the circuit immediately after switching operation. At $t=0$ one or more switches are operated which disturb the equilibrium of the circuit. We assume that switch is operated at zero time. At this point we introduce a notation to distinguish the two states of the network. To distinguish between time immediately before & immediately after the operation of a switch, we use $t=0^-$ and $t=0^+$. The initial conditions will depend on the past history of the circuit (just before the switch is operated at $t=0$) and the network structure immediately after switching operation ($t=0^+$)

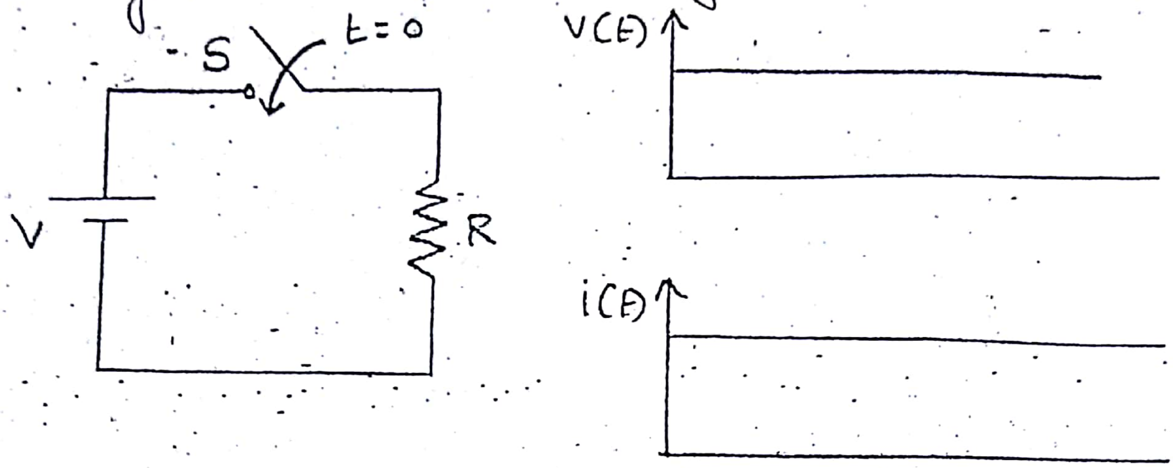
Initial Conditions of Resistor :-

In a pure resistor 'R', the Voltage - Current relationship is given by

$$v(t) = Ri(t)$$

From the above equation, it is seen that either current (or) voltage can change

Instantaneously. Considering a step change in Voltage due to closure of switch at $t = 0$ will make current also change from 0 to $\frac{V}{R}$ instantaneously. The current follows the change in Voltage with no time lag.



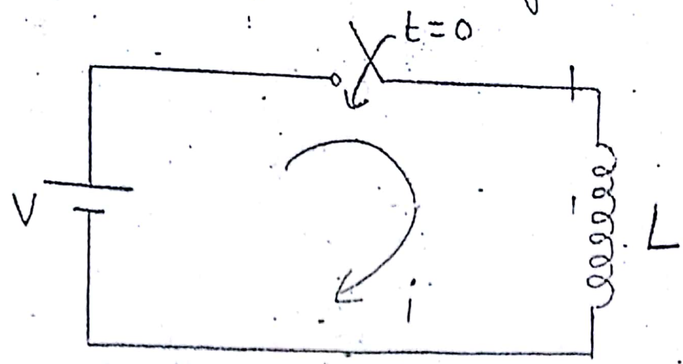
In a purely resistor circuit, switching opera[tion] immediately establish the new state and t[he] transition is occurring in zero time.
There is no Transient Period.

Initial Conditions of Inductor

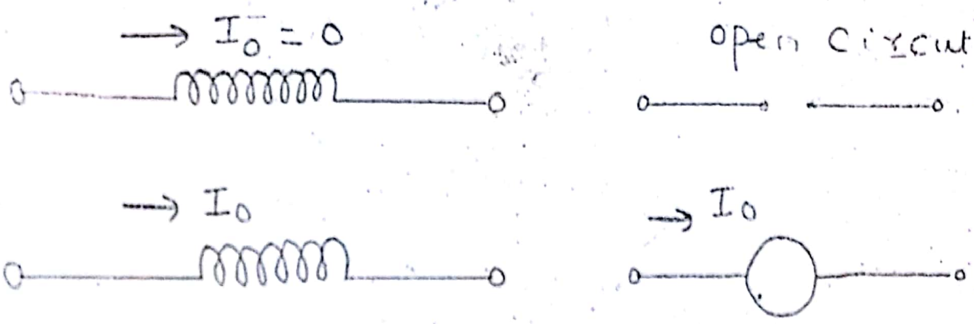
The Volt-Ampere relationship of an inducto[r] having inductance 'L' is,

$$V = L\frac{di}{dt}$$

From the above eqns, we can see that the cu[rrent] through inductor cannot change instantaneou[sly]. If it does so, $\frac{di}{dt}$ becomes infinite, hence Voltage across inductor tends to become inf[inite] which is not possible. Hence the current cannot change instantaneously when the switch is closed from it's original position. Hence inducti will act as a open circuit if there was no current prior to switching operation.



If it was carrying a current I_0 just before switchi the same current will continue to flow even after switching. i.e. $i_L(0^-) = i_L(0^+)$

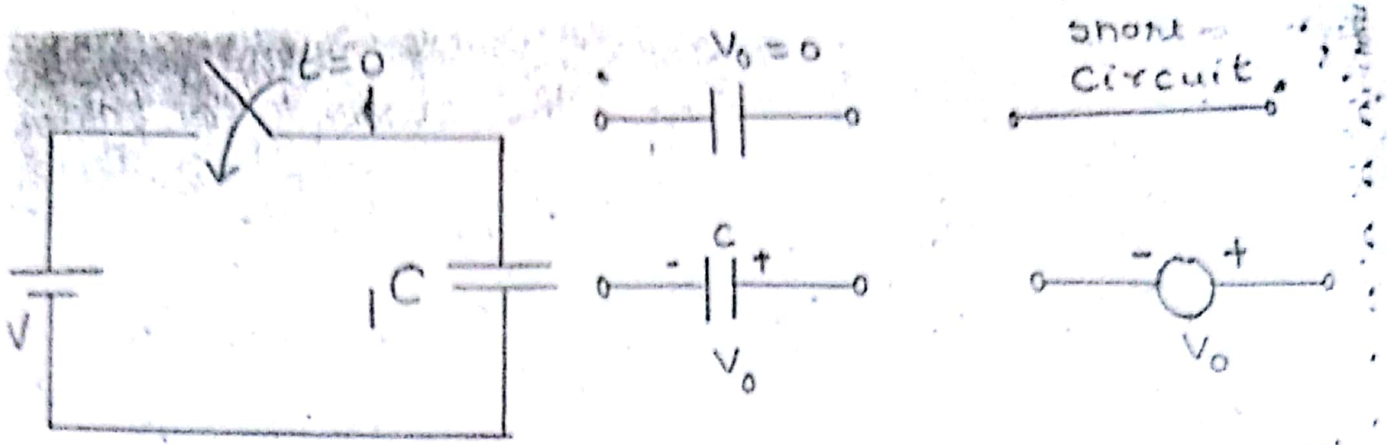


Initial conditions of Capacitor:

The Volt-Ampere equation of a capacitor with capacitance, C is given by.

$$i = C\frac{dv}{dt}$$

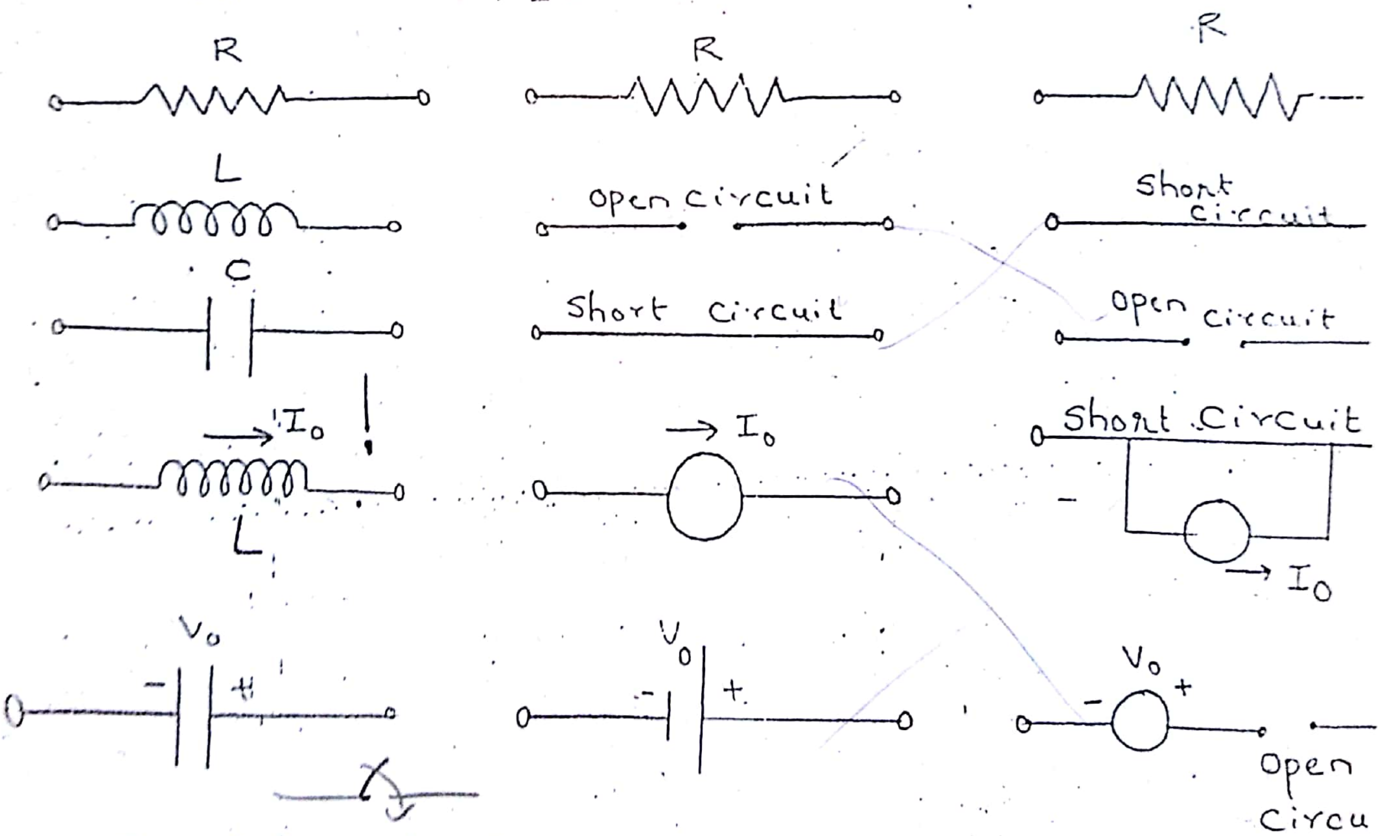
From the above equation, it is seen that the voltage across the capacitor cannot change instantaneously.



If it does so, the current becomes infinite. If uncharged capacitor is switched on to a d.c source the current flow is instantaneous and the capacitor will act as if it is short circu (V = 0). If a capacitor has an initial voltag V0, it is equivalent to an equivalent voltage source as shown in above figs.

Final Steady State Conditions

ELEMENT — EQUIVALENT CIRCUIT at $t = 0^+$ — EQUIVALENT CIR at $t = \infty$

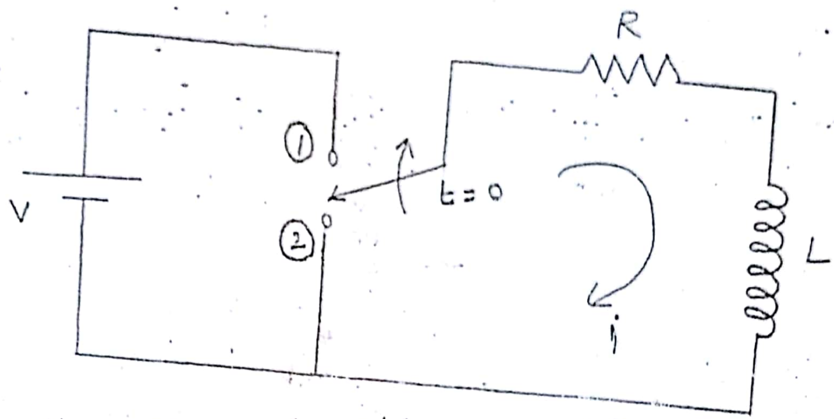


$t = 0^-$ → Before switching

DC Transients :-

In the following sections, we consider the transient response of RL, RC and RLC Series circuits excited by DC source. Using KCL or KVL, we get network equations with constant coefficients. Hence the solution of these differential equations gives the transient and steady state response of the network.

R-L Circuit :-



Consider the circuit shown in above figure in which the switch S is in position 2 for long time and is transferred from position 2 to position 1 at $t = 0$.

When switched is in Position ② i.e at $t = 0^-$, no current flows through the circuit.

Hence, $i(0^-) = 0$

When switch is transferred to position ①, at $t = 0$, the voltage V is applied to RL circuit. Since there is an inductance L, the current cannot change instantaneously, and hence

$$i(0^+) = 0 \quad \text{—— (a)}$$

By applying KVL for all $t \geq 0$, we get

$$iR + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$\Rightarrow \left(D + \frac{R}{L}\right) i = \frac{V}{L} \quad \text{--- (1)}$$

The equation (1) is a first order differential equat and the solution gives the response of the circui To get the solution we will obtain the transient Part (complementary function) and Steady state part (Particular Integral), Separat The transient part of the solution is obtained b solving the homogeneous part of the differential Equation by making forcing function to zer The homogeneous part of the differential equation

$$\left(D + \frac{R}{L}\right) i = 0$$

The auxilary equation is $D + \frac{R}{L} = 0$

The root of the auxilary equation is $D = \frac{-R}{L}$

The general solution is of form:

$$i_{transient} = A \cdot e^{-\frac{R}{L}t}$$

where constant 'A' to be evaluated using initial conditions.

Steady state part of the solution or Particular Integral is obtained from

$$\left(D + \frac{R}{L}\right) i = \frac{V}{L}$$

$$\Rightarrow \quad i = \frac{V}{L\left(D + \frac{R}{L}\right)}$$

To get the steady state part of the solution, substitute $D = 0$ (for D.C. excitation)

$$\Rightarrow \quad i_{\text{steady state}} = \frac{V}{L \cdot \frac{R}{L}} = \frac{V}{R}$$

The complete solution is given by

$$i = i_{\text{Transient}} + i_{\text{steady state}}$$

$$\boxed{i = A \cdot e^{-\frac{R}{L} \cdot t} + \frac{V}{R}}$$

To evaluate the constant A, we use initial condition

$$i(0^+) = 0 \text{ (from eqn (a)) at } t = 0^+,$$

$$i(0^+) = A \cdot e^0 + \frac{V}{R} = 0$$

$$\Rightarrow \quad A + \frac{V}{R} = 0$$

$$\Rightarrow \quad A = -\frac{V}{R}$$

The complete solution is,

$$\boxed{i(t) = \frac{V}{R} - \frac{V}{R} \cdot e^{-\frac{R}{L}t} = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

The current $i(t)$ increases exponentially starting from zero to final value.

The final value at $t = \infty$, $I_\infty = \frac{V}{R}$

time Constant :-

It is defined as the time in seconds at which the exponent of the exponential term is unity.

i.e $\frac{R}{L} t = 1$

$\Rightarrow t = \frac{L}{R} = $ Time constant (T)

$$\text{Time constant } (T) = \frac{L}{R} \text{ sec}$$

The variation of current with respect to time in a series RL circuit is,

$$i(t) = \frac{V}{R}\left(1 - e^{-t/T}\right)$$

at time $t = 0$:-

$$i(t) = \frac{V}{R}\left(1 - e^{0}\right)$$

$\Rightarrow i(t) = 0$

at time $t = T$:-

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{T}{T}}\right) = \frac{V}{R}\left(1 - e^{-1}\right)$$

$\Rightarrow$ * $i(t) = 0.632 \cdot \frac{V}{R}$

at $t = 2T$:

$$i = \frac{V}{R}\left(1 - e^{-t/T}\right) = \frac{V}{R}\left(1 - e^{-\frac{2T}{T}}\right)$$

$$i = \frac{V}{R}\left(1 - e^{-2}\right)$$

$$i = 0.864\,\frac{V}{R}$$

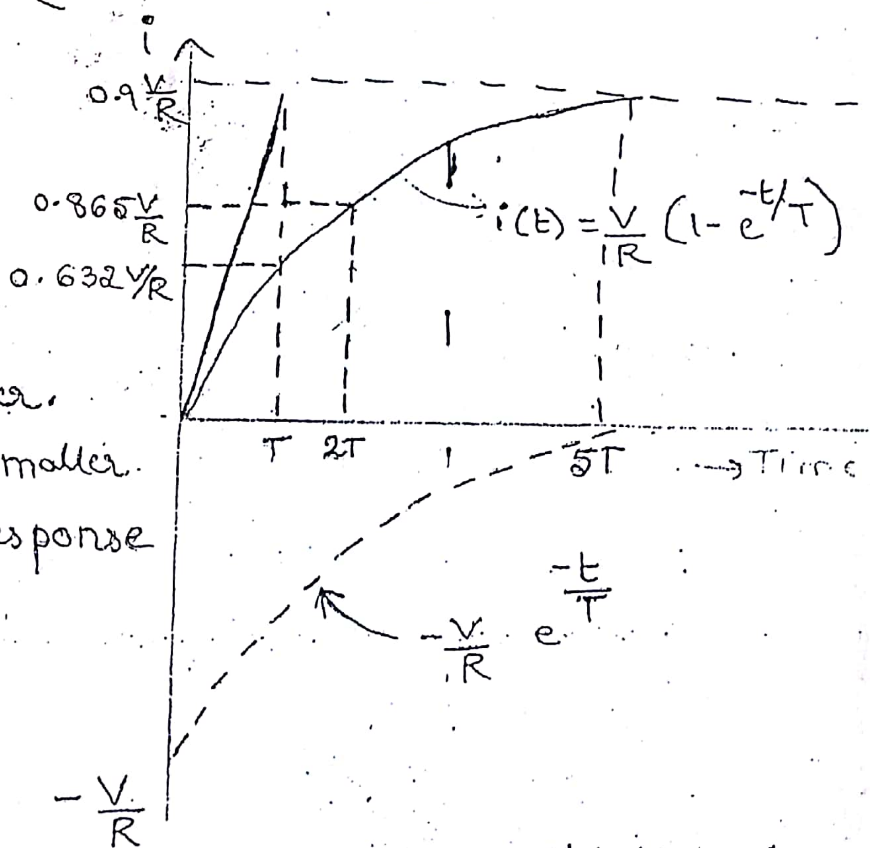
at $t = 5T$

$$i = \frac{V}{R}\left(1 - e^{\frac{-t}{T}}\right) = \frac{V}{R}\left(1 - e^{-\frac{5T}{T}}\right)$$

$$i = 0.9933\,\frac{V}{R}$$

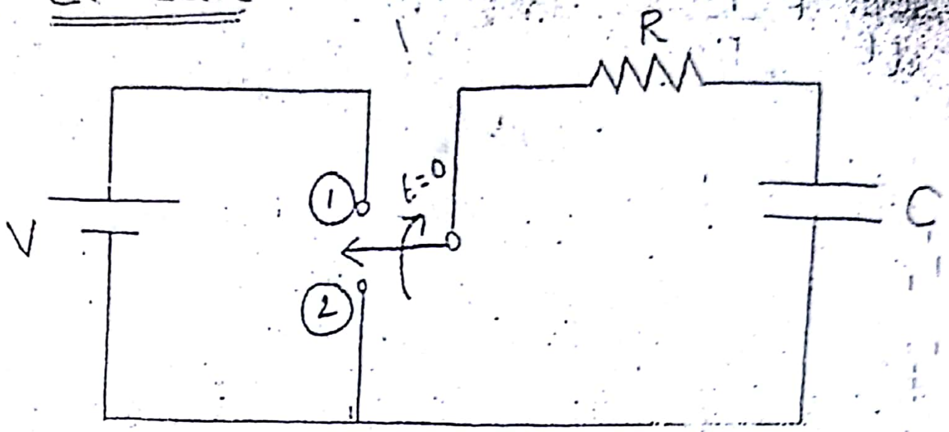
For $t \geq 5T$ the current has reached the final value. So generally, the transient period for any circuit is about 5 times (Time constant)

For any circuit, larger the time constant, the increase of current is slower or response is slower. For circuits with smaller time constant, the response is faster.



Time constant can also be defined as the time in seconds in which it will reach 0.632 times of its final value.

R-C Circuit



Let us consider an RC circuit shown in the fig. in which the switch is transferred from position 2 to position 1 at $t=0$.

This is equivalent to applying d.c voltage of V to RC circuit. The capacitor uncharged prior to $t=0$. Since the capacitor uncharged.

$v_c(0^-) = 0$, hence $v_c(0^+) = 0$, as the voltage across capacitor cannot change instantaneously.

Initially it acts as a short circuit

$$i(0^+) = \frac{V}{R}$$

By applying KVL for all $t \geq 0$, we get

$$iR + \frac{1}{C}\int i\,dt = V$$

differentiating the equation w.r.t time,

$$\cancel{R\frac{di}{dt} + \frac{1}{C}} \quad R \cdot \frac{di}{dt} + \frac{1}{C} i = 0$$

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$$\Rightarrow \frac{di}{dt} + \frac{1}{RC} i = 0$$

$$\Rightarrow \left(D + \frac{1}{RC}\right) i = 0$$

The above equation is a first order linear homogeneous equation. Hence the total solution will have only complementary function, and Particular Integral is zero.

$$\therefore I(t) = A \cdot e^{-t/RC}$$

To evaluate constant A, we have to use initial conditions. i.e. $i(0^+) = \frac{V}{R}$

$$i(0) = A = \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R} \cdot e^{-t/RC}}$$

Time constant :-

It is the time in seconds at which the expo of exponential term is unity.

$$\frac{t}{RC} = 1$$

$$\Rightarrow t = RC = T$$

Time variation

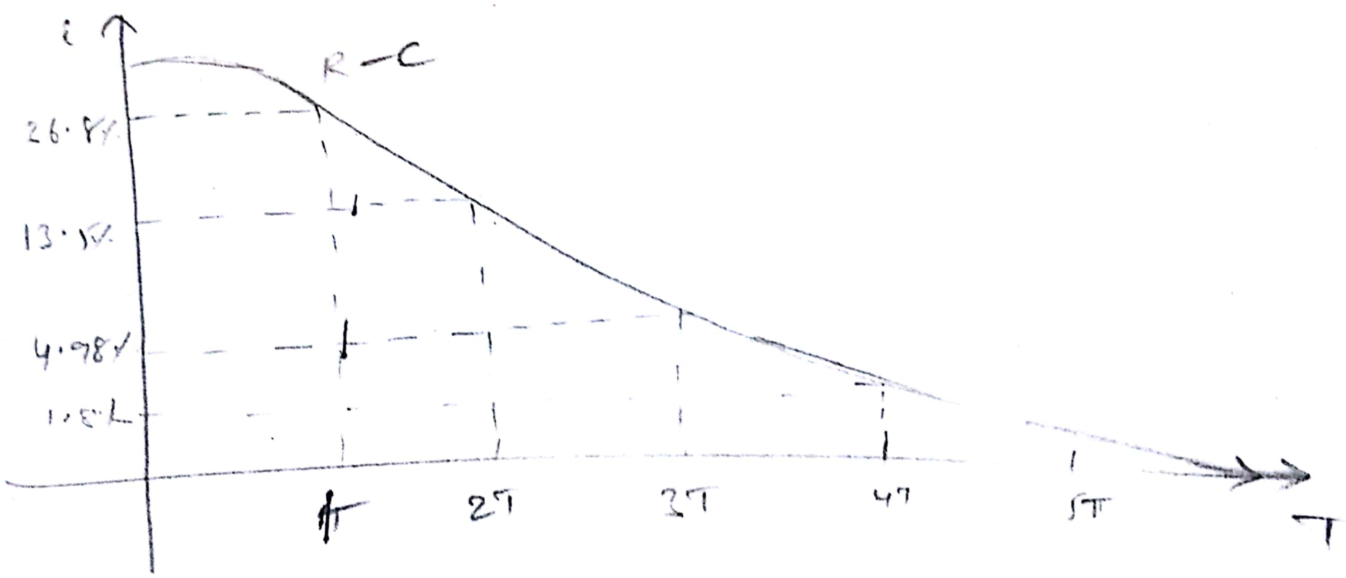
$$\boxed{\text{Time Constant } (T) = RC \text{ sec}}$$

$$i(t) = \frac{V}{R} e^{\frac{-t}{T}}$$

when $t = T \Rightarrow i(t) = \frac{V}{R} e^{-T/T} = \frac{V}{R}(e^{-1}) = 0.368 \frac{V}{R}$.

$t = 2T \Rightarrow i(t) = \frac{V}{R} e^{-2} = 0.135 \frac{V}{R}$

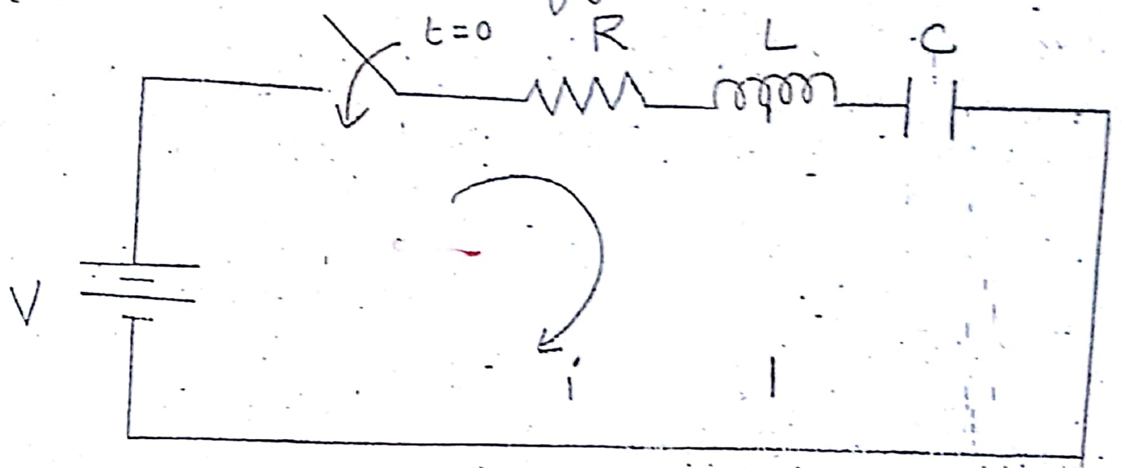
$t = 3T \Rightarrow i(t) = \frac{V}{R} e^{3} = 0.0498$



i
R – C
26.8%
13.5%
4.98%
1.8%
2T
3T
4T
5T
T

RLC Series Circuit

Consider a Series RLC circuit to which a DC Voltage V is applied by closing switch S at $t=0$ as shown in fig.



Before the switch is closed at $t=0^-$, there is no current through the circuit and the capacitor is also uncharged.

At $t=0^-$, $i_L(0^-)=0$ & $V_c(0^-)=0$

Immediately after closing the switch also, the current should be zero (Presence of Inductance) and voltage across the capacitor cannot change instantaneously.

So the initial conditions are

$$i_L(0^+)=0 \quad \text{and} \quad V_c(0^+)=0$$

By applying KVL for all $t>0$,

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V \quad \text{—— (I)}$$

This is an integro differential equation, hence by differentiating this equation w.r.t. time, we get

$$R \cdot \frac{di}{dt} + L \cdot \frac{d^2 i}{dt^2} + \frac{1}{c} i = 0 \quad \text{(Since V is constant)}$$

Arranging up the above equation, we get

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{1}{LC} i = 0;$$

$$\left(D^2 + \frac{R}{L} D + \frac{1}{LC}\right) i = 0$$

This is a second order differential equation & is a homogeneous equation.

The solution of this equation is of the form,

$$\boxed{i = A \cdot e^{m_1 t} + B \cdot e^{m_2 t}} \quad \text{——— II}$$

where A & B are constants,

m_1 & m_2 are roots of the characteristic eqn.

$$D^2 + \frac{R}{L} D + \frac{1}{LC} = 0$$

The roots of the characteristics eqn are

$$m_1, m_2 = \frac{-R}{2L} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}} = \frac{-R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

The response of the network depends on the nature of the roots m_1 & m_2.

Case i) : $R^2 > \frac{4L}{C}$, Discriminant Positive

This Condition is known as over damped

when Discriminant is +ve,
The roots m_1 & m_2 are real & different.

The roots are:

$$m_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \&$$

$$m_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

To evaluate the constants A & B, we have to substitute initial conditions.

i, At $t=0$, $i(0^+) = 0$

Substituting in eqn II, we get

$$i = A + B = 0 \Rightarrow A = -B$$

ii, At $t=0$, $i=0$ and $V_C = 0$, i.e. $\frac{1}{C}\int i\,dt = 0$

$\Rightarrow$ Substituting above values in eqn I,

$$0 + L\frac{di}{dt} + 0 = V$$

$$\Rightarrow \left.\frac{di}{dt}\right|_{t=0} = \frac{V}{L} \quad \text{—— III}$$

Differentiating eqn (II), we get,

$$\frac{di}{dt} = m_1 \cdot A \cdot e^{m_1 t} + m_2 \cdot B \cdot e^{m_2 t}$$

$$\left.\frac{di}{dt}\right|_{t=0} = m_1 A \cdot e^0 + m_2 \cdot B \cdot e^0$$

$$\Rightarrow \frac{V}{L} = m_1 A + m_2 B$$

$$\frac{V}{L} = m_1 A + m_2(-A) \quad [\text{Since } A = -B]$$

$$\frac{V}{L} = (m_1 - m_2) A$$

$$\therefore A = \frac{V}{L(m_1 - m_2)}$$

$$\Rightarrow B = \frac{-V}{L(m_1 - m_2)}$$

$\therefore$ eqn II is given by

$$i(t) = \frac{V}{L(m_1 - m_2)} \cdot e^{m_1 t} - \frac{V}{L(m_1 - m_2)} \cdot e^{m_2 t}$$

$$\therefore i(t) = \frac{V}{L(m_1 - m_2)} \left[e^{m_1 t} - e^{m_2 t}\right]$$

Substituting the roots in the above eqn, we get

$$i(t) = \frac{V \cdot \left[e^{\left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) \cdot t} - e^{\left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) \cdot t}\right]}{L\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} - \left(\frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)\right]}$$

$$= \frac{V \cdot \left(e^{-\frac{Rt}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\, t} - e^{-\frac{R}{2L} t - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \cdot t}\right)}{L\left[\cancel{-\frac{R}{2L}} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} + \cancel{\frac{R}{2L}} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right]}$$

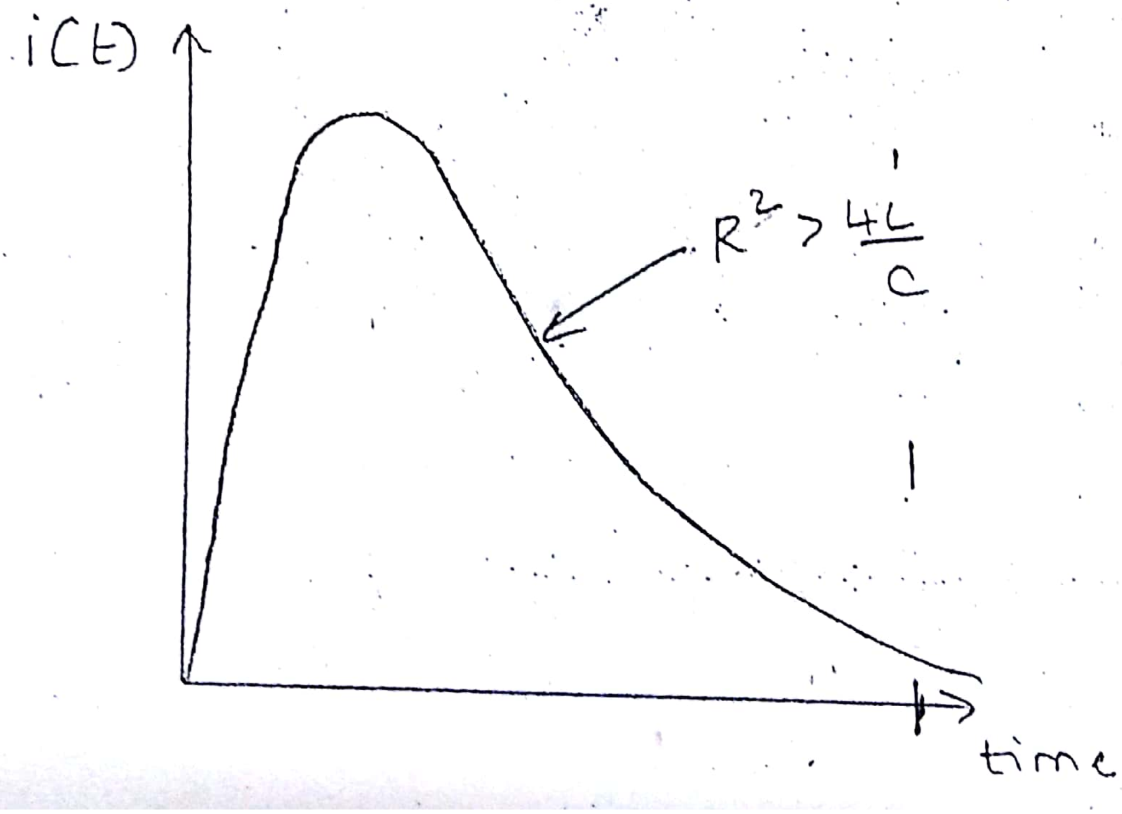
$$= \frac{V \cdot \left(e^{-\frac{Rt}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \cdot t} - e^{-\frac{Rt}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \cdot t}\right)}{L\left[2\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right]}$$

$$= \frac{V\left[e^{-\frac{Rt}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \cdot t} - e^{-\frac{Rt}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \cdot t}\right]}{\sqrt{\left(\frac{R \cdot \cancel{2L}}{\cancel{2L}}\right)^2 - \frac{4L\cancel{L}}{\cancel{L}C}}}$$

$$= \frac{V \cdot e^{-\frac{Rt}{2L}} \left[\exp\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t - \exp\left(-\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t \right]}{\sqrt{R^2 - 4\frac{L}{C}}} \quad (12)$$

$$i(t) = \frac{V \cdot e^{-\frac{Rt}{2L}}}{\sqrt{R^2 - \frac{4L}{C}}} \left[e^{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\, t} - e^{-\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\, t} \right]$$

The current response is the difference of two exponentials & is as shown in the following fig. This case is a overdamped case.



Case (iii) : $R^2 = \frac{4L}{C}$, Discriminant is zero.

This condition is known as critically damped.

As discriminant is zero, the roots are real and equal.

The roots are real and equal.

The roots are $m_1, m_2 = \frac{-R}{2L}$.

Since $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$.

The general solution of the differential equation, characteristic equation when the roots of the characteristic equation are equal and is

$$i(t) = (A + Bt)e^{mt}$$

$$i(t) = (A + Bt) \cdot e^{\frac{-Rt}{2L}} \quad \text{--- IV}$$

The constants A & B are evaluated using initial conditions.

1. At $t = 0$, $i(0^+) = 0$

$$0 = (A + 0) 1$$

$$\therefore A = 0$$

2. At $t = 0$, $\left.\frac{di}{dt}\right|_{t=0} = \frac{V}{L}$

$$\frac{di}{dt} = (A + Bt)\, e^{-\frac{R}{2L}t} \left(\frac{-R}{2L}\right) + B \cdot e^{-\frac{R}{2L}t}$$

$$\frac{di}{dt}(t = 0^+) = A \cdot e^{-\frac{R}{2L}(0)} \left(\frac{-R}{2L}\right) + B e^{0}$$

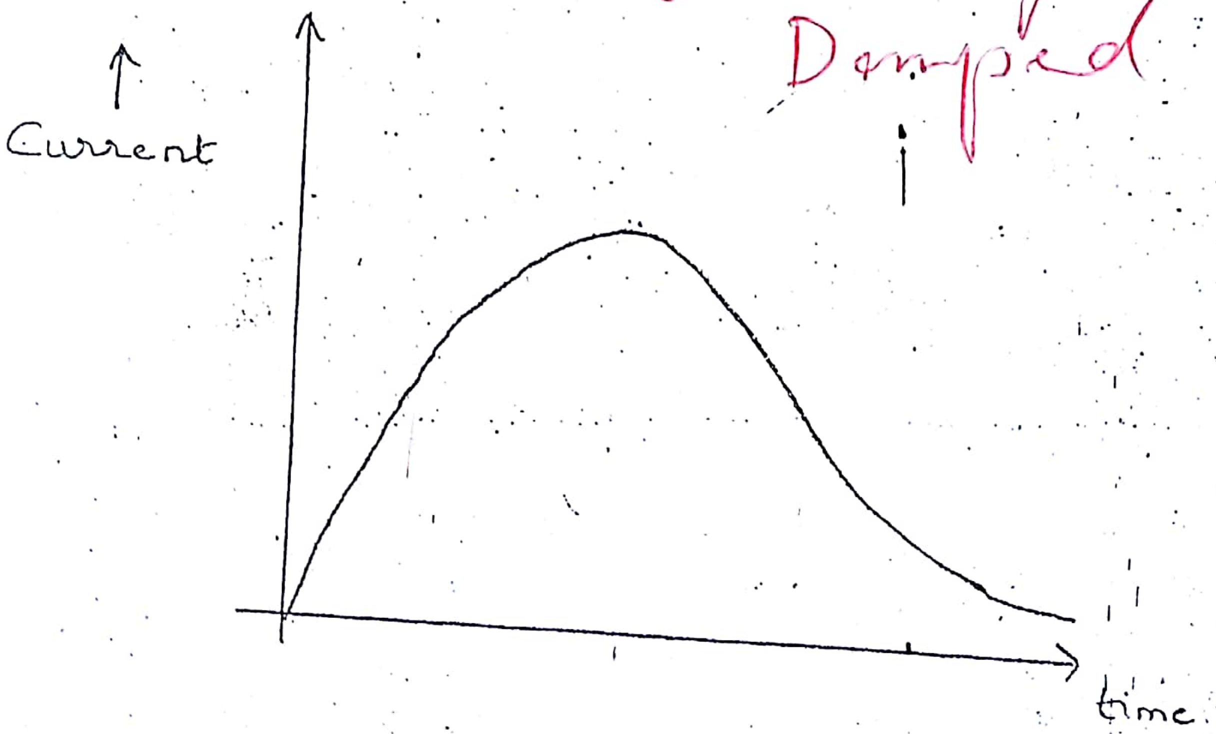
$$\Rightarrow \quad \frac{V}{L} = A \cdot \left(\frac{-R}{2L}\right) + B$$

Since $A = 0$, $B = \frac{V}{L}$

$$\therefore \quad i(t) = \frac{V}{L} t \cdot e^{-\frac{R}{2L}t}$$

The Variation of $i(t)$ with time is as shown in the following fig.

Critically Damped



Case (iii) :- $R^2 < \frac{4L}{C}$, Discriminant is negative

This Condition is known as Under Damped.

The roots m_1 and m_2 are complex conjugate

Let $m_1 = -K_1 + jK_2$

$m_2 = -K_1 - jK_2$ — G. Murali Krishna

where $K_1 = \frac{R}{2L}$, $K_2 = \frac{1}{2L}\sqrt{\frac{4L}{C} - R^2}$

The solution is given by

$$i(t) = A \cdot e^{(-K_1 + jK_2)t} + B \cdot e^{(-K_1 - jK_2)t}$$

$$= e^{-K_1 t}\left[A \cdot e^{jK_2 t} + B \cdot e^{-jK_2 t}\right]$$

$$= e^{-K_1 t}\left[(A+B)\cos K_2 t + j(A-B)\sin K_2 t\right]$$

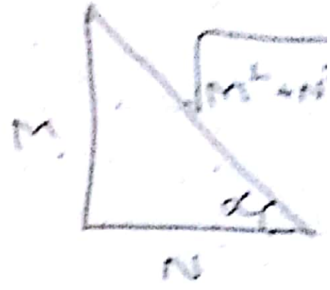
$$i(t) = e^{-K_1 t}\left[M\cos K_2 t + N\sin K_2 t\right]$$

where $M = A + B$

$N = j(A - B)$

Multiplying both Numerator & Denominator by $\sqrt{M^2+N^2}$, we get

$$i(t) = \sqrt{M^2+N^2}\cdot e^{-K_1 t}\left[\frac{M}{\sqrt{M^2+N^2}}\cos K_2 t + \frac{N}{\sqrt{M^2+N^2}}\sin K_2 t\right]$$



$$i(t) = \sqrt{M^2+N^2}\cdot e^{-K_1 t}\left[\sin\alpha \cos K_2 t + \cos\alpha \sin K_2 t\right]$$

$$= \sqrt{M^2+N^2}\cdot e^{-K_1 t}\sin(\alpha + K_2 t)$$

$$\therefore \boxed{i(t) = P\cdot e^{-K_1 t}\sin(K_2 t + \alpha)} \quad \text{— (I)}$$

where $P = \sqrt{M^2+N^2}$,

$$\alpha = \tan^{-1}\left(\frac{M}{N}\right)$$

Using initial conditions we have to evaluate P and α.

1. At $t = 0$, $i(0^+) = 0$

$$\cancel{0 = 1[P\sin\alpha]}$$

$$0 = P\cdot e^{0}\sin(0+\alpha)$$

$$\Rightarrow 0 = \underline{P}\cdot\sin\alpha$$

So either $P = 0$ (or) $\alpha = 0$

If $P=0$, the current will be zero for all t.
Hence it is not true. from (1)

So $\alpha = 0$

2. At $t=0$, $\left.\frac{di}{dt}\right|_{t=0} = \frac{V}{L}$

$$i(t) = e^{-K_1 t} P \sin K_2 t \quad \text{—— } \underline{\text{V}}$$

$$\frac{di}{dt} = -K_1 e^{-K_1 t} P \sin K_2 t + P \cdot e^{-K_1 t} \cdot \cos K_2 t \cdot K_2$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = -K \cdot e^0 \cdot P \sin(0) + P \cdot e^0 \cos(0) \cdot K_2$$

$$\Rightarrow \quad 0 + P \cdot K_2 = \frac{V}{L},$$

$$\Rightarrow \quad P = \frac{V}{LK_2} = \frac{V}{K \cdot \frac{1}{2K}\sqrt{\frac{4L}{C} - R^2}}$$

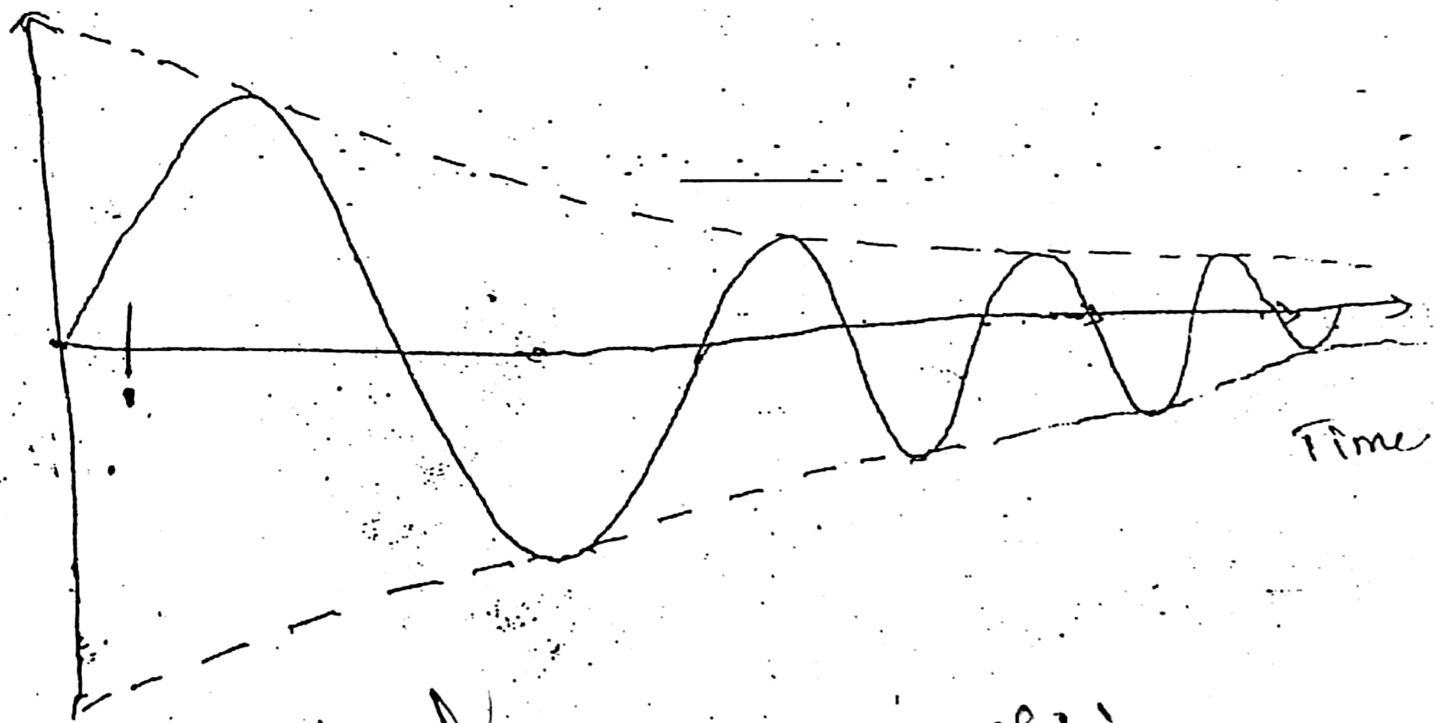
$$P = \frac{2V}{\sqrt{\frac{4L}{C} - R^2}}$$

Substituting in eqn (V), we get

$$i(t) = \frac{2V}{\sqrt{\frac{4L}{C} - R^2}} \cdot e^{-\frac{R}{2L}t} \sin\left(\frac{1}{2L}\sqrt{\frac{4L}{C} - R^2}\, t\right)$$

From the eqn of current it can be (9)

observed that the current is sinusoidal-exponen. The response is oscillatory. The final value of current in all 3 cases is zero. The current response is as shown in Fig. The peak value of sinusoidal term is decaying exponentially.



Under damped condition

...uation after 18 page.