1. Derive the Poisson equation for ideal gas.

Ideal Gas Law:

$$\mathrm{PV} = \mathrm{nRT}$$

differentiate both sides of this equation:

$$\mathrm{d(PV)} = \mathrm{nR\,dT} \Longrightarrow \mathrm{P\,dV} + V\,dT = nR\,\mathrm{dT}$$

$$\therefore \mathrm{dT} = \frac{P\,dV + V\,dP}{nR} \quad ①$$

Adiabatic Process(Q = 0) for ideal gas:

$$\begin{cases} dU = -dW = -P\,dV \\ dU = C_v\,dT = nC_{V,n}\,dT \end{cases}$$

$$\Rightarrow \mathrm{dT} = -\frac{P\,dV}{nC_{V,n}} \quad ②$$

Relation between heat capacities:

$$C_{P,n} - C_{V,n} = R \quad ③$$

By combining equations ①, ②and ③, we obtain:

$$PC_{P,n}\,\mathrm{dV} + VC_{V,n}\,\mathrm{dP} = 0 \quad ④$$

Define r:

$$\mathrm{r} \overset{\text{def}}{=} \frac{C_{P,n}}{C_{V,n}} \quad ⑤$$

By combining equations ④and ⑤, we obtain:

$$\frac{dP}{P} + \frac{dV}{V}r = 0$$

$$\Rightarrow \int \frac{dP}{P} = -r\int \frac{dV}{V}$$

$$\Rightarrow \mathrm{lnP} = -\mathrm{rlnV} + \mathrm{C}$$

$$\Rightarrow \ln(\mathrm{P}V^r) = \mathrm{C}$$

$$\Rightarrow \mathrm{P}V^r = C_1$$

2. What is the difference between saturated vapor and superheated vapor?

In general, saturated vapor will be liquefied immediately when cooled or compressed, while superheated vapor will first become saturated vapor.

3. Explain the definitions for “saturated liquid”,“saturated vapor” and “supercritical fluid”. Explain the physical meanings for the critical point.

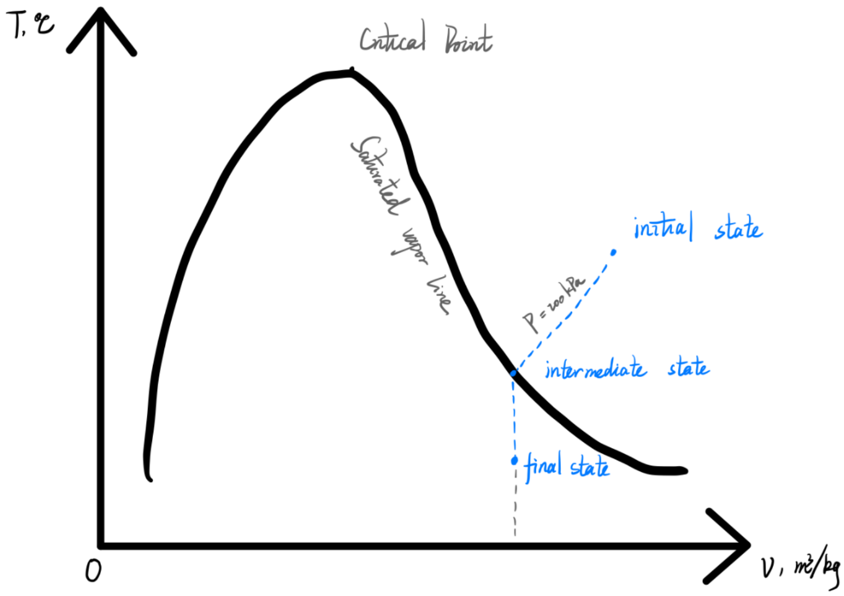
saturated liquid: A liquid that is about to vaporize.

saturated vapor: A vapor that is about to condense.

supercritical fluid: a substance in a state above the critical temperarure and critical pressure where gases and liquids can coexist.

the physical meanings for the critical point: the set of conditions under which a liquid and its vapor become identical.

4. Water initially at 200 kPa and 300℃ is contained in a piston–cylinder device fitted with stops. The water is allowed to cool at constant pressure until it exists as a saturated vapor and the piston rests on the stops. Then the water continues to cool until the pressure is 100 kPa. On the T-v diagram, sketch, with respect to the saturation lines, the process curves passing through the initial, intermediate, and final states of the water. Label the T, P, and v values **for end states** on the process curves. Find **the overall change in internal energy** between the initial and final states per unit mass of water.



Check the physical properties of saturated vapor. When $P_2 = 200\text{kPa}$, $T_2 = 120°\text{C}$.

$$PV = nRT$$

$$\Rightarrow v = \frac{RT}{PM}$$

v: specific volume, m^3/kg
M:molar mass, kg/mol

$$v_2 = \frac{8.314 \times (120 + 273)}{200 \times 10^3 \times 18.0 \times 10^{-3}} = 0.9\, m^3/kg$$

The water cool at constant volume (intermediate state to final state):

$$v_3 = v_2$$

Check the physical properties of saturated vapor. When $P_3 = 100\text{kPa}$, $T_3 = 99°\text{C}$.

The overall change in internal energy per unit mass of water:
The change in tnternal energy from the initial state to the intermediate state can be caculated by

$$\Delta U_{m1} = \int_{T1}^{T2} C_{v,m}\, dT.$$

The change in tnternal energy from the intermediate state to the final state need to consider the energy released by phase transition(liquefaction) and the energy released by reduction of the tempreature of the gas-liquid mixture. Please refer to the table for calculation if you are interested.

5. Carbon dioxide gas at 3 MPa and 500 K flows steadily in a pipe at a rate of 0.4 kmol/s. Determine (a) the **volume and mass flow rates** and the **density** of carbon dioxide at this state.
If CO2 is cooled at constant pressure as it flows in the pipe so that the temperature of CO2 drops to 450 K at the exit of the pipe, determine (b) **the volume flow rate** at the exit of the pipe.

(a)

$$\mathrm{PV} = \mathrm{nRT}$$

$$\Rightarrow V_n = \frac{RT}{P}$$

the volume flow rate:

$$q_i = 0.4 \times 10^3 \times \frac{8.314 \times 500}{3 \times 10^6} = 0.55 m^3/s$$

the mass flow rate:

$$\dot{m} = 0.4 \times 44.01 = 17.6 kg/s$$

$$\rho = \frac{\dot{m}}{q_i} = \frac{17.6}{0.55} = 32 kg/m^3$$

(b)

$$\rho_o = \frac{PM}{RT_o} = 35.28 kg/m^3$$

$$q_o = \frac{\dot{m}}{\rho_0} = \frac{17.6}{35.28} = 0.50 m^3/s$$

6. On the property diagrams indicated below, sketch (not to scale) with respect to the saturated liquid and saturated vapor lines and label the following processes and states for steam. Use arrows to indicate the direction of the process, and label the initial and final states:

(a) On the P-ν diagram, sketch the constant-temperature process through the state $P = 300\text{kPa}$, $\nu = 0.525 m^3/kg$ as pressure changes from $P_1 = 200\text{kPa}$ to $P_2 = 400\text{kPa}$. Place the value of the temperature on the process curve on the P-ν diagram.

(b) On the T-ν diagram, sketch the constant-specific-volume process through the state $T = 120°\text{C}, \nu = 0.7163 \, m^3/kg$ from $P_1 = 100\text{kPa}$ to $P_2 = 300\text{kPa}$. For this data set, place the temperature values at states 1 and 2 on its axis. Place the value of the specific volume on its axis.