# 2024-2025 学年高等代数上期末考试 2025.01

1. 求多项式 $u(x), v(x)$，使 $\big(f(x), g(x)\big) = u(x)f(x) + v(x)g(x)$，这里

$$f(x) = x^4 + 2x^3 - x^2 - 4x - 2, \qquad g(x) = x^4 + x^3 - x^2 - 2x - 2.$$

2. 将多项式 $x^4 + 4$ 分别在实数域和复数域上做因式分解.

3. 计算行列式的值：

(1) $\begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$

(2) $d_n = \begin{vmatrix} 1 & a_1 & a_1^2 & \cdots & a_1^{n-2} & a_1^{n-1} + \frac{1}{a_1} \\ 1 & a_2 & a_2^2 & \cdots & a_2^{n-2} & a_2^{n-1} + \frac{1}{a_2} \\ 1 & a_3 & a_3^2 & \cdots & a_3^{n-2} & a_3^{n-1} + \frac{1}{a_3} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & a_{n-1} & a_{n-1}^2 & \cdots & a_{n-1}^{n-2} & a_{n-1}^{n-1} + \frac{1}{a_{n-1}} \\ 1 & a_n & a_n^2 & \cdots & a_n^{n-2} & a_n^{n-1} + \frac{1}{a_n} \end{vmatrix}$

4. 已知 $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$，且

$$f(-1) = 0, f(1) = 6, f(2) = 21, f(3) = 52,$$

求 $f(x)$.

5. 求向量组

$$\alpha_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 4 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 \\ 3 \\ 1 \\ 2 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 3 \\ 0 \\ 7 \\ 14 \end{pmatrix}, \alpha_4 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \alpha_5 = \begin{pmatrix} 2 \\ 1 \\ 5 \\ 6 \end{pmatrix}$$

的秩和一个极大线性无关组，并用它表出剩下的向量.

6. 解矩阵方程：

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

7. 已知向量组 $\alpha_1, \alpha_2, \cdots, \alpha_s; \beta_1, \beta_2, \cdots, \beta_t; \alpha_1, \alpha_2, \cdots, \alpha_s, \beta_1, \beta_2, \cdots, \beta_t$ 的秩分别为 $r_1, r_2, r_3$. 证明：

$$max\{r_1, r_2\} \le r_3 \le r_1 + r_2$$

8. (1) 已知 $A, B$ 为 $n$ 阶可逆矩阵，证明：$(AB)^* = B^* A^*$；

(2) 设 $P(i,j), P(i,j(c)), P(i(c))$（$c \neq 0$）为三种初等矩阵，证明：

$$P(i,j) = P(j(-1))P(i,j(1))P(j,i(-1))P(i,j(1))$$

# 参考答案

1. $u(x) = -x - 1,\ v(x) = x + 2$.

2. 实数域：$x^4 + 4 = (x^2 - 2x + 2)(x^2 + 2x + 2)$

复数域：$x^4 + 4 = (x + 1 + i)(x - 1 + i)(x + 1 - i)(x - 1 - i)$

3. (1) $a^4$

(2)

$$d_n = \left(1 + (-1)^{n-1}\prod_{i=1}^{n}\frac{1}{a_i}\right)\prod_{1i<j\leq n}\left(a_j - a_i\right)$$

4. $f(x) = 1 + 2x + 2x^2 + x^3$

5. 秩为 4，一个极大线性无关组为 $\alpha_1, \alpha_2, \alpha_4, \alpha_5$，$\alpha_3 = 3\alpha_1 + \alpha_2$

6. $\begin{pmatrix} -9 & -8 & -2 \\ -10 & -9 & -2 \\ 13 & 11 & 3 \end{pmatrix}$