Chemical Engineering Thermodynamics

Lecture 5 Flow Processes

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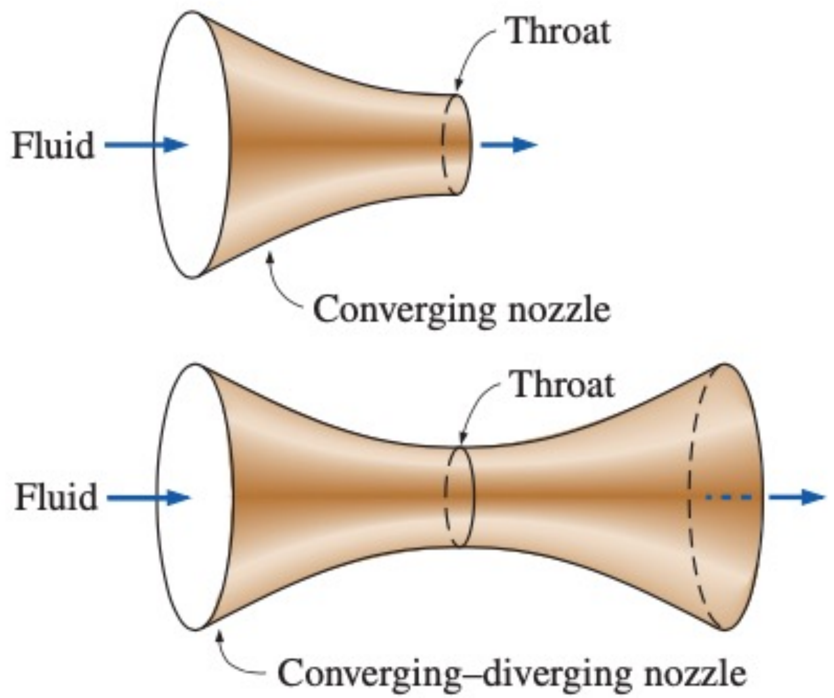
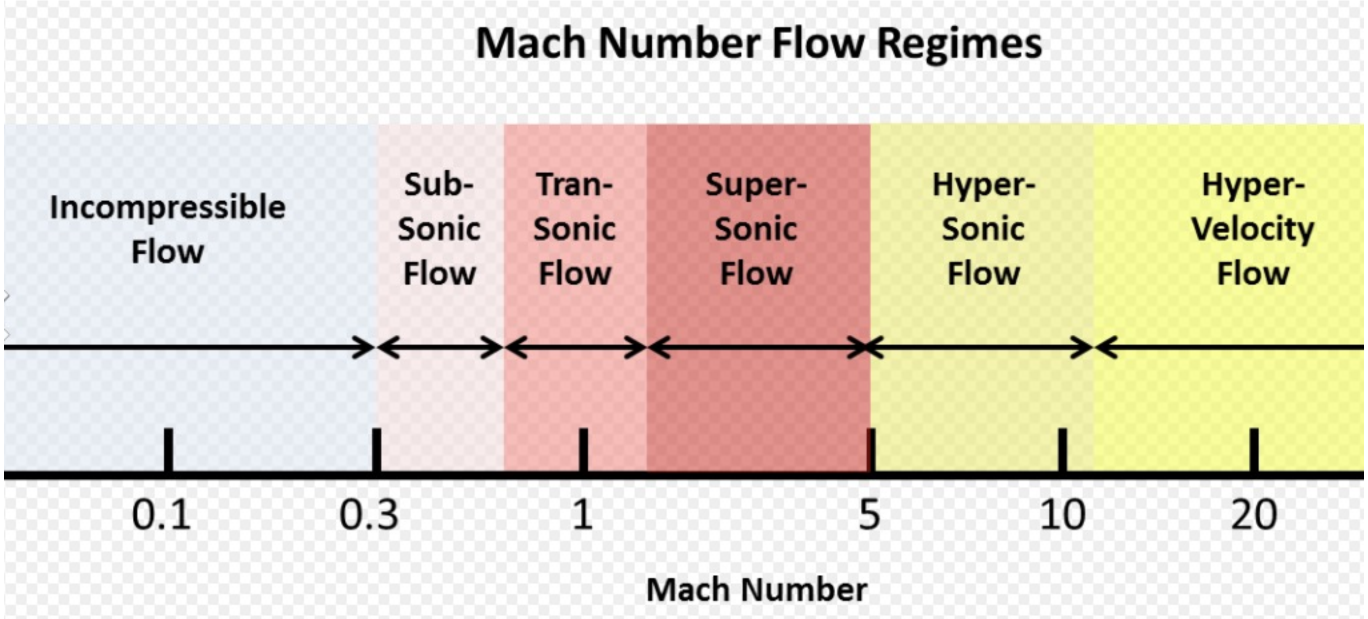


Compressible Flow

- Having significant changes in fluid density
- Flow-induced density changes
- Mach number: $M = \frac{u}{c}$
- Aircraft, jet engines, rocket motors, gas pipelines

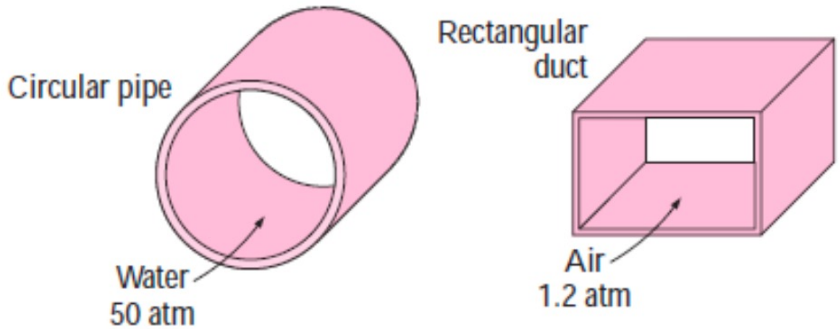


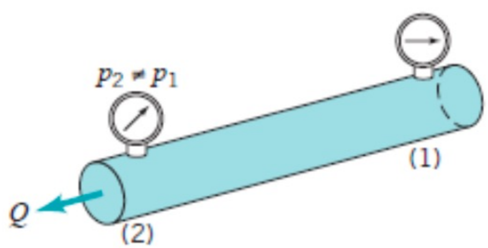
Sonic Booms

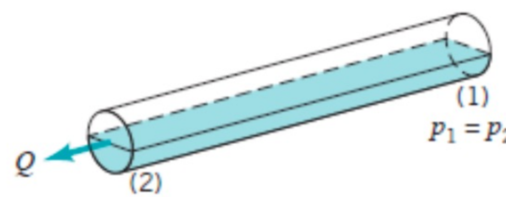
https://www.youtube.com/watch?v=gWGLAAYdbbc&list=PLyE1nk17cWv3Dvi64-DgTZjY7GXp9MtUb

Type of Flow

- Pipe flow: circular cross section; liquid flow
- Duct flow: noncircular cross section; gas flow







Pipe flow

Open-channel flow

Steady Compressible flow

- Adiabatic, steady-state, one-dimensional, compressible
- In the absence of shaft work and of changes in potential energy

$$u, A, V_m, H_m, \dot{m}$$

Mass Balance and Energy Balance

- Mass balance

$$d\left(\frac{uA}{V_m}\right) = 0$$

$$\frac{du}{u} + \frac{dA}{A} = \frac{dV_m}{V_m}$$

- Energy balance

$$\Delta H_m + \frac{\Delta u^2}{2} = 0$$

$$dH_m = -udu$$

- Fundamental property relation

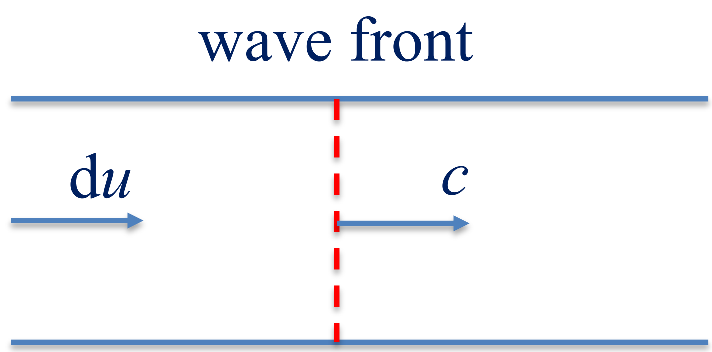
$$TdS_m + V_m dP = -udu$$

The Specific Volume

- $V_m = V_m(S, P)$
- $dV_m = \frac{\partial V_m}{\partial S_m} dS_m + \frac{\partial V_m}{\partial P} dP$
- $\left(\frac{\partial V_m}{\partial S_m}\right)_P = \left(\frac{\partial V_m}{\partial T}\right)_P \left(\frac{\partial T}{\partial S_m}\right)_P = \frac{\beta V_m T}{C_P}$
- $\frac{\partial V_m}{\partial P}$: changing ratio of specific volume (i.e. density) to pressure
- Sound: disturbance in the pressure or density

Sonic Speed

- Speed of density perturbation: c
- Velocity perturbation: du
- Mass balance: $cd\rho - \rho du = 0$
- Energy balance: $dH_m = cdu$
- Isentropic: $dH_m = \frac{1}{\rho} dP$



$$\begin{array}{ll} h + dh & h \\ p + dp & p \\ \rho + d\rho & \rho \end{array}$$

Netwon-Laplace Equation

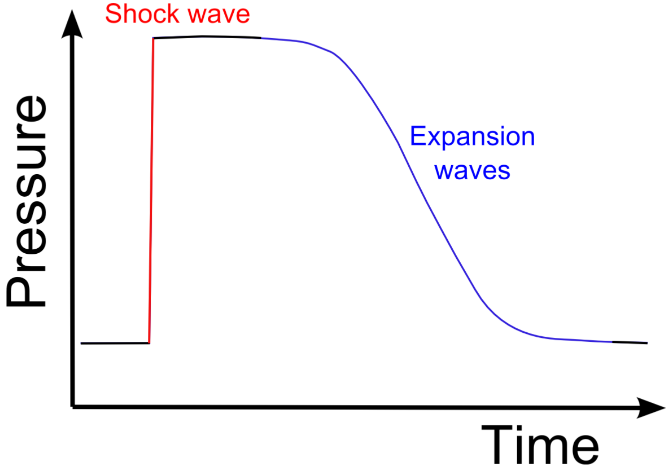
- $c^2 = \left(\frac{dp}{d\rho}\right)_S$
- $c^2 = \gamma\left(\frac{dp}{d\rho}\right)_T$
- For ideal gas: $c^2 = \gamma R_m T$

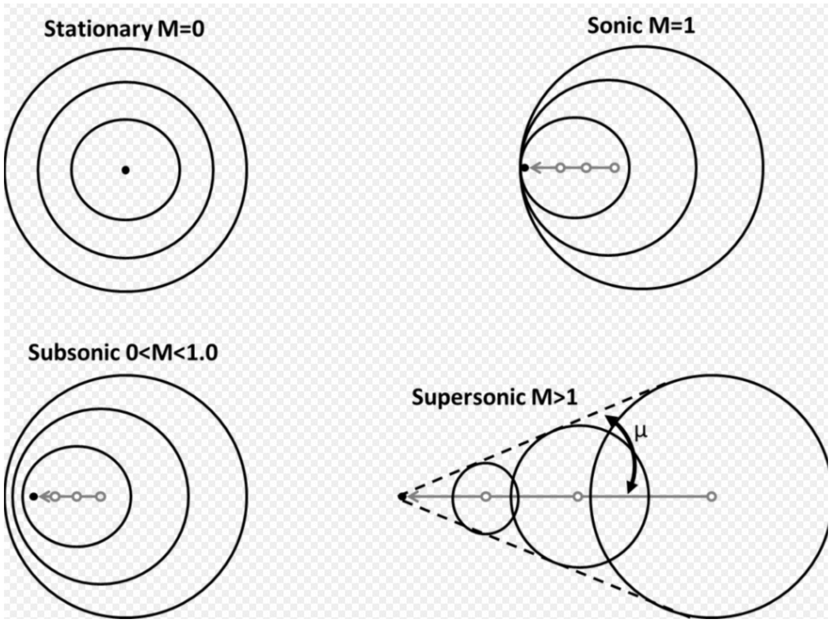
Control Equations in Compressible Flow

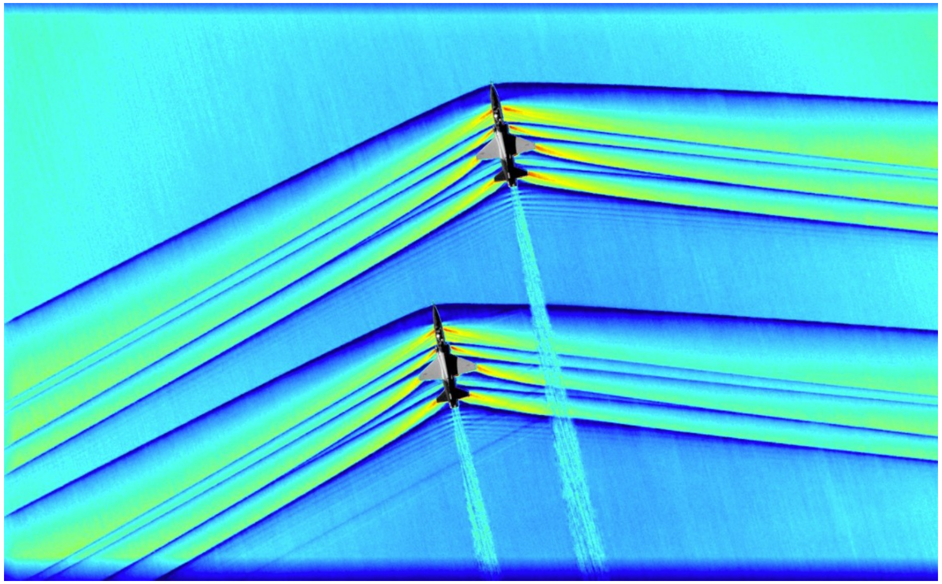
- Energy balance: $dH_m = -udu$
- Mass balance: $\frac{dV_m}{V_m} = \frac{du}{u} + \frac{dA}{A}$
- Gibbs equation： $dH_m = TdS_m + V_m dP$
- Compressible equation: $\frac{dV_m}{V_m} = \frac{\beta T}{C_p} dS_m - \frac{V_m}{c^2} dP$

$$\boxed{\begin{cases} (1-M^2)V_m dP + \left(1 + \frac{\beta u^2}{C_P}\right) TdS_m - \frac{u^2}{A} dA = 0 \\ udu - \frac{\left(\frac{\beta u^2}{C_P} + M^2\right)}{1-M^2} TdS_m + \frac{1}{1-M^2} \frac{u^2}{A} dA = 0 \end{cases}}$$

Shock Wave







NASA, 2019

Constant Cross-sectional Area

- $dA = 0$
- Subsonic flow

$$dS \geq 0, dP < 0, du > 0$$

Entropy increases; pressure decreases; velocity increases

- Supersonic flow

$$dS \geq 0, dP \geq 0, du < 0$$

Entropy increases; pressure increases; velocity decreases

$$\boxed{\begin{cases} dP = -\dfrac{T}{V_m}\dfrac{\left(1 + \dfrac{\beta u^2}{C_P}\right)}{1 - M^2} dS_m \\ udu = T\dfrac{\left(\dfrac{\beta u^2}{C_P} + M^2\right)}{1 - M^2} dS_m \end{cases}}$$

Example

For the steady-state, adiabatic, irreversible flow of an *incompressible* liquid in a horizontal pipe of constant cross-sectional area, show that:

(*a*) The velocity is constant.

(*b*) The temperature increases in the direction of flow.

(*c*) The pressure decreases in the direction of flow.

Nozzles

- $dS = 0$

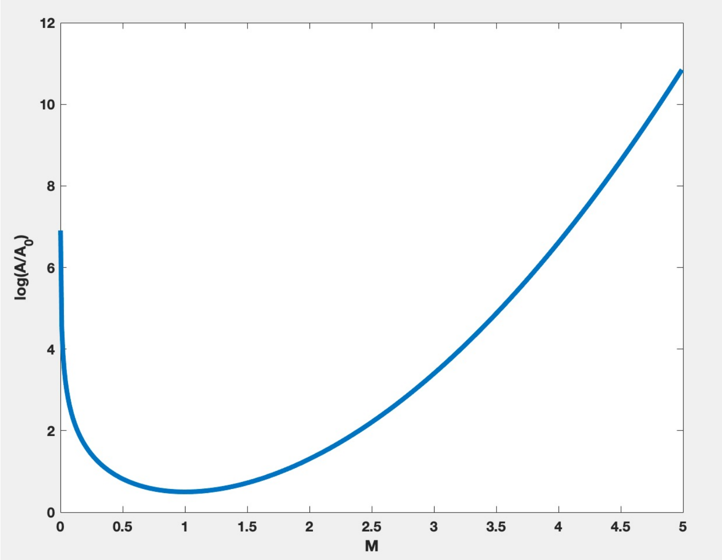
$$\boxed{\begin{cases} dP = \dfrac{u^2}{(1-M^2)AV_m} dS_m \\ du = -\dfrac{1}{1-M^2}\dfrac{u}{A} dA \end{cases}}$$

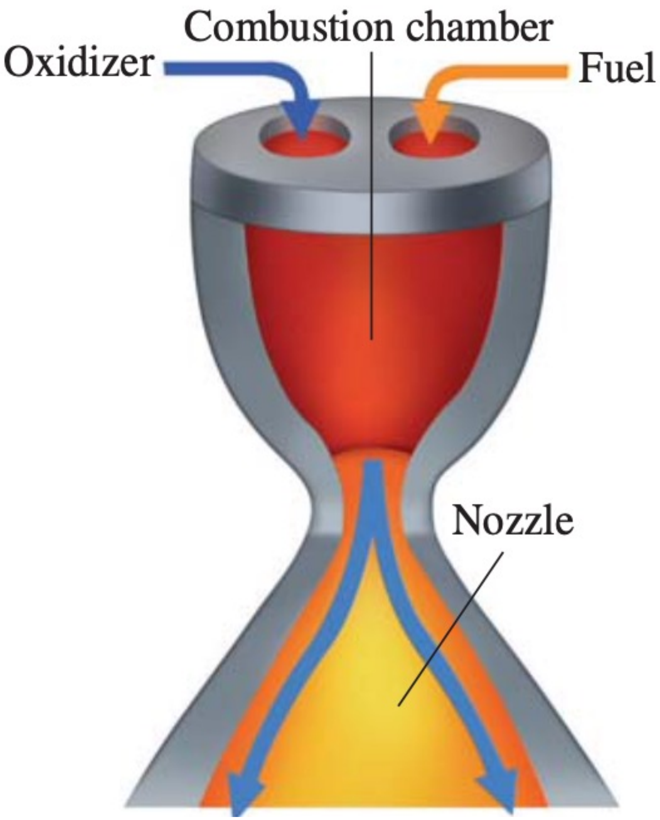
	Subsonic: **M** < 1		Supersonic: **M** > 1	
	Converging	Diverging	Converging	Diverging
$\frac{dA}{dx}$	−	+	−	+
$\frac{dP}{dx}$	−	+	+	−
$\frac{du}{dx}$	+	−	−	+

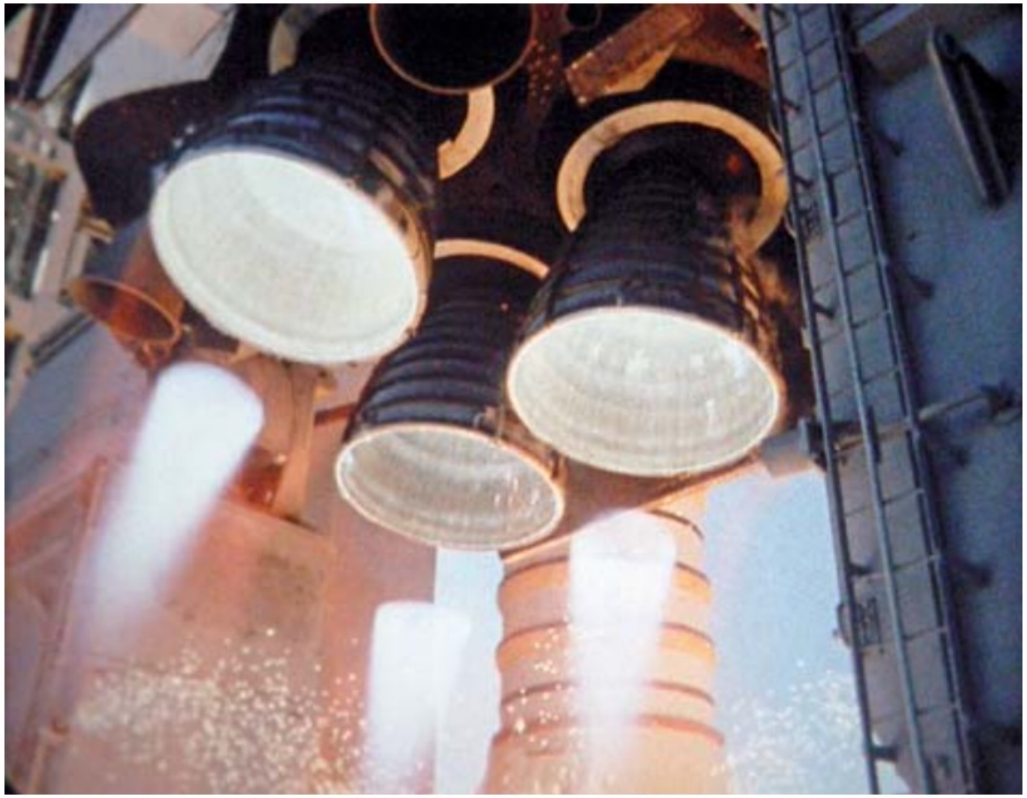
Control Equation for Nozzle

$$\frac{dA}{du} = -\frac{A}{u}(1 - M^2)$$

- Subsonic(M<1): $\frac{dA}{du} < 0$
- Supersonic(M>1): $\frac{dA}{du} > 0$
- Sonic(M=1): $\frac{dA}{du} = 0$
- Cannot reach supersonic velocity in a converging nozzle




Combustion chamber
Oxidizer
Fuel
Nozzle



Stagnation Property

- Stagnation enthalpy: $h_0 = h + \frac{1}{2}u^2$

$$h = \frac{H}{m}$$

- Stagnation temperature: $T_0 = T + \frac{u^2}{2C_p}$
- Stagnation pressure

$$P_0 = P\left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

- Stagnation density

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$$

Critical Properties at M=1 for Ideal Gas

- Critical temperature:

$$T^* = T_0 \frac{2}{k+1}$$

- Critical pressure

$$P^* = P_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

- Critical density

$$\rho^* = \rho_0 \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

Isentropic Flow of Ideal Gas

- $PV_m = R_m T$
- $PV_m^{\gamma} \equiv C$
- $udu = -V_m dP$
- $u_2^2 - u_1^2 = \frac{2\gamma P_1 V_{m,1}}{\gamma - 1}\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right)$
- As $u_2 = c$, $\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

Heat Capacity Ratios

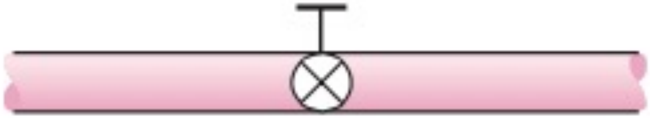
Temp.	Gas	γ	Temp.	Gas	γ	Temp.	Gas	γ
−181 °C	H_2	1.597	100 °C	Dry air	1.401	20 °C	NO	1.400
−76 °C	H_2	1.453	200 °C	Dry air	1.398	20 °C	N_2O	1.310
20 °C	H_2	1.410	400 °C	Dry air	1.393	−181 °C	N_2	1.470
100 °C	H_2	1.404	1000 °C	Dry air	1.365		N_2	
400 °C	H_2	1.387	0 °C	CO_2	1.310	20 °C	Cl_2	1.340
1000 °C	H_2	1.358	20 °C	CO_2	1.300	−115 °C	CH_4	1.410
2000 °C	H_2	1.318	100 °C	CO_2	1.281	−74 °C	CH_4	1.350
20 °C	He	1.660	400 °C	CO_2	1.235	20 °C	CH_4	1.320
20 °C	H_2O	1.330	1000 °C	CO_2	1.195	15 °C	NH_3	1.310
100 °C	H_2O	1.324	20 °C	CO	1.400	19 °C	Ne	1.640
200 °C	H_2O	1.310	−181 °C	O_2	1.450	19 °C	Xe	1.660
−180 °C	Ar	1.760	−76 °C	O_2	1.415	19 °C	Kr	1.680
20 °C	Ar	1.670	20 °C	O_2	1.400	15 °C	SO_2	1.290
−15 °C	Dry air	1.404	100 °C	O_2	1.399	360 °C	Hg	1.670
0 °C	Dry air	1.403	200 °C	O_2	1.397	15 °C	C_2H_6	1.220
20 °C	Dry air	1.400	400 °C	O_2	1.394	16 °C	C_3H_8	1.130

Example

A high-velocity nozzle is designed to operate with steam at 700 kPa and 300°C. At the nozzle inlet the velocity is 30 $m \cdot s^{-1}$. Calculate values of the ratio A/A_1 (where A_1 is the cross-sectional area of the nozzle inlet) for the sections where the pressure is 600, 500, 400, 300, and 200 kPa. Assume that the nozzle operates isentropically.

Throttling Valves

Throttle valve



(*a*) An adjustable valve

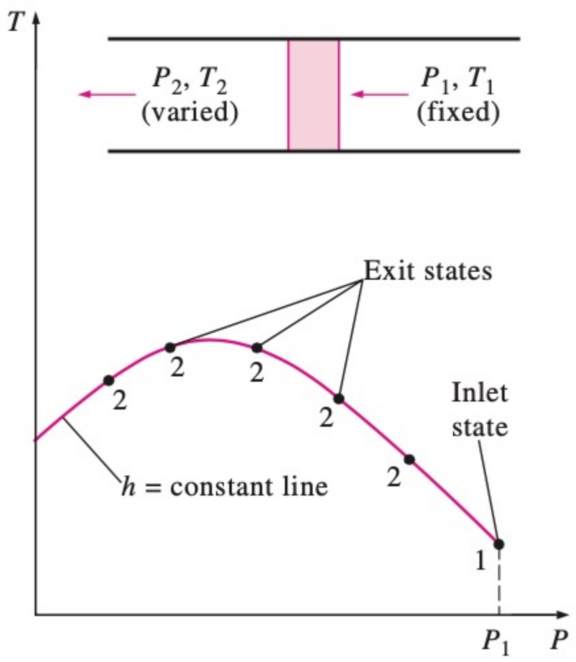
(*b*) A porous plug

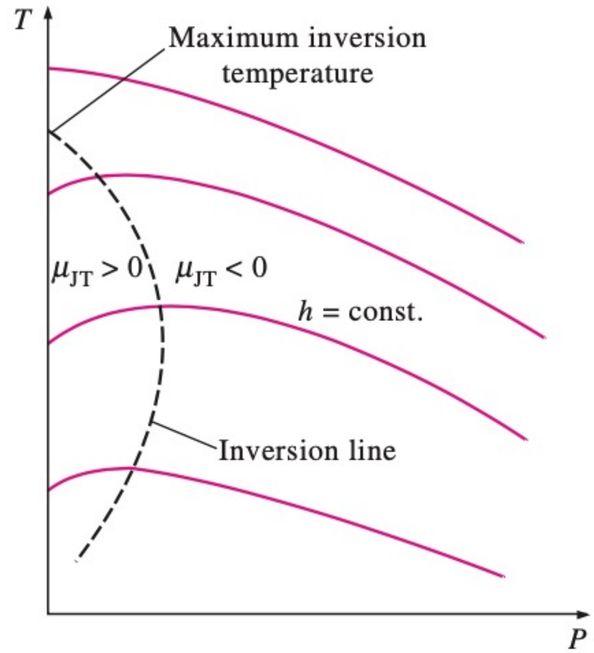
(*c*) A capillary tube

Throttling Process

- Constant enthalpy: $\Delta H = 0$
- Joule-Thomson coefficient:

$$\mu_{JT} = \frac{1}{c_{p,n}}\left[T\frac{\partial V_n}{\partial T} - V_n\right]$$





Joule-Thomson Effect

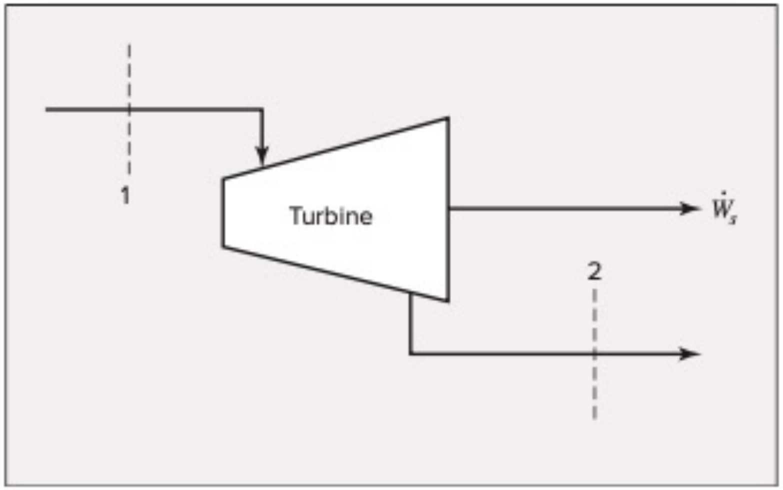
- Temperature change when the fluid is forced through a throttle
- Adiabatic
- At room temperatures, most of gases cool by the JT effect
- Except for Hydrogen, Helium, and Neon
- The gas-cooling throttling process can be used in refrigeration processes
- Most liquids will be warmed by the JT effect
- Find internally leaking valves

https://www.youtube.com/watch?v=M7h59Tg9DS4

Turbine (Expander)

- The expansion of a gas in a nozzle to produce a high-velocity stream in a process that coverts internal energy into kinetic energy.

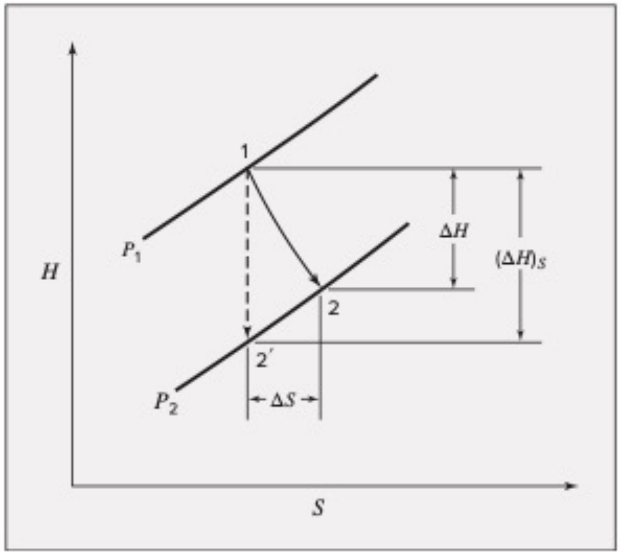
stream turbine



Turbine (Expander)

- Energy balance: $W_s = \Delta H$
- Isentropic turbine: $\mathrm{W}_{s,\mathrm{isentropic}} = (\Delta H)_{isentropic}$
- Max shaft work: reversible, adiabatic; Ientropic
- Turbine efficiency:

$$\eta \equiv \frac{W_s}{W_{s,isentropic}}$$



Example

A steam turbine with rated capacity of 56,400 kW (56,400 $kJ \cdot s^{-1}$) operates with steam at inlet conditions of 8600 kPa and 500°C, and discharges into a condenser at a pressure of 10 kPa. Assuming a turbine efficiency of 0.75, determine the state of the steam at discharge and the mass rate of flow of the steam.

Compression Processes

- Expansion: pressure reductions
- Compression: pressure increases

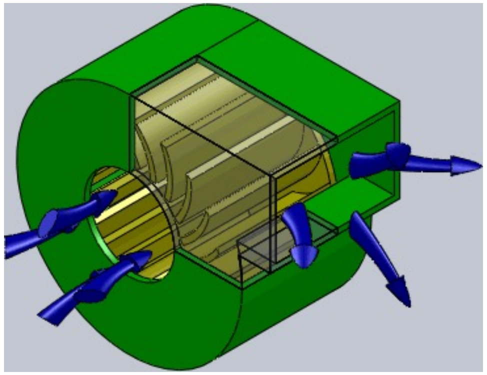
compressor

pump

fan

Transport fluid
Fluidization solid
Reduce pressure
….

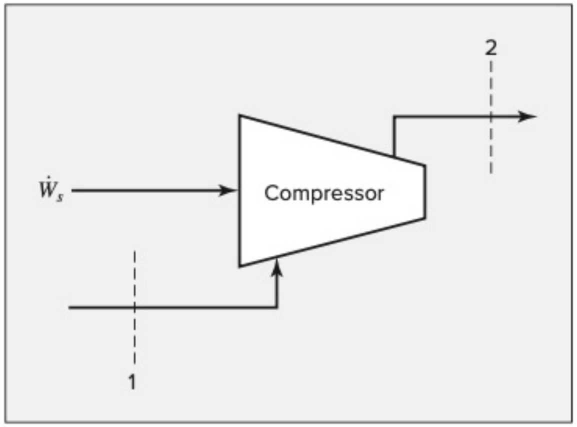
blower

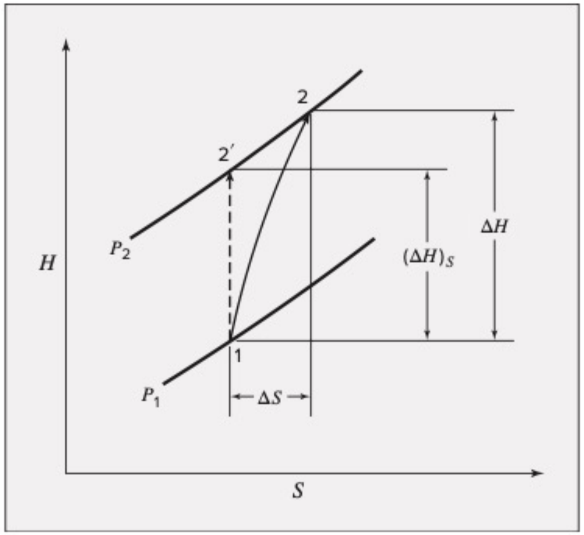


vacuum pumps

Compressor

- Energy balance: $W_s = \Delta H$
- Thermal efficicency: $\eta \equiv \frac{W_{s,isentropic}}{W_s}$





Example

Saturated-vapor steam at 100 kPa (t^{sat} = 99.63°C) is compressed adiabatically to 300 kPa. If the compressor efficiency is 0.75, what is the work required and what are the properties of the discharge stream?

Pumps

- Isentropic pumps: $W_{s,isentropic} = \int_{P_1}^{P_2} V dP$
- General cases:

$$W_s = \Delta H = C_p \Delta T + V(1 - \beta T)\Delta P$$

$$\Delta S = C_p \ln \frac{T_2}{T_1} - \beta V \Delta P$$

Example

Water at 45°C and 10 kPa enters an adiabatic pump and is discharged at a pressure of 8600 kPa. Assume the pump efficiency to be 0.75. Calculate the work of the pump, the temperature change of the water, and the entropy change of the water.

Summary Points

- Compressible flow
- Duct flow
- Pipe flow
- Nozzle flow
- 1D isentropic flow
- Throttling process
- Turbine
- Compression processes