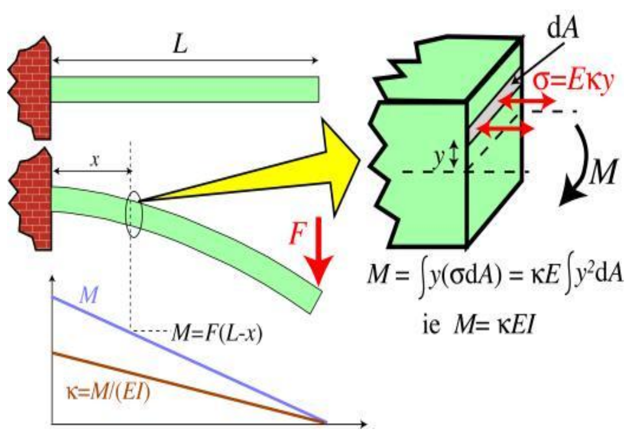
过程设备机械设计基础

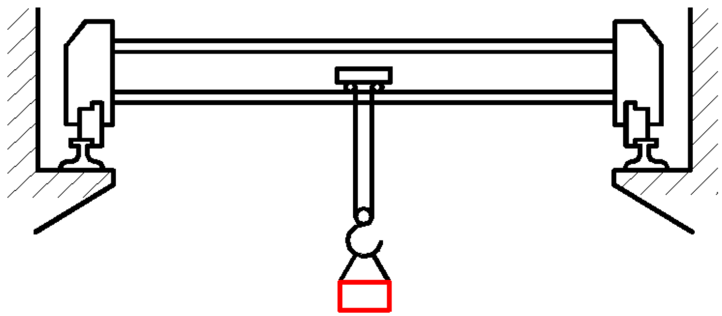
4. 平面弯曲

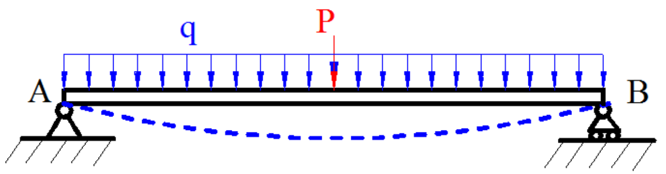


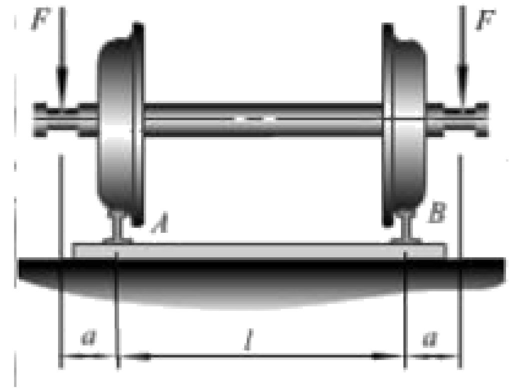
竹子为什么是中空的

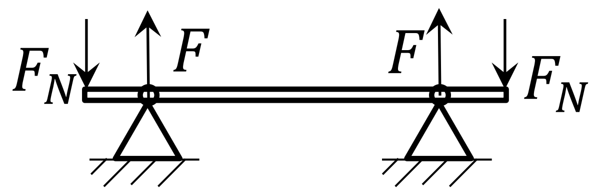
竹子是禾本科植物（与水稻、小麦、狗尾巴草等同属一科），竹子从小长到大，茎的粗细变化不大，但是成熟后竹子可高达20米。从进化史看，竹子最初是实心的，但是，后来竹子的茎渐渐演变为空心。为什么？

平面弯曲的概念



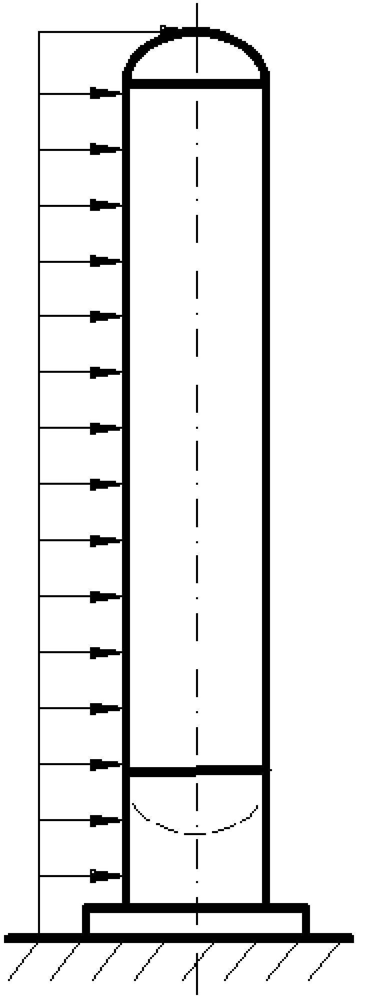


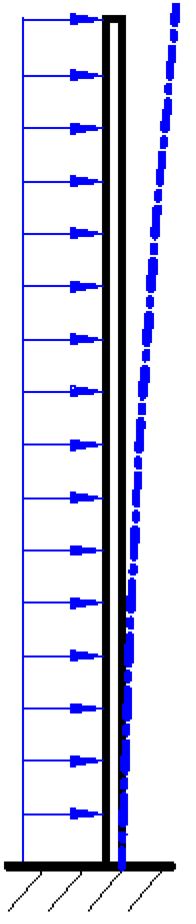




在工程实际中，受弯构件是极为常见的。例如，火车轮轴，吊车大梁、混凝土梁等。它们共同的特点是承受垂直于其轴线的外力，或在其轴线平面内作用有外力偶矩。受力后直的轴线变成了曲线，这种变形称为**弯曲变形**。

塔设备简化模型

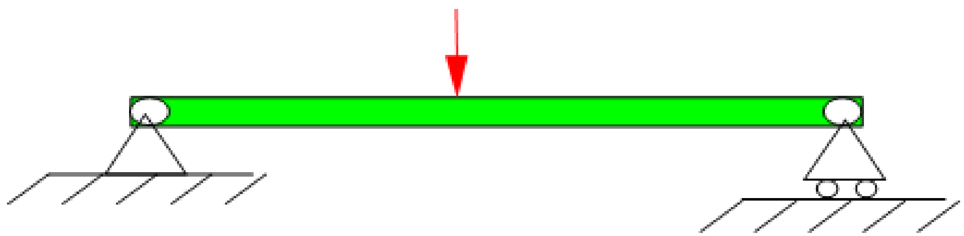




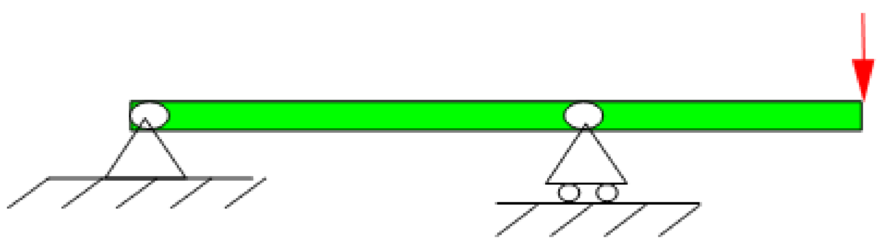
梁的分类

梁：以弯曲变形（横向力）为主的杆件

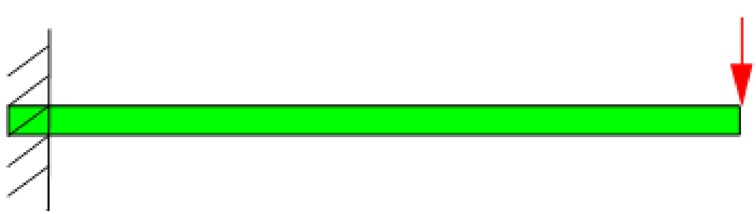
根据固定情况可分为：



简支梁

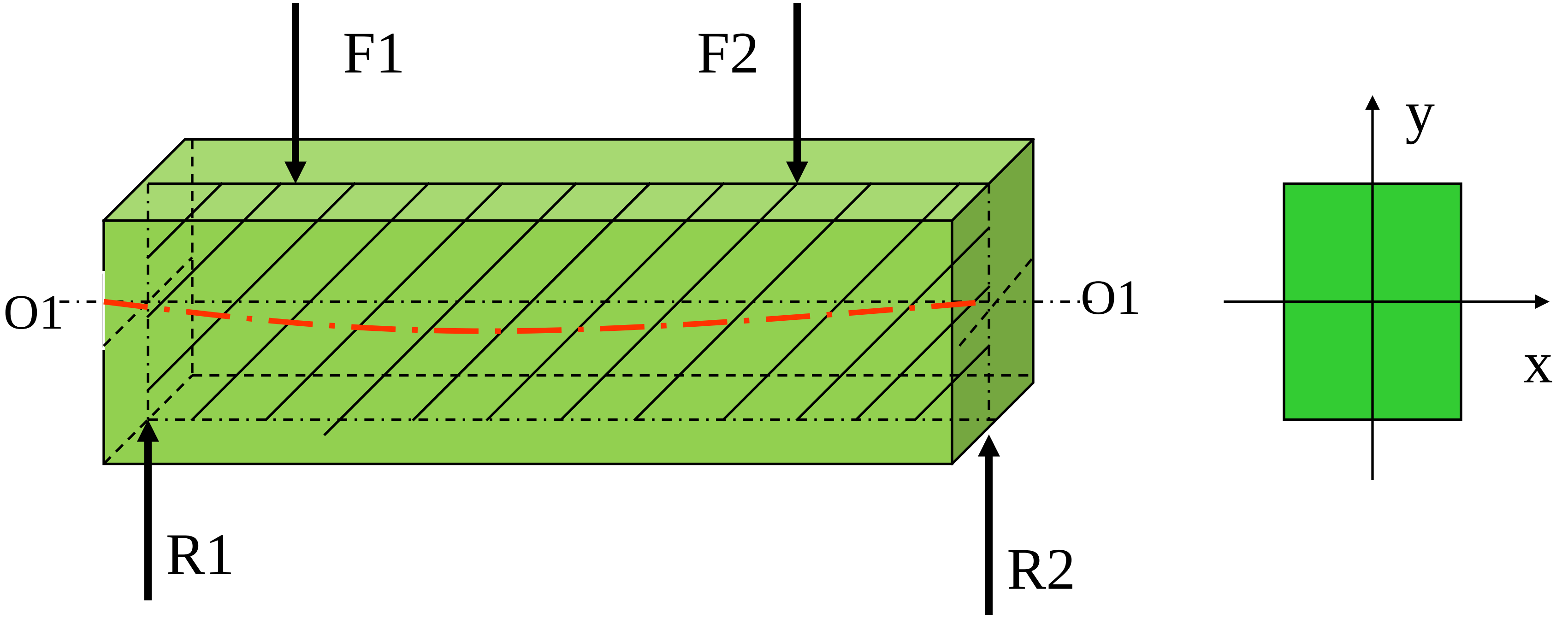


外伸梁



悬臂梁

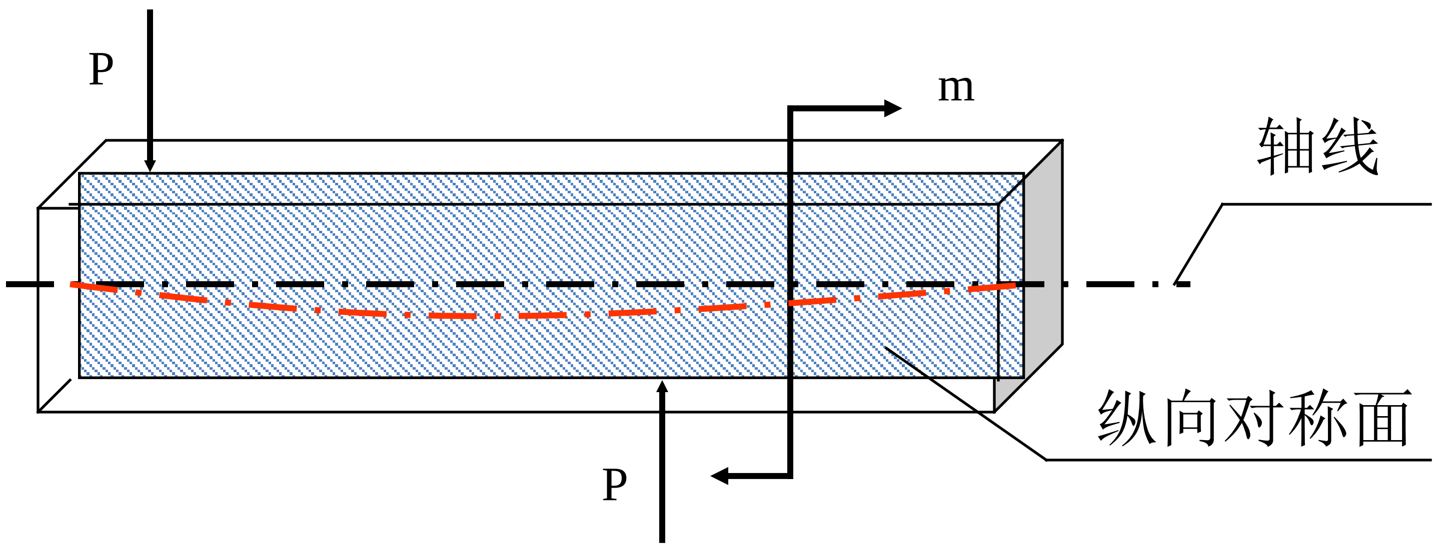
平面弯曲



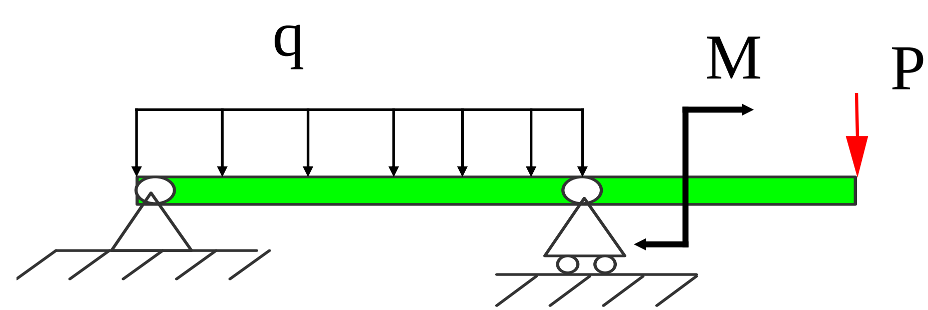
纵向对称面：　对称轴（y轴）与轴线(O1-O1)组成的平面

平面弯曲：　梁轴线弯曲成此平面内的一条曲线

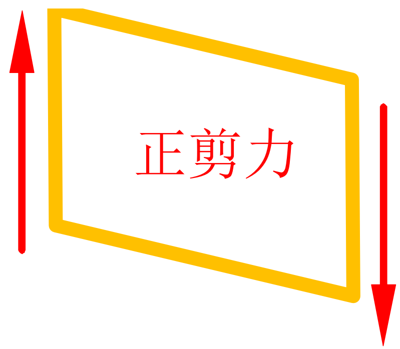
弯曲梁的简化作图



集中载荷P
分布载荷q
集中力偶M

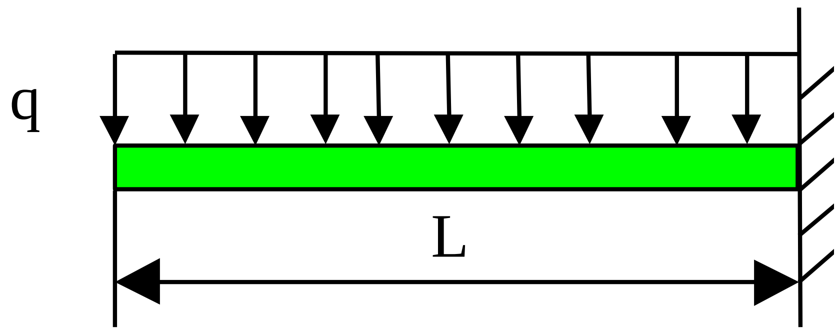


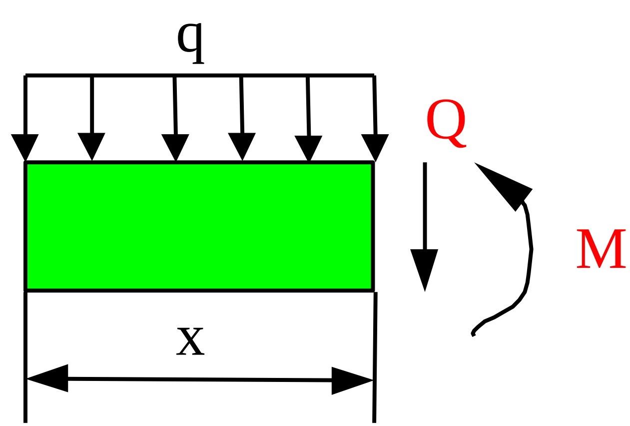
剪力和弯矩的符号规定



- 凡使微段梁发生左侧截面向上，右侧截面向下的剪力为正剪力，反之为负
- 弯矩M使梁弯曲时，凹面向上的弯矩M为正，凸面向上的弯矩为负

悬臂梁的受力分析





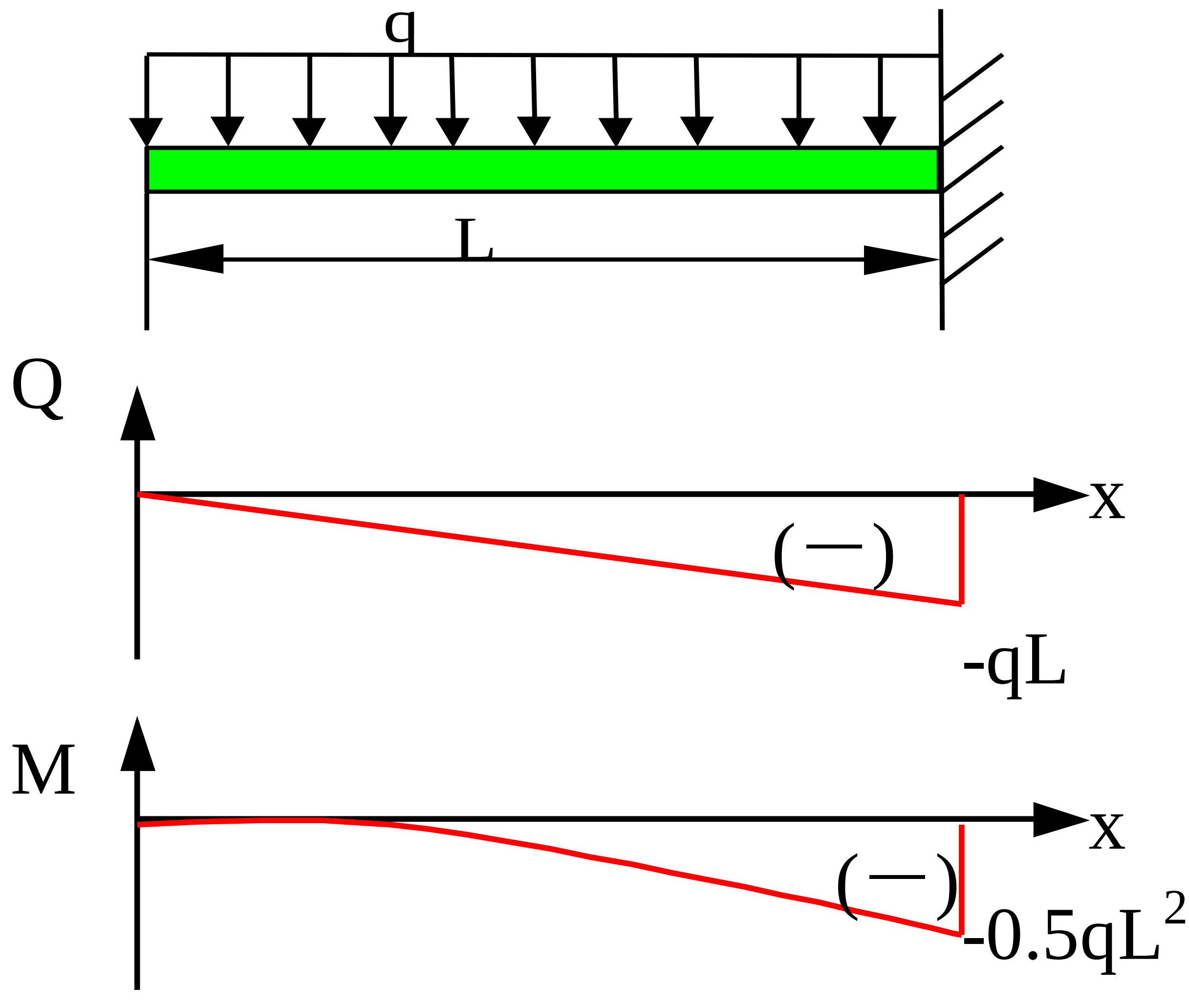
剪力方程：

$$Q = -qx \quad (0 < x < L)$$

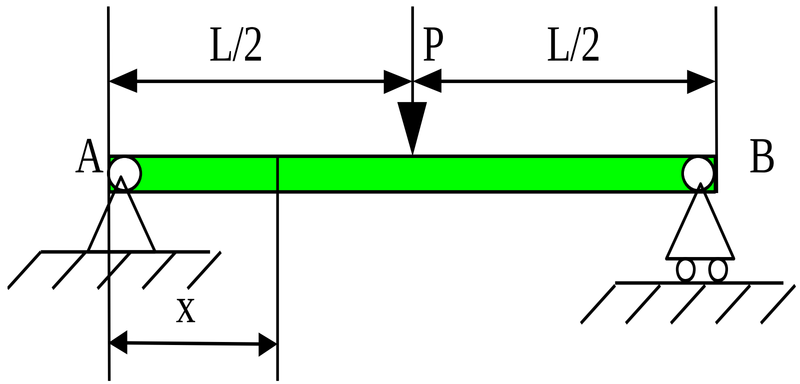
弯矩方程：

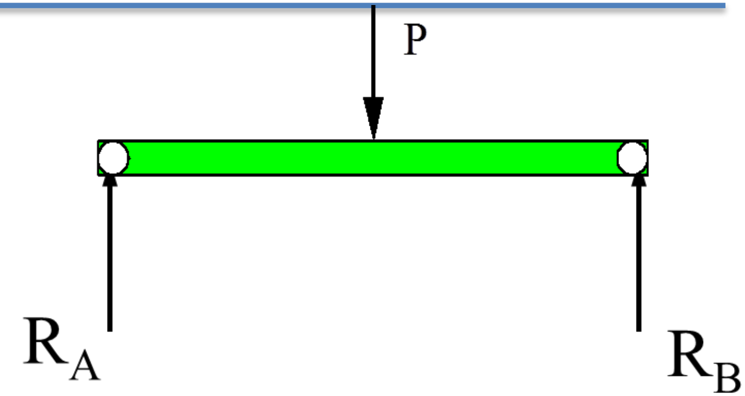
$$M = \frac{-qx^2}{2} \quad (0 < x < L)$$

悬臂梁的剪力弯矩图（Q—M图)



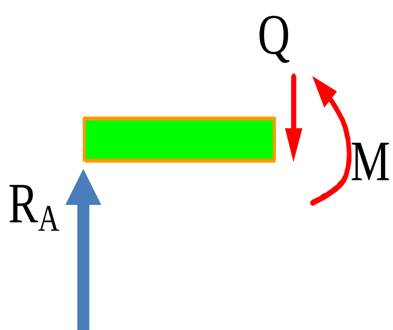
直梁平面弯曲时的内力





从左端开始分析

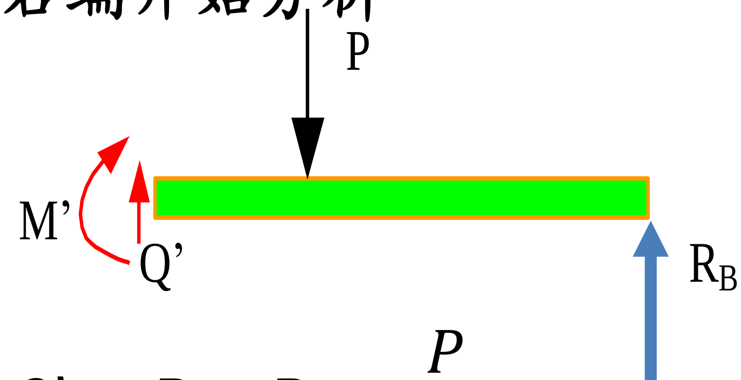
$$0 < x < L/2$$



$$Q = R_A = \frac{P}{2}$$

$$M = R_A x = \frac{Px}{2}$$

从右端开始分析



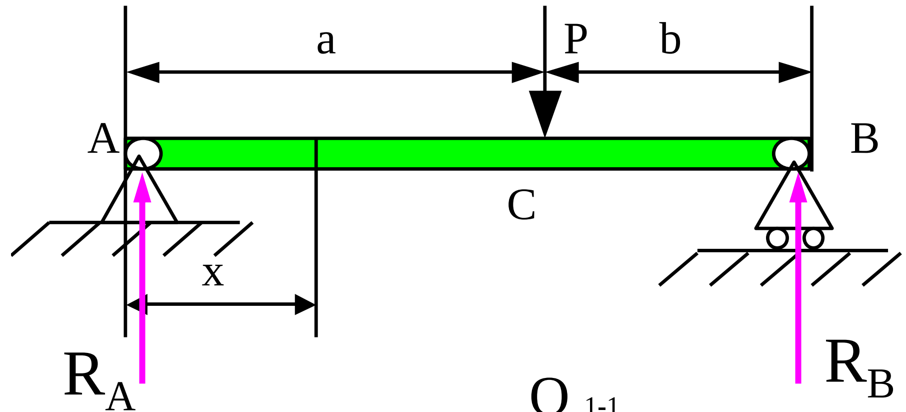
$$Q' = P - R_B = \frac{P}{2}$$

$$M' = R_B(L - x) - P\left(\frac{L}{2} - x\right)$$
$$= Px/2$$

简支梁的的受力分析

1.求出支反力：

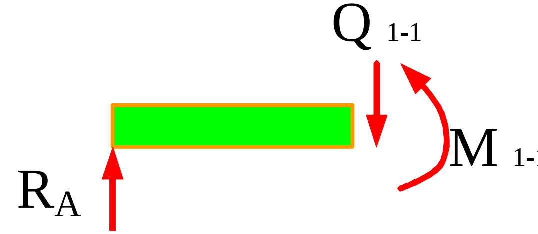
$$R_A=\frac{Pb}{a+b}\quad R_B=\frac{Pa}{a+b}$$



2.在AC段 (0<x<a)

$$Q_{1-1}=R_A=\frac{Pb}{a+b}$$

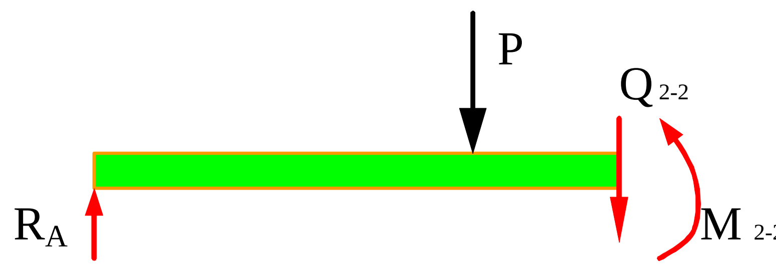
$$M_{1-1}=R_A x=\frac{Pb}{a+b}x$$



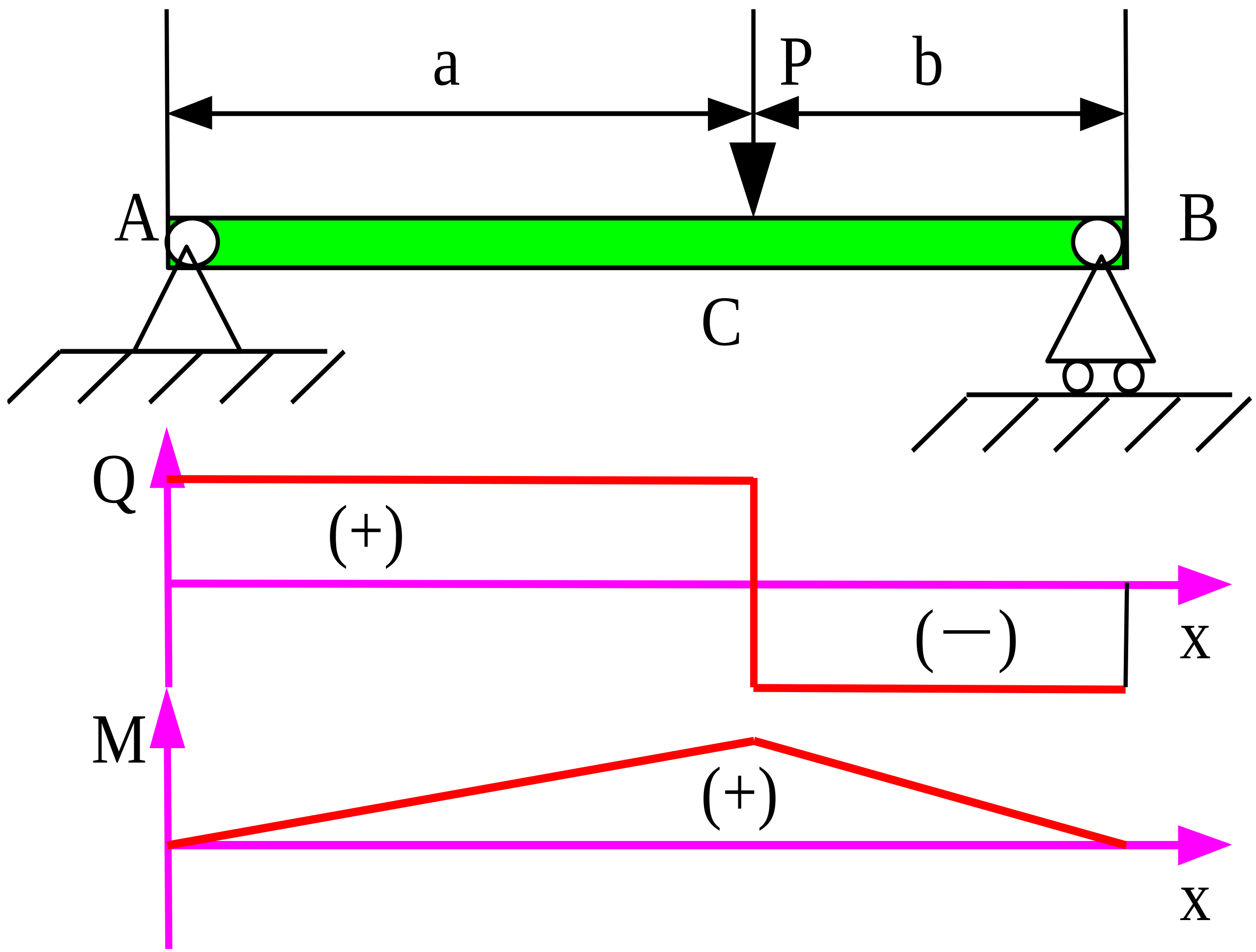
3.在BC段 (a<x<a+b)

$$Q_{2-2}=R_A-P=\frac{Pa}{a+b}$$

$$M_{2-2}=R_A x-P(x-a)=Pa-\frac{Pa}{a+b}x$$



简支梁的剪力图和弯矩图



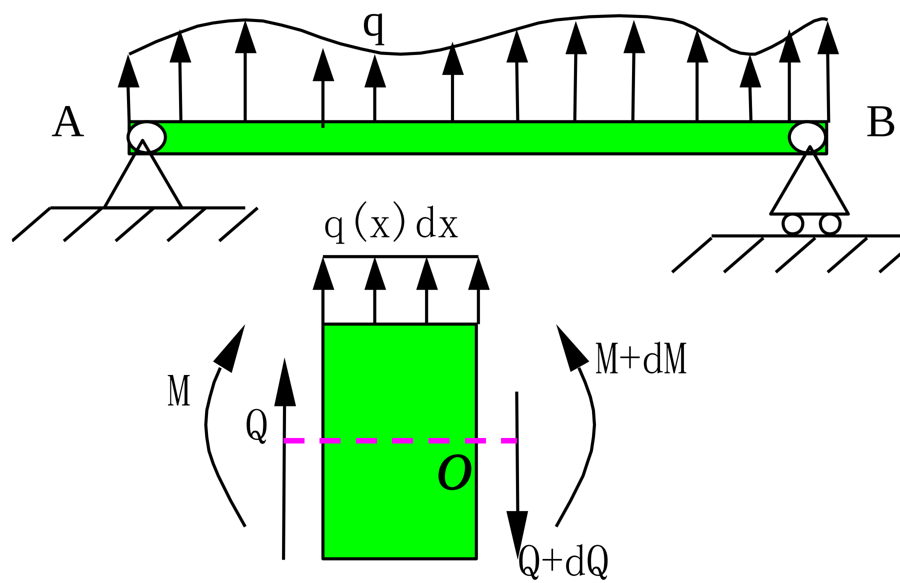
剪力/弯矩/分布载荷间的关系

$$\Sigma F_y = 0$$

$$Q + q\mathrm{d}x - (Q + \mathrm{d}Q) = 0$$

$$\Longrightarrow \mathrm{d}Q = q\mathrm{d}x$$

$$Q = \int q\mathrm{d}x$$



$$\Sigma M_o = 0$$

$$-Q\mathrm{d}x - M + (M + \mathrm{d}M) - q\mathrm{d}x\frac{\mathrm{d}x}{2} = 0$$

$$\Longrightarrow \mathrm{d}M = Q\mathrm{d}x$$

$$M = \int Q\mathrm{d}x$$

剪力/弯矩图性质

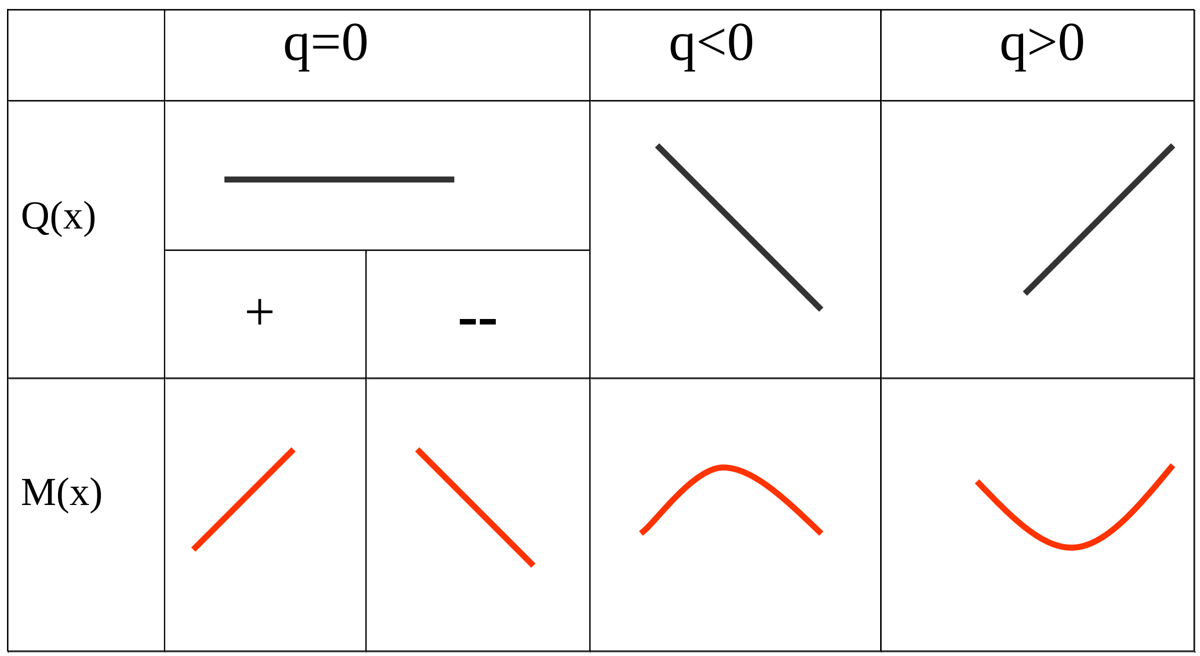
剪力图上任一点的斜率即为梁上相应横截面上的分布载荷q

弯矩图上任一点的斜率即为梁上相应横截面上的剪力Q

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = q$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = Q$$

$$\frac{\mathrm{d}^2 M}{\mathrm{d}x^2} = q$$



Q—M图的四点规律

1. 梁上某段无分布力，Q为水平线，M为斜直线

2. 有向下的分布力，Q图递减(↘)，M为上凸(⌒)

   有向上的分布力，Q图递增(↗)，M为下凹(◡)

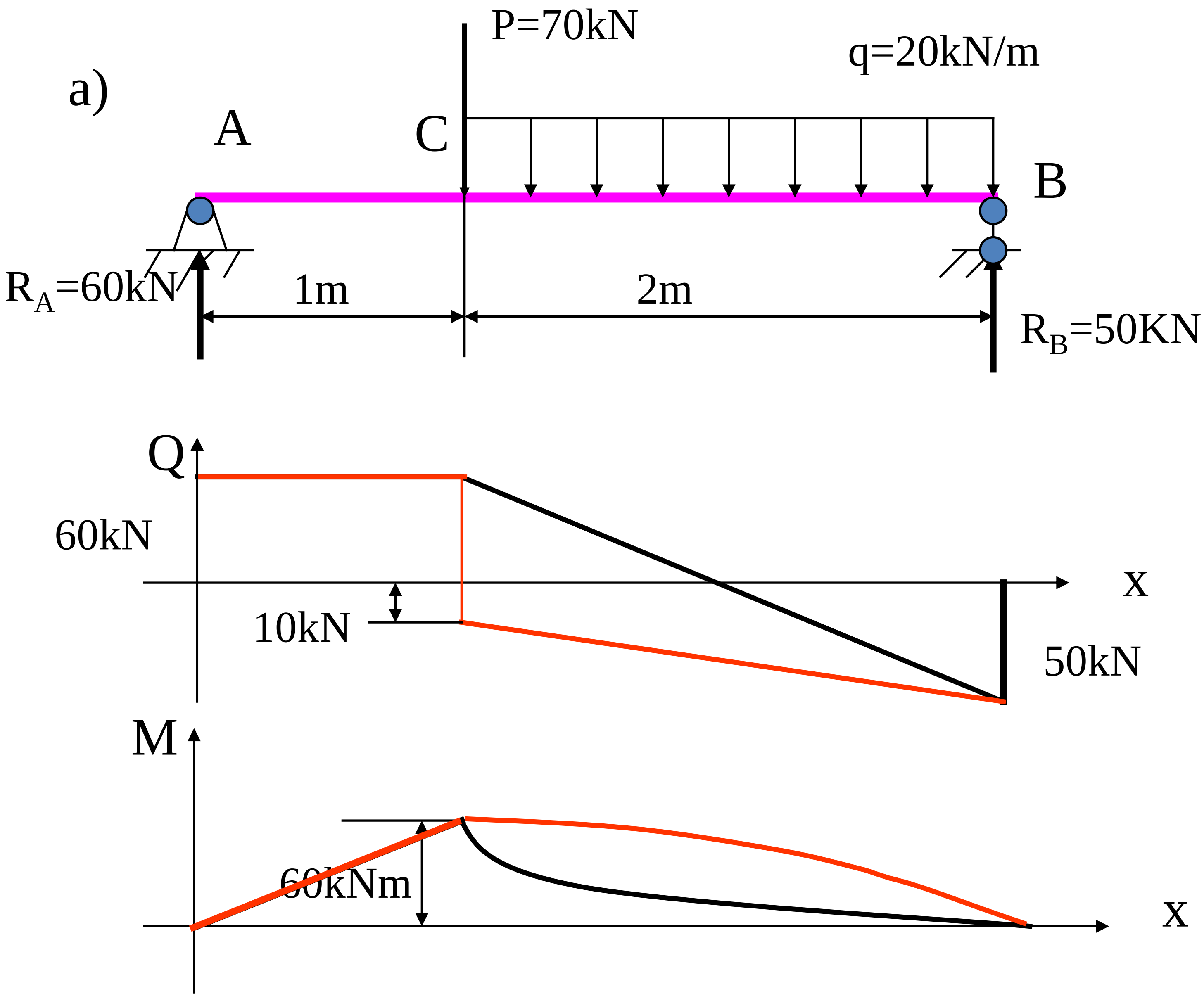
   如分布力均匀，Q为斜直线，M为二次抛物线

3 . 在集中力作用处，Q图有突变，M图有折角

   在集中力偶处，弯矩图有突变

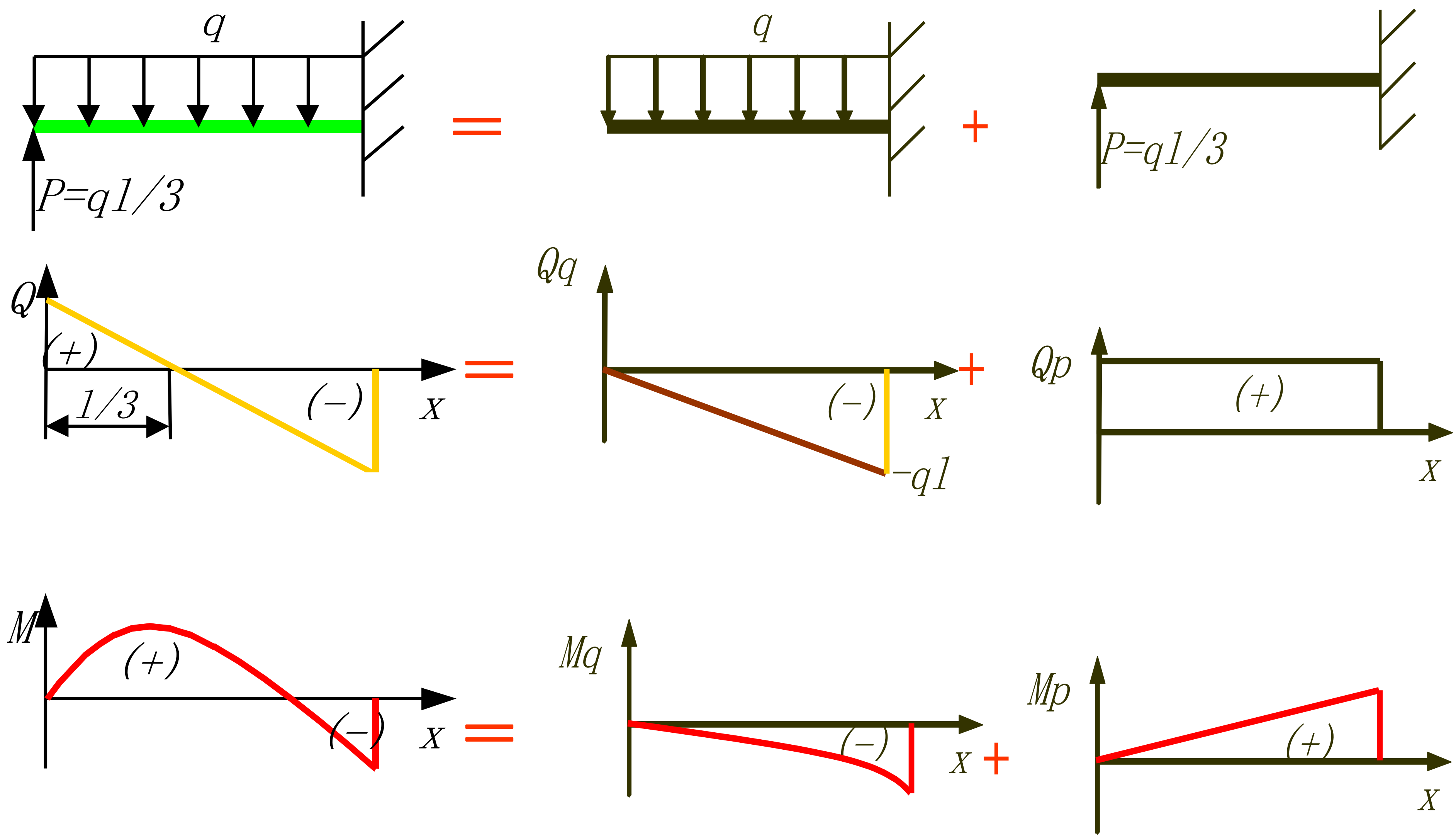
4. 某截面Q＝0，则弯矩为极值。

判断Q、M图是否有错?



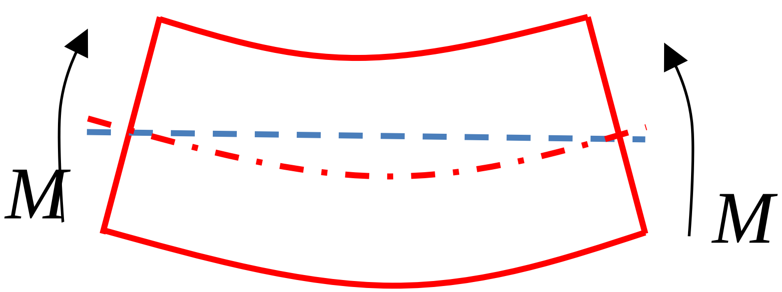
P=70kN
q=20kN/m
a)
A
C
B
R_A=60kN
1m
2m
R_B=50KN
Q
60kN
10kN
x
50kN
M
60kNm
x

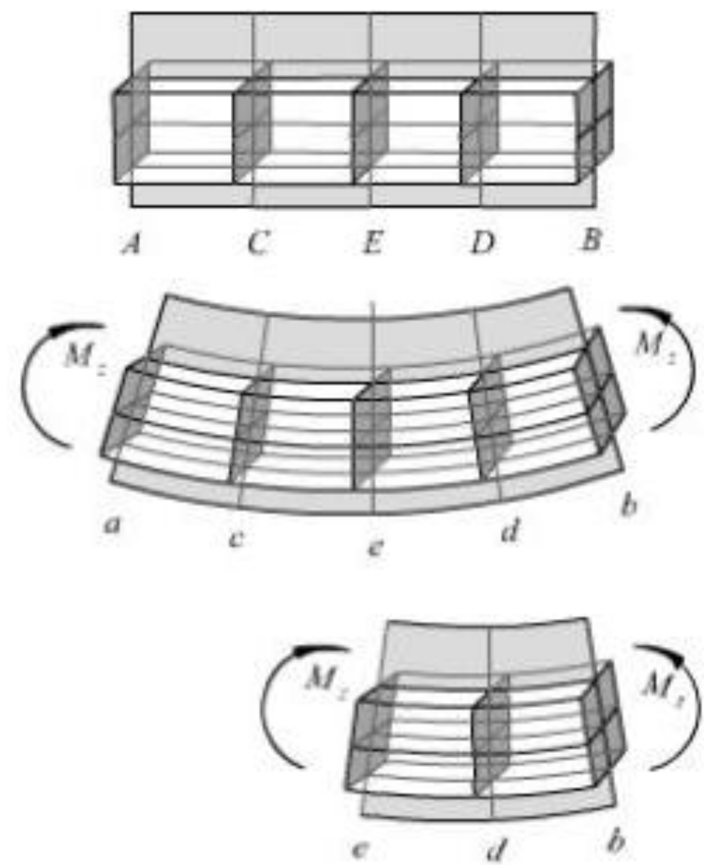
叠加法作Q－M图



平面弯曲应力

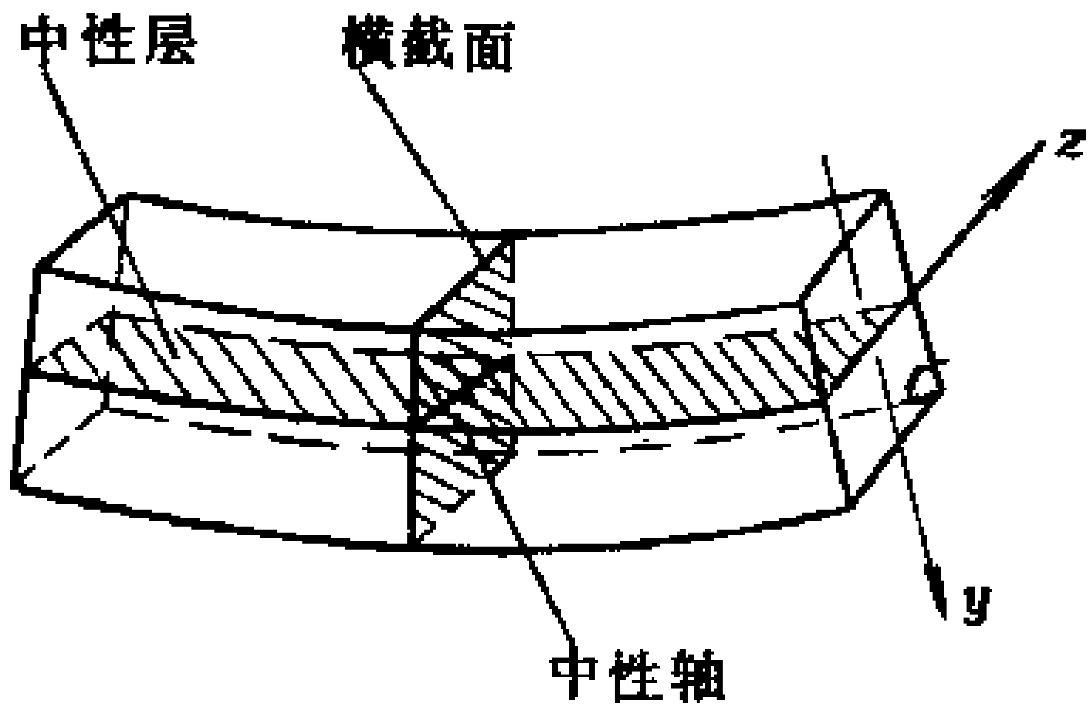
- 平面弯曲的定义：梁的轴线弯成对称平面内的一条平面曲线。





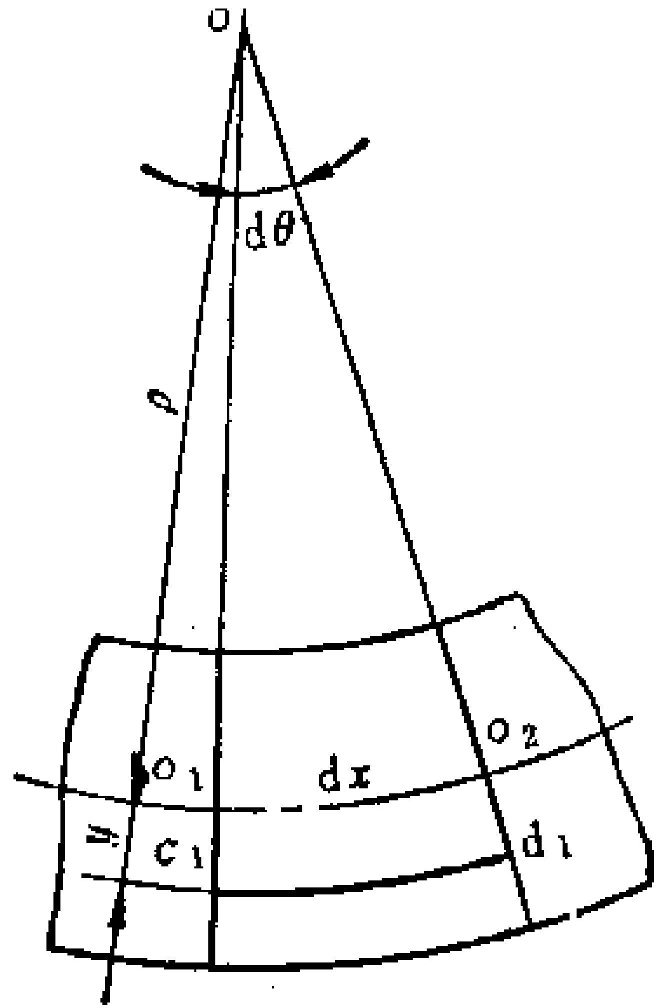
刚性平面假设

梁的所有横截面在变形过程中要发生转动，但仍保持平面，且变形后仍与梁轴线垂直。



变形量

梁的所有与轴线平行的纵向纤维都是轴向拉伸或压缩，变形量与其到中性轴的距离有关，且呈线性关系。



梁横截面上的正应力

变形几何方程：

$$\epsilon = \frac{(\rho + y)\mathrm{d}\theta - \rho\mathrm{d}\theta}{\rho\mathrm{d}\theta} = \frac{y}{\rho}$$

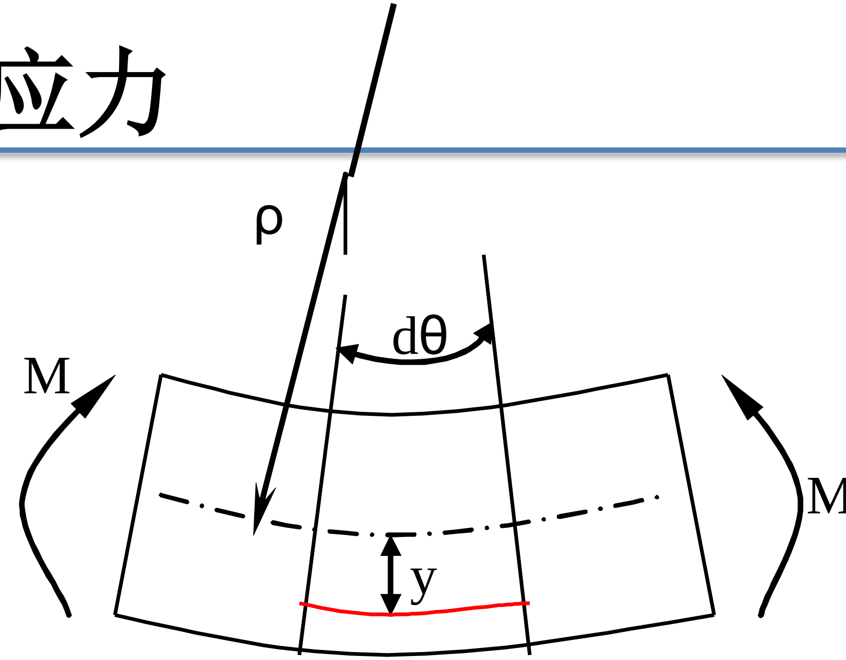
应力应变方程：

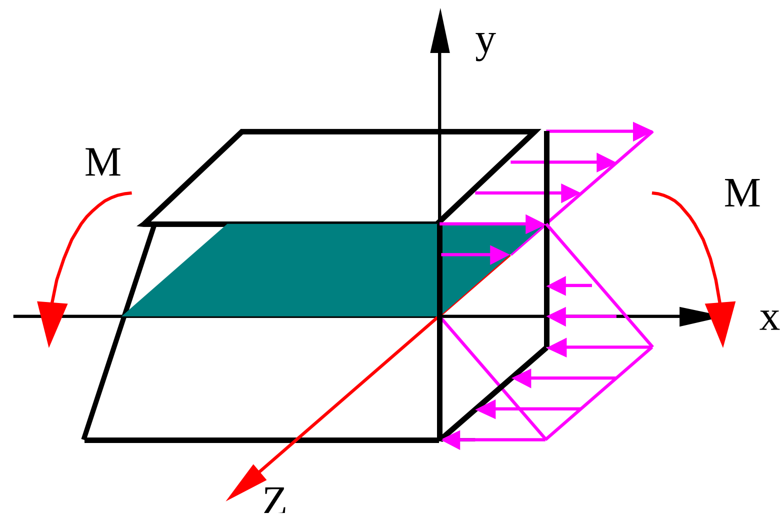
$$\sigma = E\epsilon = E\frac{y}{\rho}$$

静力平衡方程：

$$M = \int \sigma y \mathrm{d}A = \frac{E}{\rho}\int_A y^2 \mathrm{d}A$$

令： $I_z = \int_A y^2 \mathrm{d}A$





I_Z为整个横截面对中性轴的**惯性矩**
(moment of inertia)

梁横截面上的正应力

从上可推得：$\sigma = \dfrac{My}{I_z}$

截面上最大的弯曲应力：　$\sigma = \dfrac{M}{W_z}$

其中：$W_z = \dfrac{I_z}{y_{max}}$

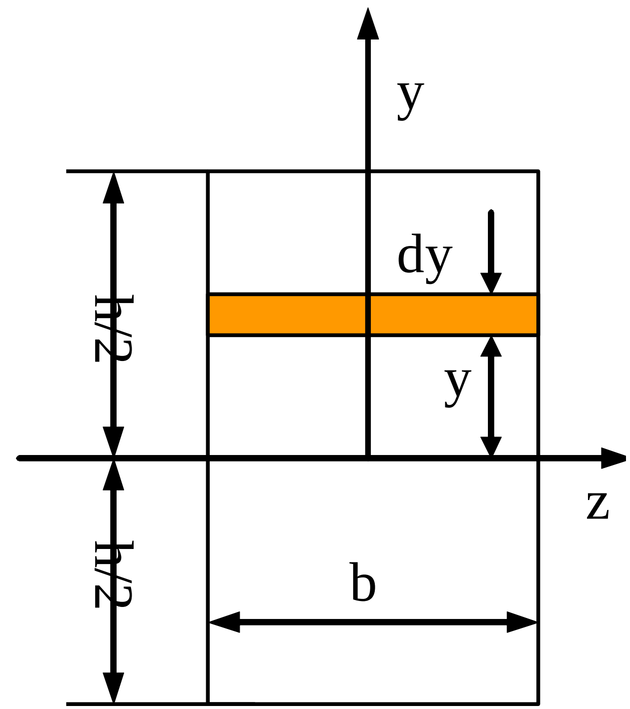
W_Z被称为抗弯截面模量（module of bending section）

常用截面惯性矩和抗弯截面模量

1. 矩形截面

惯性矩

$$I_Z = \int y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b dy = \frac{bh^3}{12}$$

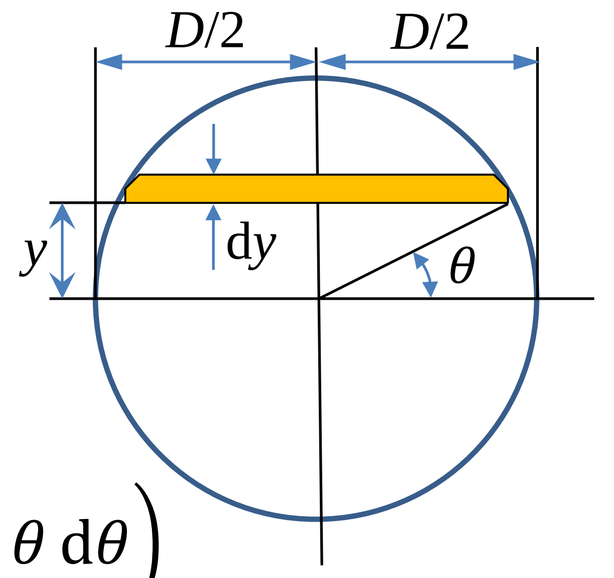


抗弯截面模量

$$W_Z = \frac{I_Z}{y_{\max}} = \frac{bh^2}{6}$$

常用截面惯性矩和抗弯截面模量

圆形截面



惯性矩

$$I_z = \int_A y^2 \mathrm{d}A = 2\int_A y^2 \frac{D}{2}\cos\theta\,\mathrm{d}y$$

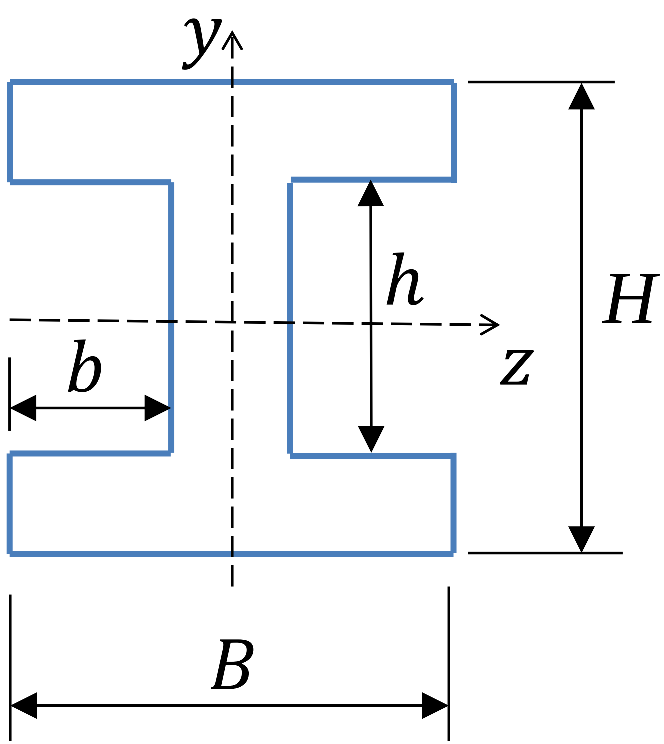
$$= 2\int_A \left(\frac{D}{2}\sin\theta\right)^2 \frac{D}{2}\cos\theta\left(\frac{D}{2}\cos\theta\,\mathrm{d}\theta\right)$$

$$= \int_{-\pi/2}^{\pi/2} \frac{D^4}{64}(1-\cos 4\theta)\mathrm{d}\theta = \frac{\pi D^4}{64}$$

抗弯截面模量

$$W_z = \frac{I_z}{y_{max}} = \frac{\pi D^3}{32}$$

工字梁截面惯性矩



$$I_z = \frac{BH^3 - 2bh^3}{12}$$

常用截面惯性矩和抗弯截面模量

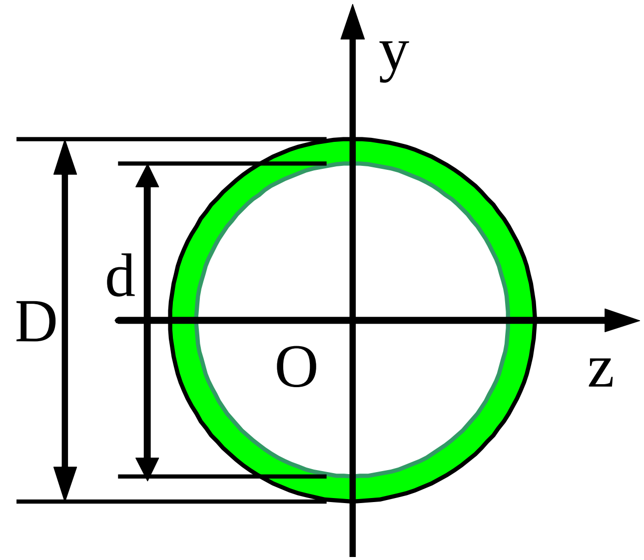
圆环截面

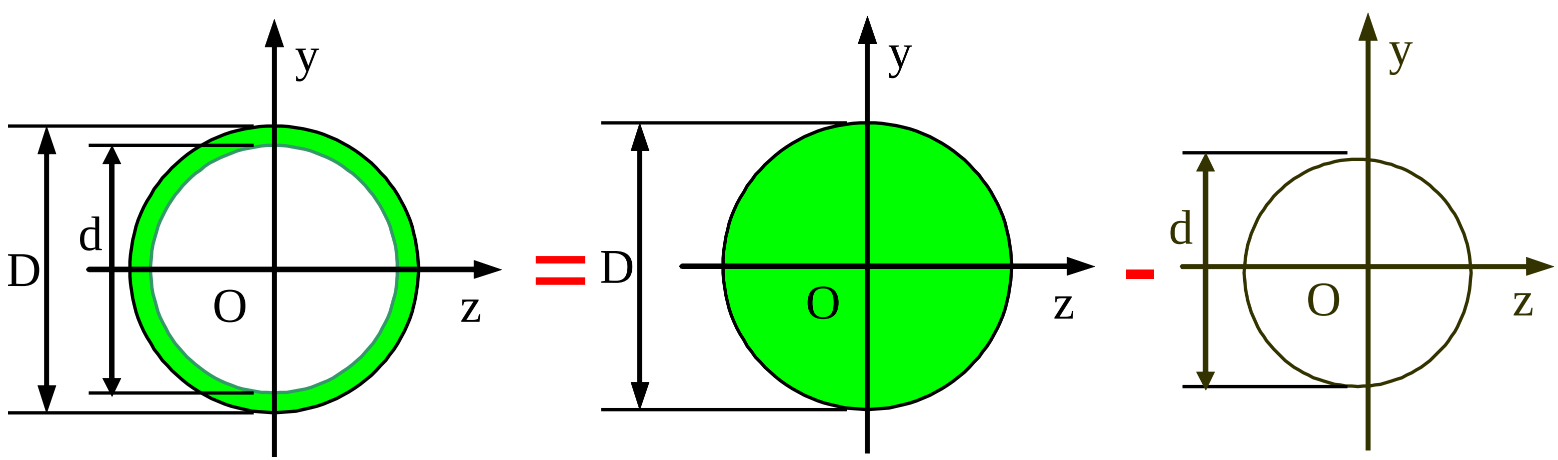
惯性矩

$$I_z = \frac{\pi D^4 - \pi d^4}{64}$$

抗弯截面模量

$$W_z = \frac{\pi(D^4 - d^4)}{32D}$$





弯曲正应力确定条件

$$\sigma_{max} = \frac{M_{max}}{W_z} \leq [\sigma]$$

根据这一确定条件可进行三项工作：

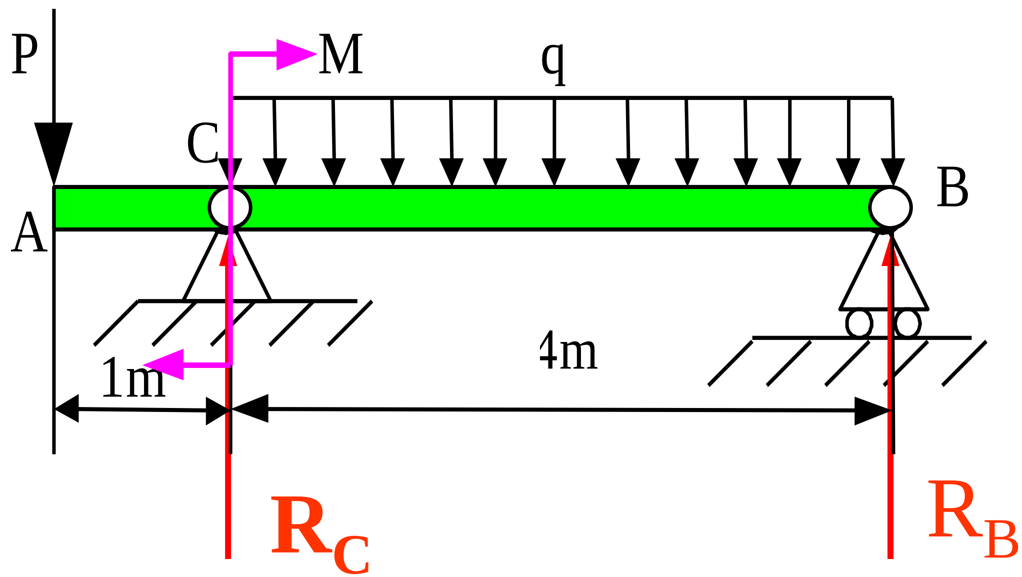
1 设计截面

2 强度校核

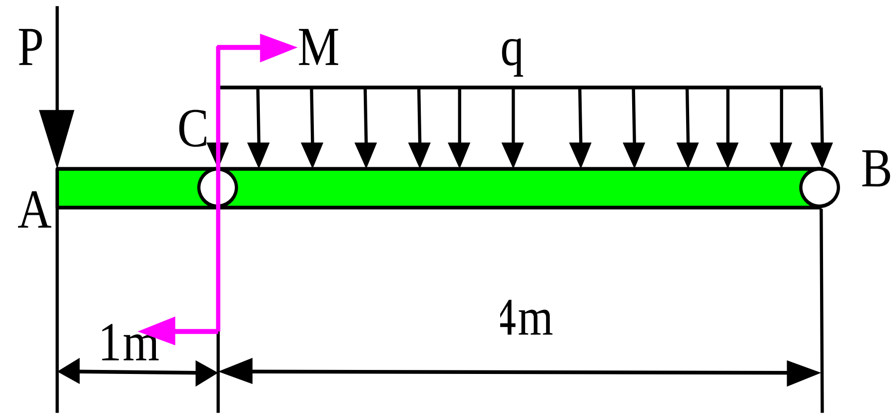
3 计算许可载荷

例

某梁由工字钢制成，材料为Q235A.F，$[\sigma]=160\text{MPa}$, $P=20\text{KN}$，$q=10\text{KN/m}$，$M=40\text{KN·m}$，试确定工字钢的型号。



求出支座反力



$$\Sigma M_c = 0$$

$$20 \times 1 - 40 + R_B \times 4 - 10 \times 4 \times 2 = 0$$

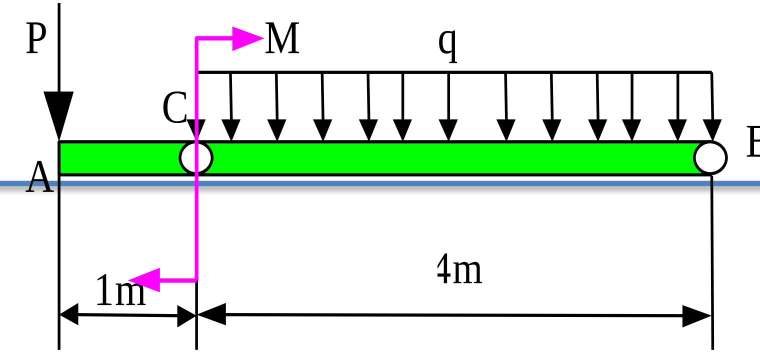
$$\Longrightarrow R_B = 25\text{KN}$$

$$\Sigma M_B = 0$$

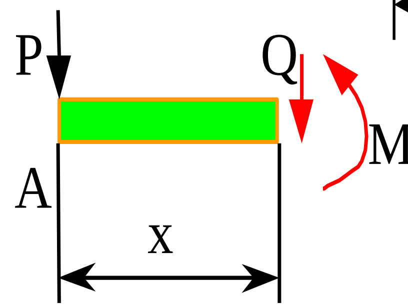
$$20 \times 5 - 40 - R_C \times 4 + 10 \times 4 \times 2 = 0$$

$$\Longrightarrow R_C = 35\text{KN}$$

求出剪力和弯矩

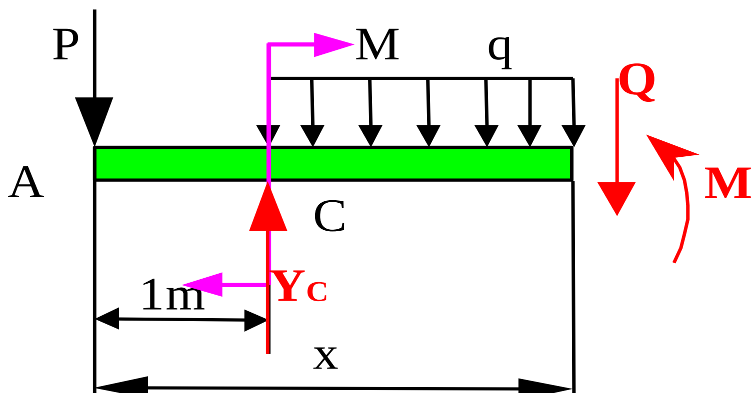


AC段：（0<x<1）



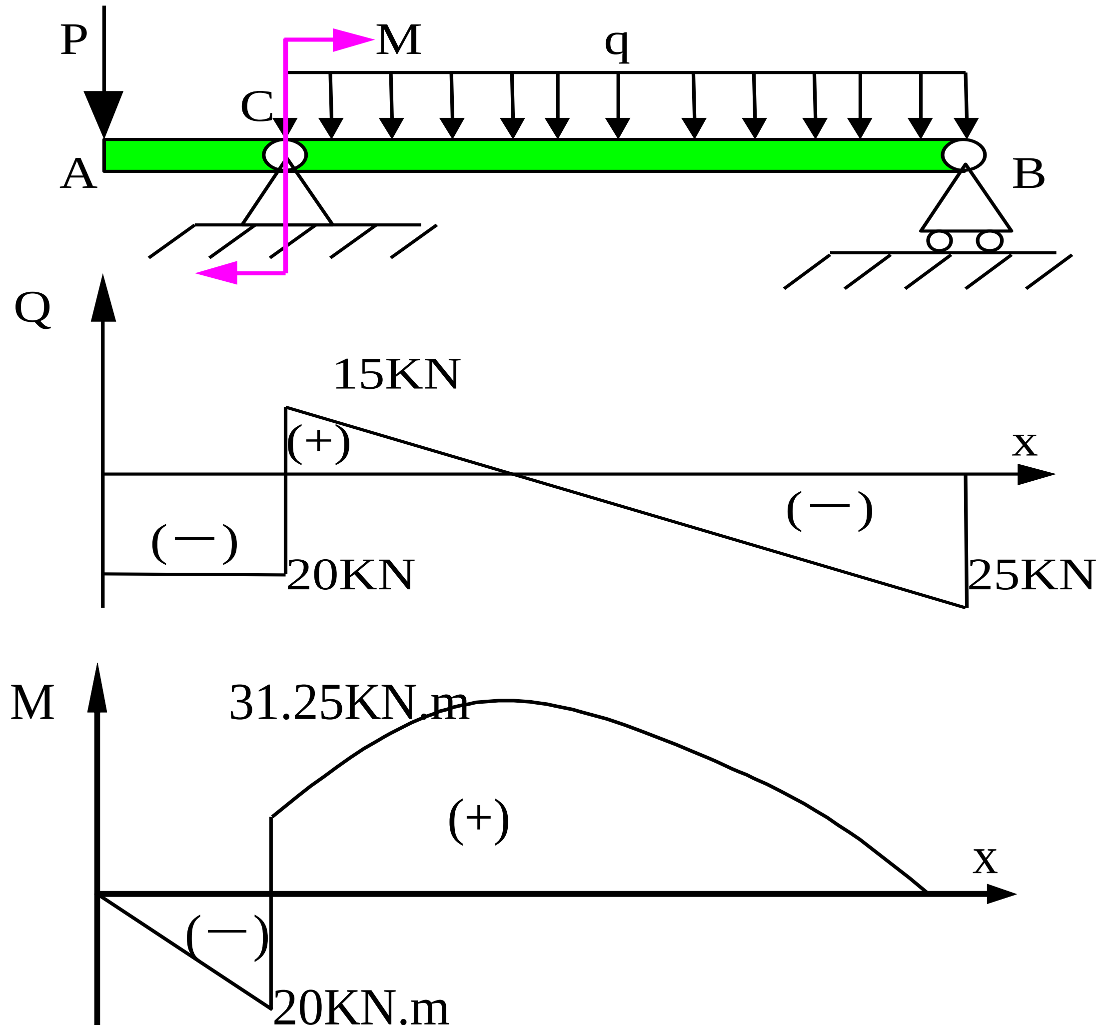
$$\begin{cases} Q_{AC} = -20 \ \ \mathrm{KN} \\ M_{AC} = -20x \ \mathrm{KN \cdot m} \end{cases}$$

CB段：（1<x<5）



$$\begin{cases} Q_{BC} = 35 - 20 - 10(x-1) = 25 - 10x \quad \mathrm{KN} \\ M_{BC} = 35(x-1) + 40 - 20x - \dfrac{5(x-1)^2}{2} \ \mathrm{KN \cdot m} \end{cases}$$

作出Q—M 图



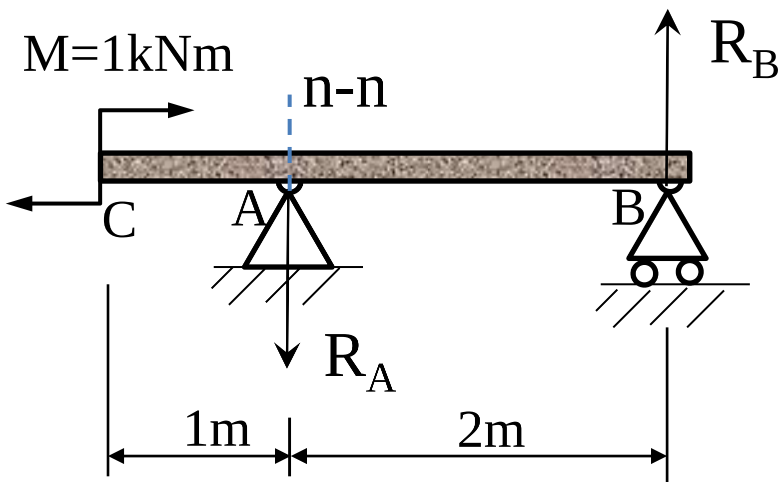
求最大W_Z

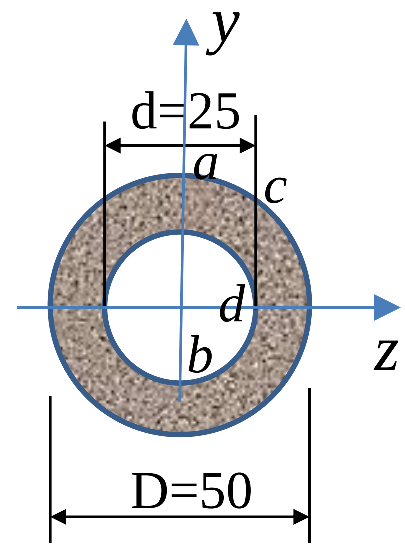
$$W_Z = \frac{M_{\max}}{[\sigma]} = \frac{31.25}{160} = 195 \times 10^3 mm^3$$

查附表，应选用20a工字钢，W_Z＝$237 \times 10^3\ mm^3$

例

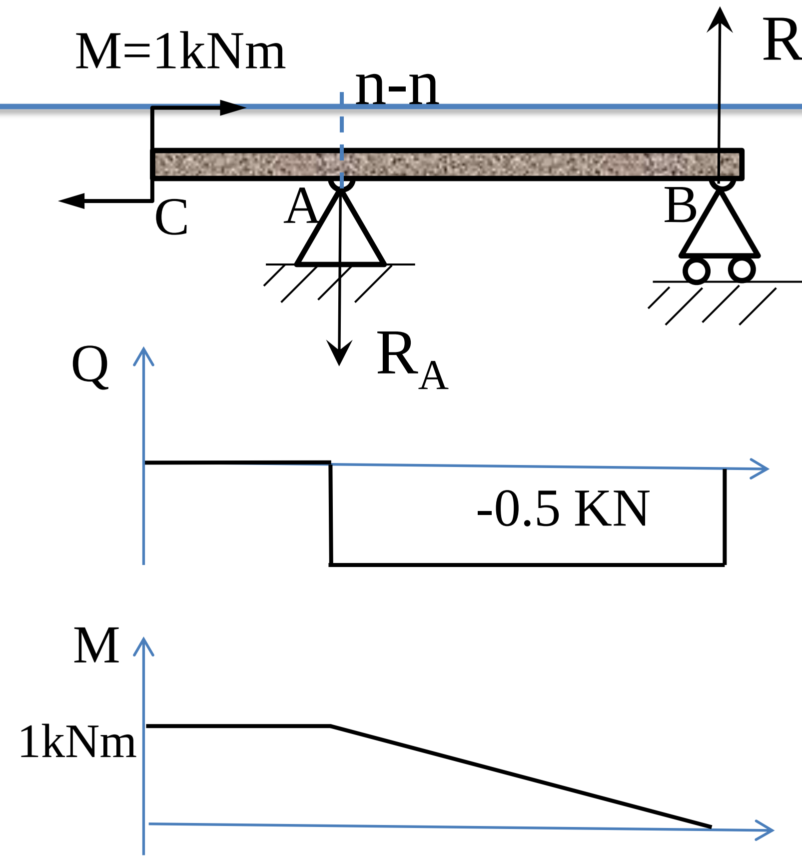
求下图所示梁*n-n*截面*a*、*b*、*c*、*d*四点的应力，长度单位mm

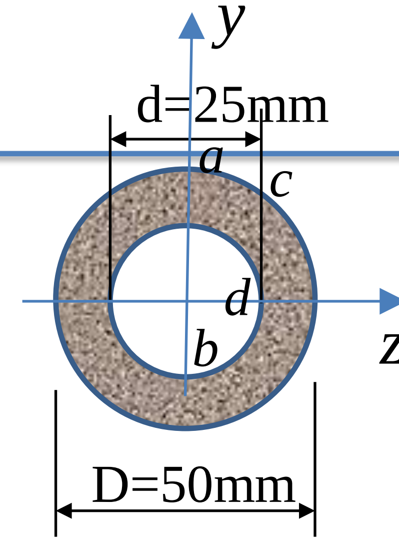




$$\Sigma M_A = 0 \Longrightarrow R_B = 0.5\ \mathrm{KN}$$

$$\Sigma F_y = 0 \Longrightarrow R_A = 0.5\mathrm{KN}$$





$$M_{n-n} = 1\ \mathrm{KN} \cdot \mathrm{m}$$

$$\sigma = \frac{My}{I_z}$$

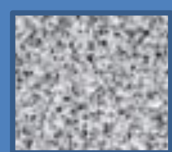
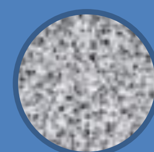
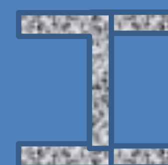
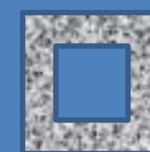
$$I_z = \frac{\pi(D^4 - d^4)}{64} = 2.88 \times 10^{-7} m^3$$

$$\sigma_a = -86.8\mathrm{MPa} \qquad \sigma_b = 43.4\mathrm{MPa} \qquad \sigma_c = -75.3\mathrm{MPa}$$

$$\sigma_d = 0\mathrm{MPa}$$

提高梁抗弯强度的主要途径

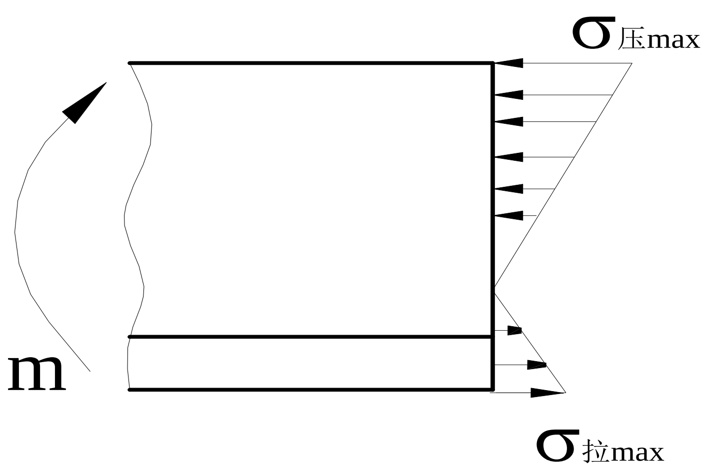
- 选择合理截面，增加抗弯模量

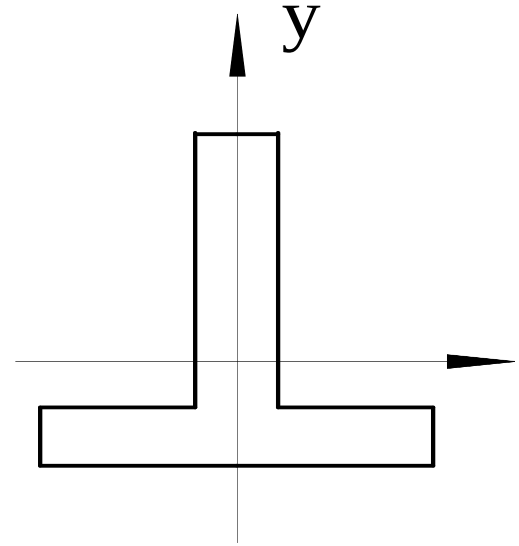
截面形状				
W_z/A	$h/6$	$d/8$	$\dfrac{BH^3-bh^3}{6H(HB-hb)}$	$\dfrac{1}{6}\left(h_1+\dfrac{h_2^2}{h_1}\right)$

$$\sigma_{max}=\frac{M_{max}}{W_z}\le[\sigma]$$

提高梁抗弯强度的主要途径

- 对于脆性材料，可采用不对称于中性轴的截面，并使中性轴更靠近受拉侧



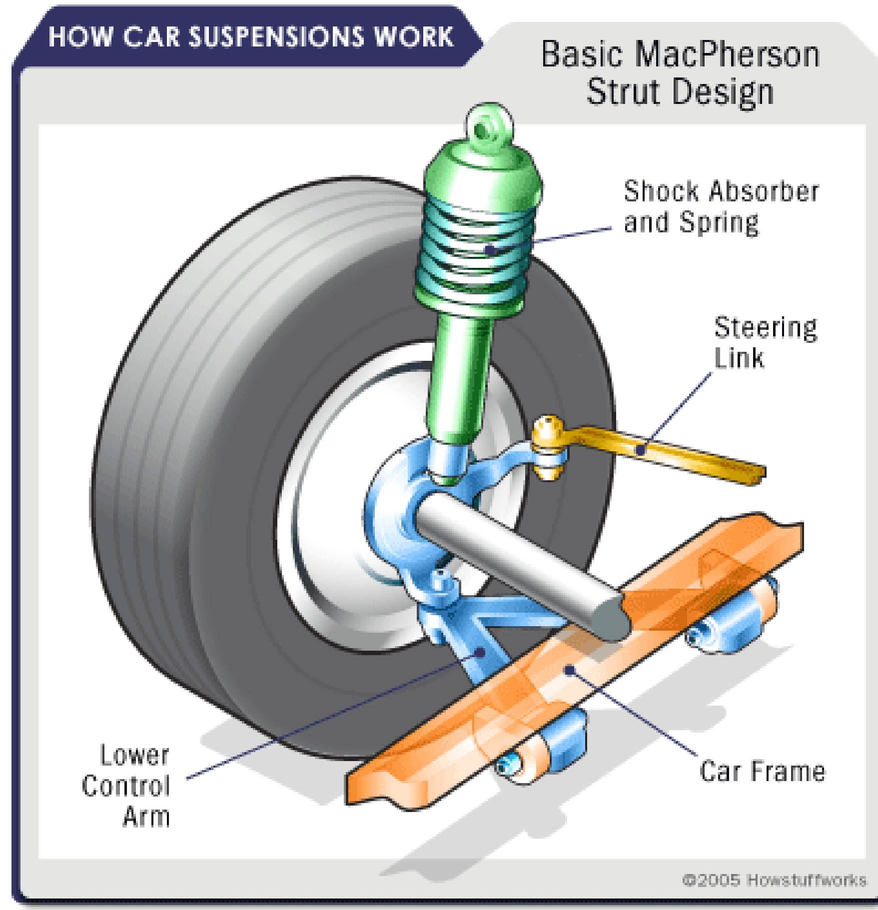


$$\frac{\sigma_{压\max}}{\sigma_{拉\max}}=\frac{\dfrac{My_1}{I_z}}{\dfrac{My_2}{I_z}}=\frac{[\sigma_{压}]}{[\sigma_{拉}]}$$

即 $$\frac{y_1}{y_2}=\frac{[\sigma_{压}]}{[\sigma_{拉}]}$$

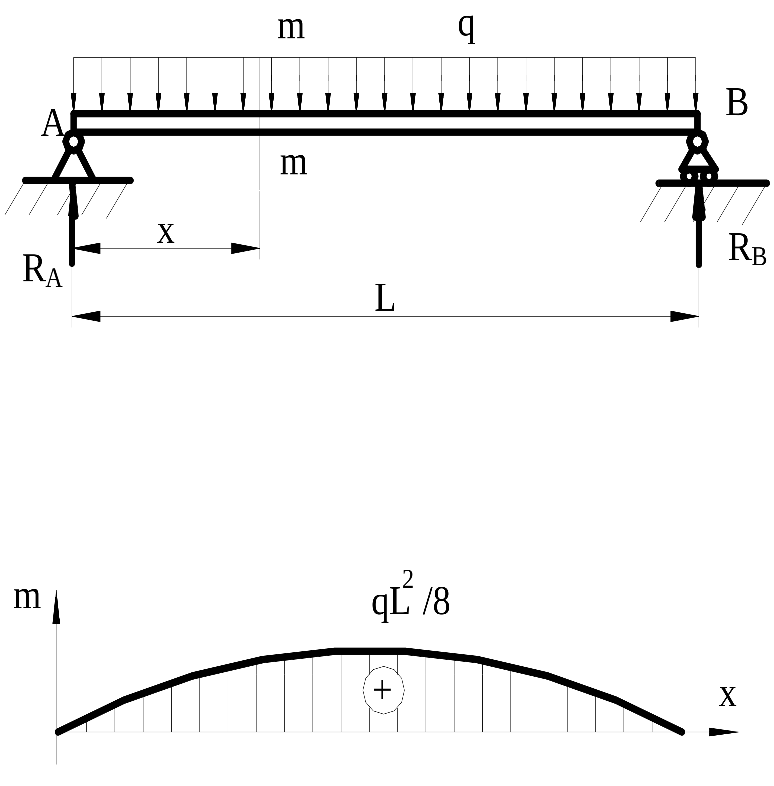
提高梁抗弯强度的主要途径

- 采用变截面梁或等强度梁

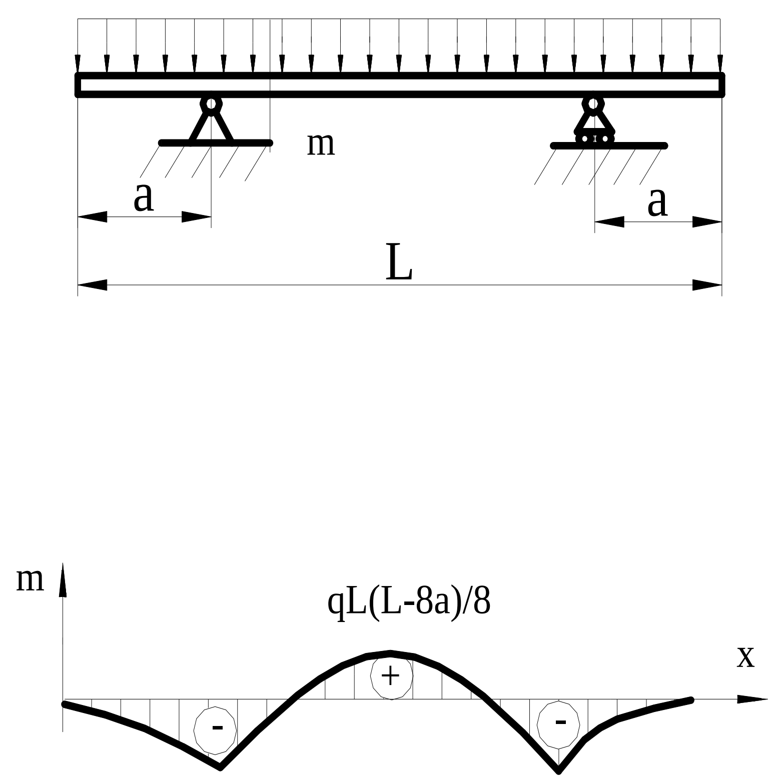


提高梁抗弯强度的主要途径

- 合理布置支座和载荷作用位置，减少梁中最大弯矩M_{max}



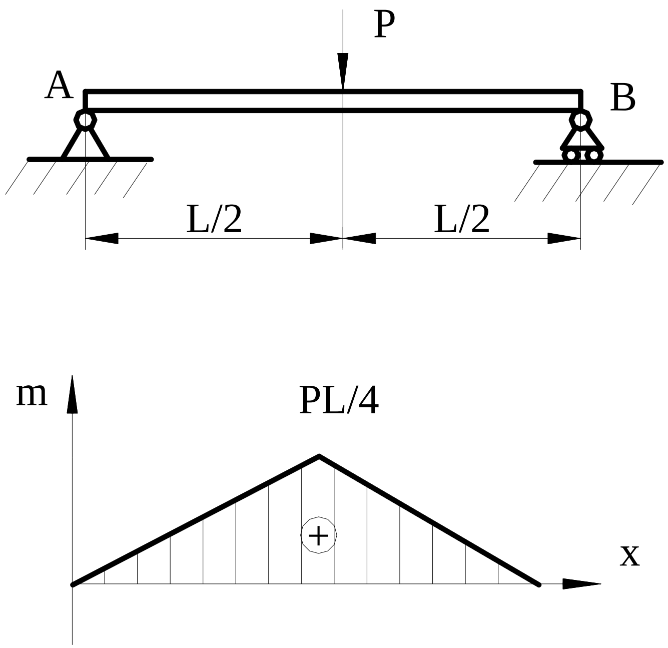
(a)



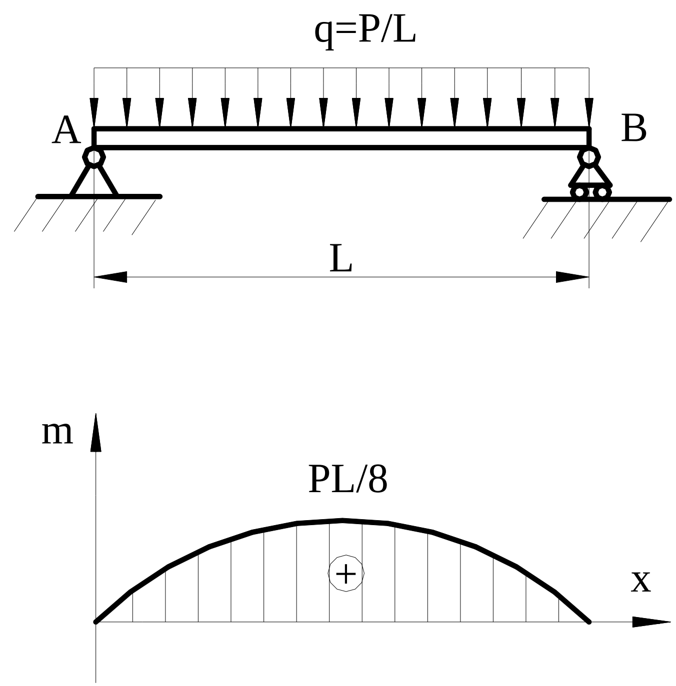
(b)

提高梁抗弯强度的主要途径

- 改变载荷的分布



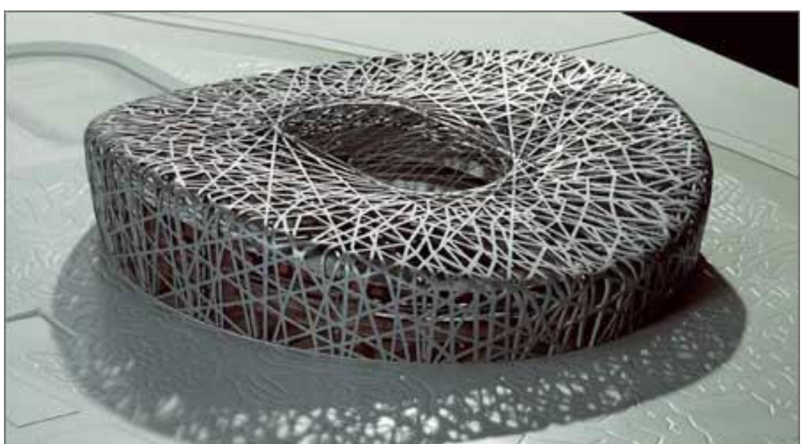
(a)



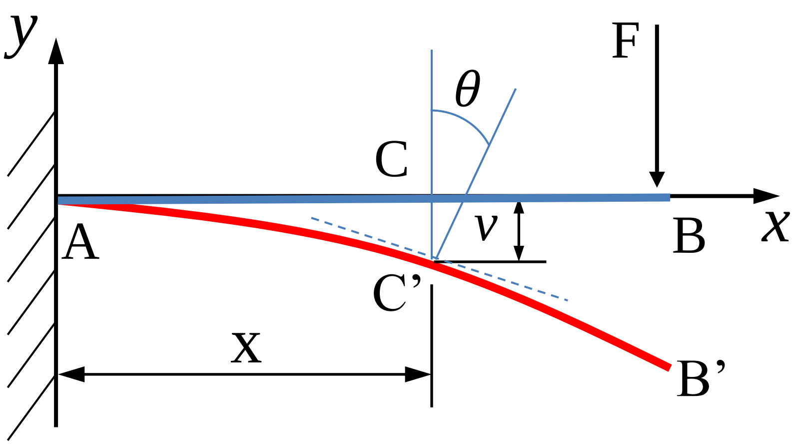
(b)

梁的弯曲

- 屋顶的设计必须考虑弯曲变形



弯曲变形



$$v = f(x)$$

$$\frac{1}{\rho} = \frac{M}{EI}$$

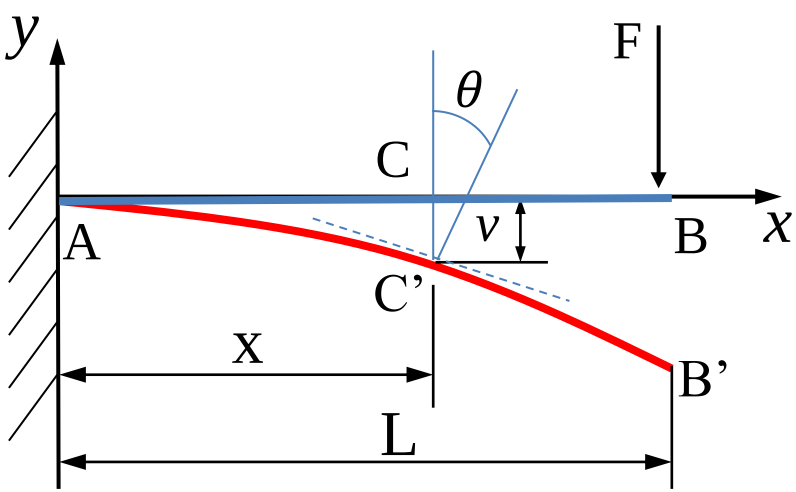
$$\frac{1}{\rho} = \frac{\mathrm{d}^2 v/\mathrm{d}x^2}{[1 + (\mathrm{d}v/\mathrm{d}x)^2]^{\frac{3}{2}}}$$

$$\frac{M}{EI} = \frac{\mathrm{d}^2 v/\mathrm{d}x^2}{[1 + (\mathrm{d}v/\mathrm{d}x)^2]^{\frac{3}{2}}} \quad \Longrightarrow \quad \frac{M}{EI} = \frac{\mathrm{d}^2 v}{\mathrm{d}x^2}$$

$$EI\frac{\mathrm{d}v}{\mathrm{d}x} = EI\theta = \int M\mathrm{d}x + C$$

$$EIv = \iint M\,\mathrm{d}x\mathrm{d}x + \int C\mathrm{d}x + D$$

例子



1. 列出弯矩方程

$$M = F(L - x)$$

2. 建立挠曲线微分方程

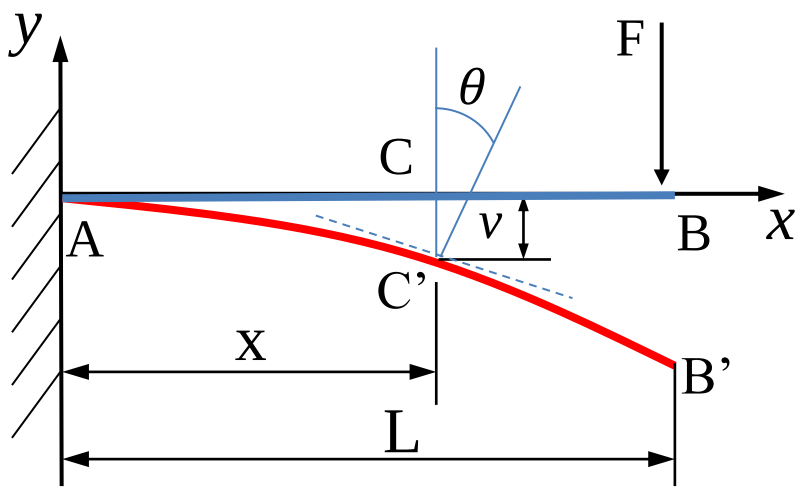
$$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = \frac{M}{EI} = \frac{F(L - x)}{EI}$$

两次积分后得

$$EI\frac{\mathrm{d}v}{\mathrm{d}x} = EI\theta = \int F(L - x)\,\mathrm{d}x + C = FLx - \frac{1}{2}Fx^2 + C$$

$$EIv = \frac{1}{2}FLx^2 - \frac{1}{6}Fx^3 + Cx + D$$

例子



3. 确定积分常数

当$x=0$时，$v=0$，

因此$C=0$

当$x=0$时，$v'=0$，

因此$D=0$

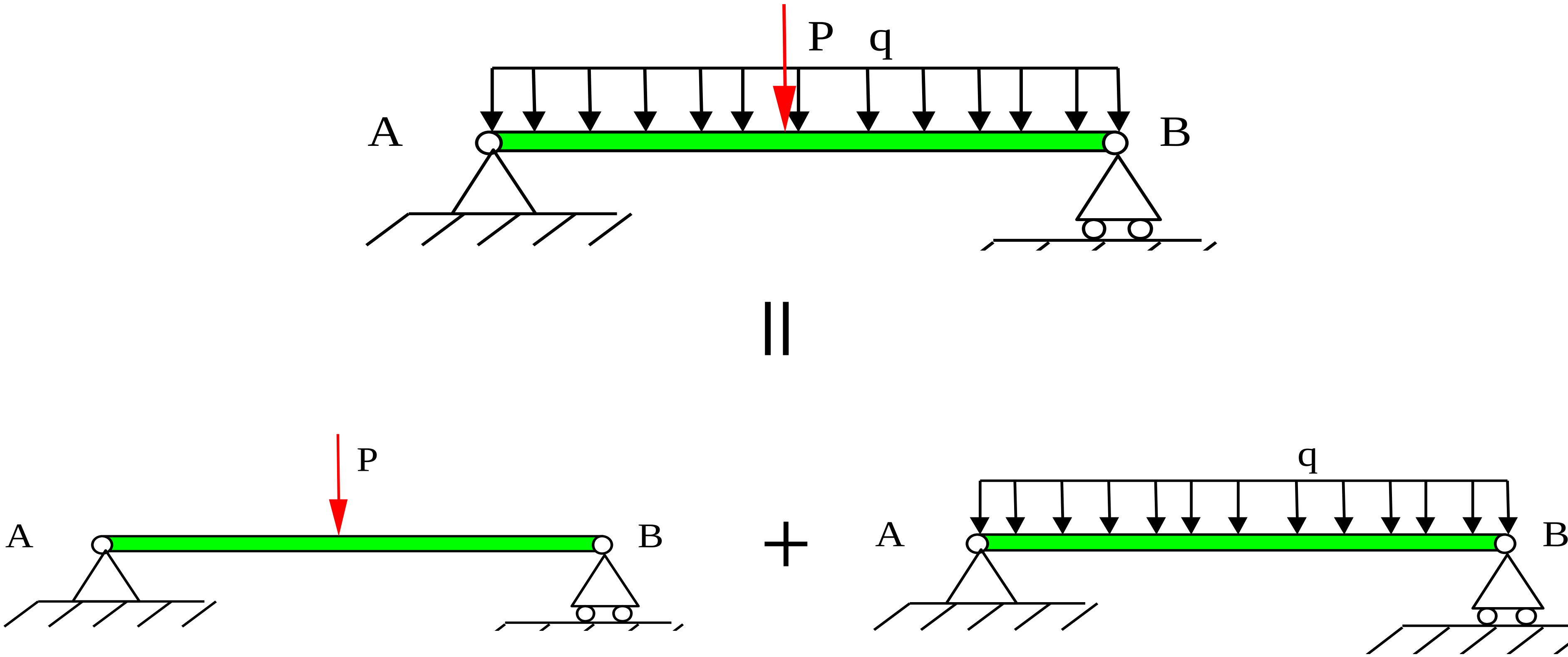
4. 建立转角和挠曲线方程

$$\theta=\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{Px}{2EI}(2L-x) \qquad v=\frac{Px^2}{6EI}(3l-x)$$

5. 求出最大转角和挠度

$$x=L\text{ 时}\quad \theta_{max}=\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{PL^2}{2EI} \qquad v_{max}=\frac{PL^3}{3EI}$$

叠加法求梁的变形



梁的刚度校核

$$v_{max} \leq [v]$$

$$\theta_{max} \leq [\theta]$$