For the flame shown in Figure 9-3, calculate the relative intensity of the 766.5-nm emission line for potassium at the following heights above the flame (assume no ionization):

(a)2.0 cm

(b) 3.0cm

(c) 4.0cm

(d) 5.0cm

step1/8

(a)Ratio between number of excited and unexcited particles is calculated using equation

$$\frac{N_j}{N_o} = \frac{g_j}{g_o} \exp\left(\frac{-E_j}{KT}\right)$$

Where N_j and N_o are number of particles in excited state and ground state. g_j and g_o are statistical weights for $4s$ and $4p$ quantum state, K is Boltzmann constant and T is absolute temperature. Wavelength for potassium emission line corresponding to $3p \to 3s$ transition is 766.5 nm. Energy corresponding to line 766.5 nm is calculated as

$$E = \frac{hc}{\lambda}$$

Substituting known values, we get;

$$\begin{aligned} E_j &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{10}}{766.5 \times 10^{-7}} \\ &= 2.59 \times 10^{-19}\ \mathrm{J} \end{aligned}$$

step2/8

Temperature of flame at 2 cm height is 1,700°C .Convert temperature in degree centigrade into kelvins as

$$\text{Temperature in kelvin} = T\ ^\circ\text{C} + 273$$

Substituting $1,700\ ^\circ\text{C}$ we get

$$\begin{aligned}\text{Temperature in kelvin} &= 1,700\ ^\circ\text{C} + 273 \\ &= 1,973\ \text{K}\end{aligned}$$

Statistical weights for 3*s* and 3*p* quantum states are 2 and 6 respectively so,

$$\begin{aligned}\frac{g_j}{g_o} &= \frac{6}{2} \\ &= 3\end{aligned}$$

Substituting values of E_j, $\frac{g_j}{g_o}$, K and temperature as 1,973 K we get

$$\begin{aligned}\frac{N_j}{N_o} &= 3\ \exp\left(\frac{-2.59 \times 10^{-19}\ \cancel{J}}{1.38 \times 10^{-23}\ \cancel{J/K} \times 1,973\ \cancel{K}}\right) \\ &= 3 \times 7.39 \times 10^{-5} \\ &= 2.22 \times 10^{-4}\end{aligned}$$

Therefore, ratio of number of particles in 3*p* excited state to ground state at 2 cm height of flame at 1,973 K for potassium atom is 2.22×10^{-4}

Relative intensity of emission line of potassium at 2 cm height of flame is assumed as 1.0 and relative intensities of emission lines at other heights are calculated by taking ratio.

Therefore, relative intensity of emission line at 766.5 nm of potassium at 2 cm height of flame is $\boxed{1.0}$.

step3/8

(b) Temperature of flame at 3 cm height is 1,863°C .Convert temperature in degree centigrade into kelvins as

$$\text{Temperature in kelvin} = T\ ^\circ\text{C} + 273$$

Substituting $1{,}863^\circ\text{C}$ we get

$$\begin{aligned}\text{Temperature in kelvin} &= 1{,}863\ ^\circ\text{C} + 273 \\ &= 2{,}136\ \text{K}\end{aligned}$$

Substituting values of E_j, $\frac{g_j}{g_o}$, K and temperature as 2136 K, we get;

$$\begin{aligned}\frac{N_j}{N_o} &= 3\ \exp\left(\frac{-2.59 \times 10^{-19}\ \cancel{J}}{1.38 \times 10^{-23}\ \cancel{J/K} \times 2{,}136\ \cancel{K}}\right) \\ &= 3 \times 1.53 \times 10^{-4} \\ &= 4.59 \times 10^{-4}\end{aligned}$$

Therefore, ratio of number of particles in 3*p* excited state to ground state at 3cm height of flame at 2,136 K for potassium atom is 4.59×10^{-4}.

step4/8

Relative intensity of emission line at 766.5 nm at 3 cm height of flame is calculated by assuming relative intensity at 2 cm height as 1.00.

$$\frac{I_x}{I_y} = \frac{\left(\frac{N_j}{N_o}\right)_{3cm}}{\left(\frac{N_j}{N_o}\right)_{2cm}}$$

$$= \frac{4.59 \times 10^{-4}}{2.22 \times 10^{-4}}$$

$$= 2.07$$

Therefore, relative intensity of emission line at 766.5 nm of potassium at 3 cm height of flame is $\boxed{2.07}$.

step5/8

(c) Temperature of flame at 4 cm height is 1820°C .Convert temperature in degree centigrade into kelvins as

$$\text{Temperature in kelvin} = T°\text{C} + 273$$

Substituting $1{,}820°\text{C}$ we get

$$\begin{aligned}\text{Temperature in kelvin} &= 1{,}820°\text{C} + 273 \\ &= 2{,}093\ \text{K}\end{aligned}$$

Substituting values of $E_j, \frac{g_j}{g_o}, K$ and temperature as 2093 K, we get;

$$\begin{aligned}\frac{N_j}{N_o} &= 3\ \exp\left(\frac{-2.59 \times 10^{-19}\ \cancel{J}}{1.38 \times 10^{-23}\ \cancel{J/K} \times 2{,}093\ \cancel{K}}\right) \\ &= 3 \times 1.275 \times 10^{-4} \\ &= 3.83 \times 10^{-4}\end{aligned}$$

Therefore, ratio of number of particles in $3p$ excited state to ground state at 4cm height of flame at 2,093 K for potassium atom is 3.83×10^{-4}

step6/8

Relative intensity of emission line at 766.5 nm at 4 cm height of flame is calculated by assuming relative intensity at 2 cm height as 1.0.

$$\begin{aligned}\frac{I_x}{I_y} &= \frac{\left(\frac{N_j}{N_o}\right)_{4\text{cm}}}{\left(\frac{N_j}{N_o}\right)_{2\text{cm}}}\\ &= \frac{3.83 \times 10^{-4}}{2.22 \times 10^{-4}}\\ &= 1.72\end{aligned}$$

Therefore, relative intensity of emission line at 766.5 nm of potassium at 4 cm height of flame is $\boxed{1.72}$

step7/8

(d)Temperature of flame at 5 cm height is 1,725°C .Convert temperature in degree centigrade into kelvins as

$$\text{Temperature in kelvin} = T°\text{C} + 273$$

Substituting $1{,}725\ °\text{C}$ we get

$$\begin{aligned}\text{Temperature in kelvin} &= 1{,}725°\text{C} + 273\\ &= 1{,}998\ \text{K}\end{aligned}$$

Substituting values of $E_j, \frac{g_j}{g_o}, K$ and temperature as 1,998K we get

$$\begin{aligned}\frac{N_j}{N_o} &= 3\ \exp\left(\frac{-2.59 \times 10^{-19}\ \cancel{J}}{1.38 \times 10^{-23}\ \cancel{J/K} \times 1{,}998\ \cancel{K}}\right)\\ &= 3 \times 8.33 \times 10^{-5}\\ &= 2.50 \times 10^{-4}\end{aligned}$$

Therefore, ratio of number of particles in 3*p* excited state to ground state at 5cm height of flame at 1,998 K for potassium atom is 2.50×10^{-4}

step8/8

Relative intensity of emission line at 766.5 nm at 5 cm height of flame is calculated by

assuming intensity of line at 2 cm height as 1.0

$$\begin{aligned}\frac{I_x}{I_y} &= \frac{\left(N_j/N_o\right)_{5\mathrm{cm}}}{\left(N_j/N_o\right)_{2\mathrm{cm}}} \\ &= \frac{2.50 \times 10^{-4}}{2.22 \times 10^{-4}} \\ &= 1.13\end{aligned}$$

Therefore, relative intensity of emission line at 766.5 nm of potassium at 5 cm height of flame is $\boxed{1.13}$.