Multi Cross-correlation Analysis in a Multi-channel EEG applied in Motor Activity (Real/Imaginary)

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Abstract

Neste artigo nós aplicamos o coeficiente de múltipla correlação cruzada para analizar múltiplas séries temporais de EEG com um total de 109 sujeitos que realizaram 4 tarefas em atividades motoras (real e imaginárias). Escolhidos 4 canais específicos (dois frontais e dois parentais), cada um deles identificado pelo sua referida série temporal, foi possível identificar suas múltiplas correlações. Como resultado, o coeficiente de múltipla correlação cruzada, mostrou que cada sujeito tem sua própria assinatura nas seus respectivas tarefas, e que na média global não há grandes diferrenças entre os tipos de tarefas (real e imagnário). Para este valor global, o coeficiente de múltipla correlação cruzada identificou que os canais frontais são aqueles que têm maiores valores e os menores desvios padrões, principalmente nas escalas temporais em torno de $0.42s$.

Portanto, pelo tipo de análise estatística envolvida (novidade), pela sua quantidade (base robusta de dados), pela qualidade dos resultados obtidos (repositório e filmes) e pela sua importância, o estudo aqui apresntado pode ser o início de uma nova era na análise de multiplas correlações cruzadas em EEG.

Introduction

The electroencephalogram (EEG) is, in short, a technic that reads electrical signals from the brain activity with the use of sensors placed in the scalp of the patient and makes sense of this data. Impulses are amplified and recorded over time in parallel, generating one time series for each sensor. The EEG equipment measure the electric potential difference from each sensor position in relation with a reference sensor, usually placed in the ear lobe. Although the EEG is almost a centenary technique, in recent decades EEG has addressed new problems as brain-triggered neurorehabilitation treatments, experimental psychology or even computational neuroscience, due to it's versatility and accessibility combined with the advances in signal processing [1].

FALAR DO PLosOne Florencio [2], e o novo aqui [3]
...importante isto!!!

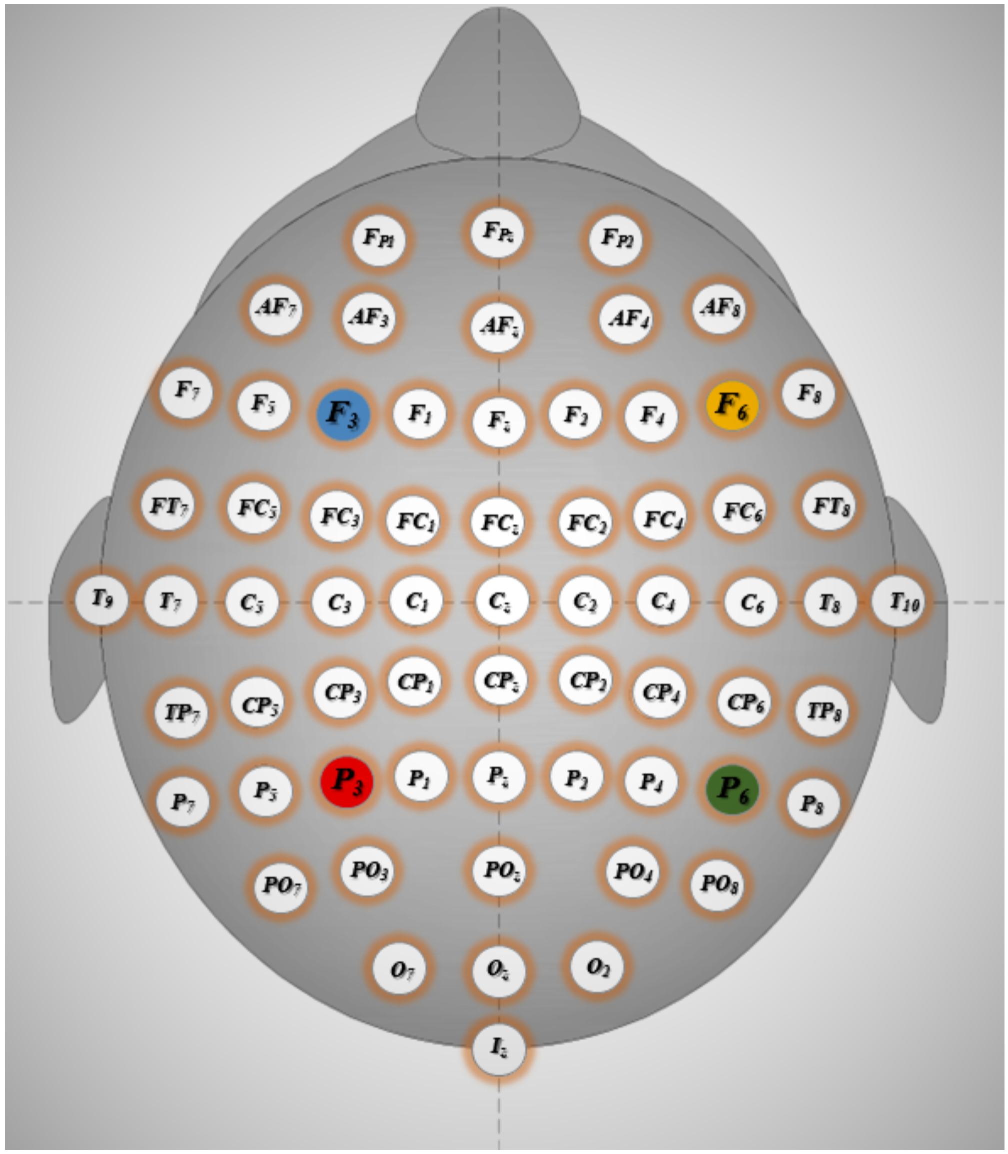

Fig 1. Electrodes positions based on international 10-20 system for 64 channel. The circles: F_3 (blue), F_6 (yellow), P_3 (red), P_6 (green) identify the channels used for multiple cross-correlation analysis.

The four channels are alternately picked as dependent variable and the correlation against the other three are calculated by the application of the Detrended Multiple Cross-Correlation Coefficient (DMC_x^2) [4]

In the next sections we will present the dataset; the methodology used to analyze the data, including pre-processing strategies and criteria, and the DMC_x^2 method; the results, statistics and data visualization for the analyzed populations and individual results for randomly selected subjects. The discussion of the results and the conclusions are presented in sequence. In the supporting materials, a link to access all the calculations and data visualization for all the experiment subjects is available.

Materials and Method

Materials

The recordings used are available at the Physionet on-line databank [5]. The data is originated from a study that perform 14 experiments on a population of 109 subjects, Using 64 electrodes to record the brain signals.

Falar do BCI2000.... [6].

The first 2 experiments are baseline references, the subjects where resting with eyes opened end the second with eyes closed (one minute for each). The other four experiments are a combination of two categories with two possible options each $2 \times 2 \ = 4$. In general, the experiments consist of making the subjects react over visual stimulus: a target that appears on a screen. One category is about the target position, one option is a target that will appear on the left or the right of the screen, in the other, the target will appear on the Down or on the Down of the screen. The second category determines if the subject will actually execute a body action related to the target position (Real) or if the corresponding action will be just imagined (Imaginary) by the subject. The tasks, with duration of two minutes, are described below:

- **Task 1**: a target appears on either the left or the right side of the screen. The subject (*real*) opens and closes the corresponding fist until the target disappears. Then the subject relaxes;
- **Task 2**: a target appears on either the left or the right side of the screen. The subject (*imaginary*) opening and closing the corresponding fist until the target disappears. Then the subject relaxes;
- **Task 3**: a target appears on either the Top or the Down of the screen. The subject (*real*) opens and closes either both fists (if the target is on Top) or both feet (if the target is on the Down) until the target disappears. Then the subject relaxes;
- **Task 4**: a target appears on either the Top or the Down of the screen. The subject (*imaginary*) opening and closing either both fists (if the target is on Top) or both feet (if the target is on the Down) until the target disappears. Then the subject relaxes.

Table 1 presents witch Task is carried in each experiment number. Every subject executes the four tasks three times. Experiments 3, 7 and 11 applies Task 1 to the subjects, experiments 4, 8 and 12, Task 2, experiments 5, 9 and 13, Task 3 and experiments 6, 10 e 14 uses Task 4.

For each Task, three experiments are performed, numbered by:

- Task 1 with experiments (3;7;11);
- Task 2 with experiments (4;8;12);
- Task 3 with experiments (5;9;13);
- Task 4 with experiments (6;10;14);

as presented in the Table 1.

The electrodes in all tasks and experiments where located as shown in Fig. 1. The colored circles in Fig. 1 correspond to the positions of the electrodes channels whose recordings were used in our analyses: F_3 (blue) and F_6 (yellow), on the frontal region of the head, and P_3 (red) and P_6 (green), on the parietal region of the head.

The Physionet on-line databank is publicly available at (with Open Data Commons Attribution License (ODC-By) v1.0):

Table 1. The experiment number and the activity executed: Two one-minute baseline runs (one with eyes open, one with eyes closed), and three two-minute of the four Tasks.

Activity\Experiment	1	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Baseline 1 (eyes open)	0													
Baseline 2 (eyes closed)		0												
Task 1: Real (L/R)			x				x				x			
Task 2: Imag (L/R)				x				x				x		
Task 3: Real (T/D)					x				x				x	
Task 4: Imag (T/D)						x				x				x

`https://physionet.org/content/eegmmidb/1.0.0/`

, presents recordings of the EEG experiments described in previous section. The data is provided in EDF (European data format) files. The files of all the experiments for every subject where downloaded using a web scraping script created by the authors using Python and the package Beautiful Soap.

The EDF files where opened using Python package *pyedflib* and translated into text files. In EEG experiments, usually the end of the recordings is filled with a sequence of zeroes, corresponding to the time gap between the shooting down of the EEG machine and the recording system. In this pre-processing stage, all the tailing zeroes sequences are eliminated from the records. To properly apply the DMC_x^2 calculations and the intended comparisons between experiments end subjects, all the time series must have the same length.

AQUI ESTAMOS FALANDO DO SUJEITO S106 (QUE FOI DESCARTADO)

The research find out the the experiment number 5 (Top/Down, Real) conducted with subject S106 has only 5808 valid recorded points.

Falar isto aqui???? The second smallest time series has 15742 valid records (subject: S100, experiment: 12 -Left/Right, Imaginary).

The interval between each recorded value in this equipment is 6.25 ms, and the recordings on experiment 5 of subject S106 is $36.3s$.

The duration is way smaller then the expected 2 minutes and the series is relatively small to the application os the DMC_x^2 method. Cutting all the subjects data to this duration implies in a great loss of data. The second smaller series of 15742 represents a total duration of 98.3875 s. The value was considered adequate to the DMC_x^2 method and the duration is about 82% of the expected duration. The decision was to eliminate subject S106 from the experiment and to cut all time series at recording point 15742. resulting in a total number of 108 subjects with all 12 experiments per subject lasting 98.3875 s.

Depois de fazermo toda a parte de mineração e tratamento dos dados, utilizamos a nova metodologia (abaixo exemplidas) para nossa análise de multi-correlação, como veremos agora a seguir....

Method

The DMC_x^2 is part of a group of new statistical methods applied to long-range time series. The DFA method [7] was proposed to identify self-affinity in of a signal. For long-memory series, the value of the F_{DFA} usually grow with the time scale n denoting a power-law, described by the auto-correlation exponent α_{DCCA}. This method was already used to evaluate the self-affinity of the same channels of this same dataset analyzed in this study, concluding xthat the amplitude of fluctuation tends to be larger in the frontal channels (F_3 and F_6), if we compare with the channels located in the parietal region of the brain (P_3 and P_6) [8]. Other works also applied DFA to evaluate different aspects of EEG signals [9] [10]....Mais citações

To analysis the dataset, this study uses the Detrended Multiple Cross-Correlation Coefficient (DMC_x^2) [4]. This coefficient, is a generalization of the ρ_{DCCA} [11], and calculates the correlation of one time serie $\{Y\}$, defined dependent variable in relation of a number k of time series $\{X_1\}$, $\{X_2\}$, $\{X_3\}$, $\ldots$, $\{X_k\}$, taken as independent variables.

$$DMC_x^2 \equiv \rho_{Y,X_i}(n)^T \times \rho^{-1}(n) \times \rho_{Y,X_i}(n) \tag{1}$$

Matrix $\rho^{-1}(n)$ is presented in Eq. 2. The main diagonal is filled with ones (1) because the ρ_{DCCA} correlation among a time serie and itself is always one. The ρ_{DCCA} correlation is independent of the order of the two time series in the equation (commutative property). Since ρ_{X_a,X_b} is equal to ρ_{X_b,X_a}, it's important to point that matrix $\rho(n)$ is symmetric with respect to the main diagonal.

$$\rho^{-1}(n) = \begin{pmatrix} 1 & \rho_{X_1,X_2}(n) & \rho_{X_1,X_3}(n) & \ldots & \rho_{X_1,X_k}(n) \\ \rho_{X_2,X_1}(n) & 1 & \rho_{X_2,X_3}(n) & \ldots & \rho_{X_2,X_k}(n) \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \rho_{X_k,X_1}(n) & \rho_{X_k,X_2}(n) & \rho_{X_k,X_3}(n) & \ldots & 1 \end{pmatrix}^{-1} \tag{2}$$

Here,

$$\rho_{Y,X_k}(n)^T = [\rho_{Y,X_1}(n), \rho_{Y,X_2}(n), \cdots, \rho_{Y,X_k}(n)] \tag{3}$$

is the vector of detrended cross-correlations between the predictor variables (independent variables) and the target variable (dependent variable).

The calculation of the inverse matrix is the most time consuming step on the method. Replacing the equal values ($\rho_{X_a,X_b} = \rho_{X_b,X_a}$) the calculus of the DMC_x^2 with 3 independent variables $[x_1, x_2, x_3]$ against one dependent Y is presented in Eq. 4.

$$\begin{aligned} DMC_x^2 \quad = \quad & \Big(\rho^2_{X_2,X_3} \times \rho^2_{Y,X_1} - \rho^2_{Y,X_1} + \rho^2_{X_1,X_3} \times \rho^2_{Y,X_2} - \rho^2_{Y,X_2} \\ & + 2 \times \rho_{X_1,X_2} \times \rho_{Y,X_1} \times \rho_{Y,X_2} - 2 \times \rho_{X_1,X_3} \times \rho_{X_2,X_3} \times \rho_{Y,X_1} \\ & + \rho^2_{X_1,X_2} \times \rho^2_{Y,X_3} - \rho^2_{Y,X_3} + 2 \times \rho_{X_1,X_3} \times \rho_{Y,X_1} \times \rho_{Y,X_3} \\ & - 2 \times \rho_{X_1,X_2} \times \rho_{X_2,X_3} \times \rho_{Y,X_1} \times \rho_{Y,X_3} \\ & - 2 \times \rho_{X_1,X_2} \times \rho_{X_1,X_3} \times \rho_{Y,X_2} \times \rho_{Y,X_3} \\ & + 2 \times \rho_{X_2,X_3} \times \rho_{Y,X_2} \times \rho_{Y,X_3}\Big) \quad \Big/ \\ & \Big(\rho^2_{X_1,X_2} + \rho^2_{X_1,X_3} + \rho^2_{X_2,X_3} - 2 \times \rho_{X_1,X_2} \times \rho_{X_1,X_3} \times \rho^{-1}_{X_2,X_3}\Big) \end{aligned} \tag{4}$$

Data Mining and Calculations

The study follow the steps presented below:

1. The data was automatically downloaded, using a script developed by the research team using Python and BBeautifulSoup package

2. Pre-processing: the data is extracted from the .EDF format to a Numpy array. The series were analyzed to eliminate the trailing zeroes, cut at the length of 15742 recorded point and saved as csv files.

3. ρ_{DCCA} and DMC_x^2 calculations: The csv files were used to calculate the coefficients. The calculations were implemented in C language by the research team and the executable was repeatedly called from a python file using the subprocess module. The C program saves the results in a spaces separated txt file.

4. The results are filtered by task. Statistics (mean, median and standard deviation) are calculated for every task for all the subjects. Results are stores in CSV files and in Python Pickle format.

5. Figures generation: To make sense of the results, visualizations are generated. Fist for every subject and task: a four figures plot with the three experiments for the same task and the mean values for each task. Then another four figures plot is generated for each subject, showing the mean value for each execution of each task. Third, for figures plot with the mean value for all the subject, one figure for each task.

6. Diferences: The diference between every subject (mean of the four tasks) and the global means calculated and a four figures plot is created for every subject. The diferences between the imaginary and real actions are calculated for the Down/down and the left/right actions, a two figure plot is generated for every subject. The mean square error is calculated resulting in four figures plots (one figure for each task, one plot for each channel as independent variable). All data is saved in CSV and Python Pickle format

Condiderando as séries temporais dos canais F_3, F_6, P_3, P_6, para cada dependent variable Y, we have 3 independent variables $[x_1, x_2, x_3]$ as, Ver tabela abaixo Table 2

Table 2. Add caption

Color	Y[Channel]	X[Channel1,Channel2,Channel3]
Blue	$\boldsymbol{Y}[F_3]$	X[F_6,P_3,P_6]
Yellow	$\boldsymbol{Y}[F_6]$	$X[F_3,P_3,P_6]$
Red	$\boldsymbol{Y}[P_3]$	$X[F_3,F_6,P_6]$
Green	$\boldsymbol{Y}[P_6]$	$X[F_3,F_6,P_3]$

Como um exemplo deste cálculo, escolhemos ao acaso o sujeito S014 para apresentar os resultados dos tres experimentos e seu valor médio, ver a Fig. 2 (subject S014, Task 2) as an example. O QUE VEMOS NESTA FIGURA????? (cada um fale aqui) Cada experimento tem a sua assinatura, porém eles são bem perecidos (vide o valor médio). Grandes escalas $n > 1000$ tendem a ter os menores valores de DMC_x^2. O valor para F_3 (Frontal no scalp) como a variável dependente [Y] tem os maiores valores...

Na seção a seguir, apresentamos os resultados para os valores médios de DMC_x^2...

Results

Assim, com um total de 108 sujeitos, 4 tarefas e 3 experimentos por tarefa, temos de 1296 valores de DMC_x^2 como função da escala temporal n. Para temos toda a estatística publica, criamos um repositório publicos em:

`https://github.com/gfzebende/RHO_DCCA-Statistics.git`

Neste caso, a estatística completa dos experimentos encontran-se acessando ***Experiments***.

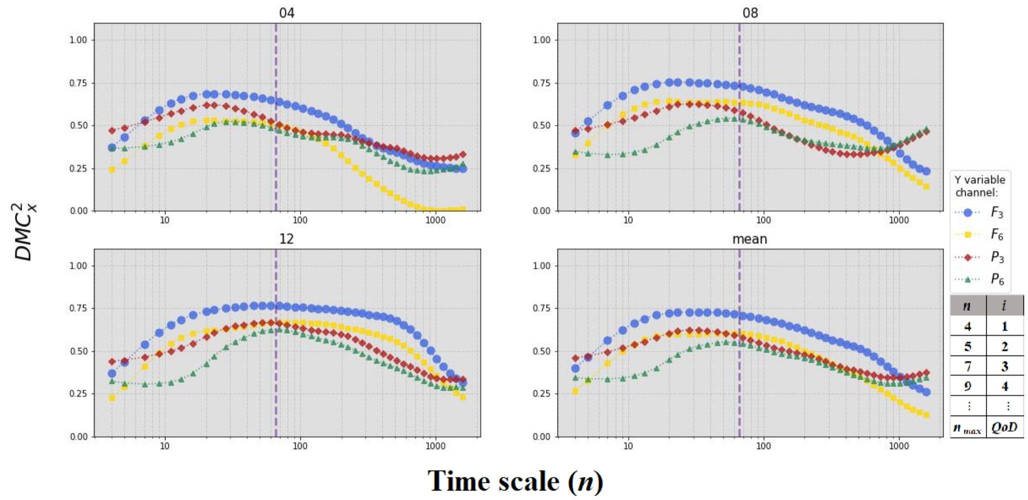

Fig 2. DMC_x^2 as a function of time scale n. Here is showing the S014 subject for Task 2 with its individuals experiments 04, 08, 12 (see Table 1), and also the mean value for these three experiments can be see. The vertical line represents $n = 67$. QoD is the total amount of time scales involved in DMC_x^2 calculations.

Do ponto de vista da média dos experimentos, além do S014, dos 107 sujeitos restantes, escolhemos mais four subjects, isto é, S036, S039, S078 and S099, para a nossa próxima análise (média das tarefas).....Apresentadas nas figuras abaixo, ou seja:

Para S014 ver Fig. 3

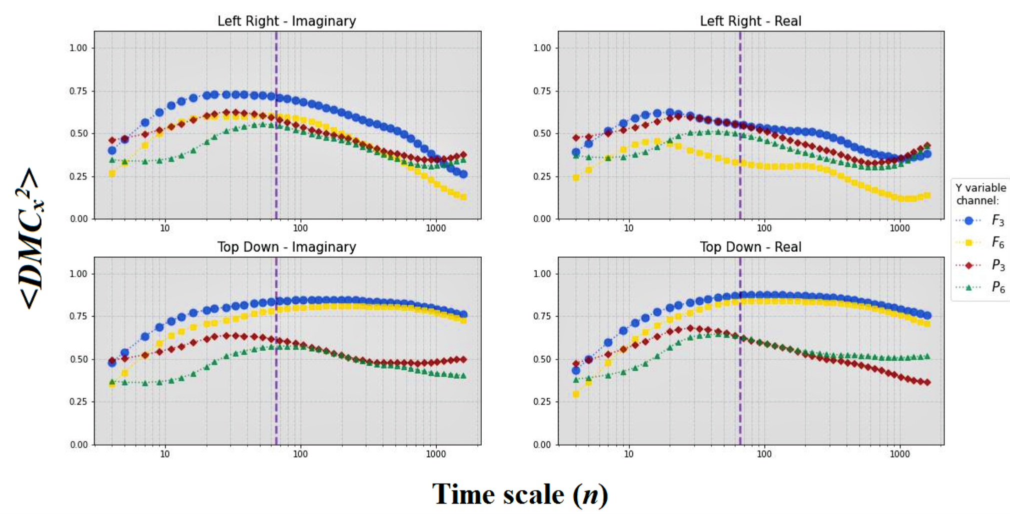

Fig 3. Mean values of $DMC_x^2 \times n$ for all Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real) for the S014 subject.

Para S036 ver Fig. 4

Para S039 ver Fig. 5

Para S078 ver Fig. 6

Para S099 ver Fig. 7

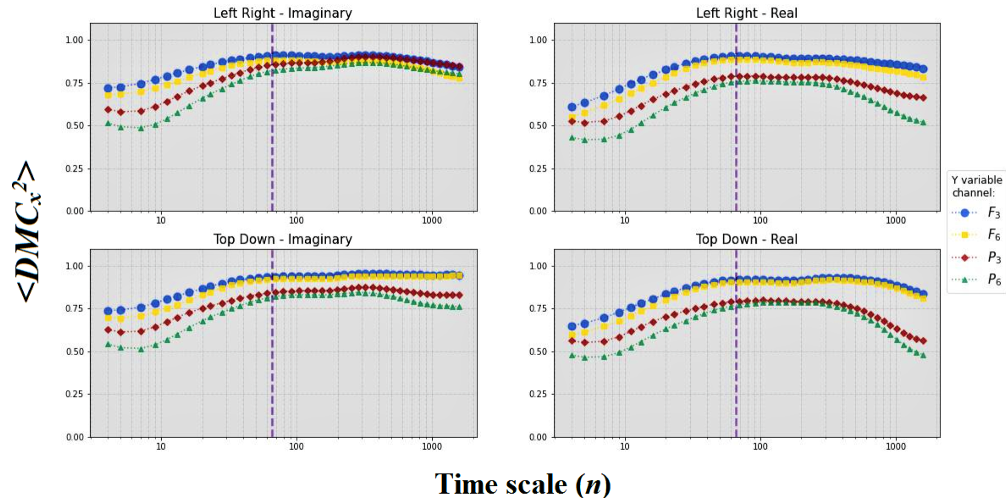

Fig 4. Mean values of $DMC_x^2 \times n$ for all Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real) for the S036 subject.

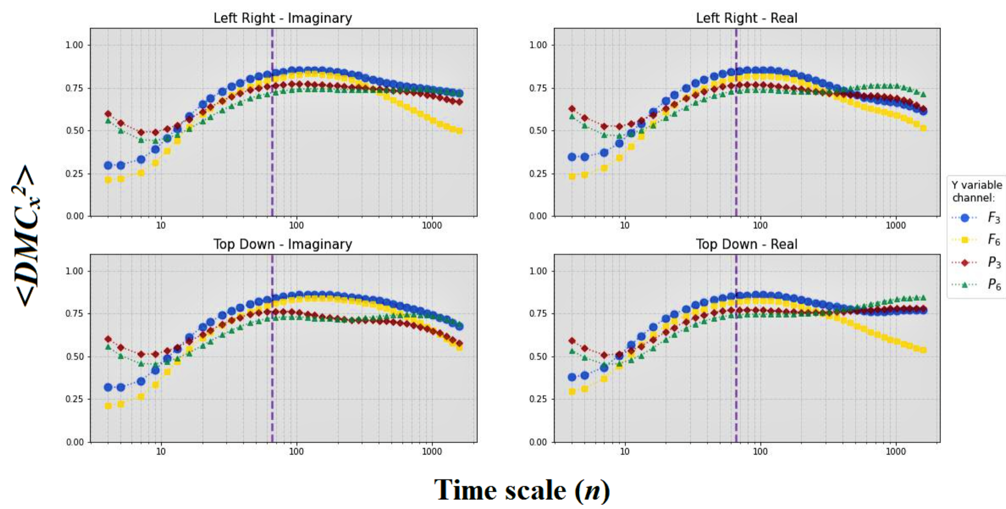

Fig 5. Mean values of $DMC_x^2 \times n$ for all Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real) for the S039 subject.

O restante das médias das tarefas, para todos os outros 103 sujeitos e experimentos encontram-se no vídeo aqui neste link ***Videos*** na pasta Mean.

Como temos 108 sujeitos, a seguir tomamos logo a média Global, isto é:

$$\overline{\langle DMC_x^2 \rangle}\,(n) = \frac{1}{108} \sum_{i=1}^{108} \left\langle DMC_x^2 \right\rangle_i (n) \qquad (5)$$

Os resultados podem ser visto na Mean Global in Fig. 8... Na média global, vemos que o canal F_3 (Frontal no scalp) como a variável dependente [Y] é aquele que apresenta em geral o maior valor de $\overline{\langle DMC_x^2 \rangle}$ principalmente para $n = 67$...vemos também que não existe grandes diferenças entre as tarefas (Left/Right and Top/Down, Real/Imaginary)....

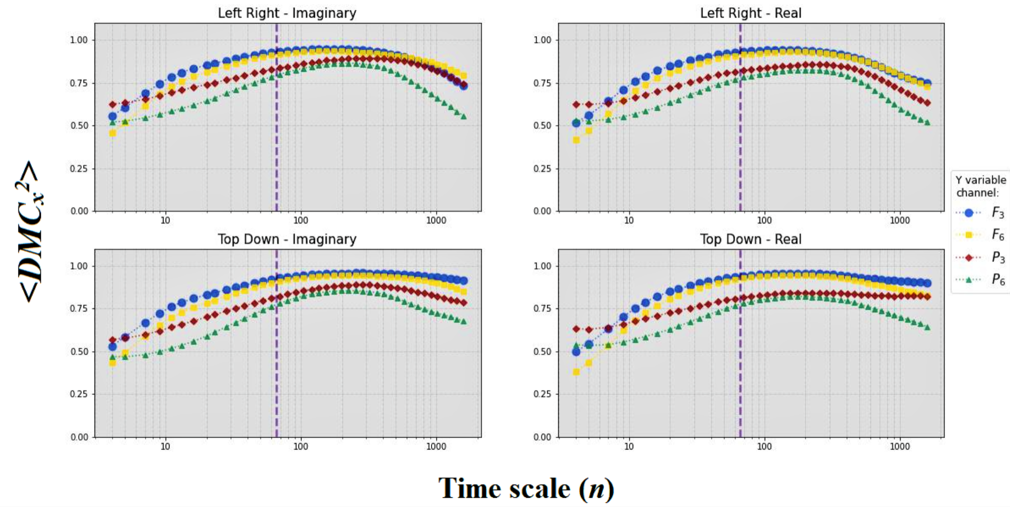

Fig 6. Mean values of $DMC_x^2 \times n$ for all Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real) for the S078 subject.

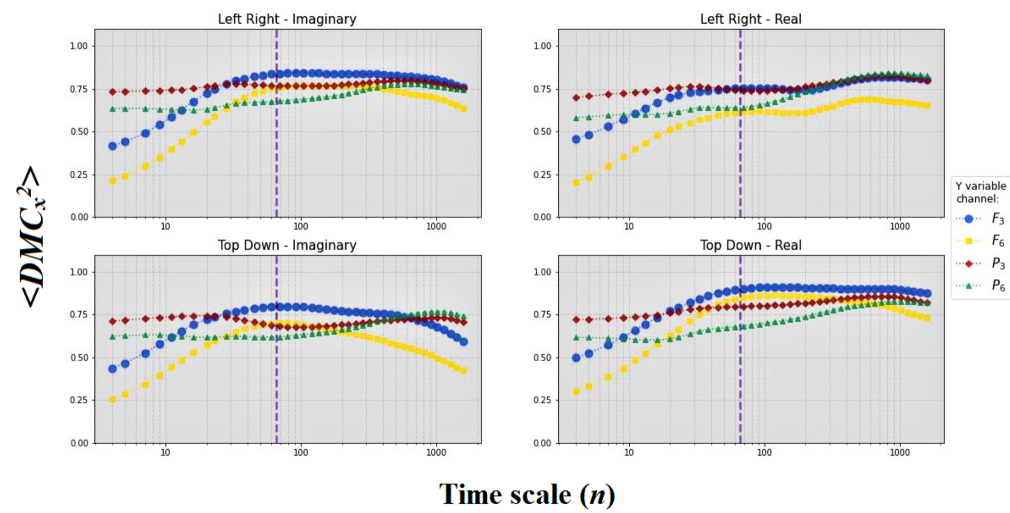

Fig 7. Mean values of $DMC_x^2 \times n$ for all Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real) for the S099 subject.

Para escalas temporais pequenas, isto é, $n \leq 7$, os canais parentais (P_3 e P_6) tem $\overline{\langle DMC_x^2 \rangle}$ maiores do que F_6

Depois, como interessa ver a flutuação em torno da média, também calculamos o Standard Deviation, sd, da média global $\overline{\langle DMC_x^2 \rangle}$, que se encontra na in Fig. 9 Nesta figura vemos que em geral o mesmo tipo de comportamento para sd em todas as tarefas. O sd com o valor mínimo está em torno de $n = 67$, assim como no caso da média para os valores máximos. O valor máximo para sd está representado em pequenas e grandes escalas temporais, e o maior deles é para o caso de [Y] para o canal F_6. No caso do cálculo da Mediana para ver....olhar o (não apresentada aqui mas no nosso repositório) que é muito parecido com o valor da média global

QUAL O MOTIVO DE UTILIZARMOS ISTO AQUI.....????

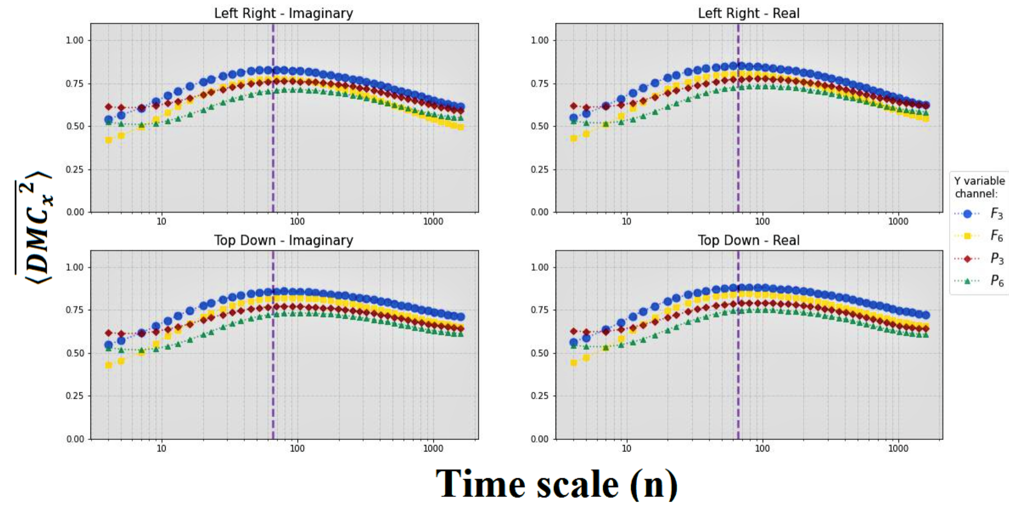

Fig 8. $DMC_x^2 \times n$ mean global for all Subjects and Tasks: Left/Right (Imaginary), Left/Right (Real), Top/Down (Imaginary), and Top/Down (Real).

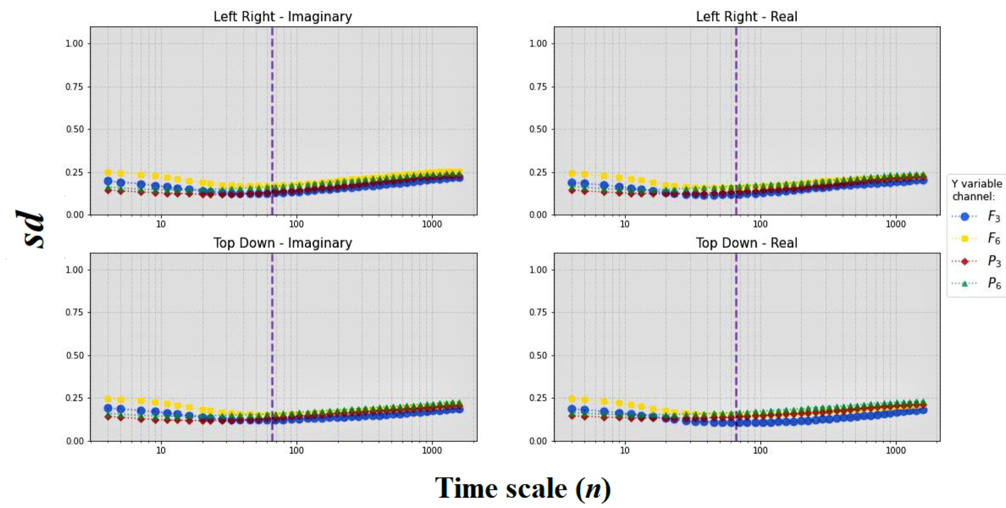

Fig 9. Standard deviation, sd, for all Subjects and Tasks.

O erro quadrático médio...MSE, para cada indivíduo S_j (médio nos três experimentos), para cada canal (como Y) e tarefa, será definido pela Eq. 6:

$$MSE(j) = \frac{1}{QoD} \sum_{i=1}^{QoD} \left(DMC_x^2[i] - \overline{\langle DMC_x^2 \rangle}[i] \right)^2 \tag{6}$$

onde QoD é a quantidade total de escalas temporais calculadas, $j = 1, 2, 3, \cdots, 108$ (número de sujeitos que realizaram o experimento)....e $\overline{\langle DMC_x^2 \rangle}[i]$ o valor médio global da Fig. 8 para cada valor de i calculado (aqui neste paper $QoD = 42$, ver inset na Fig. 2 como exemplo).

Os resultados para MSE para todos os sujeitos por canal: F_3, F_6, P_3, P_6 e todas as tarefas podem ser vistos no Appendix A..... nas Figs. 10, 11, 12, 13 do O QUE SE VÊ AQUI??????

Abaixo, definimos a diferença entre cada sujeito j em relação a média global, isto é, by Task and Channel:

$$diff_j\,(n) \equiv \left\langle DMC_x^2 \right\rangle_j (n) - \overline{\langle DMC_x^2 \rangle}\,(n) \tag{7}$$

Tal estatística das diferenças pode ver vista acessando o link ***Videos*** na pasta Difference.

Discussion

Neste trabalho apresentamos uma forma de analisar correlações cruzadas múltiplas em EEG... **AGORA AGUARDO SUAS COMTRIBUICOES**.

Conclusion

AGORA AGUARDO SUAS COMTRIBUICOES.

Supporting information

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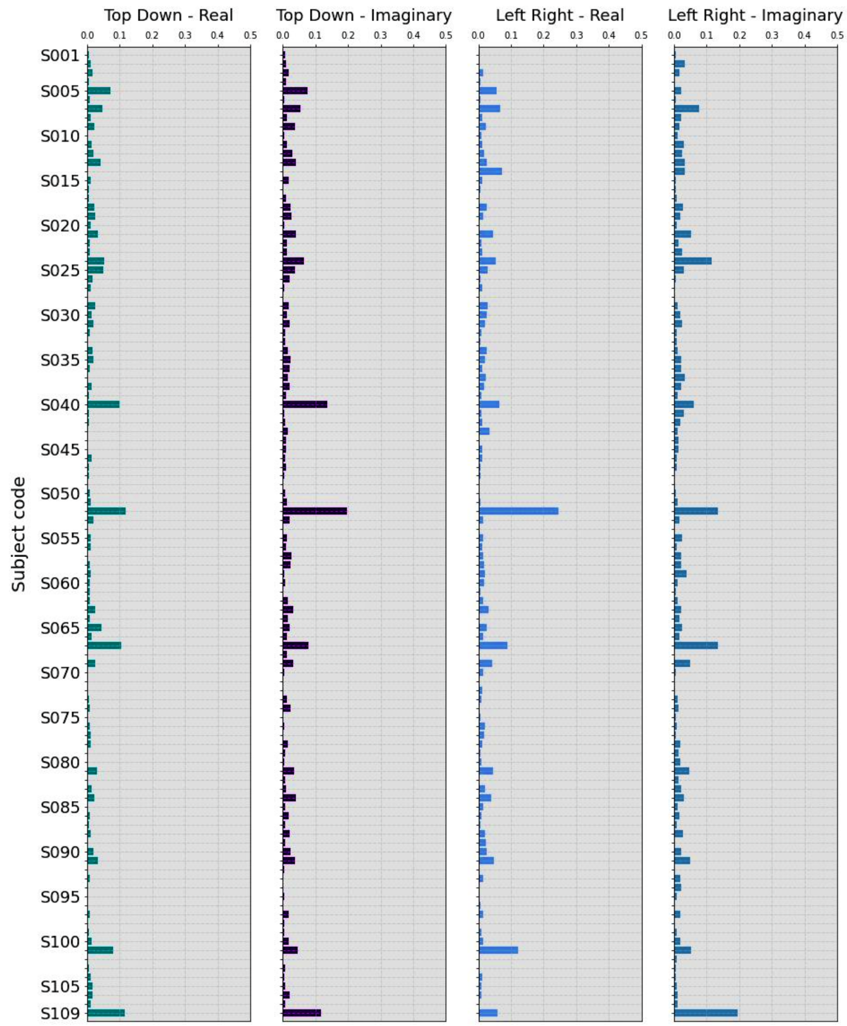

Fig 10. MSE for the Channel F_3.

Appendix A

Acknowledgments

G. F. Zebende thanks the Brazilian agency CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) (Grant 310136/2020-2). F. M. Oliveira Filho thanks the Brazilian agency CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) (Grant 150655/2022-3). We would also like to thank Paulo Murilo Castro de Oliveira, great statistical physicist (*in memoriam*).

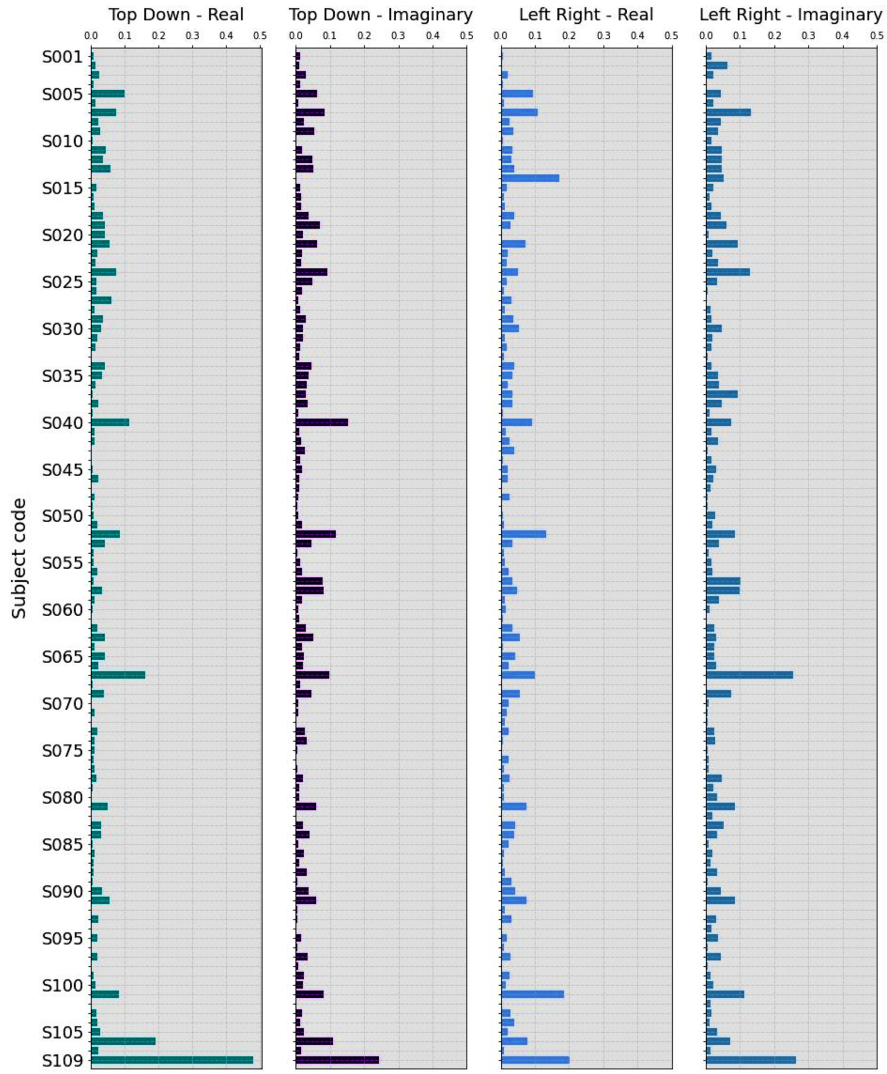

Fig 11. MSE for the Channel F_6.

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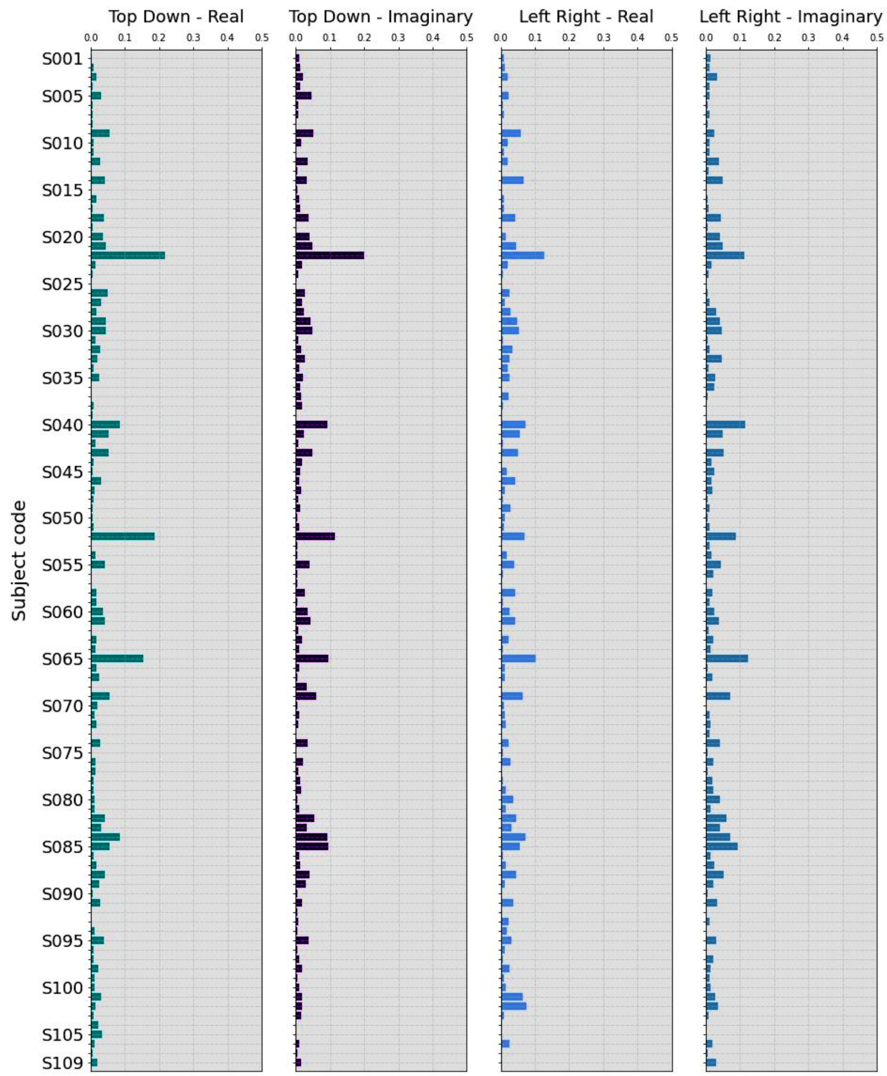

Fig 12. MSE for the Channel P_3.

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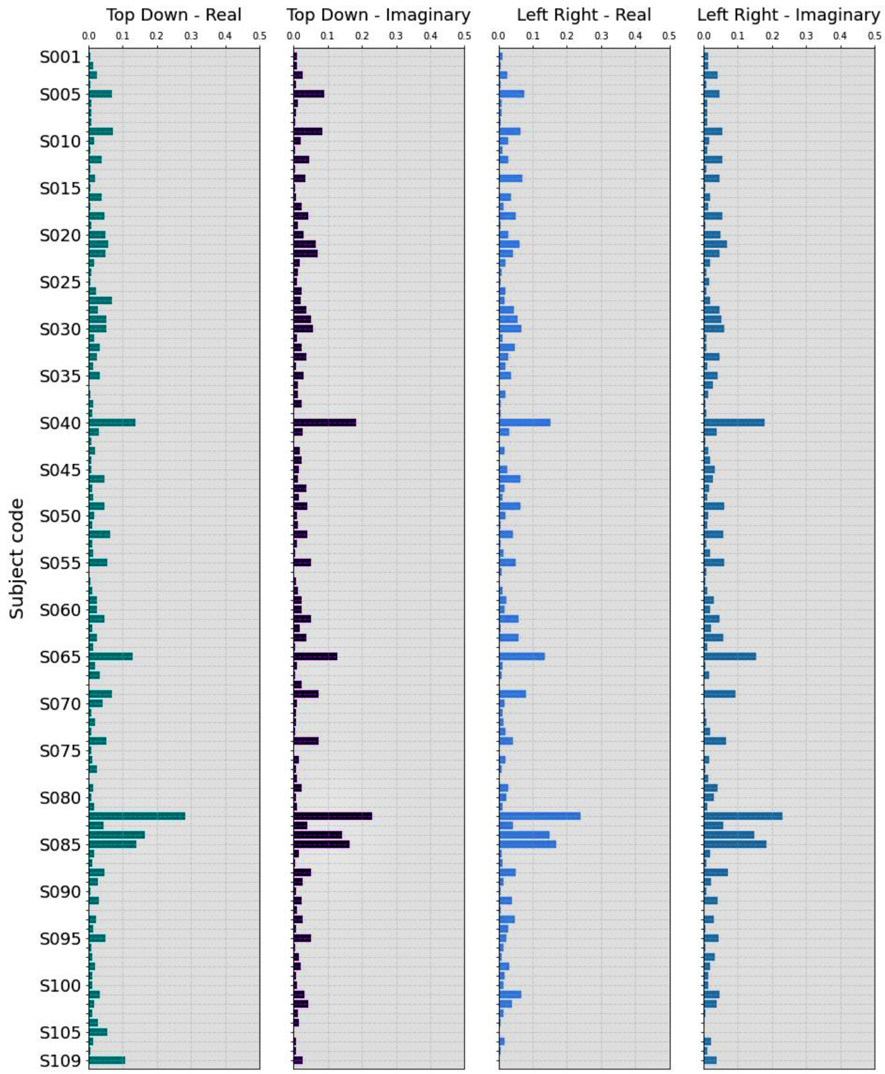

Fig 13. MSE for the Channel P_6.

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