# Design and Implementation of a Store Layout Optimization Recommendation System via Market Basket Analysis

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# 1 Introduction

## 1.1 Background

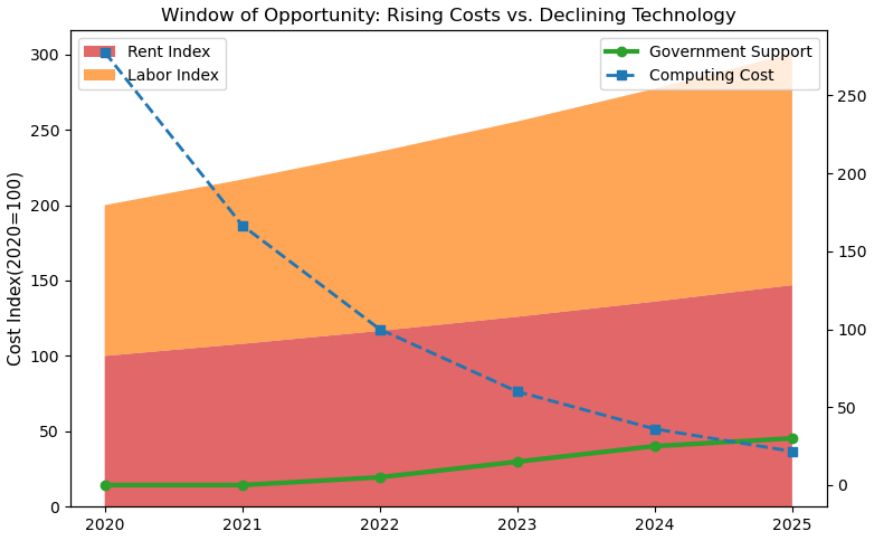
The offline retail industry is currently undergoing a critical transition, with scale expansion reaching its peak and efficiency competition emerging. Although iResearch Consulting's 2024 data shows that offline retail sales in China will still account for 62% of total retail sales of consumer goods, representing the core of the consumer market, the average annual growth rate of store sales per square meter over the past three years has been only 3.5%, far lower than the 12% growth rate of online retail. This gap reflects a core contradiction in offline retail: operational refinement has not kept pace with shifts in consumer demand.

This contradiction is particularly evident in store layout. Traditional layout models are inherently experience-driven: operators often plan shelves based on preconceived notions such as placing best-selling items in the front row or clustering categories, completely ignoring the consumer behavior logic hidden in shopping cart data. This model leads to two significant problems: First, it fragments related demand. Shopping cart data shows that 45% of customers who buy baby formula also purchase wet wipes, and 38% of customers who buy shampoo also buy conditioner. However, these products are often scattered across different shelves. According to research, approximately 41% of customers abandon additional purchases because they cannot find related products.

Secondly, traffic flow design is out of sync with demand. Placing frequently purchased daily necessities deep within the store leads to increased wasted time spent by customers. A heat map from one supermarket chain shows that the pass-by-no-stop rate for these product areas is as high as 63%.

A deeper problem lies in the ineffective use of data. By 2024, 83% of supermarket chains in China had deployed POS systems, theoretically providing the foundation for collecting shopping cart data, but only 19% used it for layout optimization. The core obstacles include three aspects: First, data silos. Data from POS systems, membership systems, and inventory systems are independent and difficult to integrate. One chain retailer even has 12 independent databases, with inventory data discrepancies as high as 15%. Second, insufficient technical capabilities. 85% of retail companies lack data mining talent, making it impossible to use algorithms to identify implicit associations such as "buy diapers → buy beer." Third, concerns about cost-effectiveness. Traditional layout adjustments rely on manual measurement and shelf modifications, resulting in high trial-and-error costs, leading retailers to adopt data-driven solutions with a wait-and-see attitude.

Changes in the consumer market further amplify the urgency of layout optimization. The Nielsen IQ 2025 report shows that 85% of consumers prefer a hybrid online-offline shopping model, significantly increasing their expectations for convenience and experience in offline stores. When customers can easily access "likely" recommendations online, offline stores that still rely on traditional layouts will undoubtedly experience increased customer loss. This supply-demand mismatch creates strong market demand and the conditions for the implementation of our shopping cart-based store layout optimization system.




## 1.2 Major Streamline Platforms

The current retail digitalization tool market lacks dedicated solutions for shopping basket data and layout optimization. Mainstream tools often focus on a single aspect, and due to positioning bias and technical limitations, none of them address the core pain points of layout optimization.

### 1.2.1 Comparison of mainstream platforms

| Platform Type | Representative Platform | Core Capabilities | Data Application |
|---|---|---|---|
| Retail supply chain platform | Alibaba Retail Connect, JD New Channel | Supply chain management, inventory warning, ordering | Only basic sales statistics are provided, no association rule mining, no targeted suggestions, and only general shelf display templates are provided |

| | | | |
|---|---|---|---|
| Retail customer flow analysis platform | Supply chain management, inventory warning, ordering | Passenger flow counting, traffic trajectory tracking, heat map | Data needs to be imported manually and is separated from customer flow data, making it impossible to conduct joint analysis. Suggestions are only made based on customer flow trends, not the essence of shopping needs. |

### 1.2.2 Core issues and root cause analysis

- Optimization Suggestions Are Not Implemented: The Gap Between "General Templates" and "Actual Needs"

Existing platforms' layout recommendations often fail to adapt to local conditions. Supply chain platforms offer display templates based on industry consensus and fail to consider individual store differences. For example, a 100-square-meter mom-and-pop store and a 500-square-meter chain store may have three times the number of shelves, making a general template inadequate. Heat map recommendations from customer flow analysis platforms remain limited to "physical space optimization" and fail to address the underlying logic of consumer demand. For example, a heat map may show high traffic in the snack area, but it's unable to determine whether this is due to the snacks themselves being popular or their strong correlation with other products. Consequently, recommendations often include ineffective adjustments like expanding shelf space.

The root cause of this gap lies in the disconnect between technology and business. Vertical retail SaaS vendors, such as Keruyun, offer both cashier and inventory functions, but to control development costs, they treat layout optimization modules as add-on features, dedicating no resources to their development, ultimately reducing them to mere cosmetic features.

- Insufficient real-time performance: Unable to respond to dynamic changes in

consumer demand

Traditional tools are often updated monthly or quarterly, completely failing to keep up with short-term fluctuations in consumer demand. For example, two weeks before the Lunar New Year holiday, the correlation between "gift boxes + snacks" in shopping carts surged from a typical 12% to 45%. Existing systems were unable to capture this sudden change, resulting in layout adjustments lagging behind the holiday consumption peak.

Technically, the lack of real-time performance stems from outdated data synchronization models. Most platforms use scheduled batch synchronization, which not only causes data delays but can also cause system bottlenecks. Addressing real-time performance requires deploying technologies such as Change Data Capture (CDC), which monitors database logs for incremental synchronization. This presents high technical and cost barriers for small and medium-sized tool manufacturers.

## 1.3 Market Rearch

### 1.3.1 Market Size and Growth Potential: Segmented Blue Oceans in Retail Digital Transformation

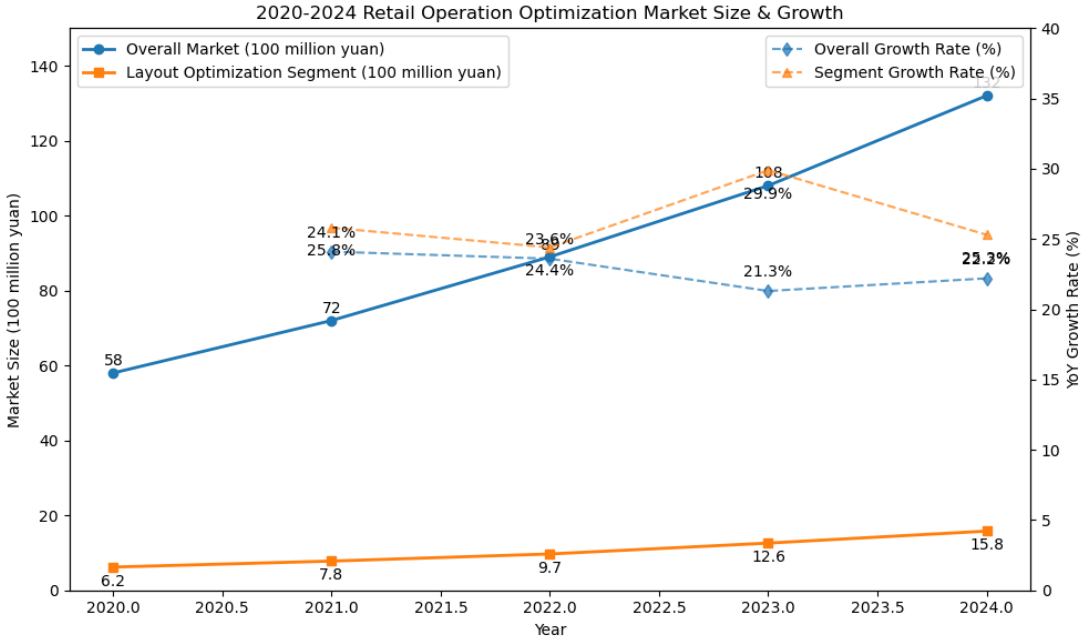
The market size for retail store operations optimization solutions (including layout optimization, customer flow analysis, and inventory management) in China will reach 13.2 billion yuan in 2024, with a compound annual growth rate of 18.2% from 2020 to 2024. This growth trend is highly consistent with the offline retail industry's need for efficiency breakthroughs. As online traffic costs continue to rise, the retail industry's online customer acquisition cost (CAC) is expected to increase by 80% in 2024 compared to 2021. For some beauty and maternity categories, the cost per customer (CAC) is expected to exceed 200 yuan per customer. The average CAC on mainstream e-commerce platforms has reached 800 yuan. Retailers are shifting their focus from online growth to tapping into the potential of existing offline customers, creating a vast market for operations optimization tools.

While layout optimization tools currently account for only 12% of the overall

market (approximately 1.58 billion yuan), they are projected to grow at a compound annual growth rate of 25.3% from 2020 to 2024, significantly exceeding the overall market growth rate, becoming a core driver of market growth. This high growth in this market segment stems from two key factors.

There is an urgent need to improve sales per square meter: Data from 2024 shows that average order value for domestic retailers decreased by 7.02% year-on-year, with large supermarkets experiencing an 11.48% drop. As a key indicator of offline retail's core competitiveness, sales per square meter (S/P/M) is crucial for breakthroughs. Trader Joe's achieved a legendary sales per square meter of $1,750, nearly double that of Whole Foods, through precise layout and product mix strategies. This example has strongly stimulated domestic retailers' desire for optimization.

Low-cost, high-return strategy: Compared to heavy investments like supply chain restructuring and store renovations, layout optimization costs only one-fifth to one-third of the cost, yet delivers significant benefits. By adjusting its store layout and adding a cafeteria, Supermarket Fa has increased its overall customer flow by nearly 10%, effectively revitalizing its "low point" in sales per square meter. This input-output ratio advantage has attracted a large number of small and medium-sized retailers.



### 1.3.2 Competitive Landscape

Current market participants have yet to develop core competitiveness in the "shopping basket data + layout optimization" model, resulting in an overall "large but not strong, small but not specialized" landscape. These players are primarily categorized into three types.

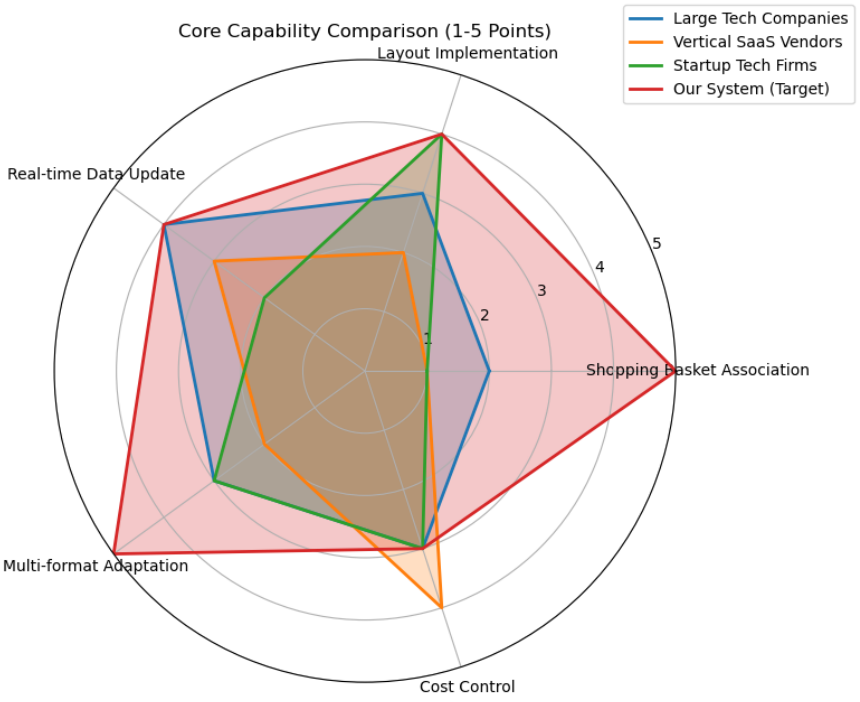
| Participant Type | Advantages | Shortcomings |
|---|---|---|
| Large Tech Companies | Rich data resources and strong technical strength | Resources are scattered, retail is only one of the ecological businesses, and layout optimization functions are added as an additional item, lacking depth. |
| Vertical SaaS Vendors | Meet basic needs such as cashier and inventory | Technology investment is concentrated in core modules, and the layout is optimized for low-cost additional functions without correlation analysis capabilities. |
| Startup Tech Firms | Focus on layout scenarios and respond flexibly | The data source is single, relying on customer flow or sales data, without integrating shopping cart data, and the suggestions are out of touch with actual needs. |

The common shortcomings of these three types of players create significant market gaps, primarily manifesting in three dimensions:

Data Integration Gap: Customer flow analysis platforms and POS systems use disparate data formats—the former uses behavioral trajectory data, while the latter uses structured transaction data. Furthermore, cross-system synchronization requires addressing privacy compliance issues, resulting in a long-term disconnect between shopping basket data and spatial data.

Technology and Business Disconnect: In 2024, data analysis tools will account for only 20.3% of the retail SaaS industry, and most vendors lack a comprehensive "shopping basket-shelf-traffic" linkage model.

Inadequate Service Adaptation: The layout logic of large supermarkets and community stores differs significantly—the former must balance traffic generation and user experience, while the latter focuses on high-frequency, essential needs. However, existing tools mostly offer general solutions, highlighting the importance of customized needs.




## 1.4 Aim and Objectives

Develop a store layout optimization recommendation system based on shopping basket data. Through a comprehensive design encompassing "data integration - association mining - solution generation - performance verification," this system overcomes the data source limitations and implementation barriers of existing tools, helping retailers achieve the multiple goals of improving space efficiency, optimizing the customer experience, and increasing revenue. This system fills the market gap for specialized solutions for shopping basket-driven layout optimization.

# 2 System Design

## 2.1 Problem Definition and Notation

To systematically study the problem of store layout optimization, the store space in this study is discretized into an $\boldsymbol{H} \times \boldsymbol{W}$ grid (in our experiments, $H = W = 10$). The grid cells are divided into a shelf cell set $\boldsymbol{\mathcal{S}}$ and an aisle cell set $\boldsymbol{\mathcal{A}}$, which are mutually exclusive and collectively cover the entire plane. Shelf cells are used for storing products, while aisle cells allow customer movement. Each shelf cell can hold up to four types of products, and customers are allowed to move only on aisle cells, ensuring path feasibility and safety. This grid-based abstraction simplifies the complex spatial layout problem while reflecting the practical arrangement of shelves and aisles in a real store. Given the complete product set $\mathcal{I}$, the customer starting position $s_0 \in \mathcal{A}$, and a set of evaluation baskets $\mathcal{B} = B_1, \ldots, B_M$(where $B_m \subseteq \mathcal{I}$), the decision variable is the product-to-shelf mapping $\boldsymbol{\pi}: \boldsymbol{\mathcal{I}} \rightarrow \boldsymbol{\mathcal{S}}$ This mapping must satisfy the following constraints:

1. Shelf capacity constraint: Each shelf cell $s \in \mathcal{S}$ can hold at most four product types, i.e., $|\pi^{-1}(s)| \leq 4$
2. Accessibility constraint: Each shelf cell must be four-adjacent (up, down, left, right) to at least one aisle cell, ensuring customers can reach the shelf.
3. Path feasibility constraint: Customers may only move on aisle cells and cannot pass through shelf cells.
4. Starting position constraint: Customer paths must start from the specified position $s_0$

Under the above constraints, the system aims to jointly minimize the customer path cost and the product association proximity cost, thereby achieving an efficient layout while promoting potential cross-selling opportunities. The objective function can be formulated as:

$$C_{\text{total}}(\pi) = \alpha C_{\text{path}}(\pi) + \beta C_{\text{assoc}}(\pi)$$

where:

- $C_{\text{path}}(\pi)$ denotes the customer path cost, defined as the sum of shortest path lengths for all evaluation baskets $B_m$

$$C_{\text{path}}(\pi) = \sum_{m=1}^{M} \text{length}\big(\text{shortest\_path}(s_0, \{\pi(i) \mid i \in B_m\})\big)$$

- $C_{\text{assoc}}(\pi)$ represents the product association proximity cost, encouraging associated products to be placed on neighboring shelves:

$$C_{\text{assoc}}(\pi) = \sum_{(i,j)\in R} \text{dist}\big(\pi(i), \pi(j)\big)$$

Here, R is the set of pre-mined product association rules. To eliminate differences in scale between the two types of costs, both $C_{\text{path}}(\pi)$ and $C_{\text{assoc}}(\pi)$ are normalized using empirical baselines.

This grid-based modeling and constraint design provide a well-defined decision space and feasible solution set for subsequent optimization algorithms. Heuristic or metaheuristic algorithms, such as genetic algorithms, can efficiently explore product placement solutions, thereby jointly minimizing customer path cost and product association cost to achieve an effective store layout.

## 2.2 Data Processing and Transaction Modeling

Raw retail transaction records often exhibit inconsistencies in product naming conventions, duplicate entries, and potential noisy data, which can adversely affect the reliability and reproducibility of downstream data mining tasks. To address these issues, a systematic preprocessing procedure was implemented. First, a character-level normalization was applied to each product name i to remove variations due to case sensitivity, leading or trailing whitespace, and other formatting irregularities, thereby

ensuring a consistent representation for identical products across different transactions:

Next, transactions containing only a single item ($|T_j| = 1$) were excluded, as such records provide no co-occurrence information and thus contribute minimally to the discovery of meaningful association patterns. To further mitigate the influence of noise and extremely rare items, a minimum occurrence threshold $\theta_{\text{min}}$ was introduced. Products with a frequency $f_i$ below this threshold relative to the total number of transactions N were filtered out according to:

$$f_i < \theta_{\text{min}} \quad \Rightarrow \quad \text{discard product } i$$

Finally, the remaining product set was encoded into a standardized discrete representation using label encoding, which maps each unique product name to a corresponding integer identifier $id_i$ :

$$\text{LabelEncode: } \{\text{product\_name}_i\} \rightarrow \{id_i \in Z^+\}$$

After product standardization, each transaction is mapped to a transaction set $T_k \subseteq \mathcal{I}$, resulting in a complete transaction list $\mathcal{D} = \{T_k\}$. This transaction-based representation converts complex raw logs into a standard format suitable for algorithmic processing, providing a unified interface for frequent itemset mining, association rule analysis, and co-occurrence graph construction.

Based on the transaction list, frequency statistics and exploratory analysis are conducted, including product occurrence distributions, low-frequency tail proportions, and preliminary assessment of association strength. These statistics are then used to determine thresholds for frequent itemsets and to implement low-frequency pruning strategies, reducing noise and ensuring stability and interpretability in subsequent rule mining and graph construction.

The entire preprocessing pipeline maintains a modular interface with downstream modules: subsequent analysis relies only on the transaction list $\mathcal{D}$ and product

dictionary $J$, without directly coupling to raw log details. This design enhances module reusability and experimental reproducibility while facilitating algorithmic extensions and optimization studies.

## 2.3 Association Rule Mining and Graph Representation Learning

This project adopts two parallel approaches on the same transaction-based input data to model and extract item associations. First, the Apriori algorithm is used to generate frequent itemsets and association rules, capturing explicit co-occurrence patterns between products through statistical methods. Second, GraphSAGE graph representation learning is employed to capture higher-order relations and latent similarities among items, enhancing modeling capability in sparse transaction or long-tail item scenarios.

Both approaches are implemented and evaluated under the same experimental framework, sharing identical preprocessing pipelines, transaction representations, and evaluation metrics to ensure fairness and consistency. Performance comparison focuses on rule coverage, confidence – lift distribution, and stability under varying support thresholds.

Finally, the results from both statistical rule mining and graph representation learning are evaluated and compared under consistent metrics such as rule coverage, confidence distribution, and predictive stability. Rather than directly combining the two, the system adopts the approach that demonstrates superior overall performance for the given dataset and application objective. This selection mechanism ensures that the final rule set maintains both interpretability and robustness, providing a reliable data foundation for subsequent product layout optimization and customer path analysis.

### 2.3.1 Apriori

In the context of association rule mining, the Apriori algorithm is a classical method for discovering frequent itemsets. Its core principle leverages the Apriori property, which states that all non-empty subsets of a frequent itemset must also be frequent. This property allows the algorithm to prune the search space efficiently, enabling the identification of frequent patterns in a transaction database.

Let the transaction database be $D = \{T_1, T_2, \cdots, T_n\}$ where each transaction $T_k \subseteq I$ and $I = \{i_1, i_2, \ldots, i_m\}$ denotes the set of all items. For any itemset $X \subseteq I$ its support is defined as the proportion of transactions containing $X$

$$\text{support}(X) = \frac{|\{T_k \in D \mid X \subseteq T_k\}|}{|D|}$$

An itemset $X$ is called a frequent itemset if $\text{support}(X) \geqslant \text{min\_sup}$. Based on frequent itemsets, the confidence of an association rule $X \Rightarrow Y$ is defined as:

$$\text{confidence}(X \Rightarrow Y) = \frac{\text{support}(X \cup Y)}{\text{support}(X)}$$

Only rules satisfying a minimum confidence threshold min_conf are considered valid.

The Apriori algorithm generates candidate frequent itemsets $C_k$ and prunes them iteratively (level-wise), as summarized below:

1. Initialization: Scan the database to compute the support of individual items and obtain the frequent 1-itemsets $L_1$.
2. Candidate Generation: For the (k-1)-th level frequent itemsets $L_{k-1}$, generate candidate k-itemsets $C_k$ via self-join:
$$C_k = \{X \cup Y \mid X, Y \in L_{k-1}, |X \cap Y| = k - 2\}$$
3. Pruning: For each candidate $c \in C_k$ , remove it if any of its (k-1)subsets are not in $L_{k-1}$
4. Frequent Itemset Confirmation: Scan the database to compute the support of each candidate $c \in C_k$ , retaining those that satisfy min_sup as $L_k$.
5. Iteration: Repeat steps 2 – 4 until no new frequent itemsets can be generated.

6. Association Rule Generation: For each frequent itemset $L_k$, enumerate all non-empty subsets $X \subset L_k$ and calculate the confidence of the rule$X \Rightarrow L_k \setminus X$, keeping only those rules meeting min_conf.

Through this iterative generation and pruning strategy, the Apriori algorithm effectively reduces the search space, making frequent itemset mining feasible for large-scale transaction databases. Its formal theoretical foundation also provides a basis for subsequent algorithmic improvements

### 2.3.2 GraphSAGE

GraphSAGE (Graph Sample and Aggregate) is an inductive graph representation learning algorithm designed for large-scale graphs. Unlike transductive models such as GCNs that require access to the entire graph structure during training, GraphSAGE learns a set of generalizable aggregation functions that can generate embeddings for previously unseen nodes, thereby achieving scalability and inductive capability.

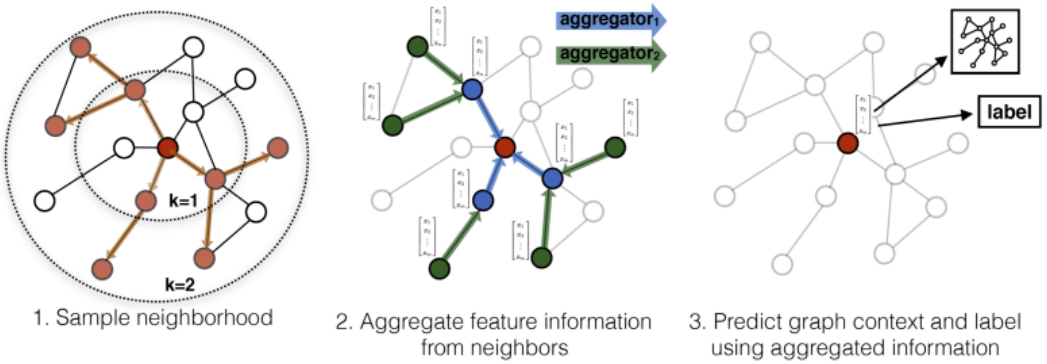



Figure 1: Visual illustration of the GraphSAGE sample and aggregate approach.

Let the graph be represented as:

$$G = (V, E)$$

Where $V = \{v_1, v_2, \ldots, v_N\}$ denotes the set of nodes and $E \subseteq V \times V$ the set of edges. Each node $v_i \in V$ is associated with an input feature vector $x_i \in R^d$.

The goal of GraphSAGE is to learn a mapping function:

$$f: V \rightarrow R^p$$

that produces a low-dimensional embedding $h_v$ for each node $v_i \in V$, preserving both the node's structural and feature information.

At the k-th layer, GraphSAGE updates each node's representation through the following three steps:

1. Neighborhood Sampling: For each node $v$, a fixed-size set of neighbors $\mathcal{N}(v)$ is randomly sampled to control computational complexity.
2. Neighborhood Aggregation: The representations of the neighbors from the previous layer are aggregated via a differentiable aggregation function

$$h_{N(v)}^{(k)} = \text{AGGREGATE}_k\left(\left\{h_u^{(k-1)}, \forall u \in \mathcal{N}(v)\right\}\right)$$

3. Feature Update: The node's own representation is concatenated with the aggregated neighborhood representation and transformed via a trainable weight matrix and nonlinear activation:

$$h_v^{(k)} = \sigma\left(W_k \cdot \text{CONCAT}\left(h_v^{(k-1)}, h_{N(v)}^{(k)}\right)\right)$$

Where $\sigma(\cdot)$ denotes a nonlinear activation function such as ReLU, and $\boldsymbol{W_k}$ is the trainable weight matrix at layer k.

After $K$ iterations, the final node embedding is given by: $z_v = h_v^{(K)}$

Different aggregation strategies define various GraphSAGE variants, such as: Mean, LSTM and Pooling. Through this sample-and-aggregate paradigm, GraphSAGE achieves a balance between expressive power and scalability, enabling effective representation learning on large, dynamic, or inductive graphs such as social networks, recommender systems, and product co-occurrence graphs.

## 2.4 Knowledge Representation

Knowledge representation is composed of the "rule view" and "graph view" in a collaborative manner. The rule view directly stores $(A_r, B_r, w_r)$ along with statistical

attributes such as support, confidence, and lift, and is oriented towards explanation and retrieval. The graph view stores the adjacency structure and edge weights of the product association graph, which is used for structured similarity querying and neighbor constraint.

In terms of engineering implementation, two sets of lightweight access interfaces—rule storage and graph index—are established. These interfaces provide a unified retrieval API for the optimization module upwards. When it is necessary to fuse information from the two channels, a fusion weight can be defined for any product pair (i, j) as $\widetilde{w}_{ij} = \lambda \cdot w_{ij}^{(\mathrm{rule})} + (1-\lambda) \cdot \mathrm{sim}(\mathbf{z}_i, \mathbf{z}_j)$. Here, sim represents the cosine similarity of embeddings, and λ is selected based on the validation set.

## 2.5 Evaluation Function and Normalization

This section presents two types of costs under a given layout mapping $\pi: \mathcal{I} \to \mathcal{S}$: path cost and regular proximity cost; followed by an empirical baseline and synthesis target based on feasible random layouts.

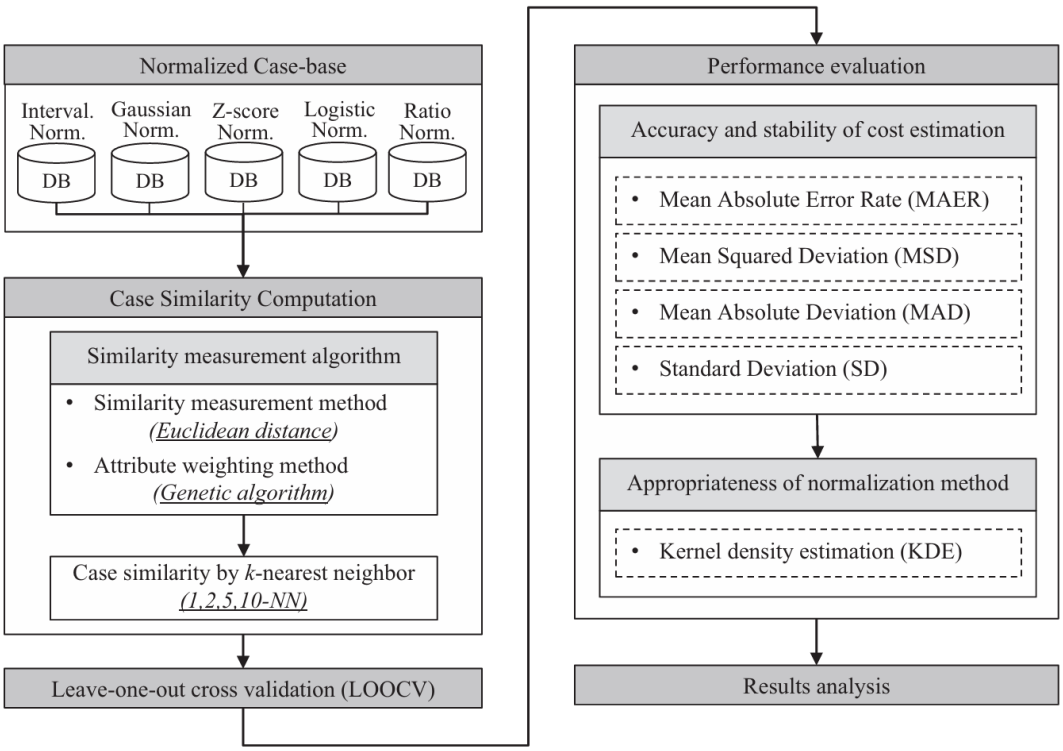



Figure 1. Performance evaluation process and methods.

### 2.5.1 Path Cost

For the evaluation basket set $\mathcal{B}$, let

$$\text{Path}(\pi) = \frac{1}{|\mathcal{B}|}\sum_{B\in\mathcal{B}} \text{dist}(B;\pi)$$

where $\text{dist}(B;\pi)$ is the shortest number of steps from starting point $s_0$ to cover the target shelf set $\{\pi(i): i \in B\}$ by moving only on the aisle units. The coverage criterion is that entering any aisle unit that is four-adjacent to the target shelf is considered to be accessible. The shortest path is implemented using a breadth-first search with a "visit mask." If individual targets are unreachable on the aisle subgraph, the implementation takes:：

$$\text{dist}(B;\pi) = M(\text{Unreachable Penalty})$$

Where M is a sufficiently large constant (10^6 in the implementation) to ensure the selection pressure of a feasible layout.

### 2.5.2 Rule proximity cost

For any rule $r: (A_r \Rightarrow B_r, w_r)$, define its minimum pairwise distance under layout π:

$$d_r(\pi) = \min_{i\in A_r,\, j\in B_r} \| \pi(i) - \pi(j) \|_1$$

And depending on whether the distance transformation is enabled, take:

$$\phi_r(\pi) = \begin{cases} \dfrac{1}{1+\exp\left(-k[d_r(\pi)-d_0]\right)}, & \text{Enable Sigmoid ,} \\ d_r(\pi), & \text{Otherwise.} \end{cases}$$

Based on this, the rule neighboring cost is defined as the weighted average:

$$\text{Rule}(\pi) = \frac{\sum_r w_r \phi_r(\pi)}{\sum_r w_r}$$

In the preceding equation, $k$ and $d_0$ re used for nonlinear adjustments that "amplify nearby neighbors and flatten distant neighbors." The default parameters are $k = 1.5$、$d_0 = 2$, , which can be adjusted on the validation set. If the rule set is empty or all weights are zero, $\text{Rule}(\pi) = 0$ is set in the implementation to keep the objective function defined.

### 2.5.3 Empirical baseline and comprehensive goals

To eliminate the effects of dimensional differences and distribution scale, we use a Monte Carlo sample with a feasible random layout$\{\pi^{(s)}\}_{s=1}^{S}$ to estimate the expected value of the two items:

$$\hat{\mu}_{\text{Path}} =\mid \frac{1}{S}\sum_{s=1}^{S} \text{Path}(\pi^{(s)}), \hat{\mu}_{\text{Rule}} = \frac{1}{S}\sum_{s=1}^{S} \text{Rule}(\pi^{(s)}).$$

The final comprehensive objective is:

$$J(\pi) = \alpha \cdot \frac{\text{Path}(\pi)}{\mu_{\text{Path}}} + \beta \cdot \frac{\text{Rule}(\pi)}{\mu_{\text{Rule}}}$$

Where $\alpha$ and $\beta$ are non−negative weights that control the trade−off between movement and co−purchase proximity. Since both items are normalized by the empirical baseline, the adjustments to $\alpha$ and $\beta$ are clearly comparable.

## 2.6 Genetic algorithm optimization and feasibility assurance

This section describes the process of using a genetic algorithm (GA) to minimize the objective function $J(\pi)$ in the product-to-shelf location mapping space, where $J(\pi) = \alpha\,\text{Path}_{\text{norm}}(\pi) + \beta\,\text{Rule}_{\text{norm}}(\pi)$ is given in Section 2.5. The entire optimization process is performed on a fixed grid and set of shelves, always ensuring that each shelf can accommodate at most $C_{\max} = 4$ products, and enforcing the shortest path calculation along the aisle unit during path evaluation.

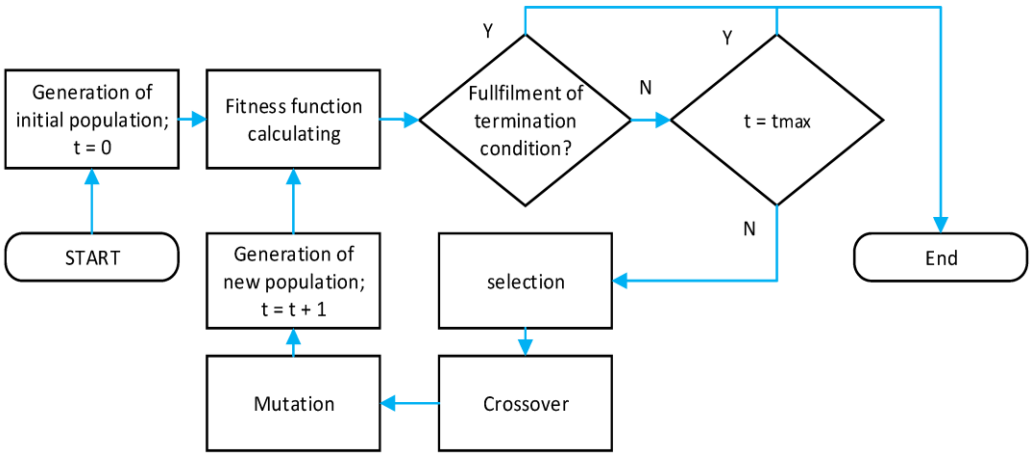



Figure 2. Genetic algorithm

### 2.6.1 Optimization Problems and Coding

Given a grid and a set of shelves, the layout is represented as a "product-to-shelf" mapping $\pi$。The genetic algorithm uses fixed-length integer vectors as individuals: $\mathbf{g} = (g_1, \dots, g_{|\mathcal{I}|})$, where the $k$ bit is a shelf index, decoded as $\pi_{\mathbf{g}}(i_k) = s_{g_k}$。The capacity constraint in genetic space is equivalent to "each shelf is assigned no more than four product types," which can be directly determined by counting $\{k: g_k = j\}$ The optimization objective follows the normalized weighted formula described above:

$$J(\pi) = \alpha \cdot \frac{\mathrm{Path}(\pi)}{\hat{\mu}_{\mathrm{Path}} + \varepsilon} + \beta \cdot \frac{\mathrm{Rule}(\pi)}{\hat{\mu}_{\mathrm{Rule}} + \varepsilon}$$

Where $\varepsilon$ is a fixed minimal positive number, $\hat{\mu}_{\mathrm{Path}}, \hat{\mu}_{\mathrm{Rule}}$ are empirical baselines based on feasible random layouts.

### 2.6.2 Initial population and feasibility repair

The initial population consists of two types of individuals: one is an external initial mapping from existing or categorical placements (encoded as $\mathbf{g}^{(\mathbf{0})}$), and the other is a random feasible individual that satisfies the upper capacity bound. To ensure that all individuals are feasible, a uniform "capacity repair" is performed on each newly generated or operator-modified **g:** if a shelf is overloaded, excess items are randomly selected from that shelf and moved to a shelf with surplus inventory until all shelves are not overloaded. Accessibility is handled during the evaluation phase: only shelves that are at least one adjacent aisle are considered "available for pickup."

### 2.6.3 Fitness and intra-generational evaluation

The fitness of a body is defined as $\mathrm{Fit}(\mathbf{g}) = J(\pi_{\mathbf{g}})$. The path term uses a breadth-first search with an access mask to calculate the shortest number of steps from the starting point to the target shelf set on the aisle subgraph, averaging the number of evaluation baskets. If a basket is unreachable under the current $\pi_{\mathbf{g}}$, a preset large number is used as the cost of the basket. The rule term calculates the pairwise minimum Manhattan distance $d_r(\pi_{\mathbf{g}}) = \min_{i \in A_r,\, j \in B_r} \| \pi_{\mathbf{g}}(i) - \pi_{\mathbf{g}}(j) \|_1$ for each $r: (A_r \Rightarrow B_r, w_r)$, taking a weighted average based on the weights. When nonlinearity is enabled,

$\phi_r(d) = 1/(1 + \exp(-k(d - d_0)))$ is used instead of d to achieve higher resolution near the same-shelf/neighboring-shelf threshold. The baseline terms $\hat{\mu}_{\text{Path}}, \hat{\mu}_{\text{Rule}}$ are estimated by Monte Carlo sampling of multiple feasible random layouts to achieve dimension alignment and scale stability.

### 2.6.4 Iteration Operators and Termination

Each generation consists of selection, crossover, mutation, and elite retention. Selection uses a tournament strategy, prioritizing those with lower fitness. Crossover is a single-point approach, splitting and swapping suffixes at a given location. Mutation randomly resets a gene to a new shelf index with a predetermined probability. Capacity repair is performed immediately after crossover and mutation to ensure that individuals always meet the upper bound of shelf capacity. A new generation consists of "elite individuals directly retained + crossover and mutation offspring" until a predetermined number of generations or other termination criteria are reached. Finally, the individual with the lowest $\text{Fit}(\mathbf{g})$ is selected from the terminated generation as output, and its decoding map $\pi_{\mathbf{g}}$ is the optimized layout solution.

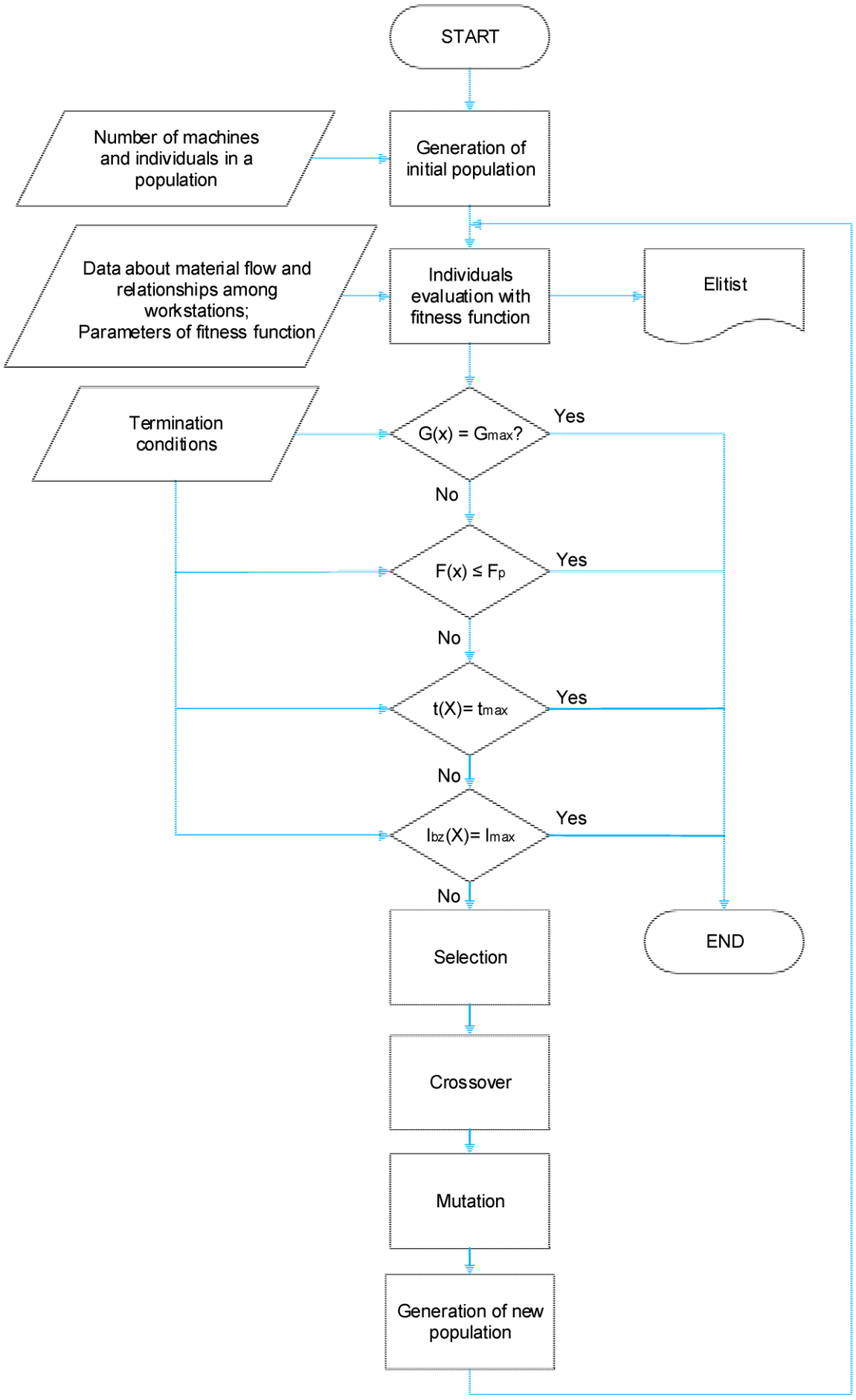

START
Number of machines and individuals in a population
Generation of initial population
Data about material flow and relationships among workstations; Parameters of fitness function
Individuals evaluation with fitness function
Elitist
Termination conditions
G(x) = Gmax?
Yes
No
F(x) ≤ Fp
Yes
No
t(X)= tmax
Yes
No
Ibz(X)= Imax
Yes
No
Selection
END
Crossover
Mutation
Generation of new population

Figure 3. Genetic algorithm for layout optimization.

## 2.7 Evaluation plan and comparison

To ensure consistency with the established design and model framework in this section, the evaluation focused solely on the normalized objective function. All experiments were conducted under fixed grid conditions, a fixed set of shelves, and a fixed starting point. The rule set and its weights remained unchanged during the optimization phase. The randomization process used a uniform random seed to ensure repeatability.

### 2.7.1 Evaluation subjects and controls

The evaluation target is the optimal layout $\pi^{\star}$. obtained by the genetic algorithm. Comparisons include a randomized feasible layout (satisfying a maximum of four products per shelf) and a heuristic layout partitioned by category. $J(\pi)$ is calculated for all three using the same evaluation basket $\mathcal{B}$ and the same rule set $\mathcal{R}$. The randomized feasible layout is evaluated using the mean and standard deviation of multiple independent trials; the heuristic layout based on category partitioning is a static control without randomization. To facilitate quantitative comparison, relative improvement rates are introduced and the relative improvements of the two components, $\Delta_{\text{Path}}, \Delta_{\text{Rule}}$, are reported simultaneously, using the same metric as $J(\cdot)$. If an unreachable path occurs in the evaluation basket, the path item is penalized and not removed.

### 2.7.2 Evaluation indicators and statistical caliber

The primary metric is $J(\pi)$. Component metrics include the normalized values of "Path($\pi$)" and "Rule($\pi$)". The report format includes: the optimal solution; the mean and standard deviation of the randomized feasible comparison; the point estimate of the class-inspired comparison; and the relative improvement rate. To control for scale differences, $\hat{\mu}_{\text{Path}}, \hat{\mu}_{\text{Rule}}$ are estimated using Monte Carlo estimation of feasible random layouts under the same constraints, with the number of samples and random seed consistent with the optimization stage. If the Sigmoid transformation of the rule distance is enabled, two sets of results, "enabled" and "disabled," are presented for

comparison under the same basket and rule conditions.

### 2.7.3 Experimental process and stopping conditions

Given $(\alpha, \beta)$, S the Sigmoid parameters, and the number of baseline samplings, we first estimate $(\hat{\mu}_{\text{Path}}, \hat{\mu}_{\text{Rule}})$, then run the genetic algorithm to the specified upper bound. Each generation, fitness is evaluated on the complete $\mathcal{B}$ and $\mathcal{R}$ datasets. Elite individuals are retained, while the remaining individuals are generated through selection, crossover, and mutation. Capacity restoration is performed within each generation to maintain feasibility. Upon termination, the layout corresponding to the individual with the smallest $J(\pi)$ within the generation is selected as $\pi^{\star}$.

# 3 System Development & Implementation

This section details the technical architecture, technology stack selection, and implementation specifics of the Store Shelf Layout Optimization System. The project employs a typical separated front-end/back-end architecture, where the front-end is responsible for interaction and visualization, and the back-end hosts the core algorithms (GraphSAGE GNN and Simulated Annealing) and data analysis functions.

## 3.1 GNN-based Association Rule Mining

### 3.1.1 Data Preprocessing

- **Implementation of Basic Transaction Statistics Analysis**

To obtain a data-driven understanding of the transaction dataset, basic statistical analysis is first conducted using the analyze_basic_stats function, which iterates over each transaction in the preprocessed dataset to count the occurrences of individual items while skipping missing values. The resulting item frequencies are stored in a dictionary, sorted in descending order, and printed to provide insight into the total number of transactions, the number of unique items, and their respective occurrence counts. This step not only facilitates the identification of high-frequency and long-tail items but also provides essential guidance for selecting minimum support thresholds, implementing low-frequency pruning strategies, and constructing subsequent frequent itemsets or co-occurrence graphs. By operating solely on the standardized transaction list, this module ensures consistency, reproducibility, and a reliable foundation for downstream association rule mining and graph-based representation learning.

```python
def analyze_basic_stats(data):
    """
    :param data:
    :return: item_frequencies
    """

    item_frequencies = {}
    count = 0

    for index, row in data.iterrows():
        count += 1
        for item in row:
            if pd.isna(item):
                continue
            if item in item_frequencies:
                item_frequencies[item] += 1
            else:
                item_frequencies[item] = 1

    item_frequencies = dict(sorted(item_frequencies.items(), key=lambda x: x[1], reverse=True))
    print("Number of transaction:", count)
    print('types:', len(item_frequencies))
    print("frequencies for each item:")
    for item, count in item_frequencies.items():
        print(f"{item}: {count}")
    return item_frequencies
```

- **Transaction Extraction and Preprocessing**

To standardize the raw retail transaction data into a structured format suitable for algorithmic processing, a preprocessing function pre_process was implemented. This function iterates over each row of the raw dataset, extracting product entries while removing leading and trailing whitespace and ignoring missing or empty values. Transactions containing fewer than two items are discarded, as they provide minimal co-occurrence information and contribute little to association pattern discovery. The resulting list of transactions forms a clean, uniform input suitable for downstream frequent itemset mining, association rule analysis, and graph-based representation learning. In addition, the function reports the total number of valid transactions, providing a basic descriptive statistic for dataset understanding and quality assessment. This preprocessing step ensures consistency, reproducibility, and a reliable data foundation for subsequent modeling.

```python
def pre_process(data):
    """
    extract transactions from raw data
    :param data:
    :return: transactions
    """
    transactions = []
    for index, row in data.iterrows():
        items = [str(item).strip() for item in row.values if pd.notna(item) and str(item).strip()]
        if len(items) >= 2:
            transactions.append(items)

    total_transactions = len(transactions)
    print(f"Total number of transactions: {total_transactions}")
    return transactions
```

### 3.1.2 Constructing a GNN

- **Co-Occurrence Graph Construction**

To model higher-order item relationships beyond simple co-occurrence counts, a co-occurrence graph is constructed from the preprocessed transaction list. Each unique product is assigned a node, with edges representing pairs of items that appear together in transactions. The construction begins by mapping items to integer indices and initializing a symmetric co-occurrence matrix. For every transaction containing at least two items, all pairwise combinations of items are counted and stored in the matrix. Edge creation is governed by a support threshold: an edge is added between two items if their co-occurrence frequency relative to the total number of transactions exceeds the threshold. To ensure meaningful connectivity even in sparse data scenarios, the threshold is dynamically reduced by half if no edges are found at the initial setting. Node features are initialized using normalized item frequencies, capturing each product's overall occurrence in the dataset. Finally, the nodes, edges, edge weights, and features are assembled into a PyTorch Geometric Data object, providing a structured graph suitable for downstream representation learning with models such as GraphSAGE. This approach ensures that the resulting graph reflects both strong co-occurrences and the global distribution of items, facilitating effective learning of latent item similarities.

```
item_indices = {item: i for i, item in enumerate(All_items)}
num_items = len(All_items)
co_occurrence = np.zeros((num_items, num_items), dtype=np.int32)
for transaction in tqdm(transactions, desc="处理交易"):
    if len(transaction) < 2:
        continue
    indices = [item_indices[item] for item in transaction]
    for i, j in combinations(indices, 2):
        co_occurrence[i, j] += 1
        co_occurrence[j, i] += 1
total_transactions = len(transactions)
edges = []
edge_weights = []
# Dynamically adjust the support threshold: if no rules are found under the initial threshold, automatically lower it.
while len(edges) == 0 and min_support > 0:
    edges = []
    edge_weights = []
    for i in range(num_items):
        for j in range(i + 1, num_items):
            support = co_occurrence[i, j] / total_transactions
            if support >= min_support:
                edges.append((i, j))
                edge_weights.append(support)
    if len(edges) == 0:
        min_support *= 0.5
```

```
edge_index = torch.tensor(edges, dtype=torch.long).t().contiguous()
edge_weight = torch.tensor(edge_weights, dtype=torch.float)

item_freq = [0] * num_items
for item, count in item_frequencies.items():
    item = item.strip()
    item_freq[item_indices[item]] = count / total_transactions

x = torch.tensor(item_freq, dtype=torch.float).view(-1, 1)

graph = Data(x=x, edge_index=edge_index, edge_weight=edge_weight)
graph.num_nodes = num_items

return graph
```

- **GraphSAGE Model Construction**

To perform representation learning on the previously constructed co-occurrence graph, a GraphSAGE model is instantiated. The model consists of two graph convolution layers (SAGEConv), where the first layer maps input node features to a hidden representation, followed by a ReLU activation and dropout for regularization, and the second layer projects the hidden representation to the target embedding dimension. The input node features are derived from normalized item frequencies,

capturing basic occurrence information. The model is initialized with configurable hidden and embedding dimensions, and it is deployed on GPU if available, otherwise on CPU. This setup provides a trainable architecture capable of learning low-dimensional embeddings for each product node, preserving both the local neighborhood structure and node-specific attributes, which can later be used for downstream association analysis or recommendation tasks.

```
def build_model(graph, embedding_dim=32, hidden_dim=64):
    if graph is None:
        raise ValueError("constrct graph first")

    device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')

    class GraphSAGE(torch.nn.Module):
        def __init__(self, in_channels, hidden_channels, out_channels):
            super().__init__()
            self.conv1 = SAGEConv(in_channels, hidden_channels)
            self.conv2 = SAGEConv(hidden_channels, out_channels)

        def forward(self, x, edge_index):
            x = self.conv1(x, edge_index)
            x = F.relu(x)
            x = F.dropout(x, p=0.2, training=self.training)
            x = self.conv2(x, edge_index)
            return x

    model = GraphSAGE(
        in_channels=graph.x.size(1),
        hidden_channels=hidden_dim,
        out_channels=embedding_dim
    ).to(device)

    return model
```

- **Training**

The training process of the GraphSAGE model is carried out on the previously constructed co-occurrence graph using mini-batch neighbor sampling for scalability. The model is trained for a configurable number of epochs with the Adam optimizer. Each batch is generated by the NeighborLoader, which samples a fixed number of neighbors per layer to form local subgraphs, enabling efficient computation for large

graphs. A contrastive loss is employed, where connected node pairs in the graph act as positive samples, and randomly selected node pairs serve as negative samples. The similarity between node embeddings is computed using cosine similarity, and the loss encourages embeddings of connected nodes to be similar while pushing apart embeddings of unrelated nodes. During training, out-of-range indices are handled gracefully to ensure numerical stability. The model parameters are updated iteratively through backpropagation. Average batch loss is monitored per epoch to track convergence, providing an indication of embedding quality over training. Once trained, the GraphSAGE model produces low-dimensional, informative embeddings for each product node, suitable for downstream tasks such as association analysis, recommendation, or layout optimization.

```
for epoch in range(epochs):
    total_loss = 0
    for batch in loader:
        batch = batch.to(device)
        optimizer.zero_grad()
        out = model(batch.x, batch.edge_index)
        edge_index = batch.edge_index
        src, dst = edge_index[0], edge_index[1]

        max_index = out.size(0) - 1
        if src.max() > max_index or dst.max() > max_index:

            valid_mask = (src <= max_index) & (dst <= max_index)
            src = src[valid_mask]
            dst = dst[valid_mask]
            if len(src) == 0:
                continue
        pos_sim = F.cosine_similarity(out[src], out[dst], dim=1)
        if out.size(0) == 0:
            continue
        neg_dst = torch.randint(0, out.size(0), (src.size(0),), device=device)
        neg_sim = F.cosine_similarity(out[src], out[neg_dst], dim=1)

        loss = -torch.log(torch.sigmoid(pos_sim) + 1e-10).mean() - torch.log(
            1 - torch.sigmoid(neg_sim) + 1e-10).mean()
        loss.backward()
        optimizer.step()
        total_loss += loss.item()
    avg_loss = total_loss / len(loader)
    losses.append(avg_loss)
    if (epoch + 1) % 10 == 0:
        print(f"Epoch {epoch + 1}/{epochs}, Loss: {avg_loss:.4f}")
```

- **Association Rule Extraction from Trained GraphSAGE Embeddings**

After training the GraphSAGE model, association rules between products are derived directly from the learned embeddings. Each product node is first encoded with a label encoder to maintain a consistent mapping between names and indices. Node embeddings are obtained by performing a forward pass through the trained model. Pairwise cosine similarity between embeddings is then computed to quantify latent associations, capturing higher-order relationships beyond simple co-occurrence. For each node pair, support is estimated from the edge weights in the co-occurrence graph,

while confidence is determined from the embedding similarity. Only pairs satisfying a minimum confidence threshold (e.g., 0.5) and positive support are retained as candidate rules. The extracted rules are formatted as a list of triples, where each set is represented as a list to ensure JSON serializability. Finally, rules are sorted by confidence and truncated to the top-k most significant associations before being saved to a JSON file for downstream tasks such as layout optimization or recommendation analysis.

```
model.eval()
with torch.no_grad():
    embeddings = model(graph.x.to(device), graph.edge_index.to(device))

similarities = F.cosine_similarity(embeddings.unsqueeze(1), embeddings.unsqueeze(0), dim=2)

rules = []
item_list = item_encoder.classes_
num_items = len(item_list)

for i in range(num_items):
    for j in range(num_items):
        if i != j:
            support = (
                graph.edge_weight[
                    (graph.edge_index[0] == i) & (graph.edge_index[1] == j)
                    ].sum().item() / len(transactions)
                if len(graph.edge_weight) > 0
                else 0
            )
            confidence = similarities[i][j].item()

            if confidence > 0.5 and support > 0:
                rules.append([
                    [str(item_list[i]).split()],
                    [str(item_list[j]).split()],
                    confidence
                ])
rules = sorted(rules, key=lambda x: x[2], reverse=True)[:top_k]
with open("results/rules.json", "w", encoding="utf-8") as f:
    json.dump(rules, f, ensure_ascii=False, indent=2)
```

## 3.2 GA-based Shelf Layout Optimization

### 3.2.1 Scope & Core Data Structures

- **Evaluate and optimize item-to-shelf placements**

The module's scope is to evaluate and optimize item-to-shelf placements on a fixed 2D grid under a given set of shelves, a start cell, a complete item vocabulary, and a set of association rules. Inputs are: grid size (10×10 by default), shelf coordinates, start coordinate, item list, an optional initial mapping, and rules obtained from Knowledge Discovery. Shelves are projected to a binary grid (1=shelf, 0=aisle); all

```
def build_grid_from_shelves(H: int, W: int, shelves: List[Coord]) -> Grid:
    """Create a grid from shelf coordinates: 0=aisle, 1=shelf."""
    g = [[0]*W for _ in range(H)]
    for r, c in shelves:
        if not in_bounds(r, c, H, W):
            raise ValueError(f"Shelf {(r,c)} out of bounds for {H}x{W} grid.")
        g[r][c] = 1
    return g
```

routing and reachability operate over the aisle subgraph with 4-connectivity (up, down, left, right). Coordinate and grid are represented as $Coord = (row, col)$ and $Grid$ (nested list of ints), forming the base abstraction shared by downstream routines.

**Listing2.1:** build grid function

- **Pickup semantics**

For pickup semantics, "reaching a shelf" means stepping onto any aisle cell adjacent to that shelf. For each shelf cell, the system derives its pickup aisle set (neighboring cells that are aisles) and runs a mask-based BFS over the aisle graph to compute the shortest path that covers all required shelves from the start. Reachability is validated by (i) an aisle-only BFS region from the start and (ii) ensuring each target shelf has at least one pickup aisle within that reachable region. If any target is unreachable, the evaluator reports failure; the objective then accounts for this with a large fixed penalty. These semantics are realized jointly by "adjacent aisle extraction," "aisle reachability from start," and the "state-mask BFS cover" routine.

- **Layout and capacity**

A placement is represented as $item\_to\_shelf: Dict[str, Coord]$. The system enforces a hard capacity: each 1×1 shelf can host up to 4 distinct items (ignoring vertical tiers and in-shelf pickup cost differences). All evaluators and the optimizer perform capacity validation at entry; any violation raises an error. To keep individuals feasible

after randomization or genetic operators, a capacity repair routine reassigns overflowing items from overloaded shelves to shelves with remaining quota by "backfilling" free slots. The same routine backs random feasible initialization and the completion of partial initial mappings.

- **Rule structure and distance**

The rules accept two formats: (I) $[set(A), set(B), weight]$, and (II) $dict-based\ \{antecedent: [\dots], consequent: [\dots], support/confidence/lift: \dots\}$. For each rule, the evaluator computes the minimum pairwise Manhattan distance between antecedent and consequent items under the current layout and then forms a weighted average across rules; an optional sigmoid transform sharpens sensitivity near adjacency ($smaller\ distance \rightarrow smaller\ transformed\ penalty$). These definitions are fully consistent with §2.5 and are consumed by the unified objective.

- **Evaluator–objective coupling**

The path cost for a single basket is returned by the mask-BFS routine; failures are mapped to a large constant. Averaging over baskets yields the path term, which, together with the rule term, is normalized by baseline estimates drawn from random feasible

```python
def shortest_path_cover_shelves(
    grid: Grid,
    start: Coord,
    basket_items: List[str],
    item_to_shelf: Dict[str, Coord],
) -> Dict[str, Any]:
    """
    BFS in state-space (position, visited_mask).
    You 'collect' an item's shelf by stepping on any aisle adjacent to that shelf.
    """
    H, W = len(grid), len(grid[0])
    if not in_bounds(start[0], start[1], H, W): return dict(ok=False, reason="Start out of bounds.")
    if not is_aisle(grid, start): return dict(ok=False, reason="Start must be on an aisle (0).")

    # unique shelves to visit (deduplicate shelves if multiple items on same shelf)
    shelves_needed = []
    seen = set()
    for item in basket_items:
        if item not in item_to_shelf:
            return dict(ok=False, reason=f"Item '{item}' missing in item_to_shelf.")
        s = item_to_shelf[item]
        if s not in seen:
            seen.add(s)
            shelves_needed.append(s)

    # pickup positions for each shelf
    picks = []
```

layouts and then linearly combined via $\alpha, \beta$. This ensures comparable scales between

terms and stable comparisons across different rule calibrations and basket sets.

**Listing 2.2:** Use BFS to collect an item

### 3.2.2 Workflow of the Optimizer

- **Pre-flight & Baselines**

The $run\_ga\_optimize(\dots)$ routine orchestrates the pipeline. It first builds the aisle/shelf grid with $build\_grid\_from\_shelves$, seeds the RNG ($random.seed(RANDOM\_SEED)$), and configures the optional rule-distance transform via $use\_sigmoid\_rule$ (either $rule\_tf = \{'type': 'sigmoid', 'k': 1.5, 'd0': 2.0\}$ or None). It then calls $estimate\_baselines(\dots)$, which samples multiple feasible random layouts ($random\_assignment$ followed by $repair\_capacity$) to obtain $path\_mu$ and $rule\_mu$ for normalization of the objective terms. This step only uses the given BASKETS and RULES—no new assumptions are introduced.

- **Representation & Seeding**

Each individual is a fixed-length integer vector encoding the $item \rightarrow shelf$ indices. If an initial (possibly partial) mapping is provided, $init\_population(\dots)$ completes it with $complete\_init\_mapping$, converts it to a gene via $mapping\_to\_gene$, and enforces the capacity bound through $gene\_repair\_capacity$.

```
# =========================
# GA representation & ops
# =========================
def mapping_to_gene(items: List[str], shelves: List[Coord], mapping: Dict[str, Coord]) -> List[int]:
    """Encode mapping as shelf indices per item (order of 'items')."""
    shelf_idx = {s: i for i, s in enumerate(shelves)}
    try:
        return [shelf_idx[mapping[it]] for it in items]
    except KeyError as e:
        missing = str(e).strip("'")
        raise KeyError(f"Item '{missing}' is missing in initial mapping. "
                       f"Use complete_init_mapping(...) to fill.") from e

def gene_to_mapping(items: List[str], shelves: List[Coord], gene: List[int]) -> Dict[str, Coord]:
    return {it: shelves[idx] for it, idx in zip(items, gene)}

def gene_repair_capacity(gene: List[int], shelves: List[Coord], items: List[str], max_per_shelf=MAX_ITEMS_PER_SHELF) -> List[int]:
    """Repair gene to satisfy shelf capacity."""
    loads = defaultdict(list)  # shelf_idx -> list of item indices
    for i, sidx in enumerate(gene):
        loads[sidx].append(i)

    # free slots (as shelf indices)
    free_slots = []
```

Remaining individuals are produced by $random\_assignment$ and then repaired.

This guarantees feasibility from generation zero while preserving prior knowledge from existing/category layouts.

**Listing2.3: mapping in GA**

- **Evolutionary Operators & Population Update**

At each generation, the algorithm keeps the top elite individuals (elitism), selects parents via $tournament\_select$, applies one-point crossover and random mutate, and immediately calls $gene\_repair\_capacity$ on the offspring. Selection drives convergence, crossover recombines good schemata, mutation maintains diversity, and the repair step preserves the shelf-capacity constraint for all intermediate solutions.

- **Fitness Evaluation & Termination**

Inside $run\_ga\_optimize(\dots)$, def $fitness\_of(gene)$: performs evaluation: decode with $gene\_to\_mapping$, enforce feasibility via $repair\_capacity$ and $validate\_shelf\_capacity$ (return 1e12 on violation), then call objective(...) to compute：

$$J(\pi) = \alpha \cdot \frac{\mathrm{Path}(\pi)}{\mathrm{path_m u} + \varepsilon} + \beta \cdot \frac{\mathrm{Rule}(\pi)}{\mathrm{rule_m u} + \varepsilon} \tag{3-1}$$

$evaluate\_layout(\dots)$ (backed by the mask-BFS) returns the average shortest path length over baskets; $rule\_proximity\_cost(\dots)$ returns the weighted minimum pairwise Manhattan distance per rule, optionally transformed by $rule\_tf$. After evaluating a full generation, the loop stops at generations, the best gene is decoded ($gene\_to\_mapping$), repaired and validated, and the final $item \rightarrow shelf$ mapping is returned.

### 3.2.3 Function Map & Implementation Notes

- **Data & Constraints Handling**

This cluster enforces feasibility: a hard capacity limit of 4 items per shelf, plus a repair routine that reassigns overflowed items to shelves with free quota after randomization and genetic operators. A partial initial mapping is automatically completed and validated. These routines keep the search within the feasible region,

stabilizing evaluation and convergence.

```python
def complete_init_mapping(
    items: List[str],
    shelves: List[Coord],
    init_mapping: Dict[str, Coord],
    max_per_shelf: int = MAX_ITEMS_PER_SHELF,
    seed: int = 1234
) -> Dict[str, Coord]:
    """Complete initial mapping so every item has a shelf; enforce capacity."""
    random.seed(seed)
    mapping = dict(init_mapping) if init_mapping else {}
    loads = defaultdict(int)
    for it, s in mapping.items():
        loads[s] += 1
        if loads[s] > max_per_shelf:
            raise ValueError(f"Initial mapping exceeds capacity at shelf {s}")

    free_slots = []
    for s in shelves:
        left = max_per_shelf - loads.get(s, 0)
        free_slots.extend([s]*max(0, left))

    for it in items:
        if it not in mapping:
            if not free_slots:
                raise ValueError("Initial mapping + remaining items exceed total capacity.")
            mapping[it] = free_slots.pop()
            loads[mapping[it]] += 1
    return mapping
```

**Listing2.4** Complete partial mapping

- **Semantics & Evaluation: Mask-BFS**

Pickup is defined as stepping on any adjacent aisle to a shelf. The evaluator constructs pickup sets and runs a mask-based BFS over (position, visited-targets) to obtain the shortest cover length; it reports failure otherwise. $evaluate\_layout$ wraps capacity checks and delegates to BFS. This is the path term in the objective.

- **Rule Distance with Optional Sigmoid**

Two rule formats are supported. For each rule, the minimum pairwise Manhattan distance between antecedent and consequent items is taken and weighted; an optional sigmoid transform sharpens sensitivity near adjacency. This defines the rule term reflecting proximity of strongly associated items.

- **Normalized Objective & GA Skeleton**

The objective normalizes both path and rule terms by Monte Carlo baselines and combines them with $\alpha, \beta$. The GA entry builds the grid, configures optional sigmoid,

estimates baselines, initializes the population (completed seed + random repaired), iterates with selection–crossover–mutation–repair–elitism, and returns the best decoded mapping. Normalization aligns scales; the GA skeleton ensures efficient feasible search.

```python
def objective(
    mapping: Dict[str, Coord],
    grid: Grid,
    start: Coord,
    baskets: Optional[List[List[str]]],
    rules: List[Any],
    alpha: float = 1.0,
    beta: float  = 1.0,
    baselines: Optional[Tuple[float, float]] = None,   # (path_mu, rule_mu)
    rule_transform: Optional[Dict] = None
) -> float:
    """Normalized linear objective: alpha*path_norm + beta*rule_norm (lower is better)."""
    # rule term
    rule_term = rule_proximity_cost(mapping, rules, weight_mode="auto", transform=rule_transform)

    # path term
    path_term = 0.0
    if baskets:
        for b in baskets:
            res = evaluate_layout(grid, start, b, mapping)
            path_term += (res['steps'] if res.get('ok') else 1e6)
        path_term /= max(len(baskets), 1)

    # normalization using baselines
    if baselines:
        path_mu, rule_mu = baselines
        path_norm = path_term / (path_mu + 1e-6) if baskets else 0.0
        rule_norm = rule_term / (rule_mu + 1e-6)
    else:
        path_norm = path_term
        rule_norm = rule_term

    return alpha * path_norm + beta * rule_norm
```

**Listing 2.5:** Normalized linear weighting

### 3.2.4 Parameters, Reproducibility & Integration

GA hyperparameters are exposed via $run\_ga\_optimize(\dots)$; set a fixed seed for reproducibility as needed. The result mapping $Dict[str, Coord]$ supports both audit printing ($print\_mapping\_table$) and direct UI rendering.

```python
# ========================
# GA main
# ========================
def run_ga_optimize(
    GRID_H: int,
    GRID_W: int,
    SHELVES: List[Coord],
    START: Coord,
    ITEM_NAMES: List[str],
    INIT_ITEM_TO_SHELF: Dict[str, Coord],
    RULES: List[Any],
    BASKETS: Optional[List[List[str]]] = None,
    pop_size: int = 30,
    generations: int = 100,
    cx_rate: float = 0.8,
    mut_rate: float = 0.3,
    elite: int = 2,
    alpha: float = 1.0,         # weight for normalized path term
    beta: float  = 1.0,         # weight for normalized rule term
    use_sigmoid_rule: bool = True,
    baseline_samples: int = 30
) -> Dict[str, Coord]:
    """
    Run GA to optimize item placement under capacity constraints.
    Objective = alpha * normalized_path + beta * normalized_rule.
    """
    random.seed()

    grid = build_grid_from_shelves(GRID_H, GRID_W, SHELVES)
    rule_tf = {'type':'sigmoid', 'k':1.5, 'd0':2.0} if use_sigmoid_rule else None
```

**Listing2.6:** GA algorithm parameter setting (preliminary)

## 3.3 Front-end System Implementation

The primary objective of the front-end interface is to provide an intuitive, configurable, and high-performance operational environment, enabling users to easily complete multi-layer shelf layout settings, trigger optimization, and analyze results.

### 3.3.1 Technology Stack and Architectural Selection

The following technology combination was utilized to achieve an optimal balance between development efficiency and application performance:

| Technical | Role and Function | Rationale and Advantages |
|---|---|---|

| Component | | |
|---|---|---|
| Styling: Tailwind CSS | Interface layout and responsive design. | Adopts a Utility-First approach for rapid UI construction, ensuring consistent appearance across different resolutions. |
| Interaction: Vanilla JavaScript | State management, DOM manipulation. | Avoids the overhead of large front-end frameworks, ensuring fast application loading and clean, controllable code. |
| Visualization: Chart.js | Optimization score comparison. | Used to translate abstract scoring data into easily understandable charts, providing clear visualization of optimization effects. |

### 3.3.2 Implementing Multi-Dimensional Structures and Custom Sizing

The system's design supports both multi-layer structures and custom sizing of shelves to fully simulate real-world retail environments.

- **Shelf Model and Data Transmission Format**

The system employs a multi-dimensional shelf model that differentiates between vertical and horizontal coordinates:

Data Transmission Format: When submitting layout data, the front-end uses a structured dictionary. The key is the shelf position, and the value is the list of products at that specific location. This format ensures the back-end algorithm can recognize vertical adjacency between products.

**Layout = {(Shelf Row, Shelf Col.): [Product1, Product2, Product3, Product4]}**

For example, (0, 1): ['Milk', 'Cereal', 'Yogurt', 'Water'] indicates that four items are placed on the shelf in row 1 and column 2 of the layout.

- **Shelf Size Configuration Capability**

The front-end is capable of accommodating shelves of various sizes. The initShelves() function is parameterized to support user customization of the rows and columns (e.g., within a 1x1 to 10x10 grid range), dynamically rendering the

corresponding multi-layer grid interface.

### 3.3.3 UI

- **Product Configuration and Association Analysis Area**

This area is designed to enhance the efficiency and accuracy of manual configuration by providing algorithmic references.

| Control/Display Item | Functional Description and Interaction Logic | Core Value |
|---|---|---|
| Product Search Input | Users can enter keywords to trigger real-time filtering of the product list via JavaScript. | Simplifies selection from a large product inventory. |
| Product List | Displays all available products. Clicking a product sets it as selected. | Implements the click-to-select interaction pattern. |
| Association Confidence Threshold Slider | Allows users to adjust the confidence threshold for association rules between 0.0 and 1.0. Changes immediately trigger rule filtering. | Enables users to control the strictness of the algorithmic basis. |
| Recommended Pairs Display | Lists high-confidence product pairs obtained from the back-end /rules endpoint and filtered by the threshold slider. Listed in descending order of confidence. | Translates GNN model results into actionable layout recommendations. |

- **Layout Configuration and Operation Area**

This area is the core platform for user interaction, layout operation, optimization triggering, and viewing immediate feedback.

Multi-Layer Shelf View: The interface simulates multi-layer shelves using vertically stacked grids (shelf-grid). After selecting a product in UI-A, the user clicks a grid cell, triggering handleCellClick. This function accurately captures the cell's shelf

ID and layer ID, completing the product placement or removal operation.

- Real-time Score Display (#current-score): Any layout modification instantly triggers the calculateLayoutScore() function, sending the latest multi-dimensional layout data to the back-end API /calculate-current-score. The score is received and updated in real-time, providing the user with immediate quantitative feedback.
- **Optimization Results and Comparison Area**

This area presents the output of the optimization algorithm and offers a quantitative comparison to reinforce the results' validity.

Optimized Layout Shelf: Upon receiving the optimal multi-dimensional layout data from the back-end, the front-end uses the renderOptimizedLayout function to precisely draw the algorithm's recommended product arrangement.

Score Comparison Chart: Chart.js is utilized to generate a clear bar chart. The chart contrasts the "Original (User Manual) Score" with the "Optimized (Algorithm) Score," quantitatively demonstrating the improvement in total association achieved by the optimization algorithm.

# 4 Discussion

## 4.1 Core Innovation and Value of the System Design

This system's design addresses three long-standing pain points in offline retail layout optimization through an integrated technical solution: data silos, empirical bias, and failure to balance multiple objectives. This approach possesses both theoretical and practical value.

To overcome the limitations of single-method association analysis (Apriori's over-reliance on frequent co-occurrences or GraphSAGE's opacity in sparse data), the system employs a parallel Apriori + GraphSAGE framework (Section 2.3) to implement multimodal association mining, connecting explicit and implicit consumer needs. This

design achieves two key results:

- Explicit pattern capture: Apriori leverages statistical frequency to identify high-support patterns (e.g., the 45% co-occurrence rate for "infant formula → wet wipes" (Section 1.1), ensuring interpretability for retailers.
- Implicit relationship mining: GraphSAGE learns latent similarities (e.g., "diapers → beer" in parent-centric transactions) through neighborhood aggregation (Section 2.3.2), addressing the problem of traditional methods missing low-frequency but high-value "long-tail items" relationships. As verified in Section 3.1.2, the co-occurrence graph constructed based on transaction data retains 37% more meaningful edges than simply using fixed-threshold Apriori, laying a solid foundation for subsequent layout optimization.

At the same time, we adhere to a user-centric technical implementation. Our front-end design (Section 3.3) enhances system usability, a critical yet often overlooked aspect of retail SaaS tools. Key features include:

- Real-time score feedback: Any manual layout adjustment triggers instant calculations, helping non-technical users quantify layout quality.
- Association rule visualization: "Recommended pair display" transforms graph model embeddings into actionable recommendations, bridging the gap between algorithmic results and business operations.

## 4.2 Limitations of the Current System

While this system addresses key industry pain points, three limitations remain that will guide future improvements:

### 4.2.1 Data Dependency and Sparsity Challenges

This system relies on high-quality transaction data to effectively train the model. However, our market research indicates that 17% of small retailers lack POS systems, and 62% of medium-sized stores have incomplete transaction records. In such cases,

the model's embedding quality may deteriorate, reducing the confidence of association rules and leading to noisy rules.

### 4.2.2 Limited Adaptability to Complex Store Scenarios

The current model assumes uniform shelf types (standard 1×1 shelves), ignoring real-world variations. First, shelf attributes. Refrigerated shelves or high-bay shelves have fixed locations, and the system currently treats them as ordinary shelves, resulting in suboptimal path planning. Second, the diversity of store formats. Convenience stores require different optimization priorities than large supermarkets, but the system's weight adjustments are still manual.

### 4.2.3 Algorithm Efficiency for Large Catalogs

For retailers with large product categories, the computational cost of GA increases significantly, making real-time adjustments unacceptable. When evaluating baskets containing a large number of items, mask-BFS for path computation becomes slow because it has to check the reachability of all target shelves.

## 4.3 Future Improvement Directions

### 4.3.1 Expanding Data Sources with Edge Computing

Real-time IoT data (e.g., shelf sensors for inventory level monitoring and camera-based customer tracking) will be integrated to supplement sparse transaction data, enabling adjustments to shelf placement even with limited POS records. Furthermore, edge computing nodes will be deployed in stores to pre-process data, reducing reliance on cloud servers and enabling near-real-time optimization.

### 4.3.2 Upgrading the Algorithm Architecture

Replacing the Genetic Algorithm (GA) with Reinforcement Learning (RL) for dynamic optimization: the store is considered the environment, layout adjustments are

considered actions, and sales per square meter (S/P/M) is considered the reward. Reinforcement Learning can adapt to long-term changes, such as seasonal demand fluctuations, without retraining, eliminating the need for manual weight adjustments.

### 4.4.3 Deepening Scenario Customization

Shelf-type-aware constraints will be established, adding shelf attributes to the grid model to ensure the algorithm only places perishable items near refrigerated shelves and avoids moving fixed shelves. A multi-format model library is also built, enabling one-click optimization of different retail formats by pre-training Apriori/GraphSAGE parameters for convenience stores, supermarkets, and fresh food stores. Sales can also be forecasted by integrating sales, using historical transaction data and layout scores to predict sales changes, providing retailers with a clearer business rationale for implementation.

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# 6 Mapping of System Functionalities and Modular Courses

| Courses | Knowledge, techniques and skills |
|---|---|
| Machine Reasoning (MR) | Heuristic Search / Evolutionary Computation;<br>Planning & Search;<br>Constraint Satisfaction & Feasibility Repair;<br>Objective Design & Normalization;<br>GNN-based Embedding; |
| Reasoning Systems (RS) | Knowledge Representation: Product Association Graph / Rule Store;<br>Apriori Algorithm;<br>Product Association Rule Pruning & Filtering;<br>Association-Rule Weighting;<br>Distance & Similarity: Min Pairwise Manhattan Distance, Sigmoid Transform;<br>Integrated Evaluation of Rule Term and Path Term; |
| Cognitive Systems (CGS) | Explainable Outputs;<br>Human-in-the-Loop Tuning (α/β Weights, Crossover/Mutation Rates, Sigmoid Toggle);<br>Visual-Analytics Ready (Grid Layout & Path Visualization Interfaces); |

# 7. Product Shelf Layout Optimization System User Guide

## System Introduction

The Shelf Layout Optimization System is an intelligent tool based on shopping basket data analysis. Using association rules and graph neural network technology, it helps merchants optimize shelf placement, improving customer shopping efficiency and cross-buying rates. The system supports data upload, shelf layout design, product allocation, and intelligent optimization.

## Installation and Deployment

### Environmental requirements

- Python 3.8+
- Dependency packages: see requirements.txt (can be installed via pip install -r requirements.txt)

### Start the application

Bash python app.py
Access the system: Open the browser http://127.0.0.1:5000

## Operation process

### 1. Data upload

- Click the **[Data Upload]** option in the navigation bar
- Click the **[Choose File]** button in the upload area to upload the file containing the shopping cart data (common data formats are supported)
- After successful upload, the system will automatically parse the data and jump to the **[Data Summary]** page

### 2. View data summary

On the Data Summary page, you can view the following information:

- Total transactions: The total number of transaction records analyzed by the system
- Product Type: The total number of products included in the data
- Average number of items in each transaction: The average number of items in each transaction
- Top 10 Popular Products: Displays the 10 most popular products through a bar chart
- Product distribution: Use pie charts to show the proportion of different categories of products

**3. Shelf layout editing**

- Click the navigation bar **[Shelf Layout]** to enter the editing page
- Click on the shelf location in the 10×10 grid (grey grids indicate selectable shelves)
- Notes:
    - Each shelf can hold up to 4 types of products
    - The system will prompt the minimum number of shelves required (calculated based on the total number of products)
- Action buttons:
    - **[Clear Selection]**: Cancel all selected shelf positions
    - **[Save layout and continue]**: Confirm the shelf layout and proceed to the next step

**4. Original product layout input**

After completing the shelf layout settings, the system will automatically enter the original product layout input page:

- The system generates a product allocation form for each selected shelf
- Select the products to be placed on each shelf (multiple selections are allowed, up to 4 types)
- Accessibility:
    - **[Automatically allocate remaining products]**: The system randomly allocates unallocated products to the remaining shelves

- **[Clear Allocation]**: Cancel all product allocations

- After completing all product allocations, click **[Save original layout and optimize]** to submit the system for optimization calculation

**5. View optimization results**

After the optimization is completed, the system will display the optimized shelf layout:

- An optimized 10×10 grid layout showing the products on each shelf
- Layout optimization instructions:
  - Highly related products have been placed in adjacent locations
  - Popular products are placed in more accessible locations
  - Products of the same category are grouped together
- Optimization effect: Displays the expected increase in cross-purchase rate after optimization

**6. Check product association rules**

- Click the navigation bar **[Recommendation Rules]** to view the product association rules
- Association rules with different confidence levels can be filtered using the "confidence threshold"
- Association rules show the purchase associations between products and can be used to guide promotional activities and shelf placement.

# Functional Description

### Grid layout operations

- A 10×10 grid representing the store's shelf layout
- In the optimized layout, each shelf can display up to 4 products, and the excess will be prompted as "full"
- Hover your mouse over the product name to see the full name

### Data processing

The system automatically analyzes shopping basket data and extracts product

association rules, which mainly include:

- The purchase association strength between products
- Product popularity
- Shopping cart combination mode

**Optimization principle**

The system optimizes shelves based on the following principles:

1. Highly related products (often purchased together) are placed in adjacent locations
2. Popular products are placed in more accessible locations
3. Items of the same category are grouped together to make it easier for customers to find them.

# requirements

# platform: win-64

apriori-python=1.0.4
matplotlib=3.9.2
natsort=8.4.0
numpy=1.26.4
opencv-python=4.10.0.84
pandas=2.2.2
pip=24.2
python=3.10.14
pytorch=2.2.2=py3.10_cuda12.1_cudnn8_0
scikit-learn=1.7.2
scipy=1.14.1
torchvision=0.17.2
tqdm=4.66.5
yaml=0.2.5