

Lab 08

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Representation of floating point

- The normalized representation of a floating point number is ± X * 2^E, where 1 ≤ X < 2
- IEEE-754 SP standard expresses floating point numbers in 32 bits:
 - Sign S: 0 if positive, 1 if negative.
 - Exponent E with bias 127.
 - Mantissa M: fractional part of X.

31 30 23 22 0

S E + 127 M

Exercise

- There is not any native implementation of floating point numbers in Cortex-M3: we want to implement it in software.
- If Rn contains the integer part and Rm the fractional part, write in Rd the floating point number according to IEEE-754 SP standard.

Example

- Rn =1998, Rm =142578125
- 1998.142578125 expressed in normalized scientific notation is 1.9513111145 * 2¹⁰
- E + $127 = 137 = 10001001_2$

31 30 23 22 0

0 1 0 0 0 1 0 0 1 1 1 1 1 1 0 0 1 1 1 0 0 0 1 0 0 1 0 0 1 0 0 0

You can check the result here: https://www.h-schmidt.net/FloatConverter/leee754.html

Computation of the exponent

- The exponent is equal to the position of the first bit set to 1 in the binary representation of the integer part.
- Example: $1998 = 11111001110_2$

After adding the bias (127), E = 137

Computation of the mantissa (1)

- The first bits of the mantissa are taken from the binary representation of the integer part, after removing the initial '1'.
- Example: $1998 = 11111001110_2$
 - The first 10 bits of the mantissa are 1111001110
- The remaining N bits of the mantissa are obtained by converting the fractional part.
- In the conversion, the fractional part is interpreted as an integer number.
- Example: X = 142578125

Computation of the mantissa (2)

- Example: X = 142578125
- Let P be the lowest power of 10 which is higher than X. P = 1000000000
- The following loop is repeated N times:
 - X is doubled
 - if the result is higher than P
 - the next bit of the mantissa is 1 and the new value of X is X = X – P
 - else the next bit of the mantissa is 0
 - repeat loop

Computation of the mantissa (3)

| iteration | X | 2 * X | bit |
|-----------|-----------|------------|-----|
| 1 | 142578125 | 285156250 | 0 |
| 2 | 285156250 | 570312500 | 0 |
| 3 | 570312500 | 1140625000 | 1 |
| 4 | 140625000 | 281250000 | 0 |
| 5 | 281250000 | 562500000 | 0 |
| 6 | 562500000 | 1125000000 | 1 |
| 7 | 125000000 | 250000000 | 0 |
| 8 | 250000000 | 500000000 | 0 |
| 9 | 50000000 | 1000000000 | 1 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |

Conversion to IEEE-754 SP

We call a coprocessor as follows:

```
CDP proc, imm, dest, op1, op2, sign
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- CDP: instruction to call a coprocessor.
- proc: called coprocessor. We call p0.
- imm: operation executed by coprocessor.
 - imm = 1 for conversion to the IEEE-754 SP.
- dest, op1, op2: registers containing result, integer part, fractional part, respectively.
- sign: 0 if positive, 1 if negative.

CDP encoding

Only meaningful bits are shown:

31 27 24 23 20 19 16 15 12 11 8 7 5 3 0

CDP imm op1 dest coproc. sign op2

- The registers used by a coprocessor are named c0, c1, ..., c15.
- Example: CDP p0, #1, c3, c1, c2, #1
 27 24 23 20 19 16 15 12 11 8 5 3 0
 11 10 0001 0001 0011 0000 001 0010

Software implementation

- Cortex-M3 has not any coprocessor.
- CDP raises a usage fault.
 - usage fault must be enabled, otherwise a hard fault is raised.
- The conversion to the IEEE-754 SP format is done in the exception handler.
- The following slides list the steps to be implemented in the exception handler.

1) Recognizing coprocessor fault

- Check the proper bit in the Usage Fault Status Register.
- If the exception is due to a coprocessor instruction, branch to the corresponding piece of code.
- Otherwise, write a dummy implementation of other usage faults (e.g., B .).

2) Recognizing offending instruction

- Before entering the exception handler, PC is saved in the stack (MSP or PSP) with offset 24.
 - use of MSP or PSP is determined by reading LR.
- Load 4 bytes (1 word) from the address of PC
 - when entering the exception, the operating code of the offending instruction is saved as a sequence of two distinct halfwords.
 - when the two halfwords are retrieved as a single word from the stack, they must be switched (e.g., by rotating 16 positions) to preserve the original order.

2) Recognizing offending instruction

- If the instruction is CDP p0, #1, ...
 execute code for IEEE-754 SP conversion.
 - bits are xxxx 1110 0001 xxxx xxxx 0000 xxxx xxxx

3) Changing return address

- Usage faults return to the same instructions that triggered the fault.
- Since we do not want to execute CDP again after the handler, the return address must be updated to the next instruction.
- Update the value of PC saved in the stack (with offset 24) with the new value PC + 4.

4) Accessing source registers

- Index of registers ranges between 0 and 7.
- We assume that r4 corresponds to c0, r5 = c1, r6 = c2, ..., r11 = c7.
- Save registers r4-r11 in the stack (as required by AAPCS).
 - instructions in steps 1, 2, 3 can not modify r4-r11.
- After extracting the index of a source register from the encoded instruction, you can extract its content from the stack with offset index * 4.

5) Computing the result

- Bit 5 in the encoded instruction (representing the sign) is the most significant bit of the result.
- The other bits are set by computing exponent and mantissa.
- Finally, update the destination register saved in the stack with the new computed value.

6) Concluding the handler

- Restore values of register r4-r11 with a POP.
 - The destination register will contains the IEEE-754 SP representation, since the corresponding entry in the stack was updated in step 5.
- Return to the program with BX LR.
 - the next instruction will be the one after CDP, since the value of PC (automatically retrieved from the stack) has been updated in step 3.