COMPUTER VISION

ASSIGNMENT 3

Solution to Problem (a)

Approach 1:

We can show that the **Difference of Gaussian** is equivalent to **Laplacian of Gaussian**. When we apply the Gaussian filter (which is a Low pass filter) on an image, high frequency noise is removed. On the output of the Gaussian filter, when we apply Laplacian filter (which is a High Pass Filter), Low frequency noise is removed. Hence, the **Laplacian of Gaussian** acts as a Bandpass Filter. Consequently, **Difference of Gaussian** must be a bandpass filter.

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Computer Vision

M.Tech (VIPES)

Task (a)

The given expression for DoG is,

$$DoG = \frac{1}{2\pi}\left[\frac{1}{\sigma_1^2}\exp\left(-\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma_1^2}\right)\right.$$

$$\left.-\frac{1}{\sigma_2^2}\exp\left(-\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma_2^2}\right)\right], \sigma_1<\sigma_2$$

For the sake of simplicity, consider,

$$\mu_x = 0 \quad ; \quad \mu_y = 0$$
$$\sigma_1 = \sigma \quad ; \quad \sigma_2 = \sigma + \delta\sigma$$

So, we can write,

$$DoG = \frac{1}{2\pi}\left[\frac{1}{\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) - \frac{1}{(\sigma+\delta\sigma)^2}\exp\left(-\frac{x^2+y^2}{2(\sigma+\delta\sigma)^2}\right)\right]$$

Assume that,

$$f(\sigma) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

so, $$f(\sigma+\delta\sigma) = \frac{1}{2\pi(\sigma+\delta\sigma)^2}\exp\left(-\frac{x^2+y^2}{2(\sigma+\delta\sigma)^2}\right)$$

so, we can write,

$$DoG = f(\sigma) - f(\sigma+\delta\sigma)$$

Divide and multiply by $\delta\sigma$,

$$DoG = \delta\sigma\left[\frac{f(\sigma) - f(\sigma+\delta\sigma)}{\delta\sigma}\right]$$

$$\Rightarrow DoG = -\delta\sigma\left[\frac{f(\sigma+\delta\sigma)-f(\sigma)}{\delta\sigma}\right]$$

$$\Rightarrow DoG = -\delta\sigma\left[\frac{f(\sigma+\delta\sigma)-f(\sigma)}{(\sigma+\delta\sigma)-\sigma}\right]$$

$$\Rightarrow DoG = -\delta\sigma\frac{\partial f(\sigma)}{\partial\sigma}$$

Now; we will solve $\frac{\partial f(\sigma)}{\partial \sigma}$ first.

$$\frac{\partial f(\sigma)}{\partial\sigma} = \frac{\partial}{\partial\sigma}\left[\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right]$$

$$= -\frac{2}{2\pi\sigma^3}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)+\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times -\frac{(x^2+y^2)}{2}\times\frac{-2}{\sigma^3}$$

$$\Rightarrow \frac{\partial f(\sigma)}{\partial\sigma} = \left[-\frac{1}{\pi\sigma^3}+\frac{(x^2+y^2)}{2\pi\sigma^5}\right]\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

so, $$DoG = -\delta\sigma\frac{\partial f(\sigma)}{\partial\sigma}$$

$$\Rightarrow DoG = \delta\sigma\left[\frac{1}{\pi\sigma^3}-\frac{(x^2+y^2)}{2\pi\sigma^5}\right]\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

Now, we will look at the Laplacian of Gaussian,

$$LoG = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f(\sigma)$$

So, $$LoG = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right]$$

$$= \frac{\partial^2}{\partial x^2}\left[\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right] + \frac{\partial^2}{\partial y^2}\left[\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times -\frac{2x}{2\sigma^2}\right]$$

$$+ \frac{\partial}{\partial y}\left[\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times -\frac{2y}{2\sigma^2}\right]$$

$$\Rightarrow LoG = \frac{\partial}{\partial x}\left[\frac{-x}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right] + \frac{\partial}{\partial y}\left[\frac{-y}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right]$$

$$\Rightarrow LoG = \left[-\frac{1}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) - \frac{x}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times -\frac{2x}{2\sigma^2}\right]$$

$$+ \left[-\frac{1}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) - \frac{y}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times -\frac{2y}{2\sigma^2}\right]$$

$$\Rightarrow LoG = -\frac{2}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) + \frac{x^2}{2\pi\sigma^6} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$+ \frac{y^2}{2\pi\sigma^6} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\Rightarrow LoG = \left[-\frac{1}{\pi\sigma^4} + \frac{x^2+y^2}{2\pi\sigma^6}\right]\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\Rightarrow LoG = -\frac{1}{\sigma}\left[\frac{1}{\pi\sigma^3} - \frac{(x^2+y^2)}{2\pi\sigma^5}\right]\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

On comparing the expressions of DoG and LoG, it is clear that,

$$LoG = -\frac{1}{\sigma} \times \frac{DoG}{\delta\sigma}$$

or, $$\boxed{LoG = -\frac{1}{\sigma\delta\sigma} DoG}$$

or, $$\boxed{DoG = -\sigma\delta\sigma . LoG}$$

As LoG works as bandpass filter, DoG should also be a bandpass filter.

Approach 2:

We know that the sum of all the elements of the kernel of a Lowpass filter is 1 and the sum of all the elements of the kernel of a High Pass filter is 0 . We can see in the Task_a.ipynb file that the sum of all the elements of DoG kernel is between 0 and 1, so DoG filter must be a Bandpass filter.

Solution to Problem (b)

At lower values of sigma, we get more details in the image and as we increase the value of sigma, it's similar to looking at a broader area and the details in the image are less visible.

Case 1: $\sigma 1 \ll \sigma 2$; $\sigma 1$ is small, about a few pixel widths.

For the first case, we have considered the following values of sigma,

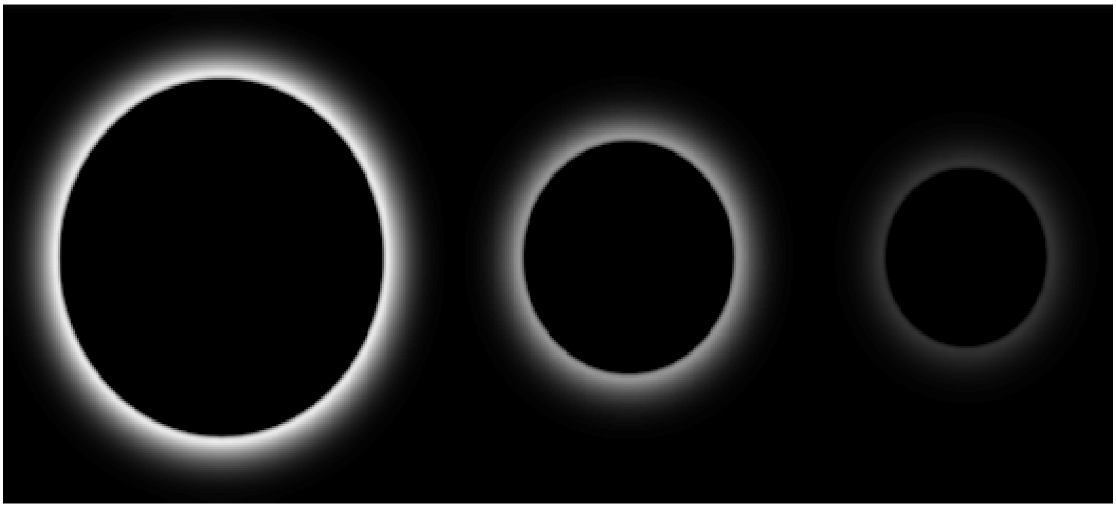
sigma1 = 1.6 sigma2 = 12.8

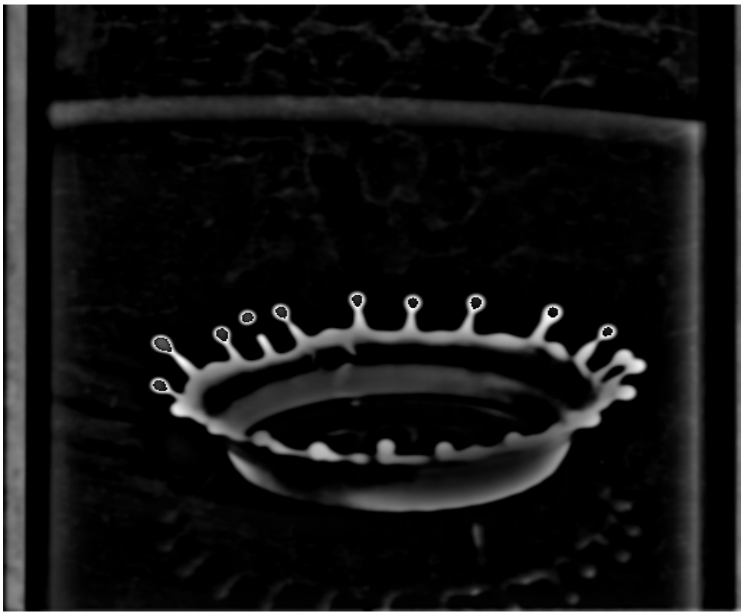
When we take the difference of the kernels corresponding to the two values of the sigma (i.e. 1.6 and 12.8) and convlove it with the given images , the resulting output images are as shown below.

- **Blob Detection:** Blobs in the image refers to binary large objects.

  When sigma = 1.6 , zero crossings in the image are more visible.

  When sigma = 12.8, only the strong edges will remain in the image and the weak edges will be vanished. Higher values of sigma are more suitable for blob detection.





As the blobs are the groups of connecting pixels, the balls (in one of the images) can be considered as the blobs. We can observe that blobs in the images are more visible and detectable in this case (when sigma1 << sigma2).

- **Edge detection:** Lower values of sigma are more suitable for the edge detection. But in this case, as we convolve the image with the difference of the gaussian kernel, we see that no weak edges are visible in the image (only strong ones are there). So edge detection is not perfect in this case.

- **Edge Localization:** Edge localization refers to the thinning of the detected edges. In this case, we can see that the localization of edges is poor. No thin edges are visible.

Case 2: $\sigma 1 \approx \sigma 2$; $\sigma 1$ is large, about many pixel widths.

For the second case, we have considered the following values of sigma,

sigma1 = 15   sigma2 = 17

Difference of Gaussian corresponding to the above values of sigma, when applied on given images, the output images resulted are as follows:

- **Blob Detection:** As values of both the sigma1 and sigma2 are higher, blobs would be more visible if we had taken only the gaussian; but as we have taken the difference between the two, blobs are not visible.
- **Edge Detection:** Edges in the above output images are also not visible. Only the strong changes in the intensity can be observed but there is no trace of the weak edges.
- **Edge Localization:** Edge localization in this case is also poor.

Case 3: $\sigma 1 \approx \sigma 2$; $\sigma 1$ is small, about a few pixel widths.

For the third case, we have considered the following values of sigma,

sigma1 = 1.6 sigma2 = 2.26

Difference of Gaussian corresponding to the above values of sigma, when applied on given images, the output images resulted are as follows:

- **Blob Detection:** Smaller values of sigma are not suitable for detecting the blobs of large size. Here the values of both the sigma1 and sigma2 are smaller. So, blobs in the images are not that obvious.
- **Edge Detection:** Smaller values of sigma are better for edge detection. When sigma is 1.6, even weak edges are detected but as we increase sigma a little, only relatively stronger images remain. Now, as we have convolved the image with the DoG kernel, the edges in the image are quite visible. So, edge detection is better in this case.

- **Edge Localization:** We can observe that the detected edges in the image are thin and describe the shape of the object in a much better way. So, the edge localization in this case is better.