

Differentiation & Integration

Part 1: Differentiation Rules

1. Power Rule

Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Example

$$\frac{d}{dx}(x^4) = 4x^3$$

2. Constant Rule

Rule

$$\frac{d}{dx}(c) = 0$$

Example

$$\frac{d}{dx}(7) = 0$$

3. Constant Multiple Rule

Rule

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

Example

$$\frac{d}{dx}(5x^3) = 15x^2$$

4. Sum and Difference Rule

Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Example

$$\frac{d}{dx}(x^2 + 3x) = 2x + 3$$

5. Exponential Functions

(a) Natural Exponential

Rule

$$\frac{d}{dx}(e^x) = e^x$$

Example

$$\frac{d}{dx}(e^x) = e^x$$

(b) General Base

Rule

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Example

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

6. Logarithmic Functions

(a) Natural Logarithm

Rule

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Example

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

(b) Log Base a

Rule

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

7. Trigonometric Functions

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

8. Chain Rule

Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example

$$\frac{d}{dx}(x^2 + 1)^5$$

Let $u = x^2 + 1$

$$= 5(x^2 + 1)^4 \cdot 2x = 10x(x^2 + 1)^4$$

9. Product Rule

Rule

$$\frac{d}{dx}(uv) = u'v + uv'$$

Example

$$\begin{aligned} & \frac{d}{dx}(xe^x) \\ &= 1 \cdot e^x + x \cdot e^x = e^x(x + 1) \end{aligned}$$

10. Quotient Rule

Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Example

$$\begin{aligned} & \frac{d}{dx} \left(\frac{x^2}{x+1} \right) \\ &= \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} \end{aligned}$$

11. Implicit Differentiation

Rule

Differentiate both sides, treating y as a function of x .

Example

$$x^2 + y^2 = 25$$

$$2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

12. Higher-Order Derivatives

Rule

$$\frac{d^n y}{dx^n}$$

Example

$$y = x^3$$

$$y' = 3x^2, \quad y'' = 6x$$

13. Derivative as Rate of Change

$$\text{Rate of change} = \frac{dy}{dx}$$

Example

If $s(t) = t^2$, velocity:

$$v(t) = \frac{ds}{dt} = 2t$$

14. Common Mistakes to Avoid

- Forgetting chain rule
 - Missing negative signs
 - Treating constants as variables
 - Using quotient rule unnecessarily
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Part 2: Integration Rules

1. Power Rule

Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

Example

$$\int x^3 dx = \frac{x^4}{4} + C$$

2. Constant Multiple Rule

Rule

$$\int kf(x) dx = k \int f(x) dx$$

Example

$$\int 5x^2 \, dx = 5 \cdot \frac{x^3}{3} = \frac{5x^3}{3} + C$$

3. Sum and Difference Rule

Rule

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

Example

$$\int (x^2 + 3x) \, dx = \frac{x^3}{3} + \frac{3x^2}{2} + C$$

4. Integral of a Constant

Rule

$$\int c \, dx = cx + C$$

Example

$$\int 7 \, dx = 7x + C$$

5. Logarithmic Rule

Rule

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

Example

$$\int \frac{1}{x} dx = \ln|x| + C$$

6. Exponential Rules

(a) Natural Exponential

Rule

$$\int e^x dx = e^x + C$$

Example

$$\int e^x dx = e^x + C$$

(b) General Base

Rule

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$$

Example

$$\int 2^x dx = \frac{2^x}{\ln 2} + C$$

7. Trigonometric Integrals

Function	Integral
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Function	Integral
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$

8. Substitution Rule (Reverse Chain Rule)

Rule

If $u = g(x)$, then

$$\int f(g(x))g'(x) dx = \int f(u) du$$

Example

$$\int 2xe^{x^2} dx$$

Let $u = x^2$, $du = 2xdx$

$$= \int e^u du = e^u + C = e^{x^2} + C$$

9. Integration by Parts

Rule

$$\int u dv = uv - \int v du$$

Example

$$\int xe^x dx$$

Choose $u = x \Rightarrow du = dx$, $dv = e^x dx \Rightarrow v = e^x$

$$= xe^x - \int e^x dx = xe^x - e^x + C$$

10. Definite Integrals (Evaluation Rule)

Rule

$$\int_a^b f(x) dx = F(b) - F(a)$$

Example

$$\int_1^3 x^2 dx = \left[\frac{x^3}{3} \right]_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$$

11. Zero and Symmetry Properties

Zero width

$$\int_a^a f(x) dx = 0$$

Odd function

$$\int_{-a}^a f(x) dx = 0$$

Even function

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

12. Constant of Integration

Key Rule: Every indefinite integral must include $+C$

Part 3: Side-by-Side Comparison

Differentiation vs Integration Table

Operation	Differentiation	Integration
Power Rule	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
Constant	$\frac{d}{dx}(c) = 0$	$\int c dx = cx + C$
Exponential (e^x)	$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
Natural Log	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
Sine	$\frac{d}{dx}(\sin x) = \cos x$	$\int \sin x dx = -\cos x + C$
Cosine	$\frac{d}{dx}(\cos x) = -\sin x$	$\int \cos x dx = \sin x + C$
General Base (a^x)	$\frac{d}{dx}(a^x) = a^x \ln a$	$\int a^x dx = \frac{a^x}{\ln a} + C$
Constant Multiple	$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x) dx = k \int f(x) dx$
Sum Rule	$\frac{d}{dx}[f \pm g] = f' \pm g'$	$\int [f \pm g] dx = \int f dx \pm \int g dx$
Product	$\frac{d}{dx}(uv) = u'v + uv'$	$\int u dv = uv - \int v du$
Composition	Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	Substitution: $\int f(g(x))g'(x) dx = \int f(u) du$

Key Relationships

Concept	Differentiation	Integration
Purpose	Find rate of change / slope	Find area / accumulation
Result	Instantaneous rate	Total accumulated value
Geometric	Slope of tangent line	Area under curve
Physics	Velocity from position	Position from velocity
Notation	$\frac{d}{dx}$ or $f'(x)$	$\int dx$ or $\int f(x)dx$
Constant	Disappears ($\frac{d}{dx}(c) = 0$)	Appears ($\int f dx$ includes $+C$)
Reversal	Differentiating undoes integration	Integrating undoes differentiation



Memory Tips

Differentiation

- **Breaks things down** — reduces power, finds rate
- **Speed / Slope / Sensitivity**
- **Chain Rule** — work from outside to inside

Integration

- **Builds things back up** — increases power, accumulates
- **Accumulation / Area / Total**
- **Don't forget $+C$** for indefinite integrals

Common Mistakes to Avoid

Differentiation Mistakes

1. Forgetting the chain rule when differentiating composite functions
2. Missing negative signs (especially with trig functions)
3. Treating constants as variables
4. Using the quotient rule when product rule would be simpler

Integration Mistakes

1. Forgetting the constant of integration $+C$
 2. Forgetting to substitute back after u -substitution
 3. Sign errors in integration by parts
 4. Not checking if the integral requires a technique (substitution, by parts, etc.)
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Quick Reference

Fundamental Theorem of Calculus

$$\int_a^b f(x) dx = F(b) - F(a)$$

where $F'(x) = f(x)$

This connects differentiation and integration as inverse operations!
