Experiment No. 05

Aim: Analysing Data trends through Curve fitting techniques.

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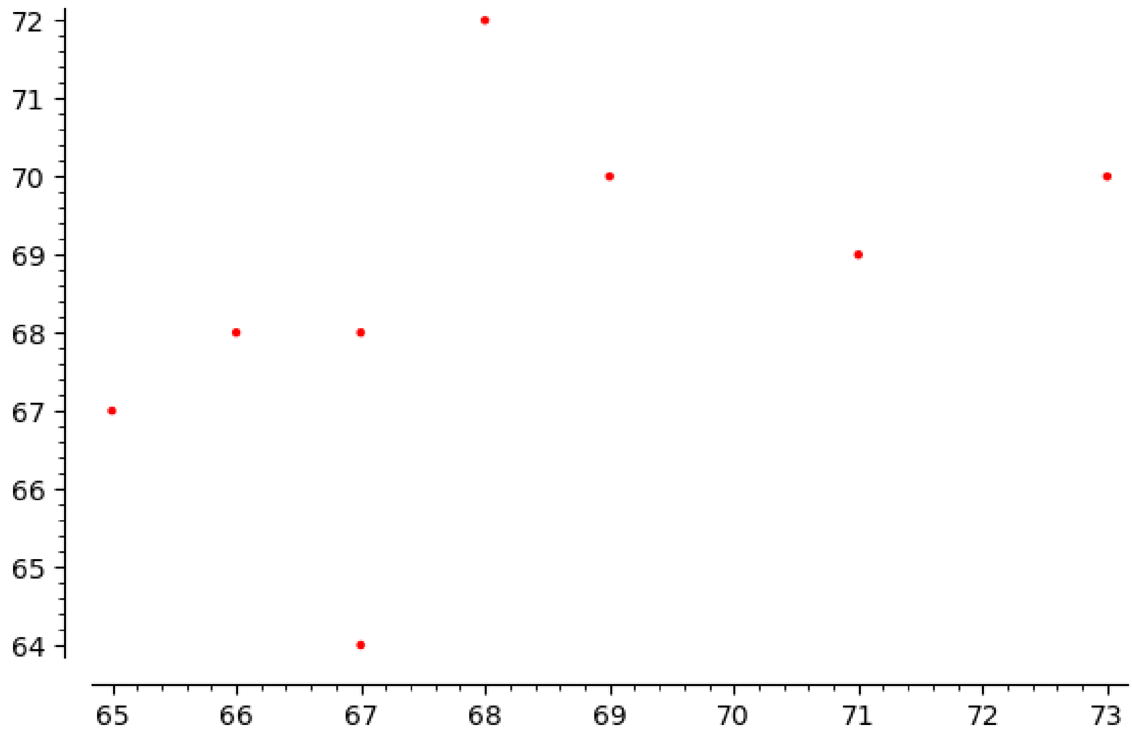
In [276]:

```
data=[(71,69), (68,72), (73,70), (69,70), (67,68), (65,67), (66,68), (67,64)]
point(data, color="red")
```

Out[276]:



In [277]:

```
a,b = var('a,b') #defining variable
y(x)=a+b*x # Writing equation to fit point
```

In [278]:

```
f1=find_fit(data, y) #fitting data in eq
show(f1)
```

$$[a = 39.54545454553873, b = 0.42424242424116665]$$

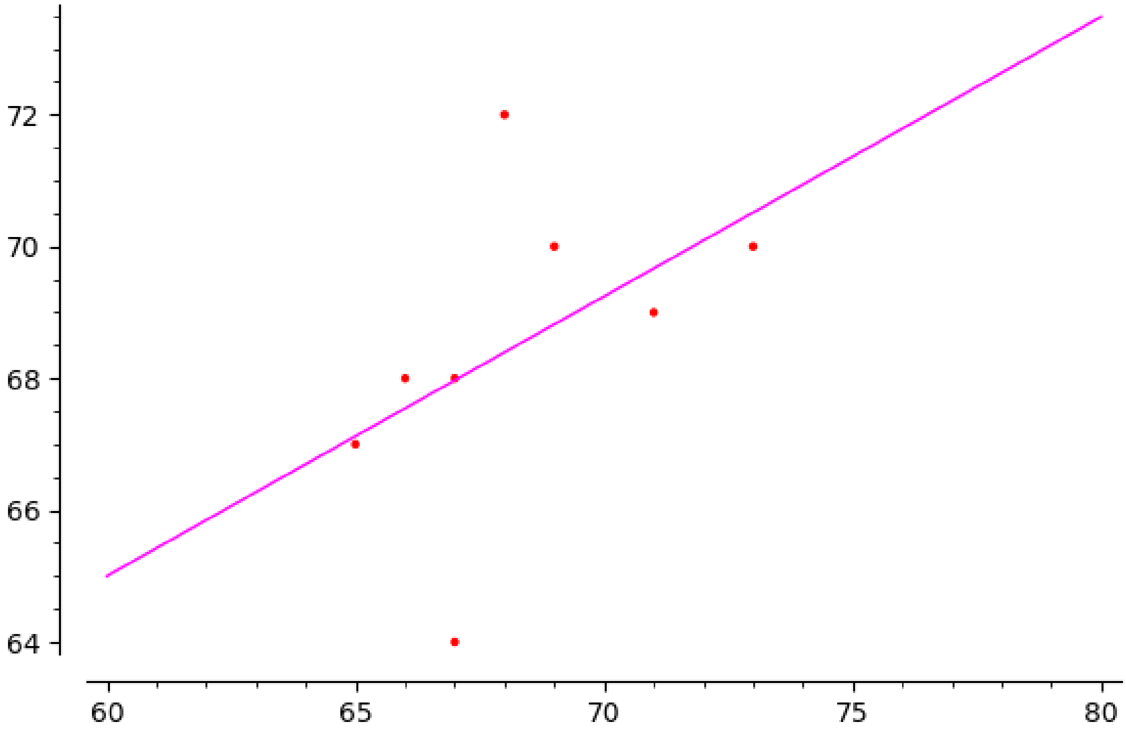
In [279]:

```
model=y.subs(f1) # substituting value of a and b in eq
show(model)
```

$$x \mapsto 0.42424242424116665\,x + 39.54545454553873$$

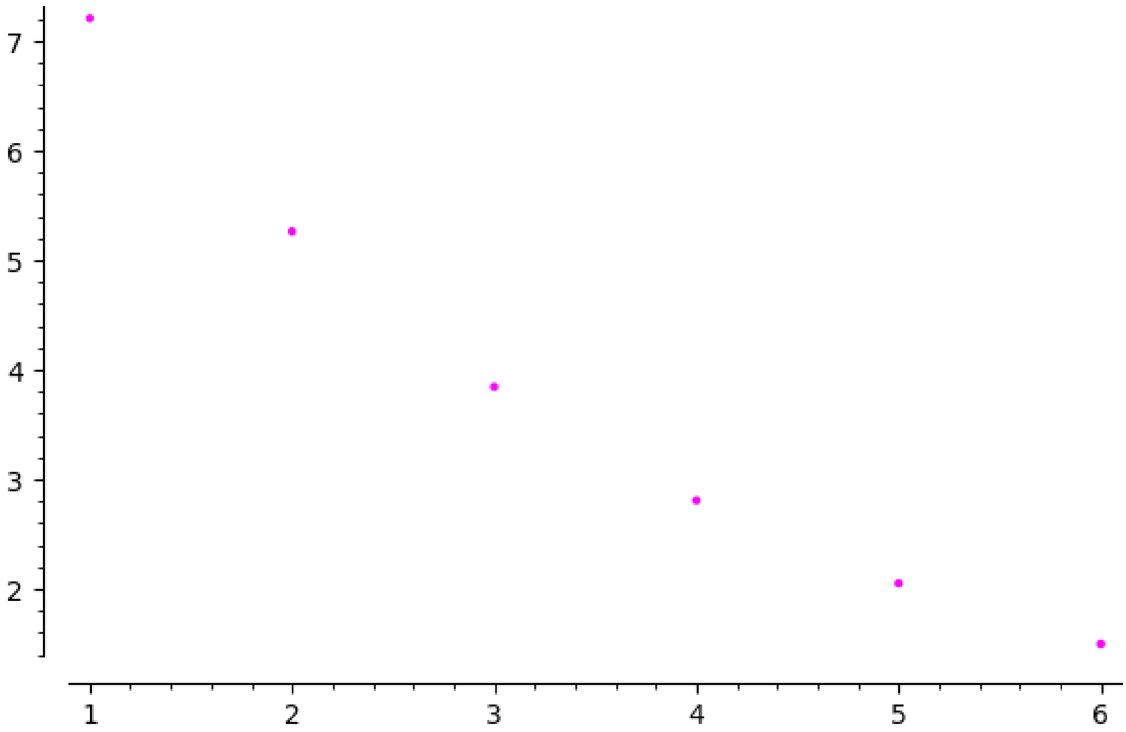
In [280]:
```
point(data, color="red") + plot(model, (x, 60,80), color='magenta') #plotin
g both data and model
```

Out[280]:



In [281]:
```
data1= [(1,7.209), (2, 5.265), (3,3.846), (4,2.809), (5,2.052), (6, 1.499)]
point(data1, color="magenta")
```

Out[281]:



In [282]:
```
a,b = var('a,b') #defining variable
y(x)=a*exp(b*x) # Writing equation to fit point
```

In [283]:
```
f2=find_fit(data1, y) #fitting data in eq
show(f2)
```

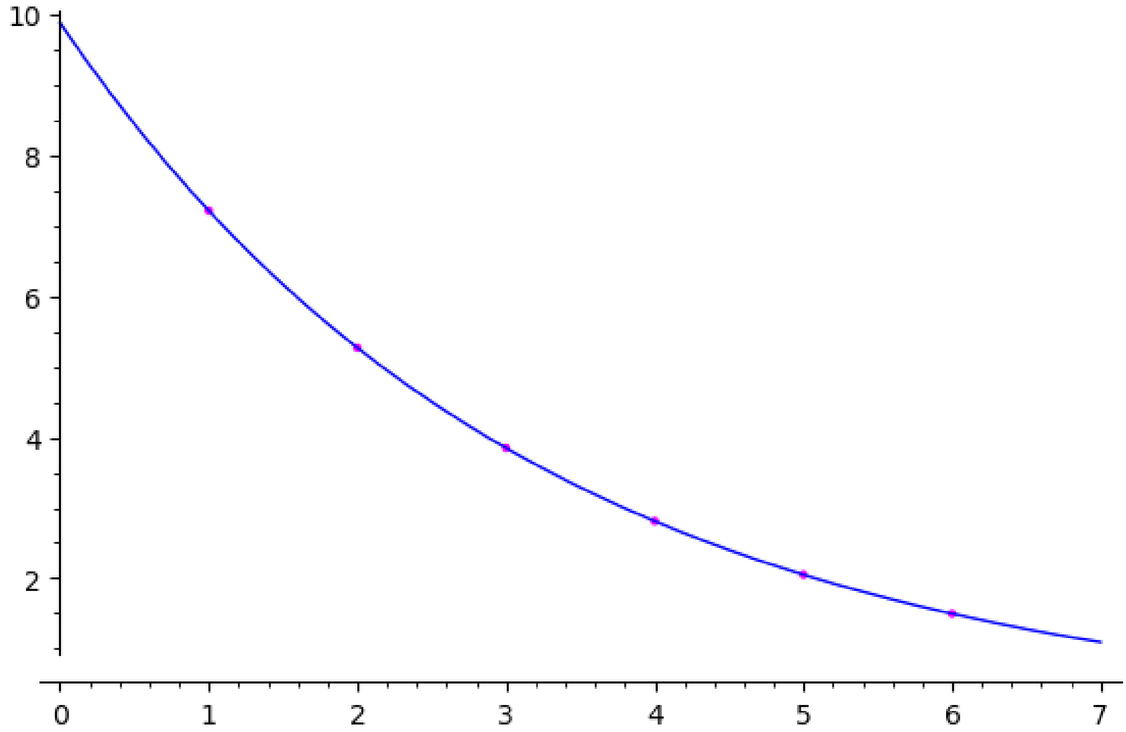
$$[a = 9.869341396282927, b = (-0.3141368303447278)]$$

In [284]:
```
model2=y.subs(f2) # substituting value of a and b in eq
show(model2)
```

$$x \mapsto 9.869341396282927\, e^{(-0.3141368303447278\, x)}$$

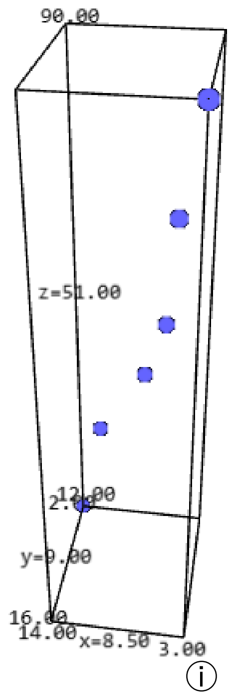
In [285]:
```
point(data1, color="magenta") + plot(model2, (x,0,7)) #ploting both data an
d model
```

Out[285]:



In [286]:
```
data3= [(3,16,90), (5,10,72), (6,7,54), (8,4,42), (12,3,30), (14,2,12)]
point(data3, size=300)
```

Out[286]:



In [287]:
```
a,b,c = var('a,b,c') #defining variable
z(x,y)=a*x+b*y+c # Writing equation
```

In [288]:
```
f3=find_fit(data3, z) #fitting data in eq
show(f3)
```

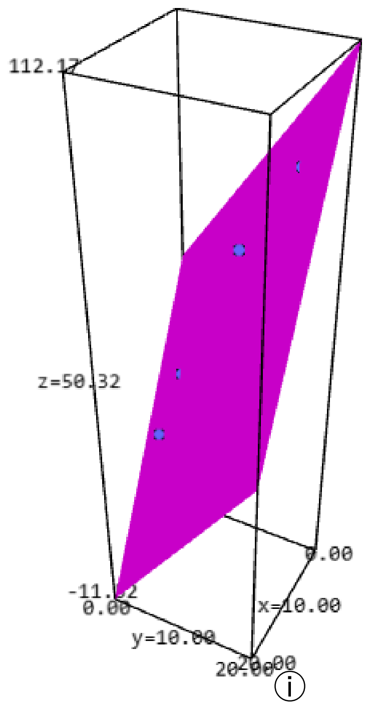
$$[a = (-3.646153846163976)\,, b = 2.5384615384648987, c = 61.4000000001317]$$

In [289]:
```
model3=z.subs(f3) # substituting value of a and b in eq
show(model3)
```

$$(x, y) \mapsto -3.646153846163976\,x + 2.5384615384648987\,y + 61.4000000001317$$

In [290]:
```
point(data3, size=300) + plot3d(model3, (x,0,20), (y,0,20), color='magenta')
```

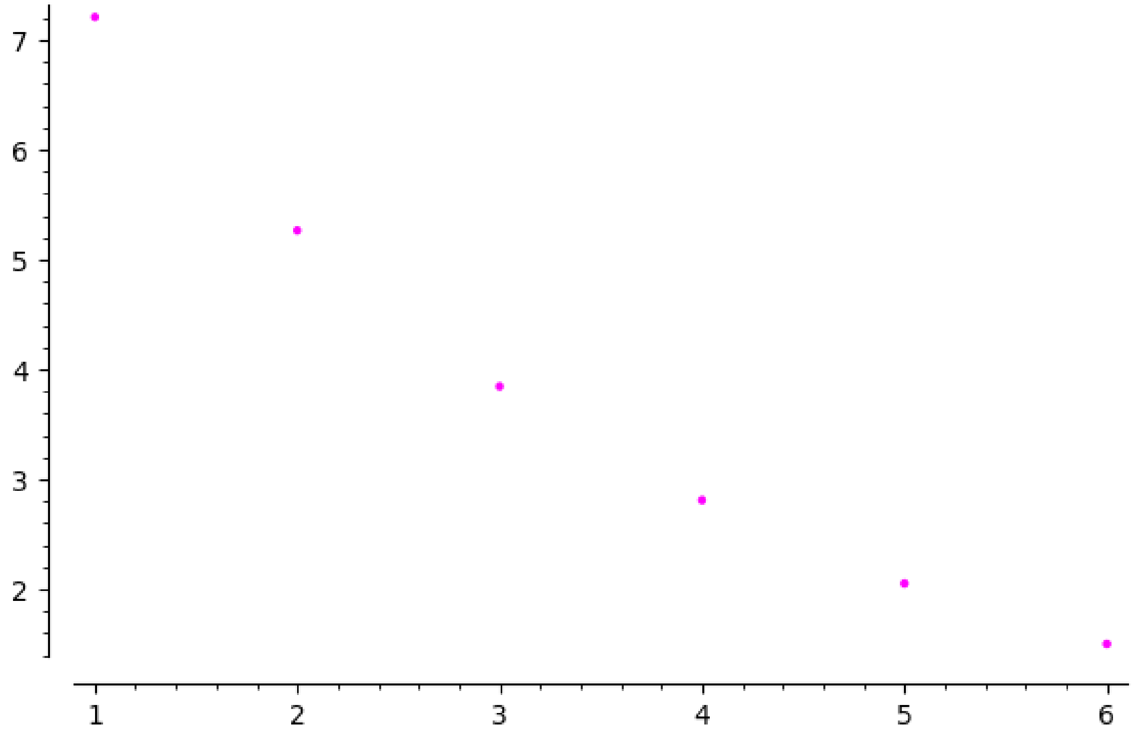
Out[290]:



In [291]:
```
## Comparing 2 fittings
```

In [292]:
```
data1= [(1,7.209), (2, 5.265), (3,3.846), (4,2.809), (5,2.052), (6, 1.499)]
point(data1, color="magenta")
```

Out[292]:



In [293]:
```
a,b,c,d,e = var('a,b,c,d,e') #defining variable
y(x)=a*x*x+b*x+c
z(x)=d*x+e
show(y)
show(z)
```

$$x \mapsto ax^2 + bx + c$$

$$x \mapsto dx + e$$

In [294]:
```
f1=find_fit(data1, y)
f2=find_fit(data1, z)
show(f1)
show(f2)
```

$[a = 0.17148214211021806, b = (-2.3211178517224806), c = 9.303099992357042$

$[d = (-1.1207428571474831), e = 7.702600000014619]$

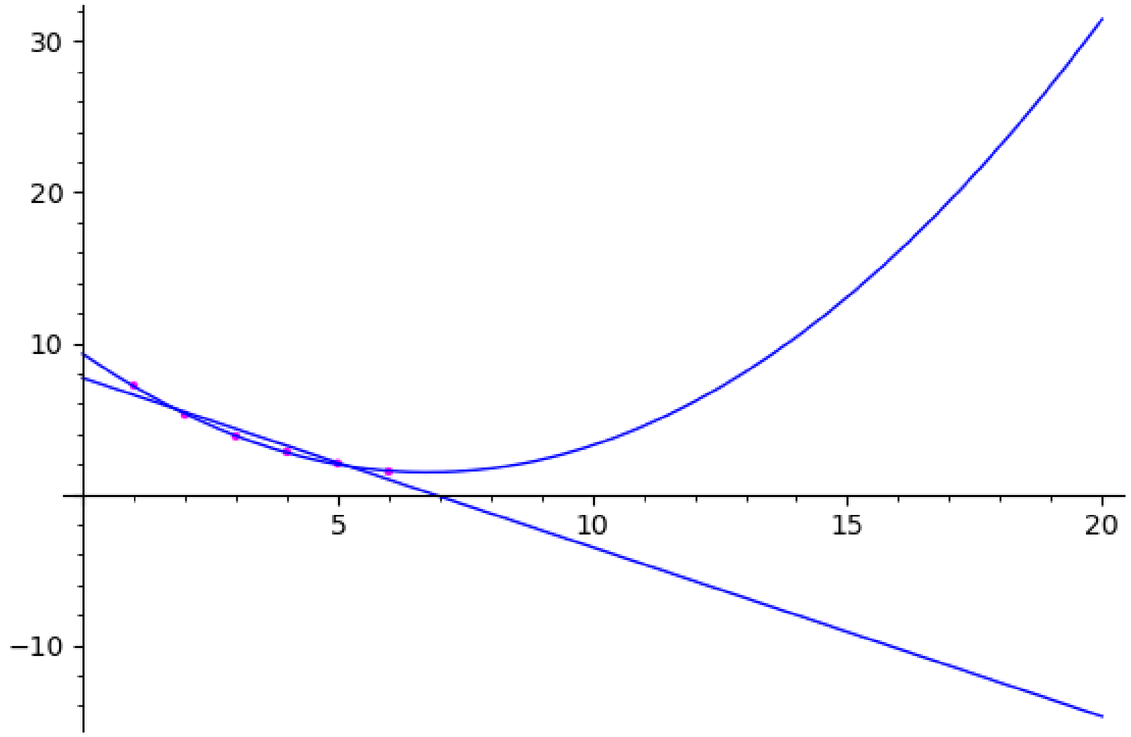
In [295]:
```
model4=y.subs(f1)
model41=z.subs(f2)
show(model4)
show(model41)
```

$$x \mapsto 0.17148214211021806\,x^2 - 2.3211178517224806\,x + 9.303099992357042$$

$$x \mapsto -1.1207428571474831\,x + 7.702600000014619$$

In [296]:
```
point(data1, color="magenta") + plot(model4, (x,0,20)) + plot(model41, (x,
0,20))#ploting both data and model
```

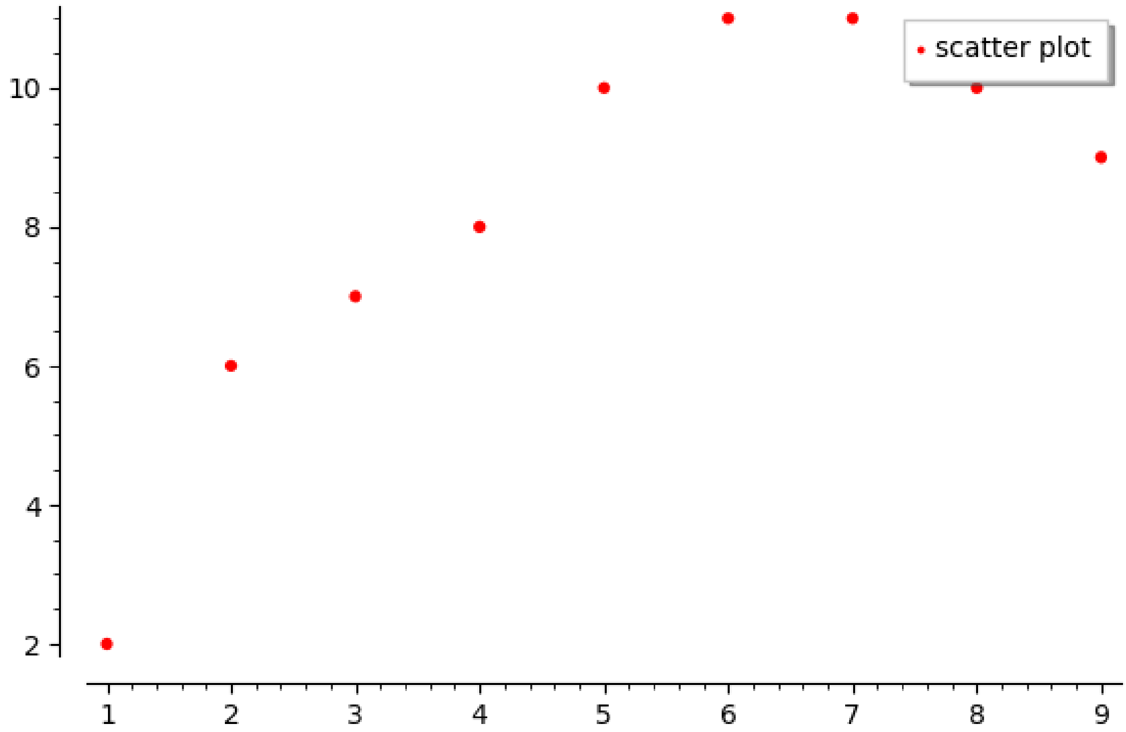
Out[296]:



Problem

In [297]:
```
data=[(1,2),(2,6),(3,7),(4,8),(5,10),(6,11),(7,11),(8,10),(9,9)]
point(data,color = "red",size=20,legend_label="scatter plot")
```

Out[297]:



In [298]:
```
a,b,c=var('a,b,c')
y(x)=a+b*x+c*x^2
```

In [299]:
```
f2=find_fit(data,y)
f2
```

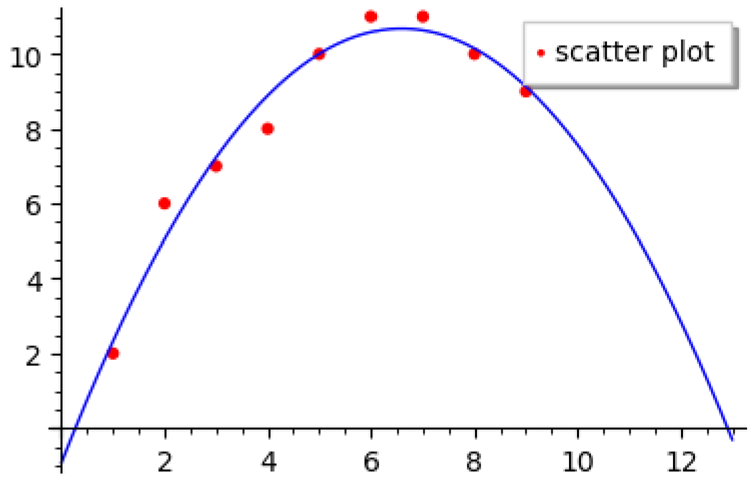
Out[299]:
```
[a == -0.928571428575653, b == 3.5231601731656825, c == -0.267316017318781
85]
```

In [300]:
```
model2= y.subs(f2)
model2
```

Out[300]:
```
x |--> -0.26731601731878185*x^2 + 3.5231601731656825*x - 0.928571428575653
```

In [301]:
```
point(data,color = "red",size=20,legend_label="scatter plot") + plot(model
2,(x,0,13),figsize=4)
```

Out[301]:



Comparing of two fittings

In [302]:
```
data = [(0, -0.1834),
(0.2, -0.1311),
(0.4, 0.0268),
(0.8, 0.1105),
(1.0, 0.2539),
(1.2, 0.2571),
(1.4, 0.5318),
(1.6, 0.5790),
(2.0, 0.9351),
(2.2, 0.9166),
(2.60, 1.1332),
(2.8, 1.2689),
(3.0, 1.1020),
(3.40, 1.1339)]
a,b,c,d,e = var('a,b,c,d,e')
y(x) = a*(x^2)+b*x+c
z(x)=d*(x)+e
show(y)
show(z)
```

$$x \mapsto ax^2 + bx + c$$

$$x \mapsto dx + e$$

In [303]:
```
f1 = find_fit(data, y, solution_dict=True)
f2 = find_fit(data, z,solution_dict=True )
show(f1,f2)
```

$\{a : -0.05822240152651912, b : 0.6538604551790161, c : -0.273358991734433\}$ {

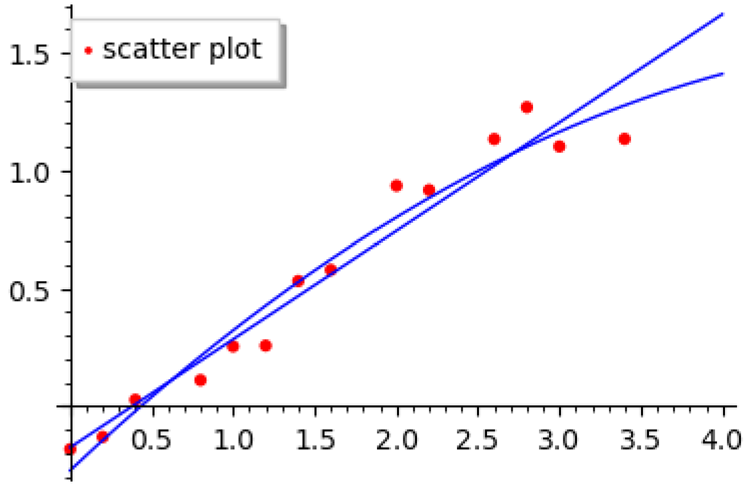
In [304]:
```
model1 = y.subs(f1)
model2 = z.subs(f2)
show(model1)
show(model2)
```

$$x \mapsto -0.05822240152651912\,x^2 + 0.6538604551790161\,x - 0.273358991734433$$

$$x \mapsto 0.4599573762683372\,x - 0.1757669076389894$$

In [305]:
```
point(data,color = "red",size=20,legend_label="scatter plot") + plot(model
1,(x,0,4))+ plot(model2,(x,0,4),figsize=4)
```

Out[305]:

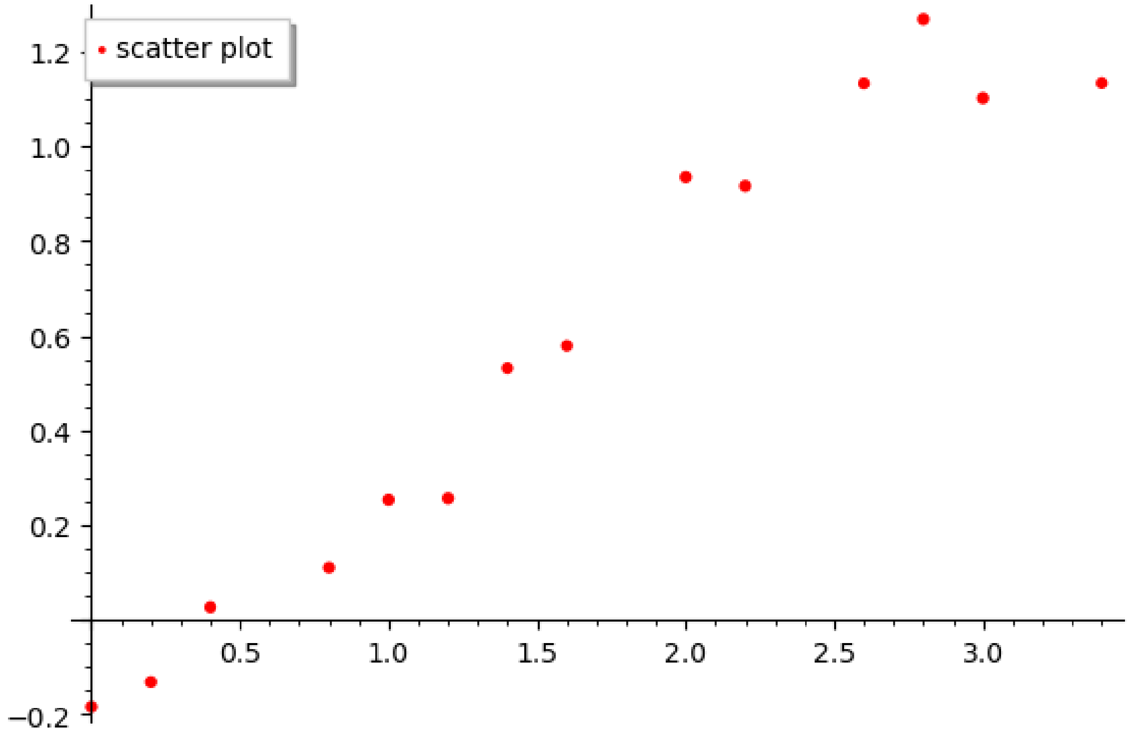


Problem:

For the following data fit curve upto fifth degree polynomial and Decide which curve is a best fit.

In [306]:
```
data = [(0, -0.1834),
(0.2, -0.1311),
(0.4, 0.0268),
(0.8, 0.1105),
(1.0, 0.2539),
(1.2, 0.2571),
(1.4, 0.5318),
(1.6, 0.5790),
(2.0, 0.9351),
(2.2, 0.9166),
(2.60, 1.1332),
(2.8, 1.2689),
(3.0, 1.1020),
(3.40, 1.1339)]
point(data,color = "red",size=20,legend_label="scatter plot")
```

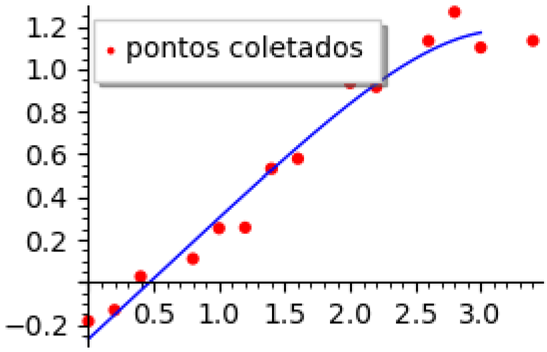
Out[306]:



In [307]:
```
a, b, c = var('a b c')
x = var('x')

for n in range(1, 6):
    z(x) = a*(x^n) + b*x + c
    d1 = find_fit(data, z)
    modelo2 = z.subs(d1)
    # print(modelo2)

show(point(data,color = "red",size=20,legend_label="pontos coletados",figsize=3) + plot(modelo2,(x,0,3),figsize=3))
```



Higher Dimensional Fitting

Fit a curve $ax^2 + bxy + cy^2 + dz$ for the following data.

In [308]:
```
data = [(0, -0.183440428023042,1,2),

(0.2, -0.1311,1,2),
(0.40, 0.0268,1,2),
(0.8, 0.1105,1,2),
(1.0, 0.2539,1,2),
(1.200, 0.2571,1,2),
(1.40, 0.5318,1,2),
(1.6, 0.5790,1,2),
(2.0, 0.9351,1,2),
(2.2, 0.9166,1,2),
(2.6, 1.1332,1,2),
(2.8, 1.2689,1,2),
(3.0, 1.1020,1,2),
(3.4, 1.1339,1,2)]
```

In [309]:
```
a,b,c,d=var('a b c d')
model(x,y,z)=a*x^2+b*x*y+c*y^2+d*z
l=find_fit(data,model)
show(l)
```

$\left[a = \left(-2.579901598207274 \times 10^{-08}\right), b = \left(1.3258440990631034 \times 10^{-07}\right), c = \left(\right.\right.$

In [310]:
```
modelo2 = model.subs(l)
print(modelo2)
```

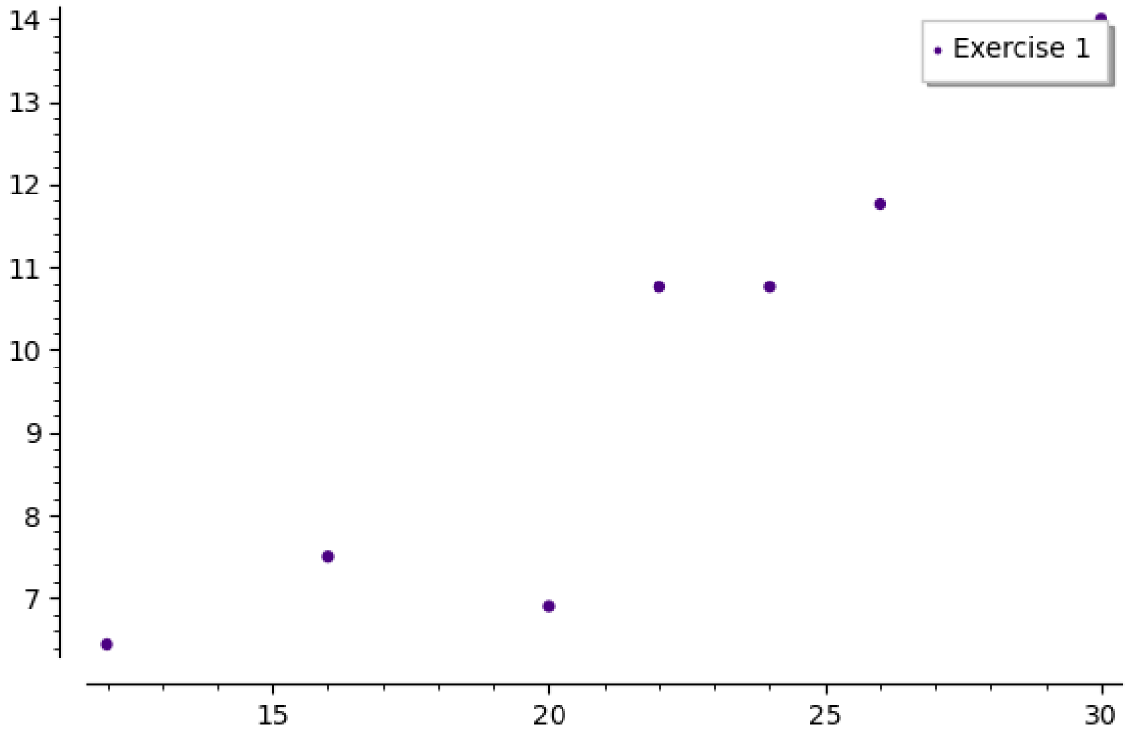
```
(x, y, z) |--> -(2.579901598207274e-08)*x^2 + (1.3258440990631034e-07)*x*y
- (1.7063132611372988e-07)*y^2 + 2.0000000044845447*z
```

Exercise

1. Fit a curve $y = ax + b$ for the following data

In [311]:
```
data1 = [(12,6.44),(16,7.5),(20,6.9),(22,10.76),(24,10.76),(26,11.76),(30,14.0)]
point(data1,color = "indigo",size=20,legend_label="Exercise 1")
```

Out[311]:



In [312]:
```
a,b=var('a,b')
y(x)=a+b*x
```

In [313]:
```
f1=find_fit(data1,y)
show(f1)
```

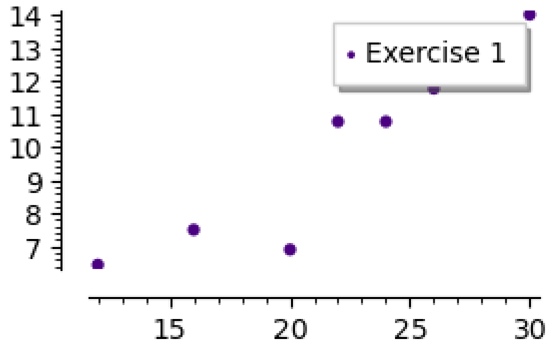
$$[a = 0.4254639175245253, b = 0.4342783505142298]$$

In [314]:
```
model1 = y.subs(f1)
show(model1)
```

$$x \mapsto 0.4342783505142298\, x + 0.4254639175245253$$

In [315]:
```
point(data1,color = "indigo",size=20,legend_label="Exercise 1") + plot(mode
l1,(x,30,15),figsize=3)
```

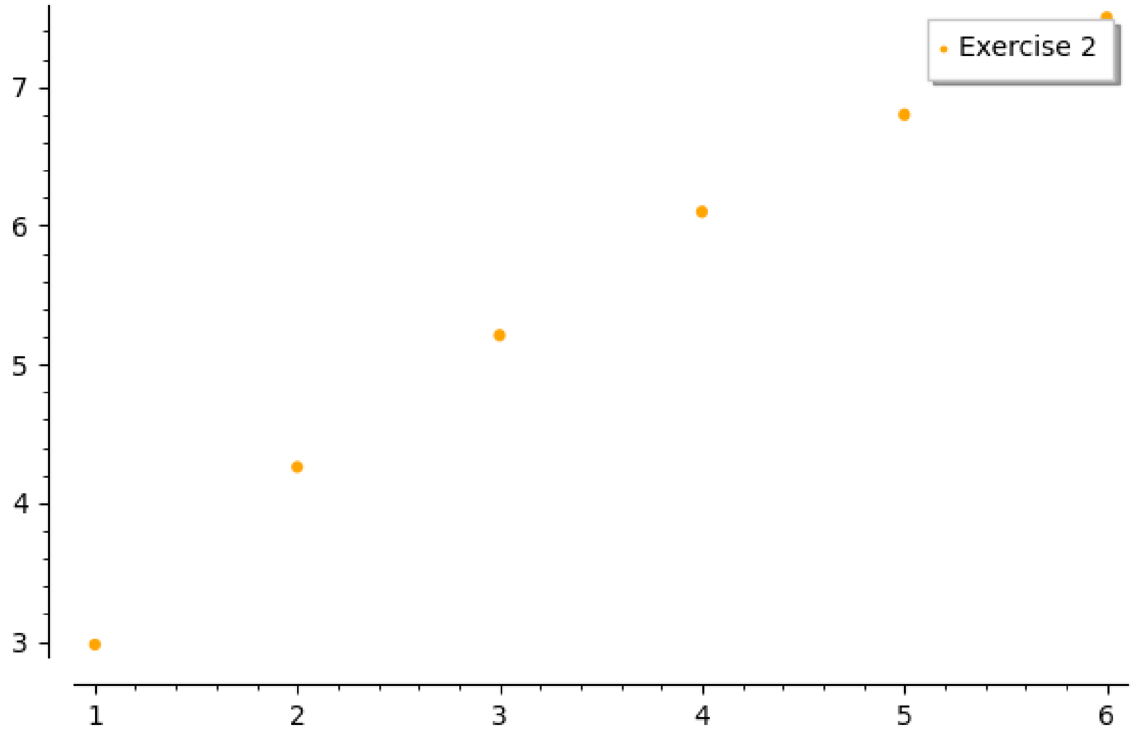
Out[315]:



2. Fit a curve $y = ax^b$ for the following data

In [316]:
```
data2 = [(1,2.98),(2,4.26),(3,5.21),(4,6.1),(5,6.8),(6,7.5)]
point(data2,color = "orange",size=20,legend_label="Exercise 2")
```

Out[316]:



In [317]:
```
a,b=var('a,b')
y(x)=a*x^b
f2=find_fit(data2,y)
show(f2)
```

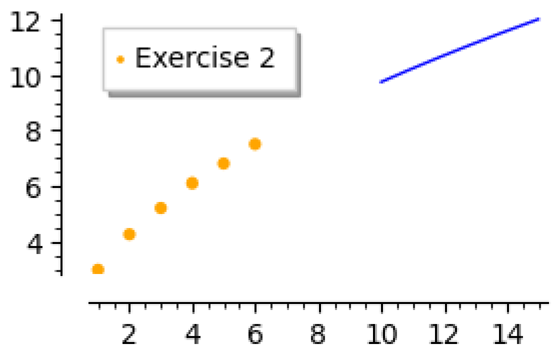
$$[a = 2.974134552834811, b = 0.515433145594774]$$

In [318]:
```
model2 = y.subs(f2)
show(model2)
```

$$x \mapsto 2.974134552834811\, x^{0.515433145594774}$$

In [319]:
```
point(data2,color = "orange",size=20,legend_label="Exercise 2") + plot(mode
l2,(x,10,15),figsize=3)
```

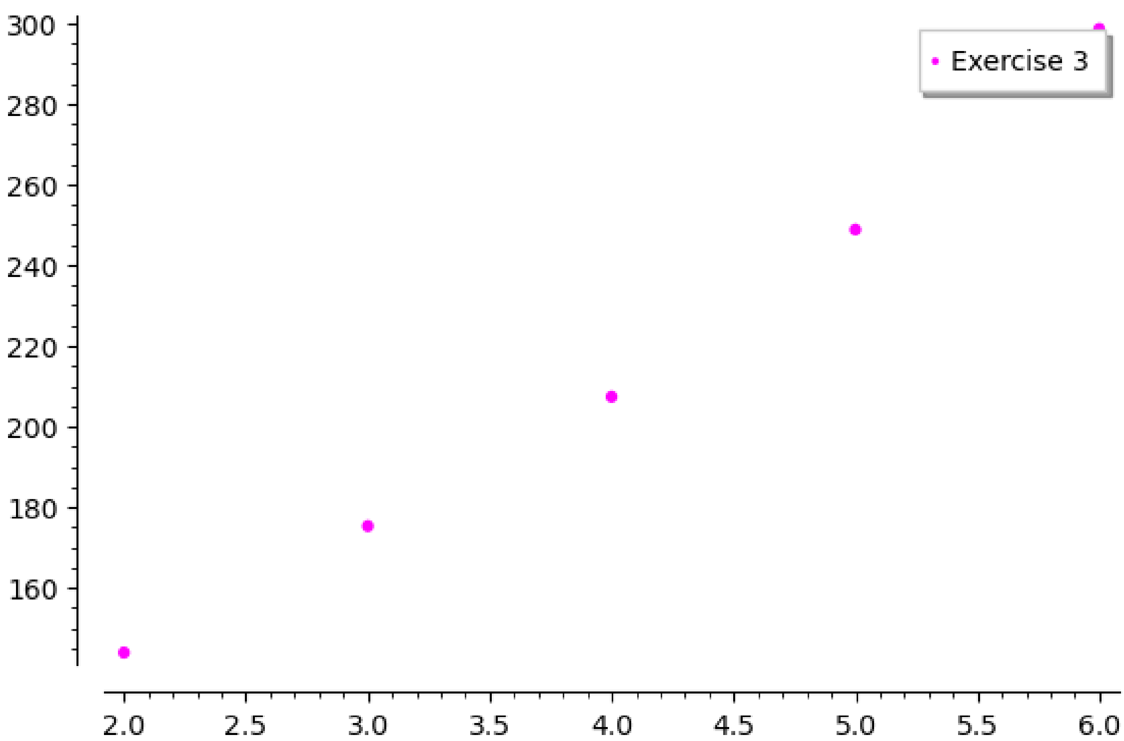
Out[319]:



3. Fit a curve $y = ab^x$ for the following data

In [320]:
```
data3 = [(2,144),(3,175.3),(4,207.4),(5,248.8),(6,298.5)]
point(data3,color = "fuchsia",size=20,legend_label="Exercise 3")
```

Out[320]:



In [321]:
```
a,b=var('a,b')
y(x)=a*b^x
f3=find_fit(data3,y)
show(f3)
```

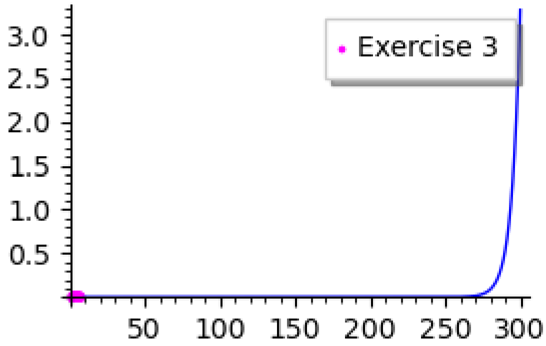
$$[a = 101.04573024140554, b = 1.1977607830183203]$$

In [322]:
```
model3 = y.subs(f3)
show(model3)
```

$$x \mapsto 101.04573024140554 \cdot 1.1977607830183203^x$$

In [323]:
```
point(data3,color = "fuchsia",size=20,legend_label="Exercise 3") + plot(mod
el3,(x,10,300),figsize=3)
```

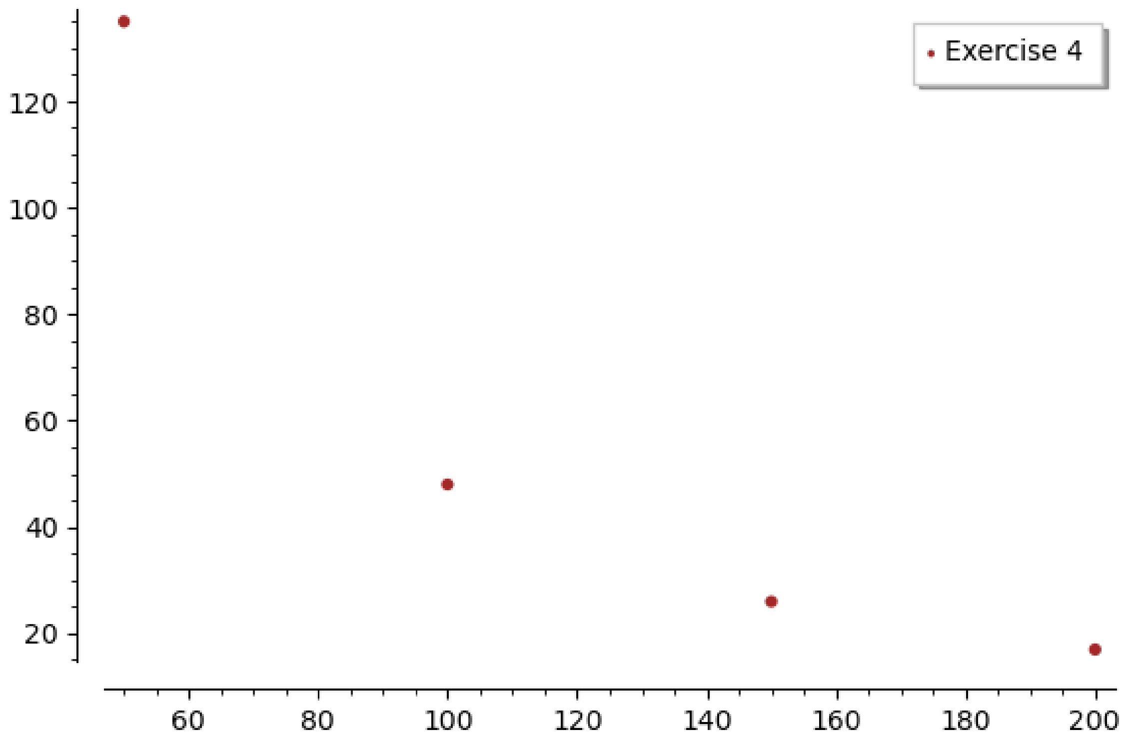
Out[323]:



4. Fit an exponential curve obeying the gas equation $pv^r = k$ for the following data

In [324]:
```
data4 =[(50,135),(100,48),(150,26),(200,17)]
point(data4,color = "brown",size=20,legend_label="Exercise 4")
```

Out[324]:



In [325]:
```
r,k= var('r,k')
p(v)=k/v^r
f4=find_fit(data4,p)
show(f4)
```

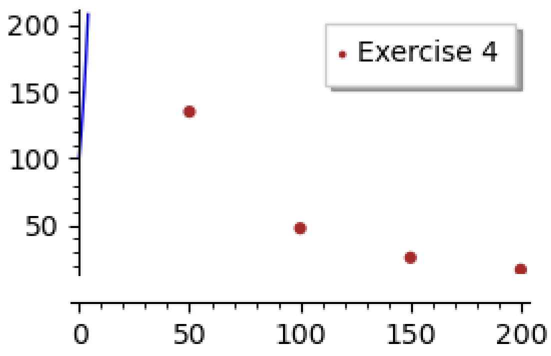
$$[k = 46822.26221207778, r = 1.4950684099693468]$$

In [326]:
```
model4 = p.subs(f4)
show(model4)
```

$$v \mapsto \frac{46822.26221207778}{v^{1.4950684099693468}}$$

In [327]:
```
point(data4,color = "brown",size=20,legend_label="Exercise 4") + plot(model
3,(x,0,4),figsize=3)
```

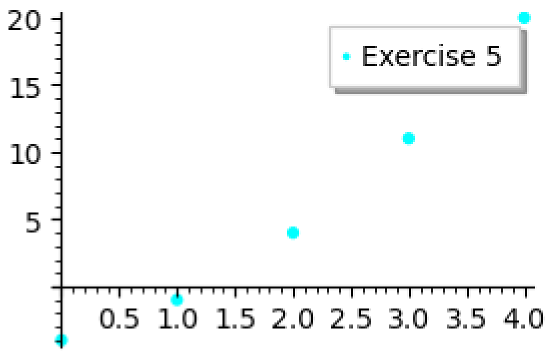
Out[327]:



5. Find the polynomial approximation of degree two to the data

In [328]:
```
data5 = [(0,-4),(1,-1),(2,4),(3,11),(4,20)]
point(data5,color = "cyan",size=20,legend_label="Exercise 5",figsize=3)
```

Out[328]:



In [329]:
```
a,b,c = var('a,b,c')
y(x) =a*x^2+b*x+c
f5 = find_fit(data5,y)
show(f5)
```

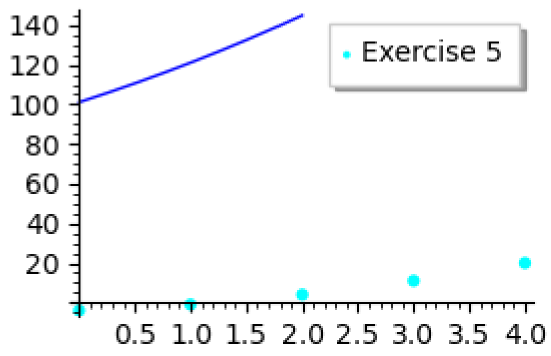
$$[a = 1.0, b = 2.0, c = (-4.0)]$$

In [330]:
```
model5 = y.subs(f5)
show(model5)
```

$$x \mapsto 1.0\,x^2 + 2.0\,x - 4.0$$

In [331]:
```
point(data5,color = "cyan",size=20,legend_label="Exercise 5") + plot(model
3,(x,0,2),figsize=3)
```

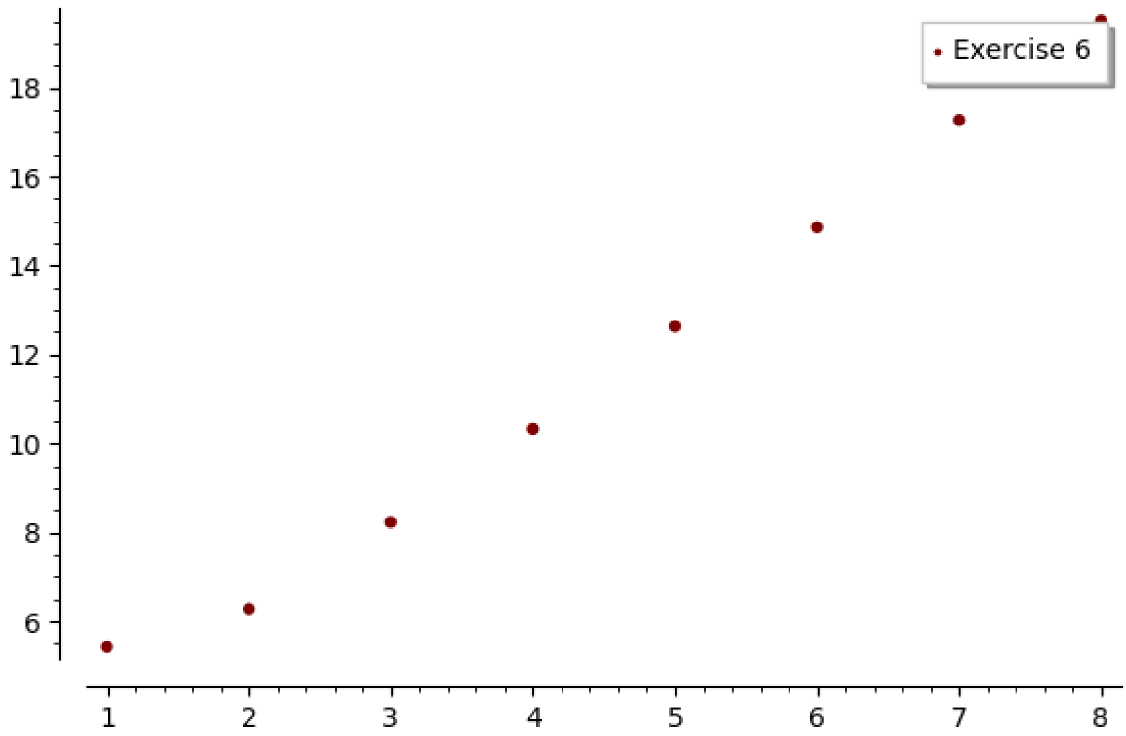
Out[331]:



6. Fit a curve $y = ax+b/x$ to the following data

In [332]:
```
data6 = [(1,5.43),(2,6.28),(3,8.23),(4,10.32),(5,12.63),(6,14.86),(7,17.2
7),(8,19.51)]
point(data6,color = "maroon",size=20,legend_label="Exercise 6")
```

Out[332]:



In [333]:
```
a,b = var('a,b')
y(x) =a*x+b/x
f6 = find_fit(data6,y)
show(f6)
```

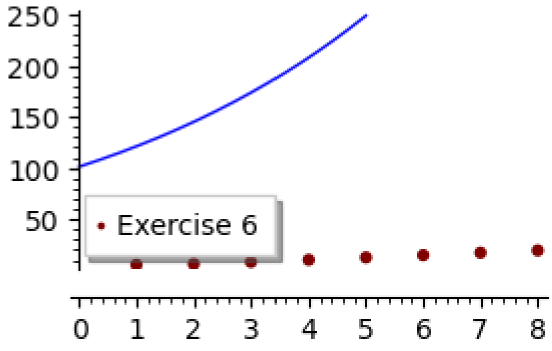
$$[a = 2.3971449823057016, b = 3.027802950899304]$$

In [334]:
```
model6 = y.subs(f6)
show(model6)
```

$$x \mapsto 2.3971449823057016\, x + \frac{3.027802950899304}{x}$$

In [335]:
```
point(data6,color = "maroon",size=20,legend_label="Exercise 6") + plot(mode
l3,(x,0,5),figsize=3)
```

Out[335]:

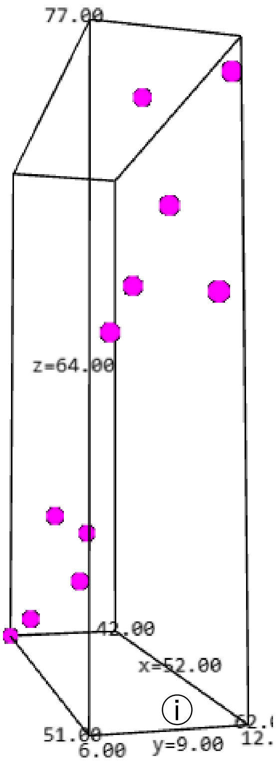


7. Table shows the Weight to the nearest pounds, Height to the nearest inches and age to the nearest years of boys. Find the best fit for the data.

Weight (z)	66	71	53	67	55	58	77	57	56	51	76	68
Height (x)	57	59	49	62	51	50	55	48	52	42	61	57
Age (y)	8	10	6	11	8	7	10	9	10	6	12	9

In [336]:
```
data7 = [(57,8,66),(59,10,71),(49,6,53),(62,11,67),(51,8,55),(50,7,58),(55,
10,77),(48,9,57),(42,6,51),(42,6,51),(61,12,76),(57,9,68)]
point(data7,color = "magenta",size=200,legend_label="Exercise 7")
```

Out[336]:



In [337]:
```
a,b,c = var('a,b,c') #defining variable
z(x,y)=a*x+b*y+c # Writing equation
f7=find_fit(data7, z) #fitting data in eq
show(f7)
```

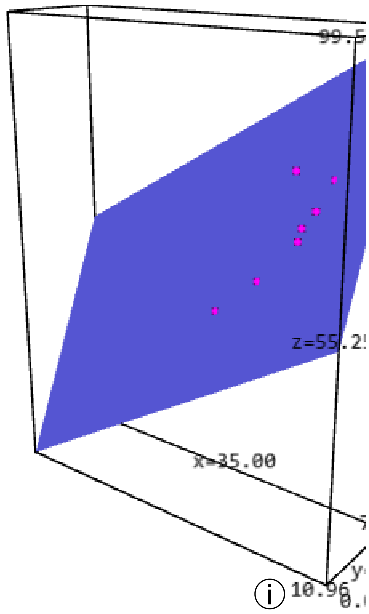
$$[a = 0.6038863976075073, b = 2.315396113605259, c = 10.964125560559834]$$

In [338]:
```
model7 = z.subs(f7)
show(model7)
```

$$(x, y) \mapsto 0.6038863976075073\, x + 2.315396113605259\, y + 10.964125560559834$$

In [339]:
```
point(data7,color = "magenta",size=300,legend_label="Exercise 7") + plot3d
(model7,(x,0,70), (y,0,20),figsize=3)
```

Out[339]:



What do you learn from this activity?

The activity shows me how to take raw data and turn it into a mathematical model. I learn how to organize data into pairs or triples, choose an appropriate type of curve depending on the situation, and apply SageMath's `find_fit` function to calculate the coefficients. I also see how to substitute those coefficients back into the model to get the explicit equation of best fit. By plotting the data points together with the fitted curve or surface, I can visually judge how well the model matches the observations. Comparing different models demonstrates that some fits are more accurate or meaningful than others. Overall, the activity teaches me the process of curve fitting as a way to connect theoretical mathematics with practical applications, making it possible to predict and interpret unknown values.