Experiment No. 03

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Aim: Basic of Linear Algebra.

In [4]:
```
A=matrix([[1,2,3],[4,5,6],[7,8,9]])
A
```

Out[4]:
```
[1 2 3]
[4 5 6]
[7 8 9]
```

In [35]:
```
A=matrix([[1,2,3],[4,5,6],[7,8,9]])
show(A)
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

In [36]:
```
B=matrix(3,3,[5,2,3,8,4,3,2,1,9])
show(B)
```

$$\begin{pmatrix} 5 & 2 & 3 \\ 8 & 4 & 3 \\ 2 & 1 & 9 \end{pmatrix}$$

In [37]:
```
show(A+B)
```

$$\begin{pmatrix} 6 & 4 & 6 \\ 12 & 9 & 9 \\ 9 & 9 & 18 \end{pmatrix}$$

In [38]:
```
show(A-B)
```

$$\begin{pmatrix} -4 & 0 & 0 \\ -4 & 1 & 3 \\ 5 & 7 & 0 \end{pmatrix}$$

In [39]: `show(A*B)`

$$\begin{pmatrix} 27 & 13 & 36 \\ 72 & 34 & 81 \\ 117 & 55 & 126 \end{pmatrix}$$

In [40]: `show(B.transpose())`

$$\begin{pmatrix} 5 & 8 & 2 \\ 2 & 4 & 1 \\ 3 & 3 & 9 \end{pmatrix}$$

show(A.adjoint())

In [41]: `show(A.adjugate())`

$$\begin{pmatrix} -3 & 6 & -3 \\ 6 & -12 & 6 \\ -3 & 6 & -3 \end{pmatrix}$$

In [42]: `show(A.inverse())`

```
---------------------------------------------------------------------------
ZeroDivisionError                         Traceback (most recent call last)
<ipython-input-42-6fcec01c1398> in <module>
----> 1 show(A.inverse())

/opt/sagemath-9.3/local/lib/python3.7/site-packages/sage/matrix/matrix2.pyx in sage.matrix.matrix2.Matrix.inverse (build/cythonized/sage/matrix/matrix2.c:69711)()
   9688
   9689             """
-> 9690             return ~self
   9691
   9692     def adjugate(self):

/opt/sagemath-9.3/local/lib/python3.7/site-packages/sage/matrix/matrix_integer_dense.pyx in sage.matrix.matrix_integer_dense.Matrix_integer_dense.__invert__ (build/cythonized/sage/matrix/matrix_integer_dense.cpp:33806)()
   4095                 ZeroDivisionError: matrix must be nonsingular
   4096             """
-> 4097             A, d = self._invert_flint()
   4098             return A / d
   4099

/opt/sagemath-9.3/local/lib/python3.7/site-packages/sage/matrix/matrix_integer_dense.pyx in sage.matrix.matrix_integer_dense.Matrix_integer_dense._invert_flint (build/cythonized/sage/matrix/matrix_integer_dense.cpp:33642)()
   4056             fmpz_clear(fden)
   4057             if res == 0:
-> 4058                 raise ZeroDivisionError('matrix must be nonsingular')
   4059             if den < 0:
   4060                 return -M, -den

ZeroDivisionError: matrix must be nonsingular
```

In [ ]: `A.det()`

In [ ]: `show(B.inverse())`

In [ ]: `B.det()`

In [ ]: `show(B.inverse().det())`

Verify the $B.\,inverse.\,() * B.\,det() = B.\,adjugate()$

In [43]: `(B.inverse())*(B.det())==B.adjugate()`

Out[43]: True

```
In [44]: var('x,y,z')
         D=matrix(3,3,[1,z,-y,-z,1,x^2,y*x,-z,1])
         show(D)
```

$$\begin{pmatrix} 1 & z & -y \\ -z & 1 & x^2 \\ xy & -z & 1 \end{pmatrix}$$

```
In [45]: E=D.det()
         show(E)
```

$$\left(x^2 z + y\right)xy + x^2 z - (yz - z)z + 1$$

```
In [46]: show(E.full_simplify()) #For Simplified Version
```

$$xy^2 - (y - 1)z^2 + \left(x^3 y + x^2\right)z + 1$$

```
In [47]: show(diff(D,x)) # For differentiation of matrix
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2x \\ y & 0 & 0 \end{pmatrix}$$

```
In [48]: show(diff(D,y))
```

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ x & 0 & 0 \end{pmatrix}$$

```
In [49]: show(diff(D,z))
```

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

```
In [50]: show(A)
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

```
In [51]: A.minors(1) #For Minors of Matrix
```

```
Out[51]: [1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
In [52]: A.minors(2)
```

```
Out[52]: [-3, -6, -3, -6, -12, -6, -3, -6, -3]
```

```
In [53]: A.minors(3)
```

```
Out[53]: [0]
```

In [54]:
```
F=ones_matrix(10,10) #Matrix of 1
show(F)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [55]:
```
F[1,2]=500
show(F)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 500 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [56]:
```
S=zero_matrix(10,10) # Matrix of 0's
show(S)
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In [57]:
```
show(identity_matrix(2)) #Identity Matrix of order 2
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In [148]:
```
show(identity_matrix(3))
```
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [59]:
```
show(random_matrix(ZZ,2,2,x=100,y=100)) #For Random matrix, ZZ  for intege
r's and of 2*2 order with range of upto 100
```
$$\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$$

In [60]:
```
A[0] #For 1st row element since indexing start from 0
```
Out[60]: (1, 2, 3)

In [61]:
```
A[1]
```
Out[61]: (4, 5, 6)

In [62]:
```
A[:,0] #For 1st column, no need for row element therefor ':'
```
Out[62]:
```
[1]
[4]
[7]
```

In [63]:
```
show(A.echelon_form()) #For checking Rank
```
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{pmatrix}$$

In [64]:
```
show(B.echelon_form())
```
$$\begin{pmatrix} 1 & 0 & 18 \\ 0 & 1 & 6 \\ 0 & 0 & 33 \end{pmatrix}$$

Solve $x - 2y + 3z = 2, 2x - 3z = 3, x + y + z = 0$

In [65]:
```
I=matrix(3,3,[1,-2,3,2,0,-3,1,1,1])
show(I)
```
$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

```
In [66]:  J=matrix(3,1,[2,3,0])
          show(J)
```

$$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

```
In [67]:  C=I.augment(J) # To augment I matrix in J
          show(C)
```

$$\begin{pmatrix} 1 & -2 & 3 & 2 \\ 2 & 0 & -3 & 3 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

```
In [68]:  show(C.echelon_form())
```

$$\begin{pmatrix} 1 & 0 & 8 & -1 \\ 0 & 1 & 12 & -4 \\ 0 & 0 & 19 & -5 \end{pmatrix}$$

```
In [69]:  rank(I) #for Rank
```

Out[69]: 3

```
In [70]:  rank(C)
```

Out[70]: 3

```
In [71]:  rank(I)==rank(C)
```

Out[71]: True

Linearly dependent and independent vectors

```
In [72]:  P(x)=I.characteristic_polynomial(x) # [A-Lamda*Identity]=0 equation
          show(P)
```

$$x \mapsto x^3 - 2x^2 + 5x - 19$$

```
In [73]:  show(P.roots())
```

$$\left[\left(-\frac{1}{6} \left(\frac{1}{2} \right)^{\frac{1}{3}} \left(9\sqrt{815}\sqrt{3} + 439 \right)^{\frac{1}{3}} \left(i\sqrt{3} + 1 \right. \right. \right.$$

$$\left(-\frac{1}{6} \left(\frac{1}{2} \right)^{\frac{1}{3}} \left(9\sqrt{815}\sqrt{3} + 439 \right)^{\frac{1}{3}} \left(-i\sqrt{3} + 1 \right) + \frac{11 \left(\frac{1}{2} \right)^{\frac{2}{3}} \left(i\sqrt{3} + 1 \right)}{3 \left(9\sqrt{815}\sqrt{3} + 439 \right)^{\frac{1}{3}}} + \frac{2}{3}, 1 \right.$$

In [74]:
```
A=matrix(3,3,[-2,1,1,-6,1,3,-12,-2,8])
show(A)
```

$$\begin{pmatrix} -2 & 1 & 1 \\ -6 & 1 & 3 \\ -12 & -2 & 8 \end{pmatrix}$$

In [75]:
```
B=A.characteristic_polynomial(x)
show(B)
```

$$x^3 - 7x^2 + 14x - 8$$

In [76]:
```
B.roots()
```

Out[76]: [(4, 1), (2, 1), (1, 1)]

In [77]:
```
C=A.eigenvectors_right()
show(C)
```

$$\left[\left(4, \left[(1, 2, 4)\right], 1\right), \left(2, \left[\left(1, \frac{3}{2}, \frac{5}{2}\right)\right], 1\right), (1, [(1, 1, 2)], 1) \right]$$

Another way of defining Matrix

In [78]:
```
# Defining 3 by 3 matrix
A=matrix([
[1,2,3],
[4,5,6],
[7,8,9]
])
show('A=',A)
```

$$A= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

In [79]:
```
show(matrix(3,3,[1,2,3,4,5,6,7,8,9]))
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

In [80]:
```
show(matrix(ZZ, 3,2, [1,2,3,4,5,6])) # ZZ (Integer Field) defines all entri
es in matrix is an integer
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

In [81]:
```
show(matrix(RR, 3,2, [1,2,3,4,5,6])) # RR (Real Field) defines all entriesi
n matrix is a real number
```

$$\begin{pmatrix} 1.00000000000000 & 2.00000000000000 \\ 3.00000000000000 & 4.00000000000000 \\ 5.00000000000000 & 6.00000000000000 \end{pmatrix}$$

In [82]:
```
show(matrix(CDF, 3,2, [1,2,3,4,5,6])) # CDF (Complex Dence Field) defines a
ll entries in matrix is a complex number
```

$$\begin{pmatrix} 1.0 & 2.0 \\ 3.0 & 4.0 \\ 5.0 & 6.0 \end{pmatrix}$$

In [83]:
```
show(matrix(QQ, 3,2, [1,2,3,4,5,6])) # QQ (Rational Field) defines all entr
ies in matrix is a rational number
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

Matrix Operations

In [84]:
```
P=matrix(2,3,[8,5,6,9,7,3])
Q=matrix(2,3,[1,2,5,3,6,4])
show('P+Q =',P+Q , 'P-Q =', P-Q)
show('P Q = ',P*Q.transpose())
```

$$\text{P+Q } = \begin{pmatrix} 9 & 7 & 11 \\ 12 & 13 & 7 \end{pmatrix} \text{P-Q } = \begin{pmatrix} 7 & 3 & 1 \\ 6 & 1 & -1 \end{pmatrix}$$

$$\text{P Q } = \begin{pmatrix} 48 & 78 \\ 38 & 81 \end{pmatrix}$$

In [85]:
```
R=matrix(2,2,[-8,5,1,-7])
show('R=',R, 'R^5 = ', R^5)
```

$$\text{R=} \begin{pmatrix} -8 & 5 \\ 1 & -7 \end{pmatrix} \text{R^5 } = \begin{pmatrix} -56313 & 94005 \\ 18801 & -37512 \end{pmatrix}$$

In [86]:
```
R^2 == R*R
```

Out[86]: True

In [87]:
```
R.det() # To find determinant of R
```

Out[87]: 51

In [88]:
```
R.adjugate()==(R.det())*(R.inverse())
```

Out[88]: True

In [89]:
```
3*R # Scalar Multiplication
```

Out[89]:
```
[-24  15]
[  3 -21]
```

In [90]:
```
R.trace()
```

Out[90]: `-15`

In [91]:
```
var('x,y,z')
S=matrix(3,3,[1,z,-y,-z,1,x,y,-z,1])
show('S=',S)
show('|S|=',S.det())
```

$$S= \begin{pmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -z & 1 \end{pmatrix}$$

$$|S|=(xz+y)y-(yz-z)z+xz+1$$

In [92]:
```
var('x,y,z')
S=matrix(3,3,[x+y,x,x,5*x+4*y,4*x,2*x,10*x+8*y,8*x,3*x])
show('S=',S)
show('|S|=',S.det())
T=S.det()
T.full_simplify()
```

$$S= \begin{pmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{pmatrix}$$

$$|S|=(5x+4y)x^2-4(x+y)x^2$$

Out[92]: `x^3`

In [93]:
```
show(diff(S,x))
show(diff(S,y))
```

$$\begin{pmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \\ 8 & 0 & 0 \end{pmatrix}$$

In [126]:
```
T=matrix(3,3,[1,2,3,4,5,6,7,8,9])
show(T)
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

```
In [127]: T[0] # First Row of T
          T[1] # Second row of T
          T[2] # Third row of T
          T[0,1] # First row second column element
          T[2,2] # Third row third column element
```

Out[127]: 9

System of equations

Solve the following system of equations

$1. x - 2y + 3z = 2;\ 2x - 3z = 3;\ x + y + z = 0$

$2. 2x - y + z = 4;\ 3x - y + z = 6;\ 4x - y + 2z = 7;\ -x + y - z = 9$

$3. 3x + y + z = 2;\ x - 3y + 2z = 1;\ 7x - y + 4z = 5$

```
In [128]: x, y, z = var('x, y, z')
          solve([x-2*y+3*z == 2, 2*x-3*z == 3,x+y+z == 0], x, y, z)
```

Out[128]: [[x == (21/19), y == (-16/19), z == (-5/19)]]

```
In [129]: A=matrix([[1,-2,3], [2,0,-3] ,[1,1,1]])
          B=vector([2,3,0])
          show('A= ', A , 'B= ', B.column())
```

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ 1 & 1 & 1 \end{pmatrix} B = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

```
In [130]: C=A.augment(B)
          show('C=',C)
```

$$C = \begin{pmatrix} 1 & -2 & 3 & 2 \\ 2 & 0 & -3 & 3 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

```
In [131]: rank(A)==rank(C)
```

Out[131]: True

```
In [139]: solve([2*x-y+z == 4, 3*x-y+z == 6, 4*x-y+2*z == 7 , -x+y-z==9], x, y, z)
```

Out[139]: []

In [134]:
```
A=matrix(4,3,[2,-1,1,3,-1,1,4,-1,2,-1,1,-1])
B=vector([4,6,7,9])
show('A= ', A , 'B= ', B.column())
```

$$A=\begin{pmatrix} 2 & -1 & 1 \\ 3 & -1 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -1 \end{pmatrix} B=\begin{pmatrix} 4 \\ 6 \\ 7 \\ 9 \end{pmatrix}$$

In [135]:
```
C=A.augment(B)
show('C=',C)
```

$$C=\begin{pmatrix} 2 & -1 & 1 & 4 \\ 3 & -1 & 1 & 6 \\ 4 & -1 & 2 & 7 \\ -1 & 1 & -1 & 9 \end{pmatrix}$$

In [136]:
```
rank(A)==rank(C)
```

Out[136]: False

In [137]:
```
show(C.echelon_form())
```

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 11 \end{pmatrix}$$

In [138]:
```
solve([3*x+y+z == 2, x-3*y+2*z == 1,7*x-y+4*z == 5], x, y, z)
```

Out[138]: [[x == -1/2*r1 + 7/10, y == 1/2*r1 - 1/10, z == r1]]

Check if the following set of vectors are linearly dependent?

$$(1, -1, 2, 4), (-3, 3, 2, 1), (-1, -2, 6, 9)$$

In [144]:
```
U=matrix(3,4,[1,-1,2,4,-3,3,2,1,-1,-2,6,9])
show(U)
```

$$\begin{pmatrix} 1 & -1 & 2 & 4 \\ -3 & 3 & 2 & 1 \\ -1 & -2 & 6 & 9 \end{pmatrix}$$

In [145]:
```
show(U.transpose().echelon_form())
```

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues and Eigenvectors

Ex. Consider the matrix A =

[-2 1 1, -6 1 3, -12 -2 8,]

(i) Find the characteristics polynolmial, p(x), of A. (ii) Find the roots of p(x), show that eigenvalues of A are roots of p(x). (iii) Find the eigenvectors of A. (iv) Show that sum of the eigenvalues of A is trace of A. (iv) Show that the product of eigenvalues of A is the determinant of A.

```
In [149]:  A=matrix(3,3,[-2,1,1,-6,1,3,-12,-2,8])
           p(x)=A.characteristic_polynomial ( x )
           show('p(x)=',p(x))
```

$$p(x)=x^3 - 7x^2 + 14x - 8$$

```
In [151]:  p(x).roots()
```

Out[151]: [(2, 1), (4, 1), (1, 1)]

```
In [152]:  E= A . eigenvalues ()
           show(E)
```

$$[4, 2, 1]$$

```
In [153]:  V= A . eigenvectors_right ()
           show('V=',V)
```

$$V= \left[\left(4, [(1, 2, 4)], 1\right), \left(2, \left[\left(1, \frac{3}{2}, \frac{5}{2}\right)\right], 1\right), (1, [(1, 1, 2)], 1) \right]$$

```
In [154]:  sum(E)==A.trace()
```

Out[154]: True

```
In [155]:  product(E)==det(A)
```

Out[155]: True

```
In [160]:  det(A)
```

Out[160]: 8

```
In [161]:  D,M=A.eigenmatrix_right() # D= Diagonal matrix and M= Modal matrix (i.e. ma
           trix obtained from eigenvectors)
           show('D=',D,  'M=',M)
```

$$D= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M= \begin{pmatrix} 1 & 1 & 1 \\ 2 & \frac{3}{2} & 1 \\ 4 & \frac{5}{2} & 2 \end{pmatrix}$$

```
In [162]: B=matrix(2,2,[cos(x) , sin(x), -sin(x) , cos(x)])
          show('B=',B)
```

$$B= \begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix}$$

```
In [163]: D,M=B.eigenmatrix_right() # D= Diagonal matrix and M= Modal matrix (i.e. ma
          trix obtained from eigenvectors)
          show('D=',D, 'M=',M)
```

$$D= \begin{pmatrix} \cos(x) - i \sin(x) & 0 \\ 0 & \cos(x) + i \sin(x) \end{pmatrix} \quad M= \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

(i) Find the characteristics polynolmial, p(x), of A.

```
In [164]: A=matrix(4,4,[1,0,-1,1,2,1,2,2,3,2,1,3,5,2,3,4])
          p=A.characteristic_polynomial ( x )
          show('p(x)=',p)
```

$$p(x)=x^4 - 7x^3 - 4x^2 + 10x + 4$$

Exercise 1

1. x, y, z and t if $$

2

$$\begin{pmatrix} x & z \\ y & t \end{pmatrix}$$

- 3

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

  = 3

$$\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$$

  $$

```
In [221]: x,y,z,t = var('x y z t')
          A = matrix(2,2,[x,z,y,t])
          B = matrix(2,2,[1,-1,0,0])
          C = matrix(2,2,[3,5,4,6])

          eqns = [ (2*A + 3*B)[i,j] == (3*C)[i,j] for i in range(2) for j in range(2)
          ]
          solution = solve(eqns, x, y, z, t)
          solution
```

```
Out[221]: [[x == 3, y == 6, z == 9, t == 9]]
```

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{pmatrix} and B = \begin{pmatrix} 5 & 2 & 3 \\ 7 & 3 & 4 \\ 9 & 4 & 5 \end{pmatrix}$$

. Determine (i) determinent , adjoint , inverse and all minors of order 1,2 & 3. (ii)A+B , A-B , AB, A^6.

```
In [222]: A=matrix(3,3,[1,2,3,1,3,3,1,2,4])
          B=matrix(3,3,[5,2,3,7,3,4,9,4,5])
          show('A=',A)
          show('B=',B)
```

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 2 & 3 \\ 7 & 3 & 4 \\ 9 & 4 & 5 \end{pmatrix}$$

```
In [223]: #(i)determinent , adjoint , inverse and all minors of order 1,2 & 3. (ii)A+
          B , A-B , AB,  A^6
          det(A)
```

Out[223]: 1

```
In [224]: det(B)
```

Out[224]: 0

```
In [225]: A.adjugate()
```

Out[225]: [6 -2 -3]
           [-1 1 0]
           [-1 0 1]

```
In [226]: B.adjugate()
```

Out[226]: [-1 2 -1]
           [1 -2 1]
           [1 -2 1]

```
In [227]: A.inverse()
```

Out[227]: [6 -2 -3]
           [-1 1 0]
           [-1 0 1]

```
In [228]: A.minors(1)
```

Out[228]: [1, 2, 3, 1, 3, 3, 1, 2, 4]

In [229]: `A.minors(2)`

Out[229]: `[1, 0, -3, 0, 1, 2, -1, 1, 6]`

In [230]: `A.minors(3)`

Out[230]: `[1]`

In [231]: `B.minors(1)`

Out[231]: `[5, 2, 3, 7, 3, 4, 9, 4, 5]`

In [232]: `B.minors(2)`

Out[232]: `[1, -1, -1, 2, -2, -2, 1, -1, -1]`

In [233]: `B.minors(3)`

Out[233]: `[0]`

(ii)A+B , A-B , AB, A^6.

In [234]: `A+B`

Out[234]:
```
[ 6  4  6]
[ 8  6  7]
[10  6  9]
```

In [235]: `A-B`

Out[235]:
```
[-4  0  0]
[-6  0 -1]
[-8 -2 -1]
```

In [236]: `A*B`

Out[236]:
```
[46 20 26]
[53 23 30]
[55 24 31]
```

In [237]: `A^6`

Out[237]:
```
[13201 30912 46368]
[15456 36193 54288]
[15456 36192 54289]
```

Exercise 3

Verify whether the Vectors are linearly independent or dependent

(i) (1,2,4), (2,-1,3), (0,1,2), (-3,7,2)

(ii) (1,2,-3,4), (3,-1,2,1), (1,-5,8,-7)

In [219]:
```
X=matrix(4,3,[1,2,4,2,-1,3,0,1,2,-3,7,2])
show(X)
```

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 0 & 1 & 2 \\ -3 & 7 & 2 \end{pmatrix}$$

In [220]:
```
show(X.transpose().echelon_form())
```

$$\begin{pmatrix} 1 & 2 & 0 & -3 \\ 0 & 5 & 0 & -12 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

In [239]:
```
Y=matrix(3,4,[1,2,-3,4,3,-1,2,1,1,-5,8,-7])
show(Y)
```

$$\begin{pmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 2 & 1 \\ 1 & -5 & 8 & -7 \end{pmatrix}$$

In [240]:
```
show(Y.transpose().echelon_form())
```

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Exercise 4

Ex.4.1) Consider the matrix

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

i) Find the characteristics polynolmial, p(x), of A. (ii) Find the roots of p(x), show that eigenvalues of A are roots of p(x). (iii) Find the eigenvectors of A. (iv) Show that sum of the eigenvalues of A is trace of A. (iv) Show that the product of eigenvalues of A is the determinant of A.

```
In [241]:  A=matrix(3,3,[2,-2,2,1,1,1,1,3,-1])
           p(x)=A.characteristic_polynomial ( x )
           show('p(x)=',p(x))
```

$$\mathbf{p(x)}{=}x^3 - 2\,x^2 - 4\,x + 8$$

```
In [242]:  p(x).roots()
```

Out[242]: [(-2, 1), (2, 2)]

```
In [243]:  E= A . eigenvalues ()
           show(E)
```

$$\left[-2, 2, 2\right]$$

```
In [244]:  V= A . eigenvectors_right ()
           show('V=',V)
```

$$\mathbf{V{=}}\left[\left(-2, \left[\left(1, \frac{1}{4}, -\frac{7}{4}\right)\right], 1\right), \left(2, \left[\left(0, 1, 1\right)\right], 2\right)\right]$$

```
In [245]:  sum(E)==A.trace()
```

Out[245]: True

```
In [246]:  product(E)==det(A)
```

Out[246]: True

Ex. 4.2) For the matrix

$$\begin{pmatrix} cost & sint \\ -sint & cost \end{pmatrix}$$

Find the Eigen values and Eigen Vectors for different values of .

In [250]:
```
A=matrix(2,2,[cos(t),sin(t),-sin(t),cos(t)])
show(A)
```

$$\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

In [251]:
```
E= A . eigenvalues ()
show(E)
```

$$[\cos(t) - i\,\sin(t), \cos(t) + i\,\sin(t)]$$

In [252]:
```
V= A . eigenvectors_right ()
show('V=',V)
```

$$\mathtt{V} = [(\cos(t) - i\,\sin(t), [(1, -i)], 1), (\cos(t) + i\,\sin(t), [(1, i)], 1)]$$

Exercise 5

Generate a square Matrix of order 7 whose all entries are One. Replace the third row by 5 and 4th column by 8. Then Replace all the diagonal elements by Zero. Find the Trace and Determinant of the updated Matrix.

In [254]:
```
X=ones_matrix(7,7)
show ('X=',X)
```

$$\mathtt{X} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [257]:
```
X[5,4]=8
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 8 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [272]:
```
X[0,0]=0
X[1,1]=0
X[2,2]=0
X[3,3]=0
X[4,4]=0
X[5,5]=0
X[6,6]=0
show(X)
```

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 8 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In [273]:
```
X.trace()
```

Out[273]: 0

In [274]:
```
det(X)
```

Out[274]: 13

What you learned?

I have learned about the basics of Linear Algebra and how to perform various matrix operations using SageMath.Here is a summary of the concepts and operations demonstrated in the notebook:Matrix Creation: The notebook shows how to create matrices by either defining a nested list of rows or by specifying the dimensions and providing a flat list of elements.Basic Matrix Arithmetic: It includes examples of performing matrix addition ($A + B$), subtraction ($A - B$), and multiplication ($A \times B$).Matrix Properties and Functions:Transpose: Calculating the transpose of a matrix (B^T).Adjugate: Finding the adjugate (or adjoint) of a matrix.Determinant: Calculating the determinant of a matrix, as well as the determinant of a matrix's inverse.Inverse: Computing the inverse of a matrix, including how to handle cases where a matrix is singular (which results in a ZeroDivisionError).Trace and Minors: Finding the trace (sum of diagonal elements) and the minors of various orders for a given matrix.Symbolic Matrices: The notebook demonstrates working with matrices containing symbolic variables (like x, y, z) and performing operations such as:Simplification: Using full_simplify() to reduce complex symbolic expressions.Differentiation: Differentiating a matrix with respect to a symbolic variable.Special Matrices: It shows how to generate specific types of matrices, such as the ones matrix, zero matrix, and identity matrix.Matrix Manipulation: The code illustrates how to access and modify specific elements within a matrix using index notation (e.g., F[1,2]=500).Verification of Identities: There is a specific section dedicated to verifying the identity $B^{-1} \times \det(B) = \text{adj}(B)$, which returned True in the notebook's execution.

In [ ]: