Math 2153

Midterm 1

Autumn 2025

Name: Solution

OSU Username (name.nn): ____________

Recitation Instructor: ____________

Recitation Time: ____________

Instructions

- You have **50 minutes** to complete this exam.
- This exam consists of 9 problems and 10 pages, including this cover sheet. Page 9 may be used for extra workspace and Page 10 has some formulas.
- If you wish to have any work on the extra workspace page considered for credit, indicate in the problem that there is additional work on the extra workspace page and **clearly label** to which problem the work belongs on the extra page.
- Please **write clearly** and make sure to **justify your answers** and **show all relevant work**! Correct answers with no supporting work may receive no credit. Your solutions should only use concepts and techniques covered in this course (up to this point in the course) or prerequisite courses.
- Give **exact answers** unless instructed to do otherwise.
- Calculators of any kind are *not* permitted. No devices are permitted. Be sure all devices, including cell phones, smart watches, or other devices with internet capabilities are put away. You are not permitted to communicate with other people while taking this exam.

Improper Markings: Incomplete bubbles, x's, checkmarks, and other markings will **not** be recognized. Do not use these marks to indicate correct or incorrect answers.

- **Acceptable Marking for selected answers:** ●
- **Acceptable Marking for unselected answers:** ○
- **Unacceptable Markings:** ⓧ ◉

1. Fill in the circle next to the correct response. There is no partial credit and you do not need to show any work for this problem.

Q1: [2 pts each] Identity the following:

a. $\vec{\mathbf{v}} \cdot \vec{\mathbf{u}} + |\vec{\mathbf{v}}|$ ● Scalar (B) Vector (C) Undefined

b. $\vec{\mathbf{v}} \times \vec{\mathbf{u}} \times \vec{\mathbf{w}}$ (A) Scalar ● Vector (C) Undefined

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Q2: True or False [2 pts each] Determine whether each statement below is **true** or **false**.

(T) ● If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are unit vectors, then $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$ is a unit vector.

● (F) If $\vec{\mathbf{u}}$ is a vector then $\vec{\mathbf{u}} \cdot \vec{\mathbf{u}} = |\vec{\mathbf{u}}|^2$.

(T) ● The lines $x = 3 + t, y = 4 + 2t, z = 2 - t$ and $x = 2t, y = 4t, z = t$ are parallel.

● (F) It is possible for non-parallel lines to be non-intersecting in $\mathbb{R}^3$.

● (F) If f_{xy} and f_{yx} are continuous then $f_{xy} = f_{yx}$.

● (F) If $\vec{\mathbf{u}} \neq 0$ and $\vec{\mathbf{u}} + \vec{\mathbf{v}} = 0$, then $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are parallel.

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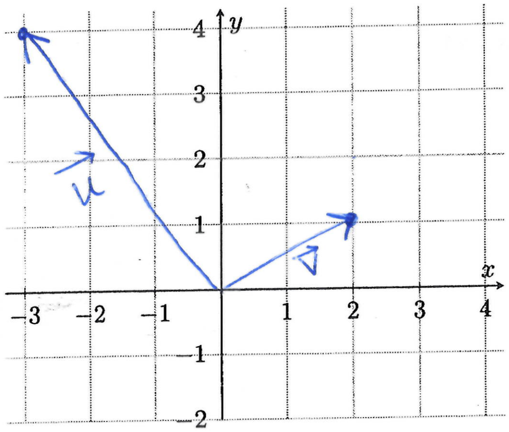
2. [8 pts] Let $\vec{u} = -3\vec{i} + 4\vec{j}$ and $\vec{v} = 2\vec{i} + \vec{j}$. Sketch these vectors and compute $\text{proj}_{\vec{v}}\vec{u}$ and $\text{scal}_{\vec{v}}\vec{u}$.



$$\text{scal}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{(-3)(2)+(4)(1)}{\sqrt{(2)^2+1}}$$

$$= \frac{-6+4}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

$$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta \cdot \frac{\vec{v}}{|\vec{v}|}$$

$$= -\frac{2}{\sqrt{5}}\,\frac{\langle 2,1\rangle}{\sqrt{4+1}} = -\frac{2}{\sqrt{5}}\langle 2,1\rangle$$

3. [8 pts] Find the parametric equation of the line segment joining $(2, 4, 8)$ and $(7, 5, 3)$.

$$\vec{v} = \langle 5, 1, -5\rangle$$

$$\vec{r}(t) = \langle 2,4,8\rangle + t\langle 5,1,-5\rangle$$
$$= \langle 2+5t,\ 4+t;\ 8-5t\rangle$$

$$\left.\begin{aligned} x &= 2+5t \\ y &= 4+t \\ z &= 8-5t \end{aligned}\right\} \quad 0 \le t \le 1$$

4. [7 pts] Find both the parametric and vector equation of the line passing through $(1, -3, 4)$ and parallel to the line $x = 3 + 4t, y = 5 - t, z = 7$.

parallel lines have same direction.

$\vec{v} = \langle 4, -1, 0 \rangle$

$\vec{r}(t) = \langle 1, -3, 4 \rangle + t\langle 4, -1, 0 \rangle$

$= \langle 1 + 4t, -3 - t, 4 \rangle$

$$\left.\begin{aligned} x &= 1 + 4t \\ y &= -3 - t \\ z &= 4 \end{aligned}\right\} \quad -\infty < t < \infty$$

5. [10 pts] Explain why the following limit does not exist at $(0, 0)$.

$$\lim_{(x,y)\to(0,0)} \frac{4x^2y}{x^3 + xy^2}$$

Use 2-path test $y = mx$

$$\lim_{x \to 0} \frac{4mx(x^2)}{x^3 + x(mx)^2}$$

$$= \lim_{x \to 0} \frac{4mx^3}{x^3 + m^2x^3} = \lim_{x \to 0} \frac{\cancel{x^3}(4m)}{\cancel{x^3}(1+m^2)}$$

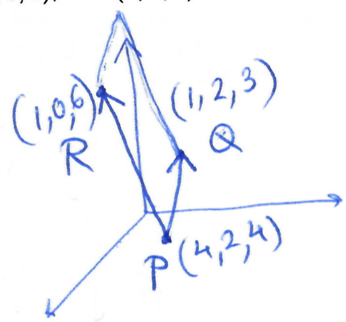
$$= \frac{4m}{1+m^2}$$

~~For line $x = 0$, m~~

this value depends on m.

Thus for different lines (slopes will different) limiting value will be different. So limit doesn't exist.

6. [10 points] Find the area of a parallelogram with vertices $(1,2,3)$, $(1,0,6)$, and $(4,2,4)$.



$\overrightarrow{PQ} = \langle -3, 0, -1 \rangle$

$\overrightarrow{PR} = \langle -3, -2, 2 \rangle$

Area of parallelogram = Magnitude of cross product of $\overrightarrow{PR}$ and $\overrightarrow{PQ}$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 0 & -1 \\ -3 & -2 & 2 \end{vmatrix}$$

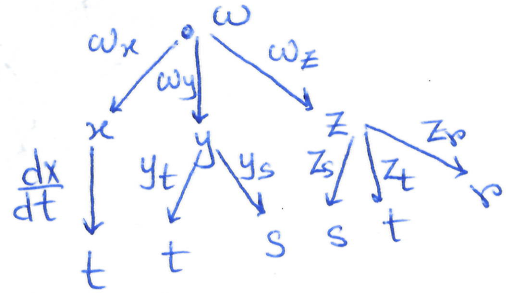
$$= \vec{i}(0-2) - \vec{j}(-6-3) + \vec{k}(6)$$

$$= -2\vec{i} + 9\vec{j} + 6\vec{k}$$

$$\text{Area} = \sqrt{(-2)^2 + (9)^2 + (6)^2}$$

$$= \sqrt{4 + 81 + 36} = \sqrt{121} = \boxed{11}$$

7. a. [8 pts] Given that $w = F(x, y, z)$, and x is a function of t and y is a function of s, t and z is a function of s, t and r. Sketch a Chain Rule tree diagram with branches labeled with the appropriate derivatives. Then write the expression for $\frac{\partial \cancel{z} w}{\partial t}$.



$$\frac{\partial w}{\partial t} = w_x \frac{dx}{dt} + w_y y_t + w_z z_t.$$

b. [10 pts] Using chain rule find the following derivatives:

z_s and z_t where $z = \sin(2x + y), x = s^2 - t^2$, and $y = s^2 + t^2$.

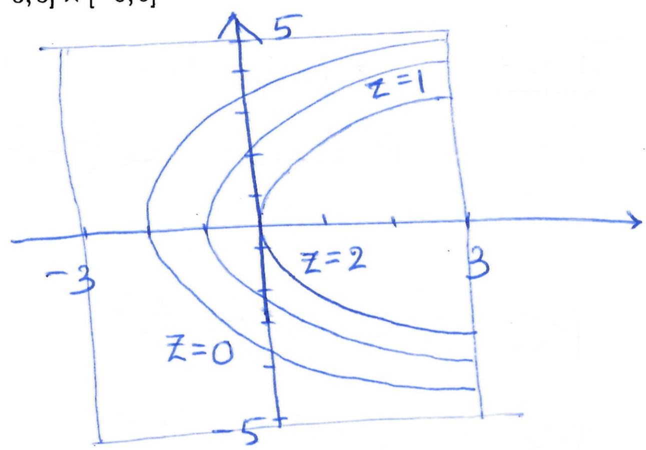
$$z_x = 2\cos(2x+y)$$
$$z_y = \cancel{2}\cos(2x+y)$$
$$x_s = 2s \qquad x_t = -2t$$
$$y_s = 2s \qquad y_t = 2t$$

$$z_s = 2\cos(2x+y) \cdot 2s + \cos(2x+y) \cdot 2s$$
$$= 4s\cos(2(s^2-t^2)+s^2+t^2) + 2s\cos(2(s^2-t^2)+s^2+t^2)$$
$$= 6s\cos(3s^2-t^2)$$

$$z_t = 2\cos(2x+y)(-2t) + \cos(2x+y)(2t)$$
$$= -2t\cos(3s^2-t^2)$$

8. [8 pts] Draw 3 level curves for the following function. Label the appropriate z values.

$$z = x + 2 - y^2; [-3, 3] \times [-5, 5]$$



$Z = 0$

$x + 2 - y^2 = 0$

$y^2 = x + 2$

$Z = 1$

$x + 2 - y^2 = 1$

$y^2 = x + 1$

$Z = 2$

$x + 2 - y^2 = 2$

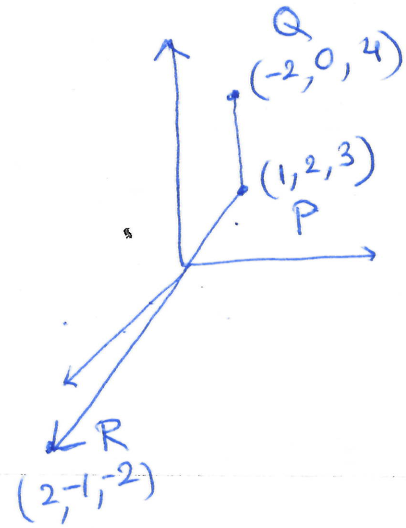
$x = y^2$

All are parabolas that opens to the right.
You must label the domain and make sure to indicate the vertices (either by plotting them or writing the co-ordinate)

9. [15 pts] Find a plane which contains the points $(1, 2, 3)$, $(-2, 0, 4)$, and $(2, -1, -2)$. You may use the fact that these points are noncollinear. Write your answer in the form $ax + by + cz = d$.

Step 1 Form the correct vectors

$P(1,2,3)$ $Q(-2,0,4)$ $R(2,-1,-2)$



$\vec{PQ} = \langle -3, -2, 1 \rangle$

$\vec{PR} = \langle 1, -3, -5 \rangle$

Step 2 Find the normal vector.

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -2 & 1 \\ 1 & -3 & -5 \end{vmatrix} = \vec{i}(10+3) - \vec{j}(15-1) + \vec{k}(9+2)$$

$$= 13\vec{i} - 14\vec{j} + 11\vec{k}$$

Step 3

Equation of the plane

$$13(x-1) - 14(y-2) + 11(z-3) = 0$$

$$13x - 13 - 14y + 28 + 11z - 33 = 0$$

$$13x - 14y + 11z = 18$$

(You must get this form).