

MA374 : Financial Engineering Laboratory
Lab 11

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Question 1:

➤ In the Vasicek Model the risk neutral dynamics of r can be expressed as -

$$dr = (b - ar)dt + \sigma dW$$

➤ On comparing with the model we get $a = \beta$ and $b = \beta u$.

➤ Price of the bond is calculated using following formulae -

$$B(t, T) = \frac{1 - e^{-a(T-t)}}{a}$$

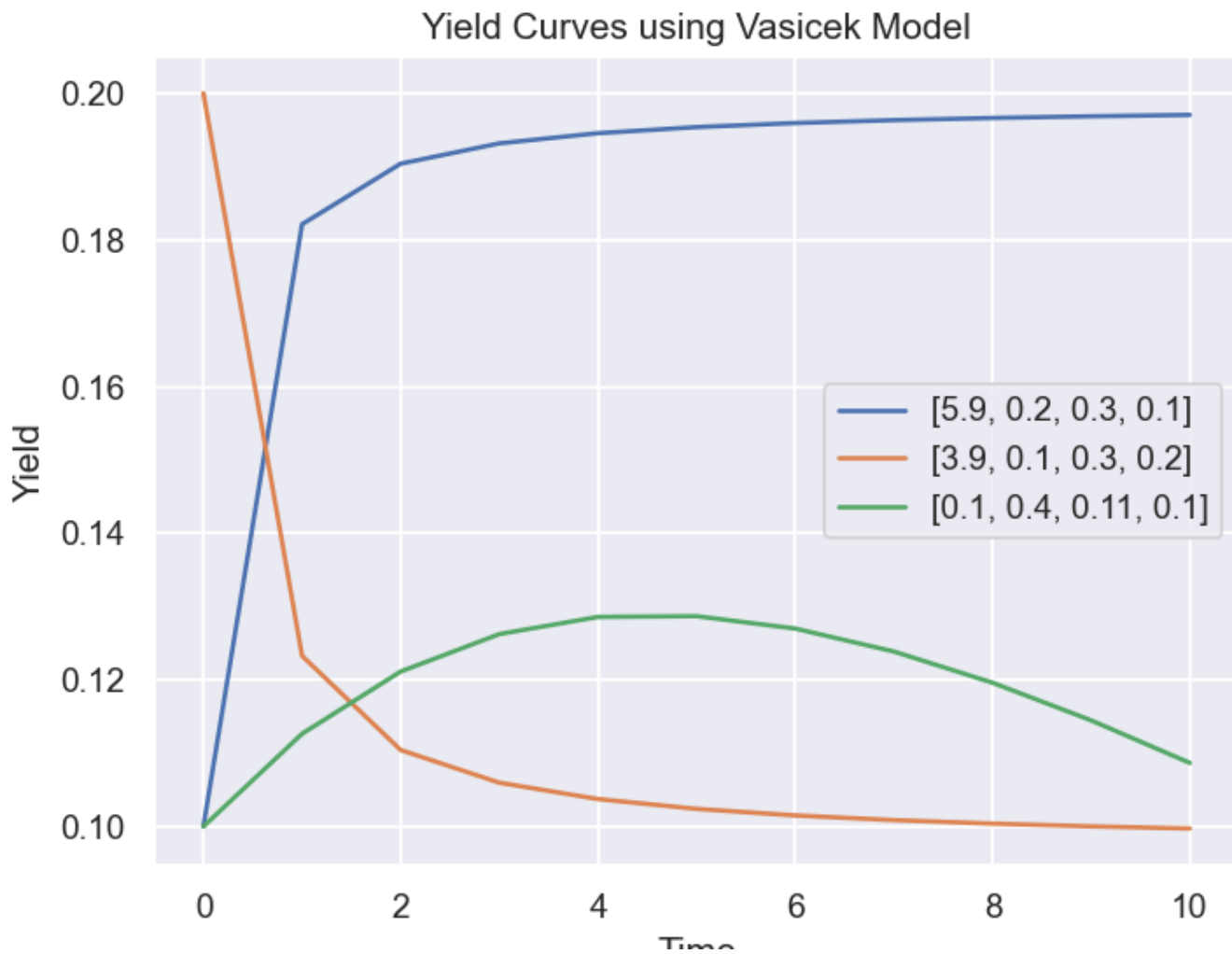
$$A(t, T) = \frac{(B(t, T) - T + t)(ab - \frac{\sigma^2}{2})}{a^2} - \frac{\sigma^2 B^2(t, T)}{4a}$$
$$\rho(t, T) = e^{A(t, T) - B(t, T)r(t)}$$

➤ Yield can be calculated from the price using following formula—

$$Yield = \frac{-\log(p(t, T))}{T - t}$$

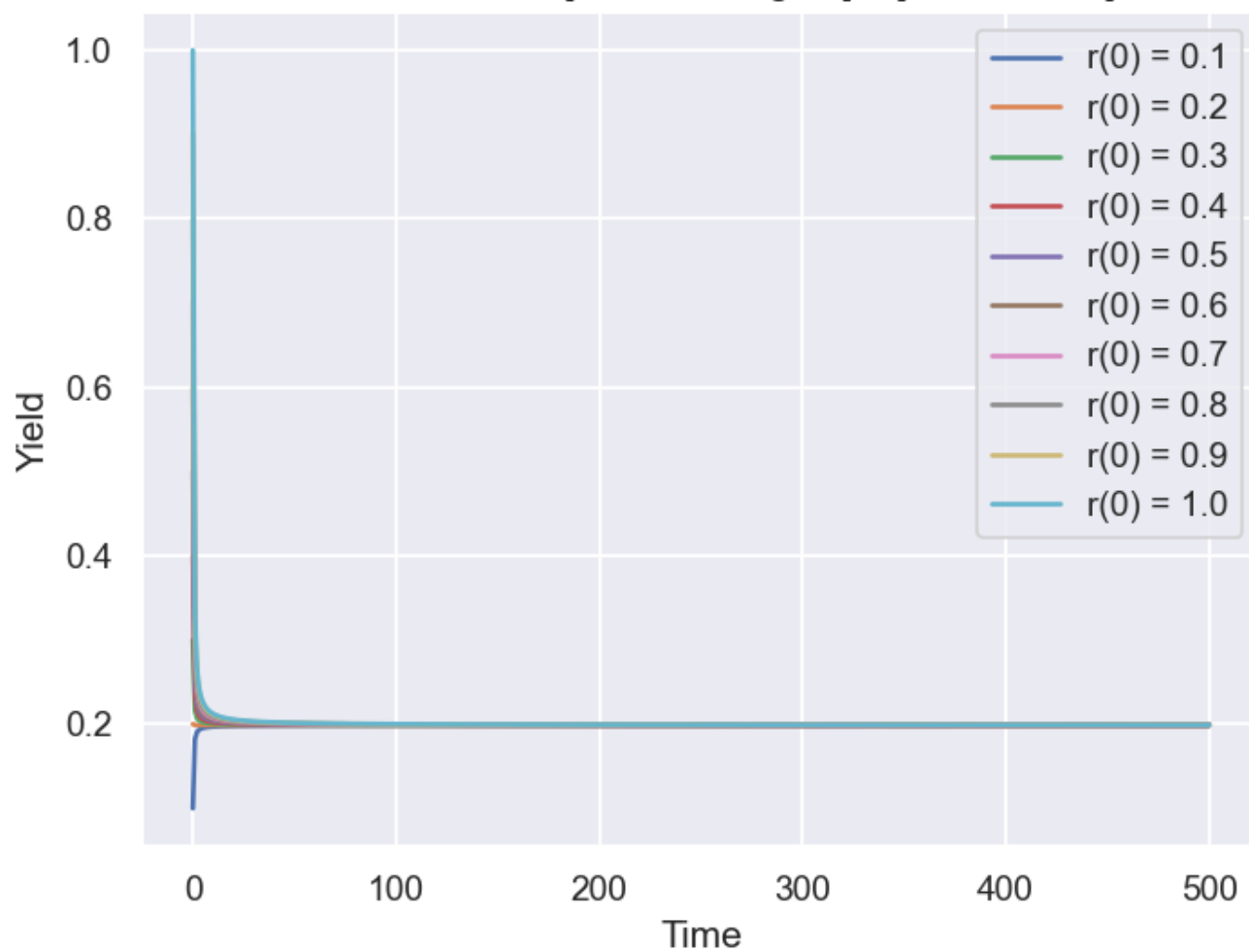
➤ $t = 0$ in our case

Term structure for the given parameters is plotted using 10 time units.

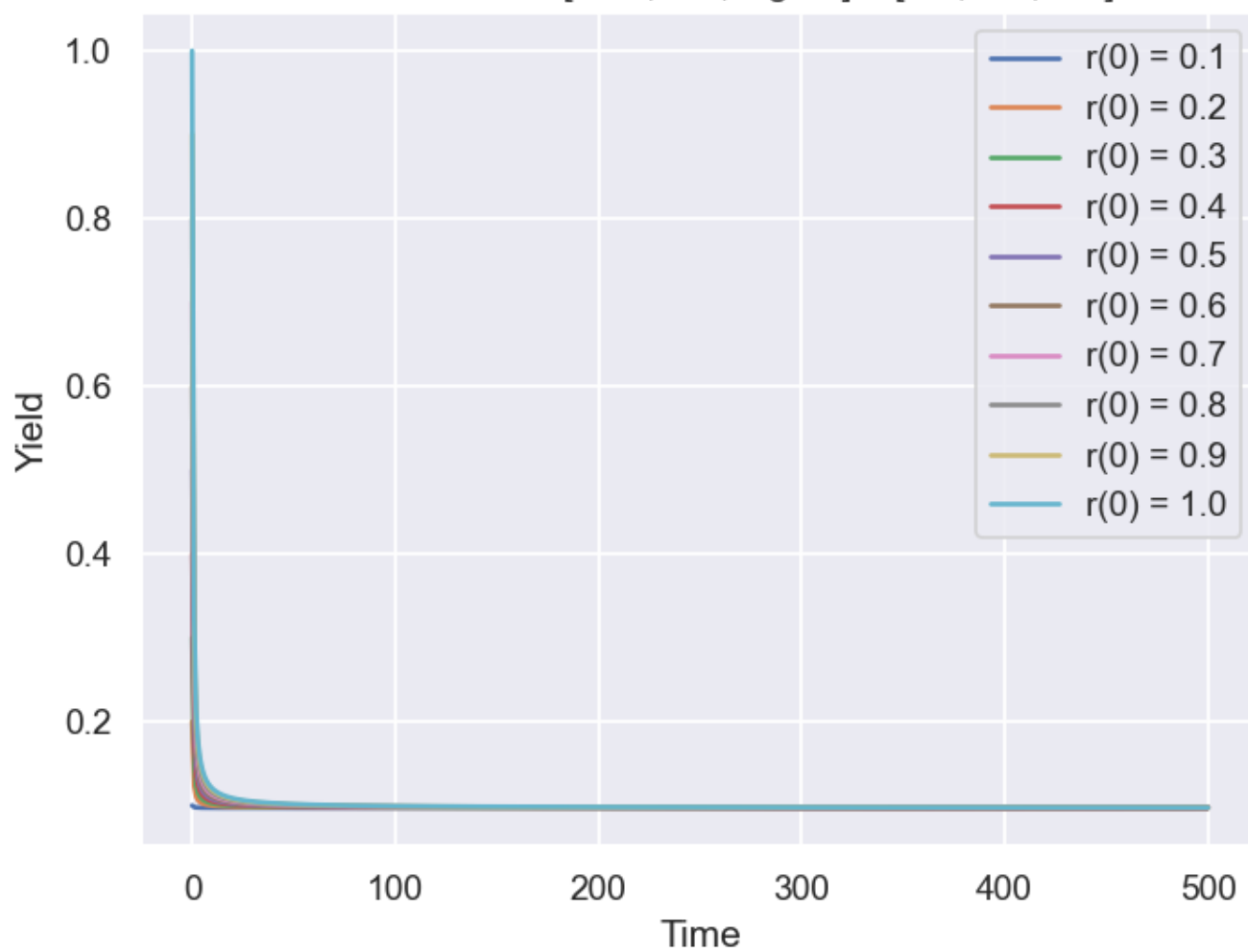


Now, yield curves versus maturity up to 500 time units for 10 different values of $r(0)$ are plotted for all the three sets of parameters.

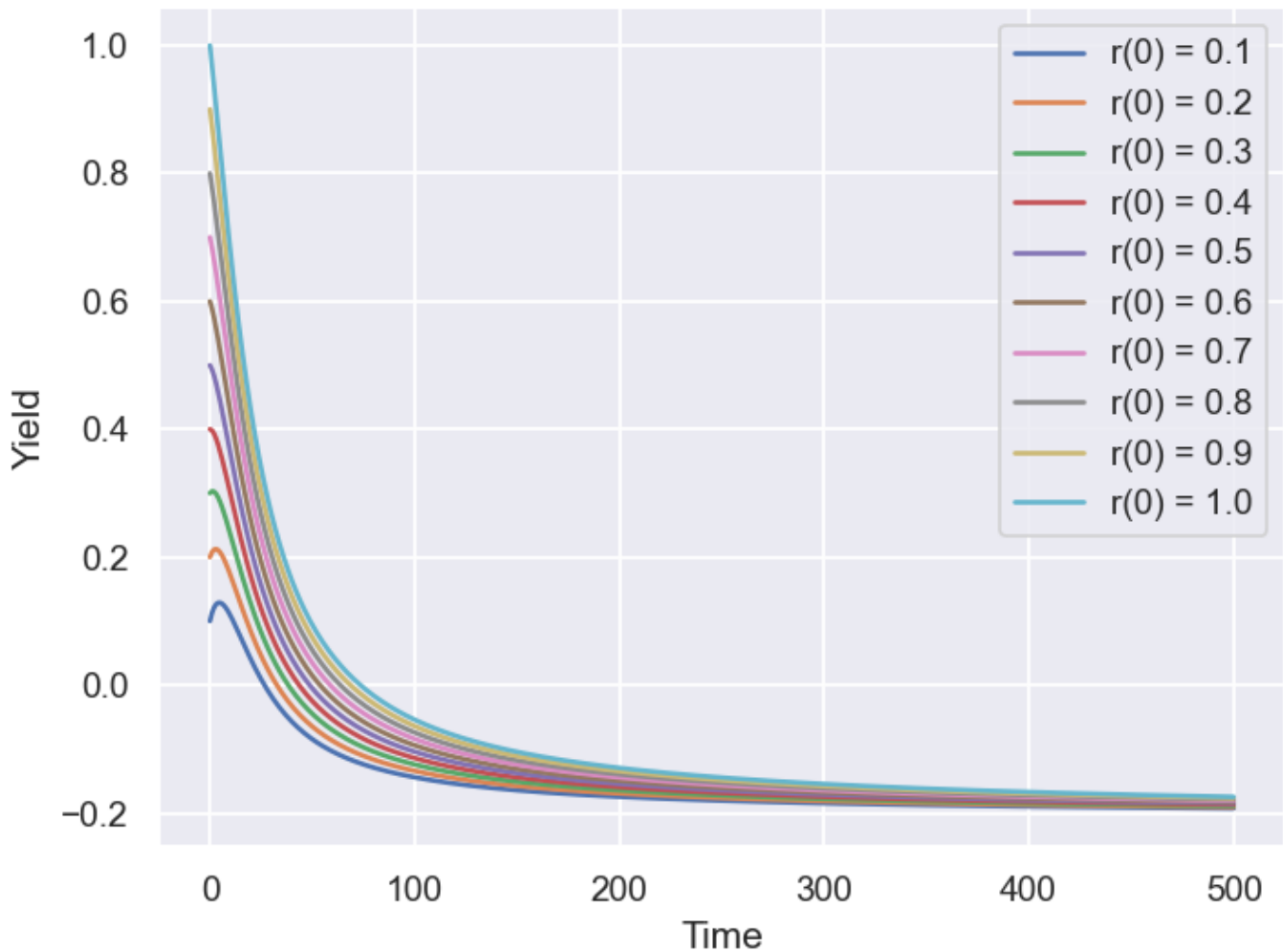
Yield Curves for [beta, mu, sigma] = [5.9, 0.2, 0.3]



Yield Curves for [beta, mu, sigma] = [3.9, 0.1, 0.3]



Yield Curves for $[\text{beta}, \mu, \sigma] = [0.1, 0.4, 0.11]$



Observations :

- For higher $r(0)$, yield is higher
- Yield converges to a limit for all the parameters.
- Yield can increase or decrease with time to maturity. It depends on the prediction made using the current parameters about the future interest rates.

Question 2:

- In the CIR(Cox-Ingersoll-Ross) model the risk neutral dynamics of r can be expressed as –

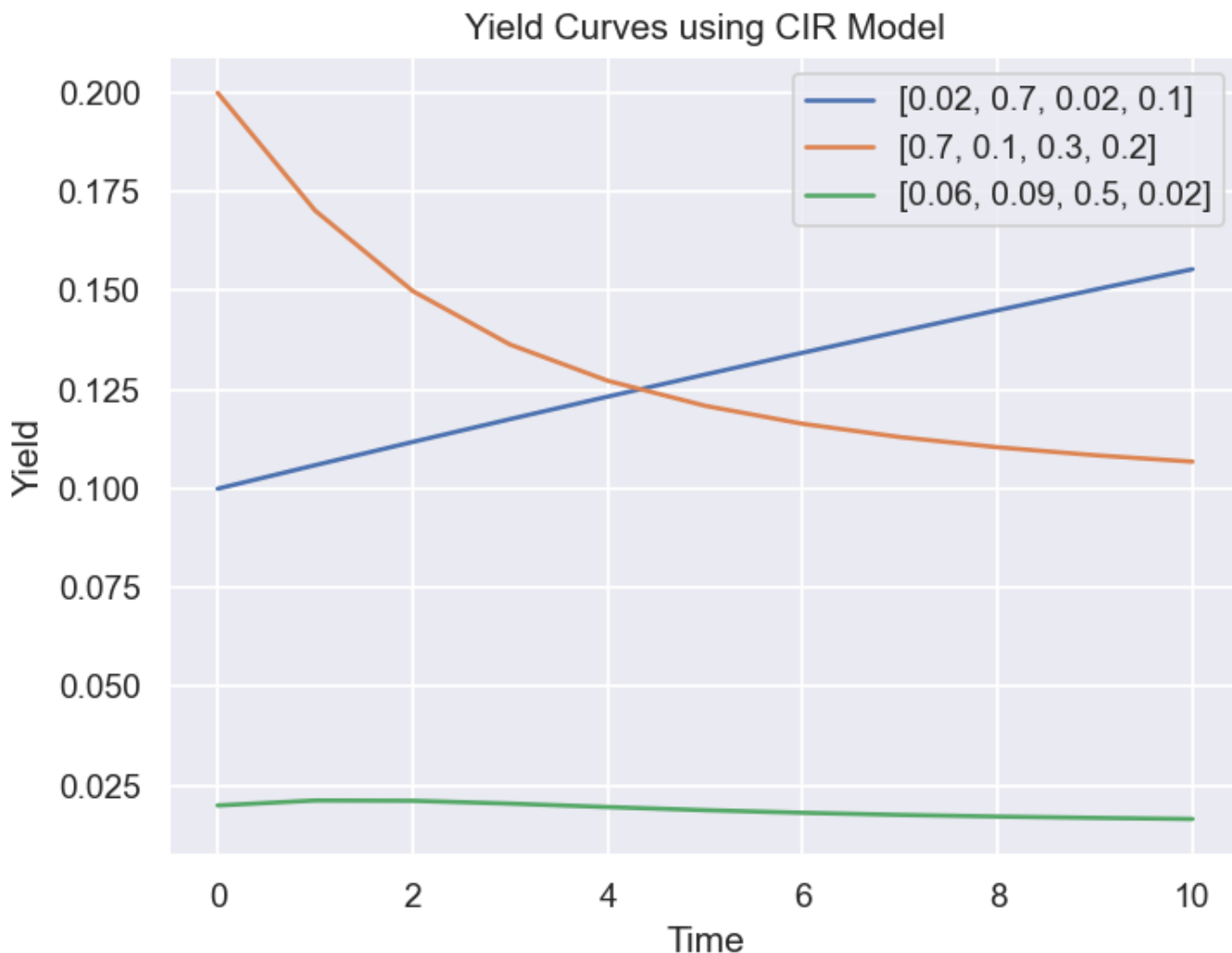
$$dr = a(b - r)dt + \sigma\sqrt{r}dW$$

- On comparing with the model we get $a = \beta$ and $b = \mu$.
- Price of the bond is calculated using following formulas –
- Yield can be calculated from the price using following formula –

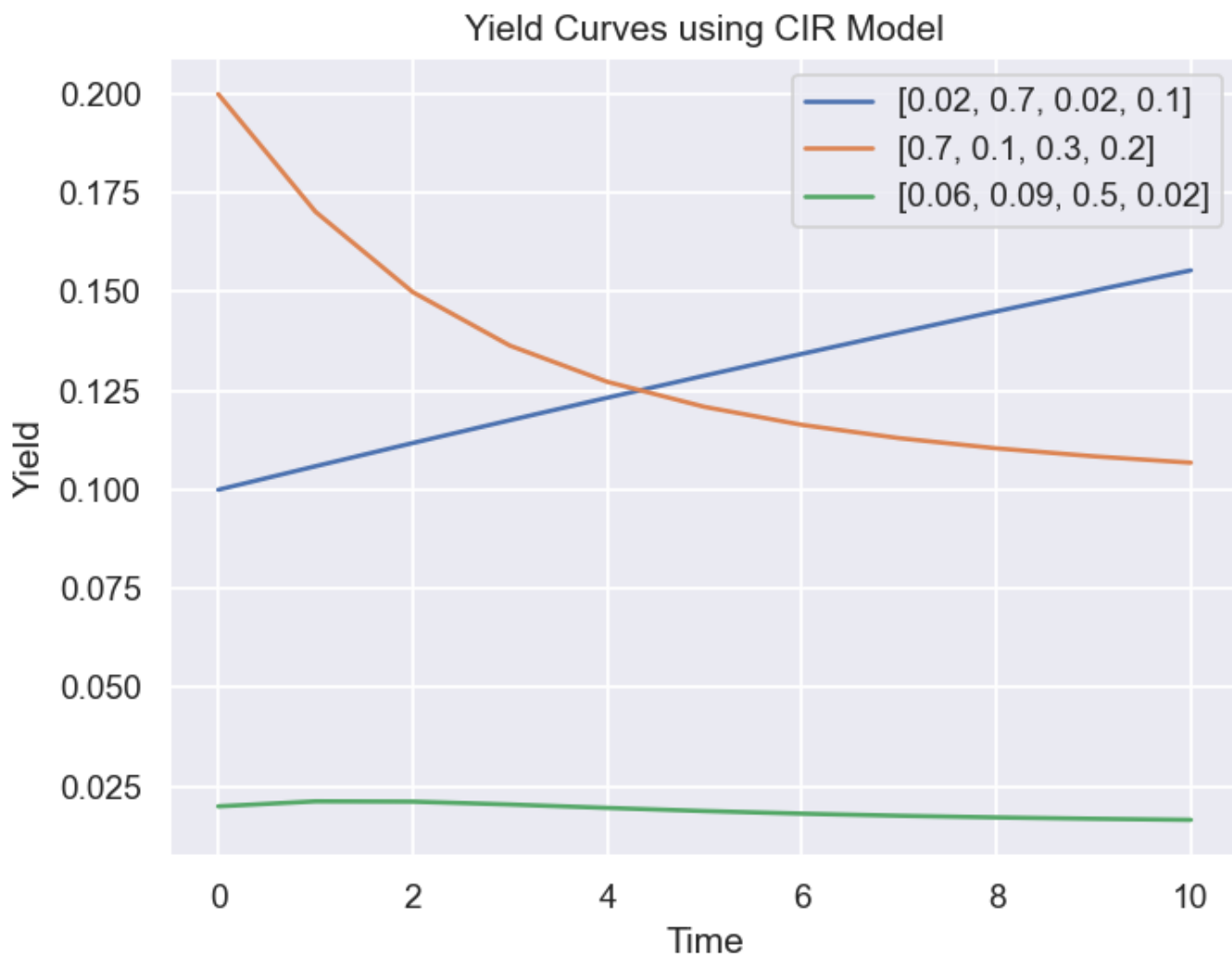
$$Yield = \frac{-\log(p(t, T))}{T - t}$$

- $t = 0$ in our case.

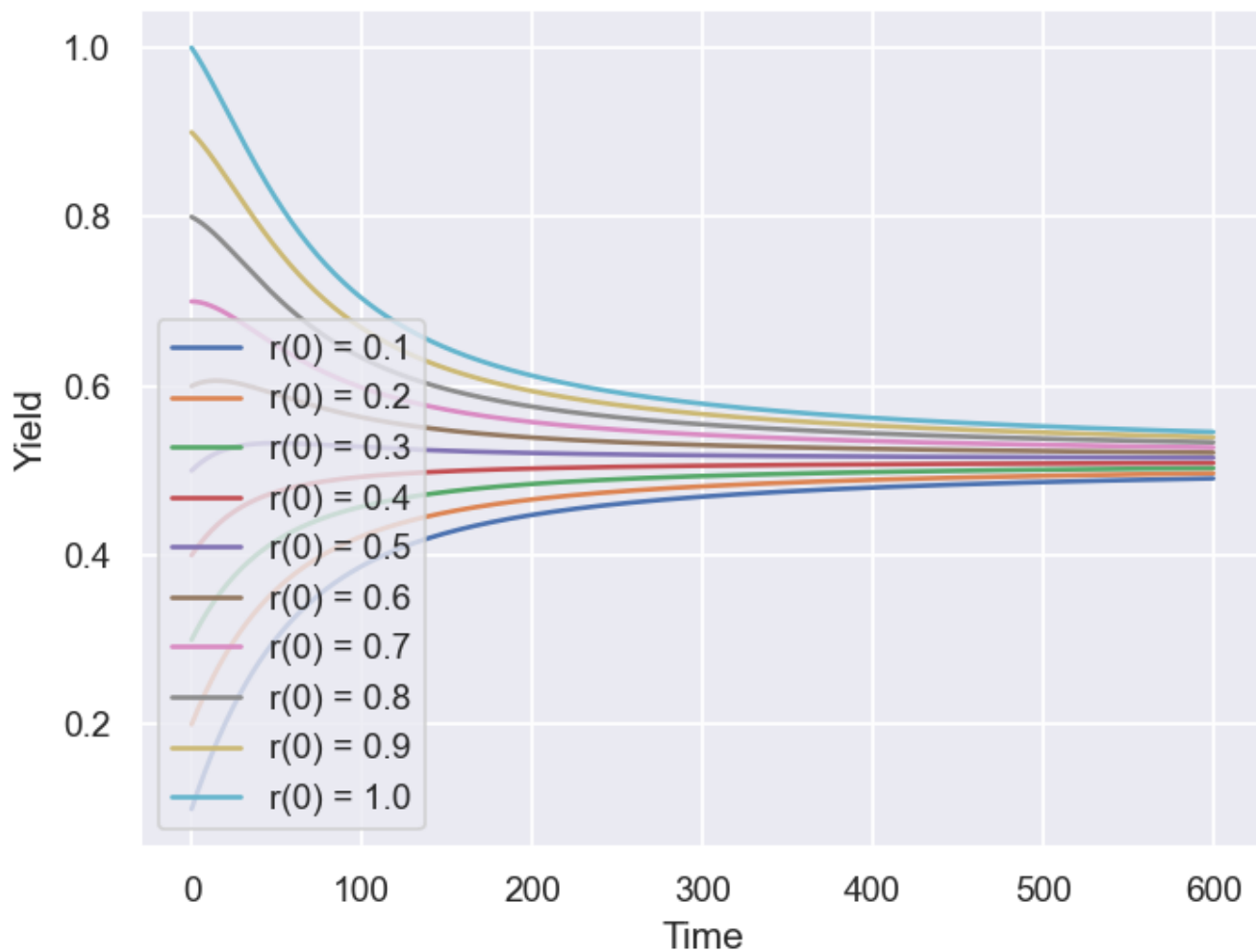
Term structure for the given parameters is plotted using 10 time units.



Now, yield curves versus maturity up to 600 time units for $r(0) = 0.1:0.1:1$ is plotted for $[\text{beta}, \text{mu}, \text{sigma}] = [0.02, 0.7, 0.02]$.



Yield Curves using CIR model for $[\text{beta}, \mu, \sigma] = [0.02, 0.7, 0.02]$



Observations :

- For higher $r(0)$, yield is higher.
- Yield converges to a limit.
- Yield can increase or decrease with time to maturity. It depends on the prediction made using the current parameters about the future interest rates.