MA374 LAB 03

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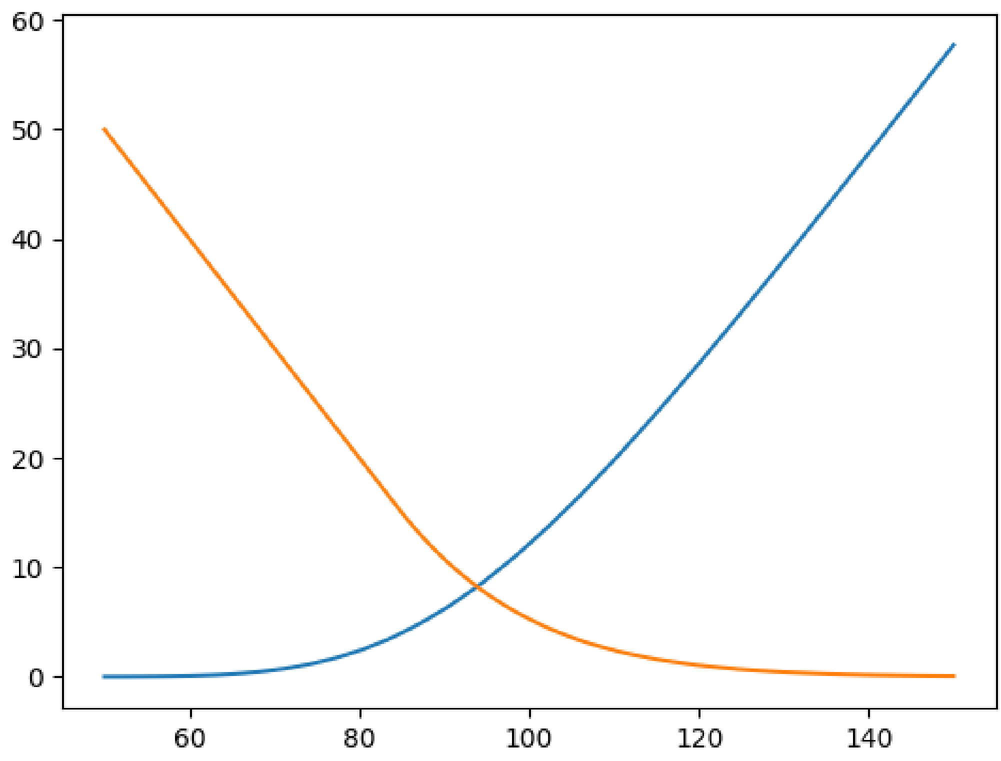
Question 1:-

Given expression for u and d are:

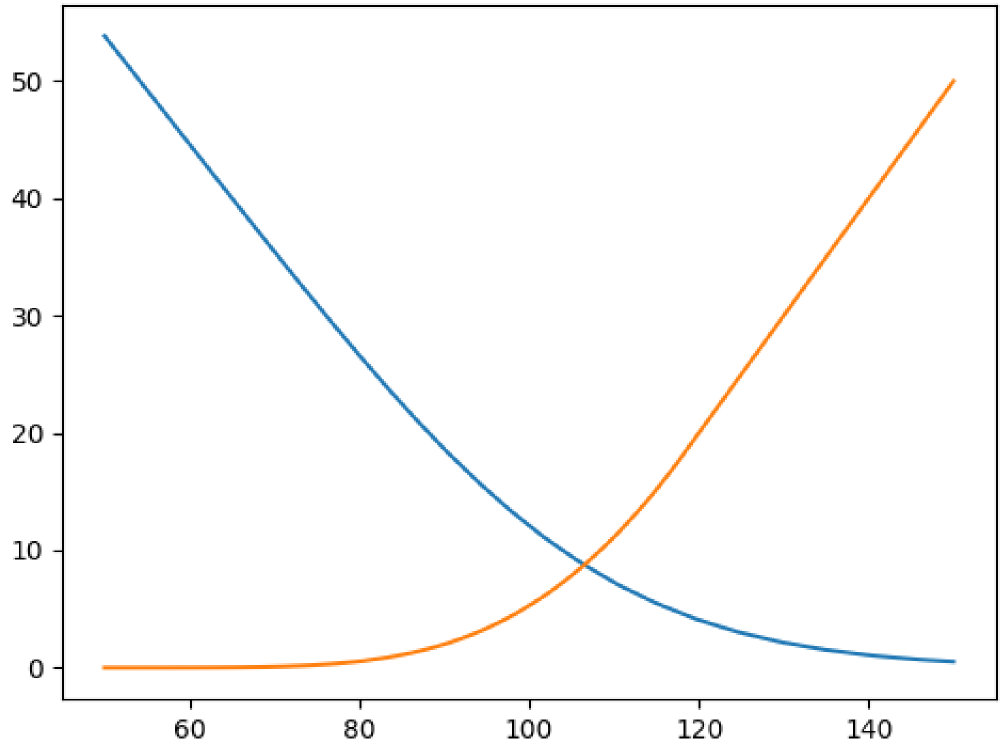
$$C_i^A = max(e^{-\Delta t}(\hat{p} * C_{i+1}^A(H)) + \hat{q} * C_{i+1}^A(T), max((S(t_i) - K), 0)), where\, \Delta t = T/M$$
$$u = e^{\sigma\sqrt{(\Delta)} + (r - \sigma^2/2)t}, d = u = e^{-\sigma\sqrt{(\Delta)} + (r - \sigma^2/2)t}$$
$$\hat{p} = e^{r\Delta} - d/u - d \quad \text{and q = 1-p}$$

Initial price of the American Call option price = 12.12304707401241 and Put option price = 5.27983714598915.

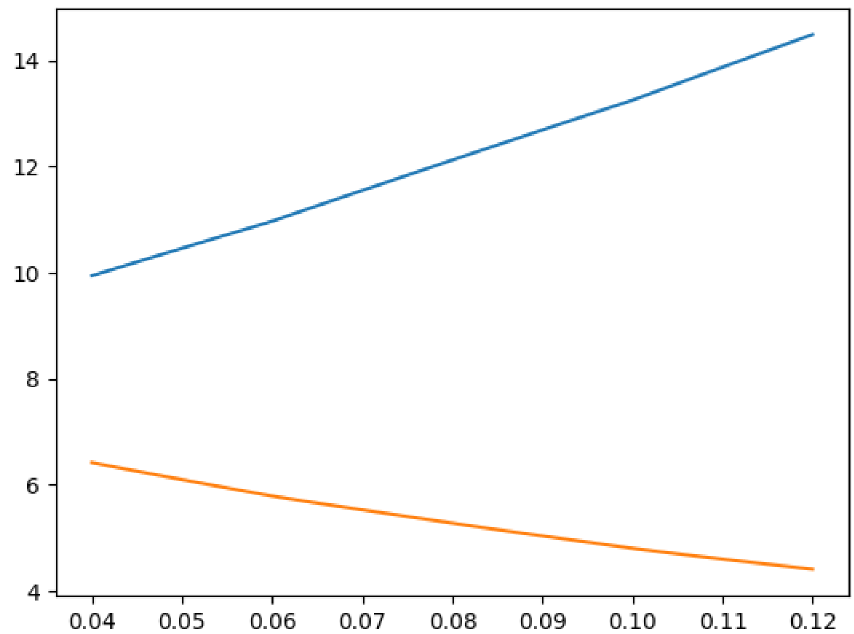
Price of American call vs put option for different S0



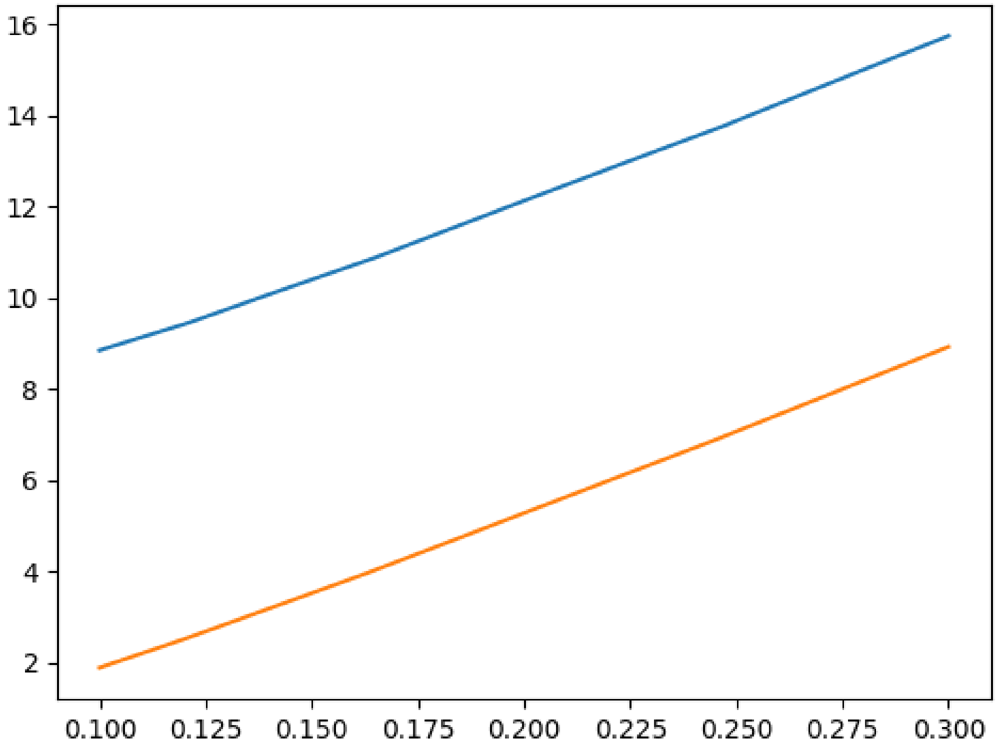
Price of American call vs put option for different K



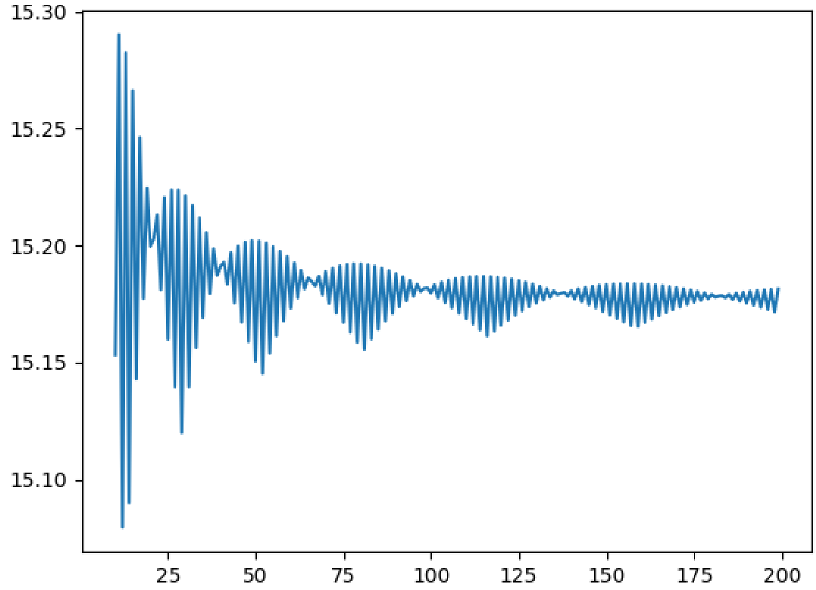
Price of American call vs put option for different r

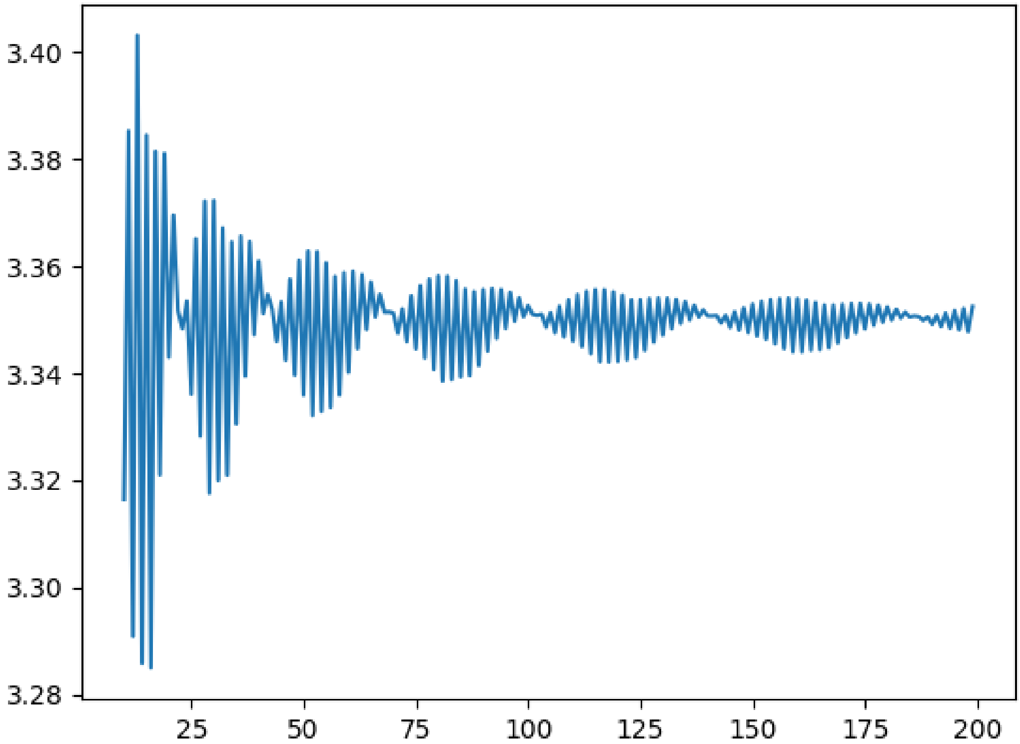


Price of American call vs put option for different sigma

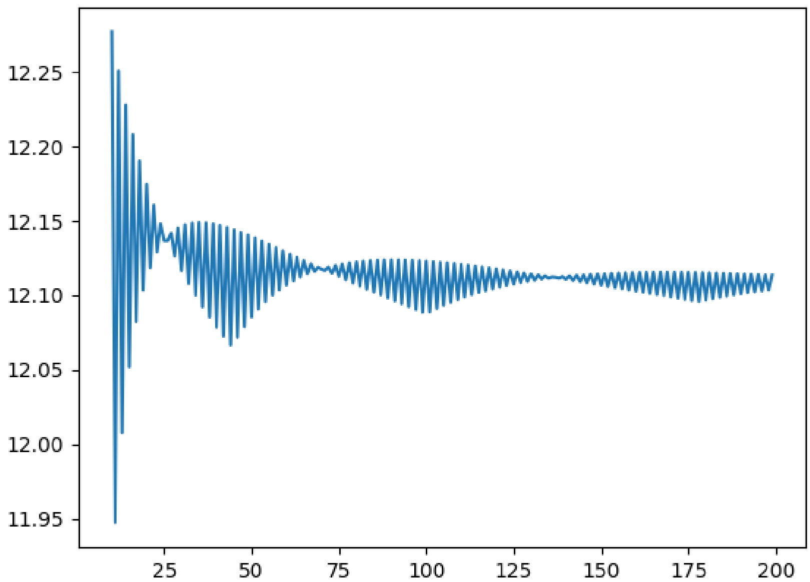


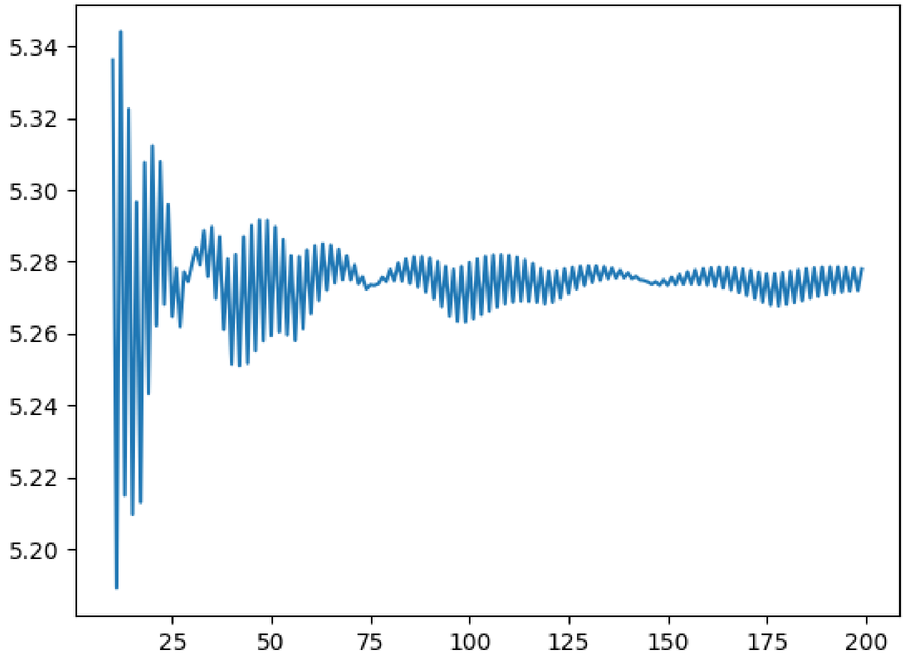
Price of American call vs put option for different M's K=95



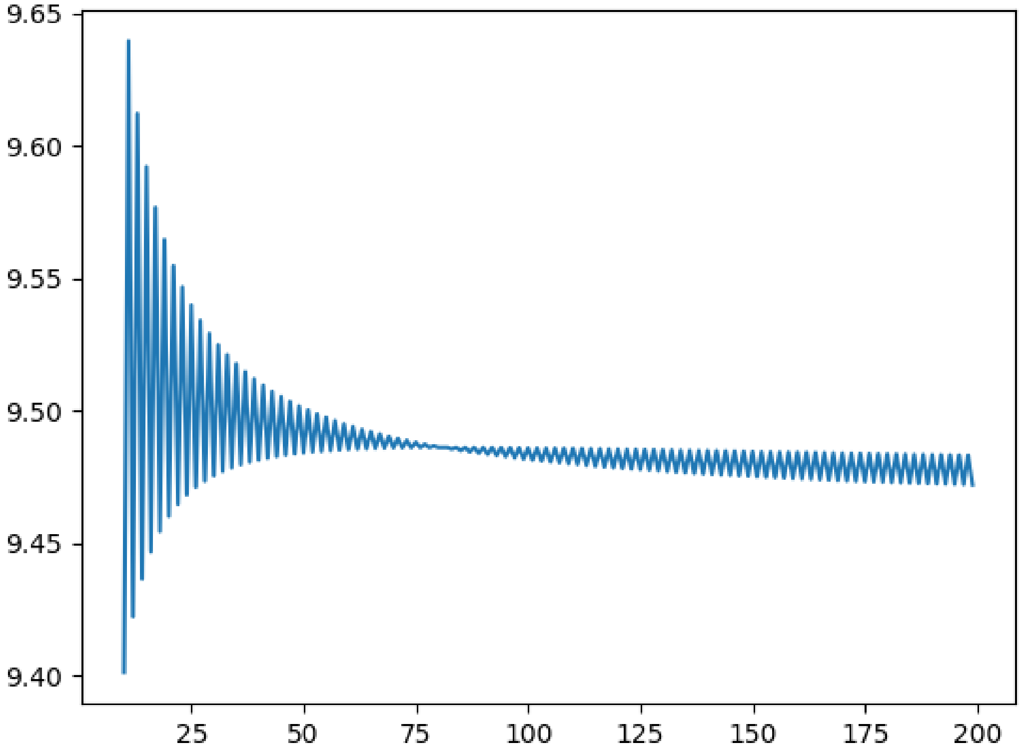


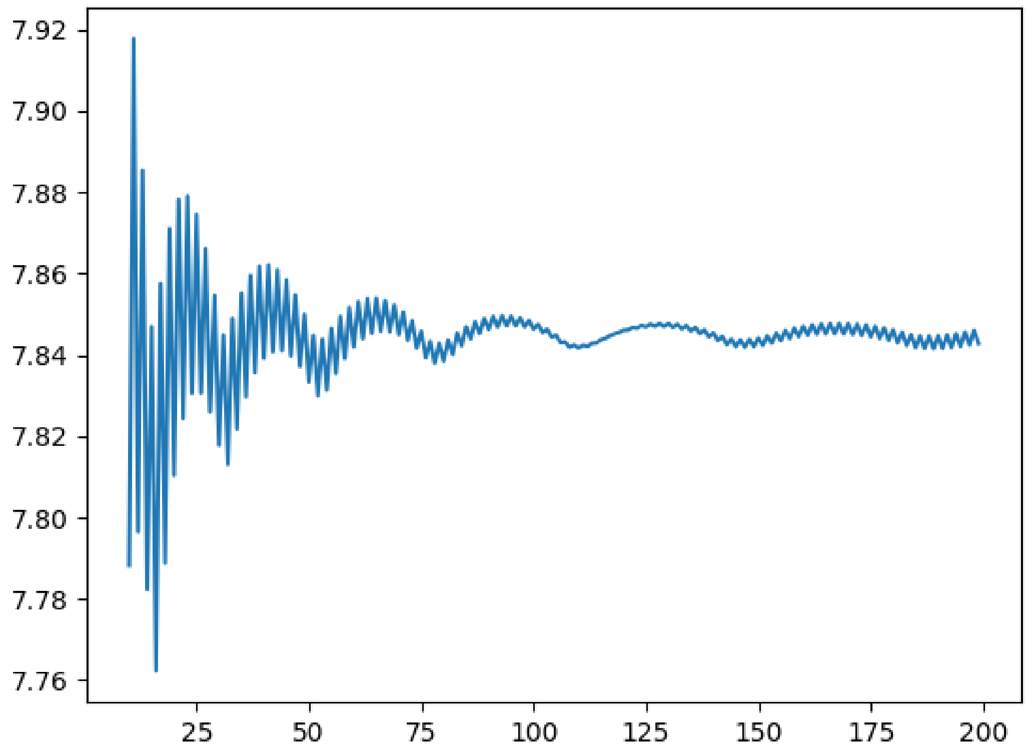
Price of American call vs put option for different K=100 M’s





Price of American call vs put option for different K=105, M’s





Question 2:-

For the European Option, we use the following data,

$$S(0) = 100,\ T = 1,\ r = 8\%,\ \sigma = 20\%$$

The payoff of the lookback option is given as,

$$V = S(i) - S(M)$$

$S(i) = S(i\Delta t)$ and

$u = e^{\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$ $and\ d = e^{-\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$, $where\ \Delta t = \frac{T}{M}$, $M\ here\ is\ the\ number\ of\ su$

(a) Initial price of the option for –

M = 5 is 9.119299

M = 10 is 10.080583

Using binomial algorithms, calculating option price for M = 25 and M = 50 is computationally infeasible.

(b)

	M	Loopback Option Price
0	5	9.119299
1	6	9.415434
2	7	9.609088
3	8	9.806368
4	9	9.936758
5	10	10.080583
6	11	10.175899
7	12	10.286896
8	13	10.367182
9	14	10.452999
10	15	10.519165

(c)

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Values of the options for M = 5 is -
u*d = 1.0242903178906213
----------------  -----------------  ------------------  ------------------  ------------------  --------
9.11929898586469  9.504839866450858  12.168664659721797  17.582062714095425  25.05122945703703   32.1054
                  9.027951165547757  7.1479157567747444  7.1484182081901215  10.680904426029972  18.8059
                                     9.79911875354703    8.32461466963142    10.680904426029972  18.8059
                                     8.548076183576446   6.201916453882752   3.846928884415608    2.90135
                                                         13.712862965988537  13.07138097092879   18.8059
                                                         6.201916453882752   3.846928884415608    2.90135
                                                         9.95527127295782    8.003613780975444    7.81842
                                                         7.416771005131012   4.600479677676438    0
                                                                             21.188089345345652  21.235
                                                                             6.680842999256647    5.33038
                                                                             8.003613780975444    7.81842
                                                                             4.600479677676438    0
                                                                             15.631851880479829  16.2664
                                                                             4.600479677676438    0
                                                                             9.57139153170023     9.34992
                                                                             5.501638813873981    0
                                                                                                 29.4826
                                                                                                 13.578
                                                                                                 13.578
                                                                                                  0
                                                                                                 16.2664
                                                                                                  0
                                                                                                  9.34992
                                                                                                  0
                                                                                                 25.3946
                                                                                                  6.37452
                                                                                                  9.34992
                                                                                                  0
                                                                                                 19.4527
                                                                                                  0
                                                                                                 11.1814
                                                                                                  0
```

<u>Question 3:-</u>

(a)

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------------------------  sub-part(a)  ------------------------


*********  Executing for M = 5  *********

No arbitrage exists for M = 5
Initial Price of Loopback Option        = 9.119298985864683
Execution Time                          = 5.412101745605469e-05 sec



*********  Executing for M = 10  *********

No arbitrage exists for M = 10
Initial Price of Loopback Option        = 10.080582906831
Execution Time                          = 0.000453948974609375 sec



*********  Executing for M = 25  *********

No arbitrage exists for M = 25
Initial Price of Loopback Option        = 11.00349533564633
Execution Time                          = 0.031832218170166016 sec



*********  Executing for M = 50  *********

No arbitrage exists for M = 50
Initial Price of Loopback Option        = 11.510862222177286
Execution Time                          = 1.8234710693359375 sec
```
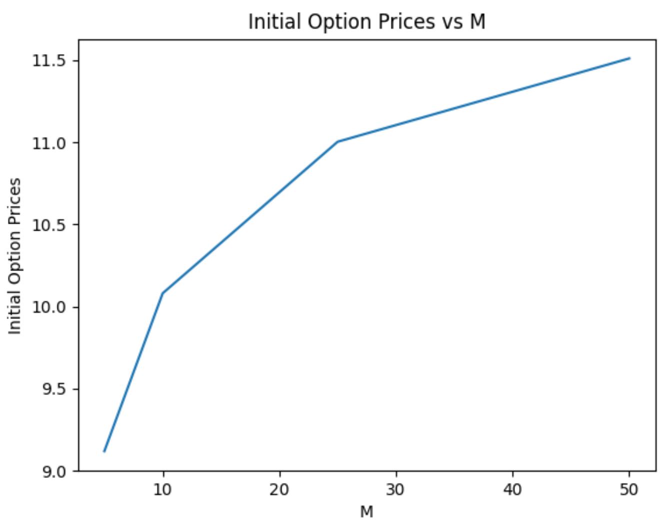

(b)



(c)

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-----------------------  sub-part(c)  -----------------------
At t = 0
Intermediate state = (100, 100)         Price = 9.119298985864683

At t = 1
Intermediate state = (110.676651999383, 110.676651999383)               Price = 9.027951165547751
Intermediate state = (92.54800352077254, 100)           Price = 9.504839866450853

At t = 2
Intermediate state = (122.49321297792528, 122.49321297792528)           Price = 8.548076183576441
Intermediate state = (102.42903178906215, 110.676651999383)             Price = 9.799118753547026
Intermediate state = (102.42903178906214, 102.42903178906214)           Price = 7.147915756774744
Intermediate state = (85.65132955680926, 100)           Price = 12.168664659721792

At t = 3
Intermediate state = (135.57138705044142, 135.57138705044142)           Price = 7.416771005131011
Intermediate state = (113.3650230595177, 122.49321297792528)            Price = 9.955271272957816
Intermediate state = (113.3650230595177, 113.3650230595177)             Price = 6.201916453882752
Intermediate state = (94.79602394643446, 110.676651999383)              Price = 13.712862965988533
Intermediate state = (113.36502305951768, 113.36502305951768)           Price = 6.201916453882752
Intermediate state = (94.79602394643445, 102.42903178906214)            Price = 8.32461466963314
Intermediate state = (94.79602394643445, 100)           Price = 7.14841820819012
Intermediate state = (79.26859549382432, 100)           Price = 17.582062714095418
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At t = 4
Intermediate state = (150.04587225655362, 150.04587225655362)          Price = 5.501638813873981
Intermediate state = (125.46861206060268, 135.57138705044142)          Price = 9.571391531700229
Intermediate state = (125.46861206060268, 125.46861206060268)          Price = 4.600479677676438
Intermediate state = (104.91706553244704, 122.49321297792528)          Price = 15.631851880479827
Intermediate state = (104.91706553244704, 113.3650230595177)           Price = 8.003613780975444
Intermediate state = (104.91706553244704, 110.676651999383)            Price = 6.6808429992566465
Intermediate state = (87.73182757949854, 110.676651999383)             Price = 21.18808934534565
Intermediate state = (125.46861206060267, 125.46861206060267)          Price = 4.600479677676438
Intermediate state = (104.91706553244703, 113.36502305951768)          Price = 8.003613780975444
Intermediate state = (104.91706553244701, 104.91706553244701)          Price = 3.8469288844156075
Intermediate state = (87.73182757949853, 102.42903178906214)           Price = 13.071380970928788
Intermediate state = (87.73182757949853, 100)          Price = 10.68090442602997
Intermediate state = (73.36150254849147, 100)          Price = 25.051229457037028

At t = 5
Intermediate state = (166.06574787682462, 166.06574787682462)          Price = 0.0
Intermediate state = (138.86445913876912, 150.04587225655362)          Price = 11.181413117784501
Intermediate state = (138.8644591387691, 138.8644591387691)            Price = 0.0
Intermediate state = (116.118695507311, 135.57138705044142)            Price = 19.452691543130413
Intermediate state = (116.118695507311, 125.46861206060268)            Price = 9.349916553291678
Intermediate state = (116.11869550731102, 122.49321297792528)          Price = 6.374517470614265
Intermediate state = (97.09864950286031, 122.49321297792528)           Price = 25.39456347506497
Intermediate state = (116.11869550731102, 116.11869550731102)          Price = 0.0
Intermediate state = (97.09864950286031, 113.3650230595177)            Price = 16.266373556657385
Intermediate state = (97.09864950286031, 110.676651999383)             Price = 13.578002496522686
Intermediate state = (81.1940548771124, 110.676651999383)              Price = 29.48259712227059
Intermediate state = (116.11869550731099, 125.46861206060267)          Price = 9.349916553291678
Intermediate state = (116.11869550731099, 116.11869550731099)          Price = 0.0
Intermediate state = (97.0986495028603, 113.36502305951768)            Price = 16.266373556657385
Intermediate state = (116.11869550731097, 116.11869550731097)          Price = 0.0
Intermediate state = (97.0986495028603, 104.91706553244701)            Price = 7.8184160295867144
Intermediate state = (97.0986495028603, 102.42903178906214)            Price = 5.330382286201839
Intermediate state = (81.19405487711239, 102.42903178906214)           Price = 21.234976911949744
Intermediate state = (97.0986495028603, 100)           Price = 2.9013504971397026
Intermediate state = (81.19405487711239, 100)          Price = 18.805945122887607
Intermediate state = (67.89460596146952, 100)          Price = 32.10539403853048
```

Maximum value of M for the algorithm to run in reasonable in time:

- For binomial: 15
- For Markov: 50

Time Complexity

- Time complexity for binomial algorithm is O(2^M) because we are exploring every path of the binomial tree.
- Markov algorithm depends on 2 states, the current stock price and maximum stock price encountered along the path till now. Time complexity of this algorithm is O(M^4), because number of unique paths is bounded by O(M^2) and hence, maximum stock prices is also bounded by O(M^2).

Question 4:-

Similar to above questions, pricing of European call option (assuming strike price$(K) = 100$) is being performed using binomial algorithm and markov algorithm and following computational difference is observed –

Time Complexity

The binomial algorithm is$O(2^M)$, because we are exploring every path.

For the markov algorithm we are using step number and count of upstep taken in the path till now. Step number is bounded by $O(M)$ and count of up steps taken would also be bounded by $O(M)$. Therefore the number of unique states would be bounded by $O(M^2)$. In each state we are doing $O(1)$ work, therefore time complexity of the algorithm is $O(M^2)$. Maximum M allowed Maximum value of M that can be handled in reasonable time for –

Binomial algorithm – 20

Markov algorithm – around 1000 (My python code ran successfuly in reasonable time for M till 995, after that it gives maximum recursion depth exceeded error)

(a)

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----------------------  sub-part(a)  ----------------------

######################  Unoptimised Binomial Algorithm executing  ######################
No arbitrage exists for M = 5
European Call Option            = 12.16318594676458
Execution Time                  = 4.506111145019531e-05 sec

No arbitrage exists for M = 10
European Call Option            = 12.27732781922299
Execution Time                  = 0.0009112358093261719 sec

No arbitrage exists for M = 25
European Call Option            = 12.136745963232949
Execution Time                  = 46.69279384613037 sec


######################  Efficient Binomial Algorithm executing (Markov Based)  ######################
No arbitrage exists for M = 5
European Call Option            = 12.163185946764584
Execution Time                  = 0.00010704994201660156 sec

No arbitrage exists for M = 10
European Call Option            = 12.277327819222982
Execution Time                  = 2.193450927734375e-05 sec

No arbitrage exists for M = 25
European Call Option            = 12.136745963232947
Execution Time                  = 7.390975952148438e-05 sec

No arbitrage exists for M = 50
European Call Option            = 12.0853615100722
Execution Time                  = 0.0002570152282714844 sec


######################  Most Efficient Binomial Algorithm executing (Markov Based) ######################
No arbitrage exists for M = 5
European Call Option            = 12.163185946764584
Execution Time                  = 7.867813110351562e-05 sec

No arbitrage exists for M = 10
European Call Option            = 12.277327819222986
Execution Time                  = 1.5974044799804688e-05 sec

No arbitrage exists for M = 25
European Call Option            = 12.136745963232956
Execution Time                  = 3.600120544433594e-05 sec

No arbitrage exists for M = 50
European Call Option            = 12.085361510072197
Execution Time                  = 8.58306884765625e-05 sec
```
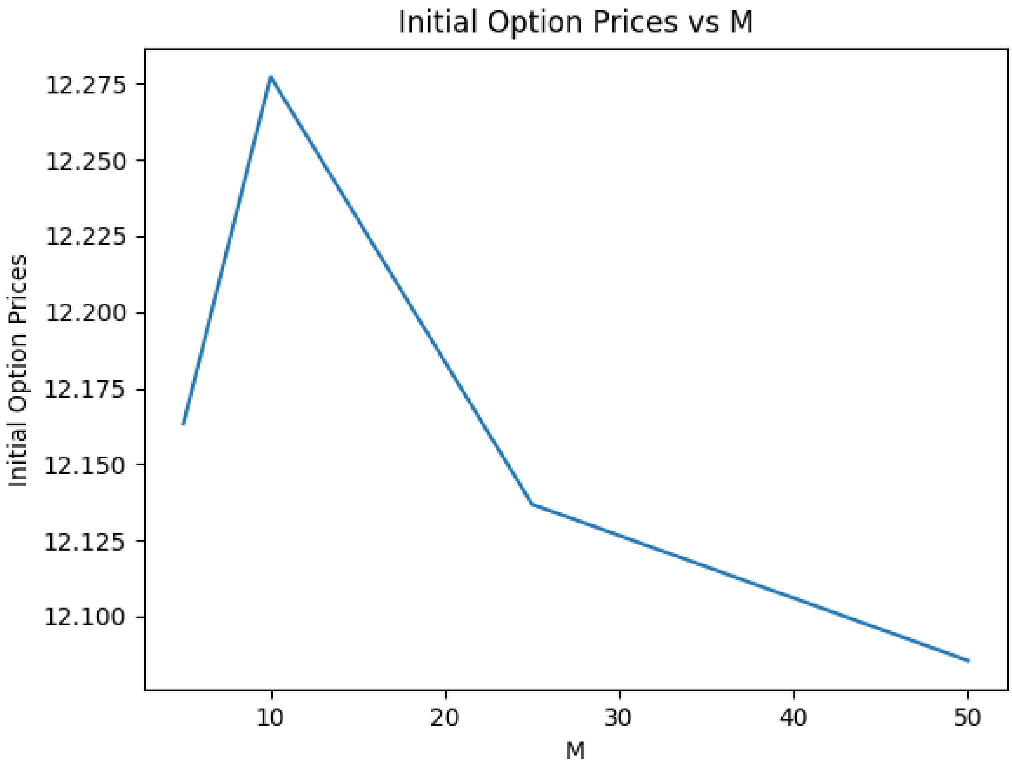
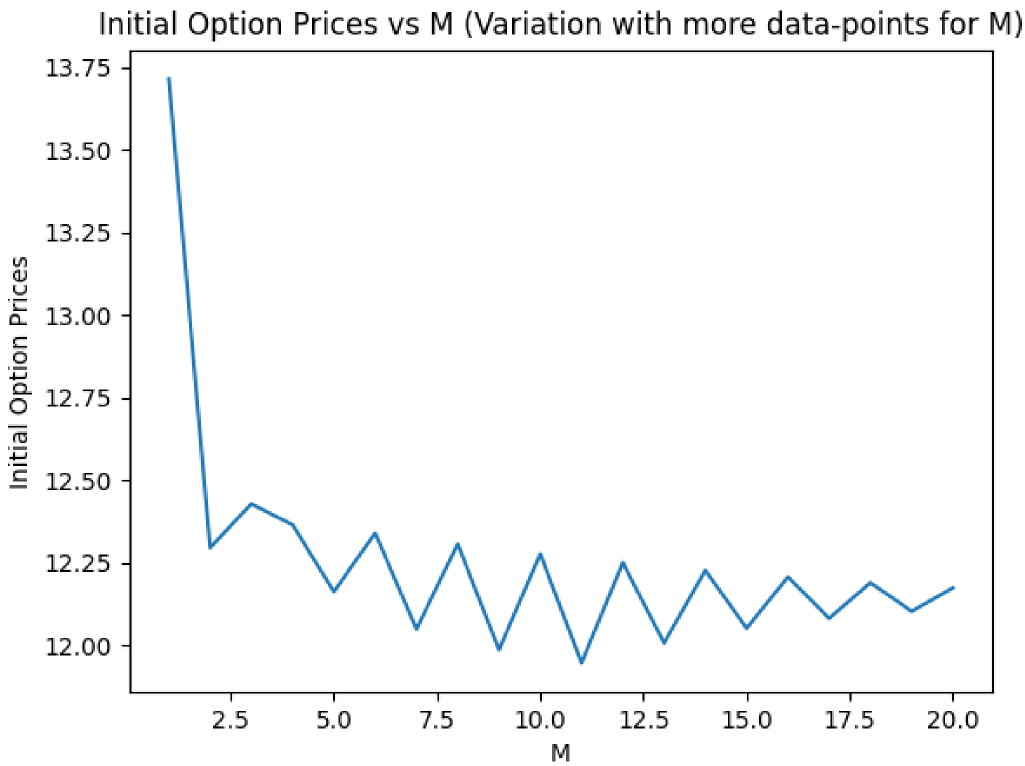

(b)


Initial Option Prices vs M
Initial Option Prices
12.275
12.250
12.225
12.200
12.175
12.150
12.125
12.100
10
20
30
40
50
M


Initial Option Prices vs M (Variation with more data-points for M)
Initial Option Prices
13.75
13.50
13.25
13.00
12.75
12.50
12.25
12.00
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
M

(c)

```
-----------------------  sub-part(c)  -----------------------
At t = 0
Index no = 0    Price = 12.163185946764584

At t = 1
Index no = 0    Price = 18.65868251160212
Index no = 1    Price = 6.0592900974208455

At t = 2
Index no = 0    Price = 27.525444303544514
Index no = 1    Price = 10.392778619897372
Index no = 2    Price = 1.9207528986659217

At t = 3
Index no = 0    Price = 38.72072884252166
Index no = 1    Price = 17.21677529537563
Index no = 2    Price = 3.9032313677700126
Index no = 3    Price = 0.0

At t = 4
Index no = 0    Price = 51.633140251025104
Index no = 1    Price = 27.055880055074176
Index no = 2    Price = 7.9318974975518906
Index no = 3    Price = 0.0
Index no = 4    Price = 0.0

At t = 5
Index no = 0    Price = 66.06574787682459
Index no = 1    Price = 38.86445913876909
Index no = 2    Price = 16.118695507311017
Index no = 3    Price = 0
Index no = 4    Price = 0
Index no = 5    Price = 0
```