# ASSIGNMENT 1

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- By first fundamental theorem of asset pricing, if a market model has a risk neutral measure, then it does not admit arbitrage. Therefore, we will check for the existence of risk neutral measure for checking no-arbitrage condition in the model.
- Under risk neutral measure, the discounted portfolio is a martingale. Therefore

$$E[e^{-rt_1}S(t_1)|S(t_0)] = e^{-rt_0}S(t_0)$$

- For one step binomial model taking $t_0 = 0$,

$$E[S(t_1)|S(0)] = e^{rt_1}S(0) \qquad \text{...eq1}$$

- In one step binomial model,

$$E[S(t_1)|S(0)] = quS(0) + (1-q)dS(0) \qquad \text{...eq2}$$

where q is the risk neutral probability

- By eq1 and eq2 –

$$quS(0) + (1-q)dS(0) = e^{rt_1}S(0)$$

$$q = \frac{e^{rt_1} - d}{u - d}$$

- q will be valid risk neutral probability if $0 \le q \le 1$. Therefore, for a binomial model no arbitrage condition boils down to checking if $0 \le q \le 1$. The case of multistep binomial model is equivalent to applying the same single step binomial model multiple times. Therefore checking for $0 \le q \le 1$ is sufficient.

# QUESTION 1

Price of the option can be calculated by using multistep binomial model.

Given Data –

S(0) = Initial stock price = 100

K = Strike price = 105

T = Time to maturity = 5 years

r = Risk free rate = 0.05%

σ = Volatility of the stock.

At each time step prices of the stock can go up by a factor of u or go down by a factor of d.

Formula used for u and d –

$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$$

$$d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$$

where $\Delta t = T/M$ with M being the number of subintervals in the time interval [0,T]. Final payoff can be calculated using following formula –

for call option – max(S(T) - K, 0)

for put option – max(K - S(T),0)

After calculating the final payoff, we have to discount it to time 0 to get the price of the option. Discounting at each step i can be done using following formula –

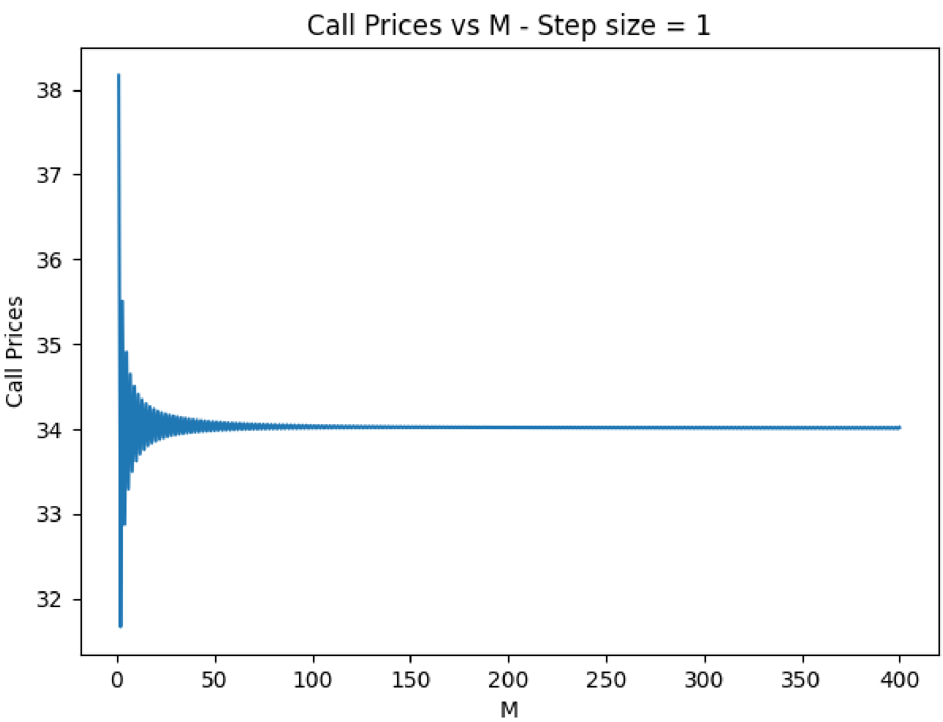
$$C(i) = e^{-r\Delta t}(qC_u + (1-q)C_d)$$

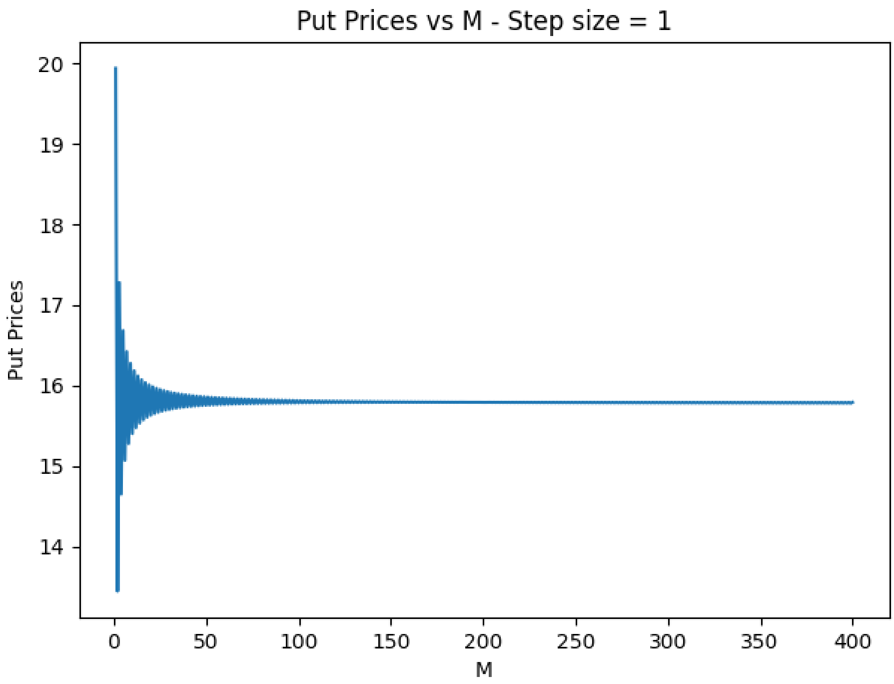
where C(i) is value of the option in step i, $C_u$ is payoff in up state and $C_d$ is payoff in down state.

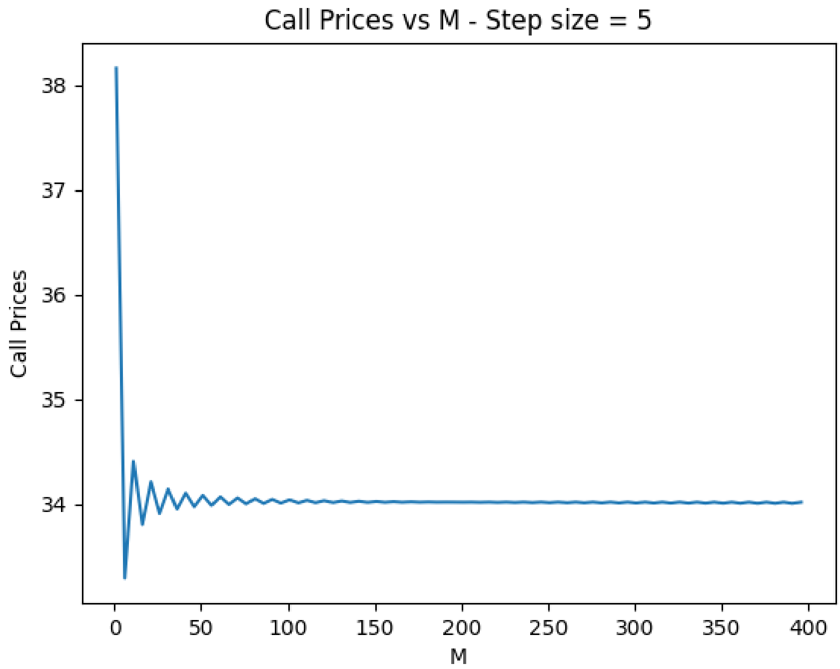
On running the program for various values of M, following initial option prices were obtained.

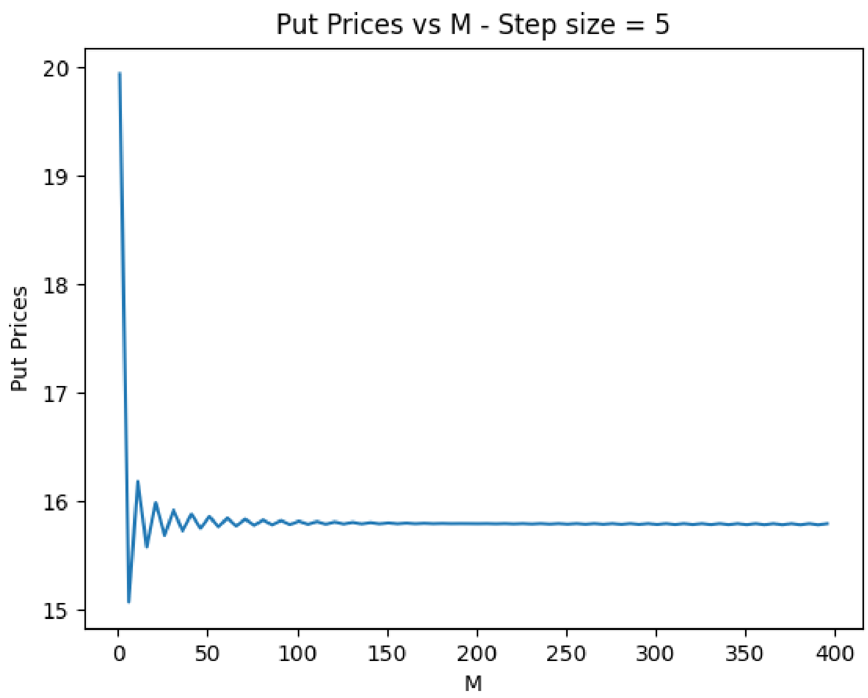
```
harsh@hi aise_heen % cd 200123022_lab1
harsh@hi 200123022_lab1 % python3 q1.py
   Step Size  Call Option Price  Put Option Price
0          1          38.167635         19.941717
1          5          34.906533         16.680615
2         10          33.625022         15.399104
3         20          33.859449         15.633532
4         50          33.981184         15.755267
5        100          34.011161         15.785243
6        200          34.019579         15.793661
7        400          34.019132         15.793214
```

## QUESTION 2




Put Prices vs M - Step size = 1
Put Prices
M
20
19
18
17
16
15
14
0
50
100
150
200
250
300
350
400


Call Prices vs M - Step size = 5
Call Prices
M
38
37
36
35
34
0
50
100
150
200
250
300
350
400




<u>Observations</u>

1. Value of the European call option converges to 34.0 and value of the put option converges to 15.7.

2. As the value of M increases deviation in the options prices decreases.

3. Option prices oscillates around the final value of convergence.

## QUESTION 3

Depending on number of up steps and down steps taken till the ith step we have (i+1) possible values of the options. Values of the options at different t values are tabulated below -

```
harsh@hi 200123022_lab1 % python3 q3.py
For t = 0 (time step = 0) -
   No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                0                  0           33.859449          15.633532


For t = 0.5 (time step = 2) -
   No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                0                  2           15.095873          24.672817
1                1                  1           31.893253          15.487143
2                2                  0           59.958769           8.479204


For t = 1 (time step = 4) -
   No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                0                  4            5.154831          35.965304
1                1                  3           13.469716          24.983287
2                2                  2           29.803955          15.269432
3                3                  1           57.699995           8.004223
4                4                  0          100.662666           3.504174


For t = 1.5 (time step = 6) -
   No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                0                  6            1.125003          48.304951
1                1                  5            4.121405          36.970072
2                2                  4           11.767497          25.270960
3                3                  3           27.573204          14.963372
4                4                  2           55.295356           7.436262
5                5                  1           98.438869           2.998250
6                6                  0          160.611388           0.942427


For t = 3 (time step = 12) -
    No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                 0                 12            0.000000          78.228223
1                 1                 11            0.000000          72.357695
2                 2                 10            0.000000          64.433311
3                 3                  9            0.118330          53.854842
4                 4                  8            1.235971          40.533314
5                 5                  7            6.148520          25.955024
6                 6                  6           19.725206          13.221829
7                 7                  5           46.976188           4.958186
8                 8                  4           91.193433           1.235702
9                 9                  3          154.841699           0.172103
10               10                  2          242.030183           0.008705
11               11                  1          359.934184           0.000000
12               12                  0          519.099689           0.000000


For t = 4.5 (time step = 18) -
    No. of up steps  No. of down steps  Call Option Values  Put Option Values
0                 0                 18            0.000000          95.534063
1                 1                 17            0.000000          93.129316
2                 2                 16            0.000000          89.883248
3                 3                 15            0.000000          85.501514
4                 4                 14            0.000000          79.586791
5                 5                 13            0.000000          71.602751
6                 6                 12            0.000000          60.825424
7                 7                 11            0.000000          46.277554
8                 8                 10            0.000000          26.639984
9                 9                  9            8.149174           8.281211
10               10                  8           36.251494           0.601546
11               11                  7           83.950577           0.000000
12               12                  6          149.149606           0.000000
13               13                  5          237.159089           0.000000
14               14                  4          355.959465           0.000000
15               15                  3          516.323199           0.000000
16               16                  2          732.791598           0.000000
17               17                  1         1024.993373           0.000000
18               18                  0         1419.424512           0.000000
```