Tutorial → 4

(1) $T(n) = 3T(n/2) + n^2$

$a = 3, \quad b = 2 \quad ; \quad k = 2$

$\log_2 3 = 1.585 < k = 2$

$\therefore$ T.C $\Rightarrow \Theta(n^2)$

(2) $T(n) = 4T(n/2) + n^2$

$a = 4, \quad b = 2 \quad k = 2$

$\log_2 4 = 2 = k = 2$

$\Theta(n^2 \log n)$

(3) $T(n) = T(n/2) + 2^n$ → pow(2,n)

$a = 1, \quad b = 2$ $n = 2, k = n, p = 0$

$\log_2 1 = 0$ $T(n) = O(\text{pow}(n,k) \log^p n$

$\therefore p = 0$

$T(n) = O(\text{pow}(n,k))$

$n = 2, k = n$

$\therefore T(n) = \text{pow}(2,n) \Rightarrow 2^n$

(4) $T(n) = 2^n T(n/2) + n^n$

Master theorem not applicable

(5) $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad k = 1$

$\log_4 16 = 2 > k = 1$

T.C $\Rightarrow \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2, \quad k = 1, \quad p = 1$

$\log_b a = \log_2 2 = 1 = k = 1, \quad p = 1$

$\therefore$ T.C $\Rightarrow \Theta(n \log^2 n)$

(7) $T(n) = 2T(n/2) + n/\log n$

$a=2, b=2, k=1, p=-1$

$\log_2 2 = 1 = k, p=-1$

$\Theta(n \log(\log n))$

(8) $T(n) = 2T(n/4) + n^{0.5}$

$a=2, b=4 \quad k=0.5$

$\log_4 2 = 0 < k=0.5$

$\Theta(n^{0.5})$

(9) $T(n) = 0.5\,T(n/2) + 1/n$

$a=0.5 \quad b=2 \quad k=-1$

$\log_2 0.5 = 0 > k=-1$

$\Theta(n^0) \Rightarrow \Theta(1)$

Master thm not applicable as $a<1$

(10) $T(n) = 16\,T(n/4) + n!$

$a=16 \quad b=4 \quad k=1$

$\log_4 16 = 2 > k=1$

$\Rightarrow \Omega(n^2)\ \Theta(n!)$

(11) $T(n) = 4T(n/2) + \log n$

$\log_2 4 = 2 > k=0$

$\Theta(n^2)$

(12) $T(n) = \sqrt{n}\,T(n/2) + \log n$

Master thm not applicable as n is not given in $\sqrt{n}$

(13) $T(n) = 3T(n/2) + n$

$\log_2 3 = 1.585 > k=1$

$\Theta(n^{1.585})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$ (2)

$\log_3 3 = 1 > k=1/2$

$\Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$\log_2 4 = 2 > k=1$

$\Theta(n^2)$.

(16) $T(n) = 3T(n/4) + n \log n$

$\log_4 3 = 0 < k=1, p=1$

$\Theta(n \log n)$

(17) $T(n) = 3T(n/3) + n/2$

$\log_3 3 = 1 = k=1$

$\Theta(n \log n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$\log_3 6 < k=2$

$\Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$\log_2 4 = 2 > k=1$

$\Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Master thm not applicable as $f(n)$ is -ve.

(21) $T(n) = 7T(n/3) + n^2$

$\log_3 7 < k=2$

$\Rightarrow \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Not applied Master thm.