

AI1103 - Assignment 1

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PROBLEM

2.12. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10 percent of girls study in class XII. What is the probability that a student chosen randomly studies in Class XII given that the chosen student is a girl ?

SOLUTION

Total number of students : 1000

Total number of girls : 430

Total number of girls in Class XII : 10 % of total girls

$$= \frac{10}{100} \times 430 = 43 \quad (1)$$

Let $X \in \{0, 1\}$ be the random variable such that 1 represents girl, 0 represents boy.

TABLE I: Probability distribution for values of X

X	P(X)
1	430/1000
0	570/1000

Let $Y \in \{0, 1\}$ be the random variable such that 1 represents chosen student is in Class XII, 0 represents chosen student is not in Class XII.

TABLE II: Probability distribution for values of Y

Y	P(Y)
1	$\frac{1}{2}$
0	$\frac{1}{2}$

We require $P(Y=1 | X=1)$ (using Baye's theorem)

$$\begin{aligned}
 &= \frac{P(X = 1|Y = 1) \cdot P(Y = 1)}{\sum_{i=0}^1 P(X = 1|Y = i) \cdot P(Y = i)} \quad (2) \\
 &= \frac{P(X=1|Y=1)P(Y=1)}{P(X=1|Y=0)P(Y=0)+P(X=1|Y=1)P(Y=1)}
 \end{aligned}$$

TABLE III: Probability for different values of X,Y

Probability	Chosen Student	Value
$P(X = 1 Y = 0)$	girl not in Class XII	$\frac{387}{1000}$
$P(X = 1 Y = 1)$	girl in Class XII	$\frac{43}{1000}$

$$= \left[\frac{\left(\frac{43}{1000} \times \frac{1}{2} \right)}{\left(\frac{387}{1000} \right) \times \frac{1}{2} + \left(\frac{43}{1000} \times \frac{1}{2} \right)} \right] \quad (3)$$

$$= \frac{1}{10} \quad (4)$$

$$= 0.1 \quad (5)$$

Hence, the probability that a student chosen randomly studies in Class XII given that the chosen student is a girl is 0.1.