AI1103 - Assignment 3

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PROBLEM

3. The mean and variance, respectively of a binomial distribution for n independent trials with the probability of success as p, are

1) $\sqrt{np}$, $np(1-2p)$
2) $\sqrt{np}$, $\sqrt{np(1-p)}$
3) np, np
4) $np, np(1-p)$

SOLUTION

Let $X_1, X_2, X_3, ..., X_n$ be the random variable for n independent trials such that

$$X = X_1 + X_2 + X_2 + X_3 + \cdots + X_n$$
$$X = \sum_{i=1}^{n} X_i$$

p = success (1) and 1 - p = failure (0)

Expected Value for n trials :

$$E(X_i) = X_i \cdot p_i$$
$$E(X_i) = 1 \cdot p + 0 \cdot (1-p)$$
$$E(X_i) = p \tag{1}$$

We know that,

$$E(X) = \sum_{i=1}^{n} E(X_i)$$
$$E(X) = np \tag{2}$$

Mean of a binomial distribution for n independent trials is **np**.

Now,

$$E(X_i^2) = X_i^2 \cdot p_i$$
$$E(X_i^2) = 1^2 \cdot p + 0^2 \cdot (1-p)$$
$$E(X_i^2) = p \tag{3}$$

For variance,

$$Var(X_i) = E(X_i^2) - E(X_i)^2$$
$$Var(X_i) = p - p^2 \tag{4}$$

We can add Var(X_i) to get Var(X) as these are independent trials

$$Var(X) = \sum_{i=1}^{n} Var(X_i)$$
$$Var(X) = n(p - p^2)$$
$$Var(X) = np(1-p) \tag{5}$$

Variance of a binomial distribution for n independent trials is **np(1-p)**.

Hence, (4) is correct option.