AI1103 - Assignment 3

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Problem :

3. The mean and variance, respectively of a binomial distribution for n independent trials with the probability of success as p, are
(A) $\sqrt{np}$, np(1-2p)
(B) $\sqrt{np}$, $\sqrt{np(1-p)}$
(C) np, np
(D) np, np(1-p)

Solution

Let $x_1, x_2, x_3, ..., x_n$ be the random variable for n independent trials

Expected Value for n trials :

$$E(x_i) = p$$

We know that,

$$X = x_1 + x_2 + \cdots + x_n \tag{1}$$
$$E(X) = E(x_1) + E(x_2) + \cdots + E(x_n)$$
$$E(X) = np \tag{2}$$

Mean of a binomial distribution for n independent trials is np.

For variance, we have

$$Var(X) = E(X^2) - E(X)^2 \tag{3}$$
$$E(x_i^2) = p \tag{4}$$

Using (1) and squaring on both sides, we get

$X^2 = (x_1^2 + x_2^2 + \cdots + x_n^2) + 2\binom{n}{k}\sum_{i\neq j}(x_i x_j)$

$$E(X^2) = \sum_{i=1}^{n} E(x_i^2) + \sum_{i\neq j} E(x_i x_j)$$
$$E(X^2) = nE(x_i^2) + n(n-1)E(x_i)E(x_j)$$
$$E(X^2) = np + n(n-1)p^2 \tag{5}$$

Now,

$$Var(X) = E(X^2) - (E(X))^2$$
$$Var(X) = np + n(n-1)p^2 - (np)^2$$
$$Var(X) = np - np^2$$
$$Var(X) = np(1-p) \tag{6}$$

Variance of a binomial distribution for n independent trials is np(1-p).
Hence, (D) is correct option.