South China University of Technology

# The Experiment Report of Machine Learning

**SCHOOL:** SCHOOL OF SOFTWARE ENGINEERING

**SUBJECT:** SOFTWARE ENGINEERING

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December 11, 2017

# Logistic Regression, Linear Classification and Stochastic Gradient Descent

**Abstract—This report tends to illustrate the experiments we have done about logistic regression, linear classification and stochastic gradient descent(SGD), with respect to understanding and comprehending the core of this mentioned topics.**

## I. INTRODUCTION

Logistic regression and linear classification are the two of most fundamental machine learning models. Additionally, gradient decent(GD) is one of the most widely-used optimizing methods to reach local optimal solution. Stochastic gradient descent(SGD), an improved version of traditional GD, accelerates the process reaching the solution. This experiment aims to compare GD to SGD, to help understanding the differences and relations between them. What's more, we also compare logistic regression to linear classification, figuring out what is and is not similar to each other. Lastly, we practice SVM on larger data to have a better command of its principles.

## II. METHODS AND THEORY

*Logistic Regression and Stochastic Gradient Descent*

1. Load the training set and validation set.
2. Initialize logistic regression model parameters, you can consider initializing zeros, random numbers or normal distribution.
3. Select the loss function and calculate its derivation, find more detail in PPT.
4. Calculate gradient G toward loss function from partial samples.
5. Update model parameters using different optimized methods(NAG, RMSProp, AdaDelta and Adam).
6. Select the appropriate threshold, mark the sample whose predict scores **greater than the threshold as positive, on the contrary as negative**. Predict under validation set and get the different optimized method loss $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$.
7. Repeat step 4 to 6 for several times, and drawing graph of $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$ with the number of iterations.

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*Linear Classification and Stochastic Gradient Descent*

1. Load the training set and validation set.
2. Initialize SVM model parameters, you can consider initializing zeros, random numbers or normal distribution.
3. Select the loss function and calculate its derivation, find more detail in PPT.
4. Calculate gradient G toward loss function from partial samples.
5. Update model parameters using different optimized methods(NAG, RMSProp, AdaDelta and Adam).
6. Select the appropriate threshold, mark the sample whose predict scores greater than the threshold as positive, on the contrary as negative. Predict under validation set and get the different optimized method loss $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$..
7. Repeat step 4 to 6 for several times, and drawing graph of $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$. with the number of iterations.

## III. EXPERIMENT

### A. Data Set

We use a9a of LIBSVM Data, including 32561/16281(testing) samples and each sample has 123/123 (testing) features.

### B. Implementation

#### 1) Logistic regression:

Loss function of logistic regression is as follows.

$$L(\theta) = -\frac{1}{m}\sum_{i=0}^{n}\left(y_i \log\left(h_\theta(X_i)\right) + (1-y_i)\log\left(1-\log\left(h_\theta(X_i)\right)\right)\right)$$

where
$$h_\theta(X) = \frac{1}{1+e^{-\theta^T \cdot X}}$$

Compute the gradient of L($\theta$), we obtain,

$$\frac{\partial L}{\partial \theta} = \frac{1}{n}\sum_{i=0}^{n} X_i\left(h_\theta(X) - y_i\right)$$

Having defined loss function and its gradient, we can use SGD to get the final solution. We use four method respectively to reach the local optimal solution including NAG, RMSProp, AdaDelta and Adam. The super parameters we select are as follows.

| | | |
|---|---|---|
| NAG | learning rate | 0.002 |
| | gamma | 0.9 |
| | iteration | 2000 |
| RMSProp | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 2000 |
| AdaDelta | learning rate | 0.005 |
| | gamma | 0.95 |

| | | |
|---|---|---|
| | epsilon | 1e-8 |
| | iteration | 2000 |
| Adam | learning rate | 0.005 |
| | beta1 | 0.9 |
| | beta2 | 0.999 |
| | epsilon | 1e-8 |
| | iteration | 2000 |

Next, we program to implement the above methods. The following are the screenshots of source code.

```
In [3]: import numpy as np
        from sklearn.datasets import load_svmlight_file
        import matplotlib.pyplot as plt
        from sklearn.model_selection import train_test_split
```

```
In [4]: data=load_svmlight_file('a9a.txt')
        X_train=data[0].todense()
        y_train=data[1]
```

```
In [5]: data=load_svmlight_file('a9a.t')
        X_validation=data[0].todense()
        y_validation=data[1]
```

```
In [6]: #x为列向量
        X_train=X_train.T
        #x0=1
        x_0=np.ones(X_train.shape[1])
        X_train=np.row_stack((X_train,x_0))
        #y为行向量
        y_train=np.mat(y_train)

        #x为列向量
        X_validation=X_validation.T
        a=np.zeros(X_validation.shape[1])
        #x0=1
        x_0=np.ones(X_validation.shape[1])
        X_validation=np.row_stack((X_validation,a))
        X_validation=np.row_stack((X_validation,x_0))
        #y为行向量
        y_validation=np.mat(y_validation)
```

```
In [7]: for i in range(y_train.shape[1]):
            if y_train[0,i]==-1.0:
                y_train[0,i]=0
        for i in range(y_validation.shape[1]):
            if y_validation[0,i]==-1.0:
                y_validation[0,i]=0
```

```
In [8]: def Loss(w,x,y):
            loss=0
            tmp=w.T*x
            loss+=-(tmp*y.T)[0,0]
            for i in range(tmp.shape[1]):
                loss+=np.log(1+np.exp(tmp[0,i]))
            return loss/float(y.shape[1])
```

```
In [9]: def SGD(w,x,y,learning_rate):
            gradient=np.mat(np.zeros(w.shape))
            for i in [np.random.randint(0,x.shape[1]) for i in range(20)]:
                gradient+=-x[:,i]*(y[0,i]-(np.exp(w.T*x[:,i])/(1+np.exp(w.T*x[:,i]))))
            w=w-learning_rate*gradient
            return w
```

```
In [10]: def SGD_NAG(w,m,x,y,learning_rate):
             momentum=m
             gradient=np.mat(np.zeros(w.shape))

             for i in [np.random.randint(0,x.shape[1]) for i in range(20)]:
                 gradient+=-x[:,i]*(y[0,i]-(np.exp((w-momentum).T*x[:,i])/(1+np.exp((w-momentum).T*
             momentum=0.9*momentum+learning_rate*gradient
             w=w-momentum
             return w,momentum
```

```
In [11]: def SGD_RMSProp(w,g,x,y,learning_rate):
             gradient=np.mat(np.zeros(w.shape))
             for i in [np.random.randint(0,x.shape[1]) for i in range(20)]:
                 gradient+=-x[:,i]*(y[0,i]-(np.exp(w.T*x[:,i])/(1+np.exp(w.T*x[:,i]))))
             g=0.9*g+0.1*(gradient.T*gradient)[0,0]
             w=w-(learning_rate/(np.sqrt(g+1e-8)))*gradient
             return w,g
```

```
In [12]: def SGD_AdaDelta(w,g,delta,x,y):
             gradient=np.mat(np.zeros(w.shape))
             for i in [np.random.randint(0,x.shape[1]) for i in range(20)]:
                 gradient=-x[:,i]*(y[0,i]-(np.exp(w.T*x[:,i])/(1+np.exp(w.T*x[:,i]))))
             g=0.95*g+0.05*(gradient.T*gradient)[0,0]

             step_length=(np.sqrt(delta+1e-8)/np.sqrt(g+1e-8))*gradient
             w=w-step_length
             delta=0.95*delta+0.05*(step_length.T*step_length)[0,0]
             return w,g,delta
```

```
In [ ]: def SGD_Adam(w,m,g,x,y,learning_rate,iter_num):
            gradient=np.mat(np.zeros(w.shape))
            for i in [np.random.randint(0,x.shape[1]) for i in range(20)]:
                gradient=-x[:,i]*(y[0,i]-(np.exp(w.T*x[:,i])/(1+np.exp(w.T*x[:,i]))))
            m=0.9*m+0.1*gradient
            g=0.999*g+0.001*gradient.T*gradient
            alpha=learning_rate*np.sqrt(1-0.999**iter_num)/(1-0.9**iter_num)
            w=w-alpha*m/np.sqrt(g+1e-8)
            return w,m,g
```

```
In [ ]: #------------------没有优化------------
        #初始化参数
        W=np.mat(np.zeros(X_train.shape[0])).T
        iter_num=2000
        learning_rate=0.002

        loss_validation=[]
        loss_validation.append(Loss(W,X_validation,y_validation))
        for i in range(iter_num):
            W=SGD(W,X_train,y_train,learning_rate)
            loss_validation.append(Loss(W,X_validation,y_validation))
```

```
In [ ]: #----------------NAG优化
        #初始化参数
        W=np.mat(np.zeros(X_train.shape[0])).T
        momentum=np.mat(np.zeros(X_train.shape[0])).T
        iter_num=2000
        learning_rate=0.002

        loss_NAG_validation=[]
        loss_NAG_validation.append(Loss(W,X_validation,y_validation))
        for i in range(iter_num):
            W,momentum=SGD_NAG(W,momentum,X_train,y_train,learning_rate)

            loss_NAG_validation.append(Loss(W,X_validation,y_validation))
```

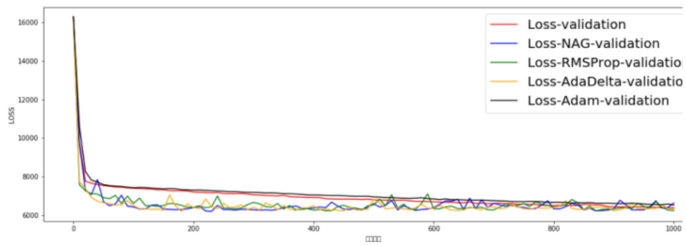
```
In [ ]: #------------RMSProp优化
        #初始化参数
        W=np.mat(np.zeros(X_train.shape[0])).T
        g=0
        iter_num=2000
        learning_rate=0.1

        loss_RMSProp_validation=[]
        loss_RMSProp_validation.append(Loss(W,X_validation,y_validation))
        for i in range(iter_num):
            W,g=SGD_RMSProp(W,g,X_train,y_train,learning_rate)
            loss_RMSProp_validation.append(Loss(W,X_validation,y_validation))
```

```
In [ ]: #-------------AdaDelta
        #初始化参数
        W=np.mat(np.zeros(X_train.shape[0])).T
        g=0.0
        delta=0.003
        iter_num=2000
        loss_AdaDelta_validation=[]
        loss_AdaDelta_validation.append(Loss(W,X_validation,y_validation))
        for i in range(iter_num):
            W,g,delta=SGD_AdaDelta(W,g,delta,X_train,y_train)
            loss_AdaDelta_validation.append(Loss(W,X_validation,y_validation))
```

```
In [ ]: #---------------------Adam优化
        #初始化参数
        W=np.mat(np.zeros(X_train.shape[0])).T
        g=0.0
        learning_rate=0.1
        momentum=np.mat(np.zeros(W.shape))
        iter_num=2000

        loss_Adam_validation=[]
        loss_Adam_validation.append(Loss(W,X_validation,y_validation))

        for i in range(iter_num):
            W,m,g=SGD_Adam(W,momentum,g,X_train,y_train,learning_rate,i+1)

            loss_Adam_validation.append(Loss(W,X_validation,y_validation))
```

```
In [ ]: plt.figure(figsize=(18,6))
        plt.plot(loss_validation,color='red',label='Loss-validation')
        plt.plot(loss_NAG_validation,color='blue',label='Loss-NAG-validation')
        plt.plot(loss_RMSProp_validation,color='green',label='Loss-RMSProp-validation')
        plt.plot(loss_AdaDelta_validation,color='orange',label='Loss-AdaDelta-validation')
        plt.plot(loss_Adam_validation,color='black',label='Loss-Adam-validation')
        plt.xlabel('迭代次数')
        plt.ylabel('LOSS')
        plt.legend(fontsize=20)
        plt.show()
```

We get the following loss graphs as results after running the program.




From the graph we find that AdaDelta reaches the local optimal solution fastest, but it also has obvious vibration. By contrast, NAG is slower than AdaDelta with a smoother curve. RMSProp and Adam are closely overlapped, which are slower than AdaDelta and faster than NAG. Four methods reaches optimal solution far faster than traditional GD.

*2) Linear classification:*

Loss function of logistic regression is as follows.

$$L(\theta) = \frac{1}{2n}\sum_{i=0}^{n}(y_i - h_\theta(X_i))^2 \quad \text{where} \quad h_\theta(X) = \sum_{i=0}^{n} \theta_i X_i$$

Compute the gradient of L($\theta$), we obtain,

$$\frac{\partial L}{\partial \theta} = \frac{1}{n}\sum_{i=0}^{n}(y_i - h_\theta(X_i)) \cdot X_i$$

Having defined loss function and its gradient, we can use SGD to get the final solution. We use four method respectively to reach the local optimal solution including NAG, RMSProp, AdaDelta and Adam. The super parameters we select are as follows.

| | | |
|---|---|---|
| NAG | learning rate | 0.005 |
| | gamma | 0.9 |
| | iteration | 3000 |
| RMSProp | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 3000 |
| AdaDelta | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 3000 |
| Adam | learning rate | 0.005 |
| | beta1 | 0.9 |
| | beta2 | 0.999 |
| | epsilon | 1e-8 |
| | iteration | 3000 |

Next, we program to implement the above methods. The following are the screenshots of source code.

From the graph we find that NAG reaches the local optimal solution fastest, but it also has obvious vibration. By contrast, AdaDelta is slower than NAG with a smoother curve. RMSProp and Adam are closely overlapped, which are slower than NAG and faster than AdaDelta. Four methods reaches optimal solution far faster than traditional GD.

## IV. CONCLUSION

SGD improves traditional GD with a faster converging speed and considerable result. Four specific methods are different improvement of plain SGD with respective advantages. Logistic regression and linear classification both solves classification problem to predict new samples. The major difference between them is the way they evaluate final super plane. Logistic regression tries to find a super plane which makes all sample points get away from it, whereas linear classification finds a plane defined by so-called ‘support vectors’. This experiment makes comparisons between GD and SGD, logistic regression and linear classification, evaluates four SGD methods in this two model and analyzes the experiment results.

```
In [1]: import numpy as np
        from sklearn.model_selection import train_test_split
        import matplotlib.pyplot as plt
        from sklearn.datasets import load_svmlight_file
```

```
In [2]: data=load_svmlight_file('a9a.txt')
        X_train=data[0].todense()
        y_train=data[1]
```

```
In [3]: data=load_svmlight_file('a9a.t')
        X_validation=data[0].todense()
        y_validation=data[1]
```

```
In [4]: #x为列向量
        X_train=X_train.T
        #x0=1
        x_0=np.ones(X_train.shape[1])
        X_train=np.row_stack((X_train,x_0))
        #y为行向量
        y_train=np.mat(y_train)

        #x为列向量
        X_validation=X_validation.T
        a=np.zeros(X_validation.shape[1])
        #x0=1
        x_0=np.ones(X_validation.shape[1])
        X_validation=np.row_stack((X_validation,a))
        X_validation=np.row_stack((X_validation,x_0))
        #y为行向量
        y_validation=np.mat(y_validation)
```

```
In [5]: def Loss(w,x,y):
            loss=0
            for i in range(x.shape[1]):
                if (1-y[0,i]*(w.T*x[:,i]))>0:
                    loss+=1-y[0,i]*(w.T*x[:,i])
            losl=loss/float(y.shape[1])
            loss+=(w.T*w)/2.0
            return loss[0,0]
```

```
In [6]: #没有优化
        def SGD(w_current,x,y,learning_rate):
            gradient=np.mat(np.zeros(w_current.shape))
            gradient=np.mat(np.zeros(w_current.shape))
            for i in [np.random.randint(0,x.shape[1]) for j in range(20)]:
                if (1-y[0,i]*(w_current.T*x[:,i]))>0:
                    gradient+=-y[0,i]*x[:,i]
            gradient+=w_current
            new_w=w_current-learning_rate*gradient
            return new_w
```

```
In [7]: #NAG优化
        def SGD_NAG(w_current,momentum,x,y,learning_rate):
            gradient=np.mat(np.zeros(w_current.shape))
            for i in [np.random.randint(0,x.shape[1]) for j in range(20)]:
                if (1-y[0,i]*(w_current.T*x[:,i]))>0:
                    gradient+=-y[0,i]*x[:,i]
            gradient+=w_current-momentum
            momentum=0.9*momentum+learning_rate*gradient

            new_w=w_current-momentum
            return new_w,momentum
```

```
In [8]: #RMSProp优化
        def SGD_RMSProp(w_current,g,x,y,learning_rate):
            gradient=np.mat(np.zeros(w_current.shape))
            for i in [np.random.randint(0,x.shape[1]) for j in range(20)]:
                if (1-y[0,i]*(w_current.T*x[:,i]))>0:
                    gradient+=-y[0,i]*x[:,i]
            gradient+=w_current
            g=0.9*g+0.1*(gradient.T*gradient)[0,0]
            new_w=w_current-(learning_rate/(np.sqrt(g+1e-8)))*gradient
            return new_w,g
```

```
In [9]: #AdaDelta优化
        def SGD_AdaDelta(w_current,g,delta,x,y):
            gradient=np.mat(np.zeros(w_current.shape))
            for i in [np.random.randint(0,x.shape[1]) for j in range(20)]:
                if (1-y[0,i]*(w_current.T*x[:,i]))>0:
                    gradient+=-y[0,i]*x[:,i]
            gradient+=w_current
            g=0.95*g+0.05*(gradient.T*gradient)[0,0]

            step_length=(np.sqrt(delta+1e-8)/np.sqrt(g+1e-8))*gradient
            new_w=w_current-step_length
            delta=0.95*delta+0.05*(step_length.T*step_length)[0,0]
            return new_w,g,delta
```

```
In [10]: #Adam优化
         def SGD_Adam(w_current,m,g,x,y,learning_rate,iter_num):
             gradient=np.mat(np.zeros(w_current.shape))
             for i in [np.random.randint(0,x.shape[1]) for j in range(20)]:
                 if (1-y[0,i]*(w_current.T*x[:,i]))>0:
                     gradient+=-y[0,i]*x[:,i]
             gradient+=w_current
             m=0.9*m+0.1*gradient
             g=0.999*g+0.001*gradient.T*gradient
             alpha=learning_rate*np.sqrt(1-0.999**iter_num)/(1-0.9**iter_num)
             new_w=w_current-alpha*m/np.sqrt(g+1e-8)
             return new_w,m,g
```

```
In [11]: #------------------没有优化------------
         #初始化参数
         W=np.mat(np.zeros(X_train.shape[0])).T
         iter_num=1000
         learning_rate=0.001

         loss_validation=[]
         loss_validation.append(Loss(W,X_validation,y_validation))
         for i in range(iter_num):
             W=SGD(W,X_train,y_train,learning_rate)
             if (i+1)%10==0:
                 loss_validation.append(Loss(W,X_validation,y_validation))
```

```
In [12]: #----------------NAG优化
         #初始化参数
         W=np.mat(np.zeros(X_train.shape[0])).T
         momentum=np.mat(np.zeros(X_train.shape[0])).T
         iter_num=1000
         learning_rate=0.001

         loss_NAG_validation=[]
         loss_NAG_validation.append(Loss(W,X_validation,y_validation))
         for i in range(iter_num):
             W,momentum=SGD_NAG(W,momentum,X_train,y_train,learning_rate)
             if (i+1)%10==0:
                 loss_NAG_validation.append(Loss(W,X_validation,y_validation))
```

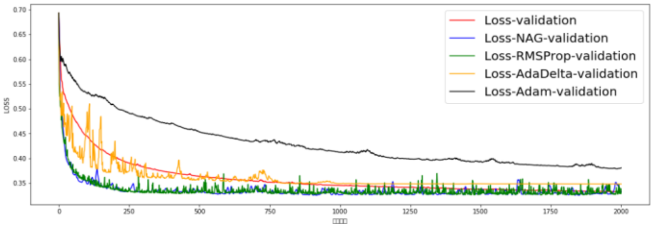
```
In [13]: #-----------RMSProp优化
         #初始化参数
         W=np.mat(np.zeros(X_train.shape[0])).T
         g=0
         iter_num=1000
         learning_rate=0.1

         loss_RMSProp_validation=[]
         loss_RMSProp_validation.append(Loss(W,X_validation,y_validation))
         for i in range(iter_num):
             W,g=SGD_RMSProp(W,g,X_train,y_train,learning_rate)
             if (i+1)%10==0:
                 loss_RMSProp_validation.append(Loss(W,X_validation,y_validation))
```

```
In [14]: #-------------AdaDelta
         #初始化参数
         W=np.mat(np.zeros(X_train.shape[0])).T
         g=0.0
         delta=0.003
         iter_num=1000
         loss_AdaDelta_validation=[]
         loss_AdaDelta_validation.append(Loss(W,X_validation,y_validation))
         for i in range(iter_num):
             W,g,delta=SGD_AdaDelta(W,g,delta,X_train,y_train)
             if (i+1)%10==0:
                 loss_AdaDelta_validation.append(Loss(W,X_validation,y_validation))
```

```
In [15]: #------------------Adam优化
         #初始化参数
         W=np.mat(np.zeros(X_train.shape[0])).T
         g=0.0
         learning_rate=0.1
         momentum=np.mat(np.zeros(W.shape))
         iter_num=1000

         loss_Adam_validation=[]
         loss_Adam_validation.append(Loss(W,X_validation,y_validation))

         for i in range(iter_num):
             W,m,g=SGD_Adam(W,momentum,g,X_train,y_train,learning_rate,i+1)
             if (i+1)%10==0:
                 loss_Adam_validation.append(Loss(W,X_validation,y_validation))
```

```
In [16]: plt.figure(figsize=(18,6))
         plt.plot([i*10 for i in range(101)],loss_validation,color='red',label='Loss-validation')
         plt.plot([i*10 for i in range(101)],loss_NAG_validation,color='blue',label='Loss-NAG-validation')
         plt.plot([i*10 for i in range(101)],loss_RMSProp_validation,color='green',label='Loss-RMSProp-validation')
         plt.plot([i*10 for i in range(101)],loss_AdaDelta_validation,color='orange',label='Loss-AdaDelta-validation')
         plt.plot([i*10 for i in range(101)],loss_Adam_validation,color='black',label='Loss-Adam-validation')
         plt.xlabel('迭代次数')
         plt.ylabel('LOSS')
         plt.legend(fontsize=20)
         plt.show()
```

We get the following loss graphs as results after running the program.


Loss-validation
Loss-NAG-validation
Loss-RMSProp-validation
Loss-AdaDelta-validation
Loss-Adam-validation
LOSS
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0
250
500
750
1000
1250
1500
1750
2000