
# The Experiment Report of Machine Learning

## SCHOOL: SCHOOL OF SOFTWARE ENGINEERING

## SUBJECT: SOFTWARE ENGINEERING

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# Logistic Regression, Linear Classification and Stochastic Gradient Descent

**Abstract—This report tends to illustrate the experiments we have done about logistic regression, linear classification and stochastic gradient descent(SGD), with respect to understanding and comprehending the core of this mentioned topics.**

## I. Introduction

Logistic regression and linear classification are the two of most fundamental machine learning models. Additionally, gradient decent(GD) is one of the most widely-used optimizing methods to reach local optimal solution. Stochastic gradient descent(SGD), an improved version of traditional GD, accelerates the process reaching the solution. This experiment aims to compare GD to SGD, to help understanding the differences and relations between them. What's more, we also compare logistic regression to linear classification, figuring out what is and is not similar to each other. Lastly, we practice SVM on larger data to have a better command of its principles.

## II. Methods and Theory

*Logistic Regression and Stochastic Gradient Descent*

1. Load the training set and validation set.
2. Initialize logistic regression model parameters, you can consider initializing zeros, random numbers or normal distribution.
3. Select the loss function and calculate its derivation, find more detail in PPT.
4. Calculate gradient G toward loss function from partial samples.
5. Update model parameters using different optimized methods(NAG, RMSProp, AdaDelta and Adam).
6. Select the appropriate threshold, mark the sample whose predict scores **greater than the threshold as positive, on the contrary as negative**. Predict under validation set and get the different optimized method loss $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$.
7. Repeat step 4 to 6 for several times, and drawing graph of $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$ and $L_{Adam}$ with the number of iterations.

*Linear Classification and Stochastic Gradient Descent*

1. Load the training set and validation set.
2. Initialize SVM model parameters, you can consider initializing zeros, random numbers or normal distribution.
3. Select the loss function and calculate its derivation, find more detail in PPT.
4. Calculate gradient G toward loss function from partial samples.
5. Update model parameters using different optimized methods(NAG, RMSProp, AdaDelta and Adam).
6. Select the appropriate threshold, mark the sample whose predict scores greater than the threshold as positive, on the contrary as negative. Predict under validation set and get the different optimized method loss $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$and $L_{Adam}$..
7. Repeat step 4 to 6 for several times, and drawing graph of $L_{NAG}$, $L_{RMSProp}$, $L_{AdaDelta}$and $L_{Adam}$. with the number of iterations.

## III. Experiment

### A. Data Set

We use a9a of LIBSVM Data, including 32561/16281(testing) samples and each sample has 123/123 (testing) features.

### B. Implementation

*1) Logistic regression:*

Loss function of logistic regression is as follows.

$$L(\theta) = -\frac{1}{m}\sum_{i=0}^{n}\left(y_i \log\left(h_\theta(X_i)\right) + (1-y_i)\log\left(1-\log\left(h_\theta(X_i)\right)\right)\right)$$

where $h_\theta(X) = \frac{1}{1+e^{-\theta^T \cdot X}}$.

Compute the gradient of L($\theta$), we obtain,

$$\frac{\partial L}{\partial \theta} = \frac{1}{n}\sum_{i=0}^{n} X_i\left(h_\theta(X) - y_i\right)$$

Having defined loss function and its gradient, we can use SGD to get the final solution. We use four method respectively to reach the local optimal solution including NAG, RMSProp, AdaDelta and Adam. The super parameters we select are as follows.

| | | |
|---|---|---|
| NAG | learning rate | 0.005 |
| | gamma | 0.9 |
| | iteration | 1000 |
| RMSProp | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 1000 |
| AdaDelta | learning rate | 0.005 |
| | gamma | 0.9 |

| | | |
|---|---|---|
| | epsilon | 1e-8 |
| | iteration | 1000 |
| Adam | learning rate | 0.005 |
| | beta1 | 0.9 |
| | beta2 | 0.999 |
| | epsilon | 1e-8 |
| | iteration | 1000 |

Next, we program to implement the above methods. The following are the screenshots of source code.

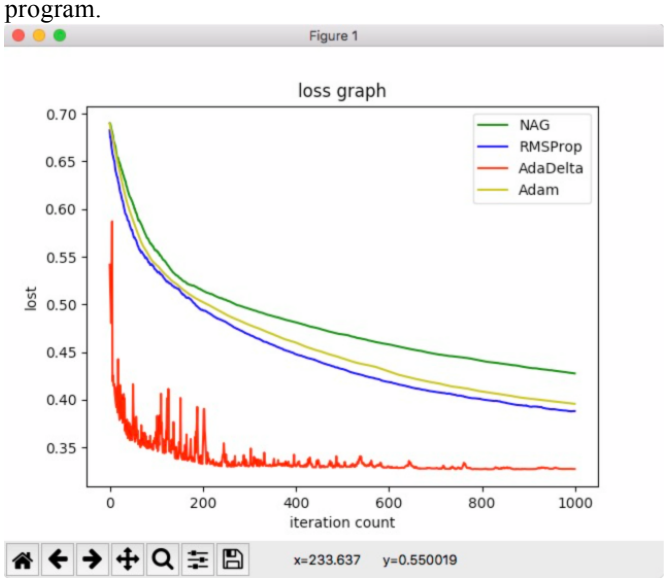
```
from sklearn import datasets, model_selection, linear_model
import numpy as np
import jupyter
import matplotlib.pyplot as plt
import math
import random

X_train, y_train = datasets.load_svmlight_file("a9atrain.txt")

# turn the csr_matrix into array for futher processing
x_ = np.array(X_train.toarray(), np.float32).reshape((-1, 123))
y_ = np.array(y_train, np.float32).reshape((-1, 1))

for i in range(y_.shape[0]):
    if y_[i,0] == -1.0 : y_[i,0] = 0.

X_ = np.hstack([x_, np.ones((x_.shape[0], 1))])


X_test, y_test = datasets.load_svmlight_file("a9atest.txt")

xt_ = np.array(X_test.toarray(), np.float32).reshape((-1, 122))
yt_ = np.array(y_test, np.float32).reshape((-1, 1))
for i in range(yt_.shape[0]):
    if yt_[i,0] == -1.0 : yt_[i,0] = 0.
# X_ = (x_;1) to fit the constent item

Xt_ = np.hstack([xt_, np.zeros((xt_.shape[0], 1)),np.ones((xt_.shape[0


# h0(X) = e^(Theta * X) / (1 + e^(Theta * X)) = 1 / (1 + e^(- Theta *
def h_theta(Xi, Theta):
    e_t = math.exp(Xi.dot(Theta.T))
    return e_t / (1 + e_t)

# L(0) = - (1/m) ∑ (yi * log(h0(Xi)) + (1 - yi) * log(1 - h0(Xi)))
def compute_loss(X, y, Theta):
    m = y.shape[0]
    loss = 0.
    for i in range(m):
        loss +=  (y[i] * math.log(h_theta(X[i, :], Theta))) + ((1 - y[
    loss /= - m
    return loss
```

```

# ∂L/∂0 = (1/m) ∑ (h0(Xi) - yi) * Xi
def compute_gradient(X, y, Theta):
    m = y.shape[0]
    gradient = np.zeros(Theta.shape)
    for i in range(m):
        gradient += (h_theta(X[i, :], Theta) - y[i]) * (X[i, :])
    gradient /= m
    return gradient


def train_model_nag(X, y, Theta, learning_rate, gamma, iteration = 10
    test_loss_history = np.zeros((iteration, 1))
    v = np.zeros(Theta.shape)
    Theta_gradient = np.zeros(Theta.shape)
    for iter in range(iteration):
        index = random.randint(0, y.shape[0]-10)
        Theta = Theta - gamma * v
        v = gamma * v - learning_rate * compute_gradient(X[index:inde
        Theta = Theta + v
        test_loss_history[iter] = compute_loss(Xt_, yt_, Theta)[-1:]
    return test_loss_history, Theta


def train_model_rmsprop(X, y, Theta, learning_rate, gamma, epsilon, i
    test_loss_history = np.zeros((iteration, 1))
    G_t = 0.
    Theta_gradient = np.zeros(Theta.shape)
    for iter in range(iteration):
        index = random.randint(0, y.shape[0]-10)
        Theta_gradient = compute_gradient(X[index:index+10,:], y[inde
        G_t = gamma * G_t + (1 - gamma) * Theta_gradient.dot(Theta_gr
        Theta = Theta - (learning_rate / np.sqrt(G_t + epsilon)) * Th
        test_loss_history[iter] = compute_loss(Xt_, yt_, Theta)[-1:]
    return test_loss_history, Theta
```

```

def train_model_adadelta(X, y, Theta, gamma, epsilon, iteration
    test_loss_history = np.zeros((iteration, 1))
    Theta_gradient = np.zeros(Theta.shape)
    G_t = 0.
    delta_theta = np.zeros(Theta.shape)
    delta_t = 0.03
    for iter in range(iteration):
        index = random.randint(0, y.shape[0]-10)
        Theta_gradient = compute_gradient(X[index:index+10,:],
        G_t = gamma * G_t + (1 - gamma) * Theta_gradient.dot(Th
        delta_theta = - (np.sqrt(delta_t + epsilon) / np.sqrt(G
        Theta = Theta + delta_theta
        delta_t = gamma * delta_t + (1 - gamma) * (delta_theta.
        test_loss_history[iter] = compute_loss(Xt_, yt_, Theta)
    return test_loss_history, Theta

def train_model_adam(X, y, Theta, learning_rate, beta1, beta2,
    test_loss_history = np.zeros((iteration, 1))
    Theta_gradient = np.zeros(Theta.shape)
    v_t = 0.
    m_t = np.zeros(Theta.shape)
    for iter in range(iteration):
        index = random.randint(0, y.shape[0]-10)
        Theta_gradient = compute_gradient(X[index:index+10,:],
        m_t = beta1 * m_t + (1 - beta1) * Theta_gradient
        v_t = beta2 * v_t + (1 - beta2) * Theta_gradient.dot(Th
        mt_estimate = m_t / (1 - pow(beta1, iter + 1))
        vt_estimate = v_t / (1 - pow(beta2, iter + 1))
        Theta = Theta - learning_rate * mt_estimate / (np.sqrt(
        test_loss_history[iter] = compute_loss(Xt_, yt_, Theta)
    return test_loss_history, Theta

iteration = 1000

be1 = 0.9
be2 = 0.999
ep = 1e-8
```

```
t_nag = np.zeros((1, 124))
t_rmsprop = np.zeros((1, 124))
t_adadelta = np.zeros((1, 124))
t_adam = np.zeros((1, 124))
# for i in range(t.shape[0]):
    # t[i] = [-0.03]

nag_loss_history, t_nag = train_model_nag(X_, y_, t_nag, 0.00
rmsprop_loss_history, t_rmsprop = train_model_rmsprop(X_, y_,
adadelta_loss_history, t_adadelta = train_model_adadelta(X_,
adam_loss_history, t_adam = train_model_adam(X_, y_, t_adam,


plt.plot(nag_loss_history, 'g', label='NAG')
plt.plot(rmsprop_loss_history, 'b', label='RMSProp')
plt.plot(adadelta_loss_history, 'r', label='AdaDelta')
plt.plot(adam_loss_history, 'y', label='Adam')


plt.legend(loc='upper right')

plt.ylabel('lost');

plt.xlabel('iteration count')

plt.title('loss graph')

plt.show()
```

We get the following loss graphs as results after running the program.




From the graph we find that AdaDelta reaches the local optimal solution fastest, but it also has obvious vibration. By contrast, NAG is slower than AdaDelta with a smoother curve. RMSProp and Adam are closely overlapped, which are slower than AdaDelta and faster than NAG. Four methods reaches optimal solution far faster than traditional GD.

*2) Linear classification:*

Loss function of logistic regression is as follows.

$$L(\theta) = \frac{1}{2n}\sum_{i=0}^{n}(y_i - h_\theta(X_i))^2$$ , where $h_\theta(X) = \sum_{i=0}^{n}\theta_i X_i$ .

Compute the gradient of L($\theta$), we obtain,

$$\frac{\partial L}{\partial \theta} = \frac{1}{n}\sum_{i=0}^{n}(y_i - h_\theta(X_i)) \cdot X_i$$

Having defined loss function and its gradient, we can use SGD to get the final solution. We use four method respectively to reach the local optimal solution including NAG, RMSProp, AdaDelta and Adam. The super parameters we select are as follows.

| | | |
|---|---|---|
| NAG | learning rate | 0.005 |
| | gamma | 0.9 |
| | iteration | 3000 |
| RMSProp | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 3000 |
| AdaDelta | learning rate | 0.005 |
| | gamma | 0.9 |
| | epsilon | 1e-8 |
| | iteration | 3000 |
| Adam | learning rate | 0.005 |
| | beta1 | 0.9 |
| | beta2 | 0.999 |
| | epsilon | 1e-8 |
| | iteration | 3000 |

Next, we program to implement the above methods. The following are the screenshots of source code.

```
import numpy
import random
import jupyter
import math
from sklearn.datasets import load_svmlight_file
from sklearn.model_selection import train_test_spl
from matplotlib import pyplot

x, y_train = load_svmlight_file("a9atrain.txt")
x_train = x.toarray()
x, y_test = load_svmlight_file("a9atest.txt")
x_test = x.toarray()

X_train = numpy.hstack([x_train, numpy.ones((x_tra
X_test = numpy.hstack([x_test, numpy.zeros((x_test
X_test = numpy.hstack([X_test, numpy.ones((x_test.

def compute_grad(x, y, w):
    gradient = x * (y - x.dot(w.T))
    return gradient

def compute_loss(x, y, w, random_i):
    loss = 0
    a = len(random_i)
    for m in range(a):
        loss += 0.5 * ((y[random_i[m]] - x[random_
    return loss/a
```

```python
def NAG_train(x, y, x_test, y_test, w, C, lr, gamma,
    vt = numpy.zeros(w.shape)
    loss_history = []
    test_loss_history = []
    random_index = []
    random_test_index = []
    for i in range(iteration):
        random_num = random.randint(0, x.shape[0]-1)
        random_test_num = random.randint(0, x_test.s
        random_index.append(random_num)
        random_test_index.append(random_test_num)
    for i in range(iteration):
        gradient = compute_grad(x[random_index[i],:]
        vt = gamma*vt - lr*gradient
        w -= vt
        loss = compute_loss(x, y, w, random_index)
        loss_history.append(loss)
        test_loss_history.append(compute_loss(x_test
        if loss < threshold :
            break
    return w, loss_history, test_loss_history
```

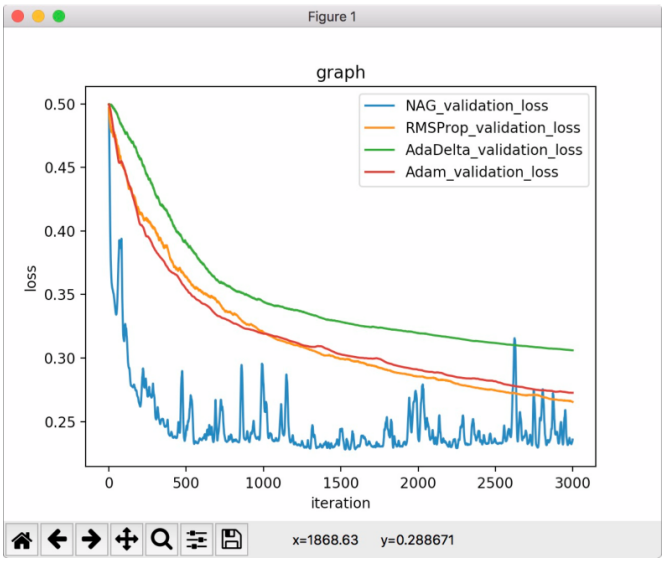
```python
def RMSProp_train(x, y, x_test, y_test, w, C, lr, ga
    Gt = 0
    loss_history = []
    test_loss_history = []
    random_index = []
    random_test_index = []
    for i in range(iteration):
        random_num = random.randint(0, x.shape[0]-1)
        random_test_num = random.randint(0, x_test.s
        random_index.append(random_num)
        random_test_index.append(random_test_num)
    for i in range(iteration):
        gradient = compute_grad(x[random_index[i],:]
        Gt = gamma*Gt + (1-gamma)*gradient.dot(gradi
        w += lr * gradient / math.sqrt(Gt+1e-8)
        loss = compute_loss(x, y, w, random_index)
        loss_history.append(loss)
        test_loss_history.append(compute_loss(x_test
        if loss < threshold :
            break
    return w, loss_history, test_loss_history
```

```python
def AdaDelta_train(x, y, x_test, y_test, w, C, lr, ga
    Gt = 0
    variable_t = 0
    loss_history = []
    test_loss_history = []
    random_index = []
    random_test_index = []
    for i in range(iteration):
        random_num = random.randint(0, x.shape[0]-1)
        random_test_num = random.randint(0, x_test.sh
        random_index.append(random_num)
        random_test_index.append(random_test_num)
    for i in range(iteration):
        gradient = compute_grad(x[random_index[i],:],
        Gt = gamma*Gt + (1-gamma)*gradient.dot(gradie
        variable_w = - math.sqrt(variable_t + 1e-8) *
        w -= variable_w
        variable_t = gamma*variable_t + (1-gamma)*var
        loss = compute_loss(x, y, w, random_index)
        loss_history.append(loss)
        test_loss_history.append(compute_loss(x_test,
        if loss < threshold :
            break
    return w, loss_history, test_loss_history
```

```python
def Adam_train(x, y, x_test, y_test, w, C, lr, gamma, thre
    Gt = 0
    moment = numpy.zeros((1, x.shape[1]))
    B = 0.9
    loss_history = []
    test_loss_history = []
    random_index = []
    random_test_index = []
    for i in range(iteration):
        random_num = random.randint(0, x.shape[0]-1)
        random_test_num = random.randint(0, x_test.shape[0
        random_index.append(random_num)
        random_test_index.append(random_test_num)
    for i in range(iteration):
        gradient = compute_grad(x[random_index[i],:], y[ra
        moment = B*moment + (1-B)*gradient
        Gt = gamma*Gt + (1-gamma)*gradient.dot(gradient.T)
        a = lr * math.sqrt(1 - pow(gamma, iteration)) / (1
        w += a * moment / math.sqrt(Gt + 1e-8)
        loss = compute_loss(x, y, w, random_index)
        loss_history.append(loss)
        test_loss_history.append(compute_loss(x_test, y_te
        if loss < threshold :
            break
    return w, loss_history, test_loss_history
```

```python
iteration = 3000
# NAG
NAG_w = numpy.zeros((1, X_train.shape[1]))
NAG_w, NAG_loss_history, NAG_test_loss_history = NAG_train

# RMSProp
RMS_w = numpy.zeros((1, X_train.shape[1]))
RMS_w, RMS_loss_history, RMS_test_loss_history = RMSProp_t

# AdaDelta
AdaDelta_w = numpy.zeros((1, X_train.shape[1]))
AdaDelta_w, AdaDelta_loss_history, AdaDelta_test_loss_hist

#Adam
Adam_w = numpy.zeros((1, X_train.shape[1]))
Adam_w, Adam_loss_history, Adam_test_loss_history = Adam_t


pyplot.plot(NAG_test_loss_history, label = 'NAG_validation
pyplot.plot(RMS_test_loss_history, label = 'RMSProp_valida
pyplot.plot(AdaDelta_test_loss_history, label = 'AdaDelta_
pyplot.plot(Adam_test_loss_history, label = 'Adam_validati
pyplot.legend(loc='upper right')
pyplot.ylabel('loss')
pyplot.xlabel('iteration')
pyplot.title('graph')
pyplot.show()
```

We get the following loss graphs as results after running the program.




From the graph we find that NAG reaches the local optimal solution fastest, but it also has obvious vibration. By contrast, AdaDelta is slower than NAG with a smoother curve. RMSProp and Adam are closely overlapped, which are slower than NAG and faster than AdaDelta. Four methods reaches optimal solution far faster than traditional GD.

## IV. CONCLUSION

SGD improves traditional GD with a faster converging speed and considerable result. Four specific methods are different improvement of plain SGD with respective advantages. Logistic regression and linear classification both solves classification problem to predict new samples. The major difference between them is the way they evaluate final super plane. Logistic regression tries to find a super plane which makes all sample points get away from it, whereas linear classification finds a plane defined by so-called ‘support vectors’. This experiment makes comparisons between GD and SGD, logistic regression and linear classification, evaluates four SGD methods in this two model and analyzes the experiment results.