Discrete Mathematics Part I Review Questions

1-1.What is the negation of “This is a boring course”?

1-2.Define (using truth tables) the disjunction, conjunction, exclusive or, conditional, and biconditional of the propositions p and q.

1-3.Let p be the proposition “I will do every exercise in this book” and q be the proposition “I will get an “A” in this course.” Express each of these as a combination of p and q.

a) I will get an “A” in this course only if I do every exercise in this book.

b) I will get an “A” in this course and I will do every exercise in this book.

c) Either I will not get an “A” in this course or I will not do every exercise in this book.

d) For me to get an “A” in this course it is necessary and sufficient that I do every exercise in this book.

1-4. Show that these compound propositions are tautologies.

a) $(\neg q \land (p \to q)) \to \neg p$

b) $((p \lor q) \land \neg p) \to q$

1-5.Show in at least two different ways that the compound propositions $\neg p \lor (r \to \neg q)$ and $\neg p \lor \neg q \lor \neg r$ are equivalent.

1-6.Given a truth table, explain how to use disjunctive normal form to construct a compound proposition with this truth table.

1-7.Find the Major Disjunctive Normal Form and Major Conjunctive Form for the compound proposition: $p \to (\neg p \land (r \to q))$.

1-8.Put the statement in prenex normal form $\neg(\forall x P(x) \to \exists y \forall z Q(y,z))$.

1-9.a) What is the difference between the quantification $\exists x \forall y P(x,y)$ and $\forall y \exists x P(x,y)$, where $P(x,y)$ is a predicate?

b) Give an example of a predicate $P(x,y)$ such that $\exists x \forall y P(x,y)$ and $\forall y \exists x P(x,y)$ have different truth values.

1-10. Use rules of inference to show that if the premises “All zebras have stripes” and “Mark is a zebra” are true, then the conclusion “Mark has stripes” is true.

1-11. Use rules of inference to show that if the premises $\forall x(P(x) \to Q(x))$, $\forall x(Q(x) \to R(x))$, and $\neg R(a)$, where a is in the domain, are true, then the conclusion $\neg P(a)$is true.

1-12. a) Describe what is meant by a direct proof, a proof by contraposition, and a proof by contradiction of a conditional statement $p \to q$.

b) Give a direct proof, a proof by contraposition and a proof by contradiction of the statement: “If n is even, then n+4 is even.”

1-13. Explain how a proof by cases can be used to prove a result about absolute values, such as the fact that $|xy|=|x||y|$ for all real numbers x and y.

2-1. Explain what it means for one set to be a subset of another set. How do you prove that one set is a subset of another set?

2-2. What is the empty set? Show that the empty set is a subset of every set.

2-3. a) Define the union, intersection, difference, and symmetric difference of two sets.

b) What are the union, intersection, difference, and symmetric difference of the set of positive integers and the set of odd integers?

2-4.Show in at least two different ways that the sets $A-(B \cap C)$ and $(A-B)\cup(A-C)$ are equal.

2-5. suppose $g:A \rightarrow B$ and $f:B \rightarrow C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,8,10\}$, and g and f are defined by $g =\{(1,b),(2,a),(3,b),(4,a)\}$ and $f =\{(a,8),(b,10),(c,2)\}$.

a) Find $f \circ g$.

b) Find f^{-1}.

c) Find $f \circ f^{-1}$.

d) Explain why g^{-1} is not a function.

2-6. a) Define the floor and ceiling functions from the set of real numbers to the set of integers. Please draw the function graph.

b) For which real numbers x is it true that $\lfloor x \rfloor = \lceil x \rceil$?

2-7. What is the sum of the terms of the geometric progression $a+ar+\cdots+ar^n$ when $r \neq 1$?

2-8. Find $A^{[n]}$ if A is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

2-9. Show that the set of odd integers is countable.

3-1. a) State the definition of the fact that f(n) is O(g(n)), where f(n) and g(n) are functions from the set of positive integers to the set of real numbers.

b) Use the definition of the fact that f(n) is O(g(n)) directly to prove or disprove that $n^2+18n+107$ is $O(n^3)$.

c) Use the definition of the fact that f(n) is O(g(n)) directly to prove or disprove that n^3 is $O(n^2+18n+107)$.

3-2. List these functions so that each function is big-O of the next function in the list: $(\log n)^3$, $n^3/1000000$, $\sqrt{n}$, $100n+101$, 3^n, $n!$, $2^n n^2$.

3-3. a) How can you produce a big-O estimate for a function that is the sum of different terms where each term is the product of several functions?

b) Give a big-O estimate for the function $f(n) = (n!+ 1)(2^n +1)+(n^{n-2} +8n^{n-3})(n^3 +2^n)$. For the function g in your estimate f(x)is O(g(x)) use a simple function of smallest possible order.

4-1. a) Define what it means for a and b to be congruent modulo 7.

b) Which pairs of the integers $-11,-8,-7,-1,0,3$, and 17 are congruent modulo 7?

c) Show that if a and b are congruent modulo 7, then $10a+13$ and $-4b+20$ are also congruent modulo 7.

4-2. Use Algorithm 5 to find the results:

a) 3^{2003} mod 99.

b)123^{1001} mod 101.

4-3. a) Define the greatest common divisor of two integers.

b) Describe at least three different ways to find the greatest common divisor of two integers. When does each method work best?

c) Find the greatest common divisor of 1,234,567 and 7,654,321.

d) Find the greatest common divisor of $2^3 3^5 5^7 7^9 11$ and $2^9 3^7 5^5 7^3 13$.

4-4. a) How can you find a linear combination (with integer coefficients) of two integers that equals their greatest common divisor?
b) Express gcd(84,119) as a linear combination of 84 and 119.
~~4-5. a) What does it mean for a to be an inverse of a modulo m?~~
~~b) How can you find an inverse of a modulo m when m is a positive integer and gcd(a,m) = 1?~~
~~c) Find an inverse of 7 modulo 19.~~
~~4-6. a) How can an inverse of a modulo m be used to solve the congruence ax ≡ b(mod m) when gcd(a,m) = 1?~~
~~b) Solve the linear congruence 7x ≡ 13 (mod 19).~~
~~4-7. a) State the Chinese remainder theorem.~~
~~b) Find the solutions to the system x ≡ 1 (mod 4), x ≡ 2 (mod 5), and x ≡ 3 (mod 7).~~

~~4-8. Suppose that $2^{n-1} \equiv 1$ (mod n). Is n necessarily prime?~~

4-9. Use Fermat's little theorem to evaluate 9^{200} mod 19.

5-1. a) For which positive integers n is $11n+17 \leqslant 2^n$?
b) Prove the conjecture you made in part (a) using mathematical induction.
5-2. a) Which amounts of postage can be formed using only 5-cent and 9-cent stamps?
b) Prove the conjecture you made using mathematical induction.
c) Prove the conjecture you made using strong induction.
d) Find a proof of your conjecture different from the ones you gave in (b) and (c).
5-3.State the well-ordering property for the set of positive integers.
5-4. Give a recursive definition of
a) the set of even integers.
b) the set of positive integers congruent to 2 modulo 3.
c) the set of positive integers not divisible by 5.
5-5. Provide a recursive definition of the function f(n) = (n+1)!.
5-6. a) Give a recursive definition of the length of a string.
b) Use the recursive definition from part (a) and structural induction to prove that l(xy)= l(x)+l(y).

6-1. Explain how the sum and product rules can be used to find the number of bit strings with a length not exceeding 10.
6-2. Explain how to find the number of bit strings of length not exceeding 10 that have at least one 0 bit.
6-3. How can you find the number of bit strings of length ten that either begin with 101 or end with 010?
6-4. a) State the pigeonhole principle.
b) Explain how the pigeonhole principle can be used to show that among any 11 integers, at least two must have the same last digit.
6-5. a) State the generalized pigeonhole principle.

b) Explain how the generalized pigeonhole principle can be used to show that among any 91 integers, there are at least ten that end with the same digit.

6-6. a) What is the difference between an r-combination and an r-permutation of a set with n elements?

b) How many ways are there to select six students from a class of 25 to serve on a committee?

c) How many ways are there to select six students from a class of 25 to hold six different executive positions on a committee?

6-7. a) Explain how to find a formula for the number of ways to select r objects from n objects when repetition is allowed and order does not matter.

b) How many ways are there to select a dozen objects from among objects of five different types if objects of the same type are indistinguishable?

c) How many ways are there to select a dozen objects from these five different types if there must be at least three objects of the first type?

d) How many ways are there to select a dozen objects from these five different types if there cannot be more than four objects of the first type?

e) How many ways are there to select a dozen objects from these five different types if there must be at least two objects of the first type, but no more than three objects of the second type?

6-8. a) Let n and r be positive integers. Explain why the number of solutions of the equation $x_1 + x_2 + \cdots + x_n = r$, where x_i is a nonnegative integer for $i = 1,2,3,\ldots,n$, equals the number of r-combinations of a set with n elements.

b) How many solutions in nonnegative integers are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$?

c) How many solutions in positive integers are there to the equation in part (b)?

6-9. a) Derive a formula for the number of permutations of n objects of k different types, where there are n_1 indistinguishable objects of type one, n_2 indistinguishable objects of type two,…,and n_k indistinguishable objects of type k.

b) How many ways are there to order the letters of the word INDISCREETNESS?

6-10. a) How many ways are there to deal hands of five cards to six players from a standard 52-card deck?

b) How many ways are there to distribute n distinguishable objects into k distinguishable boxes so that n_i objects are placed in box i?