STRESS CONCENTRATION AROUND A CIRCULAR HOLE

Test name: HOLE-STRESS

Code: MEMOSA-FVM

Features tested: Spatial discretization order of structural solver

Description: Stress concentration around a circular hole in a rectangular plate subjected to unidirectional tensile load is studied. The spatial discretization order is determined by evaluating the volume-weighted RMS errors in the displacement magnitude at four successively refined mesh resolutions.

Result Summary: The spatial discretization order is found to be 2.0. This is consistent with the second-order spatial discretization of the numerical scheme.

Assessment: Passed.

References: S. Das, S. Mathur and J. Y. Murthy, "*An unstructured finite volume method for structure-electrostatics interaction in MEMS*", IMECE2010-40036, Vancouver, BC, 2010.

Test Description

Overview: A plate with a circular hole is subjected to unidirectional tensile load.
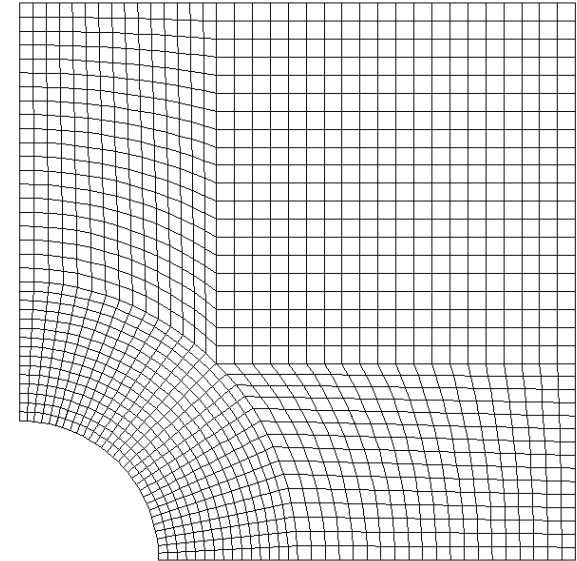
Computational Domain
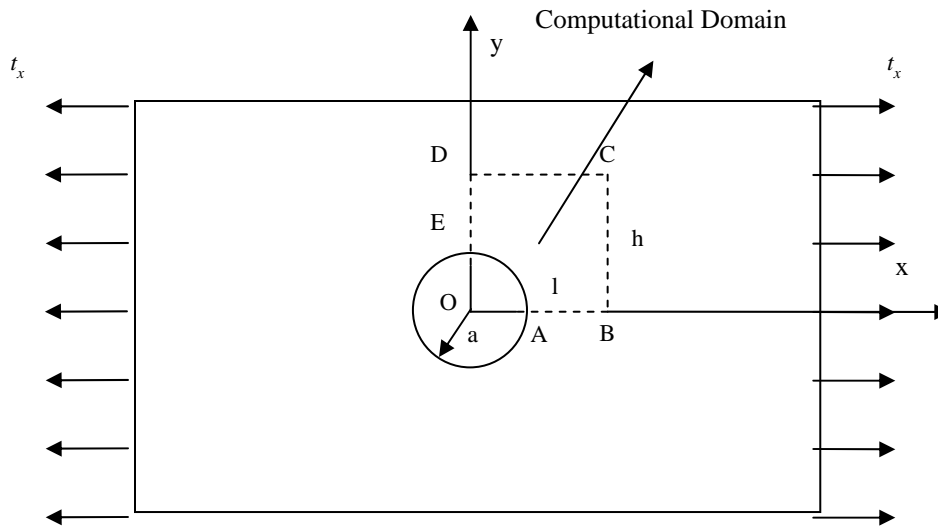
Exact Stress Solution

$$\sigma_{xx} = t_x \left[1 - \frac{a^2}{r^2} \left(\frac{3}{2} \cos 2\theta + \cos 4\theta \right) + \frac{3}{2} \frac{a^4}{r^4} \cos 4\theta \right]$$

$$\sigma_{yy} = t_x \left[- \frac{a^2}{r^2} \left(\frac{1}{2} \cos 2\theta - \cos 4\theta \right) - \frac{3}{2} \frac{a^4}{r^4} \cos 4\theta \right]$$

$$\sigma_{xy} = t_x \left[- \frac{a^2}{r^2} \left(\frac{1}{2} \sin 2\theta + \sin 4\theta \right) + \frac{3}{2} \frac{a^4}{r^4} \sin 4\theta \right]$$

- Apply exact solution as boundary conditions on the top and right faces
- Symmetry boundary condition applied on the left and bottom faces
- Hole is a traction-free surface

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Test Description

Parameter	Units	Value
Young's modulus (E)	Pa	10^7
Poisson's ratio (ν)		0.3
Hole radius (a)	m	0.5
l	m	1.5
h	m	2.0
t_x	Pa	10^4

Exact Displacement Solution

$$w_r = \frac{\sigma}{4\mu}\left\{ r\left[\frac{\kappa-1}{2}+\cos 2\theta\right] + \frac{a^2}{r}\left[1+(1+\kappa)\cos 2\theta\right] - \frac{a^4}{r^3}\cos 2\theta \right\}$$

$$w_\theta = \frac{\sigma}{4\mu}\left[(1-\kappa)\frac{a^2}{r} - r - \frac{a^4}{r^3}\right]\sin 2\theta$$
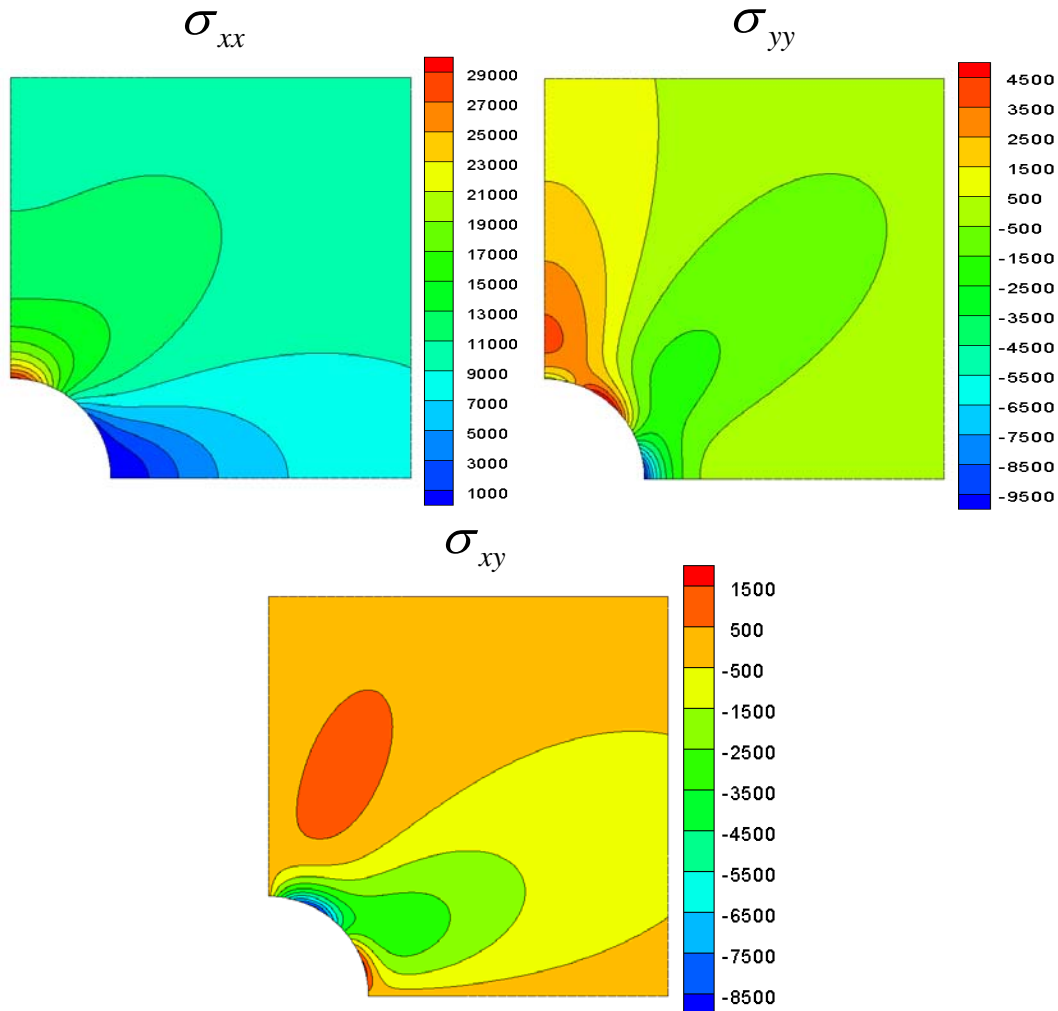
where

$$\mu = \frac{E}{2(1+\nu)}$$

$$\kappa = \begin{cases} 3-4\nu & plane\ strain \\ \dfrac{3-\nu}{1+\nu} & plain\ stress \end{cases}$$

• Plane strain condition assumed

Results

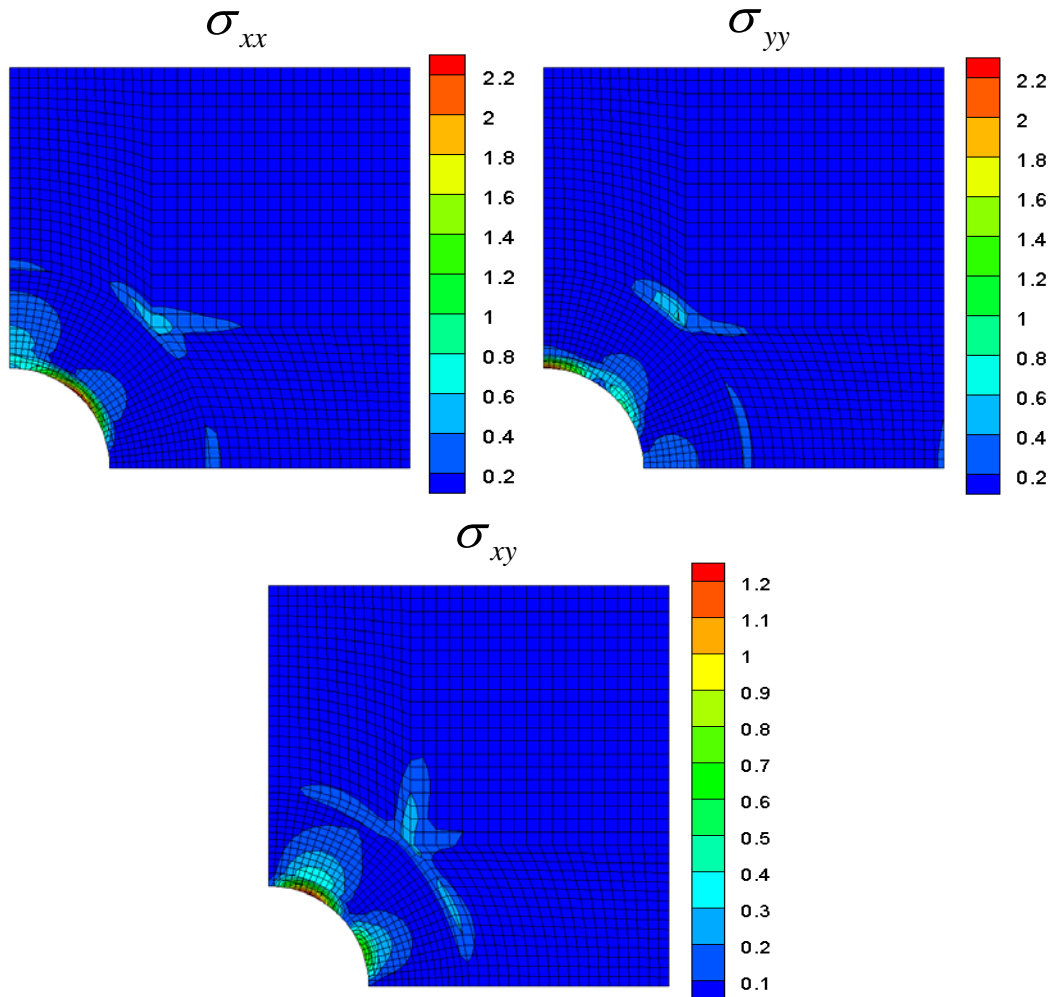## Stress Distribution

σ_{xx}

σ_{yy}

σ_{xy}

Error Analysis

Percentage error in stresses for 1450 cells

σ_{xx}

σ_{yy}

σ_{xy}

No of cells	Volume-weighted RMS error in displacement magnitude (m)
1450	1.921e-6
5800	4.811e-7
23200	1.202e-7
98000	3.001e-8
Order	2

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Assessment

- The order of accuracy is as expected. We have second-order spatial discretization.
- The test is considered passed.