

TRANSIENT VIBRATION OF CANTILEVER BEAMS

Test name: CANT-TORDER

Code: MEMOSA-FVM

Features tested: (i) Natural frequency of cantilever beams (ii) Temporal discretization order of structural solver

Description: Transient vibration of a cantilever beam subjected to shear stress at the free end is simulated. The temporal discretization order is determined by studying RMS errors in the deflection of the beam tip at four different Courant numbers.

Result Summary: The natural frequency is found to be within 0.6% of analytical results. Temporal discretization order is found to be 1.02. This is consistent with the first-order temporal discretization of the numerical scheme.

Assessment: Passed.

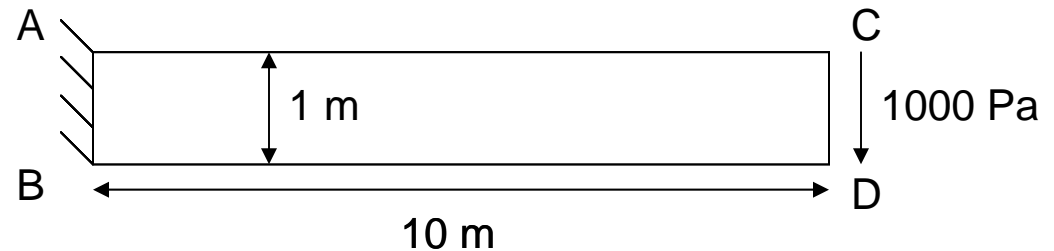
References: M. Petyt, *“Introduction to finite element vibration analysis,”* Cambridge University Press, Cambridge, UK, 1998.

S. Das, S. Mathur and J. Y. Murthy, *“An unstructured finite volume method for structure-electrostatics interaction in MEMS,”* IMECE2010-40036, Vancouver, BC, 2010.

Test Description

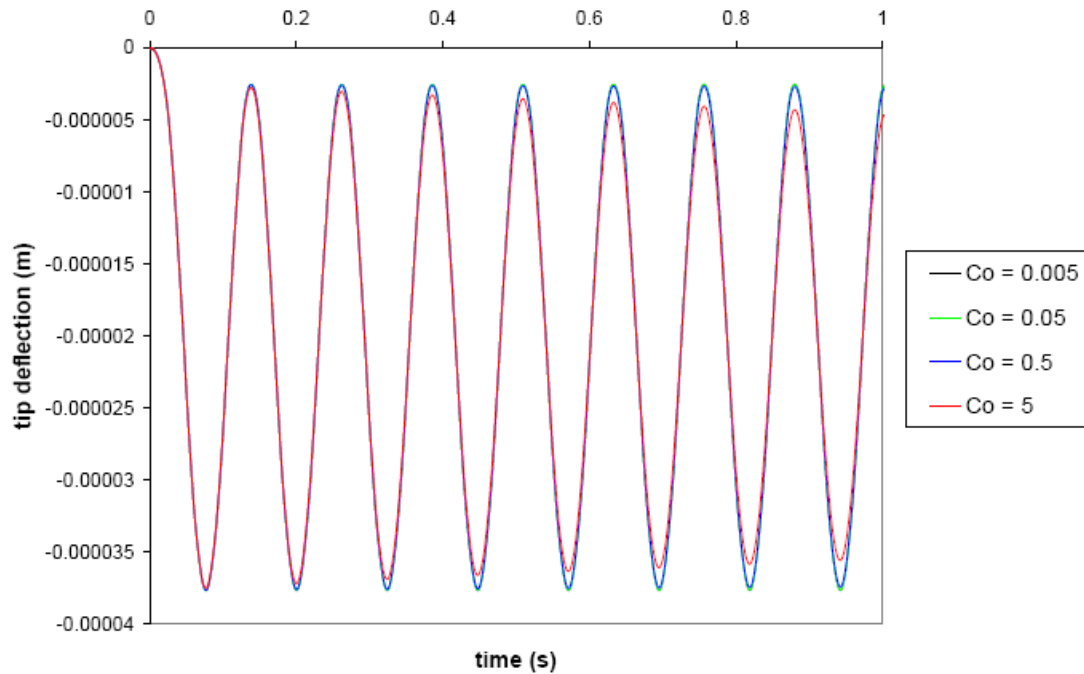
Overview: A cantilever beam is subjected to shear stress at the free end. The objective is to compute the natural frequency of the beam and the temporal discretization order.

Parameter	Value
Young's Modulus	200 GPa
Poisson's ratio	0.31
Density	7854 Kg/m ³



Description: Computations are performed in MEMOSA-FVM. The beam is assumed to have zero deflection and zero velocity as the initial conditions. At time $t = 0$, a shear stress is applied at the free end of the beam. The shear stress is linearly ramped up for the first 0.03s, and then maintained at $\tau = 1000\text{Pa}$. The top and bottom faces are traction-free surfaces. The left face is constrained to have zero displacement. An uniform quad mesh, of size 100×9 is used for the computation. The simulation is performed at Courant numbers of 0.005, 0.05, 0.5, and 5. The natural frequency is computed from the transient deflection data of the free end of the beam, and compared with the analytical result published by Petyt. RMS errors in the tip deflection at Courant numbers of 0.05, 0.5, and 5, with respect to the case with Courant number of 0.005, are used to obtain the temporal discretization order of the structural solver.

Results



	Natural Frequency (Hz)
Analytical (Petyt)	8.15
FVM	8.106

Courant Numbers	0.05	0.5	5	Order
RMS Errors	1.244e-9	1.367e-8	1.374e-7	1.02

Assessment

- The comparison of the natural frequency with the analytical result published by Petyt is good.
- The order of accuracy is as expected. We have first-order temporal discretization.
- The test is considered passed.