

World Final Template

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1 Polynomials

```
1 namespace polynomial {
2
3 // dft.h
4 template <typename T>
5 class dft {
6 public:
7     static const bool use_fast_trans = true;
8     static void trans(std::vector<T>& p) {
9         assert(__builtin_popcount(p.size()) == 1);
10         if constexpr (use_fast_trans) {
11             dif(p);
12         } else {
13             bit_reverse(p);
14             dit(p);
15         }
16     }
17
18     static void inv_trans(std::vector<T>& p) {
19         assert(__builtin_popcount(p.size()) == 1);
20         if constexpr (use_fast_trans) {
21             dit(p);
22         } else {
23             trans(p);
24         }
25         reverse(p.begin() + 1, p.end());
26         T inv = T(p.size()).inv();
27         for (T& x : p) x *= inv;
28     }
29
30 // should call dit after dif
31 static void dit(std::vector<T>& p) {
32     for (int len = 1; len < p.size(); len <= 1) {
```

```
33         auto sub_w = get_subw(len * 2);
34         for (auto sub_p = p.begin(); sub_p != p.end(); sub_p += 2 * len)
35             for (int i = 0; i < len; ++i) {
36                 T u = sub_p[i], v = sub_p[i + len] * sub_w[i];
37                 sub_p[i] = u + v;
38                 sub_p[i + len] = u - v;
39             }
40     }
41 }
42
43 static void dif(std::vector<T>& p) {
44     for (int len = p.size() / 2; len >= 1; len >= 1) {
45         auto sub_w = get_subw(len * 2);
46         for (auto sub_p = p.begin(); sub_p != p.end(); sub_p += 2 * len)
47             for (int i = 0; i < len; ++i) {
48                 T _sub_pi = sub_p[i];
49                 sub_p[i] += sub_p[i + len];
50                 sub_p[i + len] = (_sub_pi - sub_p[i + len]) * sub_w[i];
51             }
52     }
53 }
54
55 private:
56 typename std::vector<T>::iterator static get_subw(int len) {
57     static std::vector<T> w = {0, 1};
58     static const T primitive_root = T::primitive_root();
59     while (w.size() <= len) {
60         T e[] = {1, primitive_root.pow((T::modulus() - 1) / w.size())};
61         w.resize(w.size() * 2);
62         for (int i = w.size() / 2; i < w.size(); ++i) w[i] = w[i / 2] * e[i & 1];
63     }
64     return w.begin() + len;
65 }
66 };
67
68 // poly.h
69 template <typename T>
70 class poly : public std::vector<T> {
71 public:
72     using std::vector<T>::vector;
73
74     poly(std::string s) {
75         for (int i = 0; i < s.size(); ++i) {
76             auto scan_num = [&]() -> long long {
77                 int sgn = 1;
78                 if (s[i] == '-') sgn = -1, ++i;
79                 if (s[i] == '+') sgn = 1, ++i;
80                 if (i == s.size() || !std::isdigit(s[i])) return sgn;
81                 long long num = 0;
82                 while (i < s.size() && std::isdigit(s[i]))
83                     num = num * 10 + s[i++] - '0';
84                 return sgn * num;
85             };
```

```

85     };
86     auto add_item = [&](size_t exponent, T coeff) {
87         if (exponent >= this->size()) this->resize(exponent + 1);
88         this->at(exponent) = coeff;
89     };
90     T coeff = scan_num();
91     if (i == s.size() || s[i] != 'x')
92         add_item(0, coeff);
93     else {
94         size_t exponent = 1;
95         if (s[++i] == '^') {
96             ++i;
97             exponent = scan_num();
98         }
99         add_item(exponent, coeff);
100     }
101 }
102 }
103
104 int deg() const { return this->size() - 1; }
105 poly operator-() const {
106     poly ans = *this;
107     for (auto& x : ans) x = -x;
108     return ans;
109 }
110 T operator()(const T& x) const {
111     T ans = 0;
112     for (int i = this->size() - 1; i >= 0; --i) ans = ans * x + this->at(i);
113     return ans;
114 }
115 T operator[](int idx) const {
116     if (0 <= idx && idx < this->size()) return this->at(idx);
117     return 0;
118 }
119 T& operator[](int idx) {
120     if (idx >= this->size()) this->resize(idx + 1);
121     return this->at(idx);
122 }
123
124 poly rev() const {
125     poly res(*this);
126     std::reverse(res.begin(), res.end());
127     return res;
128 }
129
130 poly mulx(size_t k) const {
131     poly res = *this;
132     res.insert(res.begin(), k, 0);
133     return res;
134 }
135 poly divx(size_t k) const {
136     if (this->size() <= k) return {};

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137     return poly(this->begin() + k, this->end());
138 }
139 poly modxk(size_t k) const {
140     k = std::min(k, this->size());
141     return poly(this->begin(), this->begin() + k);
142 }
143
144 poly& operator+=(poly p) {
145     if (this->empty() || p.empty()) return *this = {};
146     constexpr int small_size = 128;
147     if (this->size() < small_size || p.size() < small_size) {
148         poly<T> t(this->size() + p.size() - 1);
149         for (int i = 0; i < this->size(); i++)
150             for (int j = 0; j < p.size(); j++) t[i + j] += this->at(i) * p[j];
151         return *this = t;
152     }
153     int len = 1 << (std::lg(this->deg() + p.deg()) + 1);
154     this->resize(len);
155     p.resize(len);
156     dft<T>::trans(*this);
157     dft<T>::trans(p);
158     for (int i = 0; i < len; ++i) this->at(i) += p[i];
159     dft<T>::inv_trans(*this);
160     return this->normalize();
161 }
162
163 poly& operator+=(const poly& p) {
164     this->resize(std::max(this->size(), p.size()));
165     for (int i = 0; i < this->size(); ++i) this->at(i) += p[i];
166     return this->normalize();
167 }
168
169 poly& operator-=(const poly& p) {
170     this->resize(std::max(this->size(), p.size()));
171     for (int i = 0; i < this->size(); ++i) this->at(i) -= p[i];
172     return this->normalize();
173 }
174
175 poly& operator/=(const poly& p) {
176     if (this->size() < p.size()) return *this = {};
177     int len = this->size() - p.size() + 1;
178     return *this = (this->rev().modxk(len) * p.rev().inv(len))
179         .modxk(len)
180         .rev()
181         .normalize();
182 }
183
184 poly& operator%=(const poly& p) {
185     return *this = (*this - (*this / p) * p).normalize();
186 }
187
188 poly& operator+=(const T& x) {
189     for (int i = 0; i < this->size(); ++i) this->at(i) += x;
190     return *this;
191 }
192
193 poly& operator/=(const T& x) { return *this *= x.inv(); }

```

```

189 poly operator*(const poly& p) const { return poly(*this) * p; }
190 poly operator+(const poly& p) const { return poly(*this) + p; }
191 poly operator-(const poly& p) const { return poly(*this) - p; }
192 poly operator/(const poly& p) const { return poly(*this) / p; }
193 poly operator%(const poly& p) const { return poly(*this) % p; }
194 poly operator*(const T& x) const { return poly(*this) * x; }
195 poly operator/(const T& x) const { return poly(*this) / x; }
196
197 // (quotient, remainder)
198 std::pair<poly, poly> divmod(const poly& p) const {
199     poly d = *this / p;
200     return std::make_pair(d, (*this - d * p).normalize());
201 }
202
203 poly deriv() const {
204     if (this->empty()) return {};
205     poly res(this->size() - 1);
206     for (int i = 0; i < this->size() - 1; ++i) {
207         res[i] = this->at(i + 1) * (i + 1);
208     }
209     return res;
210 }
211
212 poly integr(T c = 0) const {
213     poly res(this->size() + 1);
214     for (int i = 0; i < this->size(); ++i) {
215         res[i + 1] = this->at(i) / (i + 1);
216     }
217     res[0] = c;
218     return res;
219 }
220
221 // mod x^k
222 poly inv(int k = -1) const {
223     if (!k) k = this->size();
224     poly res = {this->front().inv()};
225     for (int len = 2; len < k * 2; len <= 1) {
226         res = (res * (poly{2} - this->modxk(len) * res)).modxk(len);
227     }
228     return res.modxk(k);
229 }
230
231 // mod x^k
232 poly sqrt(int k = -1) const {
233     if (!k) k = this->size();
234     poly res = {this->at(0).sqrt()};
235     for (int len = 2; len < k * 2; len <= 1) {
236         res = (res + (this->modxk(len) * res.inv(len)).modxk(len)) / 2;
237     }
238     return res.modxk(k);
239 }
240
241 // mod x^k, a0=1 should hold

```

```

241 poly log(int k = -1) const {
242     assert(this->at(0) == 1);
243     if (!k) k = this->size();
244     return (this->deriv() * this->inv(k)).integr().modxk(k);
245 }
246
247 // mod x^k, a0=0 should hold
248 poly exp(int k = -1) const {
249     assert(this->at(0) == 0);
250     if (!k) k = this->size();
251     poly res = {1};
252     for (int len = 2; len < k * 2; len <= 1) {
253         res = (res * (poly{1} - res.log(len) + this->modxk(len))).modxk(len);
254     }
255     return res.modxk(k);
256 }
257
258 // p^c mod x^k
259 poly pow(int c, int k = -1) const {
260     if (!k) k = this->size();
261     int i = 0;
262     while (i < this->size() && !this->at(i)) ++i;
263     if (i == this->size() || 1LL * i * c >= k) return {};
264     T ai = this->at(i);
265     poly f = this->divxk(i) * ai.inv();
266     return (f.log(k - i * c) * c).exp(k - i * c).mulxk(i * c) * ai.pow(c);
267 }
268
269 // evaluate and interpolate
270 struct product_tree {
271     int l, r;
272     std::unique_ptr<product_tree> lson = nullptr, rson = nullptr;
273     poly product;
274     product_tree(int l, int r) : l(l), r(r) {}
275
276     static std::unique_ptr<product_tree> build(
277         const std::vector<T>& xs, std::function<poly(T)> get_poly) {
278         std::function<std::unique_ptr<product_tree>(int, int)> build =
279             [&](int l, int r) {
280                 auto rt = std::make_unique<product_tree>(l, r);
281                 if (l == r) {
282                     rt->product = get_poly(xs[l]);
283                 } else {
284                     int mid = (l + r) >> 1;
285                     rt->lson = build(l, mid);
286                     rt->rson = build(mid + 1, r);
287                     rt->product = rt->lson->product * rt->rson->product;
288                 }
289                 return rt;
290             };
291         return build(0, xs.size() - 1);
292 }

```

```

293 poly mulT(poly p) const {
294     if (p.empty()) return {};
295     return ((*this) * p.rev()).divxk(p.size() - 1);
296 }
297
298 std::vector<T> evaluate(std::vector<T> xs) const {
299     if (this->empty()) return std::vector<T>(xs.size());
300     std::unique_ptr<product_tree> rt = product_tree::build(xs, [&](T x) {
301         return poly{1, -x};
302     });
303     return evaluate_internal(xs, rt);
304 }
305
306 static poly interpolate(std::vector<T> xs, std::vector<T> ys) {
307     assert(xs.size() == ys.size());
308     if (xs.empty()) return {};
309     std::unique_ptr<product_tree> rt = product_tree::build(xs, [&](T x) {
310         return poly{1, -x};
311     });
312     std::vector<T> coef = rt->product.rev().deriv().evaluate_internal(xs, rt);
313     for (int i = 0; i < ys.size(); ++i) coef[i] = ys[i] * coef[i].inv();
314     std::function<poly(product_tree*)> solve = [&](product_tree* rt) {
315         if (rt->l == rt->r) {
316             return poly{coef[rt->l]};
317         } else {
318             return solve(rt->lson.get()) * rt->rson->product.rev() +
319                 solve(rt->rson.get()) * rt->lson->product.rev();
320         }
321     };
322     return solve(rt.get());
323 }
324
325 // a0=0 must hold
326 poly cos(int k = -1) const {
327     assert(this->at(0) == 0);
328     if (!k) k = this->size();
329     T i = T::root().pow((T::modulus() - 1) / 4);
330     poly x = *this * i;
331     return (x.exp(k) + (-x).exp(k)) / 2;
332 }
333
334 // a0=0 must hold
335 poly sin(int k = -1) const {
336     assert(this->at(0) == 0);
337     if (!k) k = this->size();
338     T i = T::root().pow((T::modulus() - 1) / 4);
339     poly x = *this * i;
340     return (x.exp(k) - (-x).exp(k)) / (i * 2);
341 }
342
343 // a0=0 must hold
344 poly tan(int k = -1) const { return this->sin(k) / this->cos(k); }

```

```

345
346 poly acos(int k = -1) const {
347     const poly& x = *this;
348     return (-x.deriv() * (poly{1} - x * x).sqrt().inv()).integr();
349 };
350
351 poly asin(int k = -1) const {
352     const poly& x = *this;
353     return (x.deriv() * (poly{1} - x * x).sqrt().inv()).integr();
354 };
355
356 poly atan(int k = -1) const {
357     const poly& x = *this;
358     return (x.deriv() * (poly{1} + x * x).inv()).integr();
359 };
360
361 friend std::ostream& operator<<(std::ostream& os, poly p) {
362     os << "{";
363     for (auto x : p) os << x << " ";
364     os << "}";
365     return os;
366 }
367
368 private:
369 poly& normalize() {
370     while (this->size() && !this->back()) this->pop_back();
371     return *this;
372 }
373
374 std::vector<T> evaluate_internal(std::vector<T>& xs,
375                                 std::unique_ptr<product_tree>& rt) const {
376     std::vector<T> res(xs.size());
377     xs.resize(std::max(xs.size(), this->size()));
378     std::function<void(product_tree*, poly)> solve = [&](product_tree* rt,
379                                                         poly p) {
380         p = p.modxk(rt->r - rt->l + 1);
381         if (rt->l == rt->r) {
382             if (rt->l < res.size()) res[rt->l] = p.front();
383         } else {
384             solve(rt->lson.get(), p.mulT(rt->rson->product));
385             solve(rt->rson.get(), p.mulT(rt->lson->product));
386         }
387         solve(rt.get(), this->mulT(rt->product.inv(xs.size())));
388         return res;
389     };
390
391 // namespace polynomial
392
393 using poly = polynomial::poly<zint>;

```

1.1 多项式技巧

1. 若 $f(x) = \prod_i (1 + x + x^2 + \dots)^{g_i} = \prod_i \left(\frac{1}{1-x^i}\right)^{g_i}$.

两边取对数: $\ln f(x) = -\sum_i g_i \cdot \ln(1 - x^i) = \sum_i g_i \cdot \sum_{j=1}^{\infty} \frac{x^{ij}}{j}$.

两边取导数: $\frac{\sum_{i=1}^{\infty} (n-1)f_n x^{n-2}}{\sum_{i=1}^{\infty} f_n x^{n-1}} = \sum_i i g_i \cdot \sum_{j=1}^{\infty} x^{ij-1}$.

即: $\sum_{i=1}^{\infty} (n-1)f_n x^{n-2} = (\sum_{i=1}^{\infty} f_n x^{n-1}) \cdot (\sum_i i g_i \cdot \sum_{j=1}^{\infty} x^{ij-1})$.

乘以 x^2 : $\sum_{i=1}^{\infty} (n-1)f_n x^n = (\sum_{i=1}^{\infty} f_n x^n) \cdot (\sum_i i g_i \cdot \sum_{j=1}^{\infty} x^{ij})$

令 $t = ij$ 并交换求和顺序: $\sum_i i g_i \cdot \sum_{j=1}^{\infty} x^{ij} = \sum_{t=1}^{\infty} x^t \cdot \sum_{i|t} i g_i$

即: $\sum_{i=1}^{\infty} (n-1)f_n x^n = (\sum_{i=1}^{\infty} f_n x^n) \cdot (\sum_{t=1}^{\infty} x^t \cdot \sum_{i|t} i g_i)$

对应项系数相等: $(n-1)f_n = \sum_i (\sum_{j|i} j g_j) f_{n-i}$, 即

$$f_n = \frac{1}{n-1} \sum_i \left(\sum_{j|i} j g_j \right) f_{n-i}$$

若令 $ft_n = \sum_{j|i} j g_j$, 则 $f_n = \frac{1}{n-1} \sum_i ft_i f_{n-i}$.

2. 若 $F(x) = \sum_{i=0}^m f_i \cdot x^i$, 那么 $G(x) = F^k(x)$ 有

$$g_n = \frac{1}{f_0} \cdot \left(\frac{k+1}{n} \sum_{i=1}^m i f_i \cdot g_{n-i} - \sum_{i=1}^m f_i \cdot g_{n-i} \right)$$

2 Strings

```

1 //***** Lyndon *****
2
3 namespace lyndon {
4
5 std::vector<int> getfactorization(const std::string &s) {
6     std::vector<int> right_ends;
7     for (int i = 0; i < s.length(); i) {
8         int j = i, k = i + 1;
9         for (; k < s.length() && s[j] <= s[k]; j++, k++)
10             if (s[j] < s[k]) j = i - 1;

```

```

11         while (i <= j) i += k - j, right_ends.push_back(i);
12     }
13     return right_ends;
14 }
15
16 } // namespace lyndon
17
18 //***** Pam *****
19
20 template <size_t alphabet_size = 26>
21 class palindrome_automaton {
22 public:
23     struct node {
24         std::array<int, alphabet_size> to;
25         int link, len, count;
26
27         explicit node(int len = 0) : len(len), link(-1), count(0) { to.fill(0); }
28         explicit node(int len, int link) : len(len), link(link), count(0) {
29             to.fill(0);
30         }
31     };
32
33     palindrome_automaton() {
34         int even_rt = newnode(0);
35         int odd_rt = newnode(-1);
36         nodes[even_rt].link = odd_rt;
37         nodes[odd_rt].link = even_rt;
38         last = even_rt;
39     }
40
41     void extend(char c) {
42         text.push_back(c);
43         int i = text.size() - 1;
44         auto getlink = [&](int u) {
45             while (i - nodes[u].len - 1 < 0 || text[i - nodes[u].len - 1] != c) {
46                 u = nodes[u].link;
47             }
48             return u;
49         };
50         int w = c - 'a';
51         int u = getlink(last);
52         if (!nodes[u].to[w]) {
53             int v = newnode(nodes[u].len + 2, nodes[getlink(nodes[u].link)].to[w]);
54             nodes[u].to[w] = v;
55         }
56         last = nodes[u].to[w];
57         ++nodes[last].count; // should be accumulated later from fail link tree
58     }
59
60     std::string to_string() const {
61         std::ostringstream os;
62         std::function<void(int, std::string)> travel = [&](int k, std::string s) {

```

```

63     if (!k) return;
64     os << k << " "; " << s << " - " << nodes[k].count << "\n";
65     for (int c = 0; c < alphabet_size; ++c)
66         travel(nodes[k].to[c], std::string(1, c + 'a') + s + (char)(c + 'a'));
67 };
68 for (int c = 0; c < alphabet_size; ++c)
69     travel(nodes[0].to[c], std::string(2, c + 'a'));
70 for (int c = 0; c < alphabet_size; ++c)
71     travel(nodes[1].to[c], std::string(1, c + 'a'));
72 return os.str();
73 }
74
75 private:
76 int last;
77 std::string text;
78 std::vector<node> nodes;
79
80 template <typename... Args>
81 int newnode(Args... args) {
82     int res = nodes.size();
83     nodes.push_back(node{args...});
84     return res;
85 }
86 };
87
88 //***** Suffix Array *****
89
90 template <typename Container = std::vector<int>>
91 struct SuffixArray {
92     int n;
93     Container s;
94     // lc[0]=0 is meaningless
95     std::vector<int> sa, rk, lc;
96     SuffixArray(const Container& s)
97         : s(s), n(s.size()), sa(s.size()), rk(s.size()), lc(s.size()) {
98         std::iota(sa.begin(), sa.end(), 0);
99         std::sort(sa.begin(), sa.end(), [&](int a, int b) { return s[a] < s[b]; });
100         rk[sa[0]] = 0;
101         for (int i = 1; i < n; ++i)
102             rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
103         std::vector<int> tmp, cnt(n);
104         tmp.reserve(n);
105         for (int k = 1; rk[sa[n - 1]] < n - 1; k *= 2) {
106             tmp.clear();
107             for (int i = 0; i < k; ++i) tmp.push_back(n - k + i);
108             for (auto i : sa)
109                 if (i >= k) tmp.push_back(i - k);
110             cnt.assign(n, 0);
111             for (int i = 0; i < n; ++i) ++cnt[rk[i]];
112             for (int i = 1; i < n; ++i) cnt[i] += cnt[i - 1];
113             for (int i = n - 1; i >= 0; --i) sa[--cnt[rk[tmp[i]]]] = tmp[i];
114             std::swap(rk, tmp);

```

```

115         rk[sa[0]] = 0;
116         for (int i = 1; i < n; ++i) {
117             rk[sa[i]] = rk[sa[i - 1]];
118             if (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i - 1] + k == n ||
119                 tmp[sa[i - 1] + k] < tmp[sa[i] + k])
120                 ++rk[sa[i]];
121         }
122     }
123     for (int i = 0, j = 0; i < n; ++i) {
124         if (!rk[i]) {
125             j = 0;
126         } else {
127             if (j) --j;
128             int k = sa[rk[i] - 1];
129             while (i + j < n && k + j < n && s[i + j] == s[k + j]) ++j;
130             lc[rk[i]] = j;
131         }
132     }
133 }
134 };
135
136 template <typename Container = std::vector<int>>
137 class LongestCommonPrefix {
138 public:
139     LongestCommonPrefix(SuffixArray<Container>* sa) : sa(sa), st(sa->lc) {}
140
141     int lcp(int i, int j) {
142         assert(0 <= i && i <= sa->n);
143         assert(0 <= j && j <= sa->n);
144         if (i == sa->n || j == sa->n) return 0;
145         if (i == j) return sa->n - i;
146         int l = sa->rk[i], r = sa->rk[j];
147         if (l > r) std::swap(l, r);
148         return st.queryMin(l + 1, r);
149     }
150
151 private:
152     SuffixArray<Container>* sa;
153     SparseTable<int> st;
154 };
155
156 //***** SAM *****
157
158 template <size_t alphabet_size = 26>
159 class suffix_automaton {
160 public:
161     struct node {
162         std::array<int, alphabet_size> to;
163         int link, len, count;
164
165         explicit node(int len = 0) : len(len), link(-1), count(0) { to.fill(-1); }
166         explicit node(int len, int link, const std::array<int, alphabet_size>& to)

```

```

167         : len(len), link(link), to(to), count(0) {}
168     };
169
170     suffix_automaton() : nodes() { newnode(); }
171
172     explicit suffix_automaton(const std::string& s) : suffix_automaton() {
173         insert(s);
174     }
175
176     void insert(const std::string& s) {
177         nodes.reserve(size() + s.size() * 2);
178         int last = 0;
179         for (int i = 0; i < s.size(); ++i) {
180             last = extend(last, s[i] - 'a');
181         }
182     }
183
184     int extend(int k, int c) {
185         if (~nodes[k].to[c] && nodes[nodes[k].to[c]].len == nodes[k].len + 1) {
186             return nodes[k].to[c];
187         }
188         int leaf = newnode(nodes[k].len + 1);
189         for (; ~k && !nodes[k].to[c]; k = nodes[k].link) nodes[k].to[c] = leaf;
190         if (!~k) {
191             nodes[leaf].link = 0;
192         } else {
193             int p = nodes[k].to[c];
194             if (nodes[k].len + 1 == nodes[p].len) {
195                 nodes[leaf].link = p;
196             } else {
197                 int np = newnode(nodes[k].len + 1, nodes[p].link, nodes[p].to);
198                 nodes[p].link = nodes[leaf].link = np;
199                 for (; ~k && nodes[k].to[c] == p; k = nodes[k].link)
200                     nodes[k].to[c] = np;
201             }
202         }
203         return leaf;
204     }
205
206     void build_ancestors() {
207         for (int i = 1; i < size(); ++i) ancestors[i] = {nodes[i].link};
208         for (int j = 1; (1 << j) < size(); ++j) {
209             for (int i = 0; i < size(); ++i)
210                 if (~ancestors[i][j - 1]) {
211                     ancestors[i][j] = ancestors[ancestors[i][j - 1]][j - 1];
212                 } else {
213                     ancestors[i][j] = -1;
214                 }
215         }
216     }
217
218     std::vector<int> mark_count(const std::string& s) {

```

```

219         std::vector<int> ends;
220         int k = 0;
221         for (char c : s) {
222             k = nodes[k].to[c - 'a'];
223             assert(~k);
224             ends.push_back(k);
225             ++nodes[k].count;
226         }
227         return ends;
228     }
229
230     void addup_count() {
231         std::vector<int> ids(size()), bucket(size());
232         for (int i = 0; i < size(); ++i) ++bucket[nodes[i].len];
233         for (int i = 1; i < bucket.size(); ++i) bucket[i] += bucket[i - 1];
234         for (int i = 0; i < size(); ++i) ids[--bucket[nodes[i].len]] = i;
235         for (int i = size() - 1; i; --i)
236             nodes[nodes[ids[i]].link].count += nodes[ids[i]].count;
237     }
238
239     const node& operator[](int v) const { return nodes[v]; }
240
241     int maxlen(int v) const { return nodes[v].len; }
242
243     int minlen(int v) const { return v ? nodes[nodes[v].link].len + 1 : 0; }
244
245     int size() const { return nodes.size(); }
246
247     std::string to_string() const {
248         std::ostringstream os;
249         std::function<void(int, std::string)> travel = [&](int k, std::string s) {
250             if (!~k) return;
251             os << k << ": " << s << " - " << nodes[k].count << "\n";
252             for (int c = 0; c < alphabet_size; ++c)
253                 travel(nodes[k].to[c], s + (char)(c + 'a'));
254         };
255         travel(0, "");
256         return os.str();
257     }
258
259 private:
260     std::vector<node> nodes;
261     std::vector<std::vector<int>> ancestors;
262
263     template <typename... Args>
264     int newnode(Args... args) {
265         int res = nodes.size();
266         nodes.push_back(node{args...});
267         return res;
268     }
269 };
270

```

```

271 //***** ACauto *****
272
273 int go[maxtri][26], sum, count[maxtri], d[maxtri], fail[maxtri];
274 void make_tri(int k) {
275     fo(i, 0, nb-1) {
276         int index=sb[i]-'a';
277         if (!go[k][index]) go[k][index]++;
278         kgo[k][index];
279     }
280     count[k]++;
281 }
282 void make_fail() {
283     int i=0, j=0;
284     fo(p, 0, 25) if (go[0][p]) d[++j]=go[0][p];
285     while (i++<j) {
286         int now=d[i];
287         fo(p, 0, 25) if (go[now][p]) {
288             int son=go[now][p];
289             fail[son]=go[fail[now]][p];
290             count[son]+=count[fail[son]];
291             d[++j]=son;
292         } else go[now][p]=go[fail[now]][p];
293     }
294 }
295 void find(int k) { //sa中出现了多少次sb
296     fo(i, 1, n) ans+=count[k=go[k][sa[i]-'a']];
297 }
298
299 //***** manacher *****
300
301 int f[2*maxn];
302 void manacher()
303 {
304     int lim=0, mid=0;
305     fo(i, 1, m) // m=2*n+1
306     {
307         f[i]= (i<=lim) ? min(f[mid*2-i], lim-i+1) : 1 ;
308         while (i-f[i]>0 && i+f[i]<=m && s[i-f[i]]==s[i+f[i]]) f[i]++;
309         if (i+f[i]-1>lim) lim=i+f[i]-1, mid=i;
310     }
311 }
312
313 //***** exkmp *****
314
315 int next[maxn], ex[maxn];
316 void exkmp() {
317     next[1]=nb;
318     int k=0;
319     fo(i, 2, nb) {
320         int lim=k+next[k]-1, L=next[i-k+1];
321         if (i+L<=lim) next[i]=L; else {
322             next[i]=max(lim-i+1, 0);

```

```

323         while (i+next[i]<=nb && sb[i+next[i]]==sb[1+next[i]]) next[i]++;
324         k=i;
325     }
326 }
327
328 k=1;
329 fo(i, 1, na) {
330     int lim=k+ex[k]-1, L=next[i-k+1];
331     if (i+L<=lim) ex[i]=L; else {
332         ex[i]=max(lim-i+1, 0);
333         while (i+ex[i]<=na && 1+ex[i]<=nb && sa[i+ex[i]]==sb[1+ex[i]]) ex[i]++;
334         k=i;
335     }
336 }
337 }
338
339 //***** min_representation *****
340
341 int s[maxn];
342 int min_representation(int *s, int len) { // index from 0
343     int i=0, j=1;
344     while (i<len && j<len) {
345         int k=0;
346         for (; k<len && s[(i+k)%len]==s[(j+k)%len]; k++);
347         if (k==len) break;
348         (s[(i+k)%len]<s[(j+k)%len]) ? j+=k+1 : i+=k+1;
349         i+=(i==j);
350     }
351     return min(i, j);
352 }

```

3 Geometry

```

1 namespace geometry2d {
2
3 constexpr long double eps = 1e-7;
4 constexpr long double pi = std::acos(-1);
5
6 int fsign(long double x) { return (x > eps) - (x < -eps); }
7 long double sqr(long double x) { return x * x; }
8
9 // Ax^2+Bx+c=0
10 std::vector<long double> solveEquationP2(long double A, long double B,
11                                         long double C) {
12     long double delta = B * B - 4 * A * C;
13     if (fsign(delta) < 0) return {};
14     if (fsign(delta) == 0) return {-0.5 * B / A};
15     long double sqrt_delta = std::sqrt(delta);

```



```

16     long double x1 = -0.5 * (B - sqrt_delta) / A,
17         x2 = -0.5 * (B + sqrt_delta) / A;
18     if (fsign(x1 - x2) > 0) std::swap(x1, x2);
19     return {x1, x2};
20 }
21
22 struct Point;
23 struct Point3D {
24     long double x, y, z;
25
26     explicit Point3D(long double x = 0, long double y = 0, long double z = 0)
27         : x(x), y(y), z(z) {}
28
29     explicit Point3D(const Point& p);
30
31     Point3D operator-(const Point3D& p) const {
32         return Point3D(x - p.x, y - p.y, z - p.z);
33     }
34     long double innerProd(const Point3D& p) const {
35         return x * p.x + y * p.y + z * p.z;
36     }
37     Point3D crossProd(const Point3D& p) const {
38         return Point3D(y * p.z - z * p.y, -x * p.z + z * p.x, x * p.y - y * p.x);
39     }
40 };
41
42 struct Point {
43     long double x, y;
44     explicit Point(long double x = 0, long double y = 0) : x(x), y(y) {}
45     Point operator+(const Point& p) const { return Point(x + p.x, y + p.y); }
46     Point operator-(const Point& p) const { return Point(x - p.x, y - p.y); }
47     Point operator/(const long double& l) const { return Point(x / l, y / l); }
48     Point operator*(const long double& l) const { return Point(x * l, y * l); }
49     Point unit() const {
50         long double l = len();
51         assert(fsign(l) > 0);
52         return *this / l;
53     }
54     long double crossProd(const Point& p) const { return x * p.y - y * p.x; }
55     long double innerProd(const Point& p) const { return x * p.x + y * p.y; }
56     long double lenSqr() const { return sqr(x) + sqr(y); }
57     long double len() const { return std::sqrt(lenSqr()); }
58     long double distanceSqr(const Point& p) const {
59         return sqr(x - p.x) + sqr(y - p.y);
60     }
61     long double distance(const Point& p) const {
62         return std::sqrt(distanceSqr(p));
63     }
64     long double angleWith(const Point& p) const {
65         return std::atan2(crossProd(p), innerProd(p));
66     }
67     // return  $(-pi, pi]$ 

```

```

68     long double angle() const { return std::atan2(y, x); }
69     int angleSign() const {
70         if (!y) return fsign(x) < 0;
71         return fsign(y);
72     }
73     Point rotate90() const { return Point(-y, x); }
74
75     static long double crossProd(const Point& o, const Point& a, const Point& b) {
76         return (a - o).crossProd(b - o);
77     }
78     static long double innerProd(const Point& o, const Point& a, const Point& b) {
79         return (a - o).innerProd(b - o);
80     }
81
82     friend std::ostream& operator<<(std::ostream& os, Point p) {
83         return os << "(" << p.x << ", " << p.y << ")";
84     }
85
86     friend bool operator==(const Point& lhs, const Point& rhs) {
87         return !fsign(lhs.x - rhs.x) && !fsign(lhs.y - rhs.y);
88     }
89
90     struct HorizontalComparer {
91         bool operator()(const Point& lhs, const Point& rhs) const {
92             return lhs.x < rhs.x || (lhs.x == rhs.x && lhs.y < rhs.y);
93         }
94     };
95
96     struct PolarComparer {
97         bool operator()(const Point& lhs, const Point& rhs) const {
98             int angleSign_diff = lhs.angleSign() - rhs.angleSign();
99             if (angleSign_diff) return angleSign_diff < 0;
100             int crossProd_sign = fsign(lhs.crossProd(rhs));
101             if (crossProd_sign) return crossProd_sign > 0;
102             return lhs.lenSqr() < rhs.lenSqr();
103         }
104     };
105 };
106
107 Point3D::Point3D(const Point& p) : Point3D(p.x, p.y, p.x * p.x + p.y * p.y) {}
108
109 struct Line {
110     Point pivot, unit_direction;
111     Line(Point pivot, Point direction)
112         : pivot(pivot), unit_direction(direction.unit()) {}
113     long double angle() const { return unit_direction.angle(); }
114     // 1: on the left
115     // 0: in the line
116     // -1: on the right
117     int side(const Point& p) {
118         return fsign(unit_direction.crossProd(p - pivot));
119     }

```

```

120 bool isParallel(const Line& l) {
121     return fsign(unit_direction.crossProd(l.unit_direction)) == 0;
122 }
123 bool isSame(const Line& l) {
124     return isParallel(l) &&
125         fsign(unit_direction.crossProd(l.pivot - pivot)) == 0;
126 }
127 bool isSameDirection(const Line& l) {
128     return isParallel(l) &&
129         fsign(unit_direction.innerProd(l.unit_direction)) > 0;
130 }
131 Point intersection(const Line& l) {
132     assert(!isParallel(l));
133     return pivot + unit_direction *
134         (l.unit_direction.crossProd(l.pivot - pivot)) /
135         l.unit_direction.crossProd(unit_direction);
136 }
137
138 friend std::ostream& operator<<(std::ostream& os, Line l) {
139     return os << "(" << l.pivot << ", " << l.unit_direction << ")";
140 }
141 };
142
143 struct Segment {
144     Point a, b;
145     Segment(Point a, Point b) : a(a), b(b) {}
146     bool hasIntersection(const Segment& s) const {
147         return fsign(Point::crossProd(a, b, s.a)) *
148             fsign(Point::crossProd(a, b, s.b)) <
149             0 &&
150             fsign(Point::crossProd(s.a, s.b, a)) *
151             fsign(Point::crossProd(s.a, s.b, b)) <
152             0;
153 }
154 bool isPointIn(const Point& p) const {
155     return fsign(Point::innerProd(p, a, b)) <= 0 &&
156         fsign(Point::crossProd(p, a, b)) == 0;
157 }
158
159 Point lerp(const long double& ratio) const { return a + (b - a) * ratio; }
160
161 friend std::ostream& operator<<(std::ostream& os, Segment s) {
162     return os << "(" << s.a << ", " << s.b << ")";
163 }
164 };
165
166 struct Polygon {
167     static bool isConvexInCCW(const std::vector<Point>& p) {
168         for (int i = 0; i < p.size(); ++i) {
169             int l = (i + p.size() - 1) % p.size();
170             int r = (i + 1) % p.size();
171             if (Point::crossProd(p[i], p[l], p[r]) > 0) return false;

```

```

172     }
173     return true;
174 }
175 static std::vector<int> convexHullId(const std::vector<Point>& p) {
176     if (p.size() == 0) return {};
177     if (p.size() == 1) return {0};
178     std::vector<int> ids(p.size());
179     std::iota(ids.begin(), ids.end(), 0);
180     sort(ids.begin(), ids.end(),
181         [&, comp = Point::HorizontalComparer()](int i, int j) {
182             return comp(p[i], p[j]);
183         });
184     std::vector<int> res;
185     for (int i : ids) {
186         while (res.size() > 1 &&
187             fsign(Point::crossProd(p[res.end()[-2]], p[res.end()[-1]],
188                 p[i])) <= 0)
189             res.pop_back();
190         res.push_back(i);
191     }
192     ids.pop_back();
193     std::reverse(ids.begin(), ids.end());
194     int lower_size = res.size();
195     for (int i : ids) {
196         while (res.size() > lower_size &&
197             fsign(Point::crossProd(p[res.end()[-2]], p[res.end()[-1]],
198                 p[i])) <= 0)
199             res.pop_back();
200         res.push_back(i);
201     }
202     res.pop_back();
203     return res;
204 }
205 static std::vector<Point> convexHullPoint(const std::vector<Point>& p) {
206     std::vector<int> ids = convexHullId(p);
207     std::vector<Point> res;
208     for (int i : ids) res.push_back(p[i]);
209     return res;
210 }
211
212 // should be guaranteed that convex is a convex hull in cww
213 static bool isPointInConvexCCW(const Point& p,
214     const std::vector<Point>& convex) {
215     assert(Polygon::isConvexInCCW(convex));
216     for (int i = 0; i < convex.size(); ++i) {
217         Point a = convex[i];
218         Point b = convex[(i + 1) % convex.size()];
219         if (Point::crossProd(a, b, p) < 0) return false;
220     }
221     return true;
222 }
223 };

```

```

224
225 struct Circle {
226     Point o;
227     long double r;
228     Circle() : Circle(Point(0, 0), 0) {}
229     Circle(Point o, long double r) : o(o), r(r) {}
230     Point pointInDirection(long double angle) {
231         return Point(o.x + r * std::cos(angle), o.y + r * std::sin(angle));
232     }
233
234     std::vector<Point> intersection(const Line& l) const {
235         long double A = 1;
236         long double B = l.unit_direction.innerProd(l.pivot - o) * 2;
237         long double C = o.distanceSqr(l.pivot) - sqr(r);
238         std::vector<long double> roots = solveEquationP2(A, B, C);
239         std::vector<Point> intersects;
240         for (long double x : roots)
241             intersects.push_back(l.pivot + l.unit_direction * x);
242         return intersects;
243     }
244     std::vector<Point> intersection(const Segment& s) const {
245         std::vector<Point> line_intersects = intersection(Line(s.a, s.b - s.a));
246         std::vector<Point> intersects;
247         for (const Point& p : line_intersects)
248             if (s.isPointIn(p)) {
249                 intersects.push_back(p);
250             }
251         return intersects;
252     }
253
254     // triangle oab
255     long double overlapAreaWithTriangle(const Point& a, const Point& b) const {
256         if (!fsign(Point::crossProd(o, a, b))) return 0;
257         std::vector<Point> key_points;
258         key_points.push_back(a);
259         for (const Point& p : intersection(Segment(a, b))) key_points.push_back(p);
260         key_points.push_back(b);
261         long double res = 0;
262         for (int i = 1; i < key_points.size(); ++i) {
263             Point mid_point = (key_points[i - 1] + key_points[i]) / 2;
264             Point ray1 = key_points[i - 1] - o;
265             Point ray2 = key_points[i] - o;
266             if (o.distanceSqr(mid_point) <= sqr(r)) {
267                 res += std::abs(ray1.crossProd(ray2));
268             } else {
269                 res += sqr(r) * std::abs(ray1.angleWith(ray2));
270             }
271         }
272         dbg(a, b, key_points, res);
273         return 0.5 * res;
274     }
275

```

```

276 long double overlapAreaWithPolygon(const std::vector<Point>& p) const {
277     long double res = 0;
278     for (int i = 0; i < p.size(); ++i) {
279         int j = (i + 1) % p.size();
280         res += overlapAreaWithTriangle(p[i], p[j]) *
281             fsign(Point::crossProd(o, p[i], p[j]));
282     }
283     dbg(res);
284     return res;
285 }
286
287 // 1: outside
288 // 0: edge
289 // -1: inside
290 // equal to solving following determinant (a,b,c is counter-clockwise)
291 // | ax, ay, ax^2+ay^2, 1 |
292 // | bx, by, bx^2+by^2, 1 |
293 // | cx, cy, cx^2+cy^2, 1 |
294 // | px, py, px^2+py^2, 1 |
295 static int side(const Point& a, Point b, Point c, const Point& p) {
296     if (fsign(Point::crossProd(a, b, c)) < 0) std::swap(b, c);
297     Point3D a3(a), b3(b), c3(c), p3(p);
298     b3 = b3 - a3;
299     c3 = c3 - a3;
300     p3 = p3 - a3;
301     Point3D f = b3.crossProd(c3);
302     return fsign(p3.innerProd(f));
303 }
304
305 friend std::ostream& operator<<(std::ostream& os, const Circle& c) {
306     return os << "(" << c.o << ", " << c.r << ")";
307 }
308 };
309
310 // return the intersection convex in ccw, should be guaranteed that the
311 // intersection is finite.
312 struct HalfPlaneIntersection {
313     static std::vector<Point> solve(std::vector<Line> lines) {
314         sort(lines.begin(), lines.end(),
315             [comp = Point::PolarComparer()](auto l1, auto l2) {
316                 if (l1.isSameDirection(l2)) {
317                     return l1.side(l2.pivot) < 0;
318                 } else {
319                     return comp(l1.unit_direction, l2.unit_direction);
320                 }
321             });
322
323         std::deque<Line> key_lines;
324         std::deque<Point> key_points;
325         for (int i = 0; i < lines.size(); ++i) {
326             if (i > 0 && lines[i - 1].isSameDirection(lines[i])) continue;
327             while (key_points.size() && lines[i].side(key_points.back()) <= 0) {

```

```

328     key_lines.pop_back();
329     key_points.pop_back();
330 }
331 while (key_points.size() && lines[i].side(key_points.front()) <= 0) {
332     key_lines.pop_front();
333     key_points.pop_front();
334 }
335 if (key_lines.size()) {
336     // since it's guaranteed that the intersection is finite, therefore must
337     // be empty.
338     if (lines[i].isParallel(key_lines.back())) return {};
339     key_points.push_back(lines[i].intersection(key_lines.back()));
340 }
341 key_lines.push_back(lines[i]);
342 }
343
344 while (key_points.size() &&
345        key_lines.front().side(key_points.back()) <= 0) {
346     key_lines.pop_back();
347     key_points.pop_back();
348 }
349
350 if (key_lines.size() <= 2) return {};
351
352 std::vector<Point> convex;
353 for (int i = 0; i < key_lines.size(); ++i)
354     convex.emplace_back(
355         key_lines[i].intersection(key_lines[(i + 1) % key_lines.size()]));
356 return convex;
357 }
358 };
359
360 struct Triangulation {
361     struct Edge {
362         int v;
363         std::list<Edge>::iterator rev;
364         Edge(int v = 0) : v(v) {}
365     };
366     // delaunay triangulation
367     // should be guaranteed that all points are pairwise distinct
368     static std::vector<std::pair<int, int>> nearest(const std::vector<Point>& p) {
369         std::vector<std::list<Edge>> neighbor(p.size());
370         std::vector<int> id(p.size());
371         std::iota(id.begin(), id.end(), 0);
372         std::sort(id.begin(), id.end(),
373             [&, comp = Point::HorizontalComparer()](int i, int j) {
374                 return comp(p[i], p[j]);
375             });
376
377         auto addedge = [&](int u, int v) {
378             neighbor[u].push_front(v);
379             neighbor[v].push_front(u);

```

```

380             neighbor[u].front().rev = neighbor[v].begin();
381             neighbor[v].front().rev = neighbor[u].begin();
382         };
383         std::function<void(int, int)> divide = [&](int l, int r) {
384             if (r - l + 1 <= 3) {
385                 for (int i = l; i <= r; ++i)
386                     for (int j = l; j < i; ++j) addedge(id[i], id[j]);
387                 return;
388             }
389
390             int mid = (l + r) >> 1;
391             divide(l, mid);
392             divide(mid + 1, r);
393
394             auto get_base_LR_edge = [&]() {
395                 std::vector<int> stk;
396                 for (int i = l; i <= r; ++i) {
397                     while (stk.size() >= 2 &&
398                            fsign(Point::crossProd(p[id[stk.end()[-2]]],
399                                                    p[id[stk.end()[-1]]], p[id[i]])) < 0)
400                         stk.pop_back();
401                     stk.push_back(i);
402                 }
403                 for (int i = l; i < stk.size(); ++i)
404                     if (stk[i - 1] <= mid && stk[i] > mid)
405                         return std::make_pair(id[stk[i - 1]], id[stk[i]]);
406             };
407
408             auto [ld, rd] = get_base_LR_edge();
409
410             while (true) {
411                 addedge(ld, rd);
412                 Point ptL = p[ld], ptR = p[rd];
413                 int ch = -1, side = -1;
414                 for (auto it = neighbor[ld].begin(); it != neighbor[ld].end(); ++it) {
415                     if (fsign(Point::crossProd(ptL, ptR, p[it->v])) > 0 &&
416                         (!-ch || Circle::side(ptL, ptR, p[ch], p[it->v]) < 0)) {
417                         ch = it->v;
418                         side = 0;
419                     }
420                 }
421                 for (auto it = neighbor[rd].begin(); it != neighbor[rd].end(); ++it) {
422                     if (fsign(Point::crossProd(ptR, p[it->v], ptL)) > 0 &&
423                         (!-ch || Circle::side(ptL, ptR, p[ch], p[it->v]) < 0)) {
424                         ch = it->v;
425                         side = 1;
426                     }
427                 }
428                 if (!-ch) break;
429                 assert(side == 0 || side == 1);
430                 if (!side) {
431                     for (auto it = neighbor[ld].begin(); it != neighbor[ld].end(); {

```

```

432         if (Segment(ptL, p[it->v]).hasIntersection(Segment(ptR, p[ch]))) {
433             neighbor[it->v].erase(it->rev);
434             neighbor[ld].erase(it++);
435         } else {
436             ++it;
437         }
438     }
439     ld = ch;
440 } else {
441     for (auto it = neighbor[rd].begin(); it != neighbor[rd].end(); ) {
442         if (Segment(ptR, p[it->v]).hasIntersection(Segment(ptL, p[ch]))) {
443             neighbor[it->v].erase(it->rev);
444             neighbor[rd].erase(it++);
445         } else {
446             ++it;
447         }
448     }
449     rd = ch;
450 }
451 }
452 };
453
454 divide(0, p.size() - 1);
455
456 std::vector<std::pair<int, int>> edges;
457 for (int u = 0; u < p.size(); ++u)
458     for (auto e : neighbor[u])
459         if (u < e.v) edges.emplace_back(u, e.v);
460 return edges;
461 }
462
463 // should be guaranteed that p is strictly convex
464 static std::vector<std::pair<int, int>> furthest(
465     const std::vector<Point>& p) {
466     assert(Polygon::isConvexInCCW(p));
467     std::vector<std::pair<int, int>> edges;
468     if (p.size() < 3) {
469         for (int i = 0; i < p.size(); ++i)
470             for (int j = 0; j < i; ++j) {
471                 edges.emplace_back(i, j);
472             }
473     }
474     return edges;
475 }
476
477 std::vector<std::list<Edge>> neighbor(p.size());
478 std::vector<int> ids(p.size());
479 std::iota(ids.begin(), ids.end(), 0);
480
481 // calculate cw, ccw
482 std::vector<int> cw(p.size()), ccw(p.size());
483 for (int i = 0; i < p.size(); ++i) {
484     cw[i] = (i + p.size() - 1) % p.size();
485 }
486
487 std::random_shuffle(ids.begin(), ids.end());
488 for (int i = ids.size() - 1; i >= 2; --i) {
489     int u = ids[i];
490     std::tie(ccw[cw[u]], cw[ccw[u]]) = std::make_pair(ccw[u], cw[u]);
491 }
492
493 std::vector<std::list<Edge>> lines(p.size());
494 auto bind_rev_edge = [&](std::list<Edge>::iterator lhs,
495     std::list<Edge>::iterator rhs) {
496     lhs->rev = rhs;
497     rhs->rev = lhs;
498 };
499
500 bind_rev_edge(neighbor[ids[0]].emplace(neighbor[ids[0]].begin(), ids[1]),
501     neighbor[ids[1]].emplace(neighbor[ids[1]].begin(), ids[0]));
502
503 for (int i = 2; i < ids.size(); ++i) {
504     int u = ids[i];
505     int cur = cw[u];
506     auto cur_iter = neighbor[cur].begin();
507     while (1) {
508         while (cur_iter != neighbor[cur].end()) {
509             auto next_iter = std::next(cur_iter);
510             if (next_iter == neighbor[cur].end()) {
511                 if (cur != cw[u]) break;
512             } else {
513                 if (Circle::side(p[cur], p[cur_iter->v], p[next_iter->v], p[u]) < 0)
514                     break;
515             }
516             neighbor[cur].erase(cur_iter++);
517         }
518         bind_rev_edge(neighbor[u].emplace(neighbor[u].begin(), cur),
519             neighbor[cur].emplace(cur_iter, u));
520         if (cur == cw[u]) break;
521         std::tie(cur, cur_iter) =
522             std::make_pair(cur_iter->v, std::next(cur_iter->rev));
523     }
524 }
525
526 for (int u = 0; u < p.size(); ++u)
527     for (auto e : neighbor[u])
528         if (u < e.v) edges.emplace_back(u, e.v);
529 return edges;
530 };
531
532 struct PlanarGraphDuality {
533     struct DirectionalEdge {
534         int v, id = -1;
535         Point direction;

```

```

536     int rev;
537     DirectionalEdge(int v, Point direction) : v(v), direction(direction) {}
538 };
539
540 // return all points' id in each faces and the edges between faces
541 static std::pair<std::vector<std::vector<int>>,
542               std::vector<std::pair<int, int>>>
543 solve(const std::vector<Point>& p,
544       const std::vector<std::pair<int, int>>& edges) {
545     std::vector<DirectionalEdge> directional_edges;
546     directional_edges.reserve(edges.size() * 2);
547     std::vector<std::vector<int>> out_edges(p.size());
548     for (auto [u, v] : edges) {
549         out_edges[u].push_back(directional_edges.size());
550         directional_edges.emplace_back(v, p[v] - p[u]);
551         out_edges[v].push_back(directional_edges.size());
552         directional_edges.emplace_back(u, p[u] - p[v]);
553         directional_edges.end()[-1].rev = out_edges[u].back();
554         directional_edges.end()[-2].rev = out_edges[v].back();
555     }
556     const auto comp = [&, t_comp = Point::PolarComparer()](int lhs, int rhs) {
557         return t_comp(directional_edges[lhs].direction,
558                        directional_edges[rhs].direction);
559     };
560     for (int u = 0; u < p.size(); ++u)
561         std::sort(out_edges[u].begin(), out_edges[u].end(), comp);
562     std::vector<std::vector<int>> faces;
563     for (int u = 0; u < p.size(); ++u) {
564         for (int e_id : out_edges[u]) {
565             if (~directional_edges[e_id].id) continue;
566             std::vector<int> pids;
567             for (int cur_e_id = e_id;;) {
568                 if (~directional_edges[cur_e_id].id) break;
569                 directional_edges[cur_e_id].id = faces.size();
570                 int v = directional_edges[cur_e_id].v;
571                 pids.push_back(v);
572                 auto it = std::lower_bound(out_edges[v].begin(), out_edges[v].end(),
573                                           directional_edges[cur_e_id].rev, comp);
574                 assert(*it == directional_edges[cur_e_id].rev);
575                 if (it == out_edges[v].begin())
576                     cur_e_id = out_edges[v].back();
577                 else
578                     cur_e_id = *std::prev(it);
579             }
580             faces.push_back(pids);
581         }
582     }
583     std::vector<std::pair<int, int>> face_edges;
584     for (int u = 0; u < p.size(); ++u)
585         for (int e_id : out_edges[u]) {
586             int rev_id = directional_edges[e_id].rev;
587             if (e_id < rev_id) continue;

```

```

588             face_edges.emplace_back(directional_edges[e_id].id,
589                                     directional_edges[rev_id].id);
590         }
591     return std::make_pair(faces, face_edges);
592 }
593 };
594
595 struct Voronoi {
596     static constexpr long double kBoundaryInf = 50000;
597
598     // should be guaranteed that
599     // 1. all points are pairwise distinct
600     // 2. boundary had better to be a convex
601     // 3. all points are inside boundary
602     static std::vector<std::vector<Point>> nearest(
603         const std::vector<Point>& p,
604         const std::vector<Line>& boundary = {
605             Line(Point(-kBoundaryInf, -kBoundaryInf), Point(1, 0)),
606             Line(Point(kBoundaryInf, -kBoundaryInf), Point(0, 1)),
607             Line(Point(kBoundaryInf, kBoundaryInf), Point(-1, 0)),
608             Line(Point(-kBoundaryInf, kBoundaryInf), Point(0, -1)),
609         }) {
610         // p0 in the left
611         auto bisector = [&](const Point& p0, const Point& p1) {
612             auto dir = (p1 - p0).rotate90();
613             auto mid = (p0 + p1) / 2;
614             return Line(mid, dir);
615         };
616         auto edges = Triangulation::nearest(p);
617         std::vector<std::vector<Line>> limit(p.size(), boundary);
618         for (auto [i, j] : edges) {
619             limit[i].push_back(bisector(p[i], p[j]));
620             limit[j].push_back(bisector(p[j], p[i]));
621         }
622         std::vector<std::vector<Point>> regions(p.size());
623         for (int i = 0; i < p.size(); ++i) {
624             regions[i] = HalfPlaneIntersection::solve(limit[i]);
625         }
626         return regions;
627     }
628
629     // should be guaranteed that
630     // 1. p is strictly convex
631     // 2. boundary had better to be a convex
632     // 3. all points are inside boundary
633     static std::vector<std::vector<Point>> furthest(
634         const std::vector<Point>& p,
635         const std::vector<Line>& boundary = {
636             Line(Point(-kBoundaryInf, -kBoundaryInf), Point(1, 0)),
637             Line(Point(kBoundaryInf, -kBoundaryInf), Point(0, 1)),
638             Line(Point(kBoundaryInf, kBoundaryInf), Point(-1, 0)),
639             Line(Point(-kBoundaryInf, kBoundaryInf), Point(0, -1)),

```

```

640     } {
641     // p0 in the right
642     auto bisector = [&](const Point& p0, const Point& p1) {
643         auto dir = (p0 - p1).rotate90();
644         auto mid = (p0 + p1) / 2;
645         return Line(mid, dir);
646     };
647     auto edges = Triangulation::furthest(p);
648     std::vector<std::vector<Line>> limit(p.size(), boundary);
649     for (auto [i, j] : edges) {
650         limit[i].push_back(bisector(p[i], p[j]));
651         limit[j].push_back(bisector(p[j], p[i]));
652     }
653     std::vector<std::vector<Point>> regions(p.size());
654     for (int i = 0; i < p.size(); ++i) {
655         regions[i] = HalfPlaneIntersection::solve(limit[i]);
656     }
657     return regions;
658 }
659 };
660
661 } // namespace geometry2d
662
663 //***** 圆的面积并 *****
664
665 // Area[i] 表示覆盖次数大于等于i的面积, 复杂度  $O(n^2 \log n)$ 
666 struct P {
667     double x, y;
668     (){}(double _x, double _y) { x = _x, y = _y; }
669     operator+(const P& b) const { return P(x + b.x, y + b.y); }
670     operator-(const P& b) const { return P(x - b.x, y - b.y); }
671     operator*(double b) const { return P(x * b, y * b); }
672     operator/(double b) const { return P(x / b, y / b); }
673     double det(const P& b) const { return x * b.y - y * b.x; }
674     P rot90() const { return P(-y, x); }
675     P unit() { return *this / abs(); }
676     double abs() { return hypot(x, y); }
677 };
678 struct Circle {
679     P o;
680     double r;
681     bool contain(const Circle& v, const int& c) const { return sgn(r - (o - v.o).abs() - v.r) > c; }
682     bool disjunct(const Circle& v, const int& c) const { // 0严格, -1不严格
683         return sgn((o - v.o).abs() - r - v.r) > c;
684     }
685 };
686 //求圆与圆的交点, 包含相切, 假设无重圆
687 bool isCC(Circle a, Circle b, P& p1, P& p2) {
688     if (a.contain(b, 0) || b.contain(a, 0) || a.disjunct(b, 0)) return 0;
689     double s1 = (a.o - b.o).abs();
690     double s2 = (a.r * a.r - b.r * b.r) / s1;
691     double aa = (s1 + s2) / 2, bb = (s1 - s2) / 2;

```

```

692     P mm = (b.o - a.o) * (aa / (aa + bb)) + a.o;
693     double h = sqrt(max(0.0, a.r * a.r - aa * aa));
694     P vv = (b.o - a.o).unit().rot90() * h;
695     p1 = mm + vv, p2 = mm - vv;
696     return 1;
697 }
698 struct EV {
699     P p;
700     double ang;
701     int add;
702     EV() {}
703     EV(const P& _p, double _ang, int _add) { p = _p, ang = _ang, add = _add; }
704     bool operator<(const EV& a) const { return ang < a.ang; }
705 } eve[N * 2];
706 int E, cnt, C, i, j;
707 Circle c[N];
708 bool g[N][N], overlap[N][N];
709 double Area[N];
710 int cX[N], cY[N], cR[N];
711 bool contain(int i, int j) {
712     return (sgn(c[i].r - c[j].r) > 0 || sgn(c[i].r - c[j].r) == 0 && i < j) && c[i].contain(c[j], -1);
713 }
714 int main() {
715     scanf("%d", &C);
716     for (i = 0; i < C; i++) {
717         scanf("%d %d %d", &cX[i], &cY[i], &cR[i]);
718         c[i].o = P(cX[i], cY[i]);
719         c[i].r = cR[i];
720     }
721     for (i = 0; i <= C; i++) Area[i] = 0;
722     for (i = 0; i < C; i++)
723         for (j = 0; j < C; j++) overlap[i][j] = contain(i, j);
724     for (i = 0; i < C; i++)
725         for (j = 0; j < C; j++) g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c[j], -1));
726     for (i = 0; i < C; i++) {
727         E = 0;
728         cnt = 1;
729         for (j = 0; j < C; j++)
730             if (j != i && overlap[j][i]) cnt++;
731         for (j = 0; j < C; j++)
732             if (i != j && g[i][j]) {
733                 P aa, bb;
734                 isCC(c[i], c[j], aa, bb);
735                 double A = atan2(aa.y - c[i].o.y, aa.x - c[i].o.x);
736                 double B = atan2(bb.y - c[i].o.y, bb.x - c[i].o.x);
737                 eve[E++] = EV(bb, B, 1);
738                 eve[E++] = EV(aa, A, -1);
739                 if (B > A) cnt++;
740             }
741         if (E == 0)
742             Area[cnt] += PI * c[i].r * c[i].r;
743         else {

```

```

744     sort(eve, eve + E);
745     eve[E] = eve[0];
746     for (j = 0; j < E; j++) {
747         cnt += eve[j].add;
748         Area[cnt] += eve[j].p.det(eve[j + 1].p) * 0.5;
749         double theta = eve[j + 1].ang - eve[j].ang;
750         if (theta < 0) theta += PI * 2;
751         Area[cnt] += theta * c[i].r * c[i].r * 0.5 - sin(theta) * c[i].r * c[i].r * 0.5;
752     }
753 }
754 }
755 for (i = 1; i <= C; i++) printf(" %d % .3f\n", i, Area[i] - Area[i + 1]);
756 }

```

4 Math and Number Theory

杨表 标准杨表钩长公式: $\frac{n!}{\prod_{hook(i,j)} hook(i,j)}$, 其中 $hook(i,j) = |(i,j) \text{下方}| + |(i,j) \text{右方}| + \text{自己}$;

半标准杨表 (非严格递增) 钩长公式: $\prod_{(i,j)} \frac{n+j-i}{hook(i,j)}$

不交的 k 个上升子序列长度之和最大 \Leftrightarrow 上升杨表前 k 层长度和 (不降子序列 \Leftrightarrow 半标准杨表)。序列依次插入杨表, 若能放第 1 行末尾则放, 否则取第 1 行大于当前数的最小的一个, 取出, 插入第 2 行。

约数个数表

n	10	1e2	1e3	1e4	1e5	1e6
$d(n)$	4	12	32	64	128	240
n	1e7	1e8	1e9	1e10	1e11	1e12
$d(n)$	448	768	1344	2304	4032	6720
n	1e13	1e14	1e15	1e16	1e17	1e18
$d(n)$	10752	17280	26880	41472	64512	103680

整数拆分数

n	10	20	30	40	50
$p(n)$	42	627	5604	37338	204226
n	60	70	80	90	100
$p(n)$	966467	4087968	15796476	56634173	190569292

整数拆分 五边形数: $\phi(x) = \prod_{i=1}^{\infty} (1 - x^i) = 1 + \sum_{i=1}^{\infty} (-1)^i x^{\frac{i(3i+1)}{2}}$

整数拆分: $F(x) = \prod_{i=1}^{\infty} (1 + x^i + x^{2i} + \cdots) = \prod_{i=1}^{\infty} \frac{1}{1-x^i}$

$\phi(x)$ 与 $F(x)$ 互为逆多项式

递推: $F(n) = F(n-1) + F(n-2) - F(n-5) - F(n-7) \cdots$

单位根反演 $[i \bmod k = 0] = \frac{1}{k} \sum_{j=0}^{k-1} \omega^{ij}$, ω 为 k 次单位根

二项式反演 $a_n = \sum_{i=0}^n \binom{n}{i} b_i \Rightarrow b_n = \sum_{i=0}^n \binom{n}{i} (-1)^{n-i} a_i$

Fibonacci 通项: $f_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$

1. 相邻两项互质

2. $f_{m+n} = f_{m-1}f_n + f_mf_{n+1}$

3. $(f_{m+n}, f_n) = (f_m, f_n)$, $(f_n, f_m) = f_{(n,m)}$

4. $\sum_{i=0}^n f_i = f_{n+2} - 1$, $\sum_{i=0}^n f_i^2 = f_n f_{n+1}$

5. $f_n^2 = (-1)^{n-1} + f_{n-1}f_{n+1}$

斯特林数 第一类: 将 p 个物品排成 k 个非空循环排列

$S(p, k) = (p-1)S(p-1, k) + S(p-1, k-1)$

$S(p, 0) = 0$, $S(p, p) = 1$, $S(0, 0) = 1$

第二类: 将 p 个物品划分成 k 个非空集合 (无编号盒子)

$S(p, k) = kS(p-1, k) + S(p-1, k-1)$

$S(p, 0) = 0$, $S(p, p) = 1$, $S(0, 0) = 1$

$$S(n, m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k \binom{m}{k} (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k! (m-k)!}$$

$$x^k = \sum_{i=1}^k S(k, i) \cdot i! \cdot C(x, i)$$

二次剩余 ① $x^2 \equiv n \pmod{p}$

有解 iff $n^{\frac{p-1}{2}} \equiv 1 \pmod{p}$

解: 随机 a 使得 $w = a^2 - n$ 非二次剩余, 则 $x = \pm(a + \sqrt{w})^{\frac{p+1}{2}}$

② $ax^2 + bx \equiv c \pmod{p}$

解: 配方得 $(2ax + b)^2 \equiv b^2 - 4ac \pmod{p}$, 解 +exgcd

线性逆元 $i^{-1} = -\lfloor \frac{p}{i} \rfloor \cdot (p \pmod{i})^{-1}$

powerful number 定义: 所含质因子次数全部大于 1 的数, 只有 \sqrt{n} 个, 可以暴力求。

积性函数求和 $\sum_{i=1}^n F(i)$, 找一个积性函数 G 使得 $G(p) = F(p)$, 则 $H = \frac{F}{G}$ 只在 powerful number 下有非 0 值。

$$\sum_{i=1}^n F(i) = \sum_{i=1}^n H(i) \sum_{j=1}^{\lfloor \frac{n}{i} \rfloor} G(j) = \sum_{i \in \text{powerful num}} H(i) S_G(\lfloor \frac{n}{i} \rfloor)$$

min25 数组 $g(n, j) = \sum_{i=1}^n [i \in P \vee \min p_i > p_j] f(i)$

$$S(n, j) = \sum_{i=1}^n [\min p_i \geq p_j] f(i)$$

Sherman-Morrison

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1}u}$$

$$|A| \cdot A^{-1} = \text{adj} A, (\text{adj} A \text{ 是代数余子式矩阵的转置})$$

新数论分块 $(\lfloor \frac{n}{i} \rfloor, \lceil \frac{n}{n \% i} \rceil)$ 的有效取值只有 $O(\sqrt{n} \ln n)$

```

1  for (LL i=1, a, j; i<=n; i=j+1) {
2      j=n/(a=n/i);
3      for (LL l=1, b, r; l<=j; l=r+1) {
4          if (n%l==0) {
5              assert(l==j); //当x/n时一定出现在整除分块的末尾处。
6              //....
7          } else {
8              // c=ll(r/(n-ar))<=b -> r/(n-ar)<=b -> r<=b*(n-ar) -> r<=bn/(ab+1)
9              b=1/(n%l);

```

```

10         r=b*n/(a*b+1);
11         //....
12     }
13 }
14 }

```

```

1  //***** combinatorics mod (exLucas) *****
2
3  LL p[maxp], p0, pk[maxp], num[maxp], fac[maxp];
4  void Prime(LL P) { // P = p[1]^num[1] * ... * p[p0]^num[p0] = pk[1] * ... * pk[p0]
5      void Pre() {
6          fo(j, 1, p0) {
7              fac[j]=1;
8              fo(i, 1, pk[j]-1) if (i%p[j]) fac[j]=fac[j]*i%pk[j];
9          }
10     }
11
12     LL mo;
13     LL Pow(LL x, LL y) { // mod mo
14         LL count(LL n, LL p) {return (n)/(p+count(n/p, p)) : 0 ;}
15         LL Fc(LL n, LL j) {
16             if (!n) return 1;
17             LL re=Fc(n/p[j], j) *mi(fac[j], n/mo)%mo;
18             fo(i, 1, n%mo) if (i%p[j]) re=re*i%mo;
19             return re;
20         }
21
22         LL C(LL n, LL m, LL p) { // compute C(n+m, n). M=n+m.
23             // preprocess: Prime(P); Pre();
24             LL ans=0;
25             fo(j, 1, p0) {
26                 LL nump=count(M, p[j])-count(m, p[j])-count(n, p[j]);
27                 LL phi=pk[j]-pk[j]/p[j];
28                 mo=pk[j];
29                 LL a=(nump>num[j]) ? 0 : Fc(M, j)*Pow(Fc(n, j), phi-1)%mo*Pow(Fc(m, j), phi-1)%mo*Pow(p[j], nump)%mo;
30                 (ans+=a * (P/pk[j])%P *Pow(P/pk[j], phi-1)%P)%=P;
31             }
32
33             return ans;
34         }
35
36         //***** Miller_Rabin *****
37
38         int pr[9]={2,3,5,7,11,13,17,19,23};
39         LL mul(LL x, LL y, LL mo) {
40             LL re=0;
41             for(; y; y>>=1, x=(x+x)%mo) if (y&1) re=(re+x)%mo;
42             return re;
43         }
44         LL Pow(LL x, LL y, LL mo) {

```

```

45     LL re=1;
46     for(; y;>=1, x=mul(x,x,mo)) if (y&1) re=mul(re,x,mo);
47     return re;
48 }
49 bool Miller_Rabin(int d, LL s, LL a, LL n) {
50     a=Pow(a,s,n);
51     if (a==1) return 1;
52     fo(i,1,d) {
53         if (a==n-1) return 1;
54         if (a==1) return 0;
55         a=mul(a,a,n);
56     }
57     return 0;
58 }
59 bool isprime(LL n) {
60     if (n<2) return 0;
61     fo(i,0,8) {
62         if (n==pr[i]) return 1;
63         if (n%pr[i]==0) return 0;
64     }
65     int d=0; LL s=n-1;
66     for(; !(s&1); s>>=1, d++);
67     fo(i,0,8) if (!Miller_Rabin(d,s,pr[i],n)) return 0;
68     return 1;
69 }
70
71 //***** pollard_rho *****
72
73 inline LL ran_f(LL x, LL c, LL n) {return (mul(x,x,n)+c)%n;}
74 LL pollard_rho(LL n) {
75     for(LL c=rand()*rand()%n; ; c=rand()*rand()%n) {
76         LL x=rand()*rand()%n, y=x;
77         for(LL i=0, k=1; ; i++) {
78             x=ran_f(x,c,n);
79             LL t=_gcd(abs(x-y),n);
80             if (t==n) break;
81             else if (t>1) return t;
82             if (i==k) y=x, k<<=1;
83         }
84     }
85 }
86
87 //***** similar gcd *****
88
89 struct FGH{
90     LL f,g,h;
91 }; // f=\sum_{i=0}^n (a*i+b)/c, g=\sum_{i=0}^n i*(a*i+b)/c, h=\sum_{i=0}^n ((a*i+b)/c)^2
92
93 FGH calc(LL a, LL b, LL c, LL n) {
94     LL ac=a/c, bc=b/c, sum1=n*(n+1)%mo*inv2%mo, sum2=n*(n+1)%mo*(2*n+1)%mo*inv6%mo;
95     if (!a) return (FGH){(n+1)*bc%mo, sum1*bc%mo, (n+1)*bc%mo*bc%mo};
96     if (a>c || b>c) {

```

```

97     FGH nxt=calc(a%c,b%c,c,n);
98     LL f=(nxt.f+ac*sum1+(n+1)*bc)%mo;
99     LL g=(nxt.g+ac*sum2+sum1*bc)%mo;
100    LL h=(nxt.h+sum2*ac%mo+ac*(n+1)*bc%mo*bc%mo+2*ac*nxt.g%mo+2*bc*nxt.f%mo+n*(n+1)%mo*ac%mo*
        bc%mo)%mo;
101    return (FGH){f,g,h};
102 } else {
103     LL m=(a*n+b)/c;
104     FGH nxt=calc(c,c-b-1,a,m-1);
105     m%=mo;
106     LL f=(m*n-nxt.f+mo)%mo;
107     LL g=((n+1)*n%mo*m-nxt.f-nxt.h+mo+mo)%mo*inv2%mo;
108     LL h=((m+1)*n%mo*m-nxt.g-nxt.g-f-nxt.f-nxt.f+mo*5)%mo;
109     return (FGH){f,g,h};
110 }
111 }
112
113 //***** 第 *****
114
115 // f(n,k) 表示把 n 拆成 k 个数的积的方案数
116 LL mw[2*maxsqtrn], g[2*maxsqtrn];
117 int w0, id1[maxsqtrn], id2[maxsqtrn];
118 LL min25_g(LL n) {
119     w0=0;
120     for(LL i=1, j; i<=n; i=j+1) {
121         j=n/(n/i);
122         mw[++w0]=n/i;
123         if (mw[w0]<=sqtrn) id1[mw[w0]]=w0; else id2[j]=w0;
124         g[w0]=mw[w0]-1;
125     }
126     fo(j,1,Np[sqtrn])
127         for(int i=1; i<=w0 && (LL)p[j]*p[j]<=mw[i]; i++) {
128             int id=(mw[i]/p[j]<=sqtrn) ? id1[mw[i]/p[j]] : id2[n/(mw[i]/p[j])];
129             (g[i]-g[id]-(j-1))%=mo;
130         }
131     }
132 }
133 LL min25_S(LL x, int j, int k) {
134     if (x<=1 || p[j]>x) return 0;
135     int id=(x<=sqtrn) ? id1[x] : id2[n/x];
136     LL re=(g[id]-(j-1))*k;
137     for(int i=j; i<=Np[sqtrn] && (LL)p[i]*p[i]<=x; i++) {
138         LL pe=p[i];
139         for(int e=1; pe*p[i]<=x; e++, pe*=p[i])
140             (re+=min25_S(x/pe,i+1,k)*C[e+k-1][k-1]+C[e+k][k-1])%=mo;
141     }
142     return re;
143 }

```

5 Others

有向图最大费用循环流 思路：先让正权边全部流满，再调整做法：

1. 先强制流掉所有正权边（并加入答案）
2. 然后流量盈余的点连向汇点，源点连向流量亏损的点
3. 答案减去 MCMF

无源汇上下界网络流 记 d_i 为点 i 的入流减出流，附加网络如下：

1. 原图每条边 (u, v, l, r) ，连边 $(u, v, r - l)$
2. 若 $d_i > 0$ ，则 (ss, i, d_i)
3. 若 $d_i < 0$ ，则 $(i, tt, -d_i)$
4. 当且仅当 ss, tt 满流时有可行解

最大权闭合子图 答案 = 所有正点权的和 - 如下最小割：

1. 原图每条边 (u, v) ，连边 (u, v, inf)
2. 超级源连向正点权，负点权连向超级汇

Segment Tree Beats 维护区间最小值、区间次小值以及最小值出现的次数；将操作 (x, y) 抽象为 $a_i := \min(a_i + x, y)$ ；若 TL 允许，写成矩阵形式转移最简单。

BEST 定理 图 G 从点 s 出发的欧拉路径数 $= ts(G) \prod (deg(v) - 1)!$ ，其中 $ts(G)$ 表示以 s 为根的外向树个数（入度矩阵 - 邻接矩阵，求 \det ）。前提：每个点出度入度相同。

```

1 //***** Blossom *****
2
3 int ga[maxp];
4 int get(int x) {return (ga[x]==x) ? x : ga[x]=get(ga[x]);}
5

```

```

6 int d[maxp], di, dj, sum, pt[maxp], pf[maxp], clr[maxp], nowT, bz[maxp];
7 int lca(int x, int y) {
8     for(nowT++, x=get(x), y=get(y); bz[x]!=nowT; ) {
9         bz[x]=nowT;
10        x=get(pf[pt[x]]);
11        swap(x, y);
12    }
13    return x;
14 }
15 void shrink(int x, int y, int rt) {
16     for(; get(x)!=rt; x=pf[y]) {
17         pf[x]=y;
18         y=pt[x];
19         if (clr[y]==1) clr[ d[++dj]=y ]=0;
20         ga[x]=ga[y]=rt;
21     }
22 }
23 bool Blossom(int st) {
24     fo(i, 1, sum) ga[i]=i;
25     memset(clr, 255, sizeof(clr)); clr[st]=0;
26     d[1]=st;
27     for(di=1, dj=1; di<=dj; di++) {
28         int now=d[di];
29         for(int p=f1[now]; p; p=next[p]) if (clr[go[p]]==-1) {
30             clr[go[p]]=1;
31             pf[go[p]]=now;
32             if (!pt[go[p]]) {
33                 for(int x=now, y=go[p], t; x; y=t, x=pf[y]) {
34                     t=pt[x];
35                     pt[x]=y, pt[y]=x;
36                 }
37                 return 1;
38             } else {
39                 d[++dj]=pt[go[p]];
40                 clr[pt[go[p]]]=0;
41             }
42         } else if (clr[go[p]]==0 && get(go[p])!=get(now)) {
43             int rt=lca(go[p], now);
44             shrink(go[p], now, rt);
45             shrink(now, go[p], rt);
46         }
47     }
48     return 0;
49 }
50
51 //***** dominator tree (from CTL) *****
52
53 const int maxn = 310000;
54 const int maxm = 1050000;
55
56 int n, m, s;
57 int sdom[maxn], idom[maxn];

```

```

58 vector<int>V[maxn];
59 vector<int>g[maxn],e[maxn];
60
61 int dfn[maxn],To[maxn],id,par[maxn];
62 void build(int u){
63     To[dfn[u]++]=u;
64     for (auto v:g[u]) if (!dfn[v]) par[v]=u,build(v);
65 }
66
67 int fa[maxn],fas[maxn];
68 void find(int x)
69 {
70     if(fa[x]==x) return;
71     find(fa[x]);
72     if(dfn[sdom[fas[fa[x]]]]<dfn[sdom[fas[x]]]) fas[x]=fas[fa[x]];
73     fa[x]=fa[fa[x]];
74 }
75 int ans[maxn];
76
77 int main(){
78     scanf("%d%d%d",&n,&m,&s);
79     int nn=n;
80     rep(i,1,m) {
81         int x,y;
82         scanf("%d%d",&x,&y);
83         e[n+i].push_back(x); e[y].push_back(n+i);
84         g[x].push_back(n+i); g[n+i].push_back(y);
85     }
86     n+=m;
87     build(s);
88     rep(i,1,n) fa[i]=i,fas[i]=i,sdom[i]=idom[i]=i;
89     per(i,id,1){
90         int x=To[i],&semi=sdom[x];
91         for(auto y:e[x]) if (dfn[y]){
92             find(y);
93             if(dfn[semi]>dfn[sdom[fas[y]]]) semi=sdom[fas[y]];
94         }
95         for(auto y:V[x]){
96             find(y);
97             if(dfn[sdom[fas[y]]]<i) idom[y]=fas[y];
98             else idom[y]=x;
99         }
100         V[semi].push_back(x);
101         for (auto y:g[x]) if (par[y]==x) fa[y]=x;
102     }
103     rep(i,1,id){
104         int x=To[i];
105         if(idom[x]!=sdom[x]) idom[x]=idom[idom[x]];
106     }
107     per(i,id,2){
108         int x=To[i];
109         if (1<=x&&x<=n&&idom[x]>nn) ans[idom[x]-nn]=1;

```

```

110     }
111     int cnt=0;
112     rep(i,1,m) if (!ans[i]) cnt++;
113     printf("%d\n",cnt);
114     rep(i,1,m) if (!ans[i]) printf("%d ",i);
115     return 0;
116 }
117
118 //***** dq_mincut *****
119
120 int num,st[maxn];
121 bool vis[maxn];
122 void find(int k)
123 {
124     vis[k]=1;
125     st[++num]=k;
126     for(int p=fi[k]; p; p=nxt[p]) if (!vis[go[p]]) find(go[p]);
127 }
128
129 int bj[maxn],sum,nowh[maxn],d[maxn],bz[maxn],bzcnt;
130 // maxflow here, 稀疏图 dinic, else isap
131 void bfs(int s)
132 {
133     bz[ d[1]=s ]=++bzcnt;
134     for(int i=1, j=1; i<=j; i++)
135     {
136         for(int p=fi[d[i]]; p; p=nxt[p]) if (val2[p] && bz[go[p]]!=bzcnt)
137             bz[ d[++j]=go[p] ]=bzcnt;
138     }
139 }
140
141 int st1[maxn];
142 vector<pair<int,int>> e[maxn];
143 void Mincut(int l,int r)
144 {
145     memcpy(val2,val,sizeof(val));
146     sum=st[r];
147
148     int flow=0;
149     while (Dinic_bfs(st[l])) flow+=Dinic_dfs(st[l],inf);
150     e[st[l]].push_back(make_pair(sum,flow)), e[sum].push_back(make_pair(st[l],flow));
151
152     bfs(st[l]);
153     int newr=l-1, newl=r+1;
154     fo(i,l,r) if (bz[st[i]]==bzcnt) st1[++newr]=st[i]; else st1[--newl]=st[i];
155     fo(i,l,r) st[i]=st1[i];
156
157     if (l<newr) Mincut(l,newr);
158     if (newl<r) Mincut(newl,r);
159 }
160
161 int main()

```

```

162 {
163     fo(i,1,n) if (!vis[i] && f1[i])
164     {
165         num=0;
166         find(i);
167         Mincut(1,num);
168     }
169 }
170
171 //***** KM *****
172
173 LL lx[maxn],ly[maxn],slack[maxn];
174 int f[maxn],pre[maxn];
175 bool vis[maxn];
176 LL KM(int nl,int nr)
177 {
178     fo(i,1,nl)
179         fo(j,1,nr) lx[i]=max(lx[i],mp[i][j]);
180     fo(i,1,nl)
181     {
182         memset(slack,127,sizeof(LL)*(nr+1));
183         memset(vis,0,sizeof(bool)*(nr+1));
184         f[0]=i;
185         int py=0, nextpy;
186         for(; f[py]; py=nextpy)
187         {
188             int px=f[py];
189             LL d=inf;
190             vis[py]=1;
191             fo(j,1,nr) if (!vis[j])
192             {
193                 if (lx[px]+ly[j]-mp[px][j]<slack[j]) slack[j]=lx[px]+ly[j]-mp[px][j], pre[j]=py;
194                 if (slack[j]<d) d=slack[j], nextpy=j;
195             }
196             fo(j,0,nr) if (vis[j]) lx[f[j]]-=d, ly[j]+=d;
197             else slack[j]-=d;
198         }
199         for(; py; py=pre[py]) f[py]=f[pre[py]];
200     }
201     LL re=0;
202     fo(i,1,nl) re+=lx[i];
203     fo(j,1,nr) re+=ly[j];
204     return re;
205 }
206
207 //***** LCA *****
208
209 //倍增
210 int deep[maxn],fa[maxn][MX+1];
211 int lca(int x,int y) {
212     if (deep[x]<deep[y]) swap(x,y);
213     fd(i,MX,0)

```

```

214         while (deep[fa[x][i]]>=deep[y]) x=fa[x][i];
215     if (x==y) return x;
216     fd(i,MX,0)
217         while (fa[x][i]!=fa[y][i]) x=fa[x][i], y=fa[y][i];
218     return fa[x][0];
219 }
220
221 //tarjan
222 int totq,goq[2*maxn],num[2*maxn],nextq[2*maxn],fq[maxn];
223 void inq(int x,int y,int z) {
224     goq[++totq]=y;
225     num[totq]=z;
226     nextq[totq]=fq[x];
227     fq[x]=totq;
228 }
229 int lca[maxn],fa[maxn];
230 bool bz[maxn];
231 int get(int x) {
232     if (fa[x]==x) return x;
233     return fa[x]=get(fa[x]);
234 }
235 void tarjan(int k,int last) { //ordinary
236     fa[k]=k;
237     for(int p=f1[k]; p; p=next[p]) if (go[p]!=last) {
238         tarjan(go[p],k);
239         fa[go[p]]=k;
240     }
241     bz[k]=1;
242     for(int p=fq[k]; p; p=nextq[p])
243         if (bz[goq[p]]) lca[num[p]]=get(goq[p]); else inq(goq[p],k,num[p]);
244 }
245
246 void tarjan(int k,int last) { //支持维护值
247     fa[k]=k;
248     for(int p=f1[k]; p; p=next[p]) if (go[p]!=last) {
249         tarjan(go[p],k);
250         f[go[p]]+=val[p];
251         fa[go[p]]=k;
252     }
253     bz[k]=1;
254     for(int p=fq[k]; p; p=nextq[p]) if (bz[goq[p]]) {
255         int t=get(goq[p]);
256         ans[num[p]]=valq[p]+f[goq[p]];
257         if (t!=k) inq(t,k,f[goq[p]],num[p]); else lca[num[p]]=t;
258     } else inq(goq[p],k,0,num[p]);
259 }
260
261 //rmq
262 int fa[2*maxn][MX+5],deep[maxn],ap[2*maxn],fir[2*maxn],Log[2*maxn],er[MX+5];
263 void rmq_pre() {
264     fo(i,1,ap[0]) fa[i][0]=ap[i], Log[i]=log(i)/log(2);
265     fo(i,0,MX) er[i]=1<<i;

```

```

266     fo(j,1,MX)
267     fo(i,1,ap[0]) {
268         fa[i][j]=fa[i][j-1];
269         if (i+er[j-1]<=ap[0] && deep[fa[i+er[j-1]][j-1]]<deep[fa[i][j]])
270             fa[i][j]=fa[i+er[j-1]][j-1];
271     }
272 }
273 int lca(int x,int y) {
274     x=fir[x], y=fir[y];
275     if (x>y) swap(x,y);
276     int t=Log[y-x+1];
277     return (deep[fa[x][t]]<deep[fa[y-er[t]+1][t]]) ?fa[x][t] :fa[y-er[t]+1][t] ;
278 }
279
280 void dfs_pre(int k,int last) {
281     deep[k]=deep[last]+1;
282     ap[++ap[0]]=k, fir[k]=ap[0];
283     for(int p=f1[k]; p; p=next[p]) if (go[p]!=last) {
284         dfs_pre(go[p],k);
285         ap[++ap[0]]=k;
286     }
287 }
288
289 //***** LeftTree *****
290
291 struct node{
292     int val,l,r,fa,dis;
293 };
294
295 node lt[maxn];
296 int tot,ga[maxn];
297 int New(int val=0)
298 {
299     lt[++tot]=(node){val,0,0,0,0};
300     ga[tot]=tot;
301     return tot;
302 }
303
304 int merge(int a,int b)
305 {
306     if (!a) return b;
307     if (!b) return a;
308     if (lt[a].val>lt[b].val || lt[a].val==lt[b].val && a>b) swap(a,b);
309     lt[a].r=merge(lt[a].r,b);
310     lt[lt[a].r].fa=a;
311     ga[lt[a].r]=a;
312     if (lt[lt[a].r].dis>lt[lt[a].l].dis) swap(lt[a].l,lt[a].r);
313     lt[a].dis=(lt[a].r==0) ?0 :lt[lt[a].r].dis+1;
314     return a;
315 }
316
317 int top(int x){return (ga[x]==x) ?x :ga[x]=top(ga[x]) ;}
318 void pop(int x)
319 {

```

```

318     int t=merge(lt[x].l,lt[x].r);
319     lt[t].fa=lt[x].fa;
320     ga[x]=(top(x)==x) ?t :top(x);
321     ga[t]=ga[x];
322     for(int i=lt[x].fa; i; i=lt[i].fa) if (lt[lt[i].l].dis<lt[lt[i].r].dis)
323     {
324         swap(lt[i].l,lt[i].r);
325         lt[i].dis=lt[lt[i].r].dis+1;
326     } else break;
327     lt[x].fa=-1;
328 }
329 void push(int x,int val)
330 {
331     merge(top(x),New(val));
332 }
333
334 // init : lt[0].dis=-1;
335
336 //***** maxflow *****
337
338 //isap+gap+当前弧
339 int bj[maxsum],gap[maxsum],sum,nowh[maxsum],d[maxsum];
340 void init_Maxflow() {
341     memset(bj,0,sizeof(bj));
342     memset(gap,0,sizeof(gap)); gap[0]=sum+1;
343     memcpy(nowh,f1,sizeof(nowh));
344     d[1]=sum;
345     for(int i=1, j=1; i<=j; i++) {
346         for(int p=f1[d[i]]; p; p=e[p].nxt) if (e[p].go!=sum && !bj[e[p].go]) {
347             bj[e[p].go]=bj[d[i]]+1;
348             gap[0]--, gap[bj[e[p].go]]++;
349             d[++j]=e[p].go;
350         }
351     }
352 }
353
354 int Maxflow(int k,int flow) {
355     if (k==sum) return flow;
356     int re=0;
357     for(int &p=nowh[k]; p; p=e[p].nxt) if (e[p].val && bj[k]==bj[e[p].go]+1) {
358         int fl=Maxflow(e[p].go, (flow-re<e[p].val) ?(flow-re) :e[p].val);
359         e[p].val-=fl;
360         e[(p&1) ?p+1 :p-1].val+=fl;
361         re+=fl;
362         if (re==flow || bj[0]>sum) return re;
363     }
364     nowh[k]=f1[k];
365     if ((--gap[bj[k]])==0) bj[0]=sum+1; else bj[k]++;
366     gap[bj[k]]++;
367     return re;
368 }
369
370 //dinic+当前弧
371 bool Dinic_bfs(int s) {

```

```

370     memset(bj,255,sizeof(bj));
371     memcpy(nowh,f1,sizeof(nowh));
372     bj[ d[1]=s ]=0;
373     for(int i=1, j=1; i<=j; i++) {
374         for(int p=f1[d[i]]; p; p=e[p].nxt) if (e[p].val && bj[e[p].go]==-1) {
375             bj[e[p].go]=bj[d[i]]+1;
376             d[++j]=e[p].go;
377         }
378     }
379     return bj[sum]!=-1;
380 }
381 int Dinic_dfs(int k,int flow) {
382     if (k==sum) return flow;
383     int re=0;
384     for(int &p=nowh[k]; p; p=e[p].nxt) if (e[p].val && bj[k]+1==bj[e[p].go]) {
385         int fl=Dinic_dfs(e[p].go, (flow-re<e[p].val) ?(flow-re) :e[p].val);
386         e[p].val-=fl;
387         e[(p&1) ?p+1 :p-1 ].val+=fl;
388         re+=fl;
389         if (re==flow) return re;
390     }
391     return re;
392 }
393
394 //***** random Hung *****
395
396 int bz[maxn],tim,pt[maxn];
397 bool Hung(int x,int tim) {
398     if (bz[x]==tim) return 0;
399     bz[x]=tim;
400     random_shuffle(e[x].begin(),e[x].end());
401     for(int go:e[x]) {
402         int k=pt[go];
403         pt[k]=0, pt[x]=go, pt[go]=x;
404         if (!k || Hung(k,tim)) return 1;
405         pt[k]=go, pt[go]=k, pt[x]=0;
406     }
407     return 0;
408 }
409 int main() {
410     fo(i,1,n) pmt[i]=i;
411     random_shuffle(pmt+1,pmt+1+n);
412     int tim=0, cnt=0;
413     fo(j,1,5)
414         fo(i,1,n) if (!pt[pmt[i]]) Hung(pmt[i],++tim);
415 }
416
417 //***** round square tree *****
418
419 //广义 (任意路径都是圆方交替)
420 int sum,dfn[maxn],low[maxn],z[maxn],z0,num[2*maxn],nn;
421 vector<int> e[2*maxn];

```

```

422 void tarjan(int k,int last) {
423     dfn[k]=low[k]=++sum;
424     z[++z0]=k;
425     for(int p=f1[k]; p; p=nxt[p]) if (bh[p]!=last) {
426         if (!dfn[go[p]]) {
427             tarjan(go[p],bh[p]);
428             low[k]=min(low[k],low[go[p]]);
429         }
430         if (low[go[p]]>=dfn[k]) {
431             num[++nn]=1;
432             e[nn].push_back(k), e[k].push_back(nn);
433             do {
434                 num[nn]++;
435                 e[nn].push_back(z[z0]), e[z[z0]].push_back(nn);
436             } while (z[z0--]!=go[p]);
437         }
438     } else low[k]=min(low[k],dfn[go[p]]);
439 }
440 }
441
442 //***** virtual tree *****
443
444 int p0,p[2*maxn],z[maxn],z0;
445 bool cmpP(const int &a,const int &b) {return dfn[a]<dfn[b];}
446 void make_vtree() {
447     tot=0;
448     sort(p+1,p+1+p0,cmpP);
449     int t=p0;
450     fo(i,1,t-1) p[++p0]=lca(p[i],p[i+1]);
451     sort(p+1,p+1+p0,cmpP);
452     f1[ z[z0]=1 ]=0;
453     p[0]=1;
454     fo(i,1,p0) if (p[i]!=p[i-1]) {
455         for(;; z0 && (dfn[p[i]]<dfn[z[z0]] || en[z[z0]]<dfn[p[i]]); z0--) ins(z[z0-1],z[z0]);
456         f1[ z[++z0]=p[i] ]=0;
457     }
458     fo(i,1,z0-1) ins(z[i],z[i+1]);
459 }
460
461 //***** Tools *****
462
463 //乘法取模黑科技 Claris
464 LL mul(LL a,LL b,LL n){return (a*b-(LL) (a/(long double)n*b+1e-3)*n+n)%n;}
465
466 //split a string by whitespace
467 vector<string> split_str(string str) {
468     vector<string> result;
469     istringstream iss(str);
470     string s;
471     while ( getline( iss, s, ' ' ) ) result.push_back(s);
472     return result;
473 }

```

```

1
2 //***** RARMQ 2D *****
3 // 操作 (l1,r1,l2,r2,v) 视为在时间 l1, 对 [l2,r2]+=v, 在时间 l2+1, 对 [l2,r2]-=v,
4 // 询问 (l1,r1,l2,r2) 视为将只考虑时间 [l1,l2], [l2,r2] 上的历史最大值
5 // 每个询问套在线段树分治区间 [l,r], 分成 [l,mid],[mid+1,r] 两段询问
6 // 总复杂度 O(nlog^2n+qlogn)
7 #include<bits/stdc++.h>
8 #define maxn 500050
9 using namespace std;
10 typedef long long LL;
11
12 const LL N=65536,M=1e10,inf=1e18;
13
14 struct node {
15     LL mx,hmx;
16     LL tag,htag;
17 } T[N<<1];
18 #define mx(k) T[k].mx
19 #define tag(k) T[k].tag
20 #define hmx(k) T[k].hmx
21 #define htag(k) T[k].htag
22
23 void build(int k,int l,int r) {
24     mx(k)=hmx(k)=0;
25     tag(k)=htag(k)=0;
26     if (l==r) return ;
27     int mid=(l+r)>>1;
28     build(k<<1,l,mid);
29     build(k<<1|1,mid+1,r);
30 }
31
32 void renew(int k,LL h,LL d) {
33     h=max(h,0LL);
34     htag(k)=max(htag(k),tag(k)+h);
35     tag(k)+=d;
36     hmx(k)=max(hmx(k),mx(k)+h);
37     mx(k)+=d;
38 }
39 void godown(int k) {
40     renew(k<<1,htag(k),tag(k));
41     renew(k<<1|1,htag(k),tag(k));
42     htag(k)=tag(k)=0;
43 }
44 void update(int k) {
45     mx(k)=max(mx(k<<1),mx(k<<1|1));
46     hmx(k)=max(hmx(k<<1),hmx(k<<1|1));
47 }
48
49 void change(int k,int l,int r,int a,int b,LL d) {
50     if (a<=l&&r<=b)
51         renew(k,d,d);
52     else {

```

```

53         godown(k);
54         int mid=(l+r)>>1;
55         if (a<=mid)
56             change(k<<1,l,mid,a,b,d);
57         if (b>mid)
58             change(k<<1|1,mid+1,r,a,b,d);
59         update(k);
60     }
61 }
62
63 LL query(int k,int l,int r,int a,int b) {
64     if (a>r||l>b) return -inf;
65     if (a<=l&&r<=b) return hmx(k);
66     godown(k);
67     int mid=(l+r)>>1;
68     return max(query(k<<1,l,mid,a,b),query(k<<1|1,mid+1,r,a,b));
69 }
70
71 struct seg {
72     int l,r,x;
73     bool operator < (const seg& s) const {
74         return x<s.x;
75     }
76 };
77 vector<seg> L[maxn],R[maxn];
78
79 int n,m,q;
80 int Log2(int x) { return !x?-1:Log2(x>>1)+1; }
81 LL ans[maxn];
82 vector<pair<seg,int>> LQ[maxn],RQ[maxn];
83 void Max(LL &a,LL b) { a=max(a,b); }
84
85 int main() {
86     scanf("%d%d%d",&n,&m,&q);
87     while (m--) {
88         int l1,l2,r1,r2,x;
89         scanf("%d%d%d%d",&l1,&l2,&r1,&r2,&x);
90         L[l1].push_back(seg{l2,r2,x});
91         R[r1].push_back(seg{l2,r2,x});
92     }
93     for (int i=0;i<q;++i) {
94         int l1,l2,r1,r2;
95         scanf("%d%d%d%d",&l1,&l2,&r1,&r2);
96         int t=max(Log2(l1~r1),0);
97         LQ[t].emplace_back(seg{l2,r2,r1},i);
98         RQ[t].emplace_back(seg{l2,r2,l1},i);
99     }
100
101     for (int t=0;(1<<t)<=N;++t) {
102         build(1,0,N-1);
103         LL cnt=0,sum=0,ALL=(1<<t)-1;
104

```



```

105     sort(LQ[t].begin(),LQ[t].end());
106     for (int i=0,j=0;i<N;++i) {
107         for (seg s:L[i])
108             change(1,0,N-1,s.l,s.r,s.x),++cnt;
109         for (;j<LQ[t].size()&&LQ[t][j].first.x==i;++j)
110             Max(ans[LQ[t][j].second],query(1,0,N-1,LQ[t][j].first.l,LQ[t][j].first.r)-sum*M);
111         for (seg s:R[i])
112             change(1,0,N-1,s.l,s.r,-s.x),++cnt;
113         if (ALL&i) continue;
114         change(1,0,N-1,0,N-1,cnt*M);
115         sum+=cnt,cnt=0;
116     }
117
118     build(1,0,N-1),sum=cnt=0;
119     sort(RQ[t].rbegin(),RQ[t].rend());
120     for (int i=N-1,j=0;i>=0;--i) {

```

```

121         for (seg s:R[i])
122             change(1,0,N-1,s.l,s.r,s.x),++cnt;
123         for (;j<RQ[t].size()&&RQ[t][j].first.x==i;++j)
124             Max(ans[RQ[t][j].second],query(1,0,N-1,RQ[t][j].first.l,RQ[t][j].first.r)-sum*M);
125         for (seg s:L[i])
126             change(1,0,N-1,s.l,s.r,-s.x),++cnt;
127         if (ALL&i) continue;
128         change(1,0,N-1,0,N-1,cnt*M);
129         sum+=cnt,cnt=0;
130     }
131 }
132 for (int i=0;i<q;++i) printf("%lld\n",ans[i]);
133 return 0;
134 }

```