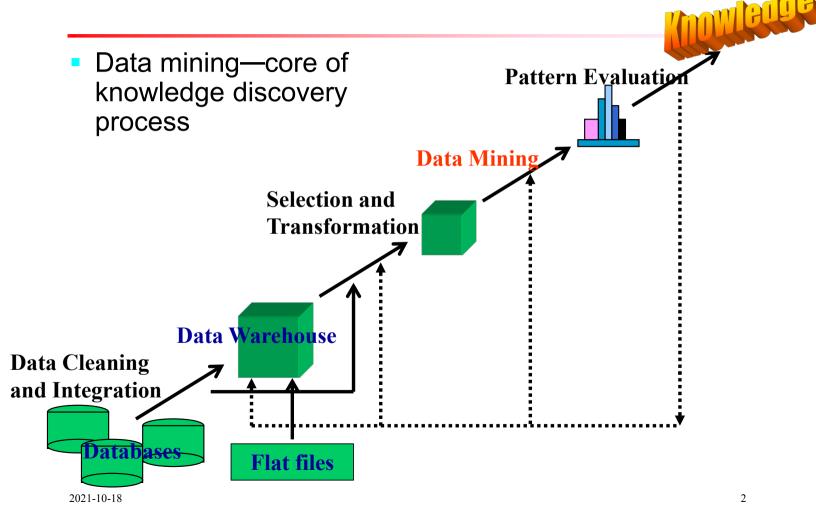
Data Mining

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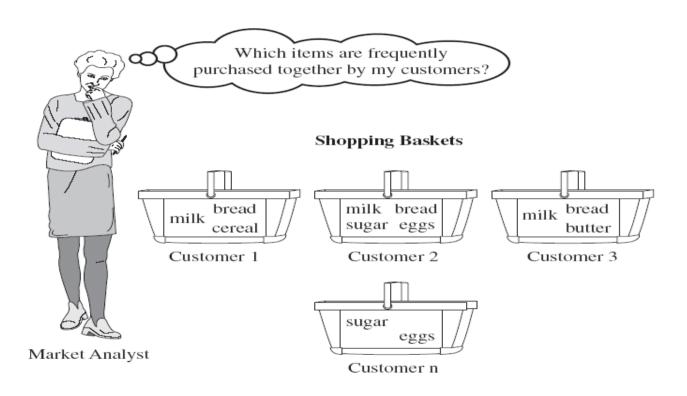
Review



Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

Market Basket Analysis



What Is Association Rules Mining?

Association rules mining

 Finding frequent patterns, associations among sets of items or objects in transaction databases, relational databases, and other information repositories.

Examples

- What products were often purchased together? Beer and diapers?!
- What DNA segments often occur together in DNA sequences?

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What Is Association Rules Mining?

- Where does the data come from?
 - supermarket transactions, e-commerce orders/shopping lists, membership cards
- Applications
 - Basket data analysis
 - Cross-marketing 捆绑锅货
 - Catalog design 日本设计

 - Web log (click stream) analysis
 - DNA sequence analysis

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Basic Concepts

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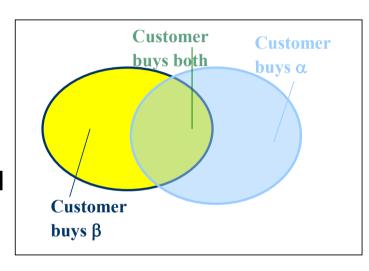
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Transaction-id	Items bought	
10	A, B, D	
20	A, C, D	
30	A, D, E	
40	B, E, F	
50	B, C, D, E, F	

- Item collection $X = \{x_1, ..., x_m\}$ Itemset: a set of items, *k*-itemset
- Transaction T ⊆ X, each T associates a unique Tid and items bought by a customer
- Rule form $\alpha => \beta$, $\alpha \subset X$, $\beta \subset X$, $\alpha \cap \beta = \emptyset$

Basic Concepts

- support, s, probability that a transaction contains α and β
 - support $(\alpha => \beta) = P(\alpha \cap \beta)$
- Frequent itemset, occurrence greater than a min_support
- Frequent itemset mining, find all the rules α => β satisfying min_support
- Let sup_{min} = 50%, frequent Itemsets {A:3, B:3, D:4, E:3, AD:3}
 support (A) = 3/5 = 60%, support (AD) = 3/5 = 60%



Basic Concepts

• confidence, c, conditional probability that a transaction having α also contains β

Confidence(
$$\alpha => \beta$$
) = P($\beta \mid \alpha$) = $\frac{P(\alpha \cap \beta)}{P(\alpha)}$ = $\frac{\text{count}(\alpha \cap \beta)}{\text{count}(\alpha)}$

- Measure of rule interestingness
- Rules satisfy min_support and min_confidence are strong
- Let sup_{min} = 50%, conf_{min} = 50%, frequent itemsets {A:3, B:3, D:4, E:3, AD:3} Association rules:

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Interestingness Measure: Correlations (Lift)

- play basketball ⇒ eat cereal [40%, 66.7%] is misleading
 - The overall % of students eating cereal is 75% > 66.7%.
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cap B)}{P(A)P(B)}$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B,C) = \frac{2000 / 5000}{3000 / 5000 * 3750 / 5000} = 0.89 \quad lift(B,\neg C) = \frac{1000 / 5000}{3000 / 5000 * 1250 / 5000} = 1.33$$

关联规则挖掘

Association Rule Mining: A Road Map

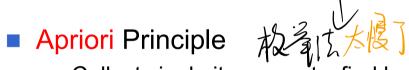
- Boolean vs. quantitative associations (based on the types of values handled)
 - Boolean association rules, only concern presence or absence of items, buys(x, "SQLServer") buys(x, "DMBook") => buys(x, "DBMiner") [0.2%, 60%]
 - Quantitative association rules, concern quantitative attributes, age(x, "30...39") ^ income(x, "42...48K") => buys(x, "high resolution TV") [1%, 75%]
- Single level vs. multiple-level analysis (based on the levels of abstraction involved)
 - age(x, "30...39") => buys(x, "laptop computer")
 - age(x, "30...39") => buys(x, "computer")
- Single dimension vs. multiple dimensional associations (based on dimensions involved)

Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

Handling Exponential Complexity

- Given *n* transactions and *m* different items:
 - Number of possible association rules: $O(2^m)$
 - Computation complexity: $O(nm2^m)$



- Collect single item counts, find large items
- Find candidate pairs, count them => large pairs of items
- Find candidate triplets, count them => large triplets of items, And so on...
- Guiding Principle: Every subset of a frequent itemset has to be frequent
 - Used for pruning many candidates 首大物等版的子前でしまる。

Apriori: A Candidate Generation-and-Test Approach

- Apriori uses prior knowledge of frequent itemsets
- Iterative approach, level-wise search
- The Apriori property (downward closure property, antimonotone) of frequent patterns
 - Any subset of a frequent itemset must be frequent
 - If any itemset is infrequent, its superset should not be generated/tested
 - If {beer, diaper, nuts} is frequent, so is {beer, diaper}, every transaction having {beer, diaper, nuts} also contains {beer, diaper}
 - If {beer, diaper} is infrequent, {beer, diaper, nut} cannot be frequent at all

Apriori: A Candidate Generation-and-Test Approach

Method:

- Initially, scan DB once to get frequent 1-itemset
- Generate length (k+1) candidate itemsets from length k frequent itemsets ② Jenerate frequent 2-itemset
- Test the candidates against DB
- Terminate when no frequent or candidate set can be generated

Apriori Algorithm — An Example Sup_{min} = 2 Itemset Sup

Database D

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

	Itemset	sup
$L_1 \mid$	{A}	2
	{B}	3
	{C}	3
	{E}	3

_		
L_2	Itemset	sup
	{A, C}	2
	{B, C}	2
	{B, E}	3
	{C, E}	2

C_2	Itemset	sup
_	{A, B}	1
	{A, C}	2
	{A, E}	1
	{B, C}	2
	{B, E}	3
	{C, E}	2

 C_2 **Itemset** 2nd scan {A, B} {A, C} {A, E} {B, C} {B, E} {C, E}

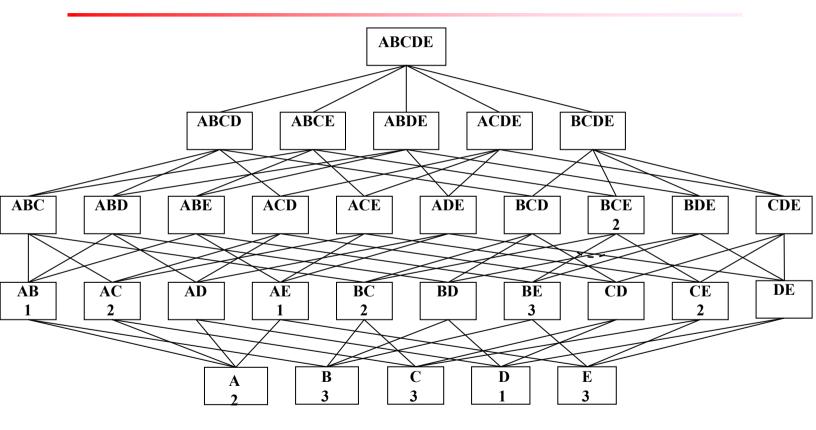
C_3	Itemse	
2021-10-18	{B, C, E}	

3 rd	scan ³
	•

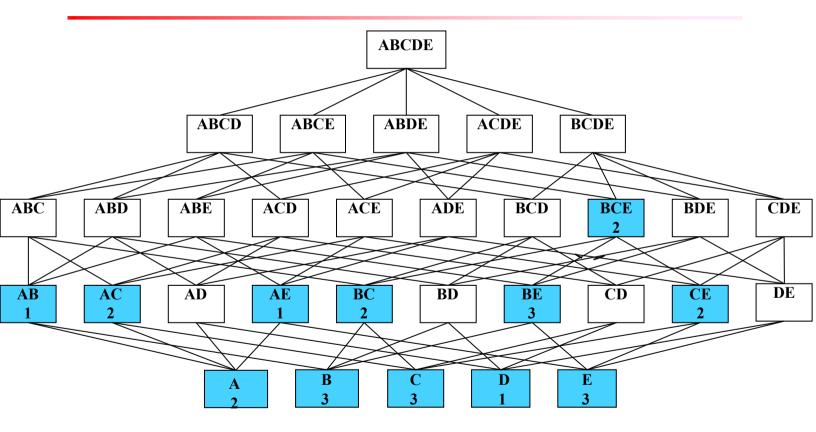
Itemset	sup
{B, C, E}	2

Itemset	sup
{B, C, E}	2

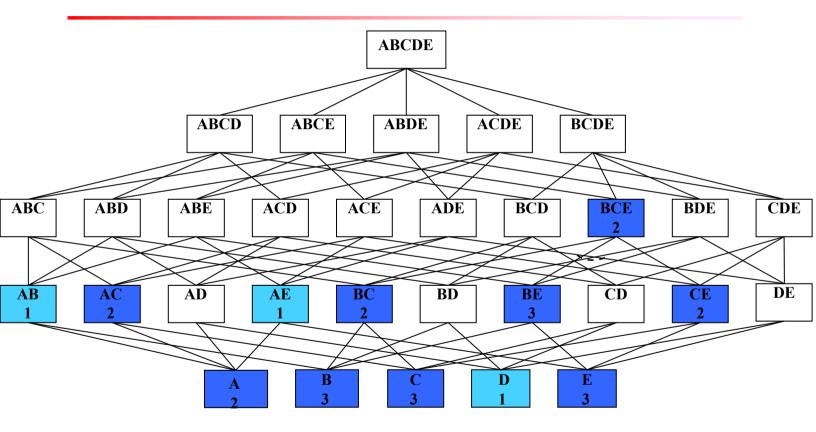
Apriori Algorithm — An Example



Apriori Algorithm — An Example



Apriori Algorithm — An Example



Apriori Algorithm

return $L = \bigcup_{k} L_{k}$;

Pseudo-code C_{k} : Candidate itemset of size k L_{k} : frequent itemset of size k Input: Database D, min sup Output: frequent itemsets L L_1 = {frequent single items from D}; for $(k = 2; L_{k-1} != \emptyset; k++)$ do begin C_k = candidates generated from L_{k-1} ; for each transaction $t \in D$ do increment the count of all candidates in C_k which are contained in t end L_k = candidates in C_k with min support end

How to Generate Candidates?

- How to generate candidates?
 - Step 1: self-joining L_k
 - Step 2: pruning
- Example
 - L₃={abc, abd, acd, ace, bcd}
 - Self-joining: L₃*L₃
 - abc and abd -> abcd, acd and ace -> acde
 - Pruning:
 - acde is pruned because ade is not in L₃
 - C₄={abcd}

How to Generate Candidates?

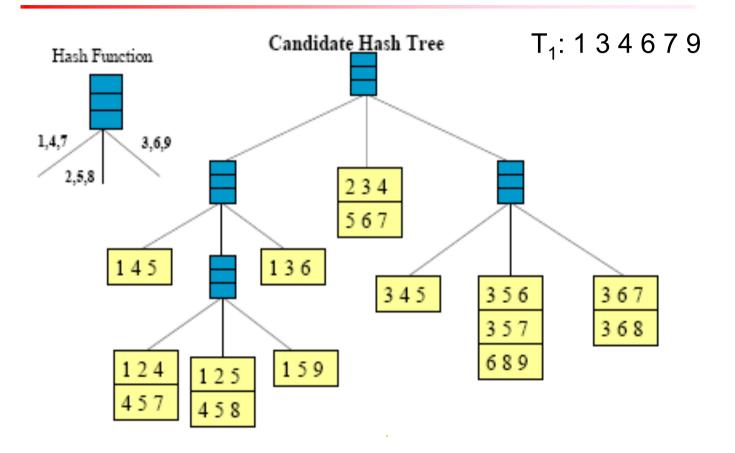
- Suppose the items in L_{k-1} are listed in order
- Step 1: self-joining L_{k-1} for each itemset $I_1 \in L_{k-1}$ for each itemset $I_2 \in L_{k-1}$ if $(I_1[1]=I_2[1])^*(I_1[2]=I_2[2])^*...^*(I_1[k-2]=I_2[k-2])$ then $c=I_1$ join I_2 pruning (c) end end
- Step 2: pruning

```
forall (k-1)-subsets s of c do
if (s is not in L_{k-1}) then delete c
```

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
 - The total number of candidates can be very huge
 - One transaction may contain many candidates
- Method:
 - Candidate itemsets are stored in a hash-tree
 - Leaf node of hash-tree contains a list of itemsets and counts
 - Interior node contains a hash table

Example: Counting Supports of Candidates



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Exercise

1. A database has 9 transactions. Let min_sup = 20%. Please present all the candidates and frequent itemsets at each iteration.

SII23	2	<u></u>
[I 1 1 2 75 I 2 73 ×	2	V
[3 [4]2 ×		
III X		

TID	List of items_IDs
T100	l1,l2,l5
T200	12,14
T300	12,13
T400	I1,I2 <mark>,</mark> I4
T500	I1,I3
T600	12,13
T700	I1, <mark>1</mark> 3
T800	11,12,1 <mark>3,</mark> 15
T900	11,12,13



Challenges of Frequent Pattern Mining

Challenges

- Multiple scans of transaction database
- Huge number of candidates
- Tedious workload of support counting for candidates
- Improving Apriori
 - Reduce passes of transaction database scans
 - Shrink number of candidates
 - Facilitate support counting of candidates

Pcc Partition: Scan Database Only Twice

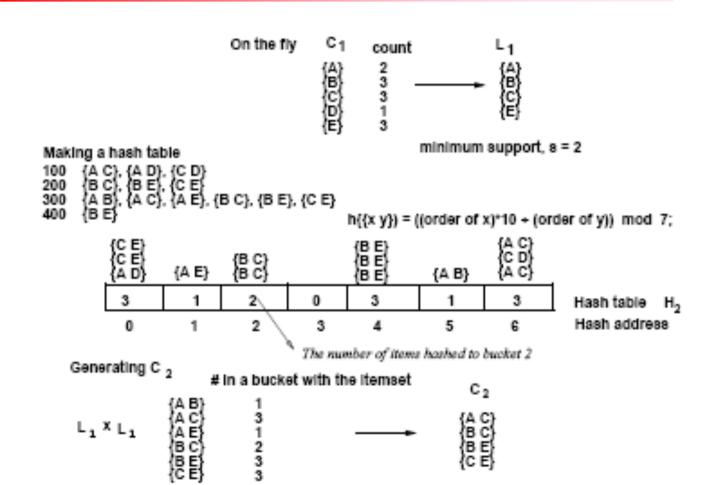
- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association in large databases. In VLDB'95.
- Partitioning technique $\frac{P(l)}{P(l)} = \frac{1}{P(l)}$
 - Partition the data into N small partitions
 - Phase 1: find local frequent itemsets on each data partition.
 Record all local frequent itemsets.
 - Phase 2: Integrate all local frequent itemsets, scan database, find global frequent itemsets.
- Correctness: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions

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Partition: Scan Database Only Twice

- Each partition can be fit into memory
- Scan database only twice! Reduce I/O cost!
- Execution time scales linearly
- Good for very large-scale database
- Applicable to parallel/distributed computing systems
 - Each processor performs FIM on its local data
 - Central server aggregates local frequent itemsets, broadcast potential global itemsets
 - Each processor scans local data to count the frequency
 - Central server aggregates the counts, find the global itemsets

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In SIGMOD'95
- Hash-based technique
 - When scanning transactions to generate frequent k-itemsets, L_k , generate all (k+1)-itemsets for each transaction
 - Hash all (k+1)-itemsets into buckets, increase bucket count
 - If a (k+1)-itemset bucket count is below min_sup , it must be removed from (k+1) candidate itemsets, C_{k+1}
- Correctness: A k-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent



Pros

- Reduce the number of candidates, C_k , especially for C_2 . Size of C_2 is usually huge, reduce C_2 is crucial
- Execution time scales linearly when varying the size of data

Comparison of time (T15.I4.D100)

	Apriori	DHP				
	number	number				
L_1	820	820				
C_2	335,790	338				
L_2	207	207				
C_3	618	618				
L_3	201	201				
C_4	184	184				
L_4	98	98				
C_{5}	30	30				
L_5	23	23				
C_6	1	1				
L_6	1	1				
total time	39.39	13.91				

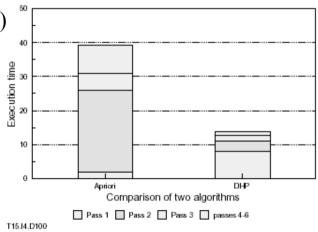
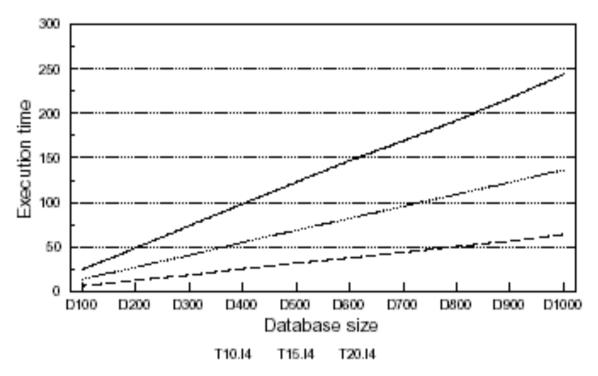


Figure 8: Execution time of Apriori and DHP

Comparison of time (T15.I4.D100)



Performance of DHP when increasing the size of database

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Cons

- Consume more memory, for hash table
- The larger the hash table, the smaller C_k and L_k

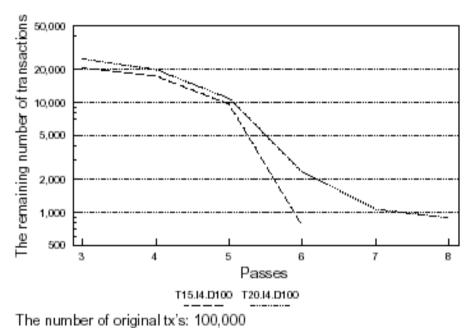
Results from varying hash table sizes (T10.I4.D100)

$ H_2 $	524,288	262,144	131,072	95,536	32,768
L_1	559	559	559	559	559
$ \{H_2 \geq s\} $	58	61.	75	96	182
C_2	81	120	199	394	1355
L_2	45	45	45	45	45
α	0.0314	0.0320	0.0345	0.0386	0.0545
size of D_3	498KB	$500 \mathrm{KB}$	507KB	539KB	603KB
$ D_3 $	19,732	19,741	19,755	20,501	21,607
total time	6.44	6.43	6.24	6.77	7.23

Transaction Reduction

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In SIGMOD'95
- Transaction reduction
 - When scanning transactions to generate frequent k-itemsets, L_k , mark the transaction that contains no k-candidate
 - Remove all the marked transaction
 - The number of transactions drops dramatically

Transaction Reduction



s=0.75%

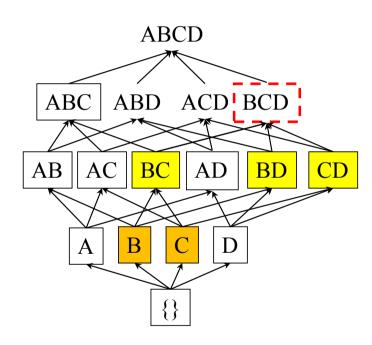
The remaining number of transaction in each pass

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DIC: Reduce Number of Scans

- S. Brin, R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In SIGMOD'97
- Sergey Brin, founder of Google!
- Partition database into blocks marked by starting points
- New candidate can be added at any starting point once all its subsets are determined frequent
- Reduce the number of database scans

DIC: Reduce Number of Scans



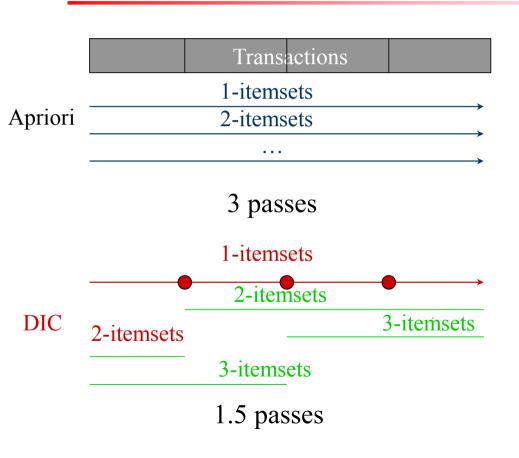
Itemset lattice

 Once both B and C are determined frequent, new candidate BC is added, the counting of BC begins at the next starting point

BC.BD.CD - 1238 tregrent, TBD B1323 BC

 Once all length-2 subsets of BCD are determined frequent, new candidate BCD is added, the counting of BCD begins at the next starting point

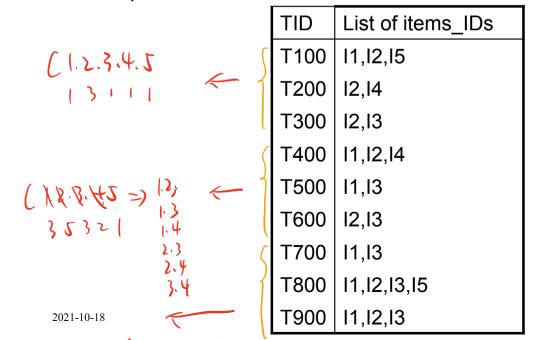
DIC: Reduce Number of Scans

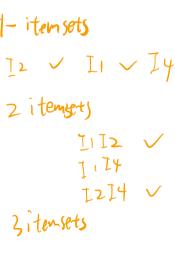


- Assume 40000 transactions, 4 partitions
- Begin counting 2-itemsets after the first 10000 have been read
- Begin counting 3-itemsets after the first 20000 have been read
- Scan database again, count 2 and 3-itemsets
- After 10000 transactions, finish counting 2-itemsets
- After 20000 transactions, finish counting 3-itemsets

Exercise

2. A database has 9 transactions. Let *min_sup* = 20%. Please present all the frequent itemsets generated by DIC in the first iteration. (Note: partition the data into 3 blocks)





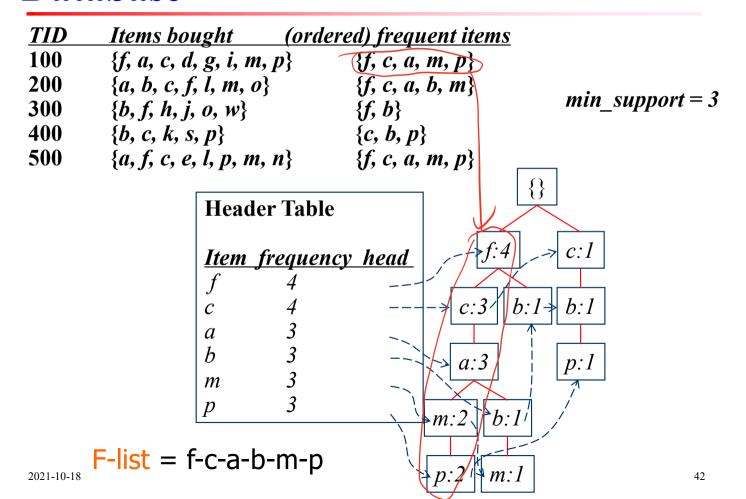
Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
 - To find frequent itemset i₁i₂...i₁₀₀
 - # of scans: 100
 - # of Candidates: $\binom{1}{100} + \binom{1}{100} + \dots + \binom{1}{1000} \binom{1}{000} = 2^{100} 1 = 1.27*10^{30}!$
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

Construct FP-tree from a Transaction Database

- Scan DB once, find frequent 1-itemset (single item pattern)
- Sort frequent items in frequency descending order L
- Create the root of the tree, labeled with "null"
- Scan DB again, sort each transaction in L order, a branch is created for each transaction
 - Increment the count of each node along a common prefix by 1
 - Create nodes for the items following the prefix
- Build a header table, connect each item point in the tree

Construct FP-tree from a Transaction Database



Construct FP-tree from a Transaction Database

- 1. Scan the transaction database D once. Collect F, the set of frequent items, and their support counts. Sort F in support count descending order as L, the *list* of frequent items.
- 2. Create the root of an FP-tree, and label it as "null." For each transaction *Trans* in *D* do the following:
 - Select and sort the frequent items in *Trans* according to the order of *L*. Let the sorted frequent item list in the *Trans* be [p|P], where p is the first element and P is the remaining list.
 - Call insert_tree ([p|P], T), which is performed as follows. If T has a child N such that N.item-name = p.item-name, then increment N's count by 1; else create a new node N, and let its count be 1, its parent link be linked to T, and its node-link to the nodes with the same item-name via the node-link structure.
 - If P is nonempty, call insert_tree(P, N) recursively.

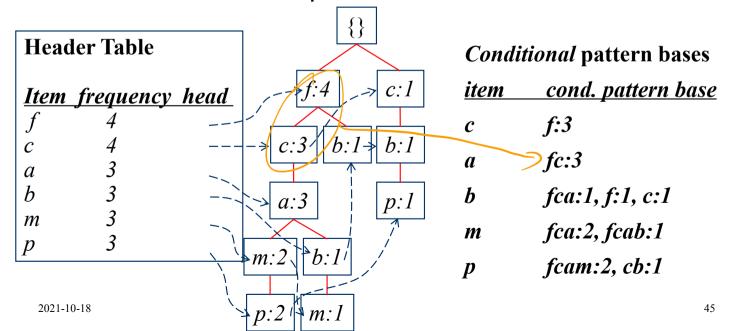
Benefits of the FP-tree Structure

- Completeness
 - Preserve complete information for frequent pattern mining
 - Never break a long pattern of any transaction
- Compactness 很为艺术,不能是整个珍珠在 可以将于一种直 Reduce irrelevant info—infrequent items are gone 在人内的

 - Items in frequency descending order: the more frequently occurring, the more likely to be shared
 - Never be larger than the original database

Construct Conditional Pattern Base

- Start at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item x
- Accumulate all of transformed prefix paths of item x into form x's conditional pattern base

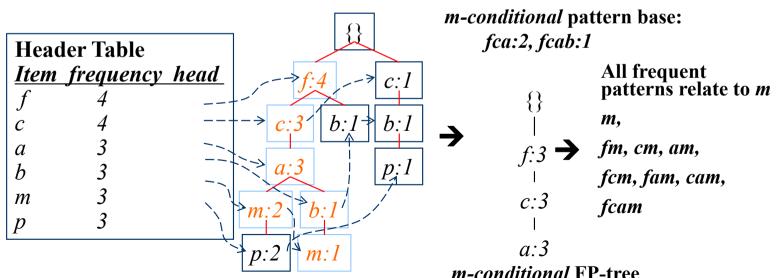


From Conditional Pattern-bases to Conditional FP-trees

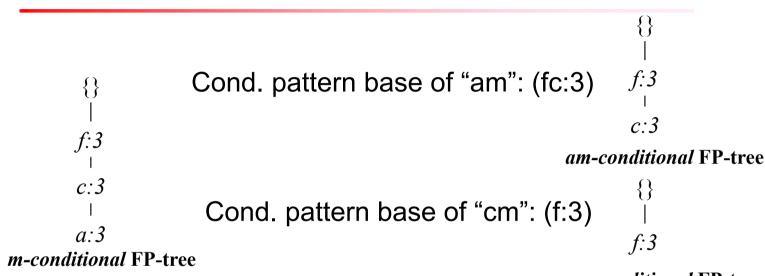
- For each pattern-base
 - Accumulate the count for each item in the base

-P-tree > (anditional P-b

 Construct the FP-tree for the frequent items of the pattern base



Recursion: Conditional FP-tree



cm-conditional FP-tree

Cond. pattern base of "cam": (f:3)

cam-conditional FP-tree

Mining Frequent Patterns With FPtrees

procedure **FP_growth**($Tree, \alpha$)

- (1) **if** *Tree* contains a single path *P* then
- (2) **for each** combination (denoted as β) of the nodes in the path *P*
- (3) generate pattern $\beta \cup \alpha$ with support_count = minimum support count of nodes in β;
- (4) **else for each** a_i in the header of *Tree* {
- (5) generate pattern $\beta = a_i \cup \alpha$ with support_count = a_i .support_count;
- (6) construct β 's conditional pattern base and then β 's conditional FP_tree $Tree_{\beta}$;
- (7) **if** $Tree_{\beta}$ then
- (8) call **FP_growth**($Tree_{\beta}$, β); }

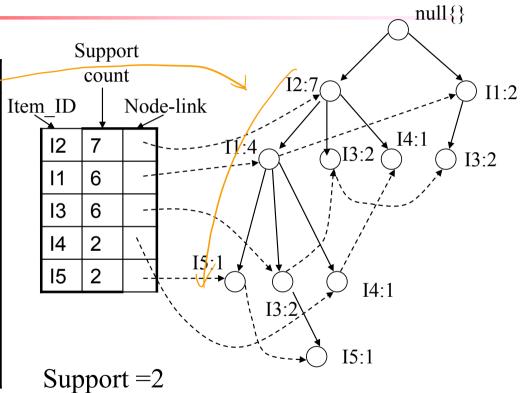
Exercise

3. A database has 9 transactions. Let *min_sup* = 20%. Please construct the FP-tree for the database, the conditional FP-trees, and all the frequent itemsets.

TID	List of items_IDs		
T100	11,12,15		
T200	12,14		
T300	12,13		
T400	11,12,14		
T500	I1,I3		
T600	12,13		
T700	I1,I3		
T800	11,12,13,15		
T900	11,12,13		

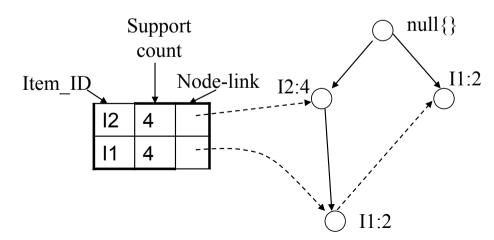
Solution

TID	List of items_IDs
T100	11,12,15
T200	12,14
T300	12,13
T400	11,12,14
T500	I1,I3
T600	12,13
T700	I1,I3
T800	11,12,13,15
T900	11,12,13

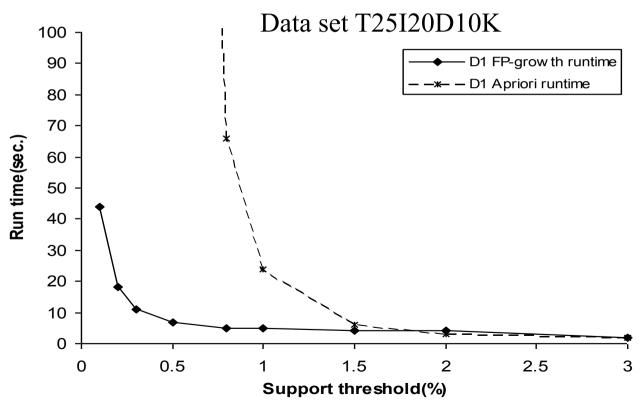


Solution

item	conditional pattern base	conditional FP-tree	frequent patterns generated
15	{{I2,I1: 1}, {I2,I1,I3: 1}}	⟨I2: 2, I1: 2⟩	{12,15: 2}, {11,15: 2}, {12,11,15: 2}
I4	{{I2,I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2,I4: 2}
13	{{I2,I1: 2}, {I2: 2}, {I1: 2}}	$\langle 12: 4, 11: 2 \rangle, \langle 11: 2 \rangle$	{I2,I3: 4}, {I1,I3: 4}, {I2,I1,I3: 2}
I1	{{I2: 4}}	⟨12: 4⟩	{I2,I1: 4}



FP-Growth vs. Apriori: Scalability With the Support Threshold



Why Is FP-Growth the Winner?

Divide-and-conquer:

- Decompose both the mining task and DB according to the frequent patterns obtained so far
- Focus searching on smaller databases

Other factors

- No candidate generation, no candidate test
- Compressed database: FP-tree structure
- Two scans of entire database
- Basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

Cons

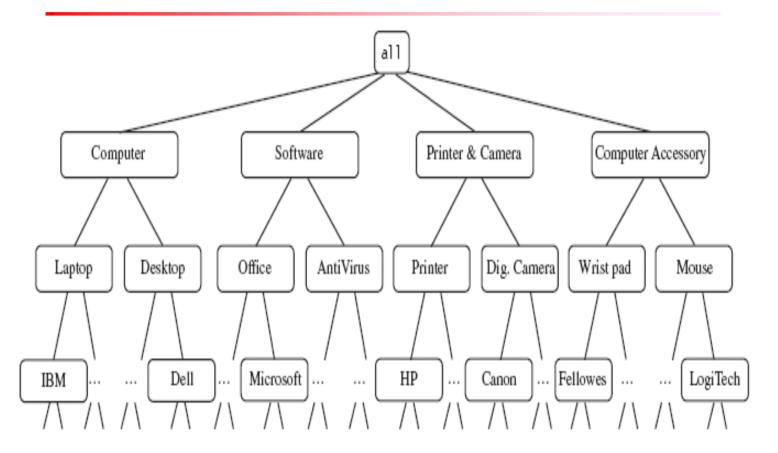
- Building FP-trees
 - A stack of FP-trees
- Redundant information
 - Transaction abcd appears in a-, ab-, abc-, ac-, c-FP-trees

Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

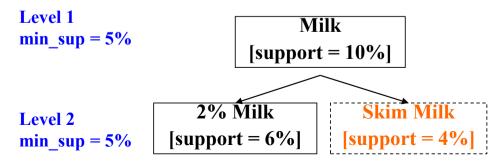
- Association rules at high concept levels may represent common sense knowledge
- Hard to find association rules at low concept level
 - Items at the lower level usually have lower support, less than min_support threshold
- Mining association rules at multiple levels of abstraction
- Example: sales in AllElectronics store computer sector

Example

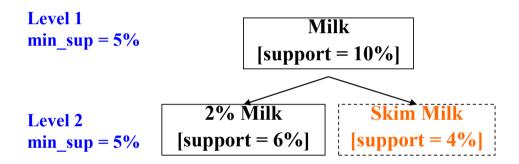


- Uniform support
 - Top-down, level-wise
 - Use uniform minimum support for each level
 - Perform Apriori at each level
 - Optimization: if an ancestor is infrequent, the search on the descendants can be avoided

uniform support



uniform support

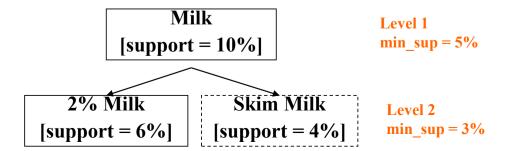


Drawbacks

- Miss interesting associations with too high threshold
- Generate too many uninteresting rules with too low threshold

- Reduced support
 - Top-down, level-wise
 - Each concept level has its own minimum support threshold
 - The lower level, the smaller threshold
 - Perform Apriori at each level

reduced support



Reduced support

- Optimization -- level-cross filtering by single item
 - An item at the *i*th concept level is examined *iff* its parent concept at the (*i*-1)th level is frequent
 - If a concept is infrequent, its descendents are pruned from the database
 - Drawbacks

2021-10-18

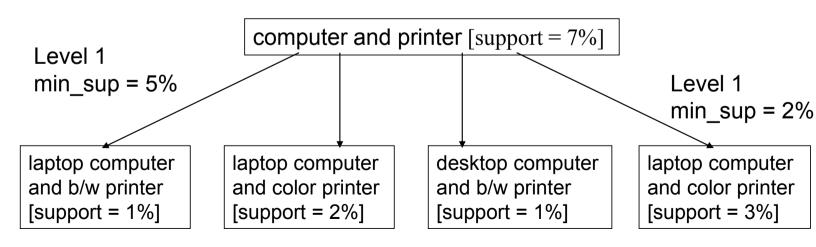
 Miss associations at low level items which are frequent based on a reduced min_support, but whose ancestors do not satisfy min_support
 reduced support

61

Milk [support = 10%]

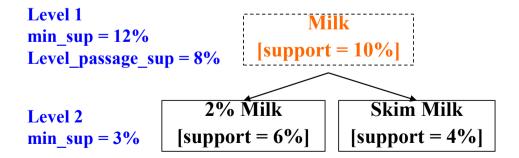
2% Milk Skim Milk Level 2
Not examined Not examined min_sup = 3%

- Reduced support
 - Optimization -- level-cross filtering by k-itemset
 - Only the children of frequent k-itemsets are examined
 - · Drawback: many valuable patterns may be filtered out



Reduced support

- Optimization -- Controlled level-cross filtering by single item
 - next level min sup < level passage threshold < min sup
 - Allow the children of items that do not satisfy the min_sup to be examined if they satisfy the level passage threshold



Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items
- Example
 - milk ⇒ wheat bread [support = 8%, confidence = 70%]
 - 2% milk ⇒ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule
- A rule is redundant if its support is close to the "expected" value, based on the rule's ancestor

Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

Mining Multi-Dimensional Association

Single-dimensional rules:

```
buys(X, "milk") ⇒ buys(X, "bread") 为特征转表位行务
```

■ Multi-dimensional rules: ≥ 2 dimensions or predicates

```
Inter-dimension assoc. rules (no repeated predicates)
age(X,"19-25") ∧ occupation(X,"student") ⇒ buys(X, "coke")
```

hybrid-dimension assoc. rules (*repeated predicates*)

age(X,"19-25") \(\) buys(X, "popcorn") \(\) buys(X, "coke")

- Categorical Attributes: finite number of possible values, no ordering among values
- Quantitative Attributes: numeric, implicit ordering among values — discretization, clustering approaches

Mining Quantitative Associations

- Techniques can be used to categorize numerical attributes
 - Static discretization based on predefined concept hierarchies
 - Dynamic discretization based on data distribution
 - Clustering: Distance-based association
 - one dimensional clustering then association

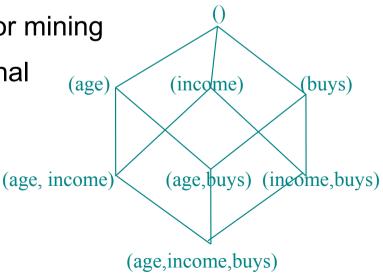
Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy
- Numeric values are replaced by ranges
- In relational database, finding all frequent k-predicate sets will require k or k+1 table scans

Data cube is well suited for mining

The cells of a n-dimensional cuboid correspond to the dimensions

Mining from data cubes can be much faster



Quantitative Association Rules



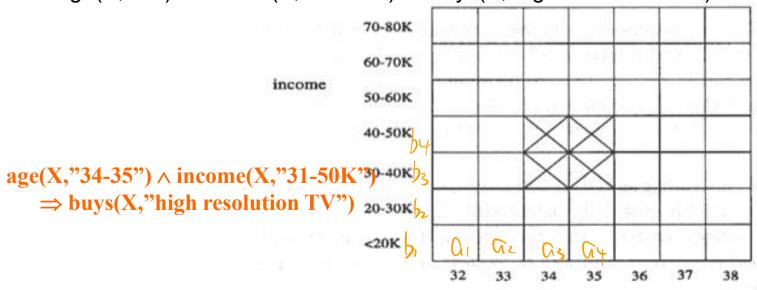
- Numeric attributes are dynamically discretized
 - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules: A_{quan1} ∧ A_{quan2} ⇒ A_{cat}
- Association rule clustering system (ARCS)
 - Binning: 2-D grid, manageable size
 - Finding frequent predicate sets: scan the database, count the support for each grid cell
 - Clustering the rules: cluster adjacent cells to form a rule

merge

Quantitative Association Rules

Example

```
age(X,"34") \land income(X,"31-40K") \Rightarrow buys(X,"high resolution TV") age(X,"35") \land income(X,"31-40K") \Rightarrow buys(X,"high resolution TV") age(X,"34") \land income(X,"41-50K") \Rightarrow buys(X,"high resolution TV") age(X,"35") \land income(X,"41-50K") \Rightarrow buys(X,"high resolution TV")
```



Mining Association Rules in Large Databases

- Basic concepts and a road map
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Summary

- Frequent pattern mining—an important task in data mining
- Scalable frequent pattern mining methods

Apriori (Candidate generation & test)

Partition, DIC, DHP, etc.

Projection-based (FP-growth)

Mining a variety of rules and interesting patterns