机器学习
Machine learning

Nonlinear Classifiers
练习题答案

授课人：周晓飞
zhouxiaofei@iie.ac.cn

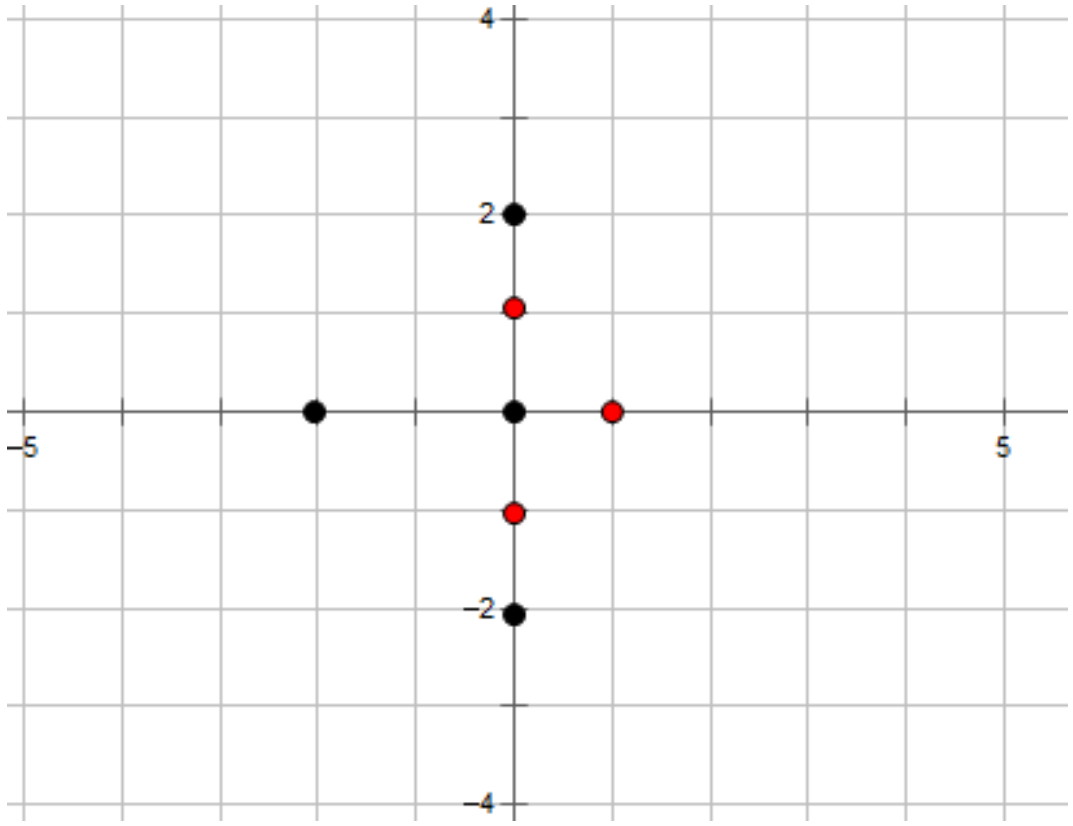
题目：有七个二维向量：$\omega_1 = \{(1,0),(0,1),(0,-1)\}$

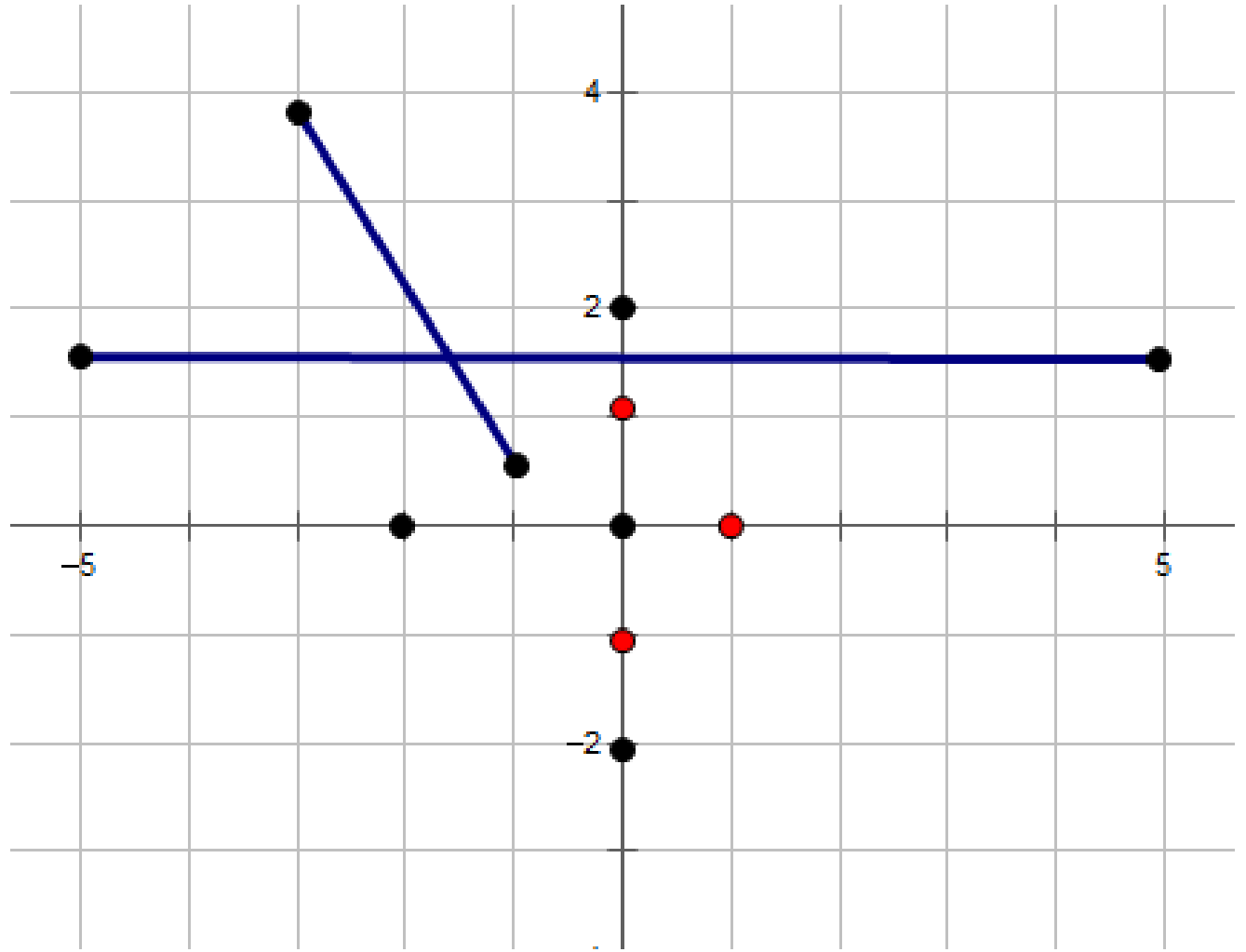
$\omega_2 = \{(0,0),(0,2),(0,-2),(-2,0)\}$

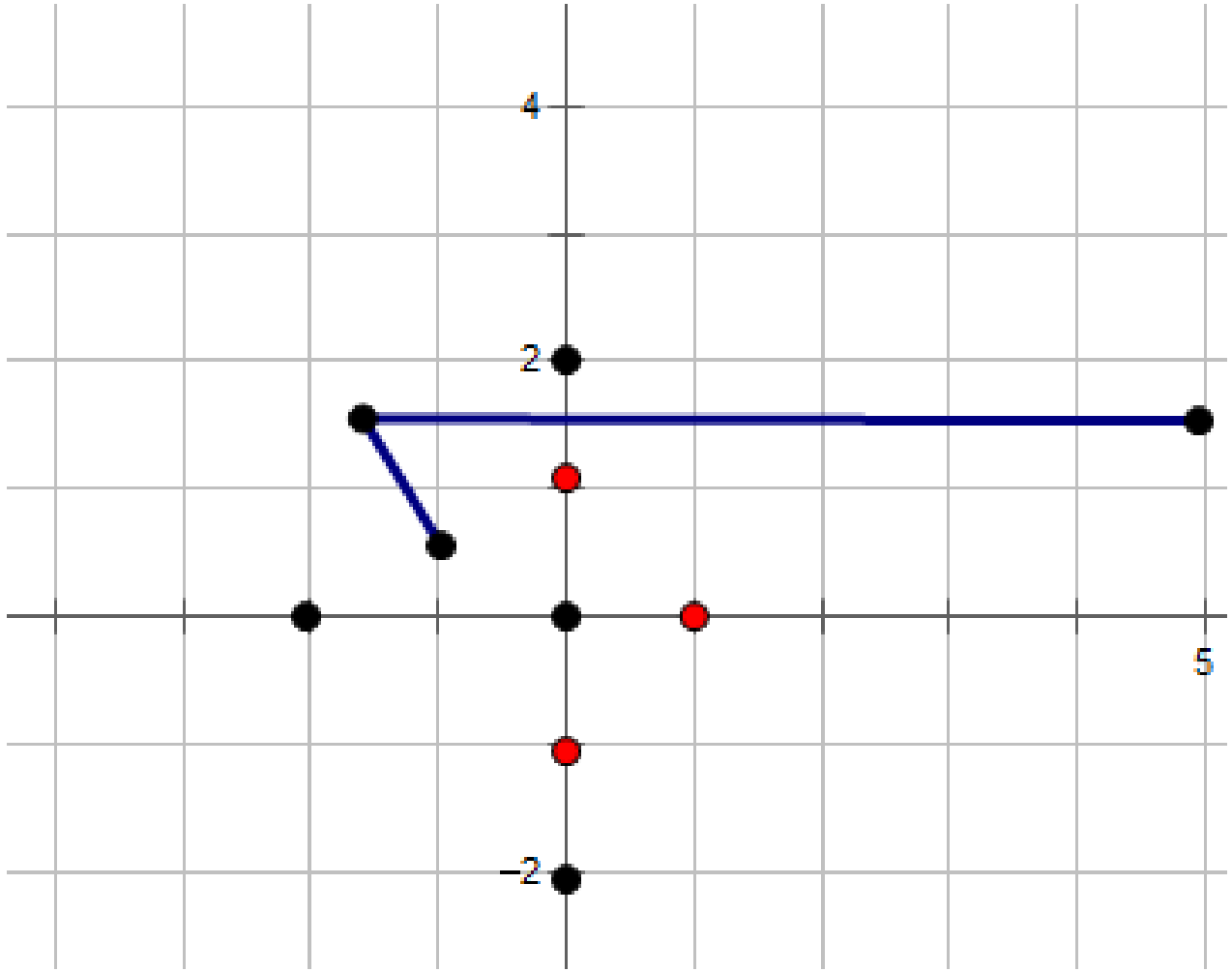
（1）画出最近邻法决策面；

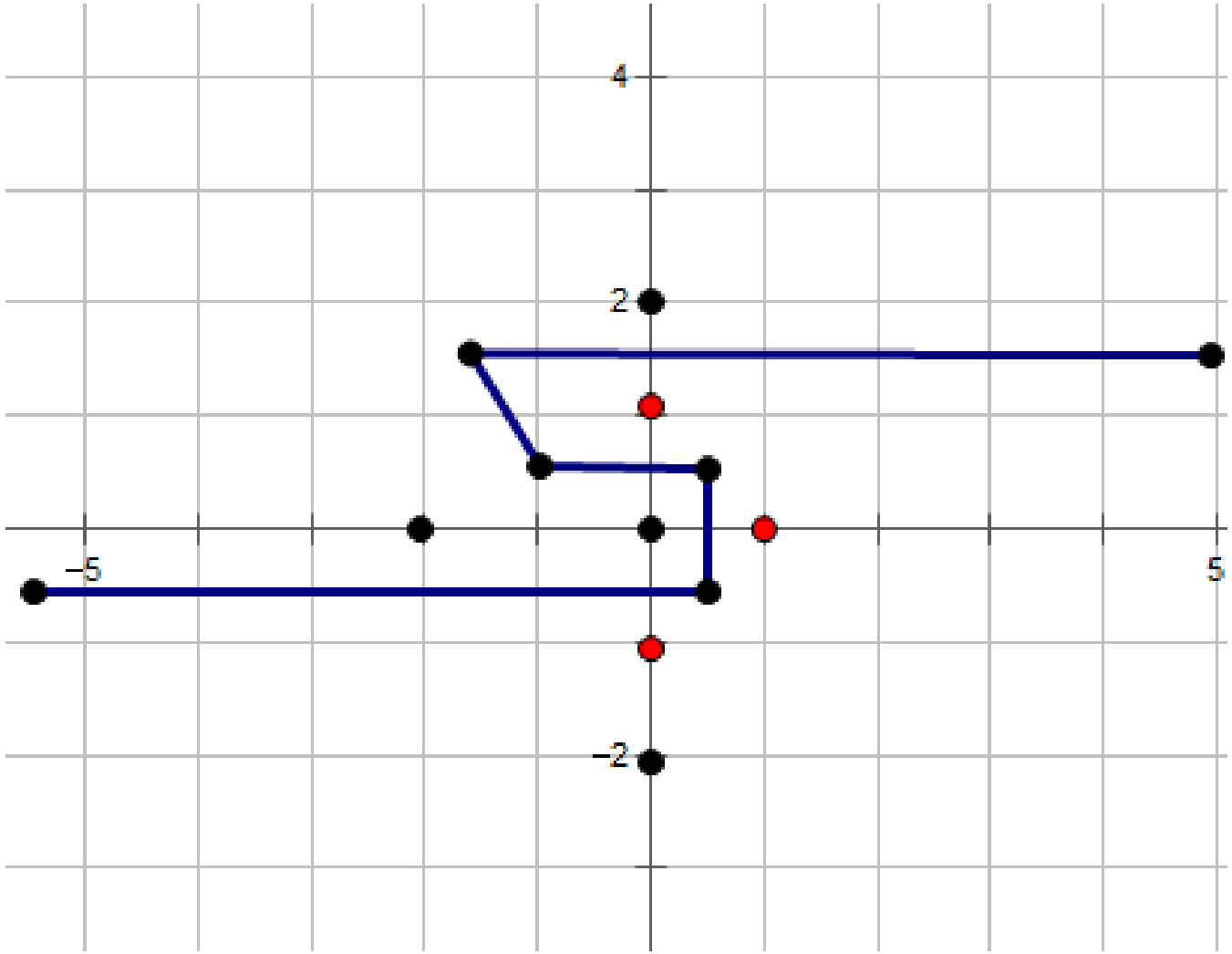
（2）求样本均值m_1, m_2，若按离样本均值距离的大小进行分类，试画出决策面。

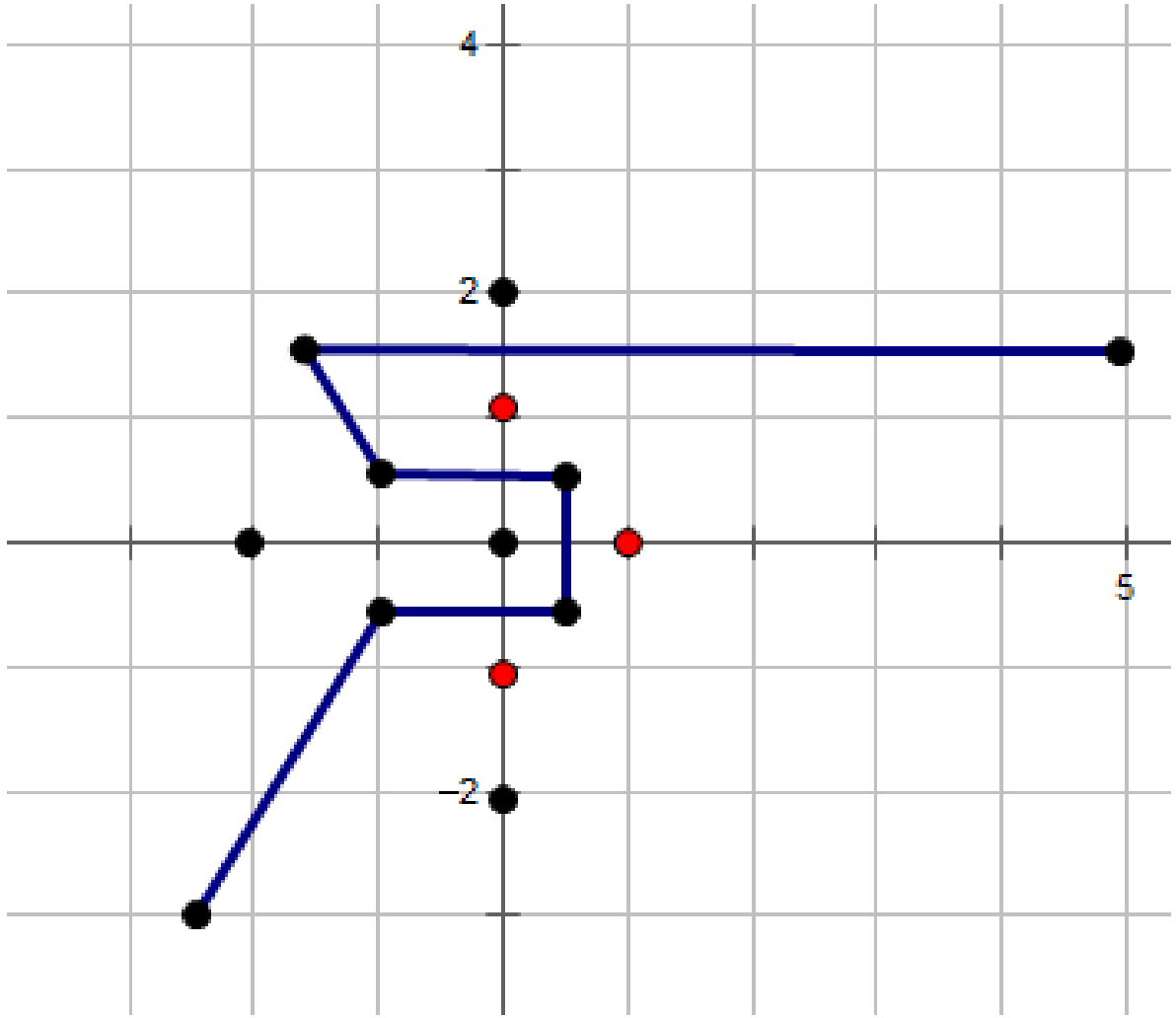
解（1）：红色为第一类点，黑色为第二类点：



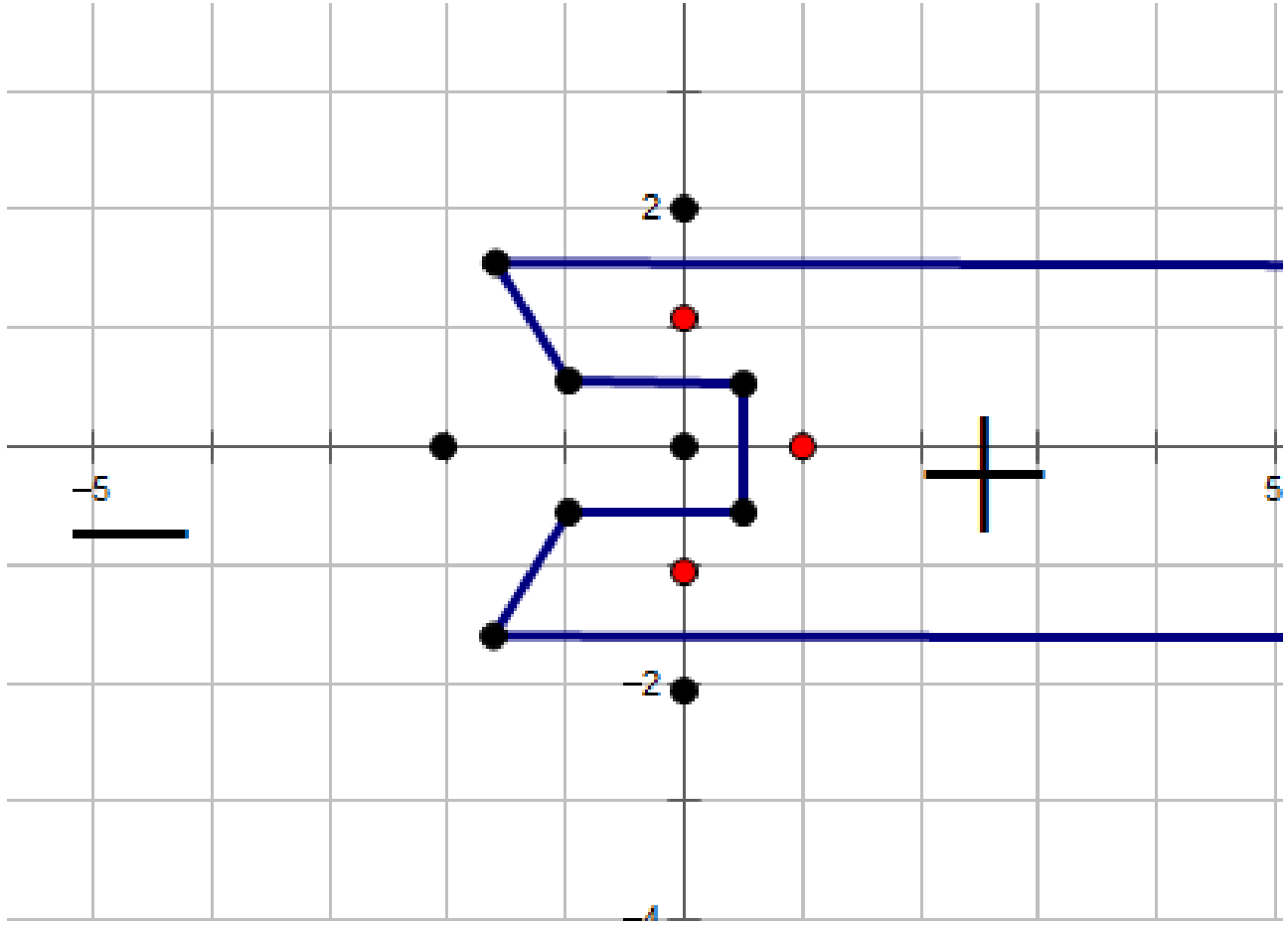

4
2
−2
−5
5


4
2
−2
5


4
2
−5
5
−2


4
2
5
−2

最近邻决策面：



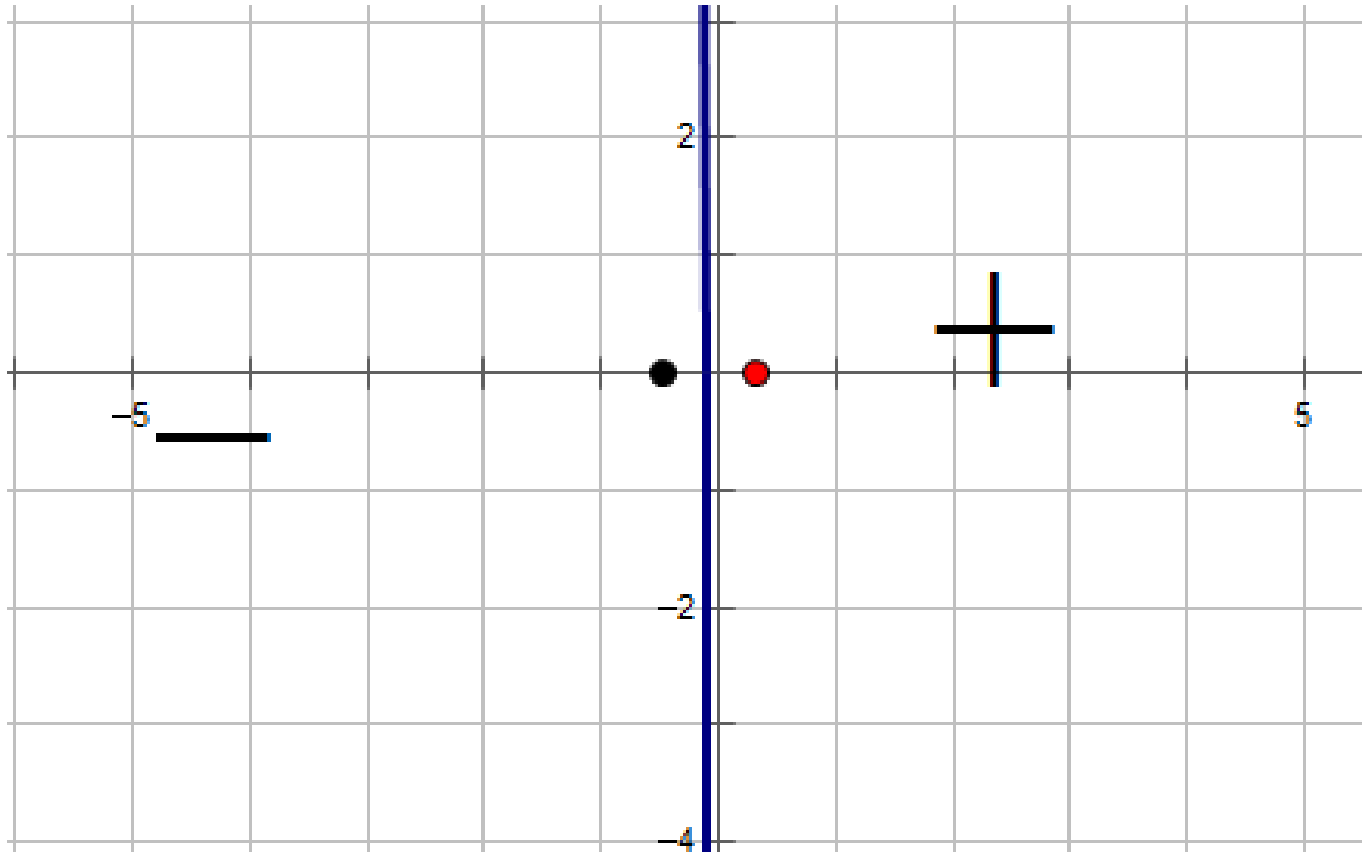
(2)样本均值为：

$$m_1 = \frac{1}{3}\sum x^{(1)} = (\frac{1}{3}, 0)$$

$$m_2 = \frac{1}{4}\sum x^{(2)} = (-\frac{1}{2}, 0)$$

决策面：

$$x = -\frac{1}{6}$$



题目：从输入空间到高维空间的映射是

$$x \in R \to y \in \phi(x) \in R^{2k+1}$$

其中，$\phi(x) = \left[\frac{1}{\sqrt{2}}, \cos x, \cos 2x, \ldots \cos kx, \sin x, \sin 2x, \ldots \sin kx \right]^T$

证明对应的内积核是

$$y_i^T y_j = K(x_i, x_j) = \frac{\sin\left(\left(k + \frac{1}{2}\right)(x_i - x_j)\right)}{2\sin\left(\frac{x_i - x_j}{2}\right)}$$

解：

$$K(x_i, x_j) = \frac{1}{2} + \sum_{m=1}^{k} (\cos x_i \cos x_j + \sin x_i \sin x_j)$$

$$= \frac{1}{2} + \sum_{m=1}^{k} \cos(x_i - x_j)$$

$$= \frac{1}{2} + \frac{2\sin(\frac{(x_i - x_j)}{2}) \sum_{m=1}^{k} \cos m(x_i - x_j)}{2\sin(\frac{(x_i - x_j)}{2})}$$

积化和差公式：

$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$

$$K(x_i,x_j)=\frac{1}{2}+\frac{2\sin(\frac{(x_i-x_j)}{2})\sum_{m=1}^{k}\cos m(x_i-x_j)}{2\sin(\frac{(x_i-x_j)}{2})}$$

$$=\frac{1}{2}+\frac{\sum_{m=1}^{k}(\sin(\frac{2m+1}{2}(x_i-x_j))-\sin(\frac{2m-1}{2}(x_i-x_j)))}{2\sin(\frac{(x_i-x_j)}{2})}$$

$$= \frac{1}{2} + \frac{\sin(\frac{2k+1}{2}(x_i - x_j)) - \sin(\frac{(x_i - x_j)}{2})}{2\sin(\frac{(x_i - x_j)}{2})}$$

$$= \frac{\sin(\frac{2k+1}{2}(x_i - x_j))}{2\sin(\frac{(x_i - x_j)}{2})}$$

证毕。