Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2222605

Optimal Strategy of Volatile Assets Based on Linear Programming

Summary

The decision to trade volatile assets is crucial for traders. Every trade in a volatile asset affects a large amount of money flowing. Sometimes a rise in an asset can lead to an influx of capital, and a fall can also lead to the withdrawal of a large amount of money. How to seek profit in the ups and downs is a very critical thing. If you can seize the opportunity, it may increase your capital hundreds or thousands of times.

We first built the observations part of the model, which collected data over a period of time in the past, including fluctuations such as interest rates and prices. The interest rate for each asset is then fitted with long-term and short-term data. The reason why we collect two kinds of data is to make short-term fluctuations in interest rates while ensuring long-term stable profits.

On this basis, we also set the expected value and loss threshold, because we want our model to make the maximum profit. But the truth is not as we thought, because no one knows the future price. We want to at least get the commission back, and if there's a bigger profit on top of that, we're going to raise the commission-only expectation to the current highest price. The loss threshold is a desired percentage, and when it is reached, we sell decisively. This not only avoids losing too much, but also sells at the fastest speed after rising to the top.

We also tried neural network forecasting and time series forecasting. After getting the forecast results, let's not talk about how to deal with the forecast and the fact if the trading strategy is specified. And the result is as expected, not as stable as our model. Our model avoids catastrophic losses in all cases, and because no one is vying with us to buy, we can smoothly earn profits in the upside area and become the best strategy.

Keywords: Mathematical Expectation, Linear Regression, Loss Threshold, Least Squares

Content

I. Introduction

1.1 Problem Background

In today's society, large numbers of investors and market traders buy and sell volatile assets and futures with the goal of maximizing their total returns, taking into account commissions and price fluctuations. However, in practice, due to the difficulty of judging the daily price rise and fall, the unpredictable future trend and the existence of commissions in transactions, it is difficult to maximize returns during the investment process, and even often lose money. In order to achieve this goal, we need to establish a mathematical model that can evaluate the future ups and downs of a volatile asset or futures, so as to give corresponding trading strategies.

1.2 Restatement of the Problem

The daily prices of gold and bitcoin are now known for the five years from November 9, 2016 to October 9, 2021. Traders need to start at $1,000 and use the five-year trading period to maximize returns on their investments. On each trading day, traders will have a portfolio consisting of cash, gold and bitcoin.The commission cost per transaction is α. Assume α_{gold} = 1% and $\alpha_{bitcoin}$ = 2%. There is no cost to holding assets. Gold only trades on days when the market is opening, which will be reflected in the pricing data file. The model should take into account this trading schedule. The following problems need to be solved.

- Develop a model that gives the best daily trading strategy based only on today's price data. And calculate the total assets in hand on October 9, 2021.
- Provide evidence that the model provides the best strategy.

- Determine the sensitivity of the strategy to transaction costs and calculate how transaction costs affect the strategy and results.
- Write a maximum of two-page memo explaining strategy, model and results to the trader.

1.3 Literature Review

Since this century, a large number of scholars have devoted themselves to the research of stock price prediction and quantitative trading strategy. Their work provides a wealth of data and mathematical models for both questions.

In the past research center of stock price prediction, scholars mainly use the ARIMA model or neural network model for short-term stock price prediction. Wu Yuxia et al. (2016)[1] used the ARIMA model to predict stock prices in the future. Peng Yan et al. (2019) [2]and Zhang Ni et al. (2021)[3] used LSTM recursive neural network to predict the stock price and stock return rate respectively. Xu Xingjun (2011)[4] used BP neural network system to judge the stock price trend. In addition, Akshit Kurani; Pavan Doshi; Aarya Vakharia; Manan Shah(2021) [5]examines various models applied to stock market forecasting. To sum up, the exploration of stock price prediction models has achieved preliminary results, and various emerging models emerge endlessly.

Although predecessors have made some progress in stock market forecasting, there are many influencing factors considered in their models, covering a comprehensive range. Whether it is applicable to this topic remains to be studied.

II. Problem analysis

2.1 Problem I

Since the issue stipulates that only the data in the pricing data files of Bitcoin and gold can be used for model development, previous prices have become the only reference for us to formulate trading strategies. Because the fluctuation range of Bitcoin's rise and fall is much greater than that of gold, and the price is also much higher than that of gold, we will be more inclined to trade Bitcoin when formulating strategies. Since the price of Bitcoin fluctuates greatly and can only be analyzed through data from the past, the conventional data prediction model is not applicable.Therefore, we decided to give up the prediction of its price, and establish a complete evaluation system based on the overall price change trend before the day and the price fluctuation in the short term, so as to determine the daily trading strategy.

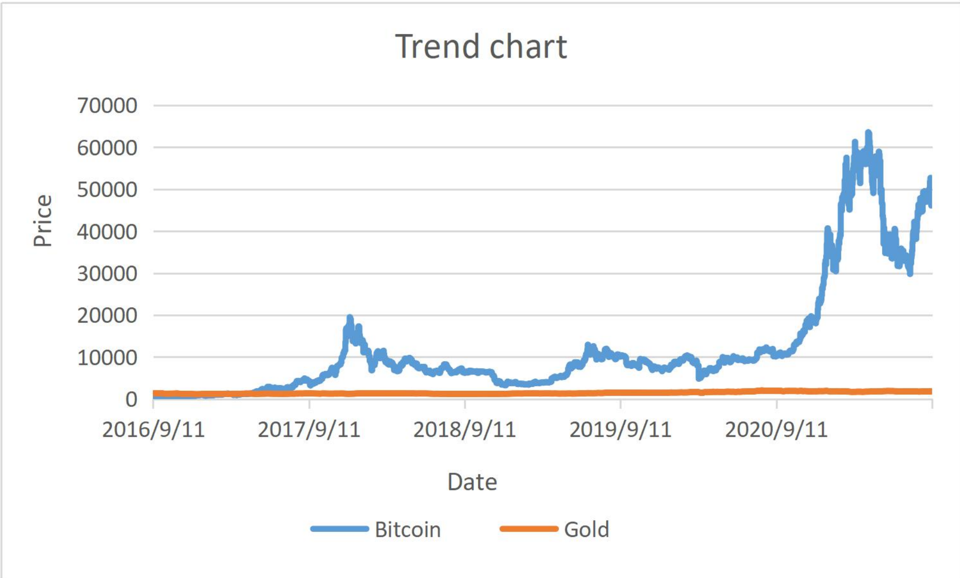

Figure 2-1: Gold vs Bitcoin Price Line Chart

2.2 Problem II

Considering the shareholder and market trading personnel on product risk, the yield of attention, as well as people's desire to buy at an undervalue and sell at a peak, we can calculate trading during the period of the model yields, according to the stability of the model of calculating the risk index, and show the policy suggestion of buying time and selling time, compared with people's expectations, illustrates its rationality. The superiority of this trading strategy is proved from the above perspectives.

2.3 Problem III

This question requires us to explore how sensitive the model is to transaction costs. The transaction costs in this question only include commissions for gold and bitcoin. Therefore, the model sensitivity can be analyzed in terms of commission and transaction times. We can use the method of controlling variables, keep other parameters unchanged, only change the commission price, observe the final income effect, and make a comprehensive evaluation of the sensitivity of the model in combination with the number of transactions.

2.4 Problem IV

This problem limited us to a two-page memo showing the results. Therefore, we need to extract important information from the conclusion of the first three questions, focus on the model, trading strategy and its use in the memo, and describe the sensitivity of this model to transaction costs. Finally, according to the results of the second question analysis, the superiority of this strategy and its application scenarios are illustrated.

III. Model

3.1 Basic Model

After our team identified the goal, we started to build mathematical models. After trying the time series prediction model and neural network prediction model, we established the following mathematical model according to the requirements of the topic and combined it with specific calculation and analysis. The model belongs to the decision model, which is divided into two parts.

The first part is to compare gold and bitcoin horizontally, to judge the risk of the two investments and the expected return effect, and then apply a certain weight to gold and bitcoin in subsequent investments according to the results. After determining the investment ratio of the two, it is necessary to determine the day's transaction based on the daily price and past price data.

The second part of the decision model is based on the long-term and short-term value growth rate, that is, the regression parameter of day i price divided by day i-1 price. Here we first set the long-term data days as γ and the short-term data days as δ. Their respective reference values, $\Theta 1$ (minimum acceptable growth in the long run) and 0(judging recent growth or decline) were assumed. At the same time, the concepts of transaction interest rate and transaction cost are also introduced in the decision-making model to further judge whether to trade on that day. An important assumption here is that we don't invest upfront, but wait for a certain number of data and then follow the model.

3.2 Terms, Definitions and Symbols

The signs and definitions are mostly generated from *queuing theory*.

Symbol	Meaning
F_n	The total value of the property owned, F_1 is the initial capital, which is $1000
α_1	Commissions on gold transactions
α_2	Commissions on bitcoin transactions
R_1	Number of gold transactions
R_2	Number of Bitcoin transactions
Ba_g	Actual value after completion of the first gold transaction (gold only)
Ba_b	Actual value after completion of the first Bitcoin transaction (Bitcoin only)
Rag_i	Growth rate of gold on day i compared to day i-1
avg_j	The average growth rate of gold in j days
Rab_i	Growth rate of Bitcoin on day i compared to day i-1
avb_j	The average growth rate of bitcoin over a period of j days
Gl_a	Regression parameters of the change rate of the overall value of gold
Bl_a	Regression parameters of the rate of change of the overall value of Bitcoin
$Gln(\tau)$	Regression parameters for the rate of change in gold value over the last τ days
$Bln(\tau)$	Regression parameters for Bitcoin value in the last τ days
H_g	Amount of gold held
H_d	Amount of cash held
H_b	Number of Bitcoin holdings
Vg_i	The price of gold on day i
Vb_i	The price of bitcoin on day i

Jg	The judgment value of gold value change, in which Jg_0 is the reference value (appreciation or depreciation)
Jb	Bitcoin value change judgment value, where Jb_0 is the reference value
Θ_1	long-term acceptable growth minimum
Θ_2	maximum acceptable loss
Θ_3	Expected earnings reference value
I_g	Gold trading interest rate
I_b	Bitcoin transaction rates
γ	Days of long-term data
δ	Days of short-term data

3.3 Assumptions

- Assume that the trader has little dependence on the principal during this period, that is, he doesn't need the principal for a short time.
- It is assumed that no economic disasters such as financial crises and systemic risks occur during this period.
- Assume all cash is spent on buying and all gold or Bitcoin is traded on selling.
- Assume not to buy any currency and keep cash in the early stages of the transaction. Wait for a certain amount of time before deciding whether to buy.

3.4 Model description

3.4.1 Reasons for choosing the value growth rate rather than the value itself in the regression parameter calculation

According to the data of the first 14 days, the first 30 days and the first 100 days, the law of value change and the corresponding change law of value growth rate are analyzed. Through data analysis, it can be concluded that the change of the value growth rate is relatively stable compared with the value itself, and through regression analysis, it can be seen that the fitted curve obtained by using the value growth rate covers more data points.

Report

	Value14	Value growth rate14	Value30	Value growth rate30	Value100	Value growth rate100
Mean	607.3057	.9977	608.7180	.9997	682.3959	1.002540445
N	14	14	30	30	100	100
Std. Deviation	6.92420	.00870	6.29647	.00745	63.77744	.0143703961
Std. Error of Mean	1.85057	.00233	1.14957	.00136	6.37774	.0014370396
Variance	47.945	.000	39.646	.000	4067.562	.000
Kurtosis	.864	1.581	.147	1.122	-1.512	6.452
Std. Error of Kurtosis	1.154	1.154	.833	.833	.478	.478
Skewness	-.206	.027	-.392	-.114	.115	.180
Std. Error of Skewness	.597	.597	.427	.427	.241	.241

Figure 3-1 Comparison of mathematical parameters of value and value growth rate

Coefficient Correlations[a]

Model			VAR00008
1	Correlations	VAR00008	1.000
	Covariances	VAR00008	3.388E-7

a. Dependent Variable:Value growth rate14

Coefficient Correlations[a]

Model			VAR00008
1	Correlations	VAR00008	1.000
	Covariances	VAR00008	2.401E-8

a. Dependent Variable: Value growth rate30

Coefficient Correlations[a]

Model			VAR00008
1	Correlations	VAR00008	1.000
	Covariances	VAR00008	.086

a. Dependent Variable: Value14

Coefficient Correlations[a]

Model			VAR00008
1	Correlations	VAR00008	1.000
	Covariances	VAR00008	.017

a. Dependent Variable: Value30

Figure 3-2 14-day, 30-day value and value appreciation rate regression fitting covariance comparison

3.4.2 Why no trades were made in the first few days?

The reason for not conducting transactions in the early stage is that the amount of data in the early stage is too small to accurately judge whether to buy or not. If the transaction is too frequent, a large number of assets will be lost due to handling fees, which will lead to less original capital in the early stage, and it will not be able to achieve a good price in the later stage. 's earnings. Therefore, we choose not to trade in the early stage and then decide whether to trade or not after the amount of data can be analyzed by the model.

Coefficient Correlations[a]

Model			VAR00008
1	Correlations	VAR00008	1.000
	Covariances	VAR00008	3.201E-8

a. Dependent Variable: Value growth rate100

Figure 3-3 100-day value growth rate regression fitting covariance

As can be seen from Figures 3-2 and 3-3, the covariance of value growth rate regression fitting within 14 days is 3.388E-7, that of value growth rate regression fitting within 30 days is 2.401E-8, and that of value growth rate regression fitting within 100 days is 3.201E-8. By comparing the three data, it can be found that the regression covariance of value growth rate at 30 days is the smallest, that is, the fitting degree is the highest. This is because too little or too many data will affect the degree of fitting, so it is necessary to calculate the most appropriate long-term data days, namely the value of γ.

3.4.3 Linear regression equation:

The average growth rate of gold in j days:

$$avg_j = \frac{\sum_1^j Rag_i}{j}$$

The average growth rate of bitcoin over a period of j days:

$$avb_j = \frac{\sum_1^j Rab_i}{j}$$

Gold regression parameter calculation formula:

$$5000 * \frac{\sum_1^j (i * Ragi) - j * avg_j * \frac{j+1}{2}}{\sum_1^j i^2 - j * \left(\frac{j+1}{2}\right)^2}$$

Bitcoin regression parameter calculation formula:

$$5000 * \frac{\sum_1^j (i * Rabi) - j * avb_j * \frac{j+1}{2}}{\sum_1^j i^2 - j * \left(\frac{j+1}{2}\right)^2}$$

Gold value change judgment value calculation:

$$Jg = Ba_g * l_g$$

Bitcoin value change judgment value calculation:

$$Jb = Ba_b * l_b$$

3.5 Model decision part

3.5.1 Determining the Investment Weight of Gold and Bitcoin

Data calculation: Collect the data of the first n days and calculate the changing trend of the value and value growth rate of gold and bitcoin and their stability degree respectively. Find out their long-term and short-term trends and combined with their actual value to determine the specific investment ratio.

3.5.2 Day Trading Strategies

Data calculation: The main judgment basis of the trading strategy of the day is the long-term and short-term change trend, and the optimal value of long-term days and short-term days is obtained through an iterative algorithm and then entered into the model. The overall trend of the last γ days and the short-term change trend of the last δ days are calculated by the regression algorithm.

The calculation of the expected return is introduced into the model, and a standard value of the transaction is initially set, that is, the value after the commission is deducted after the first transaction. After each transaction, it is guaranteed that the expected income effect can be achieved after the commission is removed, and if the expected value can be met, the transaction will be conducted. At the same time, the reference values Θ1, Θ2, and Θ3 are set, which are the minimum acceptable growth in the long term, the maximum acceptable loss, and the expected profit. The introduction of these parameters can reduce the risk, avoid catastrophic losses, and ensure that the income achieves the expected effect after the commission is removed.

3.6 Judgment of trading strategy

3.6.1When the cash held is zero

if the price of the currency held on the day is greater than the effect of the expected return and the overall value is in a state of appreciation, and the degree of decline in recent days has exceeded the maximum acceptable loss, that is, Θ2, then decisively choose to sell your currency holdings. If the price of the currency held on the day is lower than the expected return value but the overall value is in a state of appreciation, it is judged whether the loss after the transaction exceeds the acceptable maximum loss limit, and if it exceeds, choose to sell all the currencies held. If the price of the currency held on the day is greater than the effect of the expected return, but the overall depreciation process is in process, then decisively sell all the currency held. If the price of the currency held on the day is less than the expected income effect and the overall value is still in a state of depreciation, it is also chosen to sell all the currency held decisively.

3.6.2 When all assets owned are cash

if the long-term data of investment currency determined according to the first decision model is greater than the reference value of expected return, that is, Θ3. Then all the cash will be purchased according to the investment method determined by the first decision-making model.

In any other case, choose not to trade.

3.7 Calculation of the data in the model

3.7.1. Long-term data days γ :

In order to judge the trend of long-term changes in the currency, it is necessary to analyze the data of the previous γ days. We chose the initial value 15, that is, we took the regression parameter according to the price data of the first 15 days for the first time, and then iterated through 35 values to calculate the maximum profit value corresponding to the period from 15 to 49 days, and drew the line graph as follows:

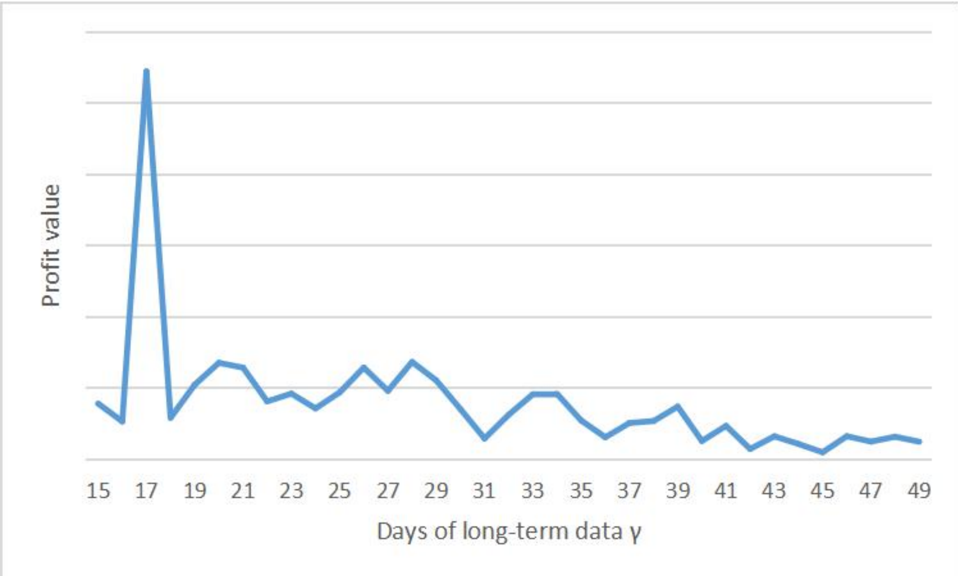

Figure 3-1 The maximum income corresponding to γ

According to the graph, it can be seen that when γ=17, the profit value is the largest.

3.7.2 Short-term data days δ:

To determine the trend of the last few days, determine whether the price of the currency you bought recently increased or decreased. According to the iteration, the optimal δ value is 3. That is, the model calculates regression parameters through long-term data and short-term data and compares them with reference values.

Long-term data is determined to be 17 days, while short-term data is determined to be 3 days.

3.7.3 Transaction limit σ:

When calculating the final return, we need to determine an expected return value, sigma, based on the long and short data days identified in 1.2. Ensure that at the end of each transaction, the return on the deducted commission is greater than the initial cost. The initial value of σ is Ba* L, where L is the trading margin. After each transaction is completed, it is compared against all the assets held and the transaction reference value.

$$\mathrm{Ba} = 1000 * (1 - \propto)$$

$$\mathrm{l} = (\mathrm{H_d} + \mathrm{H_g} + \mathrm{H_b})/\mathrm{Ba}$$

3.7.3 The minimum acceptable growth rate Θ1 in the long term in the transaction:

In decision-making and judgment, it is necessary to judge the long-term change trend of the expected purchase currency according to the long-term data days. At this time, a value needs to be determined for reference. When the obtained long-term regression parameter is greater than 0, it means that the currency is in a state of appreciation within γ days. It is still acceptable if the long-term regression parameter is less than 0 but it is greater than some threshold, Θ_1. Substitute γ=17 obtained in 1, and determine the final value of Θ_1 by dichotomy -0.7.

3.7.4 The maximum acceptable loss Θ2:

During the transaction, if the profit obtained by removing the commission after a transaction is less than a certain reference value, the transaction is determined to be unreasonable. That is, through this parameter limit, it can avoid buying and selling mistakes during the transaction process, resulting in a lot of losses. After also substituting $\gamma=17$, the final value of Θ_2 can be determined by the bisection method to be 0.882.

3.7.5 Expected return reference value Θ3:

When it is determined that the expected currency can be purchased according to the long-term and short-term change trend, in order to ensure the income, it is necessary to further judge whether the purchase can achieve the expected purpose. Introducing the parameter Θ_3, when the overall rally is greater than Θ_3, it can be determined that it is in a rising period and can be purchased. After substituting in $\gamma=17$, the final value of Θ_3 was determined by dichotomy to be 8.2.

So the parameter values are finally obtained: $\gamma=17$, $\delta=3$, $\Theta_1=-0.7$, $\Theta_2=0.882$, $\Theta_3=8.2$

IV.Conclusions

4.1 Model Analysis

4.1.1. Changes in personal assets

After substituting the above parameters, iterate. Trading was not possible as the gold market was closed for the weekend. So only make decisions on the conversion of bitcoin and cash on weekends. Through decision-making and judgment, the following personal asset distribution map can be drawn, as shown below:

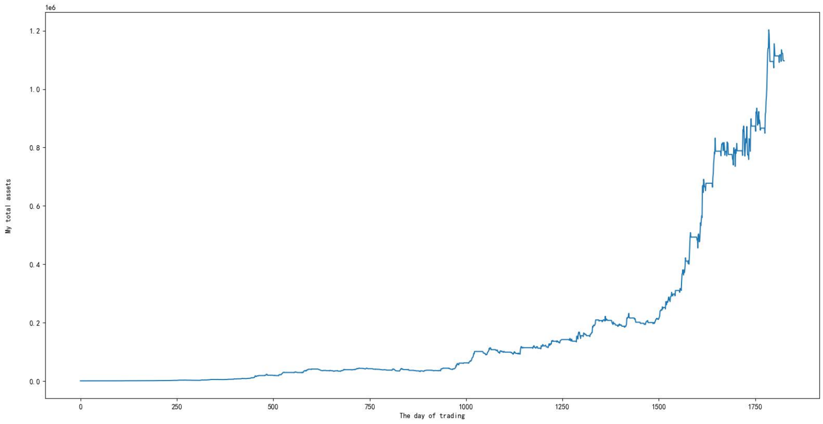

Figure 3 Trend chart of assets with transaction date

According to the analysis of the above figure, according to the model, with the progress of the transaction date, the total asset value as a whole shows an exponential increase. Among them, the maximum asset value reached $1,203,032, and the final asset was $1,096,986.

4.1.2 Analysis of buying and selling time points:

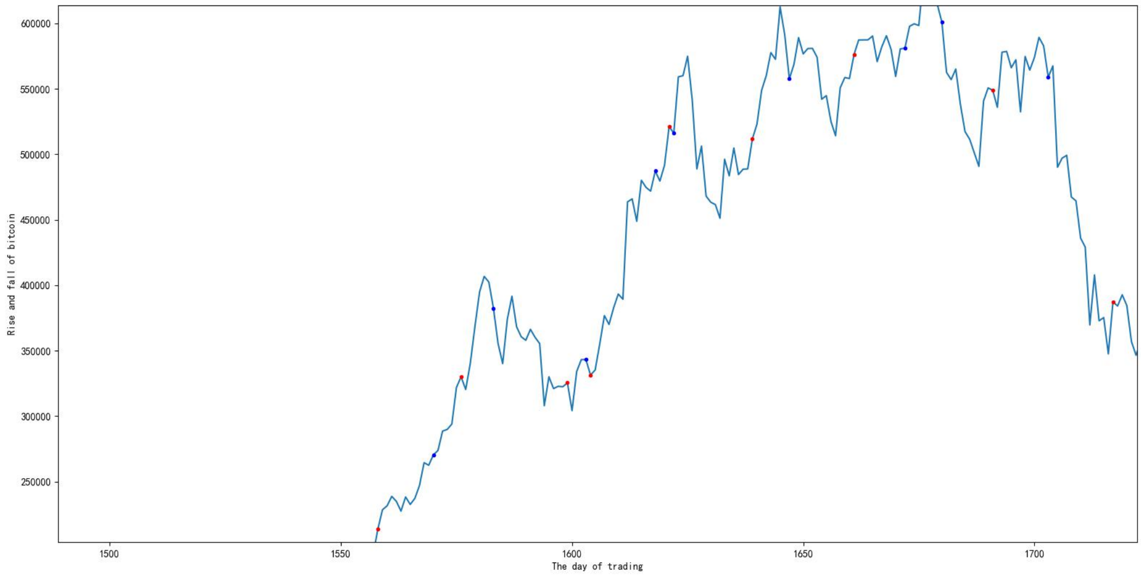

Figure 3 Trading points over a period of time (red is buy, blue is sell)

By analyzing the buying and selling points within a certain period of time, it can be seen that the buying points mostly occur around the price trough period, and the selling points are basically at the peak period. The general rule is to buy when the price is low and sell when the price is high. to maximize revenue.

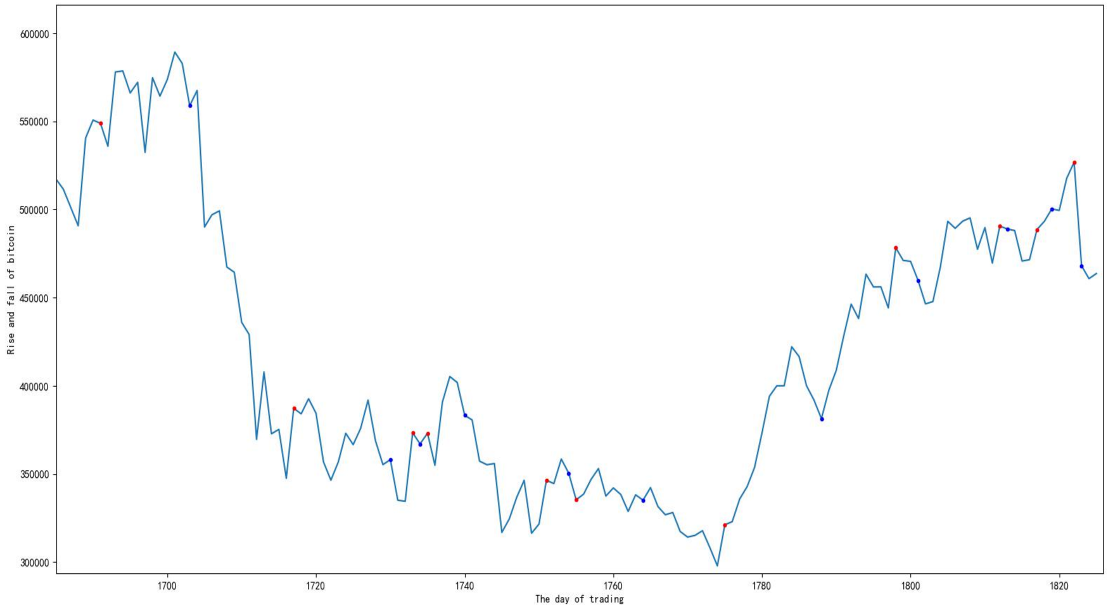

Figure 3 Buy and sell points in the last period

By analyzing the overall asset value, it can be seen that there is a certain decline at the end of the transaction. The reason is that the fluctuations of the two currencies are relatively large at this time, resulting in too frequent transactions. At the same time, some unreasonable buying and selling behaviors are caused by the drastic price fluctuations, which is also a shortcoming of the model. However, the overall benefits still meet expectations and meet the requirements.

4.2 Strength and Weakness

4.2.1 Strength:

The model divides past data into two types: long-term and short-term, to judge the trend as a whole, and whether to trade in the short-term. In this way, it can be guaranteed to buy or sell near the theoretical optimal trading time and to ensure the rationality of the transaction.

The model comprehensively guarantees the overall profit in the long-term, and at the same time has a short-term data guarantee to control the loss to a certain extent and avoid catastrophic losses.

The algorithm of regression analysis is selected when calculating the parameters, which ensures the stability of the data while the calculation accuracy is high. This is an important guarantee for successful decision-making.

The model introduces revenue expectations and loss thresholds to revise the resulting profits to maximize returns with the least risk.

4.2.2 Weakness:

The disadvantage of this model is that it cannot predict the trend of a certain period of time in the future based on past data, so the prediction part is missing in the decision-making, and only the past data may lead to wrong judgments.

The strategy is relatively conservative and cannot achieve the maximum profit.

4.3 Sensitivity analysis of transaction costs

Transaction costs mainly refer to commissions in the transaction process. Due to the existence of commissions, transactions cannot be made too frequently, because each transaction will lose a certain amount of assets, so it is necessary to try to avoid the increase in the number of transactions. Therefore, the commission has also become a factor that ultimately affects the income. To analyze the sensitivity of transaction costs is to analyze the influence of the commission on the final result. The following distribution diagram is obtained by calculation:

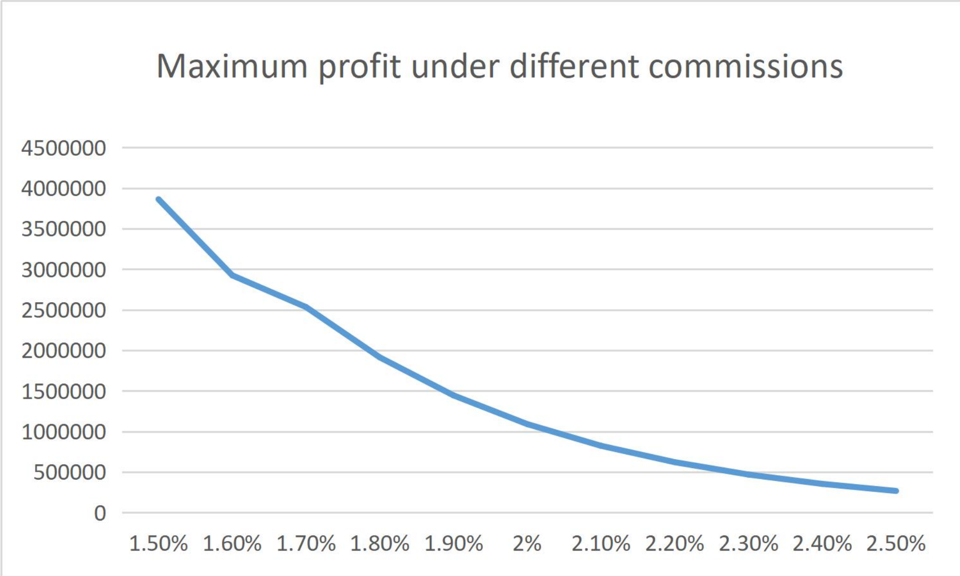

Figure 1 Distribution of maximum returns under different commissions

Through the maximum income under different commissions in the above figure, it can be seen that as the commission increases, the final maximum income shows an exponential decline. This is because there is an assumption in our model, that is, all buys or all sells for each transaction, so the maximum profit and commission have an exponential relationship. When the commission is 1.5%, the final income can reach 4,000,000, and when the commission increases by 2.5%, the final income is only 400,000. It can be seen that a difference of 1% in the commission can result in a difference of 10 times the final income, so the model is more sensitive to transaction costs.

Memorandum

Hello, trader. According to your requirements, our team has developed a complete set of strategies and models to determine how to trade on that day only through the prices of the day and the previous days.

Strategy

Since we can only get the price on the trading day and before, the price on the next day is arbitrary. We have also considered using fitting models to predict the next day's price and beyond, but the truth is that even with our most advanced forecasting models, there is no guarantee that the forecast will be 100 percent correct. In the long run, even if we can predict a high probability of future growth, we are still not sure that we are the best strategy. This is because after analyzing the data, we know that the volatility of short-term interests is much higher than that of long-term ones. If we can capture the rising interest in this to earn funds, our strategy will be optimal. Therefore, our strategies for you are as follows:

Do not trade in the first few days, and obtain data suitable for our model for inference through price statistics before trading. Because of the small size of the data, the possibility of wrong results is relatively high. But bigger isn't always better, and it might make our model more responsive. So it's safest for us to collect the right data for our model before trading. According to our calculations, the most suitable long-term data size is about seventeen days, while the short-term data size is about three days.

When we have not invested in any of them, if the long-term fitting data is calculated to be growing and substantial enough, we will choose to invest. In terms of risk, we believe that Bitcoin is the largest and cash is the smallest. Cash are risk-free, but holding it all the time is not the best strategy. Despite the risks of gold and bitcoin, in the long run, if there is profit to be made, it is obviously a smarter choice to invest. We have set the expected growth value and loss forecast for this purpose, please believe that we will make the right choice.

If we have invested, and the long-term fitting results show that the asset is growing, but the short-term fitting results are declining, it proves that although it is worth investing, it is not necessarily suitable for short-term investment. We will combine the cost of the previous investment and the expected growth value to judge whether it is currently profitable, and through the transaction cost to judge whether it is worthwhile to use the transaction to avoid the recent price trough. If asset prices have fallen slightly recently without affecting overall profits, we will choose to wait for market growth to avoid rising transaction costs. If there is a more serious decline in asset prices recently, it is better to sell them for the sake of safety and wait for the rise.

If the result of our long-term fitting shows that the price of the asset is falling, but the result of the recent fitting is rising, it means that we have most likely passed the peak of the price, and selling is the best choice. Although we cannot sell at the peak of an asset's rise through forecasting, we can sell as soon as the asset's price falls to reduce our losses.

If the long-term fit rises in line with the recent fit, then of course it is the best time to hold assets. But if it all goes down, selling is certainly the way to go.

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