$$\frac{dS}{dt} = \beta c \frac{I}{N}$$

$$\implies f := \beta c = \frac{I}{N} \left(\frac{dS}{dt}\right)^{-1}$$