Differential Equ^{n}

Runge Kutta Method

Ques Find out the approximate of the given line Segment
Given $\frac{dy}{dx} = x + y^2$, $y(0) = 1$, Find $y(0.2)$
where $h = 0.1$ using RK of order four?

Sol $f(x, y) = x + y^2$, $x_0 = 0$ $y_0 = 1$

Put $n = 0$

$x_1 = 0.1$, $y_1 = ?$
$x_2 = 0.2$, $y_2 = ?$

$x_1 = x_0 + h$

$S_1 = hf(x_n, y_n)$

$S_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{S_1}{2}\right)$

$S_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{S_2}{2}\right)$

$S_4 = hf(x_n + h, y_n + S_3)$

$K = \frac{1}{6}(S_1 + 2S_2 + 2S_3 + S_4)$

$y_{n+1} = y_n + K$ OR $y_{n+1} = y_n + \frac{1}{6}[S_1 + 2S_2 + 2S_3 + S_4]$

$S_1 = hf(x_0, y_0) \Rightarrow h(x_0 + y_0^2) \Rightarrow 0.1(0 + 1)$

$S_1 \Rightarrow 0.1$

$S_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{S_1}{2}\right) \Rightarrow h\left[\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{S_1}{2}\right)^2\right]$

x y

$\Rightarrow 0.1\left[\left(0 + \frac{0.1}{2}\right) + \left(0 + \frac{0.1}{2}\right)^2\right]$

$S_2 \Rightarrow 0.11525$

$S_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{S_2}{2}\right) \Rightarrow h\left[\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{S_2}{2}\right)^2\right]$

$\Rightarrow 0.1\left[\left(0+\frac{0.1}{2}\right)+\left(1+\frac{0.11525}{2}\right)^2\right]$

$S_3 \Rightarrow \boxed{0.1168}$

$S_4 = hf\left[\left(x_0+\frac{h}{2}\right), \left(y_0+\frac{S_3}{2}\right)\right] \Rightarrow 0.1\left[\left(0+\frac{0.1}{2}\right)+\left(1+0.1169\right)^2\right]$

$S_4 \Rightarrow \boxed{0.1347}$

$k = \frac{1}{6}\left(S_1 + 2S_2 + 2S_3 + S_4\right)$

$\Rightarrow \frac{1}{6}\left[(0.1 + 2(0.11525) + 2(0.1168) + 0.1347\right]$

$\boxed{k \Rightarrow 0.1165}$

$y_{n+1} = y_n + k$

$y_1 = y_0 + k \Rightarrow 1 + 0.1165 \Rightarrow 1.1165$

$\boxed{y_1 = 1.11659}$

Put $n = 1$

$S_1 = hf(x_1, y_1) \Rightarrow h(x_1 + y_1^2)$

$\Rightarrow 0.1\left[0.1 + (1.1165)^2\right]$

$S_1 \Rightarrow \boxed{0.1347}$

$S_2 = hf\left(x_1+\frac{h}{2}, y_1+\frac{S_1}{2}\right) \Rightarrow 0.1\left[\left(0.1+\frac{0.1}{2}\right)+\left(1.1165+\frac{0.1347}{2}\right)^2\right]$

$\boxed{S_2 \Rightarrow 0.1552}$

$S_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{S_2}{2}\right) \Rightarrow 0.1f\left[x_1+\frac{h}{2}+\left(y_1+\frac{S_2}{2}\right)^2\right]$

$\Rightarrow 0.1\left[0.1+\frac{0.1}{2}+\left(1.1165+\frac{0.1552}{2}\right)^2\right]$

$\boxed{S_3 \Rightarrow 0.1576}$

$$S_4 = hf\left(x_1 + \frac{h}{2}, y_1 + S_3\right) \Rightarrow hf\left[x_1 + \frac{h}{2} + (y_1 + S_3)^2\right]$$

$$\Rightarrow 0.1\left[0.1 + 0.1 + (1.1165 + 0.1576)^2\right]$$

$$\boxed{S_4 \Rightarrow 0.1823}$$

$$y_{n+1} = y_n + \frac{1}{6}\left[S_1 + 2S_2 + 2S_3 + S_4\right]$$

$$y_2 \Rightarrow y_1 + 0.1572$$

$$y_2 = 1.1165 + 0.1572$$

$$\boxed{y_2 = 1.2737}$$