CS201 Assignment

Submission Deadline: April 30, 2021

1. Define n-varaiate polynomials P_d and Q_d as:

$$
\begin{aligned}
P_d(x_1, x_2, \ldots, x_n) &= \sum_{\substack{J \subseteq [1,n] \\ |J| = d}} \prod_{r \in J} x_r \\
Q_d(x_1, x_2, \ldots, x_n) &= \sum_{\substack{0 \le i_1, i_2, \ldots, i_n \le d \\ i_1 + i_2 + \cdots + i_n = d}} \prod_{r=1}^{n} x_r^{i_r},
\end{aligned}
$$

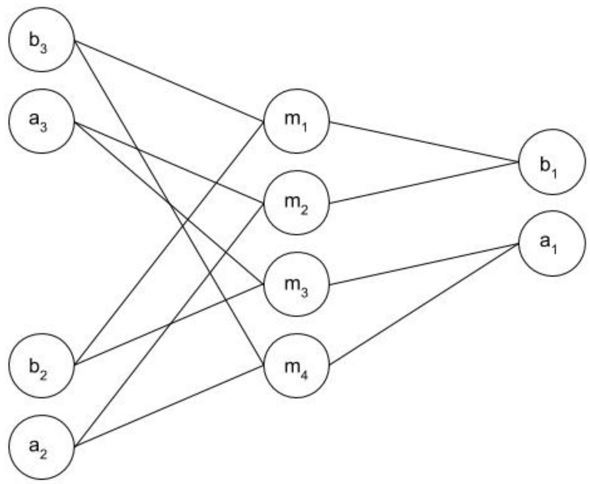
   and $P_0(x_1, x_2, \ldots, x_n) = 1 = Q_0(x_1, x_2, \ldots, x_n)$. Show that for any $d > 0$:

$$\sum_{m=0}^{d} (-1)^m P_m(x_1, x_2, \ldots, x_n) Q_{d-m}(x_1, x_2, \ldots, x_n) = 0.$$

2. Let $\alpha \in \mathbb{R}$ and N be a natural number. Using pigeon-hole principle, show that there exists integers p and q such that $1 \le q \le N$ and

$$|q\alpha - p| \le \frac{1}{N}$$

3. Let $G = (V, E)$ be a graph where V is the vertex set and E is the edge set. A bijective mapping $f : V \to V$ is an **automorphism** if it has the property that $(u, v) \in E \iff (f(u), f(v)) \in E$. Consider the following graph.



Let $A = \{a_1, a_2, a_3\}, B = \{b_1, b_2, b_3\}, M = \{m_1, m_2, m_3, m_4\}$. Then, the vertex set of the above graph is $V = A \cup B \cup M$. Consider a bijective mapping $g : A \cup B \to A \cup B$ such that $g(a_i) \in \{a_i, b_i\}$ and $g(b_i) \in \{a_i, b_i\}$ for all $i \in \{1, 2, 3\}$, i.e., g maps the ordered pair $[a_i, b_i]$ to either $[a_i, b_i]$ (no swap) or $[b_i, a_i]$ (swap).

Show that g can be extended to an automorphism f for the above graph if and only if the number of swaps performed by g is even.

4. An *endomorphism* of a ring R is a ring homomorphism $\phi : R \mapsto R$. Prove that $\phi : F_P \mapsto F_p$, $\phi(x) = x^p$ is an endomorphism where p is a prime number.