ESO207_Assignment3

Jaya Gupta(200471) Pratyush Gupta(200717)

May 3, 2022

1   Question1 → Bipartite

1.1   Pseudo Code

function $dfs(G,$u,sec)

▷ returns true if graph corresponding to vertices of the dfs tree rooted at u is
▷ bipartite else return false

    visited[u]=true
    section[u]=sec   ▷ section array store in which section $(V1, V2)$ do every vertex lies

    if sec == 1 **then**
        alt_sec=2   ▷ alt_sec stores the section in which all the vertex adjacent to u will lie in
    else
        alt_sec=1
    end if

    for all $(u, v) \in E$ **do**
        if visited[v] == false **then**
            ans=$dfs(G,$v,alt_sec)
            if ans==false **then**
                return false
            end if
        else
            if section[v]==sec **then**
                return false
            end if   ▷ if the ajacent vertex lies in the same section as u return false(cant be bipartite)
        end if
    end for

    return true
end function

function Bipartite(G)
    v_ ← G.v
    e_ ← G.e

    for all $u \in V$
     visited[u]=$false$

    return dfs(G,0,1)
end function

function main

    ans = $Bipartite$(G)
    if ans == $true$ **then**
        for all $u \in V$ **do**
            print(YES)
            print(section[u])
        end for
    else
        print(NO)
    end if

end function

2 Question2 →Uniqueness of Partition of vertices in sets $(V1, V2)$

2.1 Answer

- First of all since the above algorithm is just a slight modification of dfs (only few commands which takes constant time are added), the above algorithm works in $O(V + E)$ time.

- If the graph is connected and if it is bipartite , then the partition of vertices in the two sets $(V1, V2)$ is unique because if we decide the section of any one of the vertex then all the other vertices will automatically fall in their respective sections as the graph is connected.

- If the graph is unconnected then it will have several disconnected components . All the different components will be connected graphs individually. So if all those components are bipartite, then we can decide the section in which one vertex from each component will lie and all others will fall accordingly.

- So the choice of that section for one vertex each from each component is not unique and is totally upon us whether we want to keep it in set $V1$ or $V2$. Hence the sections $V1, V2$ are not unique in case of disconnected graph.