Fundamentals of Earth Sciences (ESO 213A)

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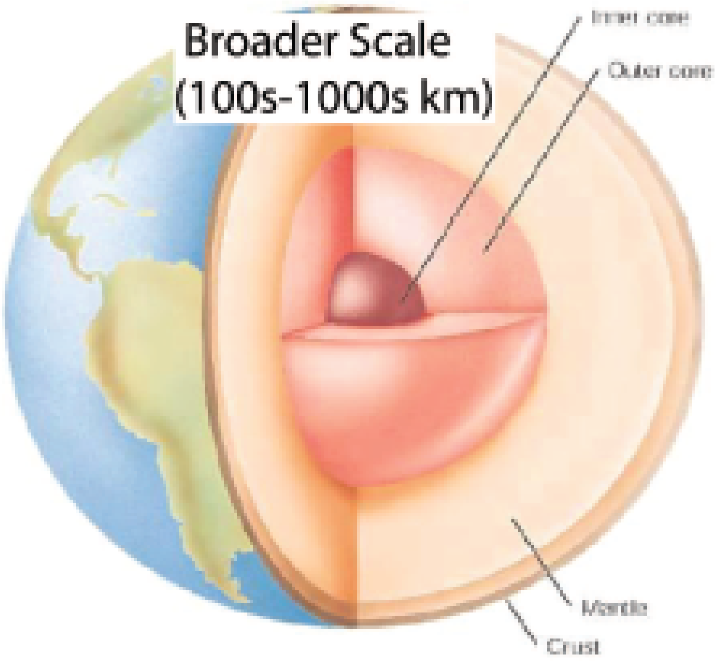
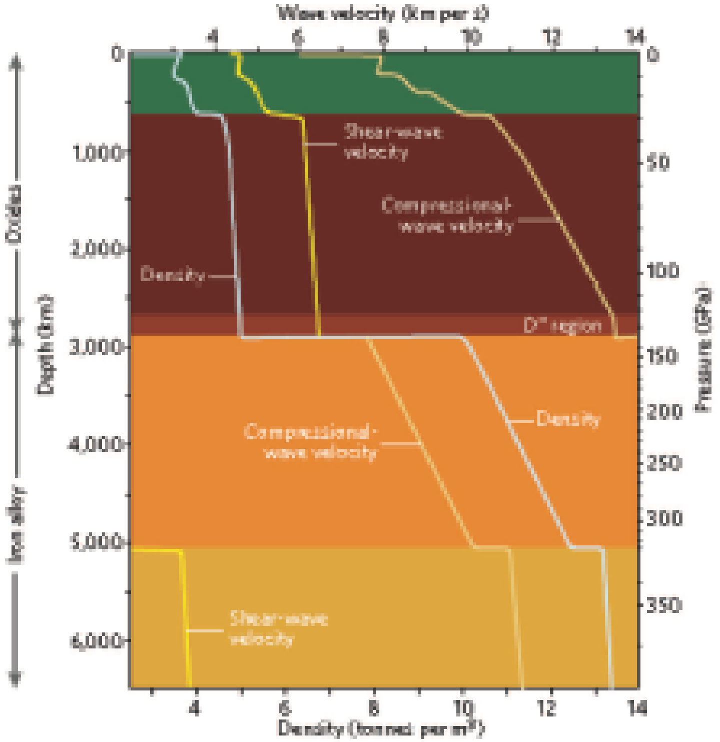
Geophysics: Seismic

Previous Class: Electrical

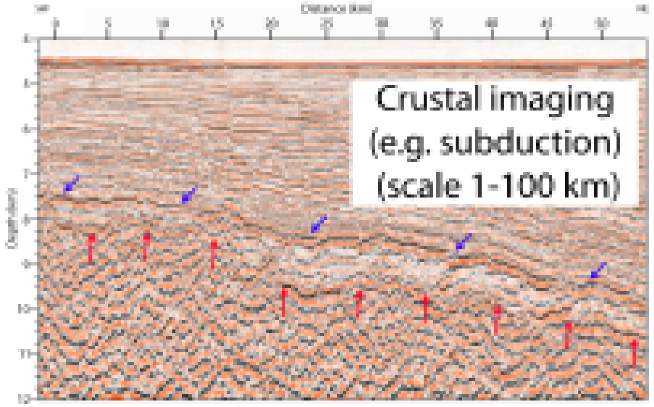
Seismology

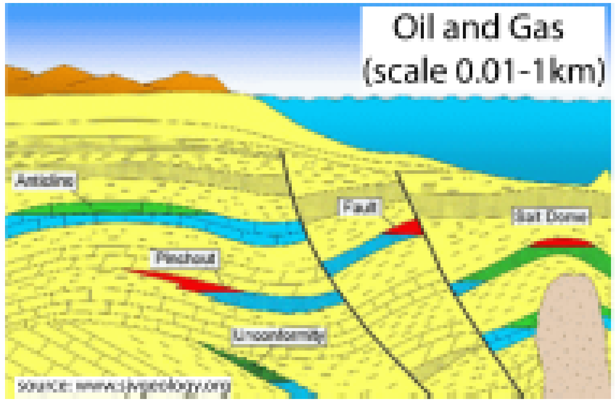
- Seismology is used to determine internal structure of the Earth
- Based on the nature of sources, seismology is classified into two branches:
  - Global (Earthquake) seismology
  - Controlled Source Seismic (CSS)
- Global seismology — images broader scale (100s to 1000s km) structures





Romanowicz, 2008, Nature





- Controlled Source Seismic — small scale structures (resolution: few meter to kilometer)

Seismic Data Acquisition

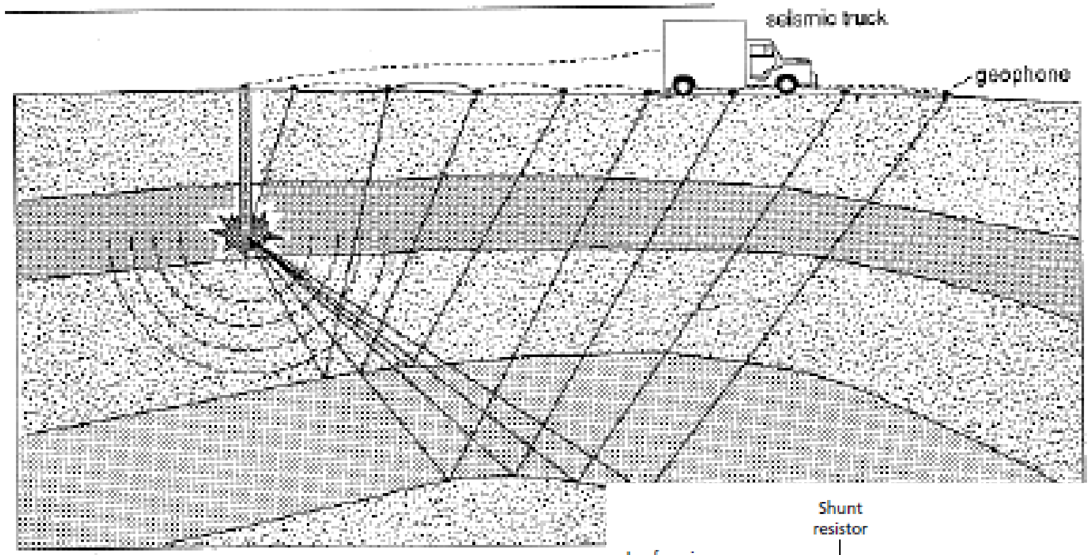
- Land survey

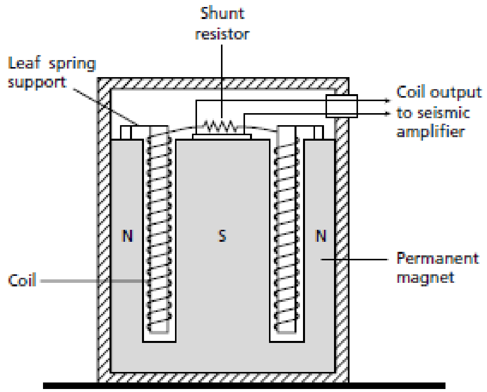
  Sources (Explosives, Weight Drop, Vibroseis); Detectors (Geophones)

- Marine survey

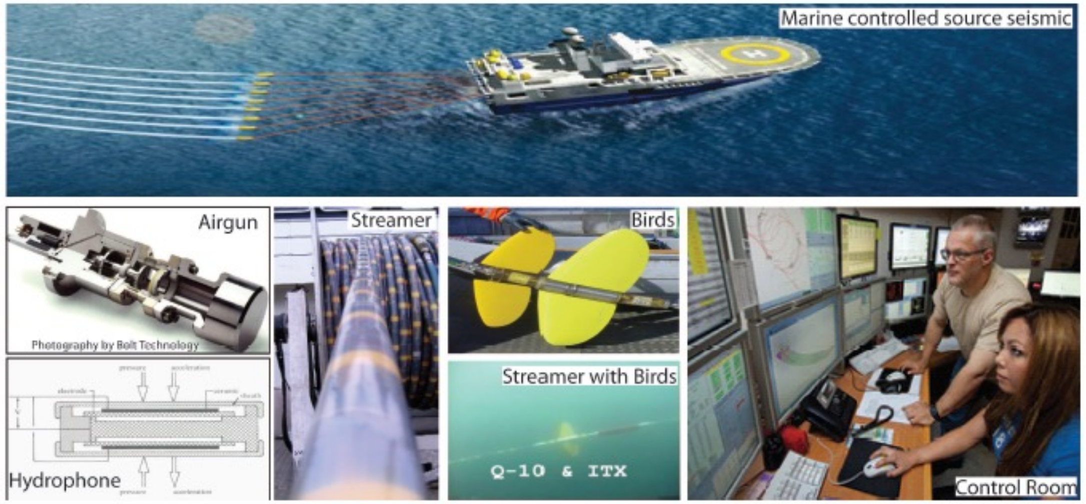
  Sources - (Airgun)

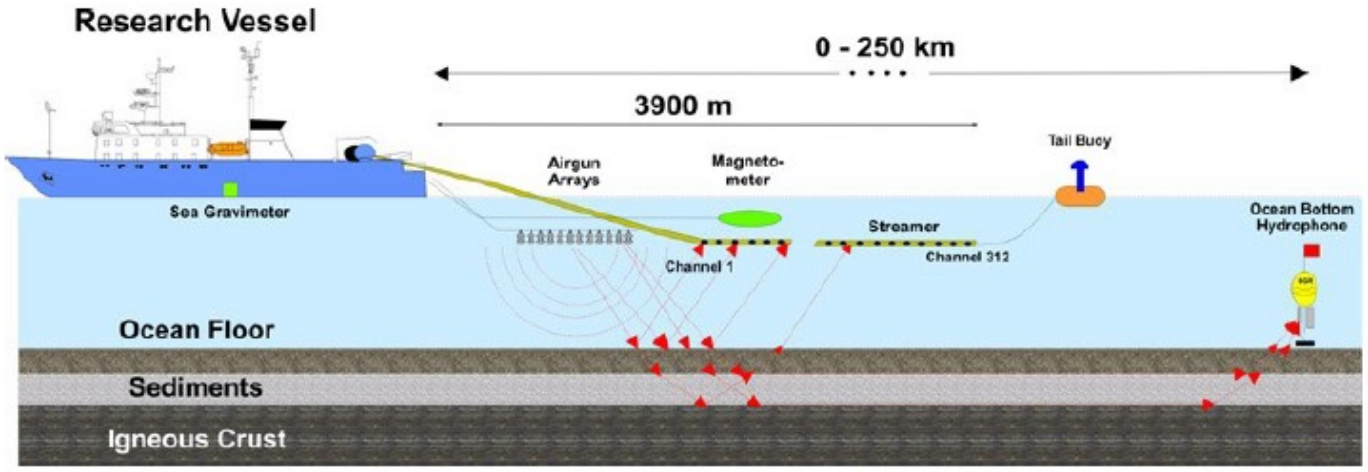
  Detectors -(Hydrophones, OBS, OBC)



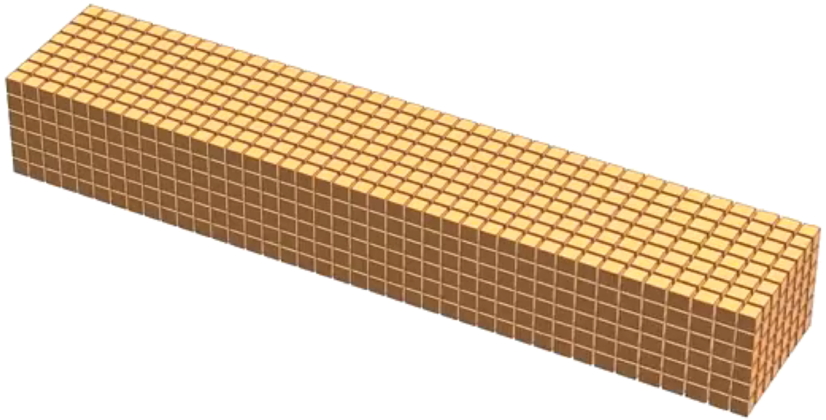


Geophone

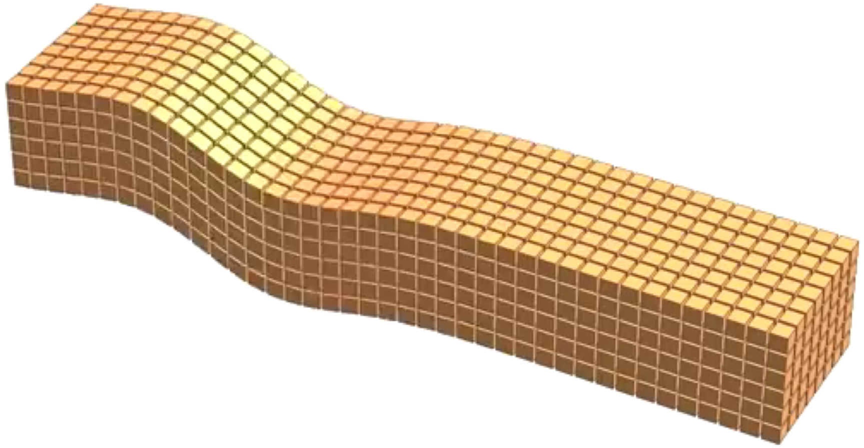




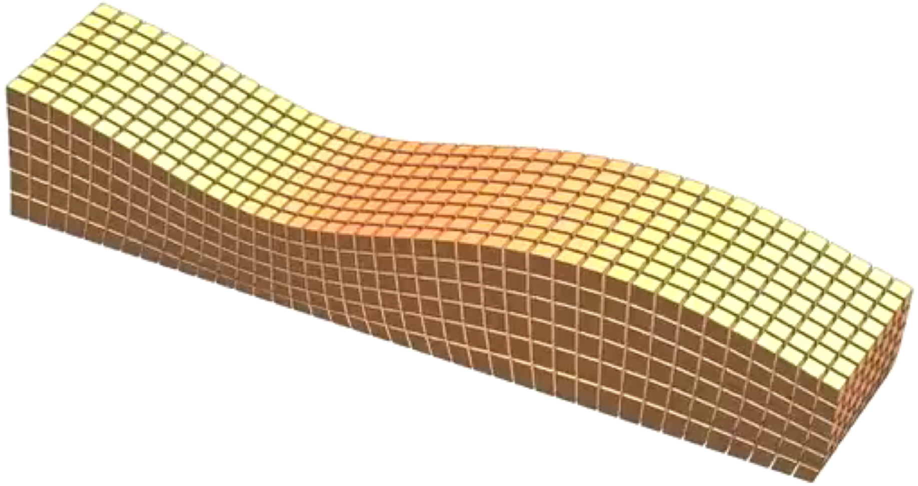
Elastic waves



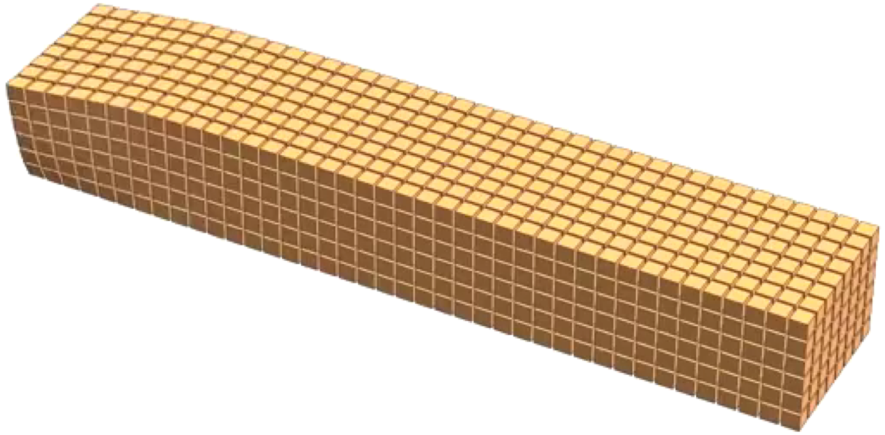
P wave



S wave



Rayleigh wave



Love wave

Scalar wave equation in spherical coordinates:

$$\nabla^2 \phi(r, t) = \frac{1}{\mathrm{v}^2} \frac{\partial^2 \phi(r, t)}{\partial t^2} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi(r, t)}{\partial r}\right)$$

Let $\phi(r, t) = \xi(r, t)/r$

to get

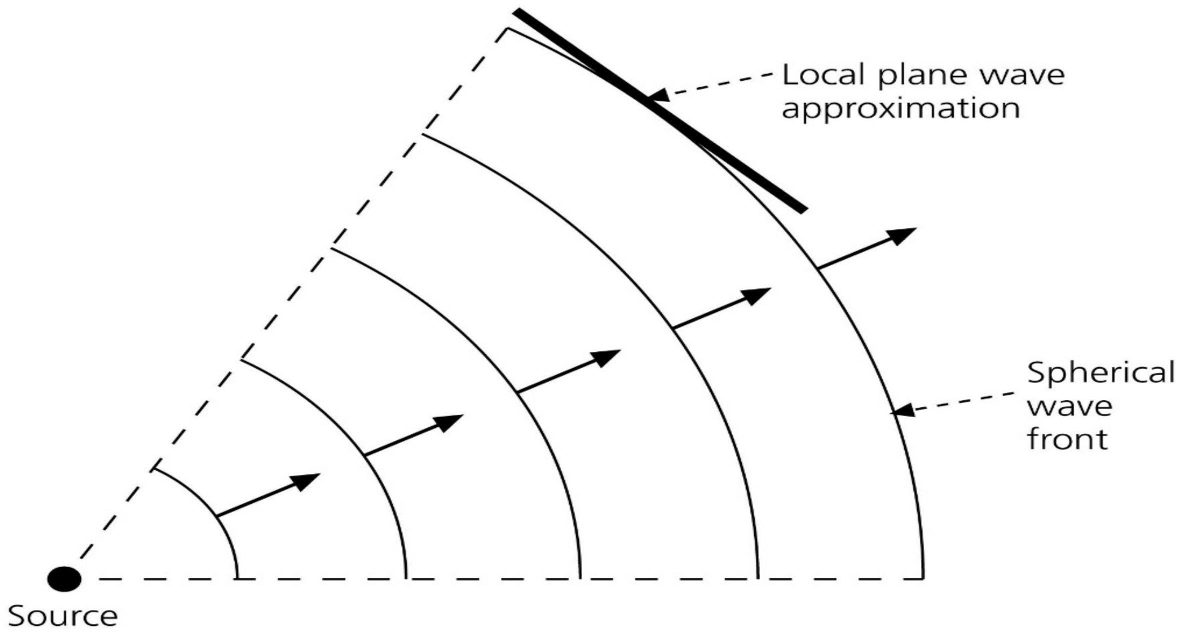
$$\frac{1}{r}\left[\frac{\partial^2 \xi}{\partial r^2} - \frac{1}{\mathrm{v}^2} \frac{\partial^2 \xi}{\partial t^2}\right] = 0$$

A solution is any function of the form:

$$\phi(r, t) = \frac{f(t \pm r/\mathrm{v})}{r}$$

Geometric spreading: amplitudes decay as $1/r$, energy decays as $1/r^2$

Figure 2.4-2: Approximation of a spherical wave front as plane waves.



Interface	Boundary Conditions
solid-solid	$T_i^+ = T_i^-$ $u_i^+ = u_i^-$
solid-liquid	$T_3^+ = T_3^-$ $T_2 = T_1 = 0$ $u_3^+ = u_3^-$
free surface	$T_i = 0$

Stein and Wysession,2003

Seismic wave velocities

Seismic wave velocities v strongly depend on

- rock type (sediment, igenous, volcanic)
- porosity
- pressure and temperature
- pore space content (gas, liquid)

$$v = \sqrt{\frac{ElasticModuli}{Density}}$$

Seismic wave velocities P-waves

Material	V_p (km/s)
Unconsolidated material	
Sand (dry)	0.2-1.0
Sand (wet)	1.5-2.0
Sediments	
Sandstones	2.0-6.0
Limestones	2.0-6.0
Igneous rocks	
Granite	5.5-6.0
Gabbro	6.5-8.5
Pore fluids	
Air	0.3
Water	1.4-1.5
Oil	1.3-1.4
Other material	
Steel	6.1
Concrete	3.6

Seismic wave velocities shear waves

The relation between P-waves velocities and shear wave velocities is often described by the v_P/v_s ratio or Poisson's ratio.

A commonly used assumption for crustal rocks is:

$v_P/v_s = \text{sqrt}(3) \approx 1.7$

This corresponds to the Poisson ratio σ

$\sigma = 0.25$

With the relation:

$$\frac{v_p}{v_s} = \left[\frac{2(1-\sigma)}{(1-2\sigma)}\right]^{1/2}$$

Fluids or gas in rocks strongly influence the v_P/v_s ratio that is one of the most important diagnostics in seismic exploration!

Seismic velocities and density Porosity

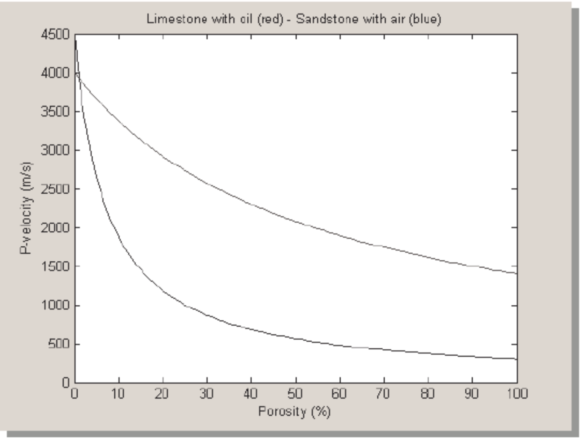
We want to quantify the effect of porosity Φ on the seismic wave velocity and density. With ρ_b the bulk density, ρ_f the pore fluid density and ρ_m the rock matrix density:

$$\rho_b = \rho_f \Phi + (1-\Phi)\rho_m$$

... and a corresponding relation exists for P-velocity

$$\frac{1}{v_b} = \frac{\Phi}{v_f} + \frac{(1-\Phi)}{v_m}$$

Seismic velocities and density Porosity



Attenuation

Propagating seismic waves loose energy due to

- geometrical spreading

  e.g. the energy of spherical wavefront emanating from a point source is distributed over a spherical surface of ever increasing size

- intrinsic attenuation

  elastic wave propagation consists of a permanent exchange between potential (displacement) and kinetic (velocity) energy. This process is not completely reversible. There is energy loss due to shear heating at grain boundaries, mineral dislocations etc.

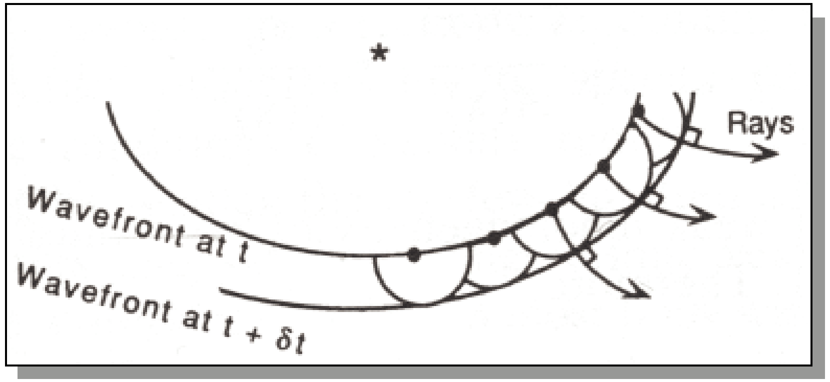
- scattering attenuation

  whenever there are material changes the energy of a wavefield is scattered in different phases. Depending on the material properties this will lead to amplitude decay and dispersive effects.

spherical spreading: For body waves (P and S), the waves spread out in spherical shells. Since the surface area of a sphere is proportional to the square of the radius, the energy per unit area (energy density) decreases as r^2 , $E=E_0/r^2$ (), and the amplitude decreases as r, $A=A_0/r$.

cylindrical spreading: Surface waves, like ground roll, are confined to the surface, so they spread out on a cylindrical shell. The area of a cylindrical shell is proportional to r, so the amplitude of a surface wave decreases only as $1/\sqrt{r}$, and the energy density decreases as 1/r.

Seismic rays



Huygens principle states that each point on the wavefront serves as a secondary source. The tangent surface of the expanding waves gives the wavefront at later times. Rays are the trajectories perpendicular to the wavefronts.

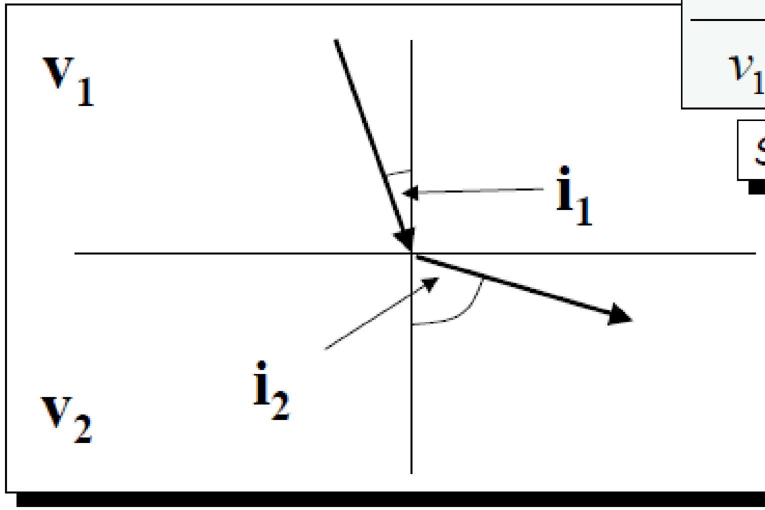
Fermat's principle and Snell's law ray transmission

Fermat's principle governs the geometry of the ray path. The ray will follow *a minimum-time* path. From Fermat's principle follows directly Snell's Law

$$\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

Snell's Law

$V_2 > V_1$



Reflection and transmission at boundaries vertical incidence

An importance notion in seismic reflection studies is the impedance. It is the product of density ρ and P-wave (or S-wave) velocity $v_{P/S}$. It is usually denoted by Z

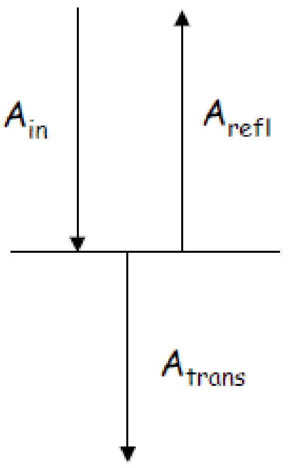
$Z = \rho * v_P$

The reflection (transmission) coefficients at an interface are given as the ratio of reflected (transmitted) to incoming wave amplitude

interface

$R = A_{refl}/A_{in}$

$T = A_{trans}/A_{in}$



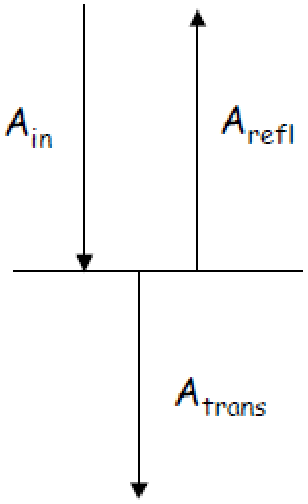
Reflection and transmission at boundaries vertical incidence

For normal (vertical) incidence the reflection coefficient is given as:

$$R = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

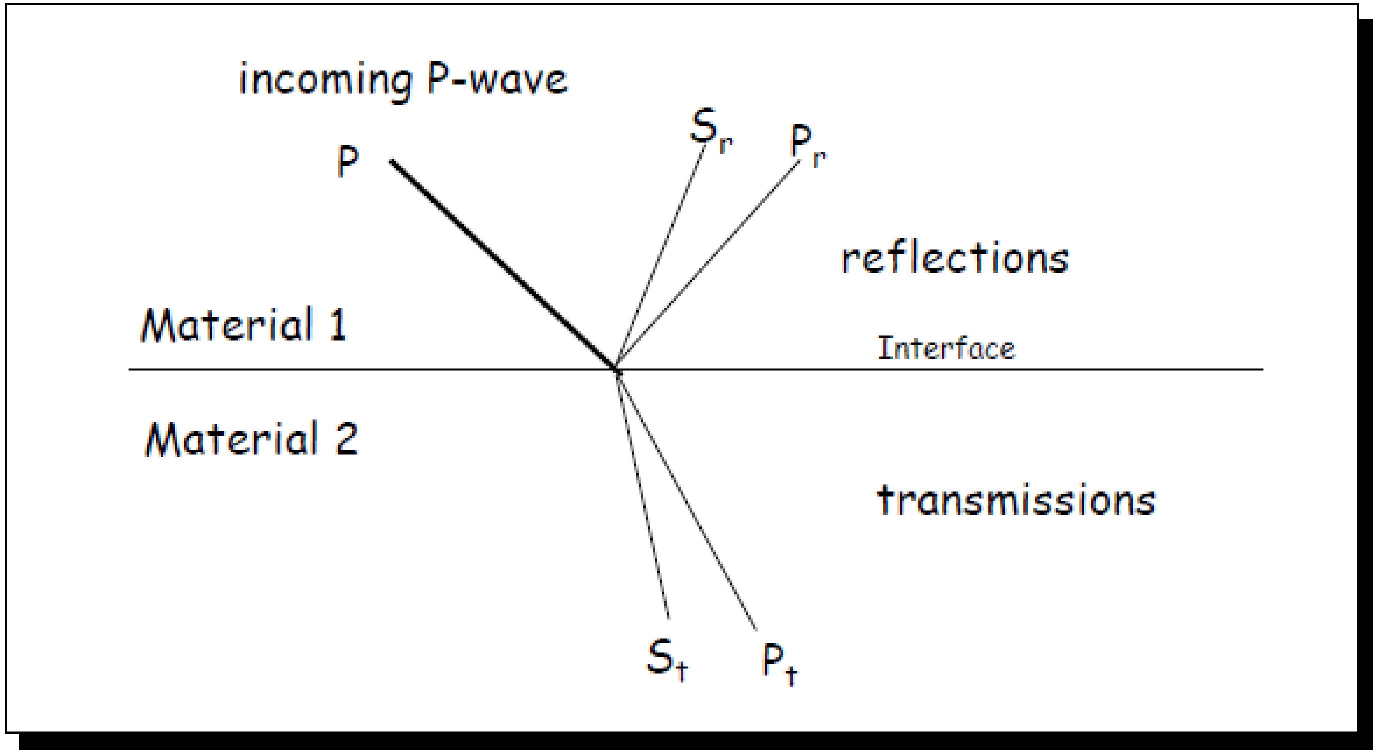
the transission coefficient is given as:

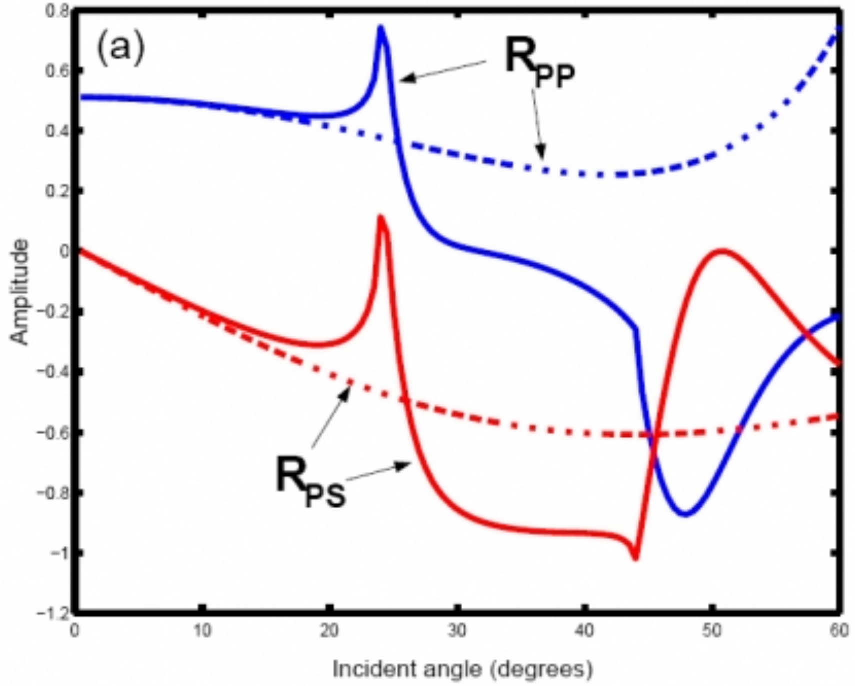
$$T = \frac{2\rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} = \frac{2Z_1}{Z_2 + Z_1}$$



Reflection and transmission at boundaries oblique incidence - conversion

P waves can be converted to S waves and vice versa. This creates a quite complex behavior of wave amplitudes and wave forms at interfaces. This behavior can be used to constrain the properties of the material interface.

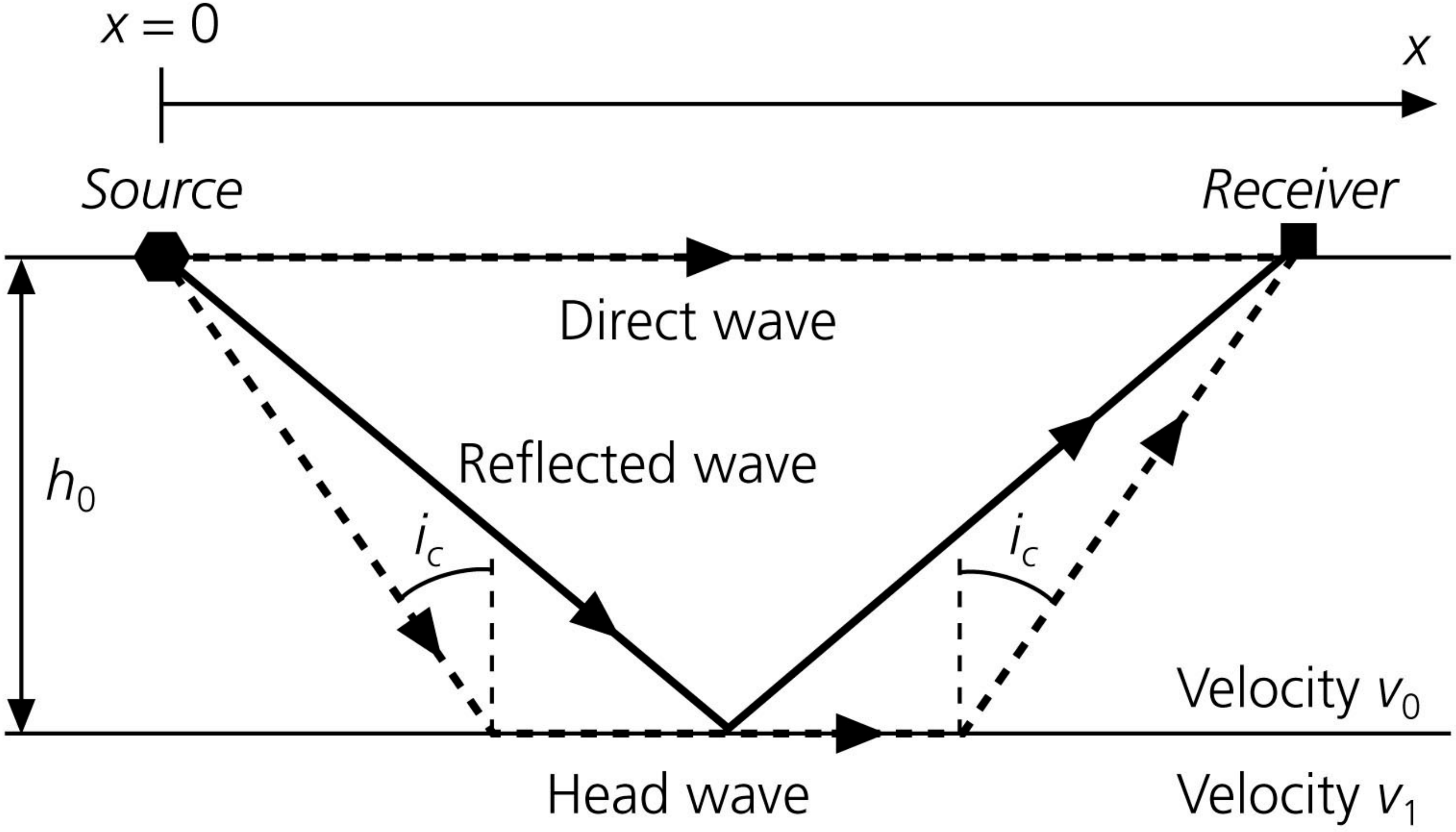




(a) The Schuey equation :

$$R(\theta) = \underbrace{Ro}_{\text{Intercept}} + \sin^2\theta \underbrace{\left[\Delta\sigma \left[\frac{9}{4}\right] - Ro\right]}_{\text{Slope}} \qquad (\theta \text{ in radians})$$

Figure 3.2-1: Ray paths for a layer over a halfspace.



Direct wave: $T_D(x) = x/\mathrm{v}_0$

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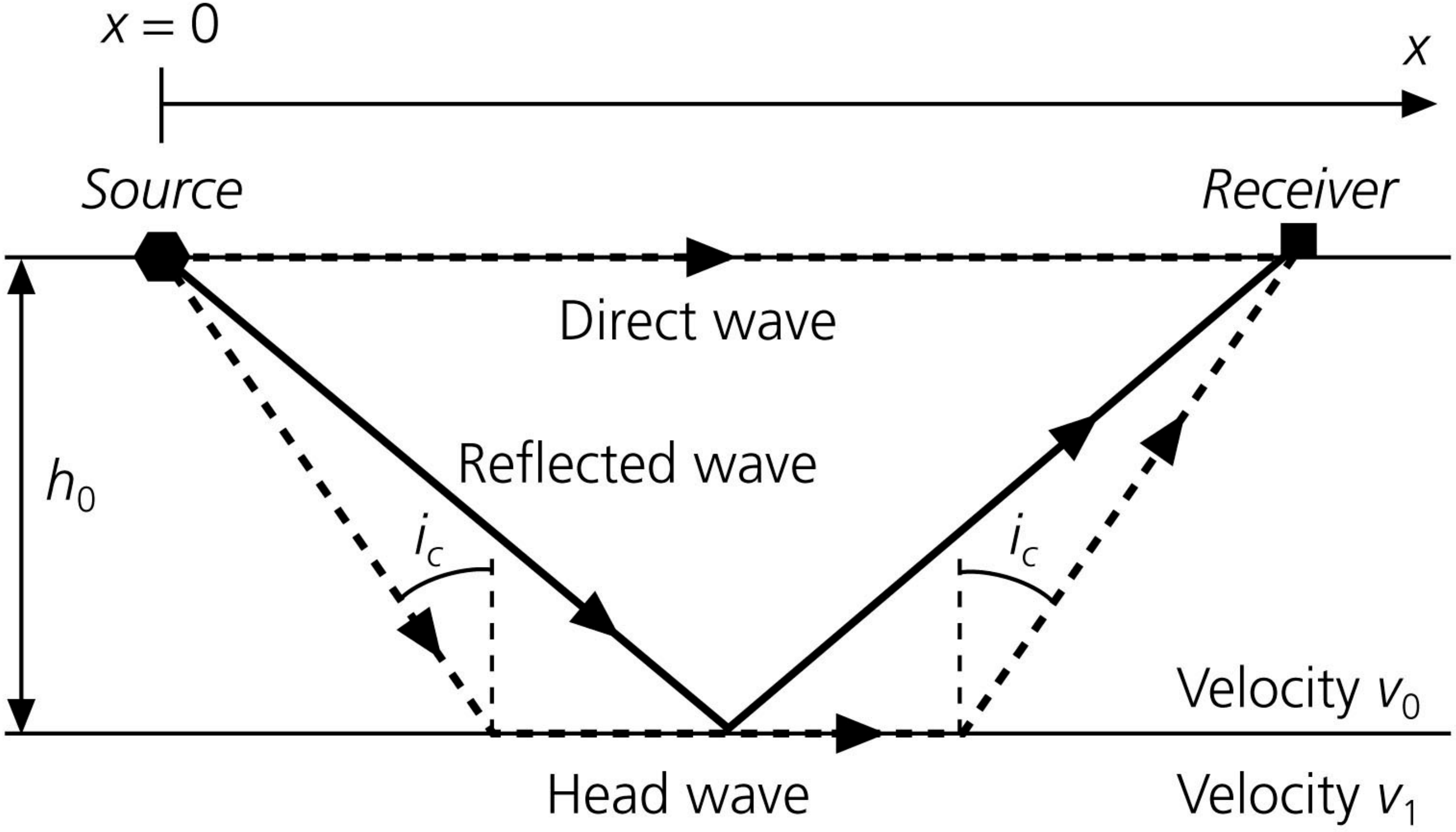

Direct wave: $T_D(x) = x/\mathrm{v}_0$

Reflected wave: $T_R(x) = 2(x^2/4 + h_0^2)^{1/2}/\mathrm{v}_0$ (This curve is a hyperbola: $T_R^2(x) = x^2/\mathrm{v}_0^2 + 4h_0^2/\mathrm{v}_0^2$)

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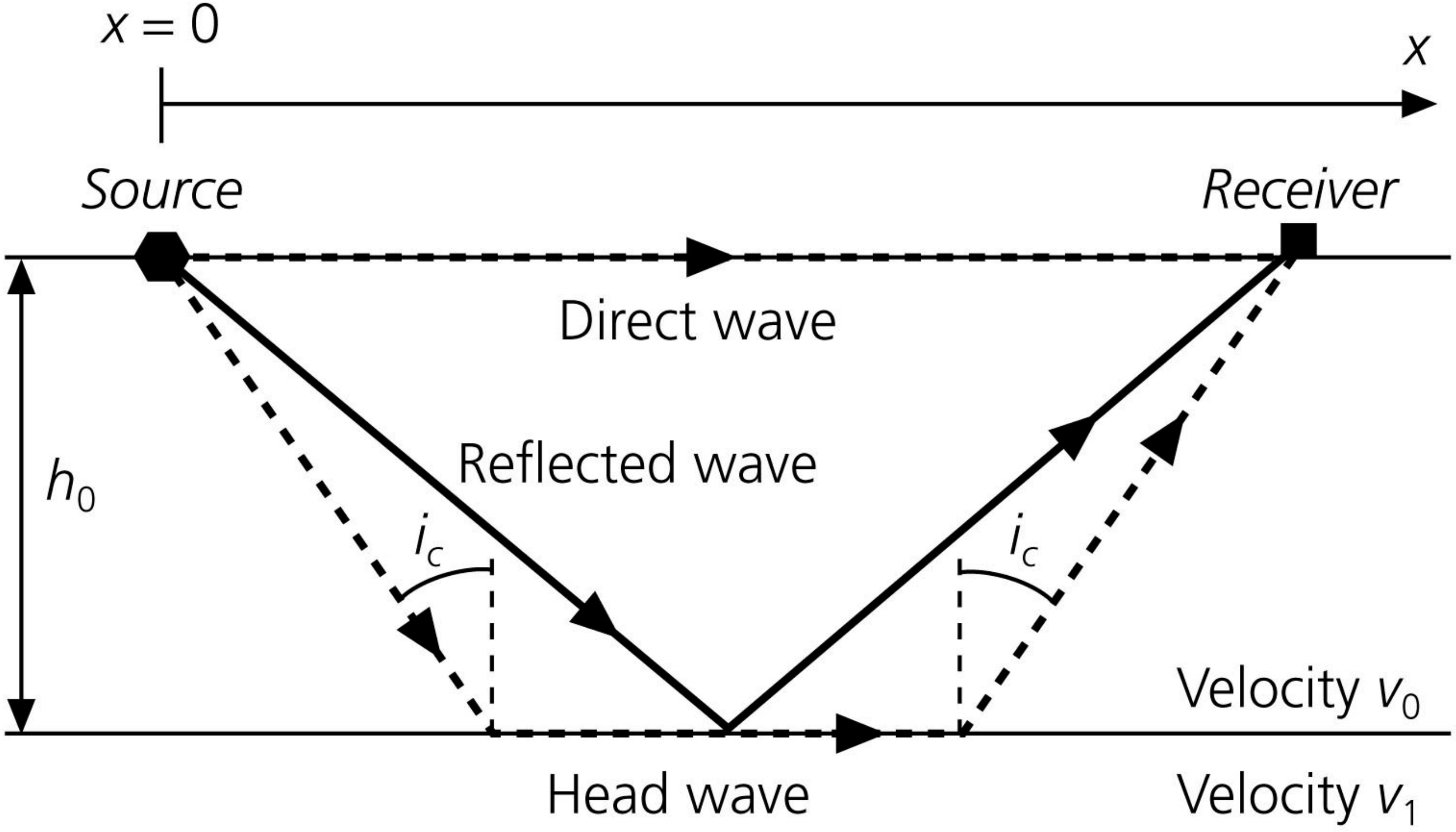

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For $x = 0$, $T_R(0) = 2h_0/\mathrm{v}_0$

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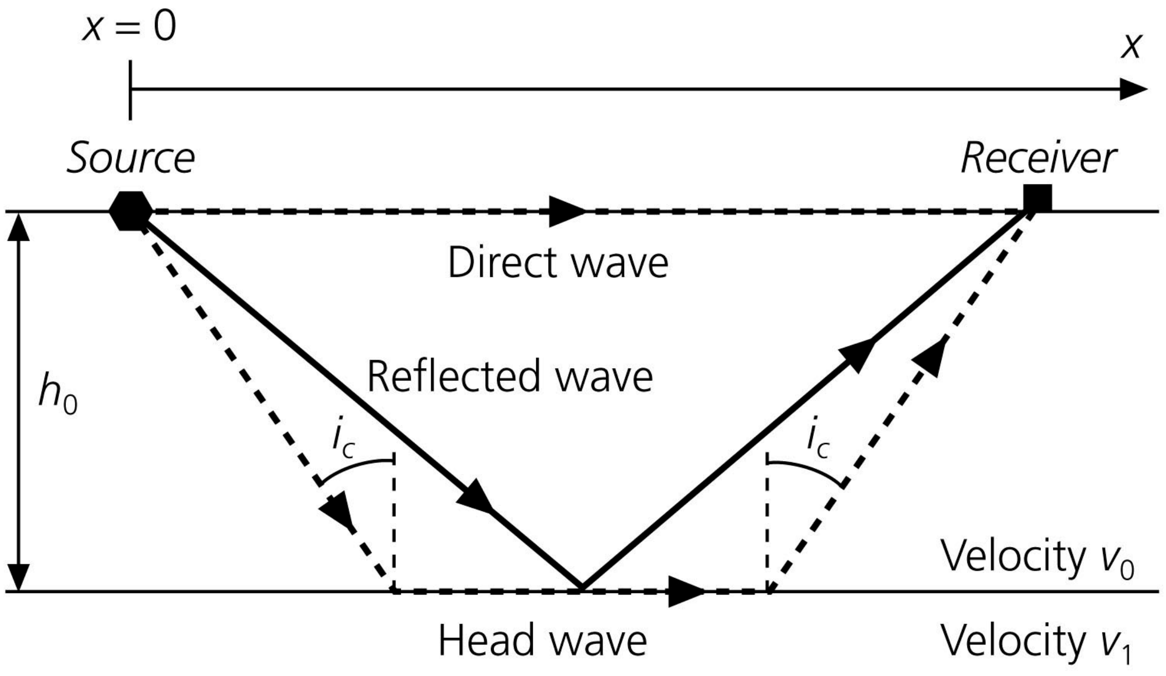

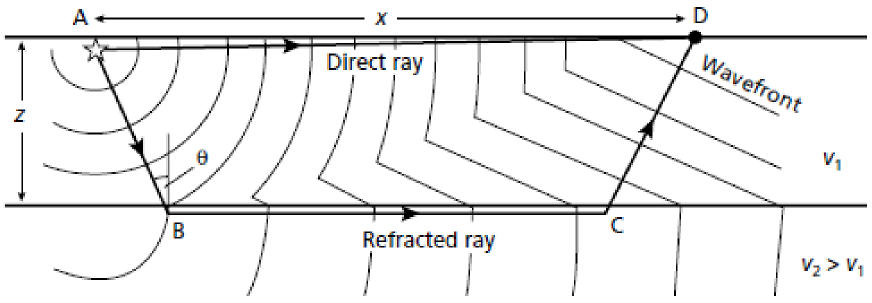

Fig. 5.1 Successive positions of the expanding wavefronts for direct and refracted waves through a two-layer model. Only the wavefront of the first arrival phase is shown. Individual ray paths from source A to detector D are drawn as solid lines.

$$\text{Head wave:} \quad T_H(x) = \frac{x - 2h_0 \tan i_c}{v_1} + \frac{2h_0}{v_0 \cos i_c} = \frac{x}{v_1} + 2h_0\left(\frac{1}{v_0 \cos i_c} - \frac{\tan i_c}{v_1}\right)$$

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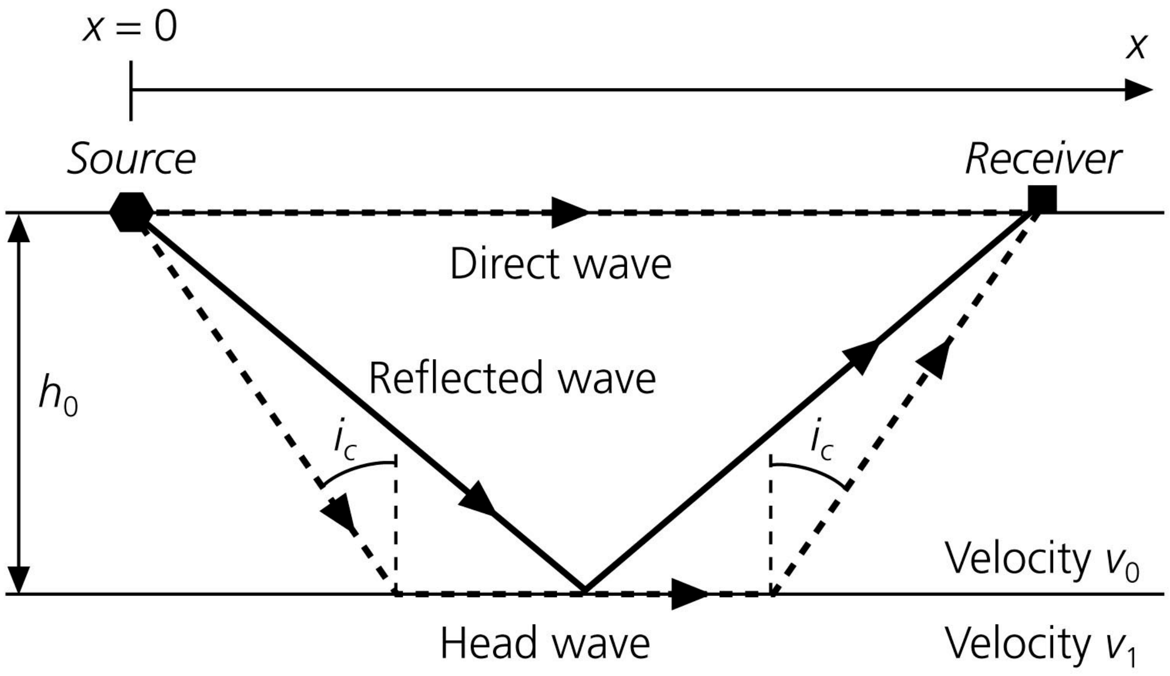

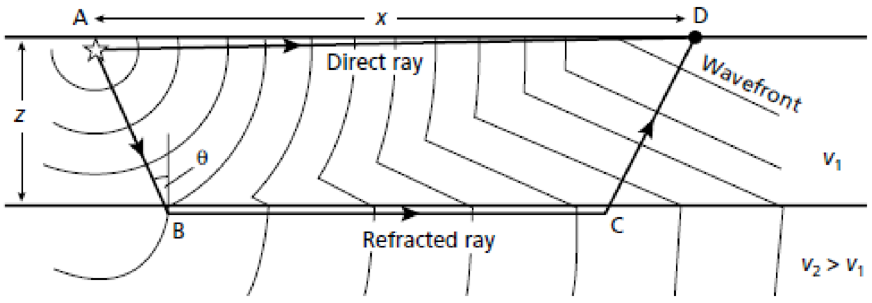

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This can be written as: $$T_H(x) = x / \mathrm{v}_1 + 2h_0\ (1 / \mathrm{v}_0^2 - 1 / \mathrm{v}_1^2\ )^{1/2} = x / \mathrm{v}_1 + \tau_1$$

where $$\tau_1 = 2h_0(1/\mathrm{v}_0^2 - 1/\mathrm{v}_1^2)^{1/2}$$

Direct wave:

$$T_D(x) = x/\mathrm{v}_0$$

Reflected wave:

$$T_R(x) = 2(x^2/4 + h_0^2)^{1/2}/\mathrm{v}_0$$

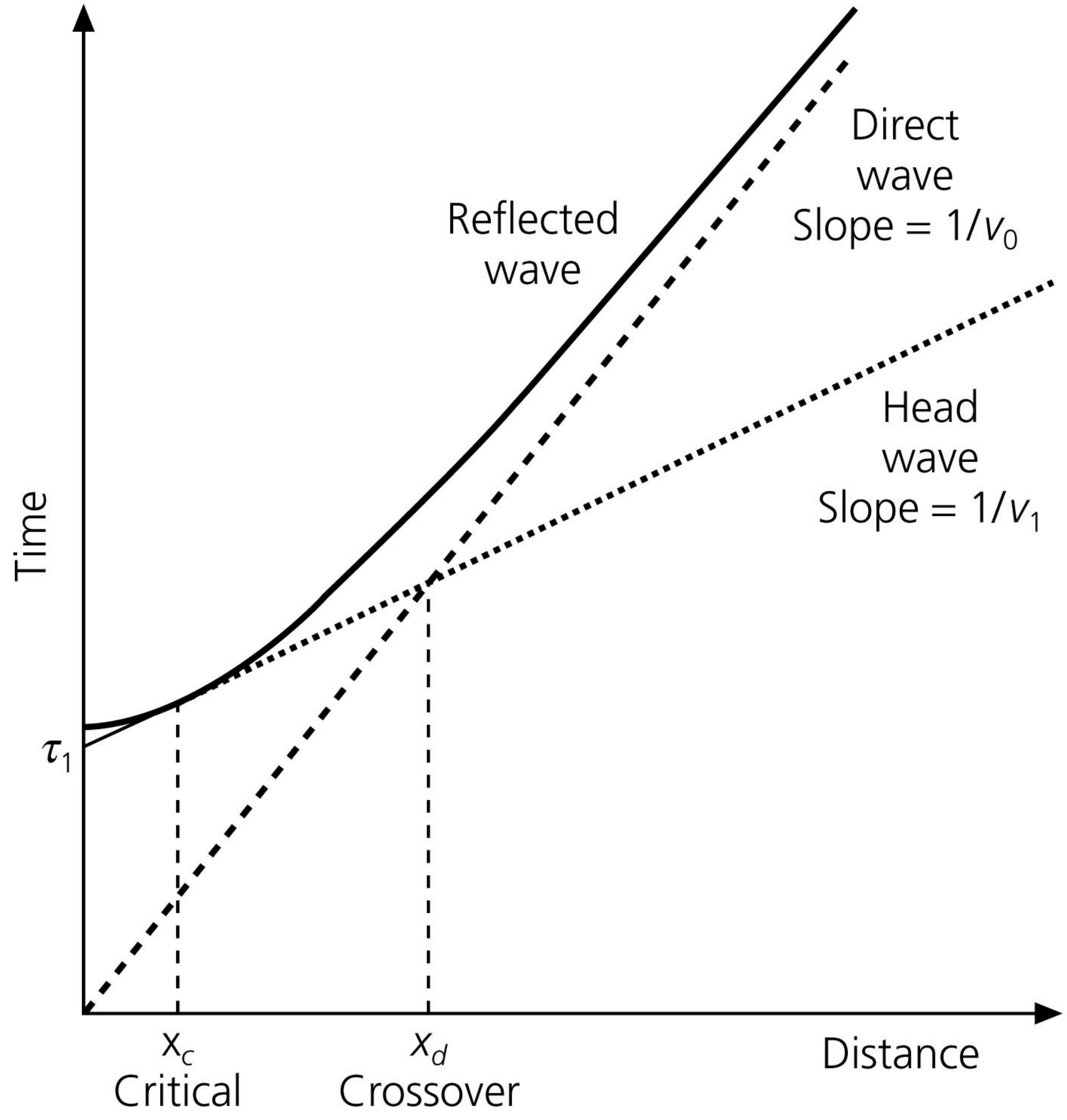
Critical distance

$$x_c = 2h_0 \tan i_c$$

Crossover distance
(setting $T_D(x) = T_H(x)$):

$$x_d = 2h_0\left(\frac{\mathrm{v}_1 + \mathrm{v}_0}{\mathrm{v}_1 - \mathrm{v}_0}\right)^{1/2}$$

Figure 3.2-2: Travel time curve for rays in a layer over a halfspace.



Refraction in Multi-layered case

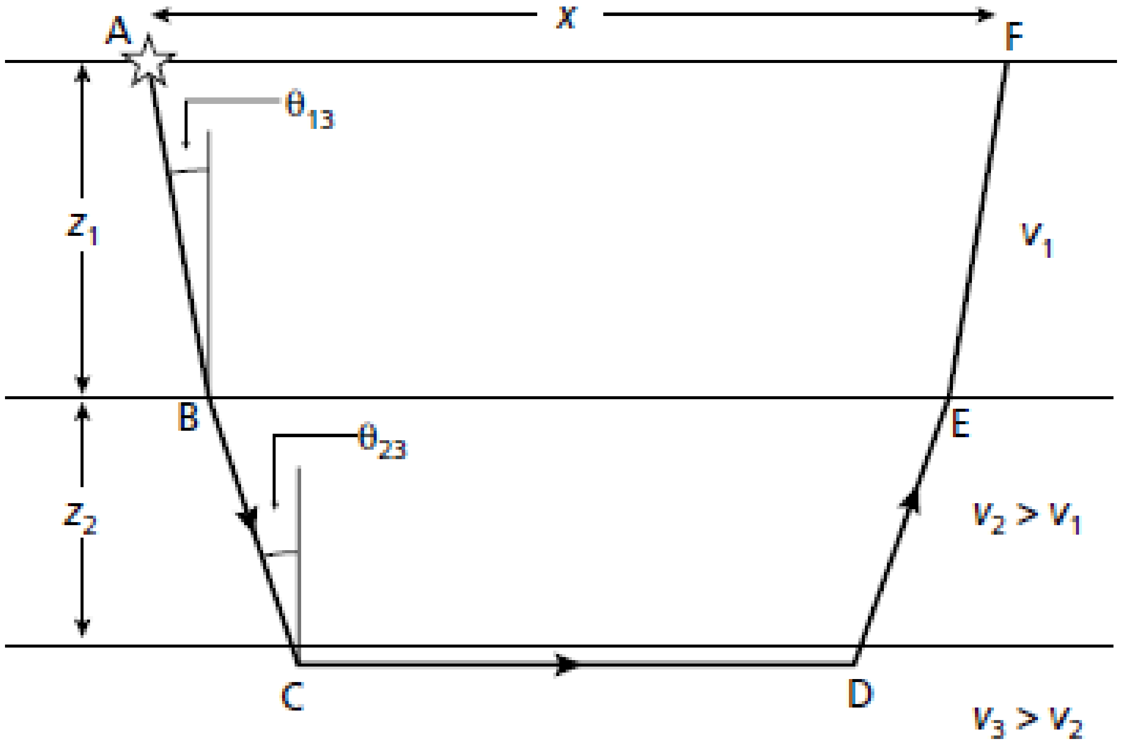


Fig. 5.3 Ray path for a wave refracted through the bottom layer of a three-layer model.

$$t = \frac{x}{v_3} + \frac{2z_1 \cos\theta_{13}}{v_1} + \frac{2z_2 \cos\theta_{23}}{v_2}$$

where

$$\theta_{13} = \sin^{-1}(v_1/v_3); \quad \theta_{23} = \sin^{-1}(v_2/v_3)$$

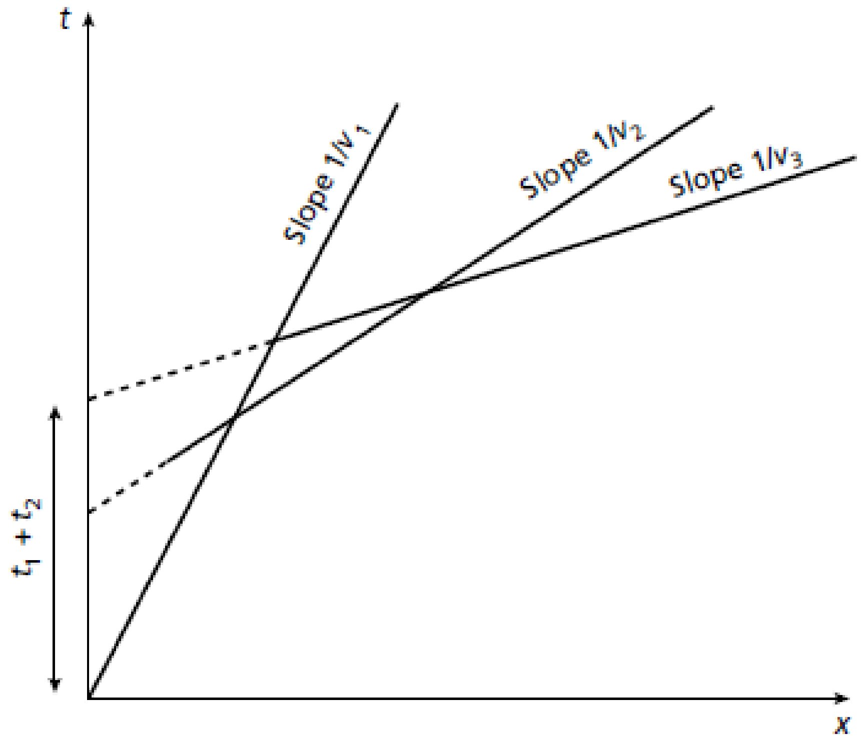


Fig. 5.4 Travel-time curves for the direct wave and the head waves from two horizontal refractors.

$$t_n = \frac{x}{v_n} + \sum_{i=1}^{n-1} \frac{2z_i \cos\theta_{in}}{v_i}$$

where

$$\theta_{in} = \sin^{-1}(v_i/v_n)$$

Multilayered case

Dipping layer with planar interface

$$1/v_{2d} = \sin(\theta_{12} + \gamma_1)/v_1$$

$$1/v_{2u} = \sin(\theta_{12} - \gamma_1)/v_1$$

$$\boxed{\begin{aligned}\theta_{12} &= \frac{1}{2}[\sin^{-1}(v_1/v_{2d}) + \sin^{-1}(v_1/v_{2u})] \\ \gamma_1 &= \frac{1}{2}[\sin^{-1}(v_1/v_{2d}) - \sin^{-1}(v_1/v_{2u})]\end{aligned}}$$

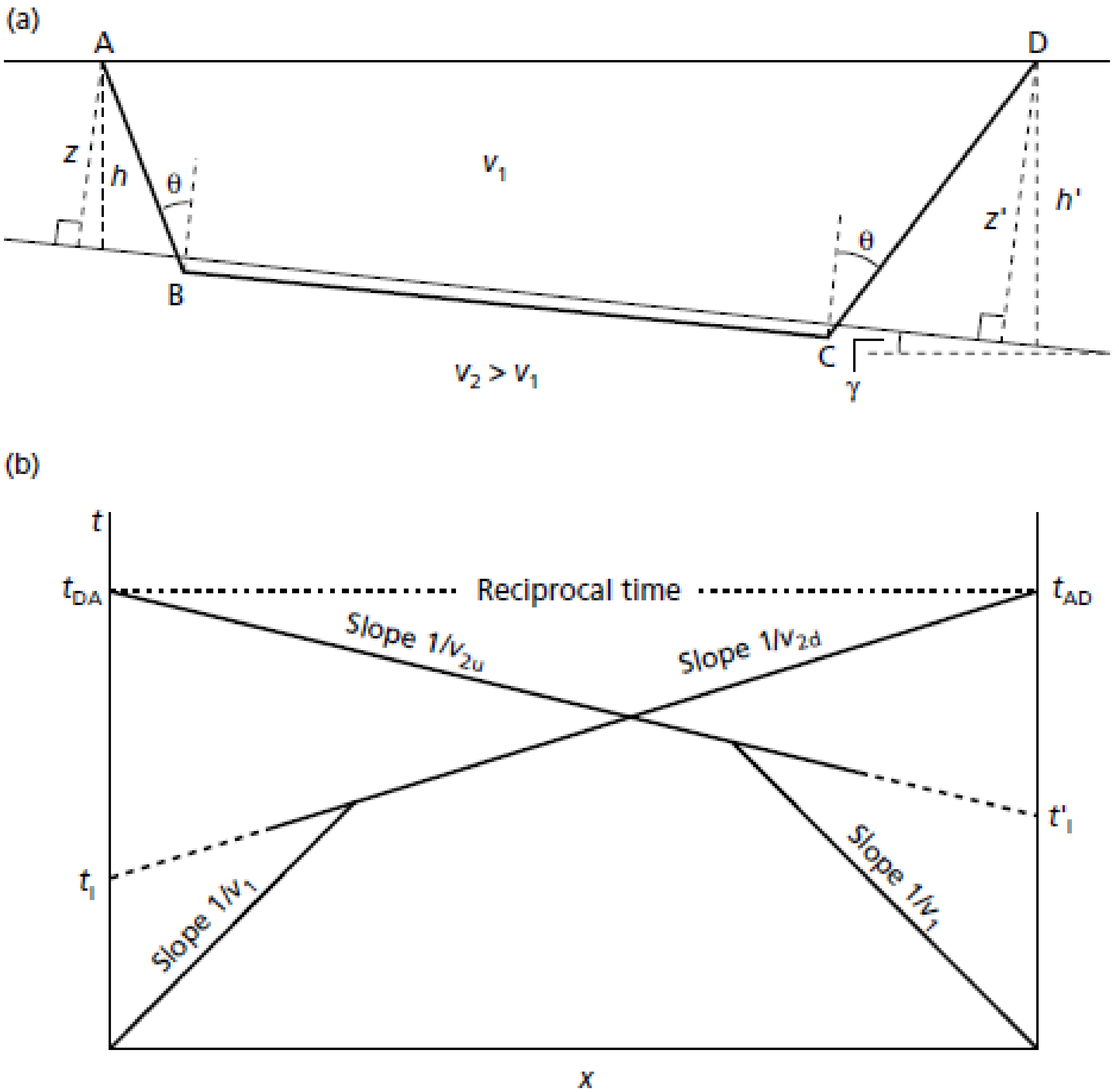
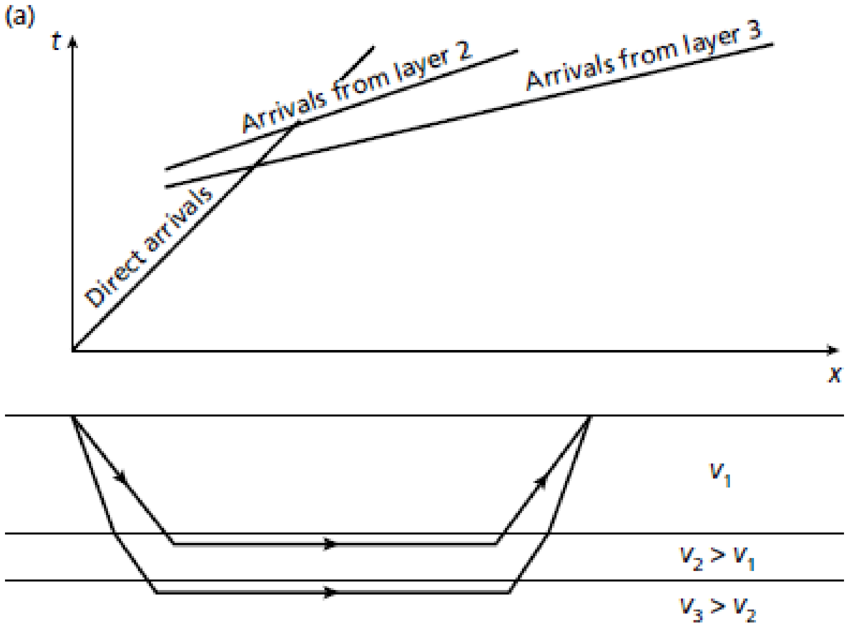
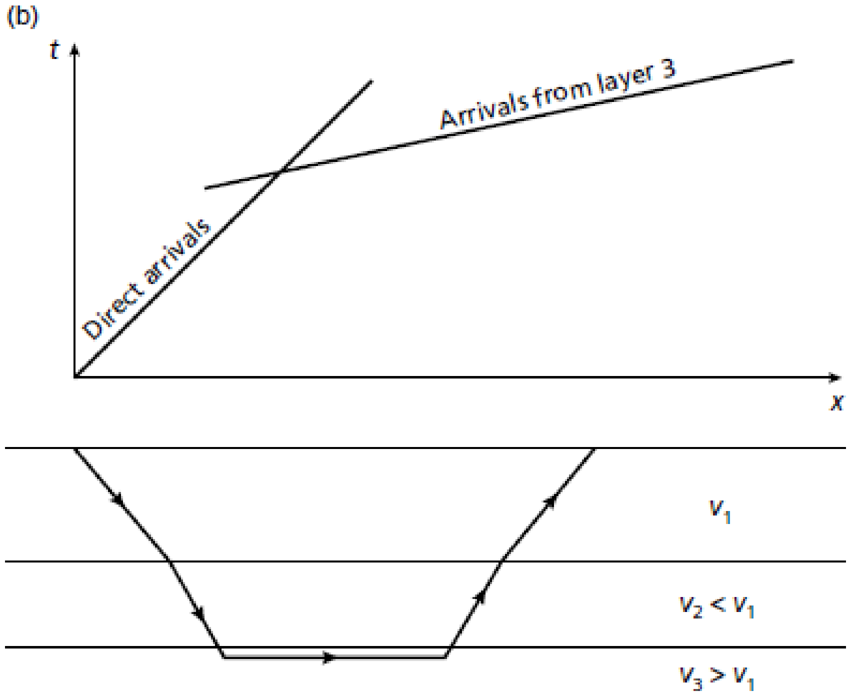


Fig. 5.5 (a) Ray-path geometry and (b) travel-time curves for head wave arrivals from a dipping refractor in the forward and reverse directions along a refraction profile line.

Hidden layer and Blind layer



Presence of a thin layer, refraction from that layer does not show up as a first arrival- hidden layer



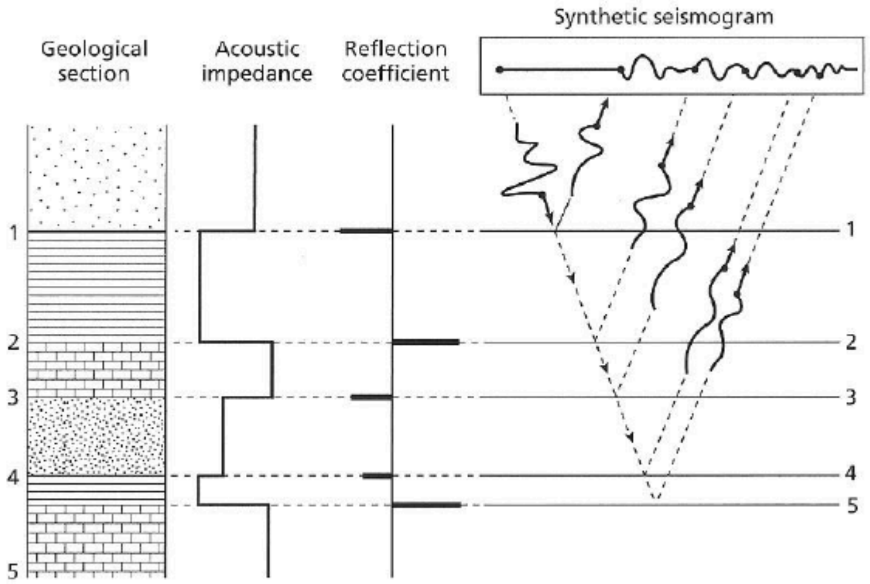
Presence of low velocity layer does not produce head waves, causing a blind layer problem.

Fig. 5.15 The undetected layer problem in refraction seismology. (a) A hidden layer: a thin layer that does not give rise to first arrivals. (b) A blind layer: a layer of low velocity that does not generate head waves.

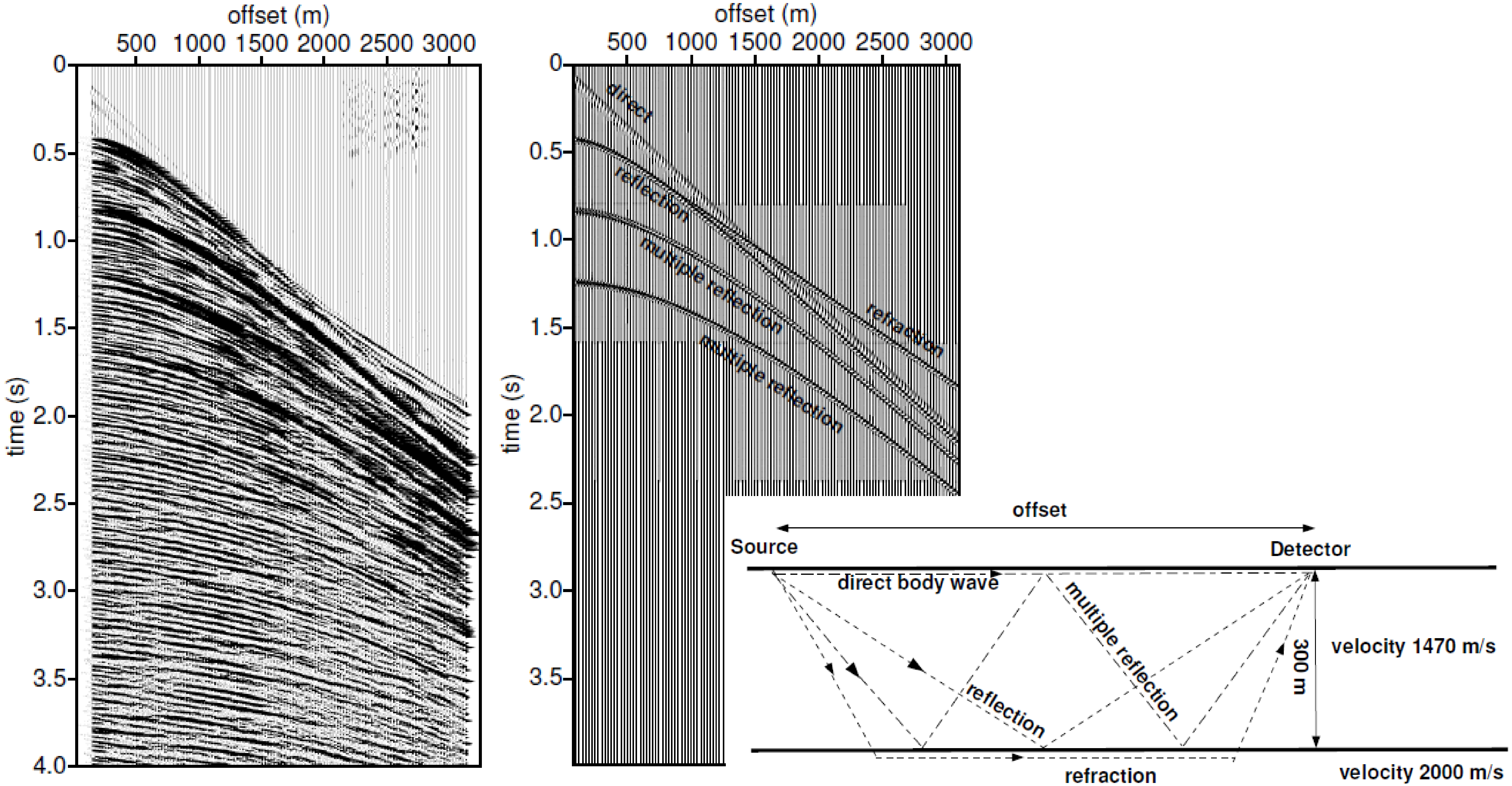
The 1D Convolutional Model

- Convolution is a superposition of a scaled, shifted copy of w for each sample of e.
- The w function is the source wavelet and e is a time series containing the earth's acoustic impedance function. Acoustic impedance is the product of the density and the compressional velocity of sound. Acoustic impedance contrasts at layer boundaries lead to echoes that are reflected back towards the surface. e is known as the reflectivity series.

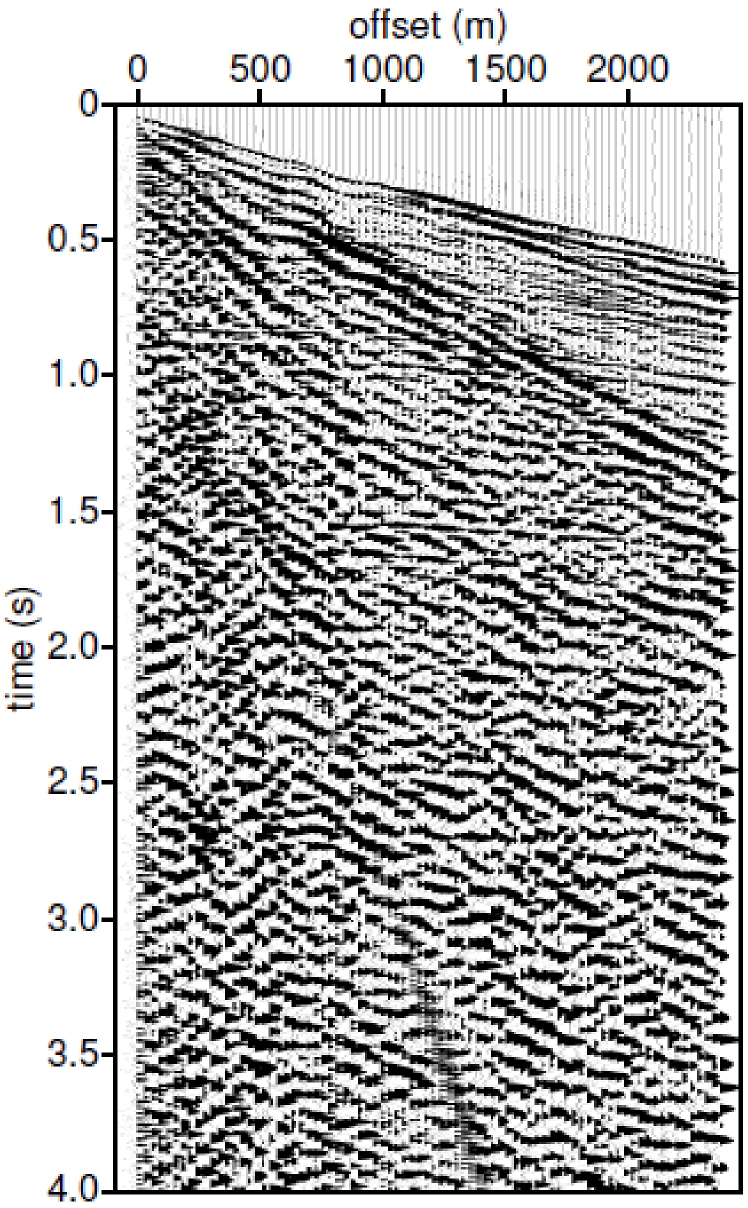
$$s(t) = w(t) * e(t) * sg(t) * rg(t) * i(t) + n(t)$$

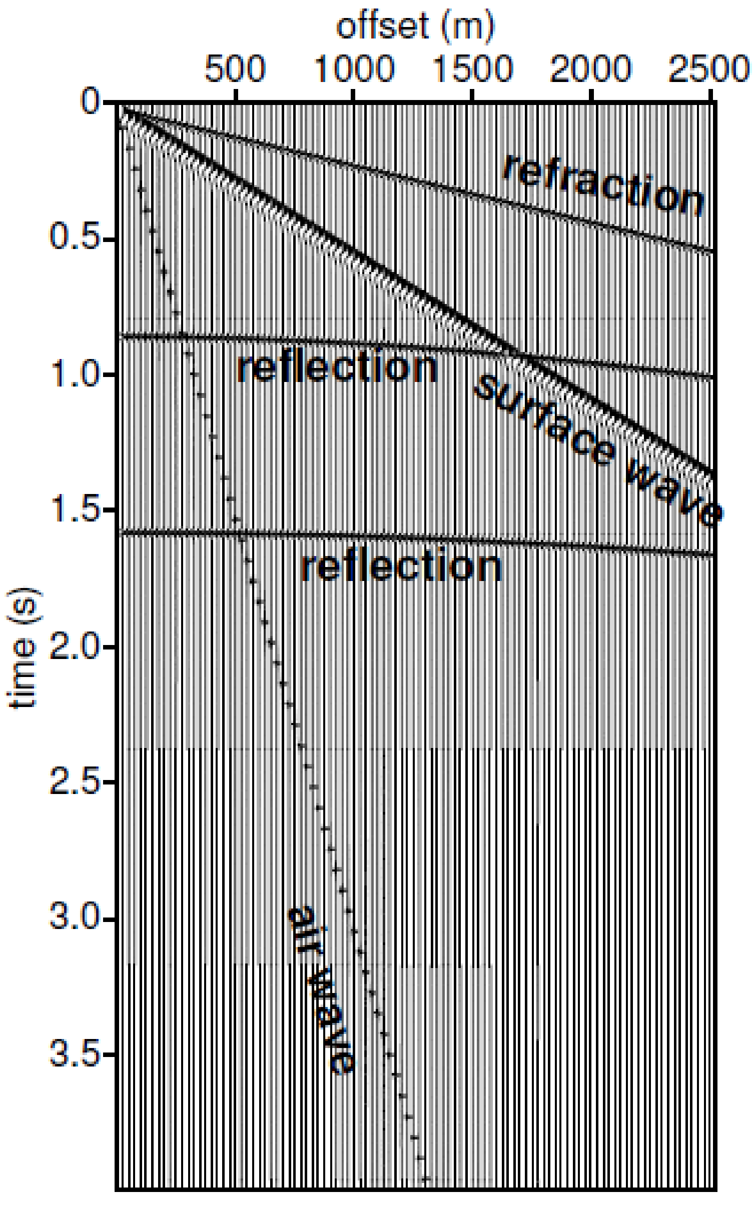


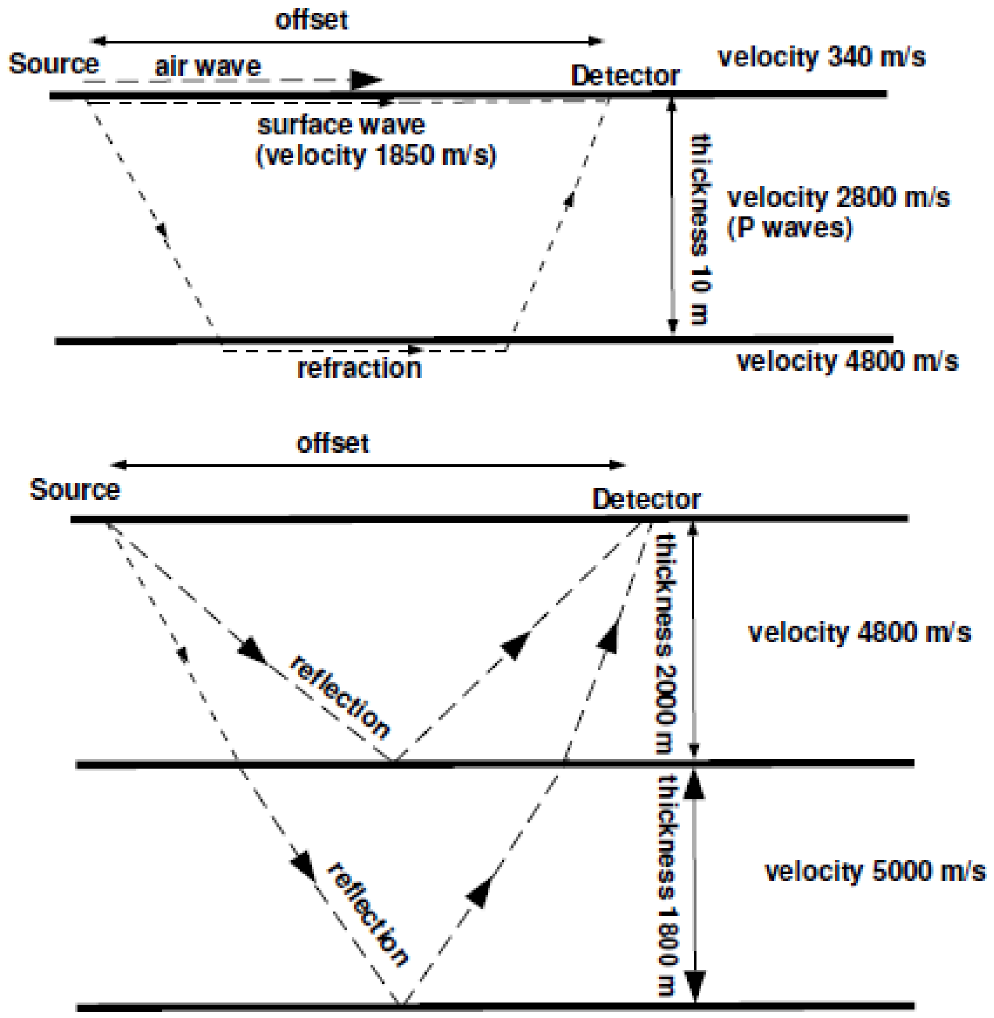
Interpretation of a Marine shot gather



Interpretation of a Land shot gather







Seismic Image from Reflection data