Fundamentals of Earth Sciences (ESO 213A)

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Crustal Deformation

Previous Class: Geological Time Scale and Radioactive Dating

Structural Geology

- **Structural geologists study the architecture and processes responsible for deformation of Earth's crust.**
- **The basic features resulting from the forces generated by the interactions of tectonic plates =** ***tectonic structures***
  - » **folds**
  - » **faults**
  - » **joints**
  - » **foliation, rock cleavage**

Deformation, Stress, and Strain

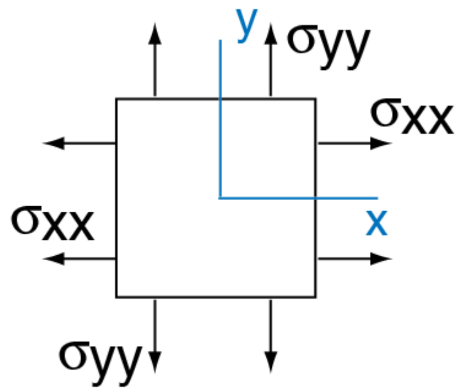
- **Deformation** is a general term that refers to all changes in the original form and/or size of a rock body.

- Most crustal deformation occurs along plate margins.

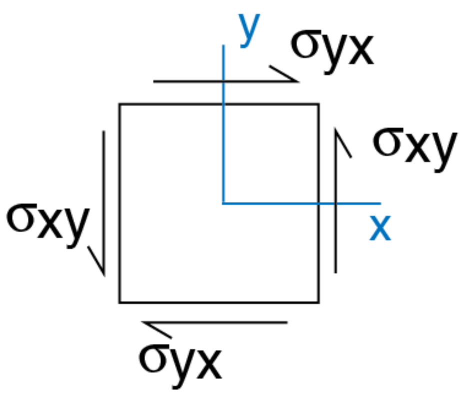
Stress

Deformation involves:

- **Stress**—Stresses refer to balanced internal "forces (per unit area)". They differ from force vectors, which, if unbalanced, cause accelerations
- **Types of stress**
  » Compressional stress shortens a rock body.
  » Tensional stress tends to elongate or pull apart a rock unit.
  » Shear stress produces a motion similar to slippage that occurs between individual playing cards when the top of the stack is moved relative to the bottom.

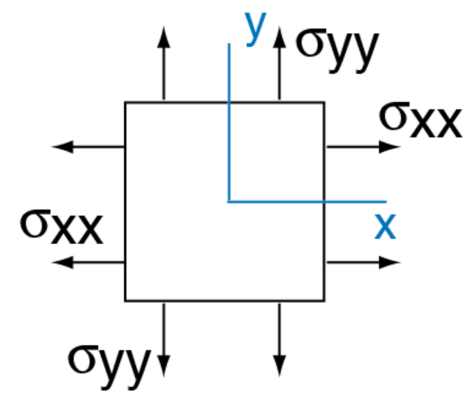


Normal stresses

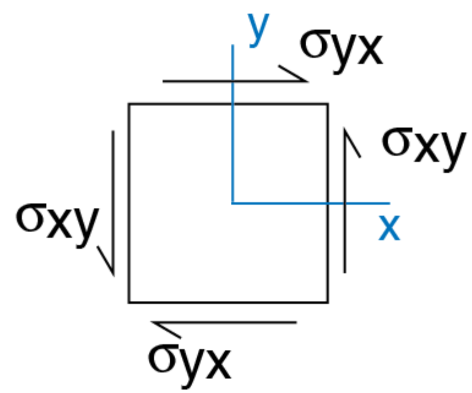


Shear stresses

- *Convention for stresses (in geology)*
    - *Tension is negative*
    - *Compression is positive*
    - *"On -in convention": The stress component σ_{ij} acts on the plane normal to the i- direction and acts in the j- direction*
    - *Counter to most mechanics books*



Normal stresses



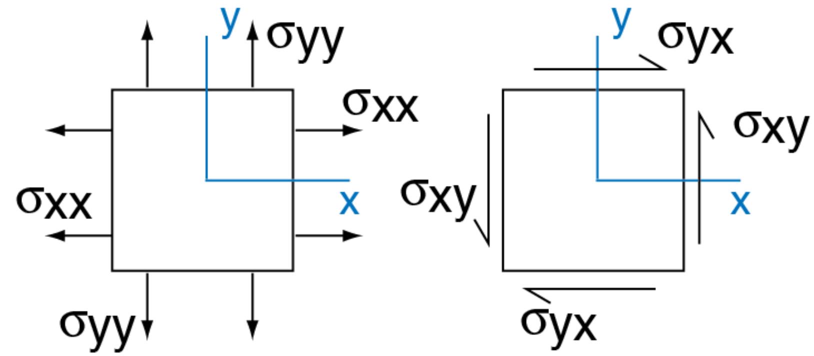
Shear stresses

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$ ***3-D (9 components)***

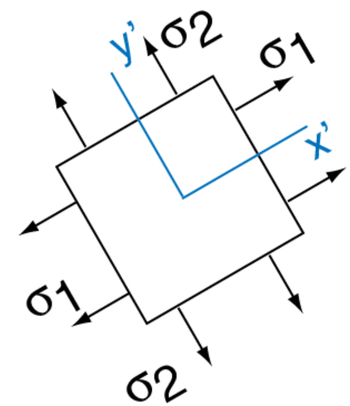
Principal Stresses

- *Have magnitudes and orientations and represents the stress state most simply*
- *Principal stresses act on planes which feel no shear stress*
- *Principal stresses are normal stresses*
- *Principal stresses act on perpendicular planes owing to symmetry of stress tensor*
- *The maximum, intermediate, and minimum principal stresses are usually designated σ_1, σ_2, and σ_3, respectively*

** If $\sigma 1 = \sigma 2 = \sigma 3$, the state of stress is called isotropic. This occurs beneath a still body of water.*



Normal stresses   Shear stresses



Principal stresses

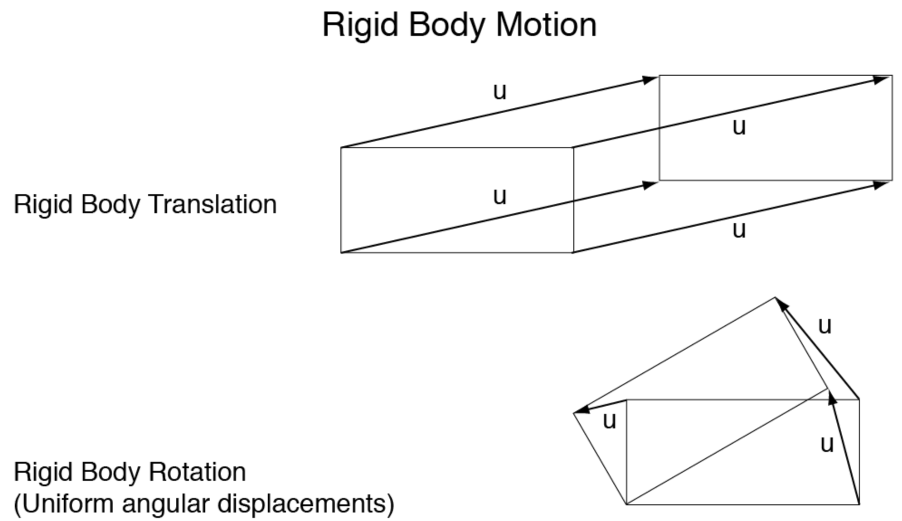
$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

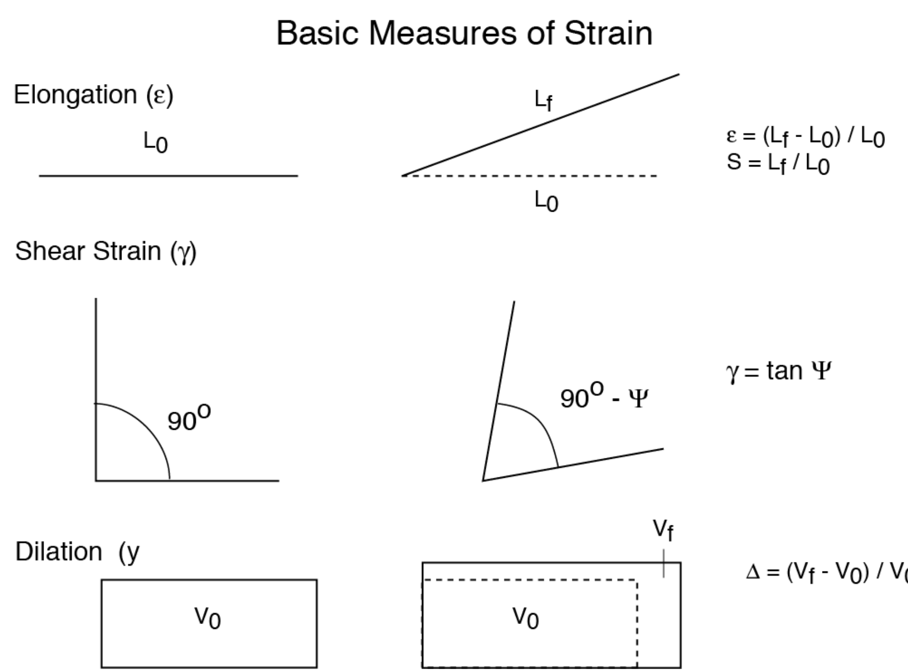
3-D components

Strain

- Strain—changes in the shape or size of a rock body caused by stress
- Strained bodies lose their original configuration during deformation
- Normal strain (ε): change in relative line length
- Shear strain (γ): change in angle between originally perpendicular lines
- Volumetric strain (Δ): change in relative volume
- Strains are dimensionless

Calculate strain from the Trilobite fossil





Rheology

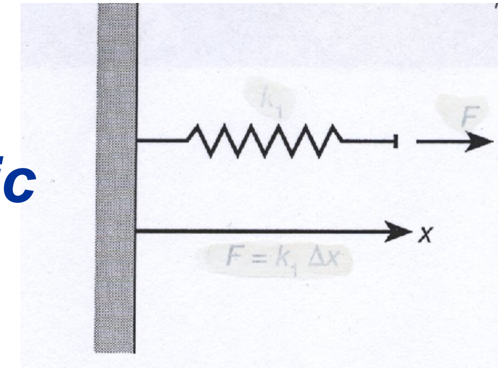
Different materials deform differently under the same state of stress. The material response to a stress is known as rheology.

Ideal materials fall into one of the following categories:

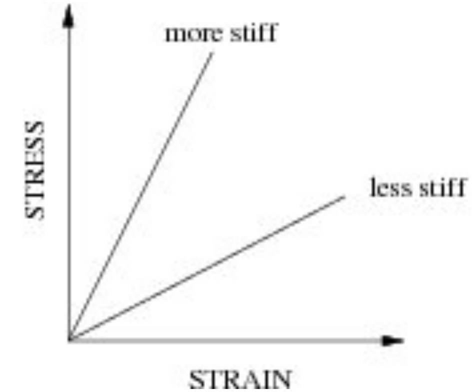
- ***Elasticity.***
- ***Viscosity.***
- ***Plasticity.***

Mechanical analogues | *Stress-Strain relations*

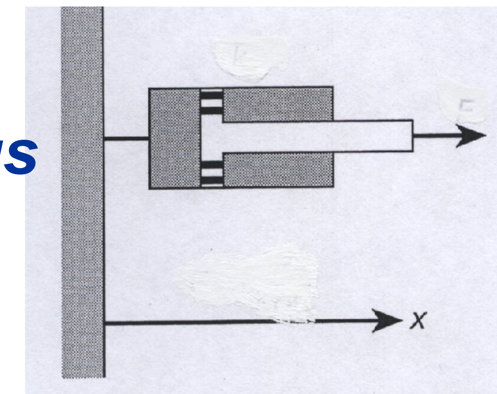
Elastic



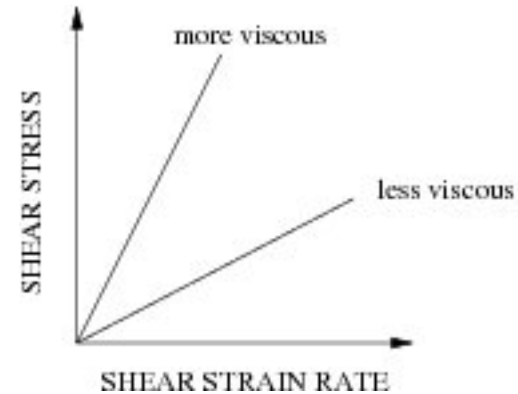
$$\sigma = C\varepsilon$$



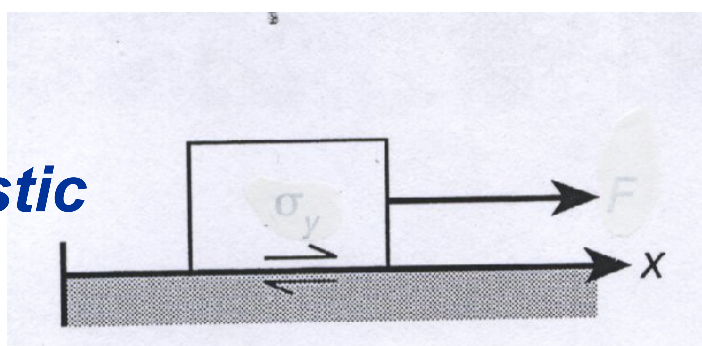
Viscous



$$\sigma_s = \eta \frac{d\varepsilon_s}{dt}$$

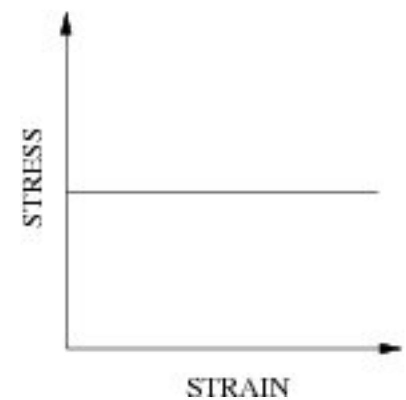


Plastic



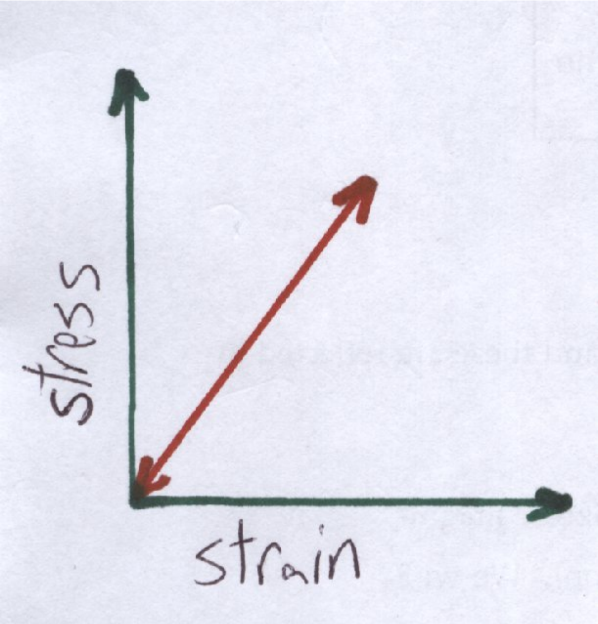
$$\text{slip} = 0 \text{ if}$$

$$\sigma_s < \mu\sigma_n$$

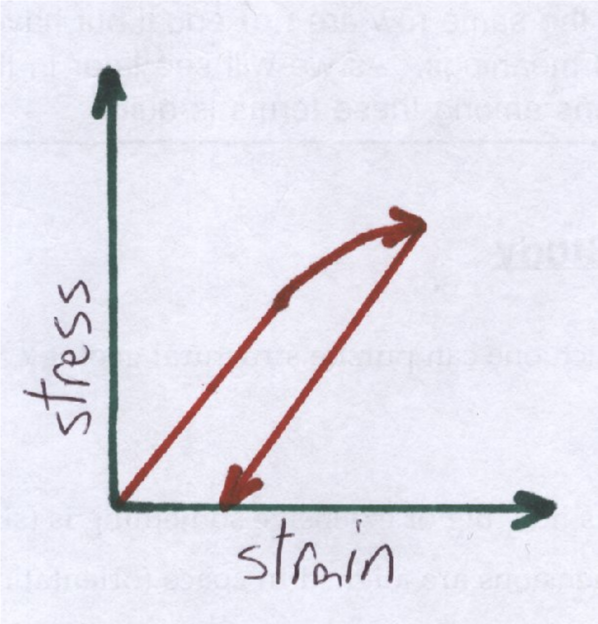


Recoverable versus permanent:

The deformation is recoverable if the material returns to its initial shape when the stress is removed.



Deformation is permanent if the material remains deformed when the stress is removed.



While elastic deformation is recoverable, viscous and plastic deformations are not.

Real materials exhibit a variety of behaviors:

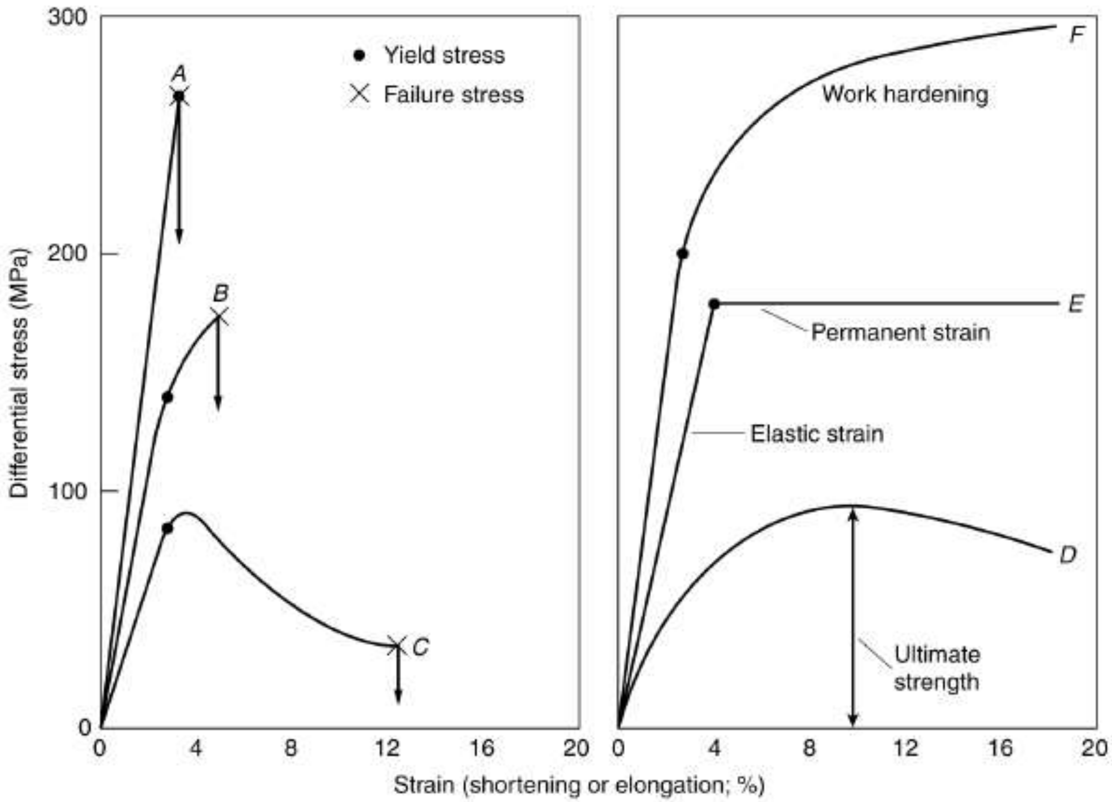
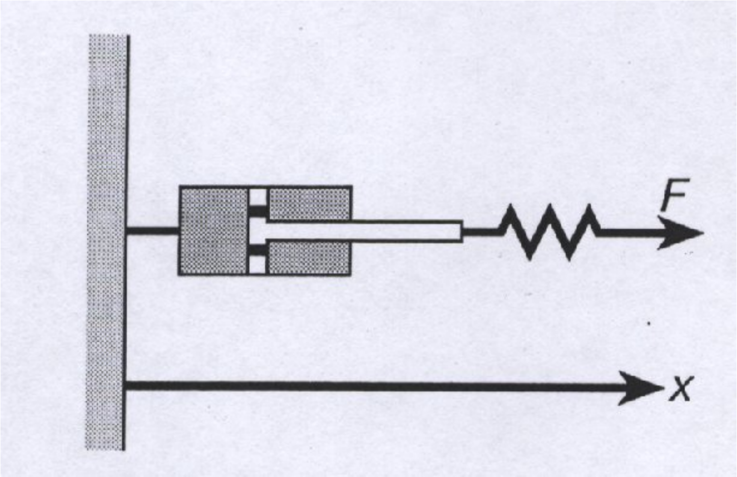


FIGURE 5.18 Representative stress–strain curves of brittle (*A* and *B*), brittle-ductile (*C*), and ductile behavior (*D*–*F*). *A* shows elastic behavior followed immediately by failure, which represents brittle behavior. In *B*, a small viscous component (permanent strain) is present before brittle failure. In *C*, a considerable amount of permanent strain accumulates before the material fails, which represents transitional behavior between brittle and ductile. *D* displays no elastic component and work softening. *E* represents ideal elastic-plastic behavior, in which permanent strain accumulates at constant stress above the yield stress. *F* shows the typical behavior seen in many of the experiments, which displays a component of elastic strain followed by permanent strain that requires increasingly higher stresses to accumulate (work hardening). The yield stress marks the stress at the change from elastic (recoverable or nonpermanent strain) to viscous (non-recoverable or permanent strain) behavior; failure stress is the stress at fracturing.

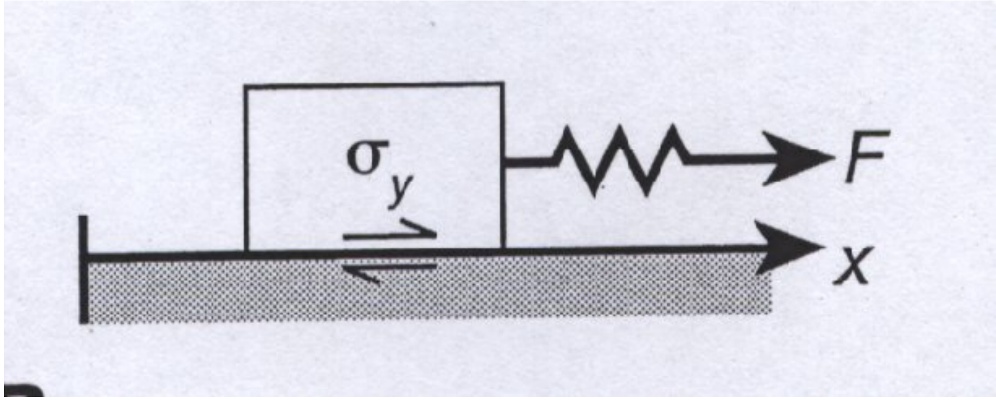
from ic.ucsc.edu/~casey/eart150

The behavior of real materials is better described by combining simple models in series or parallel. For example:

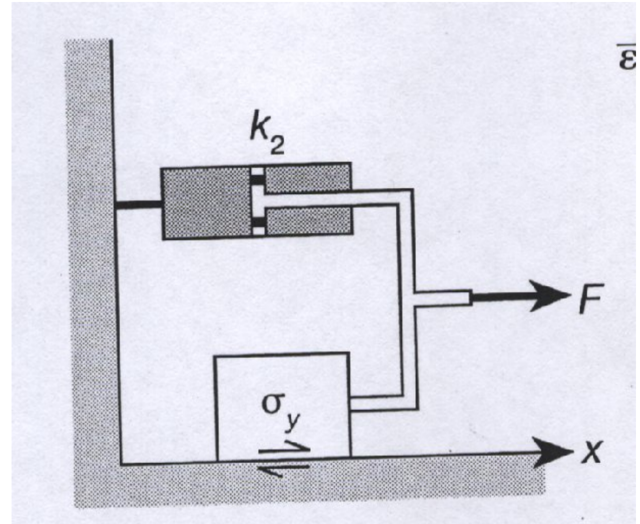
A** **visco-elastic** **(or Maxwell) solid:



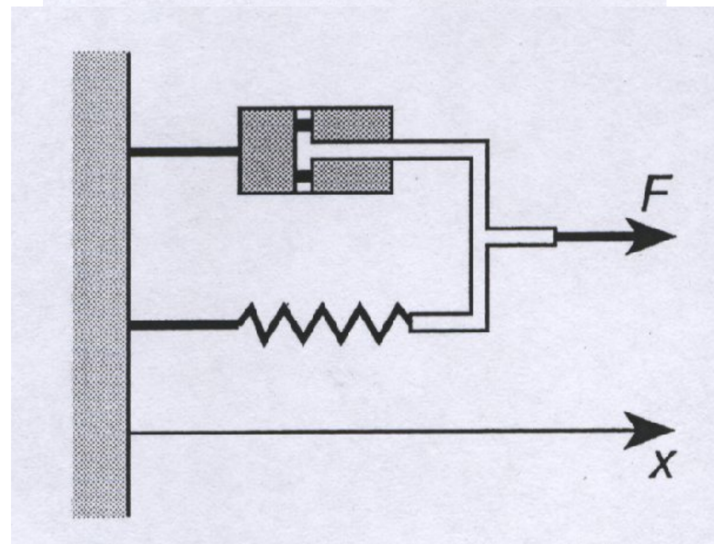
An** **elasto-plastic** **(Prandtl) material:



A visco-plastic (or Bingham) material:



A firmo-viscous (Kelvin or Voight) material:



It turns out that rocks subjected to small strains (seismic waves, slip on faults, etc.) behave as linear elastic materials.

Elasticity
:

The one-dimensional stress-strain relationship may be written as:

$$\sigma = C\varepsilon^{n}$$

where C is an elastic constant.

Note that:

- ***The response is instantaneous.***
- ***Here the strain is the infinitesimal strain.***

The material is said to be ***linear elastic*** *if n=1.*

Hooke's law:

$$\sigma = C\varepsilon$$

In three dimensions, Hooke's law is written as:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

where C_{ijkl} is a matrix whose entries are the ***stiffness coefficients****.*

It thus seems that one needs 81(!) constants in order to describe the stress strain relations.

$$\sigma_{11} = C_{1111}\varepsilon_{11} + C_{1112}\varepsilon_{12} + C_{1113}\varepsilon_{13} + C_{1121}\varepsilon_{21} + C_{1122}\varepsilon_{22} + C_{1123}\varepsilon_{23} + C_{1131}\varepsilon_{31} + C_{1132}\varepsilon_{32} + C_{1133}\varepsilon_{33}$$
$$\sigma_{12} = C_{1211}\varepsilon_{11} + C_{1212}\varepsilon_{12} + C_{1213}\varepsilon_{13} + C_{1221}\varepsilon_{21} + C_{1222}\varepsilon_{22} + C_{1223}\varepsilon_{23} + C_{1231}\varepsilon_{31} + C_{1232}\varepsilon_{32} + C_{1233}\varepsilon_{33}$$
$$\sigma_{13} = C_{1311}\varepsilon_{11} + C_{1312}\varepsilon_{12} + C_{1313}\varepsilon_{13} + C_{1321}\varepsilon_{21} + C_{1322}\varepsilon_{22} + C_{1323}\varepsilon_{23} + C_{1331}\varepsilon_{31} + C_{1332}\varepsilon_{32} + C_{1333}\varepsilon_{33}$$
$$\sigma_{21} = C_{2111}\varepsilon_{11} + C_{2112}\varepsilon_{12} + C_{2113}\varepsilon_{13} + C_{2121}\varepsilon_{21} + C_{2122}\varepsilon_{22} + C_{2123}\varepsilon_{23} + C_{2131}\varepsilon_{31} + C_{2132}\varepsilon_{32} + C_{2133}\varepsilon_{33}$$
$$\sigma_{22} = C_{2211}\varepsilon_{11} + C_{2212}\varepsilon_{12} + C_{2213}\varepsilon_{13} + C_{2221}\varepsilon_{21} + C_{2222}\varepsilon_{22} + C_{2223}\varepsilon_{23} + C_{2231}\varepsilon_{31} + C_{2232}\varepsilon_{32} + C_{2233}\varepsilon_{33}$$
$$\sigma_{23} = C_{2311}\varepsilon_{11} + C_{2312}\varepsilon_{12} + C_{2313}\varepsilon_{13} + C_{2321}\varepsilon_{21} + C_{2322}\varepsilon_{22} + C_{2323}\varepsilon_{23} + C_{2331}\varepsilon_{31} + C_{2332}\varepsilon_{32} + C_{2333}\varepsilon_{33}$$
$$\sigma_{31} = C_{3111}\varepsilon_{11} + C_{3112}\varepsilon_{12} + C_{3113}\varepsilon_{13} + C_{3121}\varepsilon_{21} + C_{3122}\varepsilon_{22} + C_{3123}\varepsilon_{23} + C_{3131}\varepsilon_{31} + C_{3132}\varepsilon_{32} + C_{3133}\varepsilon_{33}$$
$$\sigma_{32} = C_{3211}\varepsilon_{11} + C_{3212}\varepsilon_{12} + C_{3213}\varepsilon_{13} + C_{3221}\varepsilon_{21} + C_{3222}\varepsilon_{22} + C_{3223}\varepsilon_{23} + C_{3231}\varepsilon_{31} + C_{3232}\varepsilon_{32} + C_{3233}\varepsilon_{33}$$
$$\sigma_{33} = C_{3311}\varepsilon_{11} + C_{3312}\varepsilon_{12} + C_{3313}\varepsilon_{13} + C_{3321}\varepsilon_{21} + C_{3322}\varepsilon_{22} + C_{3323}\varepsilon_{23} + C_{3331}\varepsilon_{31} + C_{3332}\varepsilon_{32} + C_{3333}\varepsilon_{33}$$

Thanks to the <u>symmetry of the stress tensor</u>, the number of independent elastic constants is reduced to 54.

$$\sigma_{11} = C_{1111}\varepsilon_{11} + C_{1112}\varepsilon_{12} + C_{1113}\varepsilon_{13} + C_{1121}\varepsilon_{21} + C_{1122}\varepsilon_{22} + C_{1123}\varepsilon_{23} + C_{1131}\varepsilon_{31} + C_{1132}\varepsilon_{32} + C_{1133}\varepsilon_{33}$$

$$\sigma_{22} = C_{2211}\varepsilon_{11} + C_{2212}\varepsilon_{12} + C_{2213}\varepsilon_{13} + C_{2221}\varepsilon_{21} + C_{2222}\varepsilon_{22} + C_{2223}\varepsilon_{23} + C_{2231}\varepsilon_{31} + C_{2232}\varepsilon_{32} + C_{2233}\varepsilon_{33}$$

$$\sigma_{33} = C_{3311}\varepsilon_{11} + C_{3312}\varepsilon_{12} + C_{3313}\varepsilon_{13} + C_{3321}\varepsilon_{21} + C_{3322}\varepsilon_{22} + C_{3323}\varepsilon_{23} + C_{3331}\varepsilon_{31} + C_{3332}\varepsilon_{32} + C_{3333}\varepsilon_{33}$$

$$\sigma_{12} = C_{1211}\varepsilon_{11} + C_{1212}\varepsilon_{12} + C_{1213}\varepsilon_{13} + C_{1221}\varepsilon_{21} + C_{1222}\varepsilon_{22} + C_{1223}\varepsilon_{23} + C_{1231}\varepsilon_{31} + C_{1232}\varepsilon_{32} + C_{1233}\varepsilon_{33}$$

$$\sigma_{13} = C_{1311}\varepsilon_{11} + C_{1312}\varepsilon_{12} + C_{1313}\varepsilon_{13} + C_{1321}\varepsilon_{21} + C_{1322}\varepsilon_{22} + C_{1323}\varepsilon_{23} + C_{1331}\varepsilon_{31} + C_{1332}\varepsilon_{32} + C_{1333}\varepsilon_{33}$$

$$\sigma_{23} = C_{2311}\varepsilon_{11} + C_{2312}\varepsilon_{12} + C_{2313}\varepsilon_{13} + C_{2321}\varepsilon_{21} + C_{2322}\varepsilon_{22} + C_{2323}\varepsilon_{23} + C_{2331}\varepsilon_{31} + C_{2332}\varepsilon_{32} + C_{2333}\varepsilon_{33}$$

Thanks to the <u>symmetry of the strain tensor</u>, the number of independent elastic constants is further reduced to 36.

$$
\begin{aligned}
\sigma_{11} &= C_{1111}\varepsilon_{11} + C_{1112}\varepsilon_{12} + C_{1113}\varepsilon_{13} + C_{1122}\varepsilon_{22} + C_{1123}\varepsilon_{23} + C_{1133}\varepsilon_{33} \\
\sigma_{22} &= C_{2211}\varepsilon_{11} + C_{2212}\varepsilon_{12} + C_{2213}\varepsilon_{13} + C_{2222}\varepsilon_{22} + C_{2223}\varepsilon_{23} + C_{2233}\varepsilon_{33} \\
\sigma_{33} &= C_{3311}\varepsilon_{11} + C_{3312}\varepsilon_{12} + C_{3313}\varepsilon_{13} + C_{3322}\varepsilon_{22} + C_{3323}\varepsilon_{23} + C_{3333}\varepsilon_{33} \\
\sigma_{12} &= C_{1211}\varepsilon_{11} + C_{1212}\varepsilon_{12} + C_{1213}\varepsilon_{13} + C_{1222}\varepsilon_{22} + C_{1223}\varepsilon_{23} + C_{1233}\varepsilon_{33} \\
\sigma_{13} &= C_{1311}\varepsilon_{11} + C_{1312}\varepsilon_{12} + C_{1313}\varepsilon_{13} + C_{1322}\varepsilon_{22} + C_{1323}\varepsilon_{23} + C_{1333}\varepsilon_{33} \\
\sigma_{23} &= C_{2311}\varepsilon_{11} + C_{2312}\varepsilon_{12} + C_{2313}\varepsilon_{13} + C_{2322}\varepsilon_{22} + C_{2323}\varepsilon_{23} + C_{2333}\varepsilon_{33}
\end{aligned}
$$

The following formalism is convenient for problems in which the strains components are known and the stress components are the dependent variables:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

In cases where the strain components are the dependent parameters, it is more convenient to use the following formalism:

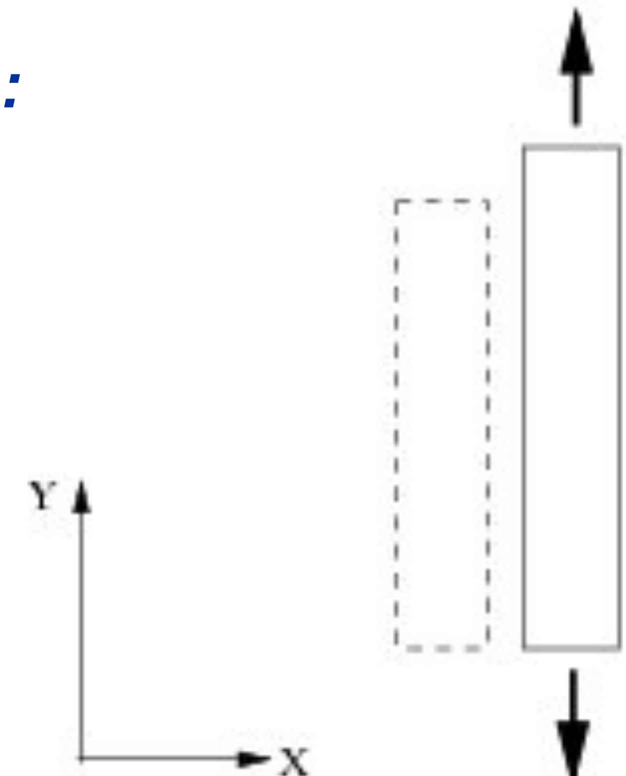
$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

Where S_{ijkl} is a matrix whose entries are the compliance coefficients.

The case of isotropic materials:

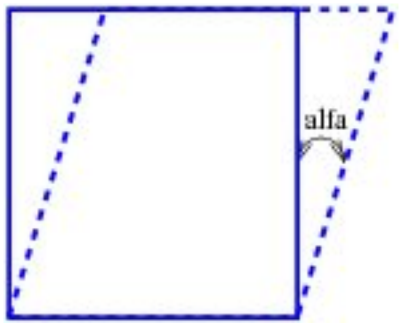
o ***A material is said to be isotropic if its properties are independent of direction.***
o ***In that case, the number of non-zero stiffnesses (or compliances) is reduced to 12, all are a function of only 2 elastic constants.***

Young modulus:



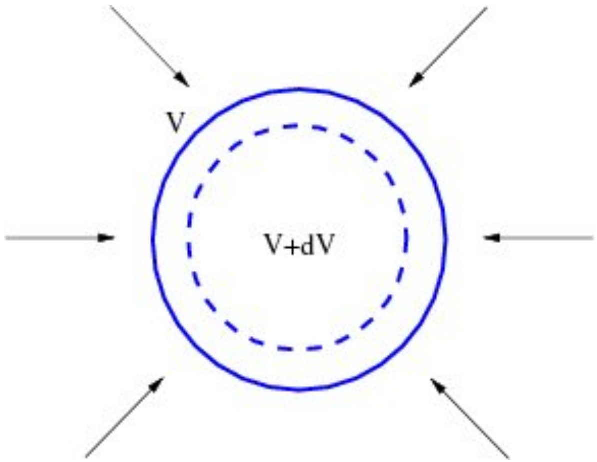
$$E = \frac{\sigma_y}{\varepsilon_y}$$

Shear modulus (rigidity):



$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

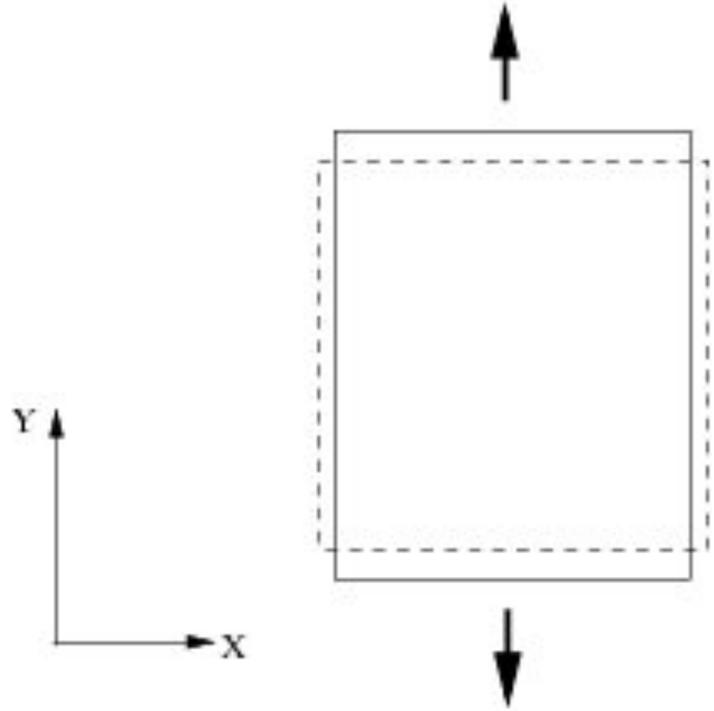
Bulk modulus (compressibility):



$$K = \frac{\text{pressure}}{\text{volumetric change}}$$

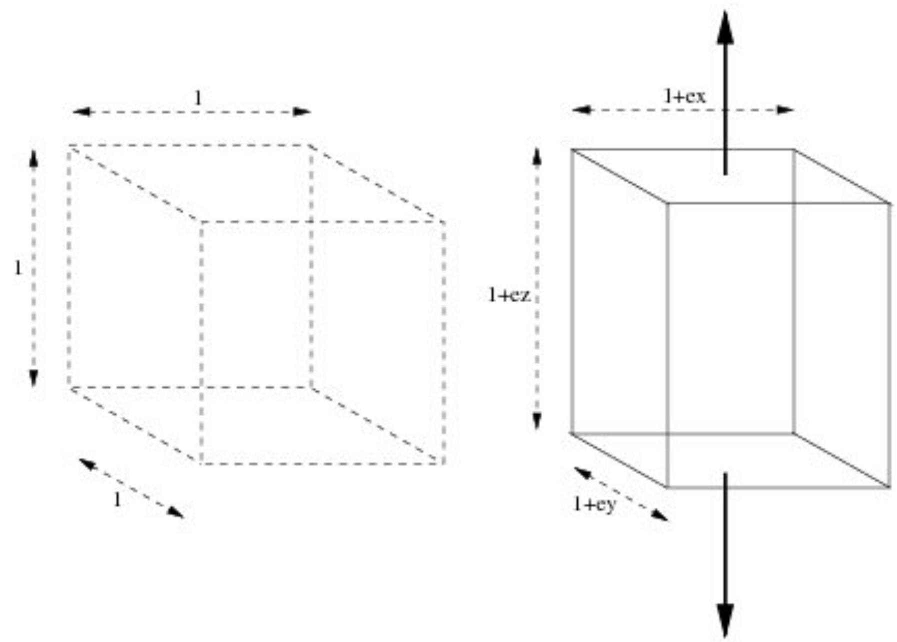
Poisson’s ratio:

Poisson’s ratio of incompressible isotropic materials equals 0.5:



$$\nu = -\frac{\varepsilon_x}{\varepsilon_y}$$

Real materials are compressible and their Poisson ratio is less than 0.5.



All elastic constants can be expressed as a function of only 2 elastic constants. Here is a conversion table:

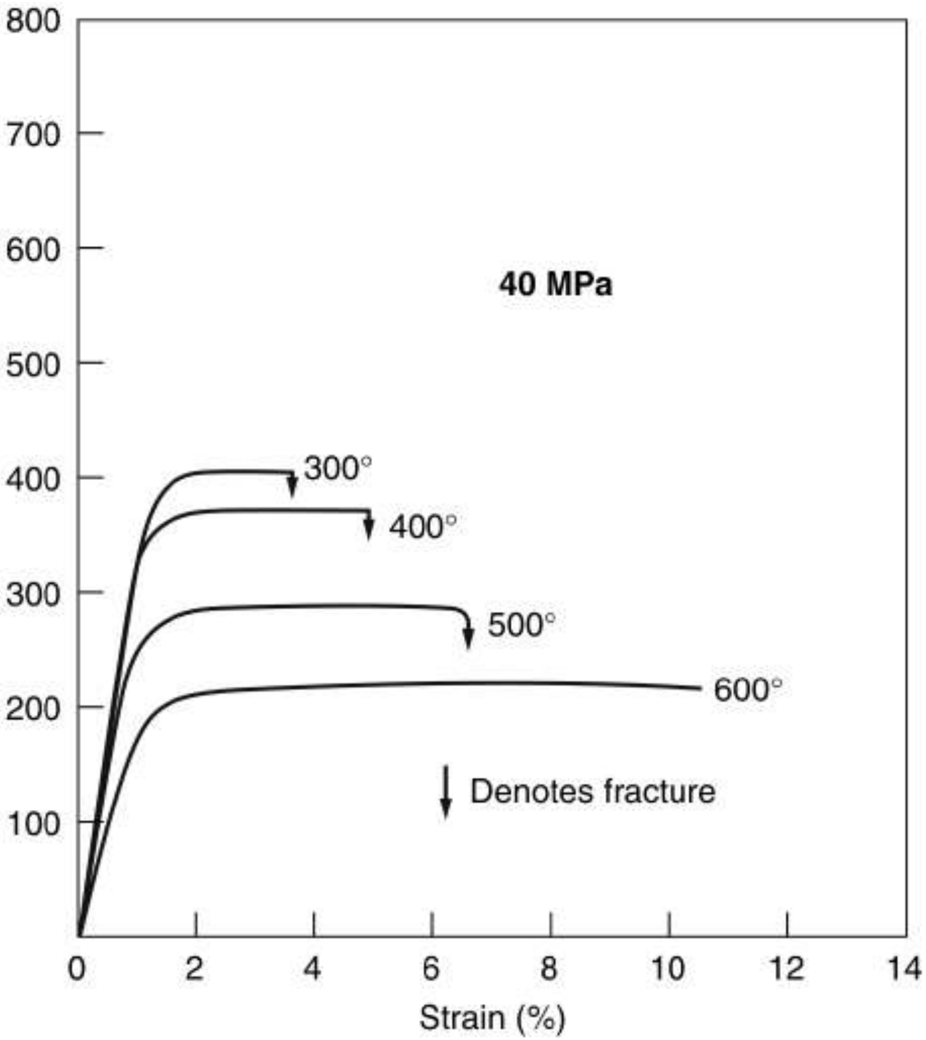
Known Elastic Constants	E	ν	μ	κ	λ
Shear modulus μ, Bulk modulus κ	$\frac{9\kappa\mu}{3\kappa+\mu}$	$\frac{3\kappa-2\mu}{6\kappa+2\mu}$	μ	κ	$\frac{3\kappa-2\mu}{3}$
Young's modulus E, Poisson's ratio ν	E	ν	$\frac{E}{2(1+\nu)}$	$\frac{E}{3(1-2\nu)}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$
Young's modulus E, Shear modulus μ	E	$\frac{E-2\mu}{2\mu}$	μ	$\frac{E\mu}{3(3\mu-E)}$	$\frac{\mu(E-2\mu)}{3\mu-E}$
Young's modulus E, Bulk modulus κ	E	$\frac{3\kappa-E}{6\kappa}$	$\frac{3\kappa E}{9\kappa-E}$	κ	$\frac{3\kappa(3\kappa-E)}{9\kappa-E}$
Shear modulus μ, Lame's constant λ	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	μ	$\frac{3\lambda+2\mu}{3}$	λ

How Rocks Deform

- Rocks subjected to stresses greater than their own strength (elastic limit) begin to deform by flowing or fracturing.

- General characteristics of rock deformation
  - Elastic deformation—The rock returns to nearly its original size and shape when the stress is removed.
  - Once the elastic limit (strength) of a rock is surpassed, it either flows (ductile deformation) or fractures (brittle deformation).

Triaxial experiments: Increasing temperature weakens rocks.



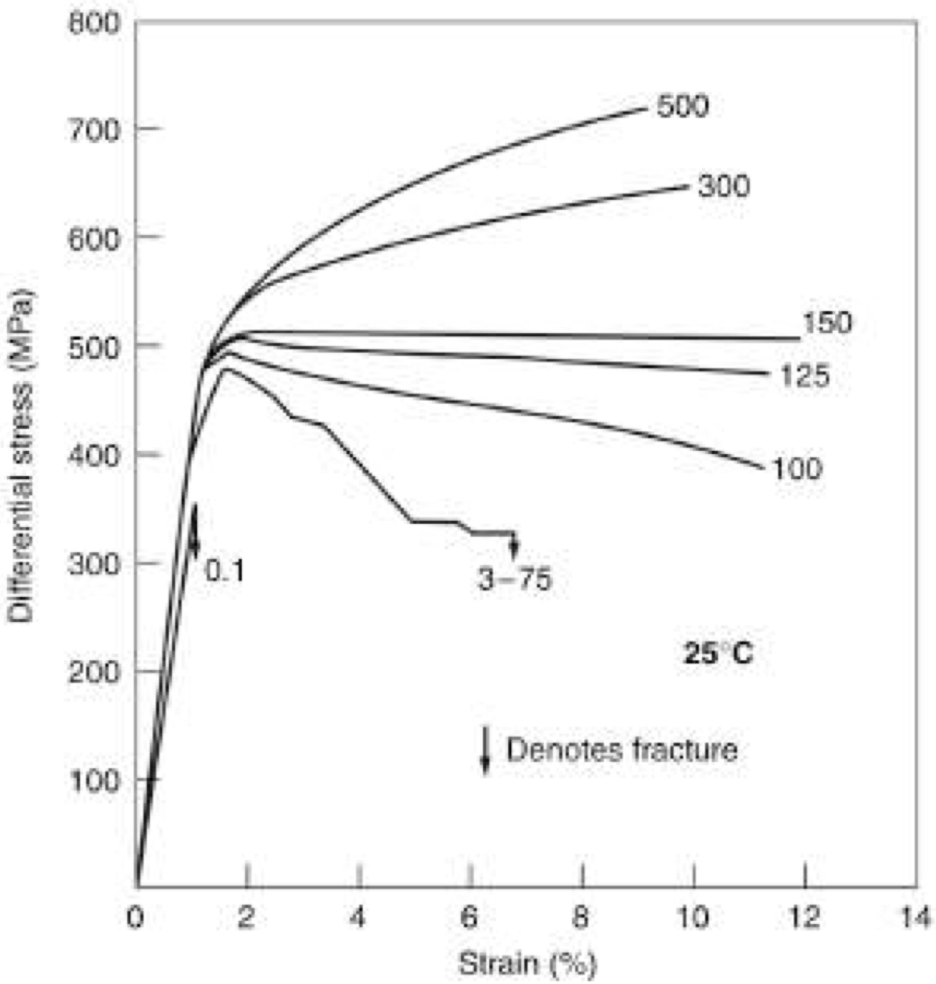
(b)

Triaxial testing machine

Weakening of fine-grained limestone with increasing temperature. Vertical axis is stress in MPa.

ic.ucsc.edu/~casey/eart150/Lectures/Rheology/13rheology.htm

Triaxial experiments: Increasing pressure strengthen rocks.



Strengthening of fine-grained limestone with increasing Pressure. Vertical axis is stress in MPa. This effect is much more pronounced at low temperatures (less than 100°) and diminishes at higher temperatures (greater than 100°)

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Triaxial experiments: In the Earth both temperature and confining pressure increase with depth and temperature overcomes strengthening effect of confining pressure resulting in generally ductile behavior at depths.

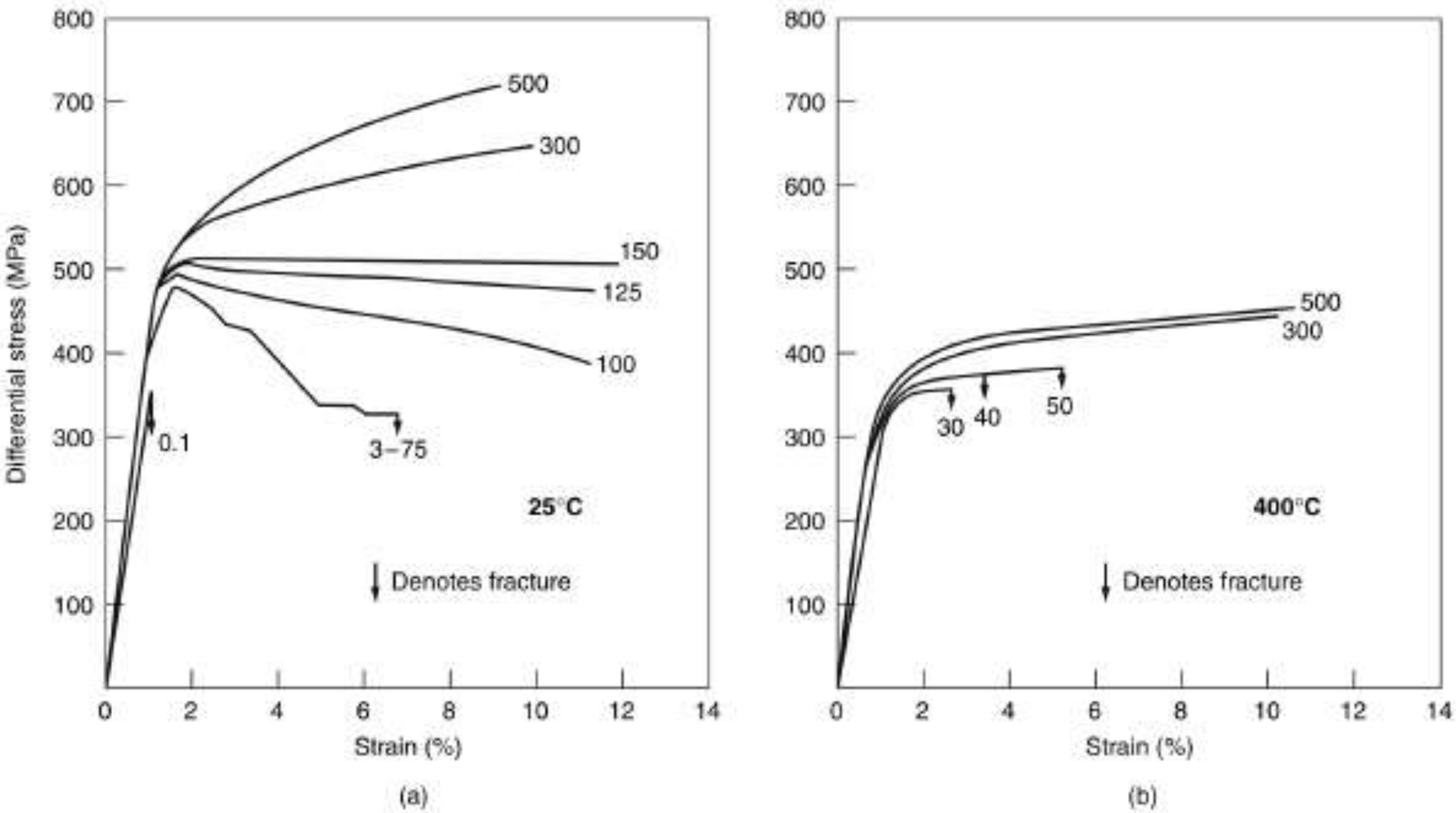

FIGURE 5.10 Compression stress–strain curves of Solnhofen limestone at various confining pressures (indicated in MPa) at (a) 25°C and (b) 400°C.

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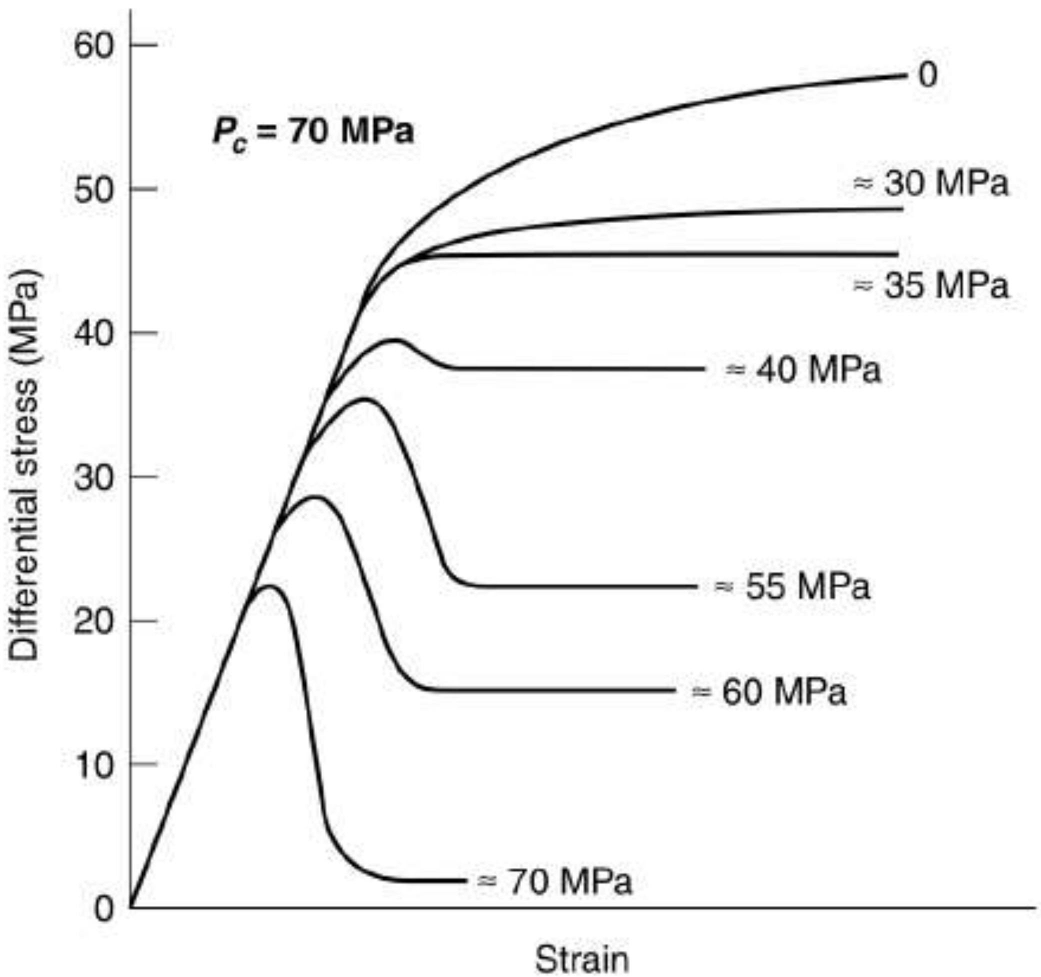
Triaxial experiments: Pore pressure weakens rocks.



(a)

- ***Fluid pressure weakens rocks, because it reduces the effective stresses.***
- ***Water weakens rocks, by affecting bonding of materials.***

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Triaxial experiments: Rocks are weaker under lower strain rates.

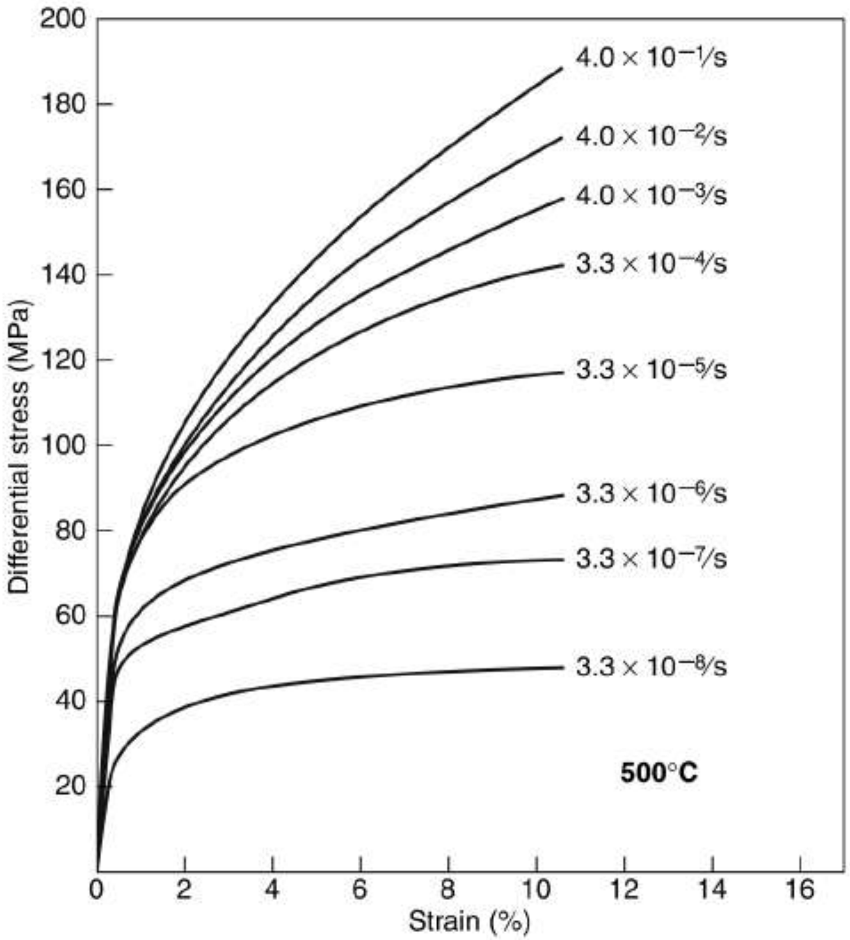

FIGURE 5.14 Stress versus strain curves for extension experiments in weakly foliated Yule marble for various constant strain rates at 500°C.

Slow deformation allows diffusional crystal-plastic processes to more closely keep up with applied stresses.

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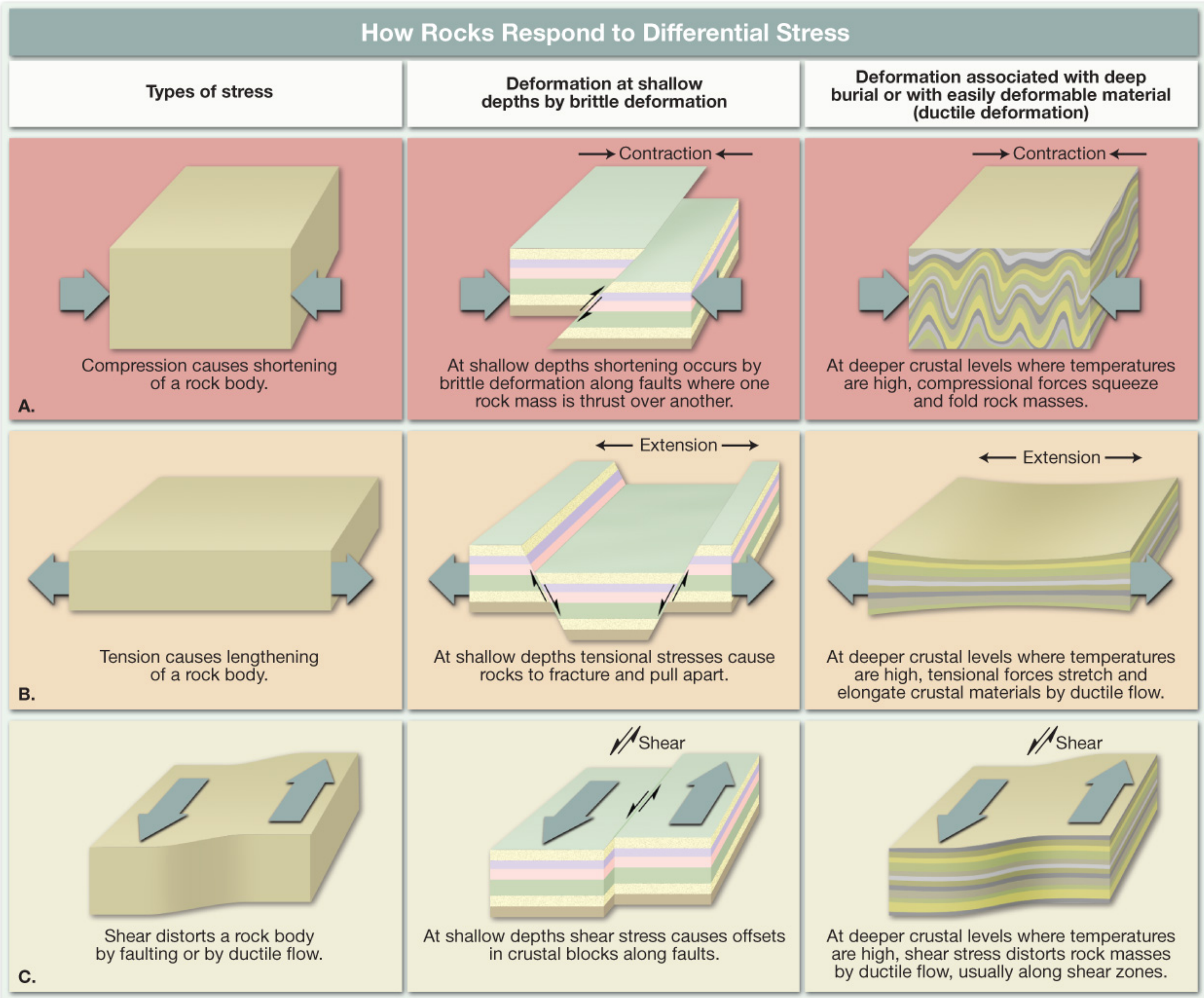
Deformation of Rocks



Mapping Geologic Structures

Describing and mapping the orientation or attitude of a rock layer involves determining the features.

- Strike (trend)
  - The compass direction of the line produced by the intersection of an inclined rock layer or fault with a horizontal plane
  - Generally expressed as an angle relative to north
- Dip (inclination)
  - The angle of inclination of the surface of a rock unit or fault measured from a horizontal plane
  - Includes both an of inclination and a direction toward which the rock is inclined.